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Optimum Commodity Taxation with a Non-Renewable Resource*

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Abstract

Optimum commodity taxation theory asks how to raise a given amount of tax revenue while minimizing distortions. We reexamine Ramsey's inverse elasticity rule in presence of Hotelling-type non-renewable natural resources. Under standard assumptions borrowed from the non-renewable-resource-extraction and from the optimum-commodity-taxation literatures, a non-renewable resource should be taxed in priority whatever its demand elasticity and whatever the demand elasticity of regular commodities. It should also be taxed at a higher rate than other commodities having the same demand elasticity and, while the tax on regular commodities should be constant, the resource tax should vary over time.

There are two basic ways to alleviate resource supply limitations; one is to produce reserves for subsequent extraction; the other one is to rely on imports. When the generation of reserves by exploration is determined by the net-of-tax rents derived during the extraction phase, reserves become a conventional form of capital and royalties tax its income; our results contradict Chamley's conclusion that capital should not be taxed at all in the very long run.

When the economy is autarkic, in the absence of any subsidy to reserve discoveries, the optimal tax rate on extraction obeys an inverse elasticity rule almost identical to that of a commodity whose supply is perfectly elastic. As a matter of fact, there is a continuum of optimal combinations of reserve subsidies and extraction taxes, irrespective of whether taxes are applied on consumption or on production. When the government cannot commit, extraction rents are completely expropriated and subsidies are maximum. In general the optimum Ramsey tax not only causes a distortion of the extraction path, as happens when reserves are given, but also distorts the level of reserves developed for extraction. When that distortion is the sole effect of the tax, it is determined by a rule reminiscent of the inverse elasticity rule applying to elastically-supplied commodities.

In an open economy, Ramsey taxes further acquire an optimum-tariff dimension, capturing foreign resource rents. For countries that import the resource, the result that domestic resource consumption is to be taxed at a higher rate than conventional commodities having the same demand elasticity emerges reinforced.

JEL classification: Q31; Q38; H21
Keywords: Optimum commodity taxation; Inverse elasticity rule; Non-renewable resources; Hotelling resource; Supply elasticity; Demand elasticity; Capital income taxation.

1 Introduction

The theory of optimal commodity taxation (OCT) addresses the following question: how should a government concerned with total welfare distribute the burden of commodity taxation across sectors in such a way as to collect a set amount of tax income while minimizing the deadweight loss? The literature originated with Ramsey (1927) and Pigou (1947) and was consolidated by Baumol and Bradford (1970), Diamond and Mirrlees (1971), Atkinson and Stiglitz (1980), and others.

Its most famous result is the "inverse elasticity rule" which says that, under simplifying conditions, the tax rate applied on each good should be proportional to the sum of the reciprocals of its elasticities of supply and of demand. The rule gives a good and general intuition to the choice of optimal commodity taxes: commodity taxes cause distortions; the distortion introduced by the tax on any specific commodity is lower, the lower its elasticities of supply and demand; hence, if the objective is to spread evenly the social cost of the distortions associated with commodity taxation, the tax should be heavier in lower elasticity markets and *vice versa*.

In this paper we reexamine "optimal commodity taxation" and the "inverse elasticity rule" in presence of non-renewable natural resources. It is often noted that energy demand, oil demand in particular, is relatively price inelastic (Berndt and Wood, 1975; Pindyck, 1979; Kilian and Murphy, 2010). According to the theory, this would call for relatively high oil taxes. Is there any other reason to devote particular attention to non-renewable resources in that context?[1]

The non-renewability of a natural resource further adds an intertemporal dimension

[1] Atkinson and Stiglitz (1976) showed that under a condition of separability of leisure and consumption choices, optimal non-linear income taxation makes commodity taxation useless. On this see also Christiansen (1984), Konishi (1995) and Kaplow (2006). However, the role of differential commodity taxation received a renewed attention recently. Cremer *et al.* (2001) showed that separability is not sufficient when individuals differ in their endowment. Saez (2002), extending the analysis to heterogeneous tastes within income levels, made clear that Atkinson-Stiglitz's result relies on the strong homogeneity of individuals. Blomquist and Christiansen (2008) showed how commodity taxes alleviate extreme self-selection constraints. Assuming non-separable but homogeneous preferences and imperfect competition in the labor market, Aronsson and Sjögren (2003) emphasized that optimum commodity taxes should depend on their specific effects on unemployment. Other considerations justifying differential commodity taxation in the absence of externalities include the production technology (Naito, 1999; Saez, 2004), tax evasion (Boadway *et al.*, 1994), uncertainty (Cremer and Gahvari, 1995), or imperfect coordination between fiscal authorities (Belan *et al.*, 2008, Footnote 1, p. 1739).

to the OCT problem. In a dynamic context, Stiglitz (1976) and Lewis *et al.* (1979) have shown the crucial role played by demand elasticity in a resource monopoly, culminating in the special isoelastic case where monopoly power becomes entirely eroded by the necessity for the monopoly to compete with itself intertemporally. Since facing a revenue constraint introduces a monopolistic revenue maximization component into the objective of Ramsey's government, one may expect the intertemporal nature of non-renewable resource taxation to confer a special role to demand elasticity as in Stiglitz's resource monopoly.

As a matter of fact, there already exists an "elasticity rule" of optimal non-renewable resource taxation. This rule is due to Bergstrom (1982) who showed that a country should set its national excise tax rate according to a "rule relating the equilibrium excise tax rates to demand elasticities and market shares" (p. 194). While Bergstrom's tax is not designed to meet revenue needs but aims at capturing resource rents otherwise enjoyed by other countries, his rule applies to the same tax instrument as Ramsey's rule and will be seen to be a particular case of the latter.

How should the Ramsey-Pigou inverse elasticity rule of optimal taxation be modified in presence of non-renewable resources? Under the standard assumptions made in the non-renewable resource extraction and the OCT literatures, it turns out that a non-renewable resource should be taxed in priority. However, it is only when the tax revenue needs of the government exceed some threshold that elasticities become relevant. While no such distinction between high and low revenue needs is to be found in usual formulations of the Ramsey-Boiteux optimum tax, the presence of a Hotelling sector in the economy introduces resource scarcity rents. Such rents are not conventional profits but they happen to be taxable by commodity taxes without distortion; this possibility to tax rents should be used before turning to other ways to generate tax income. A similar situation arises in Sandmo (1975) where government revenue needs may be covered by Pigovian taxes.[2] The distinction between low and high government-revenue needs then

[2]We thank a referee for pointing that out. Dasgupta and Stiglitz (1972) pointed out that the OCT problem is most interesting in cases "where government losses cannot be covered by the exclusive selection of optimum profit taxes." (p. 92). Sandmo (1975, 1976) expressed the same opinion.

has a clear interpretation: revenue needs are low when they can be satisfied without imposing any distortion on the economy; revenue needs are high otherwise. When revenue needs are high resource rents must be entirely taxed away. We show that the tax income raised from the resource sector can then be further increased, but not without distorting the extraction time profile; furthermore, the resource should be taxed at a higher rate than conventional commodities having the same demand elasticity.

We adhere to the conventional Ramsey-Pigou framework where commodity taxes are viewed as the sole available tax instrument. Direct taxation (of profits, of incomes, of resource rents) is not an option[3]; lump-sum transfers are impossible; indirect linear taxes or subsidies can be applied on the final consumption or on the production of any commodity or service; we assume that the resource is not used as an intermediary input[4]; taxes (or subsidies) may take the form of *ad valorem* taxes or of *unit* taxes, proportional to quantities. The government is not concerned with individual differences; in fact we assume a representative consumer. The optimal supply of public goods is not addressed either; we assume that the government faces exogenous financial needs in order to fulfill its role as a supplier of public goods so that the government's problem is to raise that amount of revenues in the least costly way, given the available tax instruments.

While this framework explicitly rules out the taxation of capital income, whether in the form considered by Chamley (1986), or in a form mimicking profit taxation as with Lucas' (1990) capital levies, or via some form of resource rent taxes as described by Boadway and Keen (2010), some results will be related to the taxation of capital income

[3]Taxes applied on the demand side are almost exclusively indirect, linear taxes. On the supply side, non-distortionary taxes such as the resource rent tax have been devised and are advocated by economists (see *e.g.* Boadway and Flatters, 1993; Boadway and Keen, 2010); however, taxing rents is certainly not easier in resource sectors than in other sectors. Royalties and other linear commodity taxes are an important form of resource taxation (Daniel *et al.*, 2010) so that adherence to the Ramsey-Pigou framework enhances rather than it reduces the empirical relevance of the analysis. Indeed, while royalties are often, usually imperfectly, modified to aim at rents (excluding quasi-rents), the resource rent tax and its cohabitation with the corporate income tax, even absent any uncertainty, raise theoretical and implementation issues (Gaudet and Lasserre, 1986; Lasserre, 1991, Chapter 5; Garnaut, 2010). For a good practical example of a relatively advanced system, see Alberta Royalty Review (2007, pp. 54-60).

[4]We focus on the resource as a consumption good. According to Diamond and Mirrlees (1971) a production input should normally not be taxed if the technology exhibits constant returns to scale. In the case of a non-renewable resource, this condition is violated so that, as anticipated by Stiglitz and Dasgupta (1971), the result only applies in the absence of restrictions on the tax instruments or when exhaustibility is not taken into account (De Miguel and Manzano, 2006; Petrucci, 2010).

because applying a commodity tax to resource extraction over time is not unlike taxing the income of the resource capital. The result that no tax should be applied on the income of Chamley's productive capital in the long run obeys the same logic as OCT: the social cost of capital taxation over the long run is so extraordinarily high that it is impossible to evenly spread distortions across sectors while having a positive capital tax. However, we show that Chamley's result does not apply to such capital as a stock of non-renewable resource, despite the fact that the generation of reserves by exploration is analogous to the generation of capital by investment.

It is conventional to establish the inverse elasticity rule of OCT under the simplifying assumption that supply elasticity is infinite, so that distortions are determined on the demand side. Such long-run perspective fits nicely with the assumption that profits are not taxed since competitive-equilibrium profits are zero under constant returns. On the other hand, the supply of a non-renewable resource is not infinitely elastic even if marginal extraction costs are constant. This is because the short-run supply of a non-renewable resource consists in allocating the production from a finite stock of reserves over time. A resource supplier that increases production at any date reduces the stock of reserves remaining for production in subsequent periods, so that the instantaneous supply elasticity is finite. An extreme example of this link between the fixity of long-run reserves and short-run supply occurs when a constant-rate commodity tax is imposed on a costlessly extracted resource, as assumed by Bergstrom (1982). Short-run supply is then insensitive to the tax.

In this paper, the commodity tax rate is allowed to vary over time. In order to facilitate comparisons with the conventional analysis involving non-resource sectors, we proceed in several steps. In the first step, presented in Section 2, we follow the traditional optimal taxation literature in assuming constant marginal costs of production. This implies that supply is infinitely elastic in non-resource sectors as should be the case in a long-run analysis when no factors are fixed. In the non-renewable resource sector, the same assumption on the technology, constant marginal extraction cost, implies that there is no limit to short-run supply; however Hotelling's long-run exhaustibility of the resource

retains its central role. It is in that setup that we obtain the result mentioned above that the resource should be taxed in priority over producible commodities.

In the rest of the investigation, we examine the role of some basic assumptions affecting resource supply and resource demand. Section 3 gathers several extensions of Section 2. It starts with the introduction of non-zero cross-price demand elasticities between the resource and other commodities. We then investigate the implications of increasing marginal costs of production/extraction, so that short-run supply elasticities are no longer assumed infinite. Finally, we relax Hotelling's resource homogeneity assumption by considering deposits of differing qualities. Exhaustibility retains the central role identified in the original setup.

There are two basic ways to alleviate resource supply limitations; one is to produce reserves for subsequent extraction; the other one is to rely on imports. In Section 4, we still assume that the economy is autarkic while the production of reserves is determined by the net-of-tax rents derived during the extraction phase, including quasi-rents, completed by subsidies or tax rebates that the owner receives toward the production of reserves. This means that resource supply is allowed to be elastic not only in the short run as in the first part of the paper, but also in the long run. A first implication is that resources should never in that case be singled out as sole targets for OCT. Reserve supply elasticity combines with demand elasticity to determine how the taxation burden should be spread across resource and non-resource sectors. As far as the resource sector is concerned, we show that there exists a continuum of mixed tax systems, combining subsidies toward reserve supply with taxes on resource production, that achieve government's objectives in terms of reserve development and tax revenues. Indeed most commonly observed extractive resource tax systems exhibit combinations of incentives to exploration and development with taxation of production; this includes the polar case of a nationalized extraction sector where the government, perhaps because it is unable to commit to less drastic a tax system, appropriates itself the totality of resource rents, including quasi-rents, during the extraction phase but also finances the totality of reserve development.

All such optimal combinations of extraction taxes with exploration and development

subsidies imply a tax load at least as high on the resource than on conventional commodities having the same demand elasticity. However the tax causes a further distortion, on induced reserves; when this is taken into account, the optimal tax on resource extraction is shown to depend, besides demand elasticity, on the long-run elasticity of reserve development. Thus the distortion induced by resource taxes is split between a distortion on the extraction profile corresponding to the inverse demand elasticity rule, and a distortion on the level of induced reserves, obeying a rule reminiscent of the inverse elasticity rule for commodities of finite supply elasticity.

Section 5 allows the country to trade the resource. A country that imports the resource cannot apply any form of resource rent taxation to foreign suppliers; however it can apply commodity taxes to home consumption as an imperfect substitute to resource rent taxation. Consequently the limits to available tax instruments implied by Ramsey's OCT framework are no longer simply reasonable as under autarky, but become compelling. For given subsidies to domestic reserve supply, the result that domestic resource consumption is to be taxed at a higher rate than conventional commodities having the same demand elasticity emerges reinforced. Furthermore, optimal resource-demand and reserve-supply taxes or subsidies reflect the rent capture motive analyzed by Bergstrom (1982) in addition to their tax revenue objective. The optimal tax formula will be seen to divide itself into components that reflect such multiplicity of objectives. Demand taxes are higher (reserve subsidies lower) when government needs are high than when they are low by an amount that reflects domestic and foreign demand elasticities, as well as the elasticity of domestic reserves.

Proofs that are economically enlightening are provided in the main text; proofs involving algebraic manipulations are relegated to Appendices.

2 OCT with a non-renewable resource: constant marginal costs

There are n produced commodities indexed by $i = 1, ..., n$, and one non-renewable resource indexed by s and extracted from a finite reserve stock S_0. The assumption of a single non-renewable resource simplifies the exposition without affecting the generality of the results. At each date $t \geq 0$, quantity flows are denoted by $x_t \equiv (x_{1t}, ..., x_{nt}, x_{st})$.

Storage is not possible, so that goods must be consumed as they are produced. Producer prices are $p_t \equiv (p_{1t}, ..., p_{nt}, p_{st})$ and goods are taxed at unit levels $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{st})$ so that the representative consumer faces prices $q_t = p_t + \theta_t$. In this section and in any situation where production equals consumption, taxes may indifferently be interpreted as falling on consumers or producers, but must be such that they leave non-negative profits to producers. In the case of the non-renewable resource, this requires that, at any date, the discounted profits accruing to producers over the remaining life of the mine be non negative. Taxes that meet these conditions will be called feasible.

Since the resource is non renewable it must be true that

$$\int_0^{+\infty} x_{st}\, dt \leq S_0, \tag{1}$$

where S_0 is the initial size of the depletable stock.

In the rest of the paper, a "~" on top of a variable means that the variable is evaluated at the competitive market equilibrium. For given feasible taxes $\Theta \equiv \{\theta_t\}_{t\geq 0}$, competitive markets lead to the equilibrium allocation $\{\widetilde{x}_t\}_{t\geq 0}$ where $\widetilde{x}_t = (\widetilde{x}_{1t}, ..., \widetilde{x}_{nt}, \widetilde{x}_{st})$. Under the set of taxes Θ, this intertemporal allocation is second-best efficient.

Defining social welfare as the cumulative discounted sum of instantaneous utilities $\widetilde{W}_t$, the OCT problem consists in choosing a feasible set of taxes Θ in such a way as to maximize welfare while raising a given level of discounted revenue $R_0 \geq 0$:

$$\max_{\Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt \tag{2}$$

$$\text{subject to } \int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0. \tag{3}$$

It is assumed that the set of feasible taxes capable of collecting R_0 is not empty.

The tax revenue constraint (3) does not bind the government at any particular date because financial markets allow expenditures to be disconnected from revenues. The government accumulates an asset a_t over time by saving tax revenues:

$$\dot{a}_t = r a_t + \theta_t \widetilde{x}_t, \tag{4}$$

where the initial amount of asset is normalized to zero and

$$\lim_{t\to+\infty} a_t e^{-rt} = R_0. \tag{5}$$

Thus the problem of maximizing (2) subject to (3) can be replaced with the maximization of (2) subject to (4) and (5), by choice of a feasible set of taxes.

As in Ramsey (1927, p. 55), Baumol and Bradford (1970), or Atkinson and Stiglitz (1980), we assume that the demand $D_i(q_{it})$ for each commodity i or s depends only on its own price, with $D_i'(.) < 0$. Moreover, following Baumol and Bradford (1970), Atkinson and Stiglitz (1980) and many other treatments of OCT, we assume in this section that the supply of each commodity is perfectly elastic, *i.e.* that marginal costs of production are constant. Let $c_i \geq 0$ be the marginal cost of producing good $i = 1, ..., n$.

In the case of the non-renewable resource, the supply is determined by Hotelling's rule under conditions of competitive extraction. Consistently with our assumption of constant marginal costs of production, we assume that the unit cost of extracting the resource is constant, equal to $c_s \geq 0$.

However, this does not imply that the producer price of the non-renewable resource reduces to this marginal cost; Hotelling's analysis shows supply to be determined in competitive equilibrium by the so-called "augmented marginal cost" condition:

$$\widetilde{p}_{st} = c_s + \widetilde{\eta}_t, \tag{6}$$

where $\widetilde{\eta}_t$ is the current-value unit Hotelling's rent accruing to producers; it depends on the tax and the level of initial reserves, and must grow at the rate of discount over time. In competitive Hotelling equilibrium,

$$\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}. \tag{7}$$

At any date, the net consumer surplus[5], producer surplus, and resource rents in competitive equilibrium are respectively

$$\widetilde{CS}_t = \sum_{i=1,...,n,s} \int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\, du - \sum_{i=1,...,n,s} (\widetilde{p}_{it} + \theta_{it})\widetilde{x}_{it}, \tag{8}$$

$$\widetilde{PS}_t = \sum_{i=1,...,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,...,n,s} c_i\widetilde{x}_{it} - \widetilde{\eta}_t\widetilde{x}_{st} \tag{9}$$

[5]Defining the consumer surplus and the welfare function in this manner implies that the utility function is assumed to be quasi-linear.

and

$$\widetilde{\phi}_t = \widetilde{\eta}_t \widetilde{x}_{st}. \tag{10}$$

Define $\widetilde{W}_t$ in problem (2) as the sum of net consumer surplus, net producer surplus, and resource rents accruing to resource owners[6,7]. The present-value Hamiltonian associated with the problem of maximizing cumulative discounted social welfare (2) under constraints (4) and (5) resulting from the budget requirement of the government is

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t), \tag{11}$$

where λ_t is the co-state variable associated with a_t while θ_t is the vector of control variables. λ_t can be interpreted as the current unit cost of levying one dollar of present-value revenues through taxes. From the maximum principle, $\dot{\lambda}_t = -\frac{\partial \mathcal{H}}{\partial a_t}$, so that $\lambda_t = \lambda e^{-rt}$, where λ is the present-value unit cost of levying tax revenues. Indeed tax revenues must be discounted according to the date at which they are collected. λ is equal to unity when there is no deadweight loss associated with taxation; it is higher than unity otherwise.

2.1 Optimal taxation of conventional goods

Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of the tax θ_{it} on good $i = 1, ..., n$ is

$$[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - c_i]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0. \tag{12}$$

Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = c_i + \theta_{it}$, it is the case that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.)}$. The optimum tax is thus $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}D_i^{-1\prime}(.)$ and the optimum tax rate is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_i}, \tag{13}$$

[6]Although changes in current taxes may affect current tax revenues, the budget constraint of the government applies only over the entire optimization period. The revenue requirements being treated as given over that period, they enter the general problem as a constant and thus no amount of redistributed taxes needs to enter the objective.

[7]This formulation has the advantage of making the value of the resource as a scarce input explicit; it would also apply if producers were not owners of the resource but were buying the resource from its owners at its scarcity price $\widetilde{\eta}_t$.

where $\varepsilon_i \equiv \frac{D_i^{-1}(.)}{x_{it} D_i^{-1\prime}(.)}$ is the elasticity of demand, negative by assumption. λ being the unit cost of levying revenues through taxes, it is strictly greater than unity when taxes are distortionary and equals unity if there is a non-distortionary way to collect revenues. Hence, the optimal tax rates on conventional goods $i = 1, ..., n$ are positive in general, lower than unity, and vanish if $\lambda = 1$.

Formula (13) is Ramsey's formula for the optimal commodity tax. It provides an inverse elasticity rule for the case of perfectly-elastic supplies. Since market conditions are unchanged from one date to the other, the taxes and the induced tax rates are constant over time.

2.2 Optimal taxation of the non-renewable resource

The first-order condition for an interior solution to the choice of the resource tax is

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0. \tag{14}$$

However, since resource supply is determined by condition (6), it follows that $D_s^{-1}(\widetilde{x}_{st}) - c_s - \theta_{st} = \widetilde{\eta}_t$, which is different from zero unlike the corresponding expression in (12). Consequently the Ramsey-type formula obtained for conventional goods does not apply.

If $\lambda = 1$, (14) reduces to $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = 0$. This means that the tax should not distort the Hotelling extraction path. Such a non-distortionary resource tax exists (Burness, 1976; Dasgupta *et al.*, 1981); it must grow at the rate of interest to keep the path of consumer prices unchanged[8]: $\theta_{st}^* = \theta_{s0}^* e^{rt}$. Since θ_{st}^* grows at the rate of interest and the resulting $\widetilde{q}_{st}$ generally grows at a lower rate, the neutral tax rate is rising over time. The only exception is when the marginal cost of extraction is zero so that $\widetilde{q}_{st}$ grows at the rate of interest and the resulting optimal tax rate is constant.

As shown earlier, when $\lambda = 1$, commodity taxes on conventional goods are zero. Hence the totality of the tax burden falls on the non-renewable resource. Since the tax on the resource is neutral in that case, then a value of unity for λ is indeed compatible

[8]Their proof goes as follows. Assume $\theta_{st} = \theta_{s0} e^{rt}$, for any θ_{s0} lower than the consumer price exclusive of the marginal cost in the absence of any resource tax. Then $\widetilde{q}_{st} = \widetilde{p}_{st} + \theta_{st} = c_s + \widetilde{\eta}_t + \theta_{st} = c_s + (\widetilde{\eta}_0 + \theta_{0t})e^{rt}$. Therefore, the price with the tax satisfies the Hotelling rule. The exhaustibility constraint must also be satisfied with equality: $\int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt = S_0$. As a result, the extraction path under this tax is the same as in the absence of tax. The reasoning survives the introduction of increasing marginal extraction costs and of cross-price effects.

with taxing the natural resource exclusively. Consequently, provided the tax on the non-renewable resource brings sufficient cumulative revenues, the government should tax the resource exclusively, and should do so while taxing a proportion of the resource rent that remains constant over time.

The maximum revenue such a neutral resource tax can extract is the totality of gross cumulative scarcity rents that would accrue to producers in the absence of a resource tax. Since unit rents are constant in present value, any reserve unit fetches the same rent, whatever the date at which it is extracted. The present value of total cumulative exhaustibility rents is thus $\widetilde{\eta}_0 S_0$ and its maximum possible value $\overline{\eta}_0 S_0$ corresponds to the absence of taxation; the maximum tax revenue that can be raised by a neutral resource tax is thus

$$\overline{R}_0 = \overline{\eta}_0 S_0.$$

This maximum is implemented with a tax equal to the unit rent in the absence of taxation: $\theta^*_{st} = \overline{\eta}_0 e^{rt}$. Both $\widetilde{\eta}_0$ and $\overline{\eta}_0$ are determined in Appendix A. If the tax revenues needed by the government are lower than $\overline{R}_0$, the level of the neutral resource tax θ^*_{st} is set in such a way as to exactly raise the required revenue: $\theta^*_{st} = \theta^*_{s0} e^{rt}$ with

$$\theta^*_{s0} = \overline{\eta}_0 - \widetilde{\eta}_0 = \frac{R_0}{S_0}. \tag{15}$$

If $R_0 > \overline{R}_0$, revenue needs cannot be met by neutral taxation of the resource sector and $\lambda > 1$; this case will be discussed further below. The following proposition summarizes our findings when government revenue needs are low in the sense that $\lambda = 1$.

Proposition 1 *(Low government revenue needs) The maximum tax revenue that can be raised neutrally from the non-renewable resource sector is $\overline{R}_0 = \overline{\eta}_0 S_0$ where $\overline{\eta}_0$ is the unit present-value Hotelling rent under perfect competition and in the absence of taxation.*

1. *If and only if $R_0 \leq \overline{R}_0$, government revenue needs are said to be low and $\lambda = 1$; if and only if $R_0 > \overline{R}_0$, government revenue needs are said to be high and $\lambda > 1$;*

2. *When $R_0 \leq \overline{R}_0$, the optimum unit tax on the non-renewable resource is positive and independent of demand elasticity while the optimum unit tax on produced goods is zero. The resource tax raises exactly R_0 over the extraction period.*

As long as the government's revenue needs are low, Proposition 1 indicates that the archetypical distortionary tax of the OCT literature should not be applied to conventional commodities; it should be applied to the sole resource according to a rule that has nothing to do with Ramsey's rule, is independent of the elasticity of demand and does not induce any distortion.[9] As Sandmo puts it (1976, p. 38), "... taxation need not be distortionary by the standard of Pareto optimality. But it seems definitely sensible to admit the unrealism of the assumption that the public sector can raise all its revenue from neutral (...) taxes, and once we admit this we face the second-best problem of making the best of a necessarily distortionary tax system. This is the problem with which the optimal tax literature is mainly concerned."

If the government revenue needs are high in the sense that $R_0 > \overline{R}_0$ and $\lambda > 1$, revenue needs cannot be met by neutral taxation; then we have shown that both the resource and the conventional goods should be taxed. Furthermore, the question arises whether the government can and should collect more resource revenues by departing from neutral taxation of the resource sector[10]. This possibility was not explored by Dasgupta, Heal and Stiglitz (1981), nor by followers.

The neutral tax that maximizes tax revenues does not leave any resource rent to producers: $\widetilde{q}_{st} = c_s + \theta_{st}$. Assume, as will be seen to be true later on, that the government can maintain its complete appropriation of producers' resource rents while further increasing tax revenues: the condition $\widetilde{q}_{st} = c_s + \theta_{st}$ remains true while θ_{st} is set so as to further extract some of the consumer surplus. This implies that, when $\lambda > 1$, $\widetilde{p}_{st} = c_s$, $\widetilde{\eta}_t = 0$, $\widetilde{x}_{st} = D_s(c_s + \theta_{st})$. With $\widetilde{\eta}_t = 0$, resource extraction is no longer determined by the Hotelling supply condition (6). The finiteness of reserves may still come as a constraint, but as a constraint faced by the government in its attempt to increase cumulative tax revenues rather than as a constraint faced by producers in maximizing cumulative profits.

[9]The fact that neutral taxation of the Hotelling commodity is possible does not mean that neutral profits taxation à la Stiglitz and Dasgupta (1971) or capital levy à la Lucas (1990), or some form of resource rent tax à la Boadway and Keen (2010) have been allowed into the model. It should be clear from the formulation that the result is reached by commodity taxation.

[10]Clearly, at each date, a non-linear tax on the resource extraction rate reaching the level of the maximum constant neutral tax at the Pareto-optimal extraction rate, would achieve such a goal. However such non-distortionary tax is ruled out in the conventional Ramsey-Pigou optimal taxation analysis. If it was feasible the Ramsey-Pigou problem would be meaningless.

Thus the government's problem is now to maximize (2), not only subject to (4) and (5), but also subject to

$$\dot{S}_t = -\widetilde{x}_{st}, \tag{16}$$

where S_t denotes the size of the remaining depletable stock at date t.

The Hamiltonian is modified to

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t, \mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t\widetilde{x}_t) - \mu_t\widetilde{x}_{st}, \tag{17}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are defined as before but with $\widetilde{\eta}_t = 0$, and μ_t is the co-state variable associated with the exhaustibility constraint. From the maximum principle, $\lambda_t = \lambda e^{-rt}$, as above, and $\mu_t = \mu \geq 0$. If the exhaustibility constraint is binding, that is to say if optimal taxation induces complete exhaustion of the reserves, $\mu > 0$; if optimal taxation leads to incomplete exhaustion, then $\mu = 0$.

The first-order condition for the choice of the tax on the resource becomes

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}. \tag{18}$$

Since no resource rent is left to producers above the marginal cost of extraction, $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = 0$, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$, and the optimum tax on the resource is thus

$$\theta_{st}^* = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}. \tag{19}$$

Provided the resource is scarce ($\mu > 0$) from the government's point of view, (19) implies that the resource is taxed at a higher rate than would be the case according to (13) for a conventional commodity having the same demand elasticity. Furthermore, while the first term on the right-hand side of (19) is neutral as it rises at the rate of discount, the presence of the second term implies that the tax is not constant in present value, so that it is distortionary in general.

Can the tax revenue collection motive cause the government to assign no scarcity value to a resource that would otherwise be extracted until exhaustion? The answer is negative. For suppose that $\mu = 0$ in (19). This implies that the tax rate is constant over time, so that the extraction rate is also constant and strictly positive, which in turn implies that the exhaustibility constraint must be violated in finite time.

The following proposition summarizes the results on the optimum taxation of the resource when neutral taxation is not sufficient to collect the revenue needs.

Proposition 2 *(High government revenue needs) If $R_0 > \overline{R}_0$, then commodity taxation is distortionary ($\lambda > 1$) and both the non-renewable resource sector and conventional sectors are subject to taxation. In that case:*

1. *Taxes on conventional commodities are given by Ramsey's rule* (13) *and the tax on the non-renewable resource is given by* (19)*, where λ is determined by the condition that total tax revenues levied from the non-resource and resource sectors equal R_0;*

2. *The non-renewable resource is taxed at a higher rate than a conventional commodity having the same demand elasticity;*

3. *The after-tax resource rent to producers is nil: $\widetilde{\eta}_t = \widetilde{\eta}_0 = 0$;*

4. *It is never optimal for the government to induce reserves to be left unexploited.*

Propositions 1 and 2 also have implications on the evolution of the total flow of tax revenues over time. When the government's revenue needs are low, the total flow of tax revenues decreases in present value. Indeed, the resource unit tax is constant in present value if (15) applies while extraction diminishes. Tax revenues from conventional sectors being nil, total tax revenues decrease in present value and vanish entirely if the resource is exhausted in finite time. When the government's revenue needs are high, the flow of tax revenues from conventional sectors is constant in current value. If the resource is exhausted in finite time, the total tax revenue flow is thus lower at and after the date of exhaustion than before exhaustion. In either case, the government's assets accumulated at resource exhaustion must be sufficient to ensure that expenditures taking place after exhaustion can be financed.

When the government cannot avoid the introduction of distortions, as when revenue needs are high, its problem acquires a revenue-maximizing dimension. This confers to OCT a resemblance with monopoly pricing (for details see Appendix D). The resource monopoly literature has shown that the exercise of market power by a Hotelling resource

monopoly is constrained by exhaustibility. The sharpest example is Stiglitz (1976) who showed that a resource monopoly facing a constant-elasticity demand and zero extraction costs must adopt the same behavior as a competitive firm; such a monopoly cannot increase its profits above the value of the mine under competition by distorting the extraction path. This limitation also applies to the OCT problem. With zero extraction cost and isoelastic demand, the tax defined by (19) is neutral and rises at the discount rate. We prove that result and make use of it in Section 4, where initial reserves are treated as endogenous.

From Propositions 1 and 2, the resource should be taxed in priority whatever its demand elasticity and whatever the demand elasticity of regular commodities. This irrelevance of demand elasticities contrasts sharply with the standard rationalization of OCT but not with Ramsey's original message. The message is "tax inelastic sectors" whether the source of inelasticity is demand or supply. Once it is realized that long-run reserve supply fixity results in short-run resource supply inelasticity, it becomes clear that the emphasis should shift from demand to supply in the case of a Hotelling resource.

In the next section, we extend the analysis to the case of increasing marginal costs of production and increasing marginal costs of extraction, so that supply elasticity is no longer infinite. While the inverse elasticity rule then acquires a supply elasticity component, the finiteness of ultimate reserves implies that non-renewable resources should be taxed in priority and at higher rates than otherwise identical conventional commodities. What matters is long-run supply inelasticity. We also examine the role of resource heterogeneity and the implications of the presence of substitutes or complements to the resource among conventional commodities. Again, the results are altered but not modified in any fundamental way.

The reader may want to skip the next section and move directly to Section 4 where the Hotelling assumption that reserves are exogenously given is relaxed. Doing away with this assumption introduces the long-run supply elasticity of the resource and also allows us to highlight the distinction between a non-renewable resource and conventional capital.

3 Interdependent demands, rising marginal costs, resource heterogeneity

One may wonder whether the sharp results of the previous section are not due to the parsimony of the model, in particular the partial-equilibrium setup ruling out any interdependence between demands, the assumption that the supply of all conventional commodities is perfectly elastic, and the assumption that marginal extraction costs are not only constant but independent of the source of resource supply. It will be shown that the basic message – tax the resource more than similar conventional commodities – is not much affected by relaxing these assumptions, although several new insights are derived from the analysis.

3.1 OCT with resource substitutes or complements

Sandmo pointed out that "In the general case, it is not easy to see the structure of taxation which follows from the general optimality conditions." (1976, p. 45). Nonetheless, it is not necessary to adopt a fully fledged general-equilibrium model to determine how substitutes or complements affect OCT. We will show that optimal tax rules then entail the same bias whether the commodity is conventional or is a non-renewable resource. However, we will show that substitutes or complements of the resource may be left untaxed while the resource is taxed, as in Proposition 1.

Assume that the demand $D_j(q_{jt}, q_{kt})$ for a conventional commodity $j \in \{1, ..., n\}$ not only depends on its own price, but also on the price of another commodity $k \in \{1, ..., j-1, j+1, ..., n, s\}$, with $\frac{\partial D_j(.)}{\partial q_j} < 0$, $\frac{\partial D_k(.)}{\partial q_k} < 0$, and $\frac{\partial D_j(.)}{\partial q_k}, \frac{\partial D_k(.)}{\partial q_j} > 0$ (< 0) if the goods are substitutes (complements). The gross consumer surplus arising from that pair of goods is not separable but should be replaced with the concave money-metric surplus[11] $\psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})$, with $\frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_j} = \widetilde{q}_j$ and $\frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_k} = \widetilde{q}_k$.

After redefining (8) and the welfare function in problem (2) accordingly, the first-order condition for θ_{jt} now takes account of the effect of that tax on the tax income raised in sector k; the first-order condition for the tax on a conventional commodity j is no longer

[11] Although, in the non-separable case, the consumer surplus cannot be written in terms of demand functions only, the utility is assumed to be quasi-linear, as in the rest of the paper.

(12) but

$$\left[\frac{\partial\psi(.)}{\partial x_{jt}}-\theta_{jt}-c_j\right]\frac{d\widetilde{x}_{jt}}{d\theta_{jt}}+\left[\frac{\partial\psi(.)}{\partial x_{kt}}-\theta_{kt}-c_k\right]\frac{d\widetilde{x}_{kt}}{d\theta_{jt}}-\widetilde{x}_{jt}$$
$$+\lambda\left(\widetilde{x}_{jt}+\theta_{jt}\frac{d\widetilde{x}_{jt}}{d\theta_{jt}}+\theta_{kt}\frac{d\widetilde{x}_{kt}}{d\theta_{jt}}\right)=0,\qquad k\neq s,\qquad(20)$$

$$\left[\frac{\partial\psi(.)}{\partial x_{jt}}-\theta_{jt}-c_j\right]\frac{d\widetilde{x}_{jt}}{d\theta_{jt}}+\left[\frac{\partial\psi(.)}{\partial x_{st}}-\theta_{st}-c_s\right]\frac{d\widetilde{x}_{st}}{d\theta_{jt}}-\widetilde{x}_{jt}$$
$$+\lambda\left(\widetilde{x}_{jt}+\theta_{jt}\frac{d\widetilde{x}_{jt}}{d\theta_{jt}}+\theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{jt}}\right)=\mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{jt}},\qquad k=s,\qquad(21)$$

where $\frac{\partial\psi(\widetilde{x}_{jt},\widetilde{x}_{kt})}{\partial x_{jt}}=c_j+\theta_{jt}$ and $\frac{\partial\psi(\widetilde{x}_{jt},\widetilde{x}_{kt})}{\partial x_{kt}}=c_k+\theta_{kt}$. The condition for $k=s$ holds because the producer rent $\widetilde{\eta}_t$ is nil whenever $\lambda>1$, as in Section 2. Moreover, $\widetilde{x}_{jt}=D_j(\widetilde{q}_{jt},\widetilde{q}_{kt})$ and $\widetilde{x}_{kt}=D_k(\widetilde{q}_{kt},\widetilde{q}_{jt})$ so that $\frac{d\widetilde{x}_{jt}}{d\theta_{jt}}=\frac{\partial D_j(.)}{\partial q_{jt}}$ and $\frac{d\widetilde{x}_{kt}}{d\theta_{jt}}=\frac{\partial D_k(.)}{\partial q_{jt}}$, whether $k\neq s$ or $k=s$. The optimum tax on a conventional commodity is thus

$$\theta^*_{jt}=\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}}-\theta_{kt}\frac{\widetilde{x}_{kt}\widetilde{\varepsilon}_{kj}}{\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}},\qquad k\neq s,\qquad(22)$$

$$\theta^*_{jt}=\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}}-\left(\theta_{st}-\frac{1}{\lambda}\mu e^{rt}\right)\frac{\widetilde{x}_{st}\widetilde{\varepsilon}_{sj}}{\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}},\qquad k=s,\qquad(23)$$

where the own-price elasticity of the demand for good j is now denoted by $\varepsilon_{jj}=\frac{q_{jt}\frac{\partial D_j(.)}{\partial q_j}}{x_{jt}}$ and where $\varepsilon_{kj}=\frac{q_{jt}\frac{\partial D_k(.)}{\partial q_j}}{x_{kt}}$ is the cross-price elasticity of the demand for commodity k with respect to the price of commodity j.

When the resource admits conventional commodity j as a substitute or complement, the first-order condition for the choice of the tax on the resource is the same as (21) except that s and j must be interchanged on the left-hand side and that the right-hand side becomes $\mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}$. Thus the optimum tax on the resource becomes, instead of (19),

$$\theta^*_{st}=\frac{1}{\lambda}\mu e^{rt}+\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_{ss}}-\theta_{jt}\frac{\widetilde{x}_{jt}\widetilde{\varepsilon}_{js}}{\widetilde{x}_{st}\widetilde{\varepsilon}_{ss}}.\qquad(24)$$

All three tax formulae are identical to their independent-demand counterparts, except for the last term on the right-hand side which is new; it reflects the change in fiscal revenues levied on the sector indirectly impacted by the tax. Remembering that the tax θ_{it} is the mark-up on top of the producer price in sector i, it is apparent that the additional term is related to monopoly pricing. Precisely, it corresponds to the term

which completes the monopoly-pricing formula when a firm holds monopoly power on a second commodity while demands are not independent and costs are separable (*e.g.* Tirole, 1988, p. 70). The adjustment to the tax is positive (negative) when commodities j and k are substitutes (complements). This adjustment is formally the same in (22) and in (24), *i.e.* whether it applies to a tax on a conventional commodity or to the tax on the resource. What matters is whether the commodity indirectly impacted by the tax is the resource or not; when it is the resource, the adjustment to the tax is smaller in absolute value, other things equal.

Comparing (22) with (23) and (24) leads to two further observations regarding the optimal taxation of the substitutes and complements of a resource. First, other things equal, a resource substitute should be taxed at a lower rate than the substitute of a conventional commodity. Second, absent cross-price effects, the stationarity of market conditions imply optimum taxes on conventional commodities to be constant over time while the resource tax varies over time (Section 2). On the contrary, (23) implies that a resource substitute or complement should be taxed in a way that depends on time. This is a noticeable difference with Sandmo's (1975) analysis of OCT when one commodity generates an externality. In his paper, when revenue needs are high in the sense that they cannot be covered by Pigovian taxation alone, the "marginal social damage (...) does not enter the formulas for the other commodities [non externality-creating], regardless of the pattern of complementarity and substitutability." (p. 92).

The neutrality of a time-varying linear tax on a Hotelling resource is not affected by demand interdependency. Hence a neutral commodity tax on the resource may suffice to meet the government's revenue needs. Then $\lambda = 1$ and no commodity other than the non-renewable resource needs to be taxed as can be verified by noting that a value of $\lambda = 1$ implies θ^*_{jt} in (23) to be nil.[12] When government revenue needs are low, conventional commodities remain isolated from the non-renewable resource for OCT purposes even when they are substitutes or complements to the resource.

[12]When $\lambda = 1$, replacing θ_{jt} in (24) by its optimal value θ^*_{jt} given by (23) yields the neutral resource tax $\theta^*_{st} = \mu e^{rt}$. Substituting into (23) gives $\theta^*_{jt} = 0$.

3.2 Rising marginal costs and heterogenous resources

The assumption of infinite supply elasticity made by so many contributors to the OCT literature may be justified on the ground that they adopt a long-run perspective, where all commodities can be produced at constant marginal costs because all inputs are variable. The natural counterpart of constant marginal production cost for conventional commodities is constant marginal extraction cost. This will be replaced by rising marginal production and extraction costs shortly.

There is another consideration. The conditions of extraction of a non-renewable resource may be quite variable over time, as resources are not necessarily homogeneous; a possibility which is ruled out by the simple Hotellian formulation adopted so far. Even with rising marginal extraction cost, the extraction technology does not provide for resource heterogeneity. Two approaches have been used in the literature to deal with this issue. The Ricardian approach considers a single stock of reserves but assumes that the extraction cost increases with cumulative extraction (See, *e.g.* Levhari and Liviatan, 1977; Pindyck, 1978); this approach has been criticized because it implicitly assumes that the economically most accessible reserves are used first, which is not always optimal.[13] The second approach consists in modeling the resource as originating from different deposits each with its own cost function and its own stock of reserves. It underlies the manner in which advanced systems such as the Alberta Oil and Gas taxation regime approach resource taxation[14] (see Slade, 1988, for a theoretical formulation, empirical considerations, and references).

We start with introducing rising marginal costs; then we further add multiple deposits.

[13] As Slade (1988) put it "The idea that the least-cost deposits will be extracted first is so firmly embedded in our minds that it is an often-made but rarely tested assumption underlying the construction of theoretical exhaustible-resource models." (p. 189). See her references.

[14] Conrad and Hool (1981) pointed at the relevance of deposits' differences for resource taxation: In the "... mining problem, (...) differences in the composition of the ore bodies cause differences in response to a given economic change. In part because of this, mineral tax policy in some countries has been negotiated on a mine-by-mine basis. Geological features must therefore be an essential part of any model that is to be used for policy or empirical analysis." (p. 18).

For example in Alberta, royalties depend on the type of resource (conventional oil, gas, oil sands) and the date at which the deposit was discovered, because exploration targets different deposits as extraction technology evolves, as oil prices increase, and as exploration prospects become exploited (Alberta Royalty Review, 2007).

Thus assume that conventional good i is supplied according to the function $S_i(p_{it})$, with $S_i'(.) > 0$, for $i = 1, ..., n$; $S_i^{-1}(x_{it})$ is the increasing marginal cost of producing a quantity x_{it}. Regarding the non-renewable resource, assume an increasing marginal cost of extraction. For notational simplicity, this marginal cost is denoted by $S_s^{-1}(x_{st})$. However, this does not denote the inverse supply function. In competitive equilibrium, the supply of resource is determined by the "augmented marginal cost" condition:

$$\widetilde{p}_{st} = S_s^{-1}\left(\widetilde{x}_{st}\right) + \widetilde{\eta}_t, \tag{25}$$

where the current-value Hotelling's rent $\widetilde{\eta}_t$ grows at the rate of discount.

The OCT problem of maximizing (2) subject to (4) and (5), and the associated Hamiltonian are only modified to the extent that the producer surplus becomes

$$\widetilde{PS}_t = \sum_{i=1,...,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,...,n} \int_0^{\widetilde{x}_{it}} S_i^{-1}(u)\, du - \int_0^{\widetilde{x}_{st}} \left(S_s^{-1}\left(u\right) + \widetilde{\eta}_t\right)\, du. \tag{26}$$

Given this change, the structure of the analysis is quite similar to that of constant marginal costs. Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of the tax θ_{it} on conventional good i is $[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - S_i^{-1}(\widetilde{x}_{it})]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0$. Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = S_i^{-1}(\widetilde{x}_{it}) + \theta_{it}$, it follows that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.) - S_i^{-1\prime}(.)}$. The optimum tax is thus such that $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}\big(D_i^{-1\prime}(.) - S_i^{-1\prime}(.)\big)$. Consequently the optimum tax rate on conventional commodity i is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\left(\frac{1 - \frac{\theta_{it}^*}{\widetilde{q}_{it}}}{\widetilde{\epsilon}_i} - \frac{1}{\widetilde{\varepsilon}_i}\right), \tag{27}$$

where $\epsilon_i \equiv \frac{S_i^{-1}(.)}{x_{it}S_i^{-1\prime}(.)}$ is the elasticity of supply, positive by assumption. As before, λ is strictly greater than unity when taxes are distortionary and equals unity if there is a non-distortionary way to collect enough revenues. Formula (27) provides an inverse elasticity rule for the case of non-perfectly-elastic supplies (Ramsey, 1927, p. 56).

The first-order condition for an interior tax on the resource is now $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st})]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0$. Since resource supply is determined by condition (25), it follows that $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st}) = \widetilde{\eta}_t$, which is different from zero. If tax

revenue needs are low, the other commodities are not taxed at all and the resource is the sole provider of tax revenues; the resource should be taxed in priority even when supply elasticities in the other sectors are not assumed to be infinite.

If the revenues needed cannot be raised neutrally so that λ exceeds unity, all sectors are taxed in such a way that the distortions are spread across sectors; the tax on the resource sector is distortionary as in the previous section. What is new however is that the distortion aims at capturing part of the consumer surplus and part of the producer surplus while no producer surplus was available when marginal extraction were assumed to be constant. In that case, as in Section 2, the government's problem is subject to the exhaustibility constraint (16); taxation completely expropriates producers' resource rents, so that $\widetilde{\eta}_t = 0$ and $\widetilde{q}_{st} = S_s^{-1}(\widetilde{x}_{st}) + \theta_{st}$; the first-order condition for the resource tax becomes $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st})]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}$, where μ is the present-value co-state variable associated with the exhaustibility constraint. The competitive equilibrium allocation satisfies $D_{si}^{-1}(\widetilde{x}_{st}) = S_s^{-1}(\widetilde{x}_{st}) + \theta_{st}$; transforming the first-order condition as for conventional goods yields the optimum tax on the resource

$$\theta_{st}^* = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\left(\frac{\widetilde{p}_{st}}{\widetilde{\epsilon}_s} - \frac{\widetilde{q}_{st}}{\widetilde{\varepsilon}_s}\right), \tag{28}$$

where $\epsilon_s \equiv \frac{S_s^{-1}(.)}{x_{st}S_s^{-1\prime}(.)}$, the reciprocal of the elasticity of marginal extraction costs, can also be interpreted as the elasticity of short-run resource supply. Consequently, the resource should be taxed at a higher rate than conventional commodities having identical elasticities.

Consider now that the resource may be extracted from m deposits using an extraction technology characterized by rising marginal costs, as above but possibly different for each deposit. Each deposit $l = 1, ..., m$ makes a contribution z_{lt} to total production so that consumption of the homogeneous final commodity is $x_{st} = \sum_{l=1,...,m} z_{lt}$. While the consumer price q_{st} is unique, producer prices and scarcity rents typically differ because extraction costs and reserves may differ from one deposit to the next: $p_{lt} = S_l^{-1}(z_{lt}) + \eta_{lt}$, $l = 1, ..., m$. However, since each deposit is homogenous, the corresponding rent satisfies Hotelling's rule and must grow at the rate of interest so that its supply is determined in competitive equilibrium by $\widetilde{p}_{lt} = S_l^{-1}(\widetilde{z}_{lt}) + \widetilde{\eta}_{lt}$ where the Hotelling rent $\widetilde{\eta}_{lt}$ corresponds

to the exhaustibility constraint applying to deposit l: $\int_0^{+\infty} z_{lt}\,dt \leq S_{l0}$. We assume that the government has the ability to tax each deposit individually[15] so that $q_{st} = p_{lt} + \theta_{lt}$, $l = 1, ..., m$. Precisely, the tax θ_{st} that could indifferently fall on demand or supply in the previous cases, is replaced with a vector of taxes that fall on the supply of individual deposits; resource demand is not taxed. For any feasible tax trajectory and Hotelling rent, the output from each deposit adjusts in such a way that marginal extraction cost plus rent equals producer price as required.

The government budget constraint is only modified by the increase in the size of the tax vector which becomes $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{n+1\ t}, ..., \theta_{n+m\ t})$ and by the replacement of consumption x_{st} by the vector of supply tax bases $(z_{1t}, ..., z_{mt})$ in the government budget constraint. Except for the increased number of variables the OCT problem is only modified to the extent that producer surplus becomes, instead of (26),

$$\widetilde{PS}_t = \sum_{i=1,...,n} \widetilde{p}_{it}\widetilde{x}_{it} + \sum_{l=1,...,m} \widetilde{p}_{lt}\widetilde{z}_{lt} - \sum_{i=1,...,n} \int_0^{\widetilde{x}_{it}} S_i^{-1}(u)\,du - \sum_{l=1,...,m} \int_0^{\widetilde{z}_{lt}} \left(S_l^{-1}(u) + \widetilde{\eta}_{lt}\right) du,$$

and the resource rents become, instead of (10),

$$\widetilde{\phi}_t = \sum_{l=1,...,m} \widetilde{\eta}_{lt}\widetilde{z}_{lt}.$$

The first-order conditions for an interior solution to the choice of the taxes on resource extraction are $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{lt} - S_l^{-1}(\widetilde{z}_{lt})]\frac{d\widetilde{z}_{lt}}{d\theta_{lt}} - \widetilde{z}_{lt} + \lambda(\widetilde{z}_{lt} + \theta_{lt}\frac{d\widetilde{z}_{lt}}{d\theta_{lt}}) = 0$. Since supply from deposit l is determined by condition $\widetilde{p}_{lt} = S_l^{-1}(\widetilde{z}_{lt}) + \widetilde{\eta}_{lt}$, it follows that $D_s^{-1}(\widetilde{x}_{st}) - \theta_{lt} - S_l^{-1}(\widetilde{z}_{lt}) = \widetilde{\eta}_{lt}$, $l = 1, ..., m$; the rest of the solution process is as above. If revenue needs are low, a combination of neutral taxes rising at the rate of interest is applied on the extraction of the deposits. If revenue needs are high, the analysis of the single-deposit case applies; denoting by μ_l the present-value co-state variable associated with the exhaustibility constraint of deposit l, one obtains the optimal tax on deposit l

$$\theta_{lt}^* = \frac{1}{\lambda}\mu_l e^{rt} + \frac{\lambda - 1}{\lambda}\left(\frac{\widetilde{p}_{lt}}{\widetilde{\epsilon}_l} - \frac{\widetilde{q}_{st}}{\widetilde{\varepsilon}_s}\right),\ l = 1, ..., m, \tag{29}$$

where $\epsilon_l \equiv \frac{S_l^{-1}(.)}{z_{lt}S_l^{-1\prime}(.)}$. Qualitative results are unchanged.

[15] See Footnote 14.

4 Endogenous reserves

In order to focus on the role of the long-run supply of reserves, we assume in this section, as in Section 2, that marginal extraction costs are constant, equal to $c_s \geq 0$. This means that the supply of the natural resource is only limited by the availability of reserves. As far as produced goods are concerned, their marginal costs of production may be either constant or rising as in Section 3, respectively implying infinite or finite supply elasticity.

The stock of reserves exploited by a mine does not become available without some prior exploration and development investment. Although exploration for new reserves and exploitation of current reserves often take place simultaneously (*e.g.* Pindyck, 1978, and Quyen, 1988), a convenient and meaningful simplification consists in representing them as taking place in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). This way to model the supply of reserves is particularly adapted to the OCT problem under study because it provides a simple and natural way to distinguish short-run supply elasticity from long-run supply elasticity. It also raises the issue of the government's ability to tax and subsidize, as well as its ability to commit.[16]

Most commonly observed extractive resource tax systems feature royalties and levies based on extraction revenues or quantities, often combined with tax incentives to exploration and development. During the extraction phase, *i.e.* once reserves are established, these systems let some Hotelling rents accrue to producers. To the extent that reserve development implied sunk costs, these Hotelling rents include quasi-rents.[17] In such systems, governments may not subsidize or otherwise directly help exploration or reserve development on a scale sufficient to compensate firms for the production of reserves. Firms rely on *ex post* extraction rents for that.

On the other hand, state-owned extraction sectors are common. A nationalized industry means that no extraction rents are left to private producers. Thus two situations are common empirically: in the first instance extraction is taxed in such a way that strictly positive rents are left to firms; in the second instance no extraction rents are left to firms.

[16] On issues of commitment and regime changes in resource taxation, see Daniel *et al.* (2010).

[17] They consist only partly of quasi-rents, first because exploration prospects also induce Hotelling rents; second because decreasing returns to exploration imply infra-marginal rents (Lasserre, 1985).

The results from the previous sections point to the importance of that distinction. Indeed, when S_0 is given as in the previous sections, if the government has high revenue needs in the sense of Proposition 2, it should tax the totality of extraction rents away from producers. If it did so when S_0 were endogenous, it would tax quasi-rents together with real scarcity rents, thus removing incentives for producers to generate reserves in the first place. If the government wants to create a tax environment allowing net extraction profits to compensate firms for the cost of reserve production, it must be able to commit, prior to extraction, to a system of *ex post* extraction taxation that leaves enough rents to producers. Alternatively, if the government taxes away extraction rents, including quasi-rents sunk into them, it must compensate firms by subsidies or tax breaks prior to extraction. In fact we will show that there exists a continuum of mixed systems, combining subsidies toward reserve supply with positive after-tax extraction rents, that achieve the government's objective. These mixed systems are feasible if the government is able to commit to leave firms the prescribed after-tax extraction rent; otherwise, an optimal system relying on reserves supply subsidies exclusively can also achieve the same objective.[18]

For simplicity assume that *ex ante* reserve producers (explorers) are the same firms as *ex post* extractors. Assume that the stock of reserves to be exploited is determined prior to extraction by a supply process that reacts to the sum of the subsidies obtained by the firms during the reserve production phase and the cumulative net present-value rents accruing to resource producers during the exploitation stage; also for simplicity, assume that reserve production is instantaneous.

Express total cumulative present-value rents from extraction as $\eta_0 S_0$. Suppose further that a linear subsidy ρ may be applied to the production of reserves, for a total subsidy

[18]The taxation of profits is compatible with exploration and reserve development expenditures. Expenditures are written against profits during the extraction phase and receive a treatment similar to that of other types of investments. This applies, for instance, to oil sand development expenditures in the Albertan system. We do not consider this option in order to keep adhering with Ramsey's commodity taxation framework. On the other hand, linear subsidies and linear commodity taxes as modeling devices also have clear practical relevance. For example, in the case of conventional oil and natural gas, the Albertan system commits to royalty rate reductions that depend on a well's discovery date; those reductions are thus linear in discovered quantities, irrespective of expenditures and irrespective of the fact that they depend on wells' discovery dates. They amount to linear exploration subsidies whose payment is postponed until extraction.

of ρS_0. Then the initial stock of reserves may be written as a function of $\eta_0 + \rho$. This function $\mathcal{S}(\eta_0 + \rho)$ can be interpreted as the long-run after-tax supply of reserves as follows. Suppose that reserves can be obtained, via exploration or purchase, at a cost $E(S_0)$. As not only known reserves but also exploration prospects are finite, the long-run supply of reserves is subject to decreasing returns, so that $E'(S_0) > 0$ for any $S_0 > 0$, and $E''(.) > 0$. Then the profit from the production of a stock S_0 of initial reserves is $(\widetilde{\eta}_0 + \rho) S_0 - E(S_0)$. Given ρ and $\widetilde{\eta}_0$, its maximization requires $\widetilde{\eta}_0 + \rho = E'(S_0)$. We define $\mathcal{S}(\widetilde{\eta}_0 + \rho) \equiv E'^{-1}(\widetilde{\eta}_0 + \rho)$, making the following assumption.

Assumption 1 *(Long-run supply) The supply of initial reserves* $\mathcal{S}(.)$ *is continuously differentiable and such that* $\mathcal{S}(0) = 0$, $\mathcal{S}(\eta_0 + \rho) > 0$ *for any strictly positive value of* $\eta_0 + \rho$, *and* $\mathcal{S}'(\eta_0 + \rho) > 0$.

The property $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$ is introduced because it is sufficient to rule out the uninteresting situation where the demand for the non-renewable resource does not warrant the production of any reserves.

4.1 Optimal resource taxation with a strictly positive producer rent

Even when the government can subsidize exploration, *i.e.* when $\rho > 0$, leaving some positive after-tax extraction rent to producers may be desirable for the government. Two reasons make it interesting to analyze situations where the government leaves positive extraction rents to producers. First, they are empirically relevant. Second, they will be shown to constitute a general case that includes no-commitment as a limiting case. In this subsection, we assume that ρ is given and is not high enough to remove the need for the government to leave producers positive after-tax extraction rents. Later on, we will analyze the choice of ρ and study whether it is desirable for the government to leave positive extraction rents to producers at all.

Ex post, once reserves have been established, producers face a standard Hotelling extraction problem. Consequently, respecting its commitment amounts for the government to choosing a tax profile that leaves producers a Hotelling rent $\widetilde{\eta}_t > 0$, with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, as defined in (6) and (7), for a total rent commitment of $\widetilde{\eta}_0 S_0$, part of which is the counterpart of exploration expenditures so that it includes quasi-rents. Clearly, given ρ, the level

of initial reserves will be determined *ex ante* by that commitment; it will be denoted $\widetilde{S}_0$, with

$$\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho), \tag{30}$$

and discussed further below.

At the extraction stage, the government chooses optimal taxes given $\widetilde{\eta}_0$, or, equivalently, given any positive $\widetilde{S}_0$. The problem is thus identical to the problem with exogenous reserves analyzed in Section 2, except that the government is now subject to its *ex ante* rent commitment. The Hamiltonian is thus (17), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$ rather than $\widetilde{\eta}_t = 0$:

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t, \mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{31}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are respectively defined by (8), (9) or (26) according to whether marginal costs are constant or rising, and (10), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. The control variables are the taxes θ_t.

Suppose, as an assumption to be contradicted, that $\lambda = 1$; then, according to Proposition 1, conventional goods are not taxed and a tax is imposed on the resource during the extraction phase to satisfy revenue needs. This reduces the rent accruing to extracting firms and, by (30), reduces the initial amount of reserves relative to the no-tax situation. Consequently, any attempt to satisfy revenue needs by taxing the resource extraction sector results in a distortion, so that, in contradiction with the initial assumption, λ is strictly higher than unity whatever the revenue needs. It follows that the tax on conventional goods is given by (13) or by (27) – depending on the assumption made on the cost structure of the conventional sectors – with $\lambda > 1$.

Consider the taxation of the resource sector now, with $\lambda > 1$. In Appendix E, we show that the optimal extraction tax differs from its value when reserves are exogenous, in that it now depends on the rent that the government is committed to as follows:

$$\theta^*_{st} = \frac{1}{\lambda}(\mu - \widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}. \tag{32}$$

The second term on the right-hand side of that expression is the familiar inverse elasticity rule; it appears in the same form as in Formula (19) describing the resource tax when reserves are exogenous. As in that case, the tax rate on the resource thus exceeds the tax

rate on a conventional good of identical demand elasticity if and only if the first term is non negative. Such is clearly the case with exogenous reserves when the first term on the right-hand side is $\frac{1}{\lambda}\mu e^{rt}$ but not so with endogenous reserves as the sign of the first term on the right-hand side of (32) depends on the sign of $(\mu - \widetilde{\eta}_0)$. Intuition suggests that the government would not commit *ex ante* to leaving a unit after-tax rent of $\widetilde{\eta}_0$ to firms if this was not at least equal to its *ex post* implicit valuation μ of a reserve unit. One can validate this intuition by analyzing the choice of $\widetilde{\eta}_0$, which we now turn to.

Let us characterize the *ex ante* choice of $\widetilde{\eta}_0$ for a given level of ρ.[19] The marginal cost of establishing reserves at a level S_0 is $E'(S_0) = \mathcal{S}^{-1}(S_0)$; the total cost of reserves evaluated at date 0 is $\int_0^{S_0} \mathcal{S}^{-1}(S)\, dS$. This cost should be deducted from the *ex ante* objective of the government. The objective should also include the total subsidy payment to producers ρS_0. The *ex ante* problem of the government is thus

$$\max_{\widetilde{\eta}_0,\ \Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{33}$$

subject to the tax revenue constraint, adapted to take account of the additional liability associated with the subsidy:

$$\int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{34}$$

Denote by $V^*\left(\widetilde{S}_0, R; \rho\right)$ the value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$ maximized with respect to θ_t as just discussed. The constant co-state variable μ in (31) can be interpreted as giving the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of reserves, while $-\lambda$ gives the marginal impact $\frac{\partial V^*}{\partial R}$ of a tightening of the budget constraint. Define $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right) \equiv V^*\left(\widetilde{S}_0, R; \rho\right)$; then $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} = \frac{\partial V^*}{\partial \widetilde{S}_0} + \rho \frac{\partial V^*}{\partial R} = \mu - \rho\lambda$. As $\widetilde{S}_0$ is a free state variable, the transversality condition that applies at $t = 0$ is $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} + \frac{\partial\left(\rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS\right)}{\partial \widetilde{S}_0} = 0$, so that at the optimum

$$\mu = \lambda\rho + \widetilde{\eta}_0. \tag{35}$$

Indeed, as hinted earlier, the marginal unit value of reserves for the government in its taxation exercise exceeds the private marginal cost $\rho + \widetilde{\eta}_0$ of developing those reserves by a factor reflecting the cost of raising funds ($\lambda > 1$) to finance the subsidy payment.

[19] Clearly the subsidy must be low enough to necessitate the presence of after-tax rents at the extraction stage. This will be addressed further below.

With $\mu - \widetilde{\eta}_0 \geq 0$, it thus follows from (32) and (13) that the tax rate on the resource is higher than the tax rate on a conventional good with the same demand elasticity. Precisely, the unit tax θ^*_{st} on the resource exceeds the common inverse-elasticity term by ρe^{rt}. This component of the unit tax grows at the discount rate so that, alone, it would leave the extraction profile unchanged. In contrast, the component that is common to the resource tax and the tax on the conventional good normally[20] causes a distortion to the extraction profile; its value is $\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}$, exactly that of a conventional Ramsey tax. This is stated in Proposition 3.

Proposition 3 *(Optimal extraction taxes; endogenous reserves) When the supply of reserves is elastic and is subsidized at the unit rate $\rho \geq 0$, while the supply of conventional goods or services is infinitely elastic,*

1. *The non-renewable resource is taxed at a strictly higher rate than a conventional good or service having the same demand elasticity if $\rho > 0$; it is taxed at the same rate if $\rho = 0$;*

2. *The tax rate on the resource is given by* (36)*; it is made up of a non-distortionary component complemented by a Ramsey inverse-elasticity component.*

Substituting (35) into (32) implies

$$\frac{\theta^*_{st}}{\widetilde{q}_{st}} = \frac{\rho e^{rt}}{\widetilde{q}_{st}} + \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_s}, \tag{36}$$

where $\widetilde{q}_{st} = c_s + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$.

Any parametric change $\Delta\rho$ exactly compensated by a one-to-one change $\Delta\widetilde{\eta}_0 = -\Delta\rho$ and by a change $\Delta\theta^*_{st} = -\Delta\widetilde{\eta}_0 e^{rt}$ ensures that (36) remains satisfied without any further adjustment. As $\widetilde{\eta}_0 + \rho$ is then unchanged, this new combination of subsidy, tax, and after-tax rent commands the same reserves level; as $\widetilde{q}_{st}$ is unchanged it generates the same extraction path; all constraints remain satisfied. In other words the optimum after-tax rent depends on the *ex ante* subsidy: $\widetilde{\eta}_0 = \widetilde{\eta}_0(\rho)$; similarly $\theta^*_{st} = \theta^*_{st}(\rho)$, with $\frac{d\widetilde{\eta}_0(\rho)}{d\rho} = -1$

[20] As already mentioned an exception arises when the demand has constant elasticity and the extraction cost is zero (Stiglitz, 1976).

and $\frac{d\theta^*_{st}(\rho)}{d\rho} = e^{rt}$. However the optimum level of reserves $\widetilde{S}_0$ and the equilibrium price profile are independent of ρ.

This is true within an admissible range for ρ. Indeed the subsidy must not exceed the threshold level above which it would not be necessary for the government to leave firms a rent during the extraction phase. That threshold can be determined as follows. The unit after-tax extraction rent induced by the optimal policy is $\widetilde{\eta}_0(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) - \rho$. Therefore, the condition ensuring that the after-tax rent $\widetilde{\eta}_0$ remains strictly positive is

$$\rho < \bar{\rho} \equiv \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0), \tag{37}$$

where $\widetilde{S}_0$ must satisfy (30), or $\mathcal{S}^{-1}(\widetilde{S}_0) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) = \bar{\rho}$.

Proposition 4 *(Tax-subsidy mix) For $0 \leq \rho \leq \bar{\rho}$, the optimum initial reserve level and the optimum extraction profile are independent of the combination of tax and subsidy by which it is achieved.*

An immediate corollary is that subsidies are not necessary to achieve the optimum if the government can commit to extraction taxes that leave sufficient rents to extractors; *vice versa* commitment is not necessary if the government is willing to subsidize sufficiently, at $\rho = \bar{\rho}$. This subsidy level corresponds to the special case of Section 2 taken with initial reserves at $\widetilde{S}_0$. By Proposition 2, the tax is then given by (19) where $\mu = \lambda\bar{\rho}$ according to (35). Thus the observed variety in non-renewable resource taxation systems is not incompatible with optimum Ramsey taxation.

4.2 The inverse elasticity rule for endogenous non-renewable resources

Let us come back to the inverse elasticity rule. Formula (36) defines the optimal tax rate *ex post*, that is given the reserves induced by the announced net-of-tax rent and the subsidy. It further satisfies (35), which means that reserves – in fact the corresponding resource rent $\widetilde{\eta}_0$ – are measured at their endogenous *ex ante* value. However this does not imply that (36) accounts for the endogeneity of initial reserves as an *ex ante* first-order condition would. Consequently, the inverse elasticity rule (36) accounts only for the first type of distortion induced by resource taxation: the distortion of the time profile

of extraction given the reserves. Ramsey taxation of a non-renewable resource further induces a distortion on reserves, which will be discussed shortly.

There is another peculiarity in (36). The usual interpretation of the inverse elasticity rule is that goods or services whose demand is relatively less elastic should be taxed at a relatively higher rate because this keeps quantities demanded as close as possible to the Pareto optimum, thus balancing the distortions across sectors in the socially least costly way. Here, this interpretation does not apply. As a matter of fact the optimal tax defined by (36) may even leave the extraction path undisturbed when the demand is isoelastic and the marginal extraction cost is zero. As underlined by Stiglitz (1976) in his analysis of monopoly pricing in the Hotelling model, this happens because the resource price at any date not only affects current extraction but also the remaining stock of reserves still to be extracted. Confronted with the dilemma of raising the price at some date while increasing supply at some other date, a zero-cost monopoly facing an isoelastic demand ends up choosing the same price as a competitive firm would. Under the same cost and demand conditions the Ramsey tax has to be neutral for the same reason. More generally, even when the tax is not neutral, its effect on current extraction cannot be given the standard interpretation in terms of distortion.

Let us turn to the second type of distortion, that affects the level of initial reserves resulting from the *ex ante* choice of the rent left to producers, given its implications on the *ex post* tax profile. Initial reserves are determined by the optimum level of the unit after-tax rent $\widetilde{\eta}_0$, as that variable determines $\widetilde{S}_0$ via (30). As a matter of fact, $\widetilde{\eta}_t$ is present in (36) since $\widetilde{q}_{st} = c_s + \widetilde{\eta}_t + \theta^*_{st}$. However, it is very difficult in general to isolate its effect or the determinants of its optimum level because there is an infinity of relationships such as (36) and it is their combined influence over the whole extraction period that determines initial reserves. An exception is the special case just discussed. With an isoelastic demand and zero extraction cost, the optimal tax does not cause any distortion to the extraction profile, which provides the ideal laboratory for the analysis of the distortion to initial reserves.

When the tax is neutral at given initial reserves, it grows at the rate of discount,

so that it can be characterized at any date by its initial level. Each initial tax level corresponds to a particular tax profile so that alternative profiles can be compared by comparing initial levels. A higher initial tax level implies a lower after-tax rent to firms which implies lower initial reserves by (30). In the spirit of Ramsey taxation, one would then expect the optimal initial tax to be inversely affected by supply elasticity. This is precisely the message of the following expression established in Appendix H for the optimum *ex ante* resource tax rate:

$$\frac{\theta_{s0}^*}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} + \frac{\lambda - 1}{\lambda}\left[\frac{1 - \frac{\theta_{s0}^*}{\widetilde{q}_{s0}}}{\widetilde{\zeta}} - \frac{1}{\widetilde{\xi}}\right], \tag{38}$$

where $\widetilde{\zeta} \equiv \frac{\widetilde{\eta}_0}{\widetilde{S}_0 \mathcal{S}^{-1\prime}(.)}$ is the *long-term* elasticity of reserve supply measured at the resource scarcity rent induced by the tax at the beginning of extraction; and where $\widetilde{\xi} \equiv \left(\frac{d\widetilde{\mathcal{D}}}{dq_{s0}}\right)\frac{\widetilde{q}_{s0}}{\widetilde{\mathcal{D}}}$ is the elasticity of the *cumulative* demand for the resource $\widetilde{\mathcal{D}} \equiv \int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt$ with respect to the initial price q_{s0}, measured over the path of equilibrium prices $\{\widetilde{q}_{st}\}_{t\geq 0}$ induced by the optimal tax.

Keeping in mind that the optimal tax has the same impact for any admissible value of ρ, let us again assume that $\rho = 0$. Then (38) is identical to (27), the expression for the optimum rate of tax that applies to conventional goods whose supply is not perfectly elastic. Its interpretation is also standard: tax more when elasticity is lower, whether the source of elasticity is on the supply or the demand side. Hence, to the extent that the supply of conventional commodities is more elastic than the supply of reserves $\left(\widetilde{\epsilon}_i > \widetilde{\zeta}\right)$, (38) implies that the resource is taxed at a higher rate than commodities of identical demand elasticity. There is an important difference between the Hotelling resource and conventional goods or services though, having to do with the notions of elasticities involved.

In (38), the supply elasticity measures the long-run adjustment of the stock of initial reserves, allowing all other inputs to adjust, relative to the percentage change in the unit producer rent. This elasticity depends on how sensitive exploration is to the rent. If exploration is relatively insensitive to the rent, then the optimal tax rate on the resource tends to be high relative to the tax rates on conventional producible goods over the entire extraction period. In (27) the concept of supply elasticity is standard; it measures the

instantaneous percentage change in production (a flow) relative to the percentage change in the unit producer price. If the elasticity is finite, it must be the case that some input, *e.g.* the stock of capital, does not fully adjust to price and tax changes, which implies decreasing returns to scale.

Similarly, while the elasticity of demand is the standard notion in (27), its counterpart in (38) is defined as the elasticity of cumulative resource demand – over the whole extraction period – with respect to the initial resource price. In the current special case, the long-run elasticity of cumulative demand is the same as the standard flow demand elasticity: $\widetilde{\xi} = \widetilde{\varepsilon}_s$.

The results are gathered in the following proposition.

Proposition 5 *(Time profile and initial reserves) When the supply of reserves is elastic and is subsidized at the unit rate* $\rho \geq 0$*,*

1. *The Ramsey tax profile described by* (36) *implies distortions in both the time profile of extraction and the level of initial reserves;*

2. *When* $\rho = 0$*, the optimal tax is described by a standard static inverse elasticity rule* (36) *at any date. That rule does not express the distortion to resource extraction at that date because it is jointly determined by the tax at all other dates;*

3. *When the demand for the non-renewable resource is isoelastic and the extraction cost is zero, the extraction tax is neutral with respect to the time profile of extraction but affects the level of initial reserves. In that case the combined influence of long-run reserve supply elasticity and demand elasticity in the determination of the tax rate is given by* (38)*, the same rule that applies to conventional goods and services whose supply is not perfectly elastic.*

The analogy underlined in Section 2 between Ramsey taxation and monopoly pricing when reserves are exogenous is even more obvious when reserves are endogenous. Take $\rho = 0$; for non-renewable resources as for conventional goods, the optimal tax rates distort the price in the direction of the monopoly price by a factor $\frac{\lambda-1}{\lambda}$ that reflects the

intensity of the government's revenue needs. Moreover, since the optimum extraction profile does not depend on ρ (by Proposition 4), this is also true when the tax is given by the unrestricted form of (38); the reserve distortion is the counterpart of the distortion highlighted in Gaudet and Lasserre (1988) for a monopoly with endogenous reserves.

5 The open economy

OCT in an open economy raises a number of issues. In a static, closed economy, commodity taxes applied on the demand side are equivalent to taxes applied on the supply side. In the closed economy taxation during the extraction phase can be interpreted to apply to resource demand while the reserve development subsidy can be interpreted to apply to resource supply. Proposition 4 then means that the equivalence of supply and demand taxation extends to the resource sector, despite the difference in timing between reserve development and resource extraction. In the open economy, domestic consumption generally differs from domestic production so that OCT must be addressed by considering taxes or subsidies on both supply and demand rather than a single tax on demand or supply indifferently. The result of Proposition 4 nonetheless allows us to simplify the taxation of domestic resource supply by focusing on the domestic reserve subsidy rather than on the taxation of domestic extraction, while combining that subsidy with a commodity tax on resource consumption, whether from domestic or foreign origin. That way, much of the model structure used in the previous sections will be preserved.

In fact, the combination of a tax or subsidy on domestic demand and a tax or subsidy on domestic supply can be designed so as to be equivalent to a tariff (Mundell, 1960, p. 96). Consequently, the use of Ramsey's traditional tax instruments in an open economy could achieve the objective pursued by optimum tariffs (Friedlander and Vandendorpe, 1968; Dornbusch, 1971). Since the OCT problem and the optimum-tariff problem then differ only by the constraint to collect a minimum revenue, the latter characterizes an optimum of Pareto from the country's point of view while optimum commodity taxes are distortionary: as Boadway *et al.* (1973) put it "domestic commodity taxes *introduce* a distortion while optimum tariffs *eliminate* a distortion" (p. 397, their italics).

For reasons that need no explanation, tariffs will not be directly available as tax in-

struments in the open-economy OCT problem. However, demand and supply commodity taxes will seek the same objective as optimal tariffs and, consequently, their first-best levels (that is, unconstrained by revenue needs) will differ from zero.[21] Besides the obvious difference in domestic versus world surplus, the ability of the government to affect national surplus differs in the closed economy, where the government has the power to affect prices as a monopoly, from the open economy, where the government is competing with other countries much like an oligopolist. Non-renewable resources are very different from conventional goods in that respect; roughly, the supply of conventional goods is elastic while the supply of the Hotelling resource is inelastic in a closed economy. In an open economy, if the country is small and trades the resource competitively, the non-renewable resource behaves just like another commodity; its supply is infinitely elastic and optimal commodity taxes on the non-renewable resource obey the conventional closed-economy inverse elasticity rule.

Consequently, the interesting setup to study Ramsey taxation in an open economy is strategic. The country trades the non-renewable resource and is big enough to affect suppliers' surplus, whether supply is domestic or foreign.[22] In this section we are going to assume that the country has no influence on the prices of other commodities. Three reasons justify this restriction. First it does not affect the generality of the results presented; second it puts the focus on the key difference between non-renewable resources and conventional goods and services: supply elasticity. Third it connects with the literature on rent capture and optimal tariffs in the presence of a non-renewable resource; more on this further below.

[21] Since the distortion results from the failure by the country to exercise market power, only "large" countries should adopt different domestic taxes when they are open to trade than when they are closed to trade. This is also true when some tariffs are set at suboptimal levels; then, as shown by Dornbusch (1971, p. 1364), domestic taxes are conferred a corrective role. Not surprisingly, if the government can freely use both tariffs and commodity taxes, it can achieve its surplus maximization objective with tariffs and satisfy its revenue collection needs using commodity taxes; then, as Boadway *et al.* (1973) showed, Ramsey optimal domestic commodity taxes are "the same as in the case of a closed economy." (p. 391).

[22] The literature on resource oligopolies and oligopsonies is relevant to the problem of OCT in an open economy. According to Karp and Newbery (1991) "the evidence for potential market power on the side of importers is arguably as strong as for oil exporters" (p. 305); the more so when suppliers and/or buyers act in concert as suggested by Bergstrom (1982).

Analysis and results

The government faces a problem similar to that of Section 4 – choose linear commodity taxes to maximize domestic surpluses subject to a minimum tax revenue constraint and to a stock of endogenously supplied mineral reserves. These reserves are located either within the country, or outside, or both but have the same constant unitary extraction cost.[23] The non-renewable resource sector is now open to trade. World scarcity rents are equalized by free trade but domestic reserve supply is determined by the sum of the rent and the domestic reserve subsidy. As in our treatment of the closed economy, we simplify and sharpen the analysis by assuming that there is an *ex ante* step where domestic and world reserve stocks are established, followed by an *ex post* extraction phase.

Although the government has less power to affect the resource price than when the economy is closed, its choice of consumption taxes applied during the extraction period and the domestic reserve subsidy applied *ex ante* determine the scarcity rent enjoyed by both foreign producers and domestic ones, if any; they amount to a rent commitment towards the latters. This rent depends on the policies implemented in the rest of the world, which are taken as given in Nash equilibrium by the home government. Unlike the closed economy, the government is restricted to leaving its suppliers a rent at least as high as they would get if the domestic market was taxed to extinction.[24] The rent commitment occurs *ex ante* and is simultaneous with the choice of the reserve subsidy. Given that market power is limited to the non-renewable resource and that the supply of conventional goods is infinitely elastic, no tax or subsidy is applied on the supply of conventional commodities. Trade in these commodities combines with resource trade as in Bergstrom in such a way that the trade balance constraint is satisfied. For simplicity, and with no consequence on the results, it is assumed that there are only two countries.

Unless otherwise mentioned all variables and functions are redefined so as to refer to the home country. Variables or functions pertaining to the rest of the world will be

[23] See Section 3 for generalizations.

[24] As justified above we do not allow the government to tax domestic extraction. If it would, domestic rents would be allowed to differ from world rents; however the sum of extraction rent and support to exploration could be kept unchanged by adjusting the reserve subsidy, implying identical domestic reserves. Thus our treatment is compatible with a continuum of domestic resource taxation systems of combining extraction taxes and support to exploration as in many observed situations.

denoted by the same symbol and identified with the superscript F. Given the absence of rents or taxes on the supply side of conventional goods, surpluses on conventional goods are defined in terms of the (domestic) demands x_{it} as before. In the case of the resource, x_{st} now denotes instantaneous domestic demand while y_t denotes instantaneous domestic supply, and θ_{st} denotes the tax on demand. The resource supply tax or subsidy ρ is applied *ex ante* as in Section 4. Given these remarks and redefinitions, the equilibrium domestic consumer surplus $\widetilde{CS}_t$ is still given by (8), the producer surplus under competitive equilibrium is identical to (9) except that $\widetilde{y}_t$ replaces $\widetilde{x}_{st}$, and the home producers' total resource rent, formerly (10) becomes $\widetilde{\phi}_t = \widetilde{\eta}_t \widetilde{y}_t$

The analysis replicates that of Section 4. Consider first the *ex post* extraction stage under the *ex ante* commitment to consumption taxes that induce a given unit rent $\widetilde{\eta}_0 > 0$. The choice of $\widetilde{\eta}_0$ and of the supply subsidy ρ will be discussed immediately thereafter. Given that the resource is traded and that its marginal extraction cost is the same in the rest of the world as in the home country, unit rents are equalized: $\widetilde{\eta}_0 = \widetilde{\eta}_0^F$. The relevant supply to the home country is the residual world supply, that is the supply remaining once demand from the rest of the world has been met. At each date, the remaining stock of reserves available for consumption in the home country is thus $\widetilde{S}_t^H \equiv \widetilde{S}_t + \widetilde{S}_t^F - \int_t^{+\infty} \widetilde{x}_{su}^F \, du$ where home and foreign reserves $\widetilde{S}_0$ and $\widetilde{S}_0^F$ are established *ex ante* so that they are given when extraction starts; and where, since $\widetilde{x}_{st}^F = D_s^F(c_s + \widetilde{\eta}_t)$, the remaining foreign demand $\int_t^{+\infty} \widetilde{x}_{su}^F \, du$ is determined by the *ex ante* rent commitment. The exhaustibility constraint relevant to the home government is thus

$$\dot{\widetilde{S}}_t^H = -\widetilde{x}_{st}. \tag{39}$$

The Hamiltonian corresponding to this open-economy problem differs from its closed-economy counterpart (31) only by the producer surplus and the resource rent:

$$\mathcal{H}(a_t, \theta_t, \lambda_t, \mu_t) = \left(\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t\right) e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{40}$$

where μ_t is now associated with (39). From the maximum principle, as in Section 4, $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu \geq 0$, with μ again given by (35); then,

$$\frac{\theta_{st}^*}{\widetilde{q}_{st}} = \rho \frac{e^{rt}}{\widetilde{q}_{st}} + \frac{\lambda - 1}{\lambda} \frac{1}{-\widetilde{\varepsilon}_s} + \frac{1}{\lambda}(1 - \widetilde{\alpha}_t)\widetilde{\eta}_0 \frac{e^{rt}}{\widetilde{q}_{st}}, \tag{41}$$

where $\widetilde{\alpha}_t \equiv \frac{d\widetilde{y}_t/d\theta_{st}}{d\widetilde{x}_{st}/d\theta_{st}}$ is the change in domestic resource production relative to the change in domestic consumption, induced by domestic taxation (Appendix J). This formula is the open-economy counterpart of (36) and differs from it by the last term; if $\widetilde{\alpha}_t$ equalled unity, this term would vanish. By the definition of $\widetilde{\alpha}_t$, this happens if any change in domestic consumption is exclusively met by domestic supply. Clearly, this includes the limit case where the rest of the world is negligible as well as situations where the foreign country does not hold any resource. In contrast, $0 < \widetilde{\alpha}_t < 1$ whenever foreign supply to the domestic resource market adjusts to a change in the tax on domestic demand in the same direction as domestic supply does. This reinforces the closed-economy result stated in Proposition 3 that the consumption of the non-renewable resource is taxed at a higher rate than the consumption of a conventional good or service having the same demand elasticity when $\rho \geq 0$.

Clearly there is an intertemporal equilibrium where $\widetilde{\alpha}_t$ is time invariant.[25] In that case the last term in (41) defines a component of the unit tax θ^*_{st} which is rising at the discount rate; hence, the extra taxation imposed upon resource consumption in the open economy relative to the closed economy is neutral. The second term, the distortionary Ramsey component, is the same as in the closed economy.

Consider now the *ex ante* open-economy problem. Given that the resource consumption taxes must satisfy (41) *ex post*, the problem of choosing $\widetilde{\eta}_0$ and ρ is

$$\max_{\widetilde{\eta}_0,\ \rho} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{42}$$

[25] This is because in any intertemporal equilibrium domestic and foreign resource supply flows are only determined to the extent that their sum is determined and that domestic and foreign exhaustibility constraints must be met. This can be shown as follows. For any given tax schedule, the rent must rise at the rate of interest: $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. The resource market must clear at each date so that $\widetilde{x}_{st} + \widetilde{x}^F_{st} = \widetilde{y}_t + \widetilde{y}^F_t$. On the demand side, $\widetilde{x}_{st}$ and $\widetilde{x}^F_{st}$ are demanded quantities for the current resource price, uniquely determined at each date by $\widetilde{\eta}_0$, thus giving the world equilibrium supply $\widetilde{y}^W_t = \widetilde{x}_{st} + \widetilde{x}^F_{st}$. On the supply side, however, producers are indifferent about when to extract since $\widetilde{\eta}_t$ rises at the rate of interest. Hence, equilibrium domestic and foreign supplies $\widetilde{y}_t$ and $\widetilde{y}^F_t$ are only determined to the extent that they must fulfill the exhaustibility constraints for established reserves, $\widetilde{S}_0 = \int_0^{+\infty} \widetilde{y}_t\, dt$ and $\widetilde{S}^F_0 = \int_0^{+\infty} \widetilde{y}^F_t\, dt$, as well as the clearing condition $\widetilde{y}_t + \widetilde{y}^F_t = \widetilde{y}^W_t$, where $\widetilde{y}^W_t$ is determined as above.

Clearly, there is an infinity of combined paths of domestic supply $\widetilde{y}_t$ and foreign supply $\widetilde{y}^F_t$ satisfying these two conditions. A simple and natural combination is the one along which relative instantaneous supplies remain constant, so that $\frac{\widetilde{y}_t}{\widetilde{y}^F_t} = \frac{\widetilde{S}_0}{\widetilde{S}^F_0} \equiv \sigma$. For a given rent-commitment $\widetilde{\eta}_0$, foreign consumption $\widetilde{x}^F_{st}$ is given, so that tax changes only affect $\widetilde{x}_{st}$. Hence, the above condition implies that the domestic supply reaction to a change $d\widetilde{x}_{st}$ must be $d\widetilde{y}_t = \frac{\sigma}{1+\sigma} d(\widetilde{y}_t + \widetilde{y}^F_t) = \frac{\sigma}{1+\sigma} d\widetilde{x}_{st}$, which defines $\widetilde{\alpha} \equiv \frac{\sigma}{1+\sigma}$, constant and lower than unity.

subject to

$$\int_0^{+\infty} \theta_t^* \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{43}$$

There is an important difference between this problem and its closed-economy counterpart (33). In the closed-economy problem the first-order condition with respect to ρ and the expression for the *ex post* tax (36) are linearly dependent. This is why an infinity of *ex post* taxes-*ex ante* subsidy combinations were shown to be optimal and equivalent: in the closed economy the equivalence of demand taxation and supply taxation extends from the static realm of conventional goods to the dynamic framework of resource extraction where ρ is applied prior to θ_{st}. This is not so in the open economy; the first-order condition for ρ in problem (42) and expression (41) for the optimal extraction tax, are not linearly dependent; they combine to determine the optimal tax path and the optimal subsidy for any feasible rent-commitment $\widetilde{\eta}_0$ by the government.[26]

Consequently, while Proposition 3 survives almost unscathered the extension from the closed economy to the open economy, Proposition 4, which states that an infinity of tax-subsidy mixes yield the optimum level of reserves and extraction path in a closed economy, does not hold in an open economy.

Proposition 6 *(Resource consumption tax in open economy) When the non-renewable resource is traded, there is an equilibrium such that the Home country and the Rest of the world contribute to world resource supply in the same proportion as they share reserves. Then,*

1. *Domestic resource consumption is taxed at a strictly higher rate than the consumption of conventional goods of the same demand elasticity when supply subsidies in the resource sector are non negative* ($\rho \geq 0$).

2. *The optimal tax rate* (41) *on resource consumption is made up of non-distortionary and distortionary components. The distortionary component is the same as in the closed economy and expresses Ramsey's inverse elasticity rule.*

[26] In Appendix L, we derive the expression for the optimal reserve subsidy ρ^* when demand is isoelastic and the unit extraction cost is zero.

In the closed economy with endogenous reserves, first-best optimum commodity taxes do not yield any fiscal revenues. In contrast, in the open economy, it is well known that optimal tariffs are not nil, so that a combination of commodity taxes mimicking optimal tariffs produces tax revenues and may meet government needs without involving any distortion. The distinction between low and high revenue needs made in Section 2 with exogenous reserves thus arises again when the economy is open in spite of the endogeneity of reserves. Low and high revenue needs should be defined according to whether government needs are below or above the amount $\overline{\overline{R}}_0$ raised when the resource tax is set so as to maximize welfare in the absence of tax-revenue constraint (Appendix M). Call this the rent-capture component of the optimal domestic consumption tax. If $R_0 > \overline{\overline{R}}_0$, the rent-capture component of the domestic resource consumption tax is not sufficient to meet revenue needs and it must be true that $\lambda > 1$; only then does the second term in (41), the distortionary component of the optimal consumption tax, become positive.

When the taxation of non-renewable resources is distortionary, the distortion may affect both the extraction path and the amount of initial reserves. Consider the international equilibrium where $\widetilde{\alpha}_t$ is time invariant; the first and third terms in (41) then rise at the rate of discount while the distortionary component is identical to its counterpart in (36). Stiglitz's (1976) special case of isoelastic domestic demand and zero extraction costs then again implies that the optimal tax on resource demand is neutral and rises at the rate of interest.

An additional interest of Stiglitz's special case is that, when extraction costs are zero, a unit resource consumption tax that is rising at the rate of interest induces the final price $\widetilde{q}_{st}$ to rise at the same rate. Hence, such a tax is tantamount to the constant *ad valorem* tax in Bergstrom (1982). Our open-economy model then differs from Bergstrom's only in the treatment of reserves, exogenous in his paper, endogenous here. Bergstrom's inverse elasticity rule maximizes the country's surplus without any constraint on tax revenues, so that it is equivalent to an optimum tariff. Stiglitz's special case then enables us to investigate how the optimal resource tax of the Ramsey government differs from a

commodity tax that would pursue the objective of an optimum tariff.

With θ^*_{st} now equal to $\theta^*_{s0}e^{rt}$, expressions (41) are determined at all dates by the initial level of the optimal resource tax. The maximization of (42) with respect to θ_{s0} is equivalent to its maximization with respect to the rent $\widetilde{\eta}_0$ induced by θ_{s0}. The resulting optimum tax rate, the open-economy counterpart of (38) is (Appendix L):

$$\frac{\theta^*_{s0}}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} \frac{\widetilde{S}_0 \widetilde{\zeta}}{\widetilde{S}_0^H \widetilde{\zeta}^H} + \frac{\lambda - 1}{\lambda} \left[\frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{\zeta}^H} + \frac{1}{-\widetilde{\xi}} \right] + \frac{1}{\lambda} \frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{S}^H \widetilde{\zeta}^H} \left[\widetilde{\mathcal{D}} - \widetilde{S}_0 \right], \tag{44}$$

where $\widetilde{S}_0^H \equiv \widetilde{S}_0 + \widetilde{S}_0^F - \widetilde{\mathcal{D}}^F$ is the residual supply of reserves available for home country consumption, whose elasticity is defined as $\widetilde{\zeta}^H \equiv \left(\frac{d\widetilde{S}_0^H}{d\eta_0} \right) \frac{\widetilde{\eta}_0}{\widetilde{S}_0^H}$.

Expression (44) simplifies to (38) when the totality of domestic consumption is met by domestic production.[27] Although complex, it brings up simple and important insights. First it shows the role of resource supply and its elasticity explicitly. It stresses the distinction between domestic production $\widetilde{S}_0$, which may be consumed locally or exported and can be taxed or subsidized in both cases, and foreign supply to the domestic market, which cannot be taxed or subsidized; $\widetilde{S}_0^H$ combines both. For a resource importer $\left(\widetilde{\mathcal{D}} - \widetilde{S}_0 > 0 \right)$ that does not tax reserve production ($\rho \geq 0$), the optimum tax rate decreases when the elasticity of residual reserve supply $\widetilde{\zeta}^H$ increases. Indeed, Pigou (1947, p. 113) attempted to extend Ramsey's principles to trading economies. Since the residual supply of internationally-traded commodities presumably has a greater elasticity than total supply, he conjectured that Ramsey's analysis would imply imposing lower tax rates on those commodities.

Second, (44) connects neatly with the literature on the capture of resource rents initiated by Bergstrom (1982) and with the question of optimal tariffs in the presence of non-renewable resources. Bergstrom treats reserves as given so he does not envisage a subsidy: $\rho = 0$. Bergstrom does not consider that the government faces any revenue constraint: $\lambda = 1$. Consequently the first and second terms disappear under his setup.

[27] The last term vanishes when $\widetilde{\mathcal{D}} - \widetilde{S}_0 = 0$, and it must then also be the case that $\widetilde{S}_0^F - \widetilde{\mathcal{D}}^F = 0$ so that $\widetilde{S}_0^H = \widetilde{S}_0$, $\widetilde{\zeta}^H = \widetilde{\zeta}$, and the first term reduces to $\frac{\rho}{\widetilde{q}_{s0}}$ as in (38).

Multiplying by $\widetilde{q}_{s0}$, substituting $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta^*_{s0}$, we obtain

$$\frac{\theta^*_{s0}}{\widetilde{\eta}_0} = \frac{1}{\widetilde{S}_0^H \widetilde{\zeta}^H} \left(\widetilde{\mathcal{D}} - \widetilde{S}_0 \right). \tag{45}$$

Since extraction costs are assumed nil, $\frac{\theta^*_{s0}}{\widetilde{\eta}_0}$ is the optimal, constant *ad valorem* tax given by Bergstrom in Expression (32), p. 198. One may wonder why Bergstrom's formula involves countries' demand elasticities and no supply elasticity. The reason is the assumption of exogenous world reserves. A country's residual supply then only depends on other countries' demands and not on the technology of reserve discovery as in this paper. Once $\widetilde{S}_0^H$ and its elasticity are written in terms of resource demands using $\widetilde{S}_0^H = \widetilde{S}_0 + \widetilde{S}_0^F - \widetilde{\mathcal{D}}^F$, we obtain Bergstrom's Expression (32).[28]

This formula is famous for it implies that a net importer should tax the resource, at least to the extent that it holds market power. This is Pareto optimal from that country's point of view and allows it to capture some of the rents otherwise falling into the hands of exporters. When reserves are endogenous this power to capture rents is attenuated: $\widetilde{S}_0^H \widetilde{\zeta}^H$ being higher than its exogenous-reserve counterpart $-\widetilde{\mathcal{D}}^F \widetilde{\xi}^F$, the importer must not tax resource consumption as much: depriving foreign suppliers of resource rents would reduce their supply of reserves.

Third, the first term in (44) shows the arbitrage between *ex ante* reserve subsidization and *ex post* taxation of resource consumption: the consumption tax increases with reserve subsidization by a factor of proportionality equal to the ratio of local production over residual supply to the home country, both weighted by their respective elasticities. This ratio is unity in the closed economy, so that the trade-off between taxing extraction or subsidizing reserves is financially neutral. The trade-off would be financially neutral in a competitive open economy if the coefficient of ρ were $\frac{\widetilde{S}_0}{\widetilde{S}_0^H}$, reflecting the fact that the tax base of domestic production is smaller than the tax base of domestic consumption; the presence of elasticities in the coefficient of ρ makes it plain that the optimum tax-subsidy combination further reflects the ability of the country to manipulate prices by its choice of the tax instruments.

[28] This being the two-country case, the summation symbol in Bergstrom disappears, so that, in our notations – we also corrected a typo in Bergstrom – the formula reads $\frac{\theta^*_{s0}}{\widetilde{\eta}_0} = \frac{\widetilde{\mathcal{D}} - \widetilde{S}_0}{-\widetilde{\mathcal{D}}^F \widetilde{\xi}^F}$.

The main results are gathered in the following proposition.

Proposition 7 *(Rent capture and Ramsey taxation) When further to the conditions of Proposition 6, domestic demand is isoelastic, and extraction is costless, the maximum revenue need* $\overline{\overline{R}}_0$ *compatible with neutral resource taxation is given by* $(M.1)$ *and the optimum taxes or subsidies on resource consumption and reserve supply are jointly determined by* (44) *and (L.4).*[29] *More precisely,*

1. *When* $R_0 \leq \overline{\overline{R}}_0$*, so that* (44) *and (L.4) hold with* $\lambda = 1$*, OCT is Pareto optimum and fulfills a resource-rent-capture objective. For an importing country, this involves taxing resource consumption while subsidizing domestic production, and* vice versa *for an exporter.*

2. *Otherwise, that is when government revenue needs are high,* (44) *and (L.4) apply with* $\lambda > 1$*. Optimum resource taxes are then higher than when* $R_0 \leq \overline{\overline{R}}_0$ *(reserve subsidies are lower) by an amount that reflects both domestic and foreign demand elasticities, as well as domestic and foreign supply elasticities.*

The formula giving the optimal level of ρ is (L.4) in Appendix L; being the sister of Formula (44), it can be read and interpreted in much the same way. When revenue needs are low, ρ is always strictly positive for importing countries, as is well understood from the optimum-tariff literature. Sufficiently high revenue needs, however, may reverse the result, implying that it may be optimal to tax reserve production, even in importing countries. Similarly, under sufficiently high revenue needs, exporters may tax consumption according to (44).

6 Final remarks

The standard Ramsey-Pigou framework used in this paper considers indirect, linear taxes or subsidies on any commodity or service. This includes linear subsidies to the production of natural resource reserves (exploration) as well as linear taxes on extraction and on consumption of the natural resource. In the Ramsey-Pigou framework, the objective of

[29] See Appendices M and L for Formulas $(M.1)$ and (L.4).

the government is to maximize the welfare of producers and consumers while securing a given level of revenues for the production of public goods. The need to secure revenues confers a profit-maximizing dimension to government taxation decisions. Optimum taxes distort consumer prices away from the Pareto optimum toward the monopoly price. For the Hotelling resource, this means that results from the resource monopoly literature are relevant to Ramsey taxation.

In a closed economy, when initial reserves are exogenous, the non-renewable resource must taxed in priority, however elastic the demands for the conventional goods and for the non-renewable resource. Precisely, the resource should be the sole taxed commodity unless the required tax revenue exceeds the totality of the rents that would be generated by the untaxed resource. When the required tax revenue is higher than the maximum that can be generated by neutral resource taxation, conventional producible goods and services should contribute to government revenues, but the resource should be taxed at a higher rate than conventional producible goods having identical elasticities.

When the supply of initial reserves is elastic and determined by the combination of after-tax rents to extraction and *ex ante* subsidies to reserve production, all sectors should be taxed simultaneously whatever the tax revenue needs of the government. In the absence of any subsidies, provided the government can commit to leaving after-tax rents to firms, the optimum tax rate on resource extraction is determined according to the inverse elasticity rule applying to any conventional good whose supply elasticity is infinite. However, this formal similarity hides a crucial difference: due to the dynamic nature of the extraction problem, a similar rule must hold at all dates during the extraction period. As a result, the distortion to extraction cannot be measured simply according to the tax applying at any particular date, however determined, but also depends on the tax applied at all other dates. If the demand for the non-renewable resource is isoelastic and the marginal extraction cost is zero, this goes as far as implying that the optimal tax, although set according to a standard inverse elasticity rule, does not cause any distortion to the extraction path. The distortion imposed on the industry then materializes at the level of reserve production rather than the extraction profile. It can be expressed by

the standard inverse elasticity rule applying to elastically supplied conventional goods and services, provided the elasticity concepts are the long-run notions defined in the paper. Both the supply and demand elasticities relevant to the Hotelling resource are elasticities of a stock in response to an after-tax asset price, rather than the flow elasticities encountered in usual Ramsey formulae.

Another remarkable result arising in a closed economy with endogenous reserves is that, although the optimal extraction tax varies according to the reserve subsidy, the optimal amount of initial reserves and the optimal extraction path of these reserves, do not depend on the tax-subsidy combination. As a result, all the tax-induced distortions just described when subsidies are absent, are insensitive to the tax-subsidy combination adopted by the government. In particular, a government that were unable to commit to leaving positive after-tax rents to firms during the extraction period, could finance reserve production by subsidies exclusively and achieve the same objective as a government that were able to commit. Similarly, a government that could not devote subsidies to reserve production could give the same incentives by committing to limit extraction taxes appropriately. Within the framework of our model, Ramsey taxation is compatible with institutional forms ranging from a nationalized industry, where the entire reserve production effort is subsidized while the total surplus from extraction is taxed away, to a system where firms finance reserve production and are paid back by future extraction rents.

When the resource sector is open to trade, Ramsey's instruments are applied domestically; since domestic supply does not necessarily meet domestic demand, optimal taxes are chosen on both domestic supply and domestic demand. This implies that the combination of domestic reserve supply subsidy and domestic natural resource consumption is no longer a matter of indifference. Although their effect on international prices is then diluted, the result that resource consumption should be taxed at a higher rate than the consumption of conventional commodities is reinforced because not only domestic supply but also foreign supply adjust to domestic consumption changes. In fact, domestic taxes in a large country are further conferred an optimum-tariff dimension which is magnified

by the existence of foreign scarcity rents. Results from the literature on tax competition in non-renewable-resource markets become relevant to Ramsey taxation; in particular Bergstrom's (1982) famous result that importing countries should tax non-renewable resource consumption arises as a particular case and comes reinforced if revenue needs constrain the importing country. The opposite holds in the case of exporters, who subsidize resource consumption in the absence of a fiscal-revenue constraint.

Natural resource reserves are a form of capital while discoveries and extraction are forms of positive and negative investments. While Ramsey taxation rules out the direct taxation of capital and profits, the linear indirect commodity taxes considered in this paper have the ability to tax natural resource rents. We found that resource rents should be taxed prior to introducing distortionary commodity taxes when the initial amount of reserves is exogenous, as anticipated by Stiglitz and Dasgupta (1971). When reserves are endogenous and resource rents include quasi-rents, the situation is close to that analyzed by Chamley (1986) in that the question whether capital should be taxed in the long run arises in a similar fashion. Chamley identified two aspects of capital revenue taxation. In the short run, capital is rigid; this makes it an attractive target for taxation if the objective is to obtain revenues while avoiding distortions. However, in the long run, the constitution of capital relies on investment, and investment becomes less profitable, the more capital is taxed. Chamley finds that the latter effect becomes dominant in the long run and the revenue from capital should not be taxed at all if the horizon of the government is long enough. We find a very different result when capital is a non-renewable natural resource. As *per* Proposition 3, the natural resource should be taxed whatever the horizon of the government in autarky, despite the fact that the supply of reserves is affected by the tax. This is also true in an open economy, although Proposition 6 indicates that the tax also seeks the capture of rents from other countries. The reason is resource scarcity. While Chamley's capital can be produced without limit under constant returns to scale, reserves, although endogenous, are produced under conditions of decreasing returns because exploration prospects are not unlimited. Whether it is traded or not, the supply of a non-renewable natural resources is not infinitely elastic in the very long run.

APPENDICES

A The Hotelling rent and the neutral tax

A Hotelling resource is a homogenous non-renewable natural asset, such as an oil deposit. As an asset it should provide the same return as any traded asset if it is to be detained. Since a unit of oil underground does not provide any return other than the value realized upon extraction, its return consists of capital gains over time. If oil was traded underground, absent any uncertainty, non arbitrage would thus require its current price to rise at the risk-free rate of interest. The value of such a non-traded asset is known as Hotelling rent and the non-arbitrage rule that it should satisfy is known as Hotelling's rule (Hotelling, 1931; Dasgupta and Heal, 1979, pp. 153-156; Gaudet, 2007).

This appendix defines the Hotelling rent with tax $\widetilde{\eta}_0$ and the Hotelling rent without tax $\overline{\eta}_0$ in competitive equilibrium. In competitive equilibrium with linear taxation, Hotelling's current-value unit rent to producers equals producer price minus marginal cost. At time zero, with constant unit extraction cost, this is $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta_{s0} - c_s$. By Hotelling's rule the rent is constant in present value so that, at any date, its present value is $\widetilde{\eta}_0$; it can be computed as follows.

If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource will be depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where $\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$: $\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st})dt = S_0$, with $\widetilde{q}_{st} - \theta_{st} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\left(\widetilde{T}-t\right)}$. At time zero, the rent is thus $\widetilde{\eta}_0(S_0) = \widetilde{q}_{s0} - \theta_{s0} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\widetilde{T}}$. If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define the present-value rent $\widetilde{\eta}_0(S_0)$ implicitly: $\lim_{T\to+\infty} \int_0^T D_s(\widetilde{\eta}_t + \theta_{st} + c_s)dt = S_0$, where $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. It can be shown that $\widetilde{\eta}_0$ is a positive and decreasing function of S_0.

The maximum value that can be raised from the mine by non-distortionary taxation is its discounted cumulative rent under competitive extraction and in the absence of taxation. That is $\overline{\eta}_0(S_0) = \widetilde{\eta}_0(S_0)$, where $\widetilde{\eta}_0$ is computed as above for the values of $\widetilde{q}_{st}$ implied by $\theta_{st} = 0$, $\forall\ t$. The present value of the mine in the absence of tax is thus $\overline{\eta}_0(S_0)\, S_0$.

If taxes are neutral, $\theta_{st} = \theta_{s0}e^{rt}$ and part of the unit scarcity rent is captured. The present value of the net-of-tax unit rent earned by the owner of the mine is thus $\widetilde{\eta}_0(S_0) = \overline{\eta}_0(S_0) - \theta_{s0}$ and the after-tax present value of the mine is $\widetilde{\eta}_0(S_0)\, S_0$.

B Proof of Proposition 1

1. We have shown in the main text that $\lambda = 1$ implies $\theta_i^* = 0$, $i = 1, ..., n$, and $\theta_{st}^* = \theta_{s0}^* e^{rt}$, so that the totality of tax revenues is raised from the resource sector. Moreover, we have argued that, if $\lambda = 1$, it must be the case that $R_0 \leq \overline{\eta}_0 S_0$. The contrapositive of that statement is that if $R_0 > \overline{\eta}_0 S_0$, then $\lambda > 1$. In that case, we have shown in the main text that $\theta_i^* > 0$, $i = 1, ..., n$, and that θ_{st}^* must be set in such a way as to raise more than $\overline{\eta}_0 S_0$ from the resource sector.

There remains to show that $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$. Assume $R_0 \leq \overline{\eta}_0 S_0$ and $\lambda > 1$. Then taxes on conventional goods θ_i^*, $i = 1, ..., n$, raise a strictly positive revenue, causing distortions. Since it is possible to generate $\overline{\eta}_0 S_0 \geq R_0$ without imposing any distortions by taxing the natural resource, this cannot be optimal. Hence, $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$.

2. Shown in the main text. ■

C Proof of Proposition 2

1. As shown in the main text, when $\lambda > 1$, the optimum tax rate on conventional good $i = 1, ..., n$ is θ_{it}^* as given in (13) and depends on λ. The optimum tax on the resource is given by (19), where $\mu > 0$ is determined to satisfy (1) with equality. Together, taxes on conventional goods and the tax on the resource must exactly raise $R_0 > \overline{\eta}_0 S_0$, which requires that $\sum_{i=1,...,n,s} \int_0^{+\infty} \theta_{it}^* \widetilde{x}_{it} e^{-rt}\, dt = R_0$. Substituting for θ_{it}^* implicitly defines λ.
$2 - 4$. Shown in the main text. ■

D OCT and monopoly pricing

If the need of tax revenues was extreme, that is to say if λ tended toward infinity, the optimum tax rate implied by (19) would be[30] $\frac{\theta_{st}^*}{\widetilde{q}_{st}} = \frac{1}{-\widetilde{\varepsilon}_s}$, corresponding to static monopoly pricing; indeed, $\frac{\theta_{st}}{\widetilde{q}_{st}} = \frac{\widetilde{q}_{st} - c_s}{\widetilde{q}_{st}}$ is the static Lerner index for the resource industry. Under such extreme condition the optimum resource tax rate would be determined by the same inverse elasticity rule as the tax rate applying to other commodities according to (13).

When revenue needs equal total rents ($\lambda = 1$), the second term in the right-hand side of (19) vanishes so that the optimal extraction tax is neutral.

Since $\frac{1}{\lambda}$ and $\frac{\lambda - 1}{\lambda}$ sum to unity, the optimum tax on the resource industry given by (19) is a weighted sum of two elements. The first element μe^{rt} can be interpreted as the neutral component of the tax since it rises at the rate of discount as does a neutral Hotelling tax. The second element was just seen to correspond to monopoly pricing.

E Proof of Expression (32)

The Hamiltonian (31) associated with the *ex post* problem is identical to (17). Hence, the application of the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. The first-order condition for the choice of the tax is also (18). However, unlike in Section 2, the first term on the left-hand side is not zero since the government is subject to its *ex ante* commitment, which determines $\widetilde{\eta}_0$ at this stage: $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st}^* - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. Therefore, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Substituting into the first-order condition and rearranging gives (32), where $\varepsilon_s \equiv \frac{q_{st}}{x_{st} D_s^{-1\prime}(.)}$.

F Proof of Proposition 3

1. Shown in the main text.
2. This is a restatement of (36), which is immediately obtained by substituting (35), shown in the main text, into (32), proven in Appendix E. The rest of the proposition summarizes findings established in the text preceding it. ■

G Proof of Proposition 4

The proof is shown in the main text. ■

[30] Although μ varies as λ changes, this scarcity rent cannot become infinite as $\lambda \to \infty$ so that the first term on the right-hand side of (19) indeed vanishes as required for this statement to be true.

H Proof of Expression (38)

Expression (38) is established under the assumption that extraction cost is zero, $c_s = 0$, and that the demand for the resource is isoelastic, $\varepsilon_s(q_{st}) = \varepsilon_s$. As mentioned in the main text, substituting $\widetilde{q}_{st} = \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$ into (19) with $\widetilde{\eta}_0 = 0$, or into (32) and into (36) with $\widetilde{\eta}_0 \geq 0$, while using the constancy of $\widetilde{\varepsilon}_s$, immediately shows that the optimal extraction unit tax then grows at the rate of interest:

$$\theta^*_{st} = \theta^*_{s0} e^{rt}, \tag{H.1}$$

where θ^*_{s0} is to be determined.

For a given ρ, the *ex ante* choice of θ^*_{s0} is equivalent to the choice of the unit rent $\widetilde{\eta}_0$ it induces, account being taken of (30). The first-order condition for the *ex ante* static maximization of (33) with respect to θ^*_{s0} subject to (34), taking the *ex post* solution (H.1) into account is

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\, dt + \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.)\frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt} = (D_s^{-1}(\widetilde{x}_{st}) - \theta^*_{s0} e^{rt}) \frac{d\widetilde{x}_{st}}{d\theta_{s0}} e^{-rt} - \widetilde{x}_{st} = \widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st}$ and where $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty} \left(\widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st} \right) dt - \widetilde{\eta}_0 \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Integrating with $\int_0^{+\infty} \widetilde{x}_{st}\, dt = \widetilde{S}_0$ and $\int_0^{+\infty} \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$ gives

$$\theta^*_{s0} = \rho - \frac{(\lambda - 1)}{\lambda} \frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\theta_{s0}}}. \tag{H.2}$$

In long-run market equilibrium $\mathcal{S}^{-1}(\widetilde{S}_0) = \widetilde{\eta}_0 + \rho$ and $\int_0^{+\infty} D_s(\widetilde{\eta}_t + \theta^*_{st})\, dt = \int_0^{+\infty} D_s\big((\widetilde{\eta}_0 + \theta^*_{s0}) e^{rt}\big)\, dt = \widetilde{S}_0$. It follows by differentiation with respect to θ_{s0} that $\mathcal{S}^{-1\prime}(.)\frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and $\left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1 \right) \int_0^{+\infty} D'_s(.) e^{rt}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$. Substituting in $\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$, one obtains $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D'_s(.) e^{rt}\, dt}{1 - \mathcal{S}^{-1\prime}(.) \int_0^{+\infty} D'_s(.) e^{rt}\, dt}$. Introducing this expression into (H.2) yields

$$\theta^*_{s0} = \rho + \frac{\lambda - 1}{\lambda} \left[\widetilde{S}_0 \mathcal{S}^{-1\prime}(.) - \frac{\widetilde{S}_0}{\int_0^{+\infty} D'_s(.) e^{rt} dt} \right], \tag{H.3}$$

from which (38) is derived after substituting the expressions for $\widetilde{\zeta}$ and $\widetilde{\xi}$ defined in the main text and using the fact that $\widetilde{q}_{st} = (\widetilde{\eta}_0 + \theta^*_0) e^{rt} = \widetilde{q}_{s0} e^{rt}$ under (H.1) so that $\frac{d\widetilde{\mathcal{D}}}{dq_{s0}} = \int_0^{+\infty} D'_s(.) e^{rt}\, dt$. Furthermore, the constancy of ε_s implies $\widetilde{\xi} = \varepsilon_s$.

I Proof of Proposition 5

The proposition summarizes findings established in the main text. ∎

J Proof of Expression (41)

The Hamiltonian associated with the *ex post* open-economy problem is (40). Applying the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. Since the government is subject to its *ex ante* commitment, $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ is determined at this stage, as well as $\widetilde{x}^F_{st}$, which depends on θ_{st} only via $\widetilde{\eta}_t$. Hence, the first-order condition for the choice of the tax is

$$\left[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s - \widetilde{\eta}_t\right] \frac{d\widetilde{x}_{st}}{d\theta_{st}} + \widetilde{\eta}_t \frac{d\widetilde{y}_t}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}.$$

Since $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, where $\widetilde{\eta}_0$ is given, the first term on the left-hand side is zero and $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Inserting into the above condition and rearranging give

$$\theta^*_{st} = \frac{1}{\lambda}(\mu - \widetilde{\alpha}_t\widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}, \tag{J.1}$$

where $\widetilde{\alpha}_t = \frac{d\widetilde{y}_t/d\theta_{st}}{d\widetilde{x}_{st}/d\theta_{st}}$ and $\widetilde{\varepsilon}_s \equiv \frac{\widetilde{q}_{st}}{\widetilde{x}_{st}D_s^{-1\prime}(.)}$.

In the open economy, the *ex post* maximized value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\,dt$, $V^*(\widetilde{S}^H_0, R; \rho)$, is a function of the residual reserves available to the home country $\widetilde{S}^H_0 \equiv \widetilde{S}_0 + \widetilde{S}^F_0 - \int_0^{+\infty} \widetilde{x}^F_{st}\,dt$. The constant co-state variable μ in (40) should be interpreted as giving the value $\frac{\partial V^*}{\partial \widetilde{S}^H_0}$ of a marginal unit of residual reserves. By definition of $\widetilde{S}^H_0$ it must be that μ is also the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of domestic reserves. The rest of the reasoning leading to (35) in Section 4 applies.

Substituting (35) into (J.1) yields (41).

K Proof of Proposition 6

The equilibrium where $\widetilde{\alpha}_t$ is time invariant is described in Footnote 25.
1. Shown in the main text: compare (41) with (13).
2. Shown in the main text: compare (41) with (36). ■

L Proof of Expressions (44) and (L.4)

This appendix assumes that Stiglitz (1976)'s conditions hold: the elasticity of domestic demand $\varepsilon_s(q_{st})$ is a constant ε_s and marginal extraction cost c_s is zero. Without any further loss of generality, we restrict attention to the equilibrium where $\widetilde{\alpha}_t = \widetilde{\alpha}$ is time invariant.

In this case the optimal extraction unit tax is given by (41) multiplied by $\widetilde{q}_{st}$; it rises at the rate of interest. This formula only differs from (36) by its last term, which is, after multiplying by $\widetilde{q}_{st}$, $\frac{1}{\lambda}(1-\widetilde{\alpha})\widetilde{\eta}_0 e^{rt}$. Recalling that the unit tax given by (36) has been shown to rise at the rate of interest in Appendix H, it remains to show that the new term does so, which is immediate since $\widetilde{\alpha}$ is constant. Hence, (H.1) is valid, where θ^*_{s0} is to be determined as follows.

The first-order condition for the *ex ante* static maximization of (42) with respect to θ^*_{s0} subject to (43), taking the *ex post* solution (H.1) into account, is, as in Appendix H,

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\,dt + \rho\frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.)\frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda\left(\int_0^{+\infty}\left(\widetilde{x}_{st} + \theta^*_{s0}\frac{d\widetilde{x}_{st}}{d\theta_{s0}}\right)dt - \rho\frac{d\widetilde{S}_0}{d\theta_{s0}}\right) = 0.$$

Furthermore, $\frac{d\widetilde{W}_t}{d\theta_{s0}}e^{-rt} = (D_s^{-1}(\widetilde{x}_{st}) - \widetilde{q}_{st})\,\frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st} + \frac{d\widetilde{\eta}_0}{d\theta_{s0}}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\theta_{s0}} - \widetilde{x}_{st}$ since $D_s^{-1}(\widetilde{x}_{st}) = \widetilde{q}_{st}$, and $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty}\left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\theta_{s0}} - \widetilde{x}_{st}\right)dt - \widetilde{\eta}_0\frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda\left(\int_0^{+\infty}\left(\widetilde{x}_{st} + \theta^*_{s0}\frac{d\widetilde{x}_{st}}{d\theta_{s0}}\right)dt - \rho\frac{d\widetilde{S}_0}{d\theta_{s0}}\right) = 0.$$

Integrating with $\int_0^{+\infty}\widetilde{x}_{st}\,dt = \widetilde{\mathcal{D}}$, $\int_0^{+\infty}\widetilde{y}_t\,dt = \widetilde{S}_0$, $\int_0^{+\infty}\frac{d\widetilde{x}_{st}}{d\theta_{s0}}\,dt = \frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}$ and $\int_0^{+\infty}\frac{d\widetilde{y}_t}{d\theta_{s0}}\,dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$, and rearranging give

$$\theta^*_{s0} = \rho\frac{\frac{d\widetilde{S}_0}{d\theta_{s0}}}{\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}} - \frac{(\lambda-1)}{\lambda}\frac{\widetilde{\mathcal{D}}}{\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}} + \frac{1}{\lambda}\frac{\frac{d\widetilde{\eta}_0}{d\theta_{s0}}}{\frac{d\widetilde{\mathcal{D}}}{\theta_{s0}}}\left[\widetilde{\mathcal{D}} - \widetilde{S}_0\right]. \tag{L.1}$$

In long-run market equilibrium, $\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho)$ and $\widetilde{\mathcal{D}} = \int_0^{+\infty} D_s\left((\widetilde{\eta}_0 + \rho)e^{rt}\right)\,dt = \widetilde{S}_0^H$, where $\widetilde{S}_0^H$ is the residual supply as defined in the main text. It follows by differentiation with respect to θ_{s0} that $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \mathcal{S}'(.)\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and that $\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}} = \left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1\right)\int_0^{+\infty} D'_s(.)e^{rt}\,dt = \frac{d\widetilde{S}_0^H}{d\eta_0}\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$. From that equality, we obtain $\frac{d\widetilde{\eta}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D'_s(.)e^{rt}\,dt}{\frac{d\widetilde{S}_0^H}{d\eta_0} - \int_0^{+\infty} D'_s(.)e^{rt}\,dt}$. Introducing these expressions in (L.1) yields

$$\theta^*_{s0} = \rho\frac{\mathcal{S}'(.)}{\frac{d\widetilde{S}_0^H}{d\eta_0}} + \frac{\lambda-1}{\lambda}\left[\frac{\widetilde{S}_0^H}{\frac{d\widetilde{S}_0^H}{d\eta_0}} - \frac{\widetilde{\mathcal{D}}}{\int_0^{+\infty} D'_s(.)e^{rt}\,dt}\right] + \frac{1}{\lambda}\frac{1}{\frac{d\widetilde{S}_0^H}{d\eta_0}}\left[\widetilde{\mathcal{D}} - \widetilde{S}_0\right], \tag{L.2}$$

from which (44) is obtained after substituting $\widetilde{\zeta}$, $\widetilde{\zeta}^H$, $\widetilde{\xi}$. For the latter, we proceed in the same way as described in Appendix H.

The first-order condition for the *ex ante* static maximization of (42) with respect to ρ subject to (43), taking the *ex post* solution (H.1) into account is

$$\int_0^{+\infty}\frac{d\widetilde{W}_t}{d\rho}e^{-rt}\,dt + \widetilde{S}_0 + \rho\frac{d\widetilde{S}_0}{d\rho} - \mathcal{S}^{-1}(.)\frac{d\widetilde{S}_0}{d\rho} + \lambda\left(\int_0^{+\infty}\theta_{s0}\frac{d\widetilde{x}_{st}}{d\rho}\,dt - \widetilde{S}_0 - \rho\frac{d\widetilde{S}_0}{d\rho}\right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\rho}e^{-rt} = (D_s^{-1}(\widetilde{x}_{st}) - \widetilde{q}_{st})\,\frac{d\widetilde{x}_{st}}{d\rho} + \frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho} = \frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho}$ since $D_s^{-1}(\widetilde{x}_{st}) = \widetilde{q}_{st}$. Substituting and using $S^{-1}(.) = \widetilde{\eta}_0 + \rho$, one has

$$\int_0^{+\infty}\left(\frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho}\right)dt + \widetilde{S}_0 - \widetilde{\eta}_0\frac{d\widetilde{S}_0}{d\rho} + \lambda\left(\int_0^{+\infty}\theta_{s0}\frac{d\widetilde{x}_{st}}{d\rho}\,dt - \widetilde{S}_0 - \rho\frac{d\widetilde{S}_0}{d\rho}\right) = 0.$$

Integrating as above and rearranging give

$$\rho^* = \theta_{s0}\frac{\frac{d\widetilde{\mathcal{D}}}{d\rho}}{\frac{d\widetilde{S}_0}{d\rho}} - \frac{(\lambda-1)}{\lambda}\frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\rho}} + \frac{1}{\lambda}\frac{\frac{d\widetilde{\eta}_0}{d\rho}}{\frac{d\widetilde{S}_0}{\rho}}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right]. \tag{L.3}$$

In long-run market equilibrium, $\widetilde{\mathcal{D}} = \int_0^{+\infty} D_s\left((\widetilde{\eta}_0 + \theta_{s0})e^{rt}\right)\,dt$ and $\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho) = \widetilde{\mathcal{D}}^H$, where $\widetilde{\mathcal{D}}^H \equiv \widetilde{\mathcal{D}} + \widetilde{\mathcal{D}}^F - \widetilde{S}_0^F$, is the residual cumulative demand of the rest of the world,

which has to be met by the supply of domestic reserves. It follows by differentiation with respect to ρ that $\frac{d\widetilde{\mathcal{D}}}{d\rho} = \frac{d\widetilde{\eta}_0}{d\rho}\int_0^{+\infty} D'_s(.)e^{rt}\,dt$ and $\frac{d\widetilde{S}_0}{d\rho} = \mathcal{S}'(.)\left(\frac{d\widetilde{\eta}_0}{d\rho}+1\right) = \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}\frac{d\widetilde{\eta}_0}{d\rho}$. From that equality, we obtain $\frac{d\widetilde{\eta}_0}{d\rho} = \frac{-\mathcal{S}'(.)}{\mathcal{S}'(.)-\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}$. Introducing these expressions into (L.3) yields

$$\rho^* = \theta_{s0}\frac{\int_0^{+\infty} D'_s(.)e^{rt}\,dt}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}} - \frac{\lambda-1}{\lambda}\left[\frac{\widetilde{S}_0}{\mathcal{S}'(.)} - \frac{\widetilde{\mathcal{D}}^H}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}\right] + \frac{1}{\lambda}\frac{1}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right].$$

Using the definition $\widetilde{\xi}^H \equiv \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}\frac{\widetilde{\eta}_0}{\widetilde{\mathcal{D}}^H} < 0$ and redefining $\widetilde{\xi} \equiv \frac{d\widetilde{\mathcal{D}}}{d\eta_0}\frac{\widetilde{\eta}_0}{\widetilde{\mathcal{D}}}$ as well as $\widetilde{\zeta} \equiv \frac{(\widetilde{\eta}_0+\rho)\mathcal{S}'(.)}{\widetilde{S}_0}$, we obtain

$$\frac{\rho^*}{\widetilde{\eta}_0+\rho^*} = \frac{\theta_{s0}}{\widetilde{\eta}_0+\rho^*}\frac{\widetilde{\mathcal{D}}\widetilde{\xi}}{\widetilde{\mathcal{D}}^H\widetilde{\xi}^H} - \frac{\lambda-1}{\lambda}\left[\frac{1}{\widetilde{\zeta}} + \frac{1-\frac{\rho^*}{(\widetilde{\eta}_0+\rho^*)}}{-\widetilde{\xi}^H}\right] + \frac{1}{\lambda}\frac{1-\frac{\rho^*}{(\widetilde{\eta}_0+\rho^*)}}{\widetilde{\mathcal{D}}^H\widetilde{\xi}^H}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right]. \quad \text{(L.4)}$$

When $\lambda = 1$, the second term on the right-hand side, the distortionary Ramsey component of the subsidy, vanishes. If $\theta_{s0} > 0$ and the home country is importing the resource, *i.e.* $\widetilde{S}_0 - \widetilde{\mathcal{D}} < 0$, ρ^* is non-ambiguously positive. Since $\frac{\widetilde{S}_0\widetilde{\zeta}}{\widetilde{S}_0^H\widetilde{\zeta}^H} < 1$ and $\frac{\widetilde{\mathcal{D}}\widetilde{\xi}}{\widetilde{\mathcal{D}}^H\widetilde{\xi}^H} < 1$ by the definitions of S_0^H and $\mathcal{D}^H$, combining (L.4) with (44), computed for $\lambda = 1$, yields a strictly positive tax $\theta^*_{s0} > 0$ and a strictly positive subsidy $\rho^* > 0$. The second term on the right-hand side of (L.4) is negative. Therefore, for sufficiently high revenue needs, ρ^* may turn negative, *i.e.* may become a tax on reserves development.

Symmetrically, if the home country is exporting the resource, *i.e.* $\widetilde{S}_0 - \widetilde{\mathcal{D}} > 0$, then θ^*_{s0} and ρ^* are strictly negative when $\lambda = 1$; the second term on the right-hand side of (44) being positive, θ^*_{s0} may turn positive for sufficiently high revenue needs, *i.e.* may become a tax on domestic resource consumption.

M Proof of Proposition 7

1. Shown in the main text and in Appendix L.
2. The proof is similar to the Proof of Proposition 1. We know that when $\lambda = 1$, $\theta^*_i = 0$, $i = 1, ..., n$, so that the totality of fiscal revenues is raised from the resource sector. In the context of Proposition 7, $\theta^*_{st} = \theta^*_{s0}e^{rt}$, where θ^*_{s0}, given by (44), is jointly determined with ρ^*, given by (L.4). Combining both expressions for $\lambda = 1$ and substituting into

$$\overline{\overline{R}}_0 \equiv \theta^*_{s0}\widetilde{\mathcal{D}} - \rho^*\widetilde{S}_0 \quad \text{(M.1)}$$

defines the net amount raised by the resource sector. Hence, when $\lambda = 1$ it must be the case that $R_0 \leq \overline{\overline{R}}_0$. The contrapositive is that any $R_0 > \overline{\overline{R}}_0$ implies $\lambda > 1$. Moreover, following the reasoning of the Proof of Proposition 1, any $R_0 \leq \overline{\overline{R}}_0$ will be raised without imposing distortion, implying $\lambda = 1$. ■

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