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The present value model of U.S. stock prices revisited: long-run evidence with structural breaks, 1871–2012

Vicente Esteve, Manuel Navarro, and María A. Prats

Abstract

According to several empirical studies, the Present Value model fails to explain the behaviour of stock prices in the long-run. In this paper, the authors consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the Present Value model of U.S. stock prices. The methodology is based on instability tests recently proposed in Kejriwal and Perron (*The limit distribution of the estimates in cointegrated regression models with multiple structural changes*, 2008, and *Testing for multiple structural changes in cointegrated regression models*, 2010) as well as the cointegration tests developed in Arai and Kurozumi (*Testing for the null hypothesis of cointegration with a structural break*, 2007) and Kejriwal (*Cointegration with structural breaks: an application to the Feldstein-Horioka Puzzle*, 2008). The results obtained are consistent with the existence of linear cointegration between the log stock prices and the log dividends. However, the empirical results also show that the cointegrating relationship has changed over time. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models suggest a model of three or two regimes.

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Keywords Present value model; stock prices; dividends; cointegration; multiple structural breaks

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1 Introduction

One of the central propositions of modern finance theory is the efficient markets hypothesis (EMH), which in its simplest formulation states that the price of an asset at time t should fully reflect all the available information at time t .¹ This has often been tested by using the present value (PV) model of stock prices, since, if stock market return are not forecastable, as implied by the EMH, stock prices should equal the present value of expected future dividends. Over the last decades, the influence of the linear PV model to explain the behavior of aggregate US stock prices has been actively investigated. According to the linear PV model, stock prices are fundamentally determined by the discounted value of their future dividends, which derive their value from future expected earnings (e.g., see Campbell et al., 1997; Cochrane, 2001).

In a series of seminal papers, Leroy and Porter (1981) and Shiller (1981a, 1981b) provide empirical evidence against the linear PV model of stock prices and, consequently, against the EMH.² In conducting their empirical analysis, however, these authors rely on the hypothesis that the underlying data, such as stock prices and dividends, are characterized by stationarity around deterministic trends.

Making use of some recent advances in the econometrics of nonstationary processes, Campbell and Shiller (1987) present new evidence that, again, seems to be unfavorable to the linear PV model of stock prices. In particular, Campbell and Shiller show that real stock prices and dividends, on the hypothesis that are difference stationary, are not cointegrated. This outcome, which effectively rules out the presence of a long-run relationship between real stock prices and dividends, clearly bears negative implications for the PV model, in which dividends are supposed to be the major determinant of stock prices in the long run.

Since the work of Campbell and Shiller (1987), empirical studies of the validity of linear PV model of stock prices have been extensively conducted in the cointegration framework. The cointegration between stock prices and dividends has implications for return predictability, cash-flow predictability and the debate on rational bubbles. However, the empirical evidence is far from conclusive (e.g., see Campbell and Shiller, 1987, Diba and Grossman (1988), Froot and Obstfeld (1991), Craine (1993), Lamont (1998), and Balke and Wohar (2002)). In most studies, standard cointegration tests do not validate the cointegration hypothesis, which implicitly supports the "rational bubbles hypothesis". From a methodological point of view, if we take the long-run validity of the PV model, non-linearities, the low power of standard unit root tests, and structural breaks, are all three possible candidates for explaining persistent deviations from the equilibrium relationship between real stock prices and dividends.

¹See Fama (1970) for a definition of weak, semi-strong and strong efficiency, and Fama (1991) for alternative definitions in terms of return predictability.

²Many recent theoretical models, however, incorporate time-varying expected returns. This means that return predictability can coexist with EMH. See Joijen and Van Nieuwerburgh (2011) for a discussion of this alternative view.

With respect to non-linearities, recent research has found that the relationship between real stock prices and dividends may best be characterized by using a nonlinear PV model; see, e.g., Gallagher and Taylor (2001), Kanas (2003, 2005), Esteve and Prats (2008, 2010), MacMillan (2009), and MacMillan and Wohar (2010). On the other hand, some researchers have argued that the dividend-stock price relationship exhibits fractional cointegration, resulting from the high persistence of temporary deviations from the long run equilibrium between real stock prices and dividends (see, e.g., Caporale and Gil-Alana, 2004, Cuñado et al., 2005, and Koustas and Serletis, 2005). Finally, some empirical studies have used Markov switching models to detect regime shifts in the dividends process (when the cointegrating vector is subject to Markov regime shifts). These models have found the existence of different phases in stock markets, (see, e.g, Bonomo and Garcia , 1994, Schaller and Van Norden, 1997, Driffill and Sola, 1998, Psaradakis, Sola and Spagnolo, 2004, and Sarno and Valente, 2005).

As regard to this last group of studies, their authors have found that the long-run relationship between real stock prices and dividends and/or the dividend-stock price ratio can be potentially subject to regime changes when the following occurs: a) changes in expectations regarding dividends, following persistent temporary shocks to output or productivity (see, e.g., Psaradakis et al, 2004); b) changes in the dividend process itself, reflecting: i) changes in business cycle conditions that determine a more accurate valuation of equity premium as a result of changes in inflation and interest rates (see, e.g., Siegel, 1999, and Fama and French, 2002); ii) changes in corporate behavior, i.e., the switch towards share repurchasing and away from dividend payments in corporate payout policy (see, e.g., Carlson et al., 2002).

The lack of control for structural breaks in the series may be reflected in the parameters of the estimated models that, when used for inference or forecasting, can induce to misleading results. In general, structural breaks are a problem for the analysis of economic series, since they are usually affected by either exogenous shocks or changes in policy regimes. As a consequence, the assumption of stability in the long-run relationship between real stock prices and dividends would seem too restrictive, so that not allowing for structural breaks would be an important potential shortcoming of the past research using cointegration techniques. In our case the long-run relationship between real stock prices and dividends has probably changed due to alterations in monetary and fiscal policy, as well as because of reforms in the financial market and in the regulation of the stock market. Thus, the information content of the linear PV model of stock prices is subject to change over time and all the empirical modeling studies that did not take into account the possible changes and instabilities will fail to explain the variations in the relationship between real stock prices and dividends. Visual examination of these variables (see Figure 1) might allow to think that the presence of some non-recurrent shocks with large magnitude might have affected the evolution of these variables, something that needs to be taken into account when assessing the stochastic properties of time series if meaningful conclusions are to be obtained (see, Perron, 2006).

Such structural changes in the long-run relationship between real stock prices

and dividends and structural breaks in the dividend-price relation have important implications for the return predictability, cash-flow predictability, and the decomposition of the variance of the dividend-price ratio (see, e.g., Lettau and Van Nieuwerburgh, 2008, and Kojen and Van Nieuwerburgh, 2011). In this context, using the approach suggested in Bai and Perron (1998), Lettau and Van Nieuwerburgh (2008) reported evidence for structural shifts in the mean of the dividend-price ratio in 1991 (one break model) and in 1954 and 1994 (two break model). Our results differ from the findings of the Lettau and Van Nieuwerburgh (2008). In particular, the Kejriwal and Perron (2008, 2010) tests for testing multiple structural breaks in cointegrated regression models suggest a model of three regimes, with the dates of the breaks estimated at 1944 and 1971, and a model of two regimes, with the date of the break estimated at 1944. Given that our paper employs longer time period and different econometric techniques than the work of Lettau and Van Nieuwerburgh (2008), our empirical analysis could add to understanding of the impact of structural breaks on stock prices movements.

The purpose of this paper is to advance the evidence on the empirical validity of the linear PV model of stock prices in two ways. In the first place, in order to avoid the econometric problems mentioned above, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use the approach proposed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. These authors develop a sequential procedure that not only enables detection of parameter instability in cointegration regression models but also allows for consistency in the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between real stock prices and dividends is analyzed using the residual based test of the null hypothesis of cointegration with multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008).

In the second place, it is well known that misspecifications due to the non consideration of structural breaks can bias the analysis that is performed using the standard Dickey-Fuller (DF) test statistics for a unit root. Consequently, the analysis of the order of integration has to consider the presence of structural breaks. Firstly, we have used the GLS-based unit root test statistics proposed in Kim and Perron (2009) and extended in Carrion-i-Silvestre et al. (2009) that allows a break at an unknown time under both the null and alternative hypothesis. The commonly used tests of unit root with a structural change in the case of an unknown break date assumes that if a break occurs it does so only under the alternative hypothesis of stationarity. The methodology developed by Kim and Perron (2009) and Carrion-i-Silvestre et al. (2009) solves many of the problems of the standard tests of unit root with a structural change in the case of an unknown break date. Secondly, Harvey et al. (2013) show that the fixed magnitude trend break asymptotic theory of Carrion-i-Silvestre et al. (2009) does not predict well the finite sample power functions of M tests, and power can be very low for the magnitudes of trend breaks typically observed in practice.

In response to this problem Harvey et al. (2013) propose a unit root test that allows for multiple breaks in trend both under the null and the alternative hypotheses, obtained by taking the infimum of the sequence (across all candidate break points in a trimmed range) of local GLS detrended augmented Dickey–Fuller-type statistics, MDF_m . They show that this procedure has power that is robust to the magnitude of any trend breaks, thereby retaining good finite sample power in the presence of plausibly-sized breaks. They also demonstrate that, unlike the OLS detrended infimum tests of Zivot and Andrews (1992), these tests display no tendency to spuriously reject in the limit when fixed magnitude trend breaks occur under the unit root null.

The rest of the paper is organized as follows. A brief description of the underlying theoretical framework is provided in section 2, the methodology and empirical results are presented in sections 3 and 4, respectively, and the main conclusions are summarized in section 5.

2 The standard present value model of stock prices

The basic theoretical framework for the analysis of the PV model of stock prices is analytically discussed in Campbell et al. (1997). Given the assumptions of rational expectations, risk-neutrality and market equilibrium the movement of shares prices over time is given by the PV of future cash flows or the arbitrage condition:

$$P_t = \frac{1}{1+R}(E_t P_{t+1} + E_t D_{t+1}) \quad (1)$$

where P_t is the real price of a share (or real stock price) at time t , D_t is the real dividend paid on the stock in time period t , $1/(1+R)$ is the discount factor, R is the constant expected stock return ($E_t[R_t + 1] = R$) and E_t is the expectations operator conditioned on information up to t .

A solution to equation (1) is provided by imposing the transversality condition (the no bubble condition) and substituting recursively for all future prices. After solving forward K periods and assuming that the expected discounted value of the stock price K periods from the present shrinks to zero as the horizon K increases, we can obtain an equation expressing the value of stock price as the expected value of future dividends out to the infinite future discounted at a constant rate and equal to the required rate of return (i.e., the so-called *dividend discount model* (DDM) of stock prices):

$$P_t = E_t \left[\sum_{k=1}^{\infty} \left(\frac{1}{1+R} \right)^k D_{t+k} \right] \quad (2)$$

The DDM can be used to illustrate the concept of cointegration. Following Campbell and Shiller (1987), if D_t follows a linear process with a unit root, so that ΔD_t is stationary, the stock price P_t will also follow a linear process with a

unit root (ΔP_t is also stationary). In this case, the DDM reflected in equation (2) relates two unit-root processes for P_t and D_t . If we subtract a multiple of the dividend from both sides of (2), we obtain:

$$P_t - \frac{D_t}{R} = \left(\frac{1}{R}\right) E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+R}\right)^i \Delta D_{t+1+i} \quad (3)$$

The left hand side of (3) reflects the difference between the stock price and $(1/R)$ times the dividend, and the right hand side reflects the expected discounted value of the future changes in dividends. If changes in dividends are stationary, then the term of the left hand side (i.e., difference between the stock price and $(1/R)$ times the dividend) should also be stationary. In this case, the DDM of stock prices should hold when stock prices and dividends are cointegrated (there is a linear combination of stock prices and dividends which is stationary), with a known cointegrating vector $(1, 1/R)'$.

Equation (3) is based on the assumption that expected stock returns are constant. However, this assumption contradicts empirical evidence since the latter suggests that stock returns are non predictable. If the expected stock return is time varying, then the exact PV model becomes nonlinear. Campbell and Shiller (1988a, 1988b) and Cochrane and Sbordone (1988) suggested an approximate loglinear PV model for use in this case:

$$p_t = \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j [(1-\rho)d_{t+1+j} - r_{t+1+j}] = \frac{k}{1-\rho} + p_{CF,t} - p_{DR,t} \quad (4)$$

where the lower case letters p , d , r denotes the logarithms of stock prices, dividends and the discount rate respectively, ρ and k are parameters of linearization, and $p_{CF,t}$ and $p_{DR,t}$ are the components of the stock price driven by cash flow (dividend) expectations and discount rate (return) expectations respectively.

We can re-write (4) in terms of the log dividend-price ratio, $d_t - p_t$, (or the log dividend yield, dy_t) as follows:³

$$d_t - p_t = -(k/1-\rho) + E_t \sum_{j=0}^{\infty} \rho^j [-\Delta d_{t+1+j} + r_{t+1+j}] \quad (5)$$

Equation (5) is used to test the DDM of stock prices when log dividends follow a unit root process, so that the log dividends and the log stock prices are nonstationary. In this case, changes in the log dividends are stationary, and from equation (5) the log dividend-price ratio is stationary provided that the expected stock return is stationary. This restriction implies that the log stock prices is a sum of a difference stationary random variable and a stationary random variable.

³The term dividend yield is used interchangeably with the price-dividend ratio, of which is the inverse, since in the literature of the PV model of stock prices the ratios D/P and P/D are both used and both are consistent within the PV model of stock prices context.

Hence the log stock prices is also difference stationary. This restriction also implies that the log stock prices and the log dividends are cointegrated with a known cointegrating vector $(1, -1)'$.

Intuitively, equation (5) states that if future dividends are expected to grow, then current stock prices will be higher and the dividend yield will be low, while if the future discount rate (rate of return) is expected to be high, then current prices will be low and the dividend yield will be high.

In the empirical section, we test the linear DDM of stock prices in the context of cointegration theory, using a log linear model such as:

$$p_t = \alpha + \gamma d_t + \varepsilon_t \quad (6)$$

3 Methodology

3.1 A linear cointegrated regression model with multiple structural changes

Issues related to structural change have received a considerable amount of attention in the statistics and econometrics literature. Bai and Perron (1998) and Perron (2006, 2008) provide a comprehensive treatment of the problem of testing for multiple structural changes in linear regression models. Accounting for parameter shifts is crucial in cointegration analysis since it normally involves long spans of data which are more likely to be affected by structural breaks. In particular, Kejriwal and Perron (2008, 2010) provide a comprehensive treatment of the problem of testing for multiple structural changes in cointegrated systems.

More specifically, Kejriwal and Perron (2008, 2010) consider a linear model with m multiple structural changes (i.e., $m + 1$ regimes) such as:

$$y_t = c_j + z'_{ft} \delta_f + z'_{bt} \delta_{bj} + x'_{ft} \beta_f + x'_{bt} \beta_{bj} + u_t \quad (t = T_{j-1} + 1, \dots, T_j) \quad (7)$$

for $j = 1, \dots, m + 1$, where $T_0 = 0$, $T_{m+1} = T$ and T is the sample size. In this model, y_t is a scalar dependent I(1) variable, $x_{ft}(p_f \times 1)$ and $x_{bt}(p_b \times 1)$ are vectors of I(0) variables while $z_{ft}(q_f \times 1)$ and $z_{bt}(q_b \times 1)$ are vectors of I(1) variables.⁴ The break points (T_1, \dots, T_m) are treated as unknowns.

The general model (7) is a partial structural change model in which the coefficients of only a subset of the regressors are subject to change. In our case, we suppose that $p_f = p_b = q_f = 0$, and the estimated model is a pure structural change model with all coefficients of the I(1) regressors and constant (slope and the intercept in (6)) allowed to change across regimes:

$$y_t = c_j + z'_{bt} \delta_{bj} + u_t \quad (t = T_{j-1} + 1, \dots, T_j) \quad (8)$$

⁴The subscript b stands for "break" and the subscript f stands for "fixed" (across regimes).

Generally, the assumption of strict exogeneity is too restrictive and therefore the test statistics for testing multiple breaks are not robust to the problem of endogenous regressors. To deal with the possibility of endogenous I(1) regressors, Kejriwal and Perron (2008, 2010) propose to use the so-called dynamic OLS regression (DOLS) where leads and lags of the first-differences of the I(1) variables are added as regressors, as suggested by Saikkonen (1991) and Stock and Watson (1993):

$$y_t = c_i + z'_{bt} \delta_{bj} + \sum_{j=-l_T}^{l_T} \Delta z'_{bt-j} \Pi_{bj} + u_t^*, \quad \text{if } T_{i-1} < t \leq T_i \quad (9)$$

for $i = 1, \dots, k+1$, where k is the number of breaks, $T_0 = 0$ and $T_{k+1} = T$.

3.2 Structural Break Tests

In this paper we test the parameter instability in cointegration regression using the tests proposed in Kejriwal and Perron (2008, 2010). They present issues related to structural changes in cointegrated models which allow both I(1) and I(0) regressors as well as multiple breaks. They also propose a sequential procedure which permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

Kejriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a *sup Wald* test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks ($m = k$):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda_\epsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \quad (10)$$

where SSR_0 denotes the sum of squared residuals under the null hypothesis of no breaks, SSR_k denotes the sum of squared residuals under the alternative hypothesis of k breaks, $\lambda = \{\lambda_1, \dots, \lambda_m\}$ is the vector of breaks fractions defined by $\lambda_i = T_i/T$ for $i = 1, \dots, m$, T_i are the break date, and where $\hat{\sigma}^2$ is:

$$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^T \tilde{u}_t^2 + 2T^{-1} \sum_{j=1}^{T-1} \varpi(j/\hat{h}) \sum_{t=j+1}^T \tilde{u}_t \tilde{u}_{t-j} \quad (11)$$

and $\tilde{u}_t (t = 1, \dots, T)$ are the residuals from the model estimated under the null hypothesis of no structural change. Also, for some arbitrary small positive number ϵ , $\Lambda_\epsilon = \{\lambda : |\lambda_{i+1} - \lambda_i| \geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \leq 1 - \epsilon\}$.

Second, they consider a test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there is an unknown number of breaks, given some upper bound $M (1 \leq m \leq M)$:

$$UD \max F_T^*(M) = \max_{1 \leq k \leq m} F_T^*(k) \quad (12)$$

In addition to the tests above, Kejriwal and Perron (2010) consider a sequential test of the null hypothesis of k breaks versus the alternative hypothesis of $k + 1$ breaks:

$$SEQ_T(k + 1|k) = \max_{1 \leq j \leq k+1} \sup_{\tau \in \Lambda_{j,\varepsilon}} T \left\{ SSR_T(\hat{T}_1, \dots, \hat{T}_k) \right\} \quad (13)$$

$$- \left\{ SSR_T(\hat{T}_1, \dots, \hat{T}_{j-1}, \tau, \hat{T}_j, \dots, \hat{T}_k) \right\} / SSR_{k+1} \quad (14)$$

where $\Lambda_{j,\varepsilon} = \left\{ \tau : \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \leq \tau \leq \hat{T}_j - (\hat{T}_j - \hat{T}_{j-1})\varepsilon \right\}$. The model with k breaks is obtained by a global minimization of the sum of squared residuals, as in Bai and Perron (1998).

3.3 Cointegration tests with structural changes

Kejriwal and Perron (2008, 2010) show that their test can reject the null of no break in a purely spurious regression. If anything, their tests have power against spurious regression. In this sense, tests for breaks in the long run relationship are used in conjunction with tests for the presence or absence of cointegration allowing for structural changes in the coefficients.

In this paper, we use the residual-based test of the null of cointegration with an unknown single break against the alternative of no cointegration proposed in Arai and Kurozumi (2007). These authors developed a *LM* test based on partial sums of residuals where the break point is obtained by minimizing the sum of squared residuals. They considered three models: i) Model 1, a level shift; ii) Model 2, a level shift with a trend; and iii) Model 3, a regime shift.

The *LM* test statistic (for one break), $\tilde{V}_1(\hat{\lambda})$, is given by:

$$\tilde{V}_1(\hat{\lambda}) = (T^{-2} \sum_{t=1}^T S_t(\hat{\lambda})^2) / \hat{\Omega}_{11} \quad (15)$$

where $\hat{\Omega}_{11}$ is a consistent estimate of the long run variance of u_t^* in (9), the date of break $\hat{\lambda} = (\hat{T}_1/T, \dots, \hat{T}_k/T)$ and $(\hat{T}_1, \dots, \hat{T}_k)$ are obtained using the dynamic algorithm proposed in Bai and Perron (2003).

The Arai and Kurozumi (2007) test may be quite restrictive since only a single structural break is considered under the null hypothesis. Hence, the test may tend to reject the null of cointegration when the true data generating process exhibits cointegration with multiple breaks. To avoid this problem, Kejriwal (2008) has extended the Arai and Kurozumi (2007) test by incorporating multiple breaks under the null hypothesis of cointegration. The Kejriwal (2008) test of the null of cointegration with multiple structural changes is denoted -with k breaks- as $\tilde{V}_k(\hat{\lambda})$.

4 Empirical results

In this section we re-examine the issue of the standard PV model of stock prices using instability tests to account for potential breaks in the long-run relationship between the log real stock prices and the log real dividends as well as the cointegration tests with multiple breaks. First, we use unit root tests to verify that the log real stock prices and the log real dividends are individually integrated of order one, and the log dividend yield ratio is stationary or integrated of order zero. Second, we test the stability of the log real stock prices and the log real dividends relationship (and select the number of breaks) using the test proposed in Kejriwal and Perron (2008, 2010). Third, we verify that the variables are cointegrated with tests for the presence/absence of cointegration allowing for a single or multiple structural changes in the coefficients as proposed by Arai and Kurozumi (2007) and Kejriwal (2008), respectively. Finally, we estimate the model incorporating the breaks in order to study if the log real stock prices and the log real dividends relationship (the slope parameter γ) have altered over time.

In our empirical analysis, we use annual data of US stock market for the period 1871-2012. The series on real stock prices and dividends are taken from Robert Shiller's website <http://www.econ.yale.edu/~shiller/data.htm>.⁵ The evolution of the log stock prices, p_t , and the log dividends, d_t , appears in Figure 1 showing a close comovement between the two series. However, the plots also suggest that the association between p_t and d_t may have altered over time. It seems clear that these series are characterized a priori by at least one shift in the slope/or intercept of the trend function.

4.1 Stationarity of time series

The first step in our analysis is to examine the time series properties of the series by testing for a unit root over the full sample. Trend breaks appear to be prevalent in macroeconomic time series, and unit root tests therefore need to make allowance for these if they are to avoid the serious effects that unmodelled trend breaks have on power.⁶ In a seminal paper, Perron (1989) shows that failure to account for trend breaks present in the data results in unit root tests with zero power, even asymptotically. Consequently, when testing for a unit root it has become a matter regular practice to allow for this kind of deterministic structural change. In order to avoid this pitfall, we run tests to assess whether structural breaks are present or not in p_t and d_t series.

Firstly, we have used the GLS-based unit root tests with multiple structural breaks both under the null and the alternative hypotheses proposed in Carrion-i-Silvestre et al. (2009). The commonly used tests of unit root with a structural change in the case of an unknown break date (Zivot and Andrews (1992), Perron (1997), Vogelsang and Perron (1998), Perron and Vogelsang (1992a, 1992b)),

⁵The series are expressed in natural logarithms. The lowercase letters denote the logs of the variables.

⁶See, *inter alia*, Stock and Watson (1996, 1999, 2005) and Perron and Zhu (2005).

assumed that if a break occurs it does so only under the alternative hypothesis of stationarity. The methodology developed by Carrión-i-Silvestre et al. (2009) solves many of the topical problems of standard unit root tests with a structural change in the case of an unknown break date.⁷

The results of applying for Model III the Carrion-i-Silvestre-Kim-Perron tests are shown in Table 1, allowing for up to one or two breaks, respectively. As can be seen in Table 1, the null hypothesis of a unit root with one or two structural breaks that affects the level and the slope of the times series cannot be rejected at the 5% level of significance, by any of the M^{GLS} and ADF^{GLS} tests.⁸ The break points are estimated: i) at 1930 (one break model) and at 1935 and 1971 (two break model) for p_t ; ii) at 1947 (one break model) and at 1895 and 1945 (two break model) for d_t .

Secondly, Harvey et al. (2013) show that the fixed magnitude trend break asymptotic theory of Carrion-i-Silvestre et al. (2009) does not predict well the finite sample power functions of M tests, and power can be very low for the magnitudes of trend breaks typically observed in practice. In response to this problem Harvey et al. (2013) propose a unit root test that allows for multiple breaks in trend both under the null and the alternative hypotheses, obtained by taking the infimum of the sequence (across all candidate break points in a trimmed range) of local GLS detrended augmented Dickey–Fuller-type statistics, MDF_m . They show that this procedure has power that is robust to the magnitude of any trend breaks, thereby retaining good finite sample power in the presence of plausibly-sized breaks. They also demonstrate that, unlike the OLS detrended infimum tests of Zivot and Andrews (1992), these tests display no tendency to spuriously reject in the limit when fixed magnitude trend breaks occur under the unit root null. Table 2 presents results for MDF_1 (only a single break in trend) and MDF_2 (two breaks in trend) Harvey et al. (2013) tests applied at the nominal asymptotic 5% significance level. As can be seen in Table 2, the null hypothesis of unit root with one or two structural breaks cannot be rejected for the two series.

Consequently, we can conclude that the p_t and d_t variables are $I(1)$ with structural breaks.

4.2 Long-run relationship

Once the order of integration of the series has been analyzed, we will estimate the long-run or cointegration relationship between p_t , and d_t . Given the relatively small sample size, we will estimate and test the coefficients of the cointegration equation by means of the Dynamic Ordinary Least Squares (DOLS) method from Saikkonen (1991) and Stock and Watson (1993), and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory

⁷See Carrión-i-Silvestre et al. (2009) for more details.

⁸The critical values were obtained by simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

variables, as well as serial correlation in the error terms of the OLS estimation. Also, in order to overcome the problem of the low power of the classical cointegration tests in the presence of persistent roots in the residuals of the cointegration regression, Shin (1994) suggested a new test where the null hypothesis is that of cointegration. therefore, in the first place, we estimate a long-run dynamic equation that includes the leads and lags of all the explanatory variables, i.e., the so-called DOLS regression:

$$p_t = c + \Phi t + \gamma d_t + \sum_{j=-q}^q \gamma_j \Delta d_{t-j} + v_t \quad (16)$$

Secondly, we use the Shin test, based on the calculation of two LM statistics from the DOLS residuals, C_μ and C_τ , in order to test for stochastic and deterministic cointegration, respectively. If there is cointegration in the demeaned specification given in (16), that occurs when $\Phi = 0$, this corresponds to a deterministic cointegration, which implies that the same cointegrating vector eliminates both deterministic and stochastic trends. But if the linear stationary combinations of $I(1)$ variables have nonzero linear trends (that occurs when $\Phi \neq 0$), as given in (16), this corresponds to a stochastic cointegration.⁹ In both cases, the parameter γ is the long-run cointegrating coefficient estimated between p_t , and d_t .

The results of Table 3 show that the null of the deterministic cointegration between p_t and d_t is not rejected at the 1% level of significance, and the estimated value for γ is 1.68. But this estimate would be significantly different from one at the 1% level, according to a Wald test on the null hypothesis $\hat{\gamma} = 1$, distributed as a χ_1^2 and denoted by W_{DOLS} in Table 3. The results obtained are consistent with the existence of linear cointegration between the log stock prices, p_t , and the log dividends, d_t , with a vector (1, -1.68). Thus, the cointegration vector is not (1, -1), as predicted by the theory.

Accounting for parameter shifts is crucial in cointegration analysis, since this type of analysis normally involves long spans of data, which for this reason are more likely to be affected by structural breaks. In particular, our data covers one hundred and forty years of the history of the U.S. stock market, and during that period of time the long-run relationship between real stock prices and dividends has probably changed due to alterations in monetary and fiscal policy, as well as because of reforms in the financial market and in the regulations of the stock market. Thus, the information content of the linear PV model of stock prices is subject to change over time and all the empirical modeling studies that did not take into account the possible changes and instabilities will fail to explain the variations in this relationship between real stock prices and dividends. Therefore, as we argued before, it is very relevant to allow for structural breaks in our cointegration relationship.

We now consider the tests for structural change that have been proposed in Kejriwal and Perron (2008, 2010). Since we have used a 20% trimming, the

⁹See Ogaki and Park (1997) and Campbell and Perron (1991) for an extensive study of deterministic and stochastic cointegration.

maximum numbers of breaks we may have under the alternative hypothesis is 3. Moreover, the intercept and the slope in equation (16) are permitted to change. Table 4 presents the results of the stability tests as well as the number of breaks selected by the sequential procedure (SP) and the information criteria BIC and LWZ proposed by Bai and Perron (2003). The test and the SP results do not suggest any instability, although the information criteria BIC and LWZ select two breaks and one break, respectively, and provide evidence against the stability of the long run relationship. Overall, the results of the Kejriwal-Perron tests suggest: i) a model with two breaks estimated at 1944 and 1971 and three regimes, 1871-1944, 1945-1971 and 1972-2012; ii) a model with one break estimated at 1944 and two regimes, 1871-1944 and 1945-2012.

Since the above reported stability tests also reject the null coefficient of stability when the regression is a spurious one, we still need to confirm the presence of cointegration among the variables. With that end in mind, we use the residual based test of the null of cointegration against the alternative of cointegration with unknown multiple breaks proposed in Kejriwal (2008), $\tilde{V}_k(\hat{\lambda})$.

Arai and Kurozumi (2007) show that the limit distribution of the test statistic, $\tilde{V}_k(\hat{\lambda})$, depends only on the timing of the estimated break fraction $\hat{\lambda}$ and the number of I(1) regressors m .¹⁰ Since we are interested in the stability of the stock prices-dividends coefficient, γ , we only consider model 3 that permits the slope shift as well as a level shift. Table 5 shows the results of the Arai-Kurozumi-Kejriwal cointegration tests allowing for both two breaks and one break. As before, the level of trimming used is 15%. As a result we find that both tests, $\tilde{V}_2(\hat{\lambda})$ and $\tilde{V}_1(\hat{\lambda})$, cannot reject the null of cointegration with two structural breaks and one break at 1% level of significance.

Therefore, we conclude that p_t and d_t are cointegrated with two structural changes estimated at 1944 and 1971 (model with two breaks) and with one structural change at 1944 (model with one break). The first break coincides approximately with the end of the Second World War and the boom in stock prices of the 1950s, while the second break coincides with the end of the Bretton Woods System, the oil price shock of 1973 and the collapse of the stock market in the early 1970s. Driffill and Sola (1998) obtain similar results using a regime-switching model that interprets the boom (the slump) as a response of the present-value stock price to a change of regime in an era of rapidly growing (declining) dividends. When a stochastic regime-switching is introduced in place of the bubble, they find that the fluctuations of stock prices that would have been explained as a bubble (e.g, Froot and Obstfeld, 1991) are now explained as breaks in the fundamental price that results from a change of regime, as we do in the present paper.

To compare the coefficients obtained from break models with those reported from models without any structural break, we estimate the cointegration equation (16) both with a two breaks model, as suggested by BIC criterion, and with

¹⁰In our case, the critical values for the test are then simulated for the corresponding break fractions using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0, 1)$ random variables.

a one break model, as suggested by the LWZ criterion. The results with the sub-samples are presented in Table 3.

First, the results of the C_μ statistics in the model with two breaks show that the null of the deterministic cointegration between p_t and d_t is not rejected at the 1% level of significance in the three regimes. The coefficient estimated between p_t and d_t (i.e, the long-run elasticity, γ) in a two-break model shows a tendency to increase over time (1.05, 2.23 and 4.03). Therefore, the coefficient in the first regime (1871-1944) is much smaller than the value obtain with the full sample (1.68); furthermore, the restriction on the estimate of γ being equal to one is clearly accepted, and the cointegration vector is (1, -1), as predicted by the theory.

Secondly, in the case of the model with one break, the results in Table 3 show that the null of the deterministic cointegration between p_t and d_t is not rejected at the 1% level of significance in the two regimes. Again, the estimated coefficient values in the first and second regimes increase over time (1.05 and 2.54). In the second regime, according to a Wald test on the null hypothesis $\hat{\gamma} = 1$, the coefficient estimated is significantly different from one.

Overall, the results suggest that ignoring structural changes in the long-run cointegration relationship may understate the extend of correlation between the log stock prices, p_t , and the log dividends, d_t , since the response of the present-value stock price to a change of dividends increases over time.

5 Conclusions

In this paper we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the Present Value model of U.S. stock prices.

To avoid the econometric problems mentioned in empirical literature, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use the approach developed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. These authors propose a sequential procedure that not only permits the detection of parameter instability in cointegration regression models but also allows for a consistent estimation of the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between the log stock prices and the log dividends is analyzed using the residual based test of the null hypothesis of cointegration with a single and/or multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008), respectively. In the empirical analysis, we use annual data of the US stock market for the period 1871-2012.

The results obtained in our study are consistent with the existence of linear cointegration between the log stock prices and the log dividends, with a vector (1, -1.68). Thus, the cointegration vector is not (1, -1), as predicted by the theory. The results for the full sample (1871-2012) only support a "weak"

version of the PV model of US stock prices.

The empirical results also show that the cointegrating relationship has changed over time, i.e., it is not stable. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models suggest a model of three regimes, with the dates of the breaks estimated at 1944 and 1971, and a model of two regimes, with the date of the break estimated at 1944. The first break coincides approximately with the end of Second World War and the boom in the stock prices of the 1950s, while the second break coincides with the end of the Bretton Woods System, the oil price shock of 1973 and the collapse in the stock market in the early 1970s.

The estimate of long-run elasticity between the log stock prices and the log dividends in both break models shows a tendency to increase over time. Finally, only the results for the period 1871-1944 support a "strong" version of the PV model of stock prices, with the long-run coefficient equal to one.

Summing up, the results obtained in this study suggest that ignoring structural changes in the long-run cointegration relationship may understate the extent of correlation between the log stock prices and the log dividends, since the response of the present-value stock price to changes in dividends increases over time.

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Table 1
M unit root tests with multiple structural breaks of Carrion-i-Silvestre et al. (2009) ^{a,b,c}

Variable	Model	MZ_{α}^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MP_T^{GLS}	Break date	
						\hat{T}_1	\hat{T}_2
p_t	III	-3.92	-1.39	0.354	40.79	1930	—
p_t	III	-11.99	-2.44	0.203	17.90	1935	1971
d_t	III	-10.82	-2.27	0.209	14.47	1947	—
d_t	III	-26.54	-3.62	0.136	8.79	1895	1945

Notes:

^a A ** denote significance at the 5% level.

^b The structural break affects the level and the slope of the time trend (Model III).

^c The critical values were obtained by simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

Table 2
 Unit root tests with multiple structural breaks using minimum DF-statistics
 of Harvey et al. (2013)

Variable	MDF_1	MDF_2
p_t	-2.76	-3.56
d_t	-3.76	-4.25

Notes:

^a * denotes rejection the null at the 5% level.

^b Structural break ($m = 1, 2$) may affect the slope of the time trend.

^c Using the Modified Akaike Information Criteria ($MAIC$) to select the order of the autoregression k .

^d The critical values for MDF_1 are taken from Harvey et al. (2013), Table

1

Table 3
 Estimation of long-run relationships: Stock-Watson-Shin^{a,b,c,d}
 cointegration tests

Parameter estimates	Model without structural breaks	Two breaks model			One break model	
	Full sample	First regime	Second regime	Third regime	First regime	Second regime
	1871-2012	1871-1944	1945-1971	1972-2012	1871-1944	1945-2012
c	1.49 (9.87)	2.87 (11.3)	-0.04 (0.09)	-5.36 (-5.27)	2.87 (11.3)	-1.00 (-2.26)
γ	1.68 (27.7)	1.05 (8.89)	2.23 (12.6)	4.03 (11.5)	1.05 (8.89)	2.54 (16.8)
C_μ	0.191	0.149	0.156	0.045	0.149	0.060
W_{DOLS}	10.81***	0.04	52.04***	23.46***	0.04	20.64***

Notes:

^a t -statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to $INT(T^{1/2})$ as proposed in Newey and West (1987).

^b We choose $q = INT(T^{1/3})$ as proposed in Stock and Watson (1993).

^c C_μ is LM statistics for cointegration using the DOLS residuals from deterministic cointegration, as proposed in Shin (1994). *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively. The critical values are taken from Shin (1994), table 1, from $m = 1$.

^d W_{DOLS} is a Wald test on the null hypothesis $\hat{\beta} = 1$, distributed as a χ_1^2 . *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Table 4
Kejriwal-Perron tests for testing multiple structural breaks
in cointegrated regression models: equation (9) and (16)

Specifications ^a			
$y_t = \{p_t\}$	$z_t = \{1, d_t\}$	$x_t = \{\emptyset\}$	$M = 3$
	$q = 2$	$p = 0$	$h = 27$
Tests ^b			
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$UD \max$
5.94	3.80	2.96	5.94
Number of Breaks			
	Selected	Breaks	
		\hat{T}_1	\hat{T}_2
SP	0	—	—
LWZ	1	1944	—
BIC	2	1944	1971

Notes:

^a y_t , z_t , q , p , h , and M denote the dependent variable, the regressors, the number of $I(1)$ variables (and the intercept) allowed to change across regimes, the number of $I(0)$ variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

^b The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Kejriwal-Perron website), trending case with $q_b = 1$.

Table 5

Arai-Kurozumi-Kejriwal cointegration tests with multiple structural breaks: equation (9) and (16)^{a,b}

Two breaks model					One break model				
Test	$\tilde{V}_2(\hat{\lambda})$	$\hat{\lambda}_1$	\hat{T}_1	$\hat{\lambda}_2$	\hat{T}_2	Test	$\tilde{V}_1(\hat{\lambda})$	$\hat{\lambda}_1$	\hat{T}_1
	0.096*	0.52	1944	0.72	1971		0.07*	0.52	1944

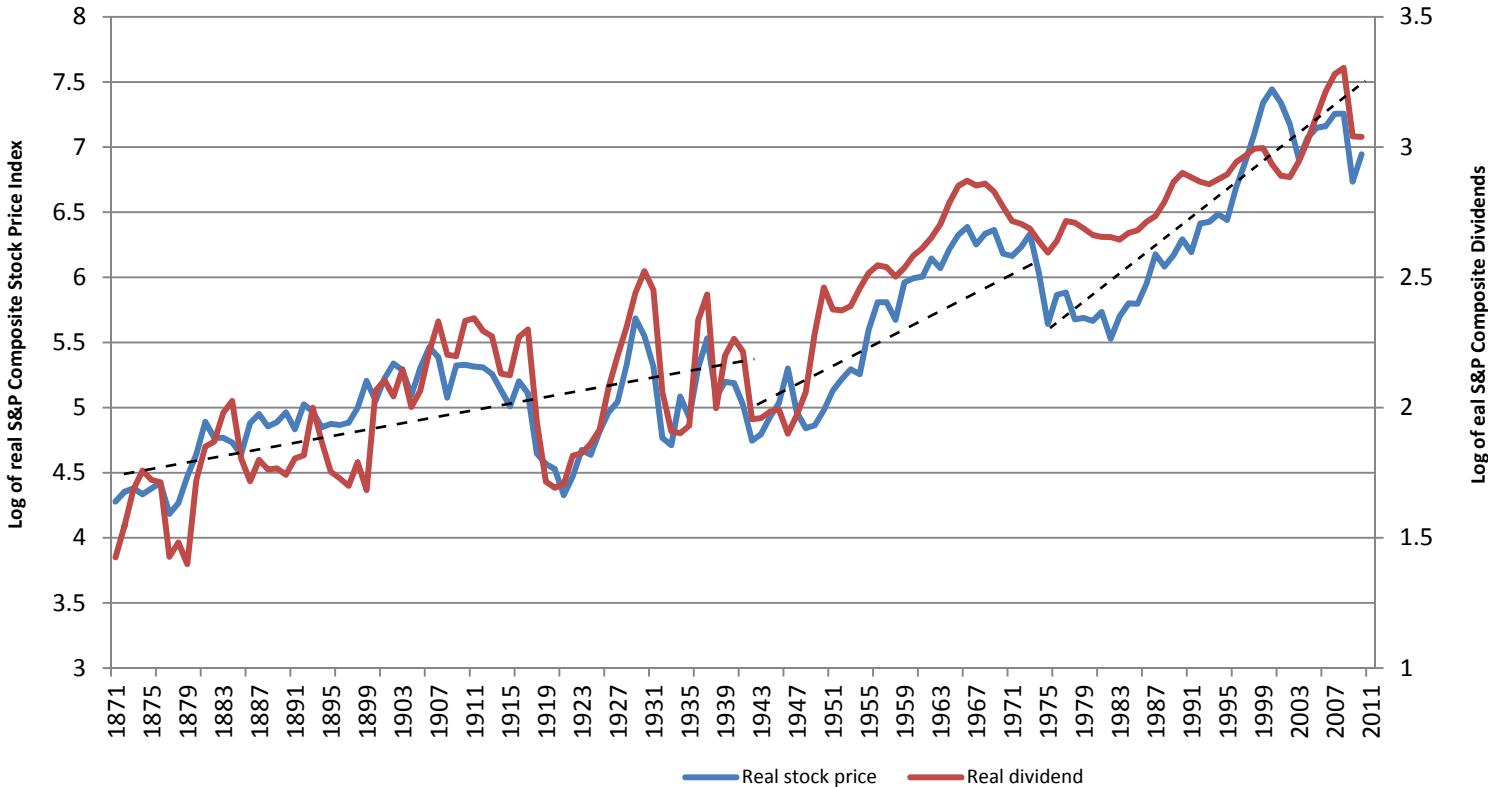
Notes:

^a *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

^b Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0,1)$ random variables.

Critical values:	10%	5%	1%
$\tilde{V}_2(\hat{\lambda})$	0.077	0.101	0.149
$\tilde{V}_1(\hat{\lambda})$	0.103	0.131	0.194

**Figure 1: U.S. real stock prices and dividends
1871-2010**



Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

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The Editor