Gersbach, Hans; Haller, Hans

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A Human Relations Paradox*

Hans Gersbach† and Hans Haller‡

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Abstract

We present a variant of a general equilibrium model with group formation to study how changes of non-consumptive benefits from group formation impact on the well-being of group members. We identify a human relations paradox: Positive externalities increase, but none of the group members gains in equilibrium. Moreover, a member who experiences an increase of positive emotional benefits in a group may become worse off in equilibrium.

Keywords: group formation, competitive markets, human relation, exit

JEL: D41, D50, D60.

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†CER-ETH – Center of Economic Research at ETH Zurich, Zürichbergstrasse 18, 8092 Zurich, Switzerland, hgersbach@ethz.ch.

‡Department of Economics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0316, USA, haller@vt.edu.

1 Introduction

Finite economies à la Walras, Arrow and Debreu can exhibit paradoxical comparative statics. For instance, if in a pure exchange economy, *ceteris paribus* a consumer's endowment bundle is increased whereas another consumer's endowment is reduced by the same amount, then in the ensuing competitive equilibrium the recipient of the positive transfer may be worse off than in the equilibrium that would have resulted without the transfer and the donor may be better off than without the transfer prior to competitive exchange. Such a transfer paradox can be easily illustrated in an Edgeworth box diagram. The paradox is usually discussed in the context of trade among countries. Its plausibility has been debated and explored by Samuelson (1952, 1954) and many authors before and after him.

In a similar vein, if *ceteris paribus*, a consumer's endowment bundle in a pure exchange economy becomes larger, then as a consequence, the consumer's equilibrium welfare may be less. This phenomenon falls under the rubric of immiserizing growth in international trade theory — which can also occur with respect to productivity gains. The seminal contributions are by Bhagwati (1958, 1968). One can look at immiserizing growth in reverse. Namely, suppose a consumer is worse off with a large endowment than with a small one. Further suppose that the consumer starts out with a large endowment. Then it would be to the consumer's benefit to destroy part of the endowment prior to competitive exchange. This constitutes an instance of manipulation via destruction of endowments or D-manipulability in the terminology of Postlewaite (1979). In all three instances, a change of the initial endowment(s) has a drastic price effect: The terms of trade (relative prices) in the subsequent competitive exchange are altered in a way that proves detrimental to the agent who is enriched prior to trade.

Here we perform comparative statics with respect to non-consumptive consumer characteristics that affect group decisions and consequently, through drastic price effects, affect market outcomes. Typically, groups (for example households) form in order to benefit from group externalities. Group externalities capture all aspects of the non-consumptive benefits of humans living together. They can represent, for instance, the emotional benefit from living together with other persons in the same group.

In isolated groups an increase in the strength of group externalities will typically benefit all group members or at least one member. For instance, if one party has all

the intra-group bargaining power, it can extract all the surplus from a relationship. Consequently, if the surplus to be shared increases, that party should benefit. We show in the present note that this may not be the case if the group is embedded in a society where groups form endogenously and trade in competitive markets for consumption goods. In particular, we identify a human relations paradox where none of the group members gains in equilibrium, although non-consumptive benefits from group formation increase.

2 The Setup

We use a simple variant of a general equilibrium model with group formation whose general properties have been studied in Gersbach and Haller (2011). We consider a population of three consumers, represented by $I = \{1, 2, 3\}$. Those consumers (agents, individuals) can form groups. Groups are denoted by g or h. A group structure is a partition of I and denoted by P, P^0 or P^*. Specifically, $P^0 = \{\{1\}, \{2\}, \{3\}\}$ describes the group structure in which everybody is single. We will frequently focus on the group structure $P^* = \{\{1, 2\}, \{3\}\}$ in which the first two individuals form a two-person group and the third individual remains single.

There are two commodities, denoted $k = 1, 2$. Preferences of an agent $i \in I$ are represented by a function of the form $U_i(x_i) + U_i^g(h) = U_i(x_i^1, x_i^2) + U_i^g(h)$ where x_i^k denotes the quantity of good k ($k = 1, 2$) consumed by individual i. $U_i^g(h)$ captures the **pure group externality** contributing to the utility of individual i if he is a member of group h. Specifically, we assume

$$\left.\begin{array}{rcl} U_1(x_1^1, x_1^2) + U_1^g(h) & = & \begin{cases} \ln x_1^1 + \ln v_1 & \text{in case } h = \{1,2\}, \text{ where } v_1 \geq 1; \\ \ln x_1^1 & \text{in all other cases;} \end{cases} \\ U_2(x_2^1, x_2^2) + U_2^g(h) & = & \begin{cases} \ln x_2^2 + \ln v_2 & \text{in case } h = \{1,2\}, \text{ where } v_2 \geq 1; \\ \ln x_2^2 & \text{in all other cases;} \end{cases} \\ U_3(x_3^1, x_3^2) + U_3^g(h) & = & \frac{1}{2} \ln x_3^1 + \frac{1}{2} \ln x_3^2. \end{array}\right\} \quad (1)$$

The variables v_1 and v_2 stand for the extent of group externalities that individual 1 and 2 experience when they live together. Hence, only individuals 1 and 2 enjoy group externalities. We further assume the individual endowments

$$w_1 = (0, 1/2), w_2 = (0, 1/2), w_3 = (1, 0).$$

3 Equilibria

We adopt the concept of efficient budget set from Gersbach and Haller (2011). For a group h, first its **budget set** at the price system $p \gg 0$ is defined as

$$B_h(p) = \left\{ (x_i)_{i\in h} \;\middle|\; p \cdot \left(\sum_{i\in h} x_i\right) \leq p \cdot \left(\sum_{i\in h} w_i\right) \right\}.$$

We further define the **efficient budget set** $EB_h(p)$ as the set of $(x_i)_{i\in h} \in B_h(p)$ with the property that there is no $(y_i)_{i\in h} \in B_h(p)$ such that

$U_i(y_i; h) \geq U_i(x_i; h)$ for all $i \in h$;

$U_i(y_i; h) > U_i(x_i; h)$ for some $i \in h$.

For a singleton $\{i\}$, $B_{\{i\}}(p) = \{x_i \in \mathbb{R}^2_+ \,|\, px_i \leq pw_i\}$ constitutes the standard budget set and $EB_{\{i\}}(p)$ is the set of utility maximizers in i's budget set. Moreover, let $V_i^0(p) = \max\{U_i(x_i) : x_i \in B_{\{i\}}(p)\}$ denote i's indirect utility at the price system p.

We look at **equilibria with free exit** in the sense of Gersbach and Haller (2011).[1] Such an equilibrium assumes the form $(p; \mathbf{x}; P)$ where (i) $p = (p_1, p_2)$ is a price system, (ii) $\mathbf{x} = (x_i)_{i\in I}$ is a feasible allocation, that is $\sum_i x_i = \sum_i w_i$, (iii) P is a group structure, and the following two conditions are satisfied:

(iv) **collective rationality:** $(x_i)_{i\in h} \in EB_h(p)$ for $h \in P$.

(v) **individual rationality:** $U_i(x_i) + U_i^g(h) \geq V_i^0$ for $i \in h \in P$.

Condition (v) means that no group member has an incentive to exit and become a single consumer.

To calculate such an equilibrium, we normalize the price of the first commodity to 1, i.e., $p_1 = 1$. To prepare the derivation of the equilibrium, we first look at equilibria where everybody would be single and this group structure is treated as fixed. Given preferences and endowments, we obtain that there exists a unique competitive equilibrium $(p^0; \mathbf{x^0}; P^0)$ with the group structure $P^0 = \big\{\{1\}, \{2\}, \{3\}\big\}$:

$$p^0 = (1, 1),\ x_1^0 = (1/2, 0),\ x_2^0 = (0, 1/2),\ x_3^0 = (1/2, 1/2).$$

[1] We could also adopt the notion of an "equilibrium with free group formation" which allows individuals to freely form new groups. In the present model, these two notions are equivalent as only two individuals benefit from forming groups.

We next calculate particular competitive equilibria with free exit for the group structure $P^* = \big\{\{1,2\},\{3\}\big\}$ where we assume that group $g = \{1,2\}$ maximizes a utilitarian social welfare function

$$\begin{aligned} W_g &= \alpha U_1(x_1) + (1-\alpha)U_2(x_2) \\ &= \alpha \ln x_1^1 + (1-\alpha)\ln x_2^2 + \alpha \ln v_1 + (1-\alpha)\ln v_2, \end{aligned}$$

subject to the budget constraint $x_1^1 + p_2 x_2^2 = p_2$, where $0 < \alpha < 1$. The parameter α can be interpreted as the weight of individual 1 in group g. Similarly, $1-\alpha$ is the weight of individual 2. A solution of this problem produces an efficient decision of group g.

Since the group externalities do not affect excess demand vectors of group $g = \{1,2\}$, the excess demand vectors of the groups g and $h = \{3\}$, denoted by z_g and z_h, are given by

$$\begin{aligned} z_g &= (\alpha p_2, -\alpha), \\ z_h &= \left(-\frac{1}{2}, \frac{1}{2p_2}\right). \end{aligned}$$

A market equilibrium without exit considerations $(p^*, \mathbf{x}^*; P^*)$ would require

$$p^* = (1, 1/(2\alpha)),\ x_1^* = (1/2, 0),\ x_2^* = (0, 1-\alpha),\ x_3^* = (1/2, \alpha).$$

The non-exit conditions (v) for group g amount to:

$$\begin{aligned} U_1(x_1^*) &= \ln\frac{1}{2} + \ln\, v_1 \geq \ln\frac{1}{4\alpha}, \\ U_2(x_2^*) &= \ln(1-\alpha) + \ln\, v_2 \geq \ln\frac{1}{2}, \end{aligned}$$

which imply $\alpha \geq \frac{1}{2v_1} = \underline{\alpha}$ and $\alpha \leq 1 - \frac{1}{2v_2} = \overline{\alpha}$. Hence, if $\alpha \in [\underline{\alpha}, \overline{\alpha}] = [\frac{1}{2v_1}, 1 - \frac{1}{2v_2}]$, then $(p^*, \mathbf{x}^*, P^*)$ is a competitive equilibrium with free exit.

It proves useful for our subsequent analysis to stress at this stage that the same equilibrium with free exit can be obtained by focussing on Nash bargaining in group g. For this purpose we denote by β and $1-\beta$ the relative bargaining power of individual 1 and 2, respectively. Furthermore, for $i = 1, 2$, let $x_i^0(p_2)$ denote individual i's demand as a single consumer at the price system $(1, p_2)$.

Let us consider then the possibility that for every price p_2, group g maximizes the

Nash product

$$\begin{aligned} N_g &= \left\{U_1(x_1) - U_1\left(x_1^0(p_2)\right)\right\}^{\beta} \cdot \left\{U_2(x_2) - U_2\left(x_2^0(p_2)\right)\right\}^{1-\beta} \\ &= \left\{\ln(x_1^1 \cdot v_1) - \ln\left(\frac{1}{2}p_2\right)\right\}^{\beta} \cdot \left\{\ln\left(x_2^2 \cdot v_2\right) - \ln\frac{1}{2}\right\}^{1-\beta} \end{aligned}$$

on g's budget set, given the relative bargaining power β and $1-\beta$. Note that the group $g = \{1,2\}$ uses as conflict outcomes the outside options available at the price p_2. The outside option values $V_i^0(1,p_2) = U_i\left(x_i^0(p_2)\right)$ amount to $\ln(\frac{1}{2}p_2)$ for individual 1 and to $\ln(\frac{1}{2})$ for the second individual. Using the group budget constraint $x_1^1 = p_2 - p_2 x_2^2$, the first-order condition for maximizing N_g amounts to:

$$\beta \cdot \frac{x_2^2}{1-x_2^2} = (1-\beta) \cdot \frac{\ln\left((1-x_2^2)\cdot 2v_1\right)}{\ln\left(x_2^2 \cdot 2v_2\right)} \tag{2}$$

This is an implicit equation for x_2^2. Now suppose the same allocation is obtained in a competitive equilibrium with free exit where the group maximizes its utilitarian welfare function, with respective weights α and $1-\alpha$. Then we have $x_2^2 = 1-\alpha$ and thus equation (2) is an implicit equation for $\beta(\alpha)$, the bargaining power of individual 1 that yields the same group decision as the group's utilitarian welfare maximum:

$$\frac{\beta}{1-\beta} = \frac{\alpha}{1-\alpha} \cdot \frac{\ln\left(\alpha\, 2v_1\right)}{\ln\left((1-\alpha)2v_2\right)} \tag{3}$$

Note that by definition of $\beta(\alpha)$, the weight α in W_g and the weight $\beta = \beta(\alpha)$ in N_g lead to the same allocation for group g. We obtain the following properties for $\beta(\alpha)$:

Fact 1 $\beta\left(\frac{1}{2v_1}\right) = 0,\ \beta\left(1-\frac{1}{2v_2}\right) = 1,\ \frac{\partial\beta}{\partial\alpha} > 0.$

Higher utilitarian power, that is, a higher weight in the group welfare function, translates into higher relative bargaining power, as long as α is in the range $\left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right]$ for which the competitive equilibrium with free exit involving group g exists. The maximal utilities of the individuals are given by

$$U_1 = \begin{cases} \ln\frac{1}{2} + \ln v_1 & \text{if } \alpha \in \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right] \\ \ln\frac{1}{2} & \text{if } \alpha \notin \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right] \end{cases} \tag{4}$$

$$U_2 = \begin{cases} \ln(1-\alpha) + \ln v_2 & \text{if } \alpha \in \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right] \\ \ln\frac{1}{2} & \text{if } \alpha \notin \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right] \end{cases} \tag{5}$$

where we have assumed that $(p^*; \mathbf{x}^*; P^*)$ prevails for $\alpha \in \left[\frac{1}{2v_1}, 1 - \frac{1}{2v_2}\right]$ while only $(p^0; \mathbf{x^0}; P^0)$ can occur for $\alpha \notin \left[\frac{1}{2v_1}, 1 - \frac{1}{2v_2}\right]$.

4 The Human Relations Paradox

Conventional wisdom has it that if a party has all the bargaining power, it can extract all the surplus from a relationship. Consequently, if the surplus increases, the party should benefit. This logic also applies here. If consumer 1 exerts total bargaining power, $\beta = 1$, then he can extract all the surplus created by the group $g = \{1, 2\}$ up to the point where consumer 2 is indifferent between staying in the group and leaving. So let us assume $\beta = 1$. Now suppose that v_2, the amount of positive group externality which consumer 1 exerts on consumer 2 increases, so that for whatever reasons consumer 2 derives more social or emotional benefit from having consumer 1 around. Would consumer 1 gain from such a change? *Ceteris paribus*, their total surplus would increase and, by the above logic, consumer 1 would be the sole beneficiary. But it turns out that neither consumer 1 nor 2 benefits because the corresponding equilibrium prices adjust. Indeed, we obtain:

Proposition 1 *Suppose $\beta \equiv 1$. Then, an increase in positive group externalities v_2 does not translate into higher utility for any member of group $g = \{1, 2\}$.*

PROOF. As $\beta \equiv 1$, we have $\alpha = \overline{\alpha} = 1 - \frac{1}{2v_2}$. From equation (4) we observe that equilibrium utility $U_1 = \ln \frac{1}{2} + \ln v_1$ is independent of α and thus an increase of v_2 does not affect 1's utility. For $\alpha = \overline{\alpha}$, the second individual's utility in equation (5) becomes $U_2 = \ln(1 - \overline{\alpha}) + \ln \frac{1}{2}$ and thus is also independent of v_2.
At a more intuitive level the upper bound $\overline{\alpha}$ is increasing in v_2. Therefore, the equilibrium price $p^* = 1/(2\overline{\alpha})$ declines in v_2. Indifference of consumer 2 between staying and leaving requires $\ln(x_2^{*2}) + \ln v_2 = \ln(1/2)$ which amounts to $x_2^{*2} = 1/(2v_2)$. As v_2 increases, more of the group endowment with good 2 will be sold in exchange for good 1. But because of the decline of the equilibrium price p_2^*, consumer 1 cannot afford more than the previous consumption level x_1^{*1}. ■

Hence, there is the paradoxical situation that an increase in positive group externalities does not translate into higher utility for any of the group members. The only one to

gain is consumer 3, whose equilibrium utility goes up.

Next let us consider the case of equal bargaining power, $\beta \equiv 1/2$, and $v_1 > 1, v_2 > 1$. Then $\frac{\beta}{1-\beta} = 1$ and by (3),

$$\frac{\alpha}{1-\alpha}\left[\ln\alpha + \ln(2v_1)\right] = \ln(1-\alpha) + \ln(2v_2).$$

Moreover, $\alpha \in (\underline{\alpha}, \overline{\alpha})$. If v_1 increases, then α must decrease to preserve the equation. As a result, consumers 1 and 2 both gain at the detriment of consumer 3. If v_2 increases, then α must increase in order to preserve the equation. Hence consumer 3 gains, consumer 2 loses in terms of utility from consumption but gains in terms of group externalities, and consumer 1 is unaffected by the increased group externality — another paradoxical outcome.

In order to keep relative bargaining power constant when v_1 or v_2 changes, the corresponding utilitarian weights have to adjust. In turn, equilibrium prices and equilibrium welfare are affected. The paradoxes occur because of a drastic price effect in response to preference changes. One might argue that in a large economy a small group can only cause negligible price effects and thus the paradox will not occur. However, a sufficiently widespread change of consumer characteristics can have drastic price effects in a large economy as well. For instance, our conclusions immediately generalize to the case of a replica economy where consumers 1, 2, and 3 are replaced by respective consumer types 1, 2, 3 and there is the same number of consumers of each type.

To conclude, we have identified a human relations paradox which complements a series of paradoxes that can occur in a market economy.

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