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Eidgenössische Technische Hochschule Zürich  
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# Sectoral Heterogeneity, Resource Depletion, and Directed Technical Change: Theory and Policy

KAREN PITTEL\* and LUCAS BRETSCHGER\*\*

## Abstract

We analyze an economy in which sectors are heterogeneous with respect to the intensity of natural resource use. Long-term dynamics are driven by resource prices, sectoral composition, and directed technical change. We study the balanced growth path and determine stability conditions. Technical change is found to be biased towards the resource-intensive sector. Resource taxes have no impact on dynamics except when the tax rate varies over time. Constant research subsidies raise the growth rate while increasing subsidies have the opposite effect. We also find that supporting sectors by providing them with productivity enhancing public goods can raise the growth rate of the economy and additionally provide an effective tool for structural policy.

Keywords: sustainable development, sectoral heterogeneity, directed technical change

JEL Classification: O4 (economic growth), O41 (multi-sector growth models), Q01 (sustainable development), Q3 (non-renewable resources)

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# 1 Introduction

The last years have witnessed a notably rising interest in natural resource scarcity and its long-run economic consequences. In the framework of capital-resource models, several papers have shown that long-run endogenous growth may be compatible with the essential use of non-renewable resources, see Barbier (1999), Scholz and Ziemes (1999), and Grimaud and Rougé (2003, 2005). This literature has analyzed the effects of endogenous technological change in a framework with a single final output. This assumption is convenient but sidesteps the possibility of analyzing sectoral composition of output and its impact on long-run development. There are strong reasons to assume a high relevance of sectoral structure in this context. First, empirical observations show that sectors differ substantially in terms of input intensities, specifically with regard to knowledge and natural resource use. Second, sectors offer different investment opportunities with aggregate growth depending on sectoral development and sector-specific innovation intensities. Third, policies directed at specific sectors are very popular and often implemented in practice. For these reasons, the impact of sectoral heterogeneity on growth emerges as a major issue, in particular when focusing on endogenous innovations, natural resource constraints, and policy.

The aim of this paper is to study the dynamic behavior of economies in which sectoral outputs are characterized by different resource intensities. We stress the role of sectoral research activities and directed technological change. By doing so, we apply the theory of factor-induced technical change, as introduced by Hicks (1932) and applied by Acemoglu (2002), to economic sectors and determine the conditions for sector-induced research. We show that long-run development is characterized by specific regularities for the direction of R&D and the composition of sectoral outputs. We analyze different types of policies employed by governments to foster sustainable development in the sense of reallocating inputs towards less resource intensive sectors, enhancing growth and reducing the speed of resource extraction. Examples include sectoral policies, e.g. subsidies or productive public good provision to resource extensive sectors, but also resource taxes. It is shown that these policies might not induce the desired effects or that the sectors to which policies are linked do not matter for the direction of policy effects.

The fact that, in practice, sectoral resource efficiency and energy-related research have recently gained in importance, may be illustrated by two examples. In the light of rapidly rising energy prices, the International Energy Agency (IEA) emphasizes the large potential for improving energy efficiency in the energy-intensive sectors, in all of which (apart from the cement industry) energy intensity is predicted to improve significantly (see IEA 2008, p. 112). The impact of energy prices on innovation activities can be illustrated with the example of Hungary, one of the few countries providing detailed

research data: from 2001 to 2005 firms' research expenditures in the field of energy (rational utilization, production, and distribution) increased by 300% while total research expenditures rose only by 48% (see OECD 2008).

The paper is related to the literature on sectoral change and sectoral reallocations that follow specific regularities. The most prominent example is the reallocation of labor from agriculture to manufacturing and services that has been pointed out by, among others, Kuznets (1957) and Chenery (1960). Recently, these regularities - often summarized under the term 'Kuznets' facts' (to complement the well-known 'Kaldor facts') - have attracted renewed interest in the debate on non-balanced growth, see Kongsamut et al. (2001) and Acemoglu and Guerrieri (2008)). Including natural resources, López et al. (2007) emphasize that sectoral change is a major issue when asking whether growth is sustainable in the long run. With the expected increase in resource prices in the future it seems natural to predict that resource-intensive sectors will grow below average and their share of total production will shrink over time. The results of this paper clarify why, when, and how sector-specific technical change can compensate for high resource dependence and how it affects long-run growth. We find, that due to resource scarcity, the quantities of goods produced in resource-intensive sectors fall compared to production in more resource-extensive sectors. Yet, due to the rising scarcity of resource-intensive goods, incentives arise to invest relatively more in R&D of the resource-intensive sector. Consequently, the composition of consumption in terms of productivity-weighted sectoral goods remains constant along a balanced growth path.

The basic technology assumptions for the different sectors in the model are based on Romer (1990). This is similar to Bretschger and Pittel (2005) who consider a multi-sector economy with sector-specific natural resource use but without directed technological change, as the substitution elasticity between sectoral outputs is assumed to be unity.<sup>1</sup> Of the recent literature on directed technological change, Smulders and de Nooij (2003) and Di Maria and Valente (2008) are closest to our approach. These papers do not assume that natural resources and labor are employed in all the different sectors of the economy as we do. Smulders and de Nooij (2003) take the supply of energy as exogenously given. We extend their approach by endogenizing the dynamics associated with the input of non-renewable natural resources. Moreover, we introduce labor reallocation between the different production and research sectors, which realistically allows for more flexible adjustments in the economy. Due to this endogeneity, policies can now affect the speed of resource extraction as well as aggregate research activities - both of which are crucial for the dynamics of the economy. Di Maria and Valente (2008)

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<sup>1</sup>Withagen (1999), Pittel (2002), and Xepapadeas (2002) provide surveys on the impact of natural resource use on economic growth. The impact of natural resource use in dynamic multi-sector models is also treated by Peretto (2008) and Bretschger (2008).

endogenize the supply of inputs, capital and resources, but again do not assume that all inputs are used in the different sectors. They conclude that long-run development is characterized by resource-augmenting technological progress only. For the case that the economic sectors employ all the inputs and only differ with respect to input intensities we are able to show that every sector conducts R&D in the long-run. The direction of technological change is endogenous and depends on the degree of heterogeneity with respect to resource intensities.

We show the existence of a balanced growth path and provide conditions for saddle-path stability of the system. In addition, we demonstrate that the share of resource-intensive sectors can be constant in the long-run as profit incentives induce a more than proportional research effort to these sectors. We also show that in an economy with heterogeneous sectors, research subsidies have positive growth effects in both sectors and that resource taxes affect dynamics only when the tax rate is varying over time. The provision of productive services by the government raises the growth rate, provided the quantity of services is steadily extended.

The paper introduces several novel features. First, it introduces a new kind of multi-sector economy suited to discuss the direction of development under natural resource constraints. Second, it derives basic characteristics of growth paths of two-sector economies with essential non-renewable resources and directed technical change. Third, we study implications of different types of policies that aim at supporting sustainability. In addition to taxes and subsidies, we consider the impact that productivity enhancing public goods have on growth, sectoral shares and resource extraction.

The remainder of the paper is organized as follows. Section 2 describes the model in detail. The short and long-run dynamics of the model are analyzed in Section 3. Section 4 deals with the effects of policies striving at abetting sustainability in this simplified setting. Finally, Section 5 concludes.

## 2 The Model

In our economy, horizontally differentiated goods are produced in two sectors – a resource-intensive and a resource-extensive sector. In each sector, the differentiated goods are assembled to sectoral outputs which are consumed by the households. Blueprints for new products are developed by sector specific research activities and sold to monopolistic producers in each sector. Besides natural resources, labor constitutes the second primary input, which is employed in research as well as in intermediate production. Sectors differ with respect to resource intensity of production. We consider infinitely living households that maximize lifetime utility. Savings are either in the form of investment in bonds or in R&D.

## 2.1 Production

**Sectoral output** The outputs of the two sectors,  $\tilde{X}$  and  $\tilde{Z}$ , each consist of a continuum of horizontally differentiated goods,  $x_i$ ,  $i \in [0, n]$ , and  $z_j$ ,  $j \in [0, m]$ , where  $n$  and  $m$  denote the number of varieties in the respective sectors.<sup>2</sup> Gains from specialization arise, i.e. the larger the variety of goods, the more productive the aggregate:<sup>3</sup>

$$\tilde{X} = \left( \int_0^n x_i^\beta di \right)^{1/\beta} \quad \text{and} \quad \tilde{Z} = \left( \int_0^m z_j^\beta dj \right)^{1/\beta}. \quad (1)$$

where  $0 < \beta < 1$ . The index of consumption  $C$  reflects households' preferences for sectoral output; it depends on  $\tilde{X}$  and  $\tilde{Z}$ , according to the following CES function:

$$C_t = \left( \tilde{X}_t^{\frac{\nu-1}{\nu}} + \tilde{Z}_t^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}}, \quad \nu > 0, \nu \neq 1 \quad (2)$$

where  $\nu$  denotes the elasticity of substitution between  $\tilde{X}$  and  $\tilde{Z}$ . To facilitate calculations without loss of generality, we choose the consumption good to be the numeraire of the system so that its price is unity, i.e.  $p_C \equiv 1$ . At each point in time, utility maximization results in:

$$\frac{\tilde{X}}{\tilde{Z}} = \left( \frac{p_{\tilde{X}}}{p_{\tilde{Z}}} \right)^{-\nu} \quad \Leftrightarrow \quad \frac{p_{\tilde{X}} \tilde{X}}{p_{\tilde{Z}} \tilde{Z}} = \left( \frac{\tilde{X}}{\tilde{Z}} \right)^{-\frac{1-\nu}{\nu}} = \frac{\phi}{1-\phi} = \tilde{\phi} \quad (3)$$

with  $\phi = \frac{p_{\tilde{X}} \tilde{X}}{C}$  and  $1 - \phi = \frac{p_{\tilde{Z}} \tilde{Z}}{C}$  denoting the expenditure shares of  $\tilde{X}$  and  $\tilde{Z}$ , such that the relative sector share of  $x$ -goods is given by  $\tilde{\phi}$ , which will prove to be a very useful variable below.

Competition in  $x$ - and  $z$ -production is monopolistic. Each type of good is produced by only one firm that has to acquire the according patent first.  $x$ - as well as  $z$ -intermediates are produced from labor  $L$  and non-renewable resources  $R$  using the following Cobb-Douglas production technologies:

$$x_i = (L_{x_i})^\alpha (R_{x_i})^{1-\alpha} \quad \text{and} \quad z_j = (L_{z_j})^\delta (R_{z_j})^{1-\delta} \quad (4)$$

with  $0 < \alpha, \delta < 1$ .  $L_k$  and  $R_k$ ,  $k = x_i, z_j$ , denote the input of labor and resources in the production of  $x_i$  and  $z_j$ . It is assumed that sectors differ with respect to their resource intensities, i.e.  $\alpha \neq \delta$ .

<sup>2</sup>For notational convenience the time index will be suppressed whenever no ambiguity arises.

<sup>3</sup>In contrast to the productivity adjusted aggregates,  $\tilde{X}$  and  $\tilde{Z}$ , we denote aggregate physical amounts of  $x_i$  and  $z_i$  by  $X = \int_0^n x_i di$  and  $Z = \int_0^m z_j dj$ . The prices for  $\tilde{X}$  and  $\tilde{Z}$  are  $p_{\tilde{X}}$  and  $p_{\tilde{Z}}$ .  $p_{x_i}$  and  $p_{z_i}$  on the other hand denote prices for individual goods.

Maximization of profits gives the first-order conditions for the input of labor and resources in the two sectors. Considering that  $x_i = x$  and  $z_j = z$  in the symmetric equilibrium gives the sectoral demands for labor and resources in terms of  $\tilde{\phi}$  and  $C$ :

$$\begin{aligned} L_X &= \alpha\beta\frac{\tilde{\phi}}{1+\tilde{\phi}}\frac{C}{w} & \text{and} & & L_Z &= \delta\beta\frac{1}{1+\tilde{\phi}}\frac{C}{w} \\ R_X &= (1-\alpha)\beta\frac{\tilde{\phi}}{1+\tilde{\phi}}\frac{C}{p_R} & \text{and} & & R_Z &= (1-\delta)\beta\frac{1}{1+\tilde{\phi}}\frac{C}{p_R} \end{aligned} \quad (5)$$

with  $R_K = \int_0^l R_k di$  and  $L_K = \int_0^l L_k dj$ ,  $(K, l, k) \in (X, n, x_i), (Z, m, z_j)$ .  $w$  and  $p_R$  denote the wage rate and the price of resources. Individual firms' demands are obtained by dividing the respective sectoral demands by the 'number' of intermediates in each sector, i.e.  $n$  and  $m$  respectively. Summing up the resource demands of the two sectors in (5) gives the aggregate extraction of resources at each point in time:

$$R = R_X + R_Z = ((1-\alpha)\tilde{\phi} + (1-\delta))\frac{\beta}{1+\tilde{\phi}}\frac{C}{p_R}. \quad (6)$$

From (5) and the production functions for  $x$  and  $z$ , (4), sectoral equilibrium profits from intermediates production can be derived:

$$\Pi_X = (1-\beta)\frac{\tilde{\phi}}{1+\tilde{\phi}}C \quad \text{and} \quad \Pi_Z = (1-\beta)\frac{1}{1+\tilde{\phi}}C. \quad (7)$$

**R&D** Blueprints for new types of goods are generated in two separate R&D sectors. The only rival input to research is labor, yet production also profits from past research activities which give rise to positive sector specific spill-overs. Production is linear in labor as well as in research experience which is, for simplicity, set equal to the 'number' of blueprints generated in the respective sector in the past ( $n$  for the  $x$ -sector and  $m$  for the  $z$ -sector):

$$\dot{n} = \frac{dn}{dt} = \frac{L_n}{a}n \quad \text{and} \quad \dot{m} = \frac{dm}{dt} = \frac{L_m}{a}m \quad (8)$$

with  $L_n$  and  $L_m$  denoting the input of labor to sectoral research.  $a$  represents the unit input coefficient of labor in research which is assumed to be equal in the two sectors.

Given the four different uses of labor, equilibrium in the labor market requires

$$L_X + L_Z + L_n + L_m = 1. \quad (9)$$

where, for simplicity, the size of the labor force is normalized to unity.

Research markets are assumed to be perfectly competitive, such that in equilibrium patent values  $V_n$ , resp.  $V_m$ , are equal to the marginal production cost

$$V_n = \frac{aw}{n} \quad \text{and} \quad V_m = \frac{aw}{m}. \quad (10)$$



Furthermore, equilibrium on the patent market requires the value of a patent to be equal to the discounted stream of profits generated by the production of the respective intermediate good which implies the following no-arbitrage conditions to hold:

$$\dot{V}_n = rV_n - \Pi_x \quad \text{and} \quad \dot{V}_m = rV_m - \Pi_z \quad (11)$$

where  $r$  is the interest rate on all assets.  $\Pi_x = \frac{\Pi_X}{n}$  and  $\Pi_z = \frac{\Pi_Z}{m}$  stand for individual intermediate firms' profits;  $\Pi_X$  and  $\Pi_Z$  are given in (7).

**Resources** Natural resources are non-renewable. The resource stock  $S$  is depleted by the extraction of resources  $R$  for production, such that the dynamics of the resource stock are

$$\dot{S} = -R. \quad (12)$$

It is assumed that resources are extracted at no cost. Resource firms maximize intertemporal profits

$$\max_R \int_0^\infty p_R(t)R(t)e^{-\int_0^t r(s)ds} dt \quad (13)$$

subject to (12) which yields the familiar Hotelling rule:<sup>4</sup>

$$g_{p_R} = \frac{\dot{p}_R}{p_R} = r. \quad (14)$$

## 2.2 Consumers

Households derive utility from consumption  $C$ . The representative household maximizes discounted lifetime utility with respect to its intertemporal budget constraint:

$$\begin{aligned} \max_c \quad & \int_0^\infty \log C(t)e^{-\rho t} dt \\ \text{s.t.} \quad & \dot{W} = rW + w - C \end{aligned} \quad (15)$$

where  $W = nV_n + mV_m + p_RS$  denotes total household asset holdings. Households supply labor inelastically. From the first-order conditions of household maximization we get the familiar Keynes-Ramsey rule

$$g_C = r - \rho. \quad (16)$$

## 3 Development in the short and long run

To analyze the dynamics of the economy we reduce the system to two first-order differential equations which are functions of the relative sector share,  $\tilde{\phi}$ , and the input of labor in intermediates production of sector  $z$ , i.e.  $L_Z$ .

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<sup>4</sup> $g_b$  denotes the growth rate of variable  $b$ , i.e.  $g_b = \frac{\dot{b}}{b}$ , where  $\dot{b}$  is the time derivative of  $b$ .

**Lemma 1.** *The dynamics of the economic system are given by*

$$\dot{L}_Z = \left( \frac{1}{2a} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}(1 + \tilde{\phi}) \right) L_Z - \rho - \frac{1}{2a} - \frac{\tilde{\phi}}{1 + \tilde{\phi}} g_{\tilde{\phi}} \right) L_Z \quad (17)$$

$$\dot{\tilde{\phi}} = \left( \frac{\left[ \frac{1}{2a} \frac{\alpha - \delta}{\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}(1 + \tilde{\phi}) \right) - \frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 (1 - \tilde{\phi}) \right] L_Z - \frac{1}{2a}(\alpha - \delta)}{\frac{1}{\nu-1}} \right) \tilde{\phi}. \quad (18)$$

*Proof.* see Appendix A1. ■

These two equations describe the dynamics of the system along the balanced growth path (BGP) as well as during the transition to the BGP.

### 3.1 Balanced growth path

A path will be called a BGP if all variables grow at constant – possibly zero or negative – rates. This implies that (i) aggregate production and production in both intermediate sectors grow at the same rate and (ii) expenditure shares, sectoral labor inputs and the interest rate are constant over time ( $\dot{\phi} = \dot{L}_Z = \dot{r} = 0$ ).

For the BGP values of  $L_Z$  and  $\tilde{\phi}$  we get (see Appendix A2):

$$L_Z^* = \frac{\delta(1 + 2a\rho)}{\delta + \tilde{\phi}^*\alpha + \frac{1-\beta}{\beta}(1 + \tilde{\phi}^*)} \quad (19)$$

$$\tilde{\phi}^* = \frac{A}{B} \quad (20)$$

with

$$A = \left( (1 - \beta)^2 (1 + 2a\rho) - a\alpha\beta^2\delta\rho \right) - a(\alpha - \delta)\beta(1 - \beta)\rho + a\delta^2\beta^2\rho \quad (21)$$

$$B = \left( (1 - \beta)^2 (1 + 2a\rho) - a\alpha\beta^2\delta\rho \right) + a(\alpha - \delta)\beta(1 - \beta)\rho + a\alpha^2\beta^2\rho \quad (22)$$

where asterisks indicate variable values or growth rates along the BGP. The equilibrium input of labor into  $x$ -intermediates can be derived from (5) which implies

$$L_X^* = \tilde{\phi}^* \frac{\alpha}{\delta} L_Z^*. \quad (23)$$

From the no-arbitrage conditions for the patent market follows that along the BGP the following relations hold (see Appendix A2):

$$L_n^* = \tilde{\phi}^* \frac{1 - \beta}{\beta} \frac{L_Z^*}{\delta} - a\rho \quad (24)$$

$$L_m^* = \frac{1 - \beta}{\beta} \frac{L_Z^*}{\delta} - a\rho. \quad (25)$$

The growth rate of resource extraction along the BGP can be determined by expressing the aggregate demand for resources, (6), in growth rates and considering the Keynes-Ramsey rule, (16). This gives

$$g_R = -\rho. \quad (26)$$

Differentiation of (2) confirms that balanced growth requires consumption and the production of  $x$ - and  $z$ -aggregates to grow at the same rate, i.e.

$$g_C^* = g_{\tilde{X}}^* = g_{\tilde{Z}}^*. \quad (27)$$

Identical constant growth rates of  $\tilde{X}$  and  $\tilde{Z}$  together with (3) imply that the sectoral expenditure shares,  $\phi$  and  $1 - \phi$ , as well as the relative expenditure share,  $\tilde{\phi}$ , are constant over time.

The condition that along the BGP aggregate production in both sectors has to grow at the same rate carries important implications for research efforts in equilibrium. Considering the production technologies for  $x$  and  $z$ , (4), as well as (26), the growth rates of  $\tilde{X}$  and  $\tilde{Z}$  along the BGP are given by

$$g_{\tilde{X}}^* = \frac{1 - \beta}{\beta} g_n^* - (1 - \alpha)\rho \quad (28)$$

$$g_{\tilde{Z}}^* = \frac{1 - \beta}{\beta} g_m^* - (1 - \delta)\rho. \quad (29)$$

**Proposition 1.** *Along the balanced growth path, research is biased towards the resource intensive sector. If, e.g., the  $z$ -sector is more resource intensive ( $\alpha > \delta$ ),  $g_m^* > g_n^*$  holds.*

*Proof.* From (28), (29) and  $g_{\tilde{X}}^* = g_{\tilde{Z}}^*$  along the BGP follows straightforwardly that the following relation holds:

$$g_m^* = (\alpha - \delta)\rho \frac{\beta}{1 - \beta} + g_n^*. \quad (30)$$

■

This condition states that for balanced growth to be feasible, differences in resource intensities between sectors have to be compensated by research. It can also easily be seen that in case that sectors are identical, innovation rates along the BGP are the same in the two sectors. If, however, sectors differ with respect to resource intensities, more research will be conducted in the sector that is more resource intensive.

While the aggregate productivity weighted amounts of goods produced in both sectors,  $\tilde{X}$  and  $\tilde{Z}$ , grow at the same, potentially positive rate in equilibrium, the physical amounts individual intermediates produced in either sector,  $x$  and  $z$ , decrease over time.

Taking into account that labor shares are constant along the BGP, it follows from (4) and  $g_{R_i} = g_R = -\rho$  that

$$g_x^* = -(1 - \alpha)\rho < 0 \quad (31)$$

$$g_z^* = -(1 - \delta)\rho < 0. \quad (32)$$

The reduction in the produced amounts is due to the decreasing input of natural resources. If the  $z$ -sector is more resource intensive than the  $x$ -sector,  $z$  falls faster than  $x$ . As economic intuition suggests, it follows from (3) that the price ratio follows a time path that is inverse to the development of quantities, i.e. prices in the more resource intensive sector rise faster due to increasing resource prices.

From (8), (28) and (29) we can express the equilibrium growth rate of consumption as

$$\begin{aligned} g_C^* &= \frac{1 - \beta}{\beta} \frac{L_n^*}{a} - (1 - \alpha)\rho \\ &= \frac{1 - \beta}{\beta} \frac{L_m^*}{a} - (1 - \delta)\rho. \end{aligned} \quad (33)$$

Overall, the sign of  $g_C$  in (33) is ambiguous. Two forces determine whether long-term development is sustainable ( $g_C > 0$ ):  $-(1 - \alpha)\rho$  and  $-(1 - \delta)\rho$  represent the negative growth effects stemming from the declining input of natural resources while  $\frac{1 - \beta}{\beta} \frac{L_n^*}{a}$  and  $\frac{1 - \beta}{\beta} \frac{L_m^*}{a}$  reflect the growth stimulating effects of research.

As to be expected, it follows from (19), (25) and (33) that consumption growth along the BGP profits from more productive research ( $\frac{dg_C}{da} < 0$ ) but suffers if impatience rises ( $\frac{dg_C}{d\rho} < 0$ ). With respect to changes in resources intensities, effects are, however, less clear. An increase in the productivity of resources (increase in  $\alpha$ , resp.  $\delta$ ) induces on the one hand a less severe drag on growth as resources are more productive, but on the other hand causes a reallocation of labor away from research. The net effect on growth depends crucially on the productivity of research and the discount rate which determine the willing to invest in research.

### 3.2 Stability

To check for the stability properties of the system, we derive the Jacobian of (17) and (18) in the proximity of the steady state,  $\tilde{\phi}^*$  and  $L_Z^*$ ,

$$D = \begin{pmatrix} \left. \frac{\partial \dot{\tilde{\phi}}}{\partial \tilde{\phi}} \right|_{\tilde{\phi}^*, L_Z^*} & \left. \frac{\partial \dot{\tilde{\phi}}}{\partial L_Z} \right|_{\tilde{\phi}^*, L_Z^*} \\ \left. \frac{\partial \dot{L_Z}}{\partial \tilde{\phi}} \right|_{\tilde{\phi}^*, L_Z^*} & \left. \frac{\partial \dot{L_Z}}{\partial L_Z} \right|_{\tilde{\phi}^*, L_Z^*} \end{pmatrix}. \quad (34)$$

**Lemma 2.** *The system given by (17) and (18) is locally saddle-path stable for  $\nu < 1$ .*

*Proof.* It can be shown that

$$\begin{aligned}\det D &= -(1-\nu) \frac{\tilde{\phi}^{*2} L_Z^*}{2a^2 \delta^2} \left( \frac{1-\beta}{\beta} \right)^2 \left( 2 \frac{1-\beta}{\beta} + \alpha + \delta \right) \\ \text{tr} D &= \frac{\frac{1}{2a\delta} L_Z^* \left[ (1+\tilde{\phi}^*) \left( \alpha \tilde{\phi}^* + \delta + (1+\tilde{\phi}^*) \frac{1-\beta}{\beta} \right) + (\nu-1) \tilde{\phi}^* \left( 4 \left( \frac{1-\beta}{\beta} \right)^2 + (\alpha-\delta)^2 \right) \right]}{1+\tilde{\phi}^*}\end{aligned}\quad (35)$$

with  $\det D \gtrless 0$  for  $\nu \gtrless 1$  and  $\text{tr} D > 0$  for  $\nu > 1$ . In a two-dimensional system the trace is equal to the sum of the eigenvalues ( $\text{tr} = EV_1 + EV_2$ ) and the determinant is given by their product, ( $\det = EV_1 \cdot EV_2$ ). Consequently, for  $\det < 0$  exactly one eigenvalue is negative while  $\det > 0$  in combination with  $\text{tr} > 0$  shows that both eigenvalues are positive.

A unique and stable trajectory in case that one eigenvalue is negative while the other is positive requires the system to contain one degree of freedom, i.e. given that households initially choose the value of one variable, the second is determined by the system. Considering the underlying model structure where the initial values of  $S_0$ ,  $n_0$  and  $m_0$  are given, one of the initial values of  $\phi$  and  $L_Z$  can be chosen by the household while the other is then determined by the system. ■

The result that for  $\nu < 1$  the system is saddle-path stable corresponds to the recent literature. We are able to show that it holds in the presence of sectoral heterogeneity with an essential non-renewable resource.

### 3.3 Generalization of results

It can be shown that our results, regarding the existence of a BGP as well as its stability properties, can be extended to economies in which sectors differ with respect to research productivity ( $a_n \neq a_m$ ) and/or gains of specialization ( $\beta_x \neq \beta_z$ ).

Let us shortly consider the case of additional heterogeneity with respect to the gains of specialization. Recall that when sectors only differ with respect to research intensities, (30) describes how BGP research is affected by sectoral heterogeneity. This equation modifies in the presence of heterogeneous gains of specialization to

$$\frac{1-\beta_z}{\beta_z} g_m^* - \frac{1-\beta_x}{\beta_x} g_n^* = (\alpha - \delta) \rho \quad (36)$$

with  $\beta_x$  and  $\beta_z$  representing the gains of specialization in sector  $x$  and  $z$  respectively. Higher gains of specialization in the sector  $x$  (i.e.  $\beta_z > \beta_x$ ) would therefore imply an even stronger bias towards  $z$ -research.

Note that research productivities do not enter (30) and (36). Sectoral differences in  $a$  affect the labor input in each research sector as well as the allocation of labor between

research and intermediates (and thereby the levels of  $g_n$  and  $g_m$ ). They do, however, not affect the functional relationship between  $g_n$  and  $g_m$ .

## 4 Policy analysis

In Subsection 3.1 we have derived that growth depends on research effort and resource use. Growth effects of policy can therefore stem from either higher innovation rates and/or slower resource extraction. Yet, alternative policies not only differ with respect to the channels through which they affect growth, but also with respect to their impact on the sectoral structure.

In the following we consider different types of policies that might constitute alternatives for a policy maker. In this section we assume throughout that  $\alpha > \delta$ , i.e. that the  $x$ -sector is less resource intensive. We check different policies with respect to their ability to foster growth, to slow down resource extraction and to affect the sectoral structure of the economy. We do not characterize optimal policies, but rather conduct a positive analysis of policy interventions. Specifically we focus on the level and dynamics, i.e. rising or falling instrument levels over time, of

- resource taxation (tax rate  $\tau$  and its growth rate  $g_\tau$ )
- labor subsidization (subsidy rate  $s_w$  and its growth rate  $g_{s_w}$ )
- differentiated research subsidization ( $s_n$  and  $g_{s_n}$ ,  $s_m$  and  $g_{s_m}$ )
- differentiated provision of productive public goods (shares  $\mu_x$ , resp.  $\mu_z$ , of consumption and growth rates  $g_{\mu_x}$ , resp.  $g_{\mu_z}$ ).

The analysis of the policy instruments is conducted in two steps. First, the traditional instruments, i.e. taxes and subsidies, are analyzed in Subsection 4.1 while the provision of public goods is treated in the Subsection 4.2.

### 4.1 Policy analysis 1: Taxes and Subsidies

In the following we consider ad valorem taxes on the input of resources as well as uniform subsidies on labor and differentiated subsidies on research. Research subsidies are in the form of wage subsidies. Coupling research and labor subsidies therefore creates a two-fold impact on research costs.

To clearly distinguish the effects of each instrument, we assume in the following that the government can balance its budget via lump-sum taxation or subsidization of households. In this case, policies are not tied together by budgetary requirements and each instrument can be analyzed independently.

Due to the policy interventions, the BGP values of  $\tilde{\phi}$  and  $L_Z$  modify to:<sup>5</sup>

$$L_Z^{p1*} = \frac{\delta(1 + 2a\rho - a(g_{\bar{s}_m} + g_{\bar{s}_n}))}{\delta + \phi^{\tilde{p}1*}\alpha + \frac{1-\beta}{\beta}\left(\frac{1}{\bar{s}_m} + \frac{\tilde{\phi}^{p1*}}{\bar{s}_n}\right)} \quad (37)$$

$$\tilde{\phi}^{p1*} = \frac{\bar{s}_n}{\bar{s}_m} \frac{A - aD_1}{B - aD_2} \quad (38)$$

with

$$D_1 = \beta(\alpha - \delta)(\rho\delta\beta(\bar{s}_m - 1) + g_{\bar{\tau}}(1 - \beta + \bar{s}_m\beta\delta)) - \bar{s}_m\delta\beta(1 - \beta)(g_{\bar{s}_m} - g_{\bar{s}_n}) + 2(1 - \beta)^2 g_{\bar{s}_n} \quad (39)$$

$$D_2 = -\beta(\alpha - \delta)(\rho\alpha\beta(\bar{s}_n - 1) + g_{\bar{\tau}}(1 - \beta + \bar{s}_n\beta\alpha)) + \bar{s}_n\alpha\beta(1 - \beta)(g_{\bar{s}_m} - g_{\bar{s}_n}) + 2(1 - \beta)^2 g_{\bar{s}_m}. \quad (40)$$

For notational convenience we denote  $\bar{\tau} = 1 + \tau$ ,  $\bar{s}_w = 1 - s_w$  and  $\bar{s}_n = 1 - s_{r_n}$ . We retrieve the no-policy BGP values of the two variables by setting  $\bar{s}_m = \bar{s}_n = 1$  and  $g_{\bar{t}} = g_{\bar{s}_m} = g_{\bar{s}_n} = 0$ .

By proceeding as in the no-policy section, it can furthermore be shown that

$$g_C^{p1*} = \frac{1 - \beta}{\beta} \frac{L_m^{p1*}}{a} - (1 - \delta)(\rho + g_{\bar{\tau}}) \quad (41)$$

$$L_m^{p1*} = \frac{1}{s_m\delta} \frac{1 - \beta}{\beta} L_Z^{p1*} + a(g_{\bar{s}_m} - \rho). \quad (42)$$

For the BGP rate of resource extraction we get

$$g_R^{p1*} = -(\rho + g_{\bar{\tau}}). \quad (43)$$

By employing the above BGP relations we can now derive the comparative statics of the different policy instruments.

**Proposition 2.** *If resource tax rates,  $\tau$ , and labor subsidies,  $s_w$ , are constant over time, they have no impact on long-run growth, resource extraction and the relative sector share. Research subsidies,  $s_m$  and  $s_n$ , affect growth as well as the relative sector share even if their rates are constant over time. The direction of the effects depends on the sector subsidized:*

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{ds_n} &< 0, & \frac{dg_C^{p1*}}{ds_n} &> 0, \\ \frac{d\tilde{\phi}^{p1*}}{ds_m} &> 0, & \frac{dg_C^{p1*}}{ds_m} &> 0. \end{aligned} \quad (44)$$

---

<sup>5</sup>For the derivation of the underlying dynamic system, see Appendix B1.

*Tax rates and labor/research subsidy rates that change over time affect long-run growth, resource extraction and the relative sector share as follows:*

$$\begin{aligned}
\frac{d\tilde{\phi}^{p1*}}{dg_\tau} &< 0, & \frac{dg_R^{p1*}}{dg_\tau} &< 0, & \frac{dg_C^{p1*}}{dg_\tau} &< 0, \\
\frac{d\tilde{\phi}^{p1*}}{dg_{sw}} &= 0, & \frac{dg_R^{p1*}}{dg_{sw}} &= 0, & \frac{dg_C^{p1*}}{dg_{sw}} &= 0, \\
\frac{d\tilde{\phi}^{p1*}}{dg_{sn}} &> 0, & \frac{dg_R^{p1*}}{dg_{sn}} &= 0, & \frac{dg_C^{p1*}}{dg_{sn}} &< 0, \\
\frac{d\tilde{\phi}^{p1*}}{dg_{sm}} &< 0, & \frac{dg_R^{p1*}}{dg_{sm}} &= 0, & \frac{dg_C^{p1*}}{dg_{sm}} &< 0.
\end{aligned} \tag{45}$$

*Proof.* Taking the derivatives of  $\tilde{\phi}^{p1*}$ ,  $g_R^{p1*}$  and  $g_C^{p1*}$  as determined by (37), (38), (41), (42) and (43) with respect to the policy variables yields either zero, or the expressions displayed in Appendix B2. ■

Resource taxation only affects long-run growth and sector structure if the rate of taxation changes over time. A constant tax rate has no impact on the economy as neither labor allocation nor resource extraction change. This result corresponds to the literature on exhaustible resources, see e.g. Groth and Schou (2007).

A rising rate of resource taxation affects growth via two channels. An increase in taxation induces the speed of resource extraction to rise. The resulting negative effect on growth is naturally stronger in the more resource intensive sector. To compensate for this stronger resource drag, labor is allocated towards research in this sector. However, the resource extraction effect dominates such that the overall effect remains negative.

Also in intermediates production, labor is reallocated towards the more resource intensive sector. Due to the tax induced faster increase of intermediates' prices in  $z$ -production, the value share of the  $z$ -sector rises which raises profitability and thereby attracts labor from the  $x$ -sector and lowers  $\tilde{\phi}$ .

Labor subsidization has no effect on growth and sector structure - neither via the level of the subsidy rate nor through changes in the subsidization rate. The intuition is, that as labor inputs in all sectors are equally affected by the subsidy, no labor reallocation is induced.

The level of research subsidy rates affects the allocation of labor in our model as it distorts the production cost ratio between intermediates production and research. So, in contrast to the effects found by Groth and Schou (2007) but in line with Grimaud and Rougé (2003) we find growth effects of time invariant policy instruments in a model with essential non-renewable resources. Research subsidies foster growth as they lead to an internalization of spill-overs from knowledge accumulation.



Yet, the positive growth effect of the subsidy rate is, at least partially, overcompensated by a direct negative growth effect of the growth rate of  $s_n$ , resp.  $s_m$ . The economic intuition behind this result follows from no-arbitrage considerations of investment in research. Consider the no-arbitrage condition, (11), for which under research subsidization  $V_i = \frac{aw\bar{s}_i}{i}$ ,  $i = n, m$  and  $g_{V_i} = g_w - g_i + g_{\bar{s}_i}$  hold. This gives for the  $x$ -sector

$$g_w - g_n + g_{\bar{s}_n} = r - \frac{1}{\bar{s}_n} \frac{\Pi_x}{aw/n} \quad (46)$$

and equivalently for sector  $z$ . Comparing (46) to (11) shows that research subsidization affects the no-arbitrage condition via two channels: the level as well as the dynamics of subsidy rates (recall that  $\bar{s}_i = 1 - s_i$  and  $g_{\bar{s}_i} < 0 \Leftrightarrow g_{s_i} > 0$ ). On the one hand, subsidization reduces research costs such that for any level of investment in R&D, more patents can be produced. The induced increase in profitability leads to more research and therefore higher growth. This is the level effect observable on the RHS of (46). On the other hand, if subsidies increase over time, research in the future would be even less costly and profitability even higher. This induces investors to postpone investment which lowers R&D and therefore affects growth negatively. This is the growth effect of subsidization which can be seen on the LHS of (46). A policy maker whose aim is to promote growth should therefore heavily subsidize early on and then reduce subsidization over time. From (37) and (38) it can be seen, however, that as long as research subsidies change over time the economy is not on a BGP.

Due to the sectoral heterogeneity of our economy we are able to study structural effects of policy. The direction in which research subsidies affect the relative market share depends on whether the more or less resource intensive sector is subsidized. Subsidies to research in the less resource intensive sector ( $s_n$ ) induce the relative sector size of this sector to decrease - and vice versa for the more resource intensive sector. The line of reasoning is equivalent to the case of resource taxation presented above.

Research subsidization exerts no effect on resource extraction in our model. Although the interest rate and therefore the growth rate of the resource price change due to subsidization, the rate of extraction remains unaltered as income and substitution effects of interest rate changes on the savings decision of households cancel.

## 4.2 Policy analysis 2: Productive public goods

As a second policy option we consider to foster sustainable development by financing activities that enhance the productivity of resources in either one or both sectors. The productivity improvement can, for example, result from investing in the public provision of sector specific infrastructure or fundamental productive knowledge.

For simplicity we again assume that the financial revenues necessary are generated via lump-sum taxation. It is further assumed that the share of consumption used for public good provision is equal to the amount of public goods  $G_k$ ,  $k = x, z$ , produced from this share, i.e.  $G_k = \mu_k C$ ,  $\mu_k < 1$ . The dynamic system is derived in Appendix C1.

From this system the new equilibrium values of the relative sector share and labor input in  $z$ -intermediates can be derived as

$$L_Z^{p2*} = \frac{\delta(1 + 2a\rho)}{\delta + \phi^{\tilde{p}2*}\alpha + \frac{1-\beta}{\beta}(1 + \tilde{\phi}^{p2*})} \quad (47)$$

$$\tilde{\phi}^{p2*} = \frac{\alpha A + aE_1}{\delta B - aE_2} \quad (48)$$

with

$$\begin{aligned} E_1 &= (1 - \beta(1 - \delta))[\beta((1 - \delta)\alpha g_{\mu_z} - (1 - \alpha)\delta g_{\mu_z}) + (\alpha - \delta)(\alpha\beta - 1)\rho] \\ E_2 &= (1 - \beta(1 - \alpha))[\beta((1 - \delta)\alpha g_{\mu_z} - (1 - \alpha)\delta g_{\mu_z}) + (\alpha - \delta)(\delta\beta - 1)\rho]. \end{aligned}$$

Note that setting  $g_\mu = 0$  does not replicate the no-policy equilibrium in this case. It can furthermore be shown by proceeding as in the no-policy section that

$$g_C^{p2*} = \frac{1}{\delta} \left( \frac{1-\beta}{\beta} \frac{L_m^{p2*}}{a} - (1 - \delta)\rho \right) + \frac{1-\delta}{\delta} g_{\mu_z} \quad (49)$$

$$L_m^{p2*} = \frac{1}{\delta} \frac{1-\beta}{\beta} L_Z^{p2*} - a\rho \quad (50)$$

where the functional forms of (49) and (50) are identical to the equilibrium conditions for  $g_C^*$  and  $L_m^*$  in the no policy scenario. For the BGP rate of resource extraction we get

$$g_R^{p2*} = -\rho. \quad (51)$$

**Proposition 3.** *The provision of public goods raises growth independently of the level of the consumption share devoted to productive public spending,  $\mu_k$ ,  $k = x, z$ .*

*Proof.* For the positive effect of public good provision on  $g_C^{p2*}$  see Appendix D. This positive effect is independent of the level of  $\mu_k$  as it follows from (47) to (50) that  $\frac{dg_C^{p2*}}{d\mu_x} = \frac{dg_C^{p2*}}{d\mu_z} = 0$ . ■

For the economic intuition behind this result, consider the case in which the policy maker provides public goods to the less resource intensive sector only. In this case, the feed-back effect of  $x$ -production on the provision of public goods is equivalent to a rise

in  $x$ -sector productivity. This increase in productivity induces a slower increase of intermediates' prices in  $x$ -production which lowers profitability and leads to a reallocation of labor from  $x$ - to  $z$ -sector research. Due to the increase in  $z$ -sector research, growth rises. In the  $x$ -sector, the reallocation of labor reduces research which affects growth negatively. But, in the aggregate this negative effect is overcompensated by the productivity increase due to public good provision.

For no-policy balanced growth (Subsection 3.1) we showed that in equilibrium the difference in research activities between the two sectors is determined by (30). This relation remained unperturbed by the taxes and subsidies considered in the previous subsection as neither affect production technologies directly. In the case of public good provision, however, the productivity of intermediate goods' production increases due to policy. The new equilibrium allocation of research efforts is determined by:

$$\frac{1-\beta}{\beta} \left[ \frac{g_m^{p2*}}{\delta} - \frac{g_n^{p2*}}{\alpha} \right] = \left( \frac{1-\delta}{\delta} - \frac{1-\alpha}{\alpha} \right) \rho - \frac{1-\alpha}{\alpha} g_{\mu_x} + \frac{1-\delta}{\delta} g_{\mu_z} \quad (52)$$

Comparing (30) and (52) shows that the gap between research in the two sectors might increase or decrease due to productive public spending, depending on the model calibration and policy rule.

Employing the BGP relations, (47) to (51), we get the comparative statics of the different policy instruments.

**Proposition 4.** *A constant share of consumption devoted to productive public spending  $\mu_k$ ,  $k = x, z$ , has no impact on long-run growth, resource extraction and the relative sector share. If, however,  $\mu_k$  changes over time, the BGP values of  $\tilde{\phi}$ ,  $g_R$  and  $g_C$  are affected as follows:*

$$\begin{aligned} \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_x}} &< 0, & \frac{dg_R^{p2*}}{dg_{\mu_x}} &= 0, & \frac{dg_C^{p2*}}{dg_{\mu_x}} &> 0, \\ \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_z}} &> 0, & \frac{dg_R^{p2*}}{dg_{\mu_z}} &= 0, & \frac{dg_C^{p2*}}{dg_{\mu_z}} &> 0. \end{aligned} \quad (53)$$

*Proof.* Taking the derivatives of  $\tilde{\phi}^{p2*}$ ,  $g_R^{p2*}$  and  $g_C^{p2*}$ , as given by (47) to (51), with respect to  $\mu_k$  and  $g_{\mu_k}$  yields either zero or the expressions displayed in Appendix C2. ■

A constant share of consumption devoted to public goods only has a level effect on consumption but does not affect growth. If, however, the share rises over time, this affects growth positively. The rising share of public goods provision lowers profitability in the respective sector which leads to a reallocation of labor to the other sector. Due to the increase in the research of this sector, growth rises. In the other sector, research efforts

decline, but in the aggregate the induced negative growth effect is again overcompensated by continuing productivity increases. The provision of public goods proves to be an effective tool to enhance growth and simultaneously induce sectoral change.

## 5 Conclusions

The paper derives the long-run consequences of sectoral heterogeneity when sectors differ with respect to resource use. We have shown that sector-specific research activities and induced innovations are crucial for the dynamic behavior of the economy. Research has to overcome the drag on growth that arises from rising resource scarcity. Moreover, resource intensive sectors can only stay competitive if they succeed to conduct faster research growth. According to our results, the markets provide the incentives that this indeed happens. Consequently, along the balanced growth path, research growth is higher in sectors that depend more on resources.

In the second part of the paper we analyzed the consequences of different policies aiming at fostering sectoral growth and sustainability, i.e. raising growth and lowering resource extraction. First, we considered the implications of traditional policy instruments: subsidies and taxes. It was shown that resource taxes only raise growth and lower resource extraction if the tax rate decreases over time. Labor subsidies, however, are allocation neutral in our model and do not generate any real effects. Subsidies on research activities proved to be more effective, with the level of subsidy rates affecting growth positively – independent of which sector receives the subsidies. Structural effects of policy arise as the effect of research subsidization on market shares depends on which sector is subsidized. Subsidies to research in one sector induce the relative sector share of this sector to decrease.

Secondly, we considered the provision of productive public goods as a possible means to raise sectoral and overall growth. We showed that the introduction of public goods affects growth directly when public good provision is tied to overall consumption. In this case, productivity in the sector in which public goods are provided rises and thereby affects growth as well as sector shares. Increasing the share of consumption devoted to public goods over time, induces a further positive effect on growth.

If, however, the share rises over time, this affects growth positively. The rising share of public goods provision lowers profitability in the respective sector which leads to a reallocation of labor to the other sector. Due to the increase in the research of this sector, growth rises. In the other sector, research efforts decline, but in the aggregate the induced negative growth effect is again overcompensated by continuing productivity increases. The provision of public goods proves to be an effective tool to enhance growth and simultaneously induce sectoral change.

The present research could be extended by assuming that the two research sectors are (incomplete) substitutes. In this case the asymmetry between, e.g., the provision of public knowledge and a disproportionate investment in sector specific research would be different and eventually smaller. This could be modeled in terms of different risks in the two sectors, which, however, is left for future research.

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## 6 Appendix

### A. No policy scenario

#### A1. Derivation of dynamic system

To derive the equation of motion for  $L_Z$ , (17), substitute intermediates profits, (7), into the no-arbitrage condition for the patent market, (11), which gives

$$g_{V_n} = r - (1 - \beta) \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{V_n n} \quad (54)$$

for sector  $x$ .

The equilibrium condition for the research sector, (10), implies  $g_{V_n} = g_w - g_n$ . Substituting the latter as well as (10) into (54) and considering furthermore that from (5) we know that  $\frac{C}{w} \frac{1 + \tilde{\phi}}{\tilde{\phi}} = \frac{L_X}{\alpha \beta}$  gives

$$g_w - g_n = r - \frac{1 - \beta}{a} \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{w} = r - \frac{1 - \beta}{a \beta} \frac{L_X}{\alpha}. \quad (55)$$

As (5) implies  $g_w = g_C - g_{L_X} + \frac{1}{1 + \tilde{\phi}} g_{\tilde{\phi}}$  and we have (16) from consumer optimization, (55) can be rewritten as

$$g_C - g_{L_X} + \frac{1}{1 + \tilde{\phi}} g_{\tilde{\phi}} - g_n = g_C + \rho - \frac{1 - \beta}{a \beta} \frac{L_X}{\alpha}. \quad (56)$$

From (5) we also know  $L_X = \frac{\alpha}{\delta} \tilde{\phi} L_Z$  which implies  $g_{L_X} = g_{\tilde{\phi}} + g_{L_Z}$ . Employing these relations as well as (8) gives for (56)

$$\frac{L_n}{a} = \tilde{\phi} \frac{1 - \beta}{a \beta} \frac{L_Z}{\delta} - \rho - \frac{\tilde{\phi}}{1 + \tilde{\phi}} g_{\tilde{\phi}} - g_{L_Z}. \quad (57)$$

Proceeding equivalently, we get from the no-arbitrage condition of sector  $z$ , (11), that

$$\frac{L_m}{a} = \frac{1 - \beta}{a \beta} \frac{L_Z}{\delta} - \rho - \frac{\tilde{\phi}}{1 + \tilde{\phi}} g_{\tilde{\phi}} - g_{L_Z}. \quad (58)$$

Adding (57) to (58) and rearranging gives

$$2g_{L_Z} = (1 + \tilde{\phi}) \frac{1 - \beta}{a \beta} \frac{L_Z}{\delta} - \frac{L_m + L_n}{a} - 2 \frac{\tilde{\phi}}{1 + \tilde{\phi}} g_{\tilde{\phi}} - 2\rho. \quad (59)$$

By considering that from the equilibrium condition for the labor market, (9), it follows that  $L_n + L_m = 1 - (1 + \frac{\alpha}{\delta} \tilde{\phi}) L_Z$ , we finally get (17):

$$\dot{L}_Z = \left[ \frac{1}{2a\delta} \left( \alpha + \delta \tilde{\phi} + (1 + \tilde{\phi}) \frac{1 - \beta}{\beta} \right) L_Z - \frac{1}{2a} (1 + 2a\rho) - \frac{\tilde{\phi}}{1 + \tilde{\phi}} g_{\tilde{\phi}} \right] L_Z. \quad (60)$$

To get an expression for  $\dot{\tilde{\phi}}$  first consider that from (3) and the production technologies in the two sectors follows

$$\tilde{\phi}^{\frac{\nu}{\nu-1}} = \left(\frac{n}{m}\right)^{\frac{1-\beta}{\beta}} \frac{L_X^\alpha R_X^{1-\alpha}}{L_Z^\delta R_Z^{1-\delta}}. \quad (61)$$

Consideration of  $L_X = \frac{\alpha}{\delta}\tilde{\phi}L_Z$  and  $R_X = \frac{1-\alpha}{1-\delta}\tilde{\phi}R_Z$  gives

$$\tilde{\phi}^{\frac{1}{\nu-1}} = \left(\frac{n}{m}\right)^{\frac{1-\beta}{\beta}} \left[ \frac{\alpha^\delta (1-\alpha)^{1-\delta}}{\delta^\delta (1-\delta)^{1-\delta}} \right] L_X^{\alpha-\delta} R_X^{\delta-\alpha}. \quad (62)$$

Differentiating (62) with respect to time and expressing the resulting expression in growth rates gives after substituting  $g_n = \frac{L_n}{a}$  and  $g_m = \frac{L_m}{a}$

$$\frac{1}{\nu-1}g_{\tilde{\phi}} = \frac{1-\beta}{a\beta}(L_n - L_m) + (\alpha - \delta)(g_{L_X} - g_{R_X}). \quad (63)$$

For the difference in the input of labor in the two types of R&D it follows from (57) and (58) that

$$L_n - L_m = (\tilde{\phi} - 1) \frac{1-\beta}{\beta} \frac{L_Z}{\delta}. \quad (64)$$

Furthermore, (5), (14) and (16) imply that  $g_{R_X} = \frac{1}{1+\tilde{\phi}}g_{\tilde{\phi}} - \rho$ . By substituting this relation as well as (64) into (63), we get

$$\frac{1}{\nu-1}g_{\tilde{\phi}} = -(1-\tilde{\phi})\frac{1}{\delta a} \left(\frac{1-\beta}{\beta}\right)^2 L_Z + (\alpha - \delta)(g_{L_X} - \frac{\tilde{\phi}}{1+\tilde{\phi}}g_{\tilde{\phi}} + \rho). \quad (65)$$

Recalling  $g_{L_X} = g_{\tilde{\phi}} + g_{L_Z}$  and (17) finally gives (18):

$$\dot{\tilde{\phi}} = \left[ \frac{\left( \frac{1}{2a\delta}(\alpha - \delta) \left( \alpha + \delta\tilde{\phi} + (1+\tilde{\phi})\frac{1-\beta}{\beta} \right) - (1-\tilde{\phi})\frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 \right) L_Z - \frac{1}{2a}(\alpha - \delta)}{\frac{1}{\nu-1}} \right] \tilde{\phi}. \quad (66)$$

## A2. Balanced growth path

From (60) and (66) the BGP values of  $L_Z$  and  $\tilde{\phi}$  can be obtained by setting  $\dot{L}_Z = \dot{\tilde{\phi}} = 0$  which gives (19) and (20). Considering furthermore that  $\dot{L}_Z = 0$ , we get the BGP labor shares in the two research sectors, (24) and (25), from (57) and (58).

## B. Policy analysis 1: subsidies and taxes

### B1. Derivation of dynamic system

The policy maker can employ three types of instruments: resource taxes, research subsidies and labor subsidies. The governmental budget constraint reads

$$\tau p_R R = s_m w L_m + s_n w L_n + s_w w + T, \quad s_n, s_m, s_w < 1 \quad (67)$$



where  $\tau$ ,  $s_n$ ,  $s_m$  denote the resource tax rate and the subsidy rates on  $x$ - and  $z$ -sector research respectively.  $s_w$  is the subsidy rate on labor.  $T$  denotes lump-sum taxation or subsidization of households that balance the government's budget at every point in time.

The profit function of the individual intermediate producer in the  $x$ -sector reads after taxation and subsidization

$$\Pi_{x_i} = p_{x_i}x_i - \bar{\tau}p_R R_{x_i} - \bar{s}_w w L_{x_i} \quad (68)$$

and equivalent for producers in sector  $z$ . Please note that for notational convenience we denote  $\bar{\tau} = 1 + \tau$  and  $\bar{s}_w = 1 - s_w$ . It is assumed that individual producers do not take account of the effect of their production on public good provision, such that the modified first-order conditions for labor and resource input are given by

$$L_{x_i} = \alpha\beta \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{\bar{s}_w w} \quad \text{and} \quad R_{x_i} = \alpha\beta \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{\bar{\tau}p_R} \quad (69)$$

and firms' equilibrium profits are still equal to (7).

The research firms' profit functions in case of labor *and* research subsidies read

$$\Pi_l = p_{V_l} \dot{l} - \bar{s}_l \bar{s}_w w L_l, \quad l = n, m \quad (70)$$

where  $\bar{s}_l = 1 - s_l$  and it is assumed that the research subsidy is paid on the basis of the wage bill after labor subsidization. In equilibrium the value of a patent has again to be equalized to marginal costs, such that

$$V_l = \frac{aw\bar{s}_l\bar{s}_w}{l}. \quad (71)$$

Proceeding as in Appendix A1 we get a modified system of differential equations that describe the system's dynamics:

$$\begin{aligned} \dot{L}_Z &= \left[ \frac{1}{2a\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} \left( \frac{1}{\bar{s}_m} + \frac{\tilde{\phi}}{\bar{s}_n} \right) \right) L_Z - \rho - \frac{1}{2a} + \frac{1}{2}(g_{\bar{s}_m} + g_{\bar{s}_n}) - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} \right] L_Z \quad (72) \\ \dot{\tilde{\phi}} &= \left[ \left( \frac{1}{2a} \frac{\alpha - \delta}{\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} \left( \frac{1}{\bar{s}_m} + \frac{\tilde{\phi}}{\bar{s}_n} \right) \right) - \frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 \left( \frac{1}{\bar{s}_m} - \frac{\tilde{\phi}}{\bar{s}_n} \right) \right) L_Z \right. \\ &\quad \left. - \frac{1}{2a}(\alpha - \delta) - \frac{1-\beta}{\beta}(g_{\bar{s}_m} - g_{\bar{s}_n}) + \frac{1}{2}(\alpha - \delta)(g_{\bar{s}_m} + g_{\bar{s}_n} + 2g_{\tilde{\tau}}) \right] (\nu - 1)\tilde{\phi}. \quad (73) \end{aligned}$$

The BGP values of  $\tilde{\phi}$  and  $L_Z$ , (37) and (38), follow from (72) and (73) by considering that along the balanced path  $g_{\tilde{\phi}} = g_{L_Z} = 0$ . The system is again saddle-path stable for  $\nu < 1$ .

## B2. Comparative statics

Using the BGP values of  $\tilde{\phi}$ ,  $g_R$  and  $g_C$  we can derive the comparative statics results for the three policy instruments where we denote<sup>6</sup>

$$\begin{aligned} G_1 &= aD_1 - A < 0 \\ G_2 &= \frac{\bar{s}_n}{\bar{s}_m} \frac{a(1-\beta)^2(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))}{G_1^2} > 0. \end{aligned}$$

As  $\bar{s}_i = 1 - s_i$ ,  $i = n, m$ , and  $\bar{\tau} = 1 + \tau$  we get  $g_{\bar{s}_i} = -\frac{s_i}{1-s_i}g_{s_i}$  and  $g_{\bar{\tau}} = \frac{\tau}{1+\tau}g_{\tau}$  such that  $\frac{d\bar{s}_i}{ds_i} = -1$  and  $\frac{d\bar{\tau}}{d\tau} = 1$  as well as  $\frac{dg_{\bar{s}_i}}{dg_{s_i}} < 0$  and  $\frac{dg_{\bar{\tau}}}{dg_{\tau}} > 0$ . The comparative statics results are given by

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{dg_{\tau}} &= -\beta\frac{\alpha-\delta}{\delta}L_Z^{p1*}\left(\delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}\left(\frac{1}{s_{\bar{s}_m}} + \frac{\tilde{\phi}^{p1*}}{s_{\bar{s}_n}}\right)\right)G_2\frac{dg_{\bar{\tau}}}{dg_{\tau}} < 0 \\ \frac{dg_R^{p1*}}{dg_{\tau}} &= -\frac{dg_{\bar{\tau}}}{dg_{\tau}} < 0 \\ \frac{dg_C^{p1*}}{dg_{\tau}} &= \left(\left(\frac{\frac{1-\beta}{\beta} + \alpha s_n}{2\frac{1-\beta}{\beta} + (\alpha s_n + \delta s_m)}\right)(\alpha - \delta) - (1 - \delta)\right)\frac{dg_{\bar{\tau}}}{dg_{\tau}} < \left(\frac{\frac{1-\beta}{\beta} + \alpha s_n}{2\frac{1-\beta}{\beta} + (\alpha s_n + \delta s_m)} - 1\right)(1 - \delta)\frac{dg_{\bar{\tau}}}{dg_{\tau}} < 0 \end{aligned}$$

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{dg_{s_m}} &= ((1-\beta)(1+2a\rho - 2ag_{\bar{s}_n}) - a\beta(\alpha-\delta)(\rho + g_{\bar{\tau}}))G_2\frac{dg_{\bar{s}_m}}{dg_{s_m}} < 0 \\ \frac{dg_C^{p1*}}{dg_{s_m}} &= \frac{(1-\beta)\delta\bar{s}_m}{2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m)}\frac{dg_{\bar{s}_m}}{dg_{s_m}} < 0 \end{aligned}$$

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{ds_m} &= (1-\beta)\frac{\bar{s}_n}{\bar{s}_m^2 G_1 G_2}\frac{d\bar{s}_m}{ds_m} > 0 \\ \frac{dg_C^{p1*}}{ds_m} &= \frac{\delta G_1}{a(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))}\frac{d\bar{s}_m}{ds_m} > 0 \end{aligned}$$

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{dg_{s_n}} &= (-(1-\beta)(1+2a\rho - 2ag_{\bar{s}_n}) - a\beta(\alpha-\delta)(\rho + g_{\bar{\tau}}))G_2\frac{dg_{\bar{s}_n}}{dg_{s_n}} > 0 \\ \frac{dg_C^{p1*}}{dg_{s_n}} &= \frac{(1-\beta)\alpha s_n}{2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m)}\frac{dg_{\bar{s}_n}}{dg_{s_n}} < 0 \end{aligned}$$

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<sup>6</sup> $G_1 = aD_1 - A < 0$  holds as it can be shown that  $L_Z^{p1*} = \frac{\beta\delta s_m}{(1-\beta)^2(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))}(-G_1)$  which is positive for an interior equilibrium, such that  $G_1 < 0$ .

$$\begin{aligned}\frac{d\tilde{\phi}^{p1*}}{ds_n} &= ((1-\beta)\frac{1}{\bar{s}_n G_1 G_2} \frac{d\tilde{\phi}^{p1*}}{d\bar{s}_n} \frac{d\bar{s}_n}{ds_n}) < 0 \\ \frac{d\tilde{g}_C^{p1*}}{ds_n} &= \frac{s_m}{s_n} \frac{\alpha \tilde{\phi}^{p1*} G_1}{a(2(1-\beta) + \beta(\alpha \bar{s}_n + \delta \bar{s}_m))} \frac{d\bar{s}_n}{ds_n} > 0.\end{aligned}$$

Note that  $M = [(1-\beta)(1+2a\rho - 2ag_{\bar{s}_n}) - a\beta(\alpha - \delta)(\rho + g_{\bar{\tau}})] > 0$ , as claimed for  $\frac{d\tilde{\phi}^{p1*}}{d\bar{s}_m} < 0$ , can be proofed as follows: It was shown that  $G_1 = aD_1 - A < 0$  for  $L_Z^{p1*} > 0$  (see Footnote 6). From  $\tilde{\phi}^{p1*} = \frac{\bar{s}_n}{\bar{s}_m} \frac{A-aD_1}{B-aD_2} > 0$ , this implies that also  $B - aD_2 > 0$ . Now it can be shown that

$$(A - aC) - (1 - \beta)M = -a\beta\delta\bar{s}_m K \quad (74)$$

$$(B - aD) - (1 - \beta)M = a(2(1 - \beta) + \alpha\beta\bar{s}_n)K \quad (75)$$

with  $K = ((1 - \beta)(g_{\bar{s}_n} - g_{\bar{s}_m}) + (\alpha - \delta)(g_{\bar{\tau}} + \rho))$ . As  $A - aD_1 > 0$  and  $B - aD_2 > 0$ , it follows from (74) and (75) that  $M < 0$  is not feasible, as in this case RHSs of the above two equations would have to be simultaneously positive.

## C. Policy analysis 2: productive public goods

### C1. Derivation of dynamic system

If public goods provided are provided to foster the productivity of resources, the modified production function for  $x_i$  and  $z_j$  are given by

$$x_i = L_{x_i}^\alpha (G_x R_{x_i})^{1-\alpha} \quad \text{and} \quad z_j = L_{z_j}^\delta (G_z R_{z_j})^{1-\delta}. \quad (76)$$

Considering that in equilibrium  $x_i = x$  and  $z_j = z$  hold, aggregate production of  $\tilde{X}$  and  $\tilde{Z}$  are

$$\tilde{X} = n^{\frac{1-\beta}{\beta}} L_X^\alpha (\mu_x C R_X)^{1-\alpha} \quad \text{and} \quad \tilde{Z} = m^{\frac{1-\beta}{\beta}} L_Z^\delta (\mu_z C R_Z)^{1-\delta}. \quad (77)$$

To endogenize  $C$ , express (2) in terms of  $\tilde{X}$ , resp.  $\tilde{Z}$ , only. Recall that  $\tilde{Z} = \tilde{\phi}^{\frac{\nu}{1-\nu}} \tilde{X}$  follows from (3), such that (2) reads

$$C = \left( \frac{1 + \tilde{\phi}}{\tilde{\phi}} \right)^{\frac{\nu}{\nu-1}} \tilde{X} = \left( 1 + \tilde{\phi} \right)^{\frac{\nu}{\nu-1}} \tilde{Z}. \quad (78)$$

Inserting (78) into (77) and rearranging gives

$$\tilde{X} = n^{\frac{1-\beta}{\beta} \frac{1}{\alpha}} L_X R_X^{\frac{1-\alpha}{\alpha}} \mu_x^{\frac{1-\alpha}{\alpha}} \left( \frac{1 + \tilde{\phi}}{\tilde{\phi}} \right)^{\frac{\nu}{\nu-1} \frac{1-\alpha}{\alpha}}. \quad (79)$$

$$\tilde{Z} = m^{\frac{1-\beta}{\beta} \frac{1}{\delta}} L_Z R_Z^{\frac{1-\delta}{\delta}} \mu_z^{\frac{1-\delta}{\delta}} \left( 1 + \tilde{\phi} \right)^{\frac{\nu}{\nu-1} \frac{1-\delta}{\delta}}. \quad (80)$$

Again proceeding as in the no-policy section we derive the modified dynamic system of this economy:

$$\dot{L}_Z = \left[ \frac{1}{2a\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} (1 + \tilde{\phi}) \right) L_Z - \rho - \frac{1}{2a} - \frac{\tilde{\phi}}{1 + \tilde{\phi}} g_{\tilde{\phi}} \right] L_Z \quad (81)$$

$$\begin{aligned} \dot{\tilde{\phi}} = & \frac{(\nu - 1)(1 + \tilde{\phi})}{\alpha(1 - 2\nu)(1 + \tilde{\phi}) + \alpha - \delta} \left[ (\alpha - \delta) \left( \frac{1}{2a} \frac{1 - \beta}{\beta} - \rho \right) + (\alpha(1 - \delta)g_{\mu_z} - \delta(1 - \alpha)g_{\mu_x}) \right. \\ & \left. - \frac{1}{2a\delta} \frac{1 - \beta}{\beta} \left( \left( \frac{1 - \beta}{\beta} (\alpha + \delta) - \alpha\delta \right) (1 - \tilde{\phi}) - (\alpha^2\tilde{\phi} - \delta^2) \right) L_Z \right] \tilde{\phi}. \end{aligned} \quad (82)$$

The BGP values of  $\tilde{\phi}$  and  $L_Z$ , (47) and (48), follow from (81) and (82) by considering that along the balanced path  $g_{\tilde{\phi}} = g_{L_Z} = 0$ . As in the policy scenario 1 and the no-policy case, the system is saddle-path stable for  $\nu < 1$ .

## C2. Comparative statics

Taking the derivatives of  $\tilde{\phi}^{p2*}$ ,  $g_R^{p2*}$  and  $g_C^{p2*}$ , as given by (47) to (51), with respect to  $\mu_k$  and  $g_{\mu_k}$  gives:

$$\begin{aligned} \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_x}} &= -(1 - \alpha)\delta \frac{a^2\beta^2(1 - \beta)(1 + 2a\rho)}{H_1H_2} < 0 \\ \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_z}} &= (1 - \delta)\alpha \frac{a^2\beta^2(1 - \beta)(1 + 2a\rho)}{H_1H_2} > 0 \\ \frac{dg_C^{p2*}}{dg_{\mu_x}} &= \frac{1 - \beta}{\beta} \frac{1 - \alpha}{a} (1 - \beta + \alpha\beta)H_2 > 0 \\ \frac{dg_C^{p2*}}{dg_{\mu_z}} &= \frac{1 - \beta}{\beta} \frac{1 - \delta}{a} (1 - \beta + \delta\beta)H_2 > 0 \end{aligned}$$

with

$$\begin{aligned} H_1 &= (\delta B - aE_2)^2 > 0 \\ H_2 &= \frac{a\beta}{(1 - \beta)((\alpha + \delta)(1 - \beta) + \beta(\alpha^2 + \delta^2))} > 0. \end{aligned}$$

## D. Proof of Proposition 3

Due to the productivity effect of public goods, labor inputs in  $x$ - and  $z$ -sector research change as follows compared to the no-policy scenario (assuming that  $g_{\mu_k} = 0$ ,  $k = x, z$ ):

$$L_n^{p2*} - L_n^* = (1 - \beta + \delta\beta)\Omega \quad (83)$$

$$L_m^{p2*} - L_m^* = -(1 - \beta + \alpha\beta)\Omega \quad (84)$$

with  $\Omega = f(a, \alpha, \beta, \delta, \rho, g_{\mu_z}, g_{\mu_x})$ . From (83) and (84) it follows that

$$\text{sgn}(L_n^{p2*} - L_n^*) = -\text{sgn}(L_m^{p2*} - L_m^*), \quad (85)$$

i.e. a policy induced rise in  $L_n$  (resp.  $L_m$ ) has to be accompanied by a decline of  $L_m$  (resp.  $L_n$ ).

Furthermore, public good provision modifies the sectoral equilibrium growth rates (28) and (29) to

$$g_{\tilde{X}}^{p2*} = \frac{1}{\alpha} \left( \frac{1 - \beta}{\beta} \frac{L_n^{p2*}}{a} - (1 - \alpha)\rho \right) \quad (86)$$

$$g_{\tilde{Z}}^{p2*} = \frac{1}{\delta} \left( \frac{1 - \beta}{\beta} \frac{L_m^{p2*}}{a} - (1 - \delta)\rho \right). \quad (87)$$

If  $L_n^{p2*}$  and  $L_m^{p2*}$  were unchanged compared to the no-policy scenario, this would imply  $g_{\tilde{X}}^* < g_{\tilde{X}}^{p2} < g_{\tilde{Z}}^{p2}$  where the relation  $g_{\tilde{X}}^{p2} < g_{\tilde{Z}}^{p2}$  is not compatible with BGP growth (see (27)). Therefore (86) and (87) together with (85) imply that a post-policy BGP with  $g_{\tilde{X}} = g_{\tilde{Z}} = g_C$  can only be compatible with  $L_n^{p2*} - L_n^* > 0$  and  $L_m^{p2*} - L_m^* < 0$ . From (86) we see that this increase in  $L_n$  raises growth.

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