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Eigenvalue Productivity: Measurement of Individual Contributions in Teams

Abstract

While the output of a team is evident, the productivity of each team member is typically not readily identifiable. In this paper we consider the problem of measuring the productivity of team members. We propose a new concept of coworker productivity, which we refer to as eigenvalue productivity (EVP). We demonstrate the existence and uniqueness of our concept and show that it possesses several desirable properties. Also, we suggest a procedure for specifying the required productivity matrix of a team, and illustrate the operational practicability of EVP by means of three examples representing different types of the available data.

JEL-Codes: D240, J240, L230.

Keywords: coworker productivity, eigenvalue productivity, centrality, team production.

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1. INTRODUCTION

Teams jointly produce a product. By judging the quality of this product, the success of the team as a whole can be easily assessed. However, the individual productivity of a worker is typically not directly observable, and thus it is difficult to conceptualize, even if a fixed task is assigned to each member of a team. This difficulty in assessing individual productivity is particularly significant if the concept is to be operationalizable with observable data. We, therefore, propose a new concept of coworker productivity to measure the individual contribution of a team member. We then demonstrate the existence and uniqueness of that concept and show that it has several desirable properties, may be applied in various situations, and is suitable for various types of data.

As is well-known, a team is more than merely the sum of its members' individual abilities. Within a team, coworkers interact and their abilities as well as their capacities for and their productivities in teamwork determine, along with other factors, the actual team output—and thus the success of the team. In this way, the interplay of a worker with their teammates depends on the ability and willingness of the other coworkers to cooperate: a worker can only interact with the teammates efficaciously if they go along with this endeavor.¹ As a consequence, the effective contribution of a worker to the team depends on the contributions of the others and thus on both the cooperative capacity and the efforts of the other coworkers.²

While this should be beyond controversy, it does not readily provide an operational concept of an individual's contribution to the productivity of the team. Similarly, the coworkers' productivities cannot be directly observed and are thus not readily derivable from the team data. While the productivity or success of a team as a whole is usually observable or relatively easy to measure, *e.g.*, by total sales, revenue, patterns, number of orders, or cases (or in team sports, by the percentage of wins, points scored, etc.), it is more difficult to measure the productivity of an individual worker within a team. This difficulty is inherently associated with the very nature of teamwork: while the team as a whole produces a joint output, the individual contributions of the team members can only be measured in terms of input, such as working time, effort, etc. Apparently, this is a consequence of the fact that “the interaction between team members is multifaceted” (Depken II and Haglund, 2011, p. 4). This multiplicity of interactions raises the question of how one can consistently define and then calculate coworker productivities.

For example, in team sports, the literature has used numerous variables to appraise the coworker productivity of a player: the number of goals scored, assists provided, duels won, ball touches, etc. Yet, each of these numbers suffers from the fact that it imputes an output (successful action) to an individual player, while this output is actually the joint product of the player and the teammates: a player can only perform well when the other teammates

¹If cooperation and coordination is not successful, problems like free-riding can arise in teams, analyzed for example by Guillen et al. (2014), Backes-Gellner et al. (2015), and Fu et al. (2015).

²The relevance of the amount of effort workers choose, based on income maximizing considerations, to put forth when on a team has been emphasized by Gould and Winter (2009).

are able and willing to put that player into the scene and lay the proper groundwork for the player’s actions to bear fruit. At the same time, professional team sports require, besides individual aptitudes and skills, complex collective moves, which constitute both a prerequisite for and a consequence of individual performance (see, for example, also Beck and Meyer, 2012). Through all of these channels, the team productivity of a player depends on the productivities of all the teammates, and in particular on the productivity of that player’s “neighbors” on the playing field³; similar arguments apply to any complex team production process in economics.

For the case of NBA teams, Berri (1999) proposed an econometric model specifically tailored to basketball along with a related concept to measure the productivity of individual players. Furthermore, he criticized the use of seemingly straightforward shortcuts to measure individual team productivity, such as the number of points scored, as they are “likely to be incorrect and misleading” (p. 415) since they neglect the specific team sport aspect: the necessary cooperation between team members. We act on the idea put forth by Berri and develop a theoretical concept of coworker productivity which is quite general and flexible enough to be used in many different economic contexts and with many different available databases—and is thus applicable in various situations. In particular, it does not require a specific model or economic situation, as does Berri’s basketball-specific approach: all it requires is some (locally) linear structure and the pairwise decomposability of total productivity. This structure suffices to obtain a measure of coworker productivity enjoying convenient properties. In this way, our work follows a suggestion of Kendall (2003), who called for more research “to derive measures of players’ ‘true’ marginal products when productivity spillovers exist” (p. 401).

In order to capture the prevalence of spillover effects and the interdependencies between the team productivities of the coworkers, a more sophisticated concept for measuring the contribution (or the relevance) of a coworker to the productivity of the team seems to be necessary. In this paper we, therefore, delineate a concept of coworker productivity which makes use of the endogenous and self-referential structure of cooperation within a team: in teams, coworkers are required to cooperate in order for the team to perform successfully, while each coworker also benefits from the cooperative abilities of the other teammates. Specifically, the more a teammate of coworker i contributes to the team, the better the conditions will be for coworker i to perform well—and thus to contribute to the success of the team. In this way, the team productivity of coworker i depends (positively) on the productivities of all other teammates and, in particular, on the productivity of those who are “adjacent to” or “central for” that player. Since this is true for any team member, coworker productivity is self-referential; as a consequence, the productivities of all coworkers on a team must be determined simultaneously.

³Accordingly, there is empirical evidence that some combinations of positions or some pairs of players are more strongly complementary than others (see, for example, Idson and Kahane, 2000).

We duly acknowledge the feature of mutual complementarity in production⁴ within teams and present a concept of coworker productivity that consistently and simultaneously defines the contributions of each coworker to the productivities of all other teammates. In this way, the concept serves to measure the cooperative productivity, and thus the significance or centrality, of a coworker within a team. Formally, our concept of coworker productivity is defined as a vector that when we apply a linear function, given by a nonnegative matrix of pairwise productivity coefficients, to it does not have its direction changed. By construction, this concept is closely related to the concept of eigenvector centrality, which is known in network analysis and has been suggested by Bonacich (1972).⁵ In order to acknowledge that origin, we refer to our concept as *eigenvalue productivity* (EVP).

In our approach, we calculate the coworker productivities for a given team assigned to a given task. We thus do not model the decision (process) of the manager on the optimal composition of the team, or, more broadly, how or why a manager selected a specific worker for a given project. This decision is not an issue here, but is considered to be exogenously determined. Yet, in a more general framework, pairwise productivities may be chosen directly by the players, thereby establishing a pairwise cooperation game or, in this sense, a network game. These possibly action-dependent pairwise productivities of the coworkers are considered as given data here, because the central purpose of the EVP is to derive a scalar productivity measure for each player when the actions have already been determined. There is, though, a significant literature that explores the activities chosen by the players for a given (possibly uncertain) network structure; we briefly discuss this in the literature review in Section 2.

While the concept of EVP is thus related to network analysis, the spirit of EVP also reflects the basic idea incorporated in the Shapley value, well known from cooperative game theory (Shapley, 1952): the distribution of the surplus between players according to each player's marginal contribution to all possible coalitions. However, since the Shapley value requires that one already knows a player's marginal contribution to all possible coalitions, it is not helpful or applicable when measuring individual productivity within a team: while we can observe both the output of the team (with a fixed composition) and individual inputs, we cannot deduce the marginal contributions unless we know the production function for each possible coalition.

After providing the formal definition of EVP, we prove the existence and uniqueness of this concept and demonstrate that it has several economically desirable properties: *symmetry*, *permutation covariance*, *null player property*, *aggregate balance*, *differentiability*, *relative*

⁴Formally, complementarity (in production) is captured by the supermodularity of a (production) function. For more details, see, for example, Milgrom and Roberts (1990) or the excellent monograph by Topkis (1998). This supermodularity may vanish in the limiting case if members work on their own, as, for example, in the case of team competition with pairwise matches, a case recently analyzed by Fu et al. (2015).

⁵The idea can even be traced back to Katz (1953), who proposed a similar index to measure the status of individuals within a group. (For a textbook presentation of centrality indexes, see, for example, Jackson, 2008, sec. 2.2.4.)

monotonicity, *absolute monotonicity*, and *duplication monotonicity*. Then we show how the EVP can be calculated. The calculation of the EVP is based on the availability of a pairwise productivity matrix. If such a matrix of the directional productivity effect between each pair of members of a team is available, the EVP can be directly employed. Frequently, though, this will not be the case, and for those cases we suggest a procedure to calculate such a matrix from the data of a team and illustrate the operational practicability of the EVP by means of three examples. (The third example is relegated to the appendix.) All examples use data for the success of a team over a specific period and the composition of the team as it varies over time. Since the data for team results (success) and team composition is often available, this procedure is arguably broadly applicable to many situations, including team sports, where the data on results and line ups during a season is readily at hand.

The first example represents a hypothetical situation that is very clear and manageable, and helps demonstrate that EVP works smoothly and brings about quite intuitive results. The second example uses real data from the movie industry. Here, the team is a group of well-known actors, directors, and writers, where subgroups join to produce a movie. The success is measured by the worldwide volume of sales of the movie. We apply this data to delineate (estimations of) pairwise productivity coefficients and use the resulting productivity matrix to calculate the EVP. In the third example, we use a team of researchers that recombine to publish papers in peer-reviewed journals. We infer the data of pairwise productivity effects from the publication record of different subgroups of the team, and with the help of the generated matrix of pairwise effects, we then calculate the EVP of all the researchers on the team.

2. RELATED LITERATURE

There is ample research on various aspects of teamwork in economics, management, psychology, and sociology. In the following, we focus on those aspects of teams and teamwork that are especially relevant for the measurement of individual contributions within a team and thus for our new measure, *eigenvalue productivity* (EVP).⁶

Investigating the decision-making process, the literature in experimental economics concludes that groups generally arrive at more rational decisions than do individuals (see the reviews of Charness and Sutter, 2012; Kugler et al., 2012), suggesting that teamwork outperforms individual actions. For example, Cooper and Kagel (2005) and Kocher and Sutter (2005) who explore the differences between the learning and adjustment process of teams and that of individuals, identify positive synergies between teammates and demonstrate the superiority of team play; Rockenbach et al. (2007) find that teams arrive at better investment decisions; Charness et al. (2007), Sutter (2009), and Maciejovsky et al. (2013) show

⁶For a broader review of the research on teams in the management literature, the reader is referred to Mathieu et al. (2008) and the reviews mentioned on pp. 410 and 411 therein; for a review of the psychological literature, to Levine and Moreland (1990), and especially on the effectiveness of teams to Kozlowski and Ilgen (2006).

that group membership has a strong effect on individual decisions; and Feri et al. (2010) demonstrate that teams coordinate more successfully than individuals.

Similarly, the significance of productivity spillovers between coworkers within teams has been demonstrated in various economic domains: examples are, among others, Idson and Kahane (2000, 2004), Kendall (2003), Franck and Nüesch (2010), and Alvarez et al. (2011). Also, several field studies demonstrate the effects of peers on the productivity of workers, see for example Mas and Moretti (2009), Bandiera et al. (2010), Chan et al. (2014), and most recently Friebe et al. (2017), who confirm the presence of significant complementarities between team members, here for the case of a retail chain.⁷ Moreover, Herbst and Mas (2015) compare results from laboratory experiments with field studies and conclude that both yield similar estimates of peer effects. Thus, even though the productivity of coworkers may significantly differ across different teams, the relevance of coworker and teammate effects on individual productivity is apparently an omnipresent phenomenon; and, as emphasized by Beck and Meyer (2012), a similar observation holds for the significance of team composition.

Within a team, the interactions between its members depend on behavioral aspects, group membership effects, employment practices, and other factors, all of which are well documented in the management literature: Ichniowski et al. (1997), Reagans and Zuckerman (2001), Apesteguia et al. (2012), and Hoogendoorn et al. (2013) evidence the (positive) significance of innovative employment practices and diversity on team production. Investigating hierarchical differentiation in pay and participation in professional sports, Halevy et al. (2012) find those differentiations may facilitate intragroup coordination and cooperation.

Beyond organizational differences and complementarities between team members, many psychological factors also affect team performance. In particular, individual performance is positively related to team performance (Chen et al., 2007); factors like motivation, empowerment, (beliefs in) collective capability, cooperative goals, team cognition, and certain personality traits (or a specific combination of these factors), all have a positive impact on team performance (see Van Kleef et al., 2010; Chen et al., 2007; Aubé et al., 2015; Alper et al., 1998; DeChurch and Mesmer-Magnus, 2010; Peeters et al., 2006). Also, things such as moods and emotions have an influence on team productivity and often play a role in the productivity of teams and organizations (see for example Kelly and Barsade, 2001; Beal et al., 2005; Barsade and Gibson, 2007). Moreover, Jordet et al. (2012), taking a look at prior team performance in sports settings and subsequent individual performances in high-pressure tasks, such as penalty shootouts, find that the history of the team has an influence on the performance of the individual.

There is also much interest in the relationship between salaries, namely the wage disparities within a team, and team performance, which has been explored by, for example, Depken II (2000), Simmons and Berri (2011). Also, intergroup competition and the payment scheme influence the performance of a team and its individuals (Erev et al., 1993). Generally, the distinction and resemblance of individual versus team financial incentives has been

⁷Kandel and Lazear (1992) provide a theoretical analysis of the influence of peers on other workers.

frequently explored, see for example Che and Yoo (2001), Barnes et al. (2011), Conroy and Gupta (2016), or the review of Garbers and Konradt (2014).

Investigating team composition and the organizational structure of a team, Humphrey et al. (2009) conclude that certain team roles are more important for team performance than others.⁸ The recognition of a single member of a team will, via social influence processes, produce positive spillover effects on the performance of the other team members, as well as on overall team performance, and this is particularly the case when the distinguished individual has a central position in the team (Li et al., 2016). More broadly, teams with more centralized structures are generally more successful: centrality is beneficial for individual and team performance (see Lin et al., 2005; Tröster et al., 2014; Sparrowe et al., 2001).

The degree and type of centralization of an organizational unit, such as a team, is frequently analyzed by means of tools well known from network analysis. For example, the centrality of a team member may be measured by degree centrality, closeness centrality, betweenness centrality, or eigenvector centrality (see Jackson, 2008).⁹ Beyond this, the network structure may also be endogenized, where team members choose their links to and intensities of cooperation with the other team members. Notable examples for network games are Ballester et al. (2006), Galeotti et al. (2010), Belhaj et al. (2014), de Marti and Zenou (2015), Allouch (2015), Calvo-Armengol et al. (2015), for theoretical; and Berninghaus et al. (2006), Berninghaus et al. (2007), Corbae and Duffy (2008), Goeree et al. (2009) and Charness et al. (2014) for experimental work on network games. Moreover, Bala and Goyal (2000), Galeotti et al. (2006), Kim and Wong (2007), Harrison and Munoz (2008), De Sinopoli and Pimienta (2010), and Baetz (2015), among others, endogenize the network structure, presenting a model of network formation where players decide upon the costly creation of pairwise links. For a comprehensive survey of the theoretical work and the recent literature on games involving economic and social networks, we refer the reader to Jackson (2008, 2014), Jackson and Zenou (2014) and the very recent survey by Jackson et al. (2017).

3. THE APPROACH

In this section, we develop a new concept of coworker productivity, *eigenvalue productivity* (EVP), and show how individual productivities may be consistently determined for all members of a team. We then show the existence and uniqueness of a *vector of eigenvalue*

⁸The importance of certain individual members of a group was also pointed out by Gruenfeld et al. (2000), using data of itinerant workers, by showing that after the return of a former group member the production of the team as a whole was of higher quality.

⁹Subsequently, versions of eigenvector centrality have been adopted in, among others, regional economics to characterize spatial structures (*e.g.* Neal, 2013; Wanzenböck et al., 2015), financial economics to measure risk exposure resulting from interconnectedness (*e.g.* Markose et al., 2012; Alter et al., 2015), the social sciences to measure social segregation (*e.g.* Echenique and Fryer, 2007; Holfve-Sabel, 2015), and network analysis to measure the effects of coauthorship on the performance of scholars (*e.g.* Yan and Ding, 2009; Li et al., 2013). However, the most well-known application of eigenvector centrality is the PageRank algorithm used by the search engine of Google to rank search results in the Web (see Page et al., 1999; Bryan and Leise, 2006).

productivities (EVP-vector), and demonstrate that this vector has convenient economic properties.

As argued above, a worker benefits from the abilities of all teammates: the more a teammate of worker i contributes to the team, the better the conditions are for worker i to perform well and thus to contribute to the success of the team. In this way, the coworker productivity of worker i depends (positively) on the productivities of their teammates. We shall now formalize this idea and show how coworker productivities may be derived.

Consider a team consisting of a fixed set of n workers $\mathcal{N} = \{1, 2, \dots, n\}$. We assume that the individual productivity of each worker $i \in \mathcal{N}$ is nonnegative. According to the argument provided above, we presume that the productivity depends linearly on the productivity of i 's teammates. The *coworker productivity of worker i in team \mathcal{N}* , $p^i(\mathcal{N})$, is then defined as

$$p^i(\mathcal{N}) = \frac{1}{\lambda} \sum_{j \in \mathcal{N}} g_{ij}(\mathcal{N}) p^j(\mathcal{N}) \quad \forall i \in \mathcal{N},$$

where $g_{ij}(\mathcal{N}) \geq 0$ denotes the extent to which worker i benefits from the coworker productivity of worker j (when both belong to team \mathcal{N}), and $\lambda > 0$ is a (strictly positive) normalization factor. In this way, the coworker productivity of i depends on the coworker productivities of all the teammates, and since this relation holds for any worker $i \in \mathcal{N}$, we arrive at (in matrix notation)¹⁰

$$\mathbf{p}(\mathcal{N}) \equiv \frac{1}{\lambda} \mathbf{G}(\mathcal{N}) \mathbf{p}(\mathcal{N}), \quad (1)$$

where the vector composed of the individual $p^i(\mathcal{N})$ s, the individual coworker's productivities, is denoted by $\mathbf{p}(\mathcal{N})$, and where $\mathbf{G}(\mathcal{N}) \equiv [g_{ij}(\mathcal{N})]_{i,j \in \mathcal{N}} \geq \mathbf{0}$ is the matrix of the coefficients measuring the extent to which the individual productivities influence each other. In order to avoid the meaningless case $\mathbf{G} = \mathbf{0}$, we shall assume that there is at least one worker whose productivity is positive when paired with another worker, so that $g_{ij} > 0$ for some pair $(i, j) \in \mathcal{N}^2$, and thus $\mathbf{G} \geq \mathbf{0}$.

For notational convenience, we subsequently suppress the team argument \mathcal{N} , but it should be kept in mind that both \mathbf{G} and \mathbf{p} depend on the team under consideration. Then, Eq. (1) may be re-written as

$$\lambda \mathbf{p} = \mathbf{G} \mathbf{p} \quad \Leftrightarrow \quad (\mathbf{G} - \lambda \mathbf{I}) \mathbf{p} = \mathbf{0}, \quad (2)$$

where \mathbf{I} denotes the identity matrix (of the proper rank, *i.e.*, of rank n in this case). For $\mathbf{p} \neq \mathbf{0}$, the homogeneous system (2) has a solution (in \mathbf{p}) if, and only if,

$$\det(\mathbf{G} - \lambda \mathbf{I}) = 0. \quad (3)$$

¹⁰We use the following notation. For any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ we write $\mathbf{x} \geq \mathbf{y} \Leftrightarrow x_i \geq y_i \forall i = 1, \dots, n$; $\mathbf{x} > \mathbf{y} \Leftrightarrow x_i > y_i \forall i = 1, \dots, n$ and $x_i > y_i$ for some i (that is $\mathbf{x} \neq \mathbf{y}$); $\mathbf{x} \geq \mathbf{0}$, we call \mathbf{x} a *nonnegative* vector; if $\mathbf{x} \geq \mathbf{0}$, a *semipositive* vector; and if $\mathbf{x} > \mathbf{0}$, a *positive* vector. The corresponding notation and wording is used for matrices. Finally, we write $\mathbb{R}_+^n := \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \geq \mathbf{0}\}$ for the nonnegative orthant of \mathbb{R}^n .

But this is equivalent to λ being an eigenvalue of \mathbf{G} , and \mathbf{p} being the corresponding eigenvector.¹¹ Since \mathbf{p} is the vector of individual productivities that we want to determine, we refer to our concept of coworker productivities as *eigenvalue productivity* (EVP). Hence, assuming that all information relevant for the productivity measure is contained in the matrix \mathbf{G} , we define:

Definition 1 (Eigenvalue productivity (EVP)). Let the set of team members be $\mathcal{N} = \{1, \dots, n\}$ with $n \in \mathcal{N}$. Let $\mathbf{G} \equiv \mathbf{G}(\mathcal{N}) \equiv [g_{ij}(\mathcal{N})]_{i,j \in \mathcal{N}}$ denote a nonnegative, nonzero (*i.e.*, semipositive) matrix, the matrix of pairwise, directional production coefficients, and denote by $\rho(\mathbf{G})$ the spectral radius of \mathbf{G} . Let $\lambda = \rho(\mathbf{G})$, we call an eigenvector associated with λ , *i.e.*, $\mathbf{p}(\lambda)$, a *vector of eigenvalue productivities* (EVP-vector).

Because individual productivities are, by assumption, nonnegative, and we assume that there is at least one worker whose coworker productivity is positive, *i.e.*, $\mathbf{G} \geq \mathbf{0}$, we are only interested in eigenvectors $\mathbf{p} \geq \mathbf{0}$. Since the matrix \mathbf{G} is thus nonnegative, we may apply (a suitable version of) the Perron–Frobenius theorem; in particular, the Perron–Frobenius theorem for nonnegative matrices (Theorem 1 in Appendix C) immediately applies without any (further) qualification. Roughly, this theorem says the following: that there is a non-negative eigenvalue $\hat{\lambda}$ of \mathbf{G} , referred to as the *Perron root* of \mathbf{G} ; that all other eigenvalues are absolutely smaller than or equal to $\hat{\lambda}$; and, most importantly, that all components of the eigenvector, $\hat{\mathbf{p}}$, associated with $\hat{\lambda}$ are nonnegative, *i.e.*, $\hat{\mathbf{p}} \geq \mathbf{0}$.¹² For this reason we are (only) interested in the largest real-valued eigenvalue $\hat{\lambda}$ of \mathbf{G} and in the associated eigenvector $\hat{\mathbf{p}}$, which we refer to as the *Perron vector*.

We may obtain even more specific results if the matrix \mathbf{G} is irreducible.¹³ The Perron–Frobenius theorem for nonnegative, irreducible matrices (see Theorem 2 in Appendix C)¹⁴ states that the Perron root is positive, $\hat{\lambda} > 0$, that it is a simple eigenvalue of \mathbf{G} , that all other eigenvalues are absolutely smaller than $\hat{\lambda}$, that the associated Perron vector is positive, $\hat{\mathbf{p}} > \mathbf{0}$,¹⁵ and that the eigenvector $\hat{\mathbf{p}}$ is unique up to a scalar multiple.

Finally, if the matrix \mathbf{G} is positive or if it is irreducible and primitive (which is a weaker requirement), then the Perron root is unique. This result is stated by the Perron–Frobenius theorem for primitive matrices, see Theorem 3 in Appendix C. In fact, if \mathbf{G} is irreducible, our assumptions will guarantee that \mathbf{G} is primitive, and thus there are no other eigenvalues of \mathbf{G} whose absolute values are equal or greater than $\hat{\lambda}$ (see Lemma 4 in Appendix C).

¹¹The eigenvalues are the roots of the characteristic polynomial (of degree n), *i.e.*, the solutions of Eq. (3).

¹²By definition, some but not all components of $\hat{\mathbf{p}}$ can be zero.

¹³A square matrix \mathbf{A} is called *reducible* if there exists a permutation matrix \mathbf{P} such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ is an upper block triangular matrix. If such a matrix \mathbf{P} does not exist, \mathbf{A} is called *irreducible*. (See Appendix C for a more detailed definition.) Chakravarti (1975, Theorem 1 and 2) provides a result that allows for a simple test of the irreducibility of a matrix. Note that some authors refer to (ir)reducible matrices as *(in)decomposable*, *e.g.*, Takayama (1985).

¹⁴For more details on different versions of the Perron–Frobenius theorem, the reader is referred to, for example, Takayama (1985, Ch. 4.B, p. 367ff), Gantmacher (2000), or Horn and Johnson (1990).

¹⁵Conversely, only if \mathbf{G} is reducible may some, but not all, components of $\hat{\mathbf{p}}$ be zero.

The Perron–Frobenius theorem thus proves to be a fundamental result to safeguard the existence and uniqueness of EVP, two results which we will prove in the next section. However, while this theorem renders our concept well defined, it does not yet provide its properties. In order to show that EVP has reasonable economic properties, we show in Section 4.3 that EVP nicely respects, among others, the following ideas: that a player’s productivity is increasing in the productivity of their teammates; that the productivity of a player who fails to contribute anything to a team’s success is zero; and that symmetric players are treated identically.

From the perspective of applications, it is straightforward to calculate coworker productivities, as measured by EVP, for the members of a given team, provided that the matrix \mathbf{G} is available. However, since frequently one cannot directly observe the (marginal) effect of worker i ’s productivity on worker j ’s productivity, the coefficient matrix \mathbf{G} is not given, but its elements have to be calculated from the data. Having defined coworker productivity in terms of the EVP, the problem thus boils down to calculating \mathbf{G} . In Section 5 we demonstrate the calculation of the matrix \mathbf{G} using two different examples of available data; a third example is provided in Appendix A.

4. EXISTENCE AND PROPERTIES OF EIGENVALUE PRODUCTIVITY

In this section, we formalize our assumptions on \mathbf{G} , demonstrate the existence and uniqueness of eigenvalue productivity, define a set of welcome properties of *some* measure of coworker productivity, and show that EVP satisfies these properties.

4.1. Normalization of the productivity matrix. For convenience we normalize $g_{ii}(\mathcal{N}) = 1$ for all $i \in \mathcal{N}$, implying $\mathbf{G} \geq \mathbf{I}$. We summarize this in the following assumption.

Assumption 1. *The matrix of pairwise productivity coefficients \mathbf{G} is semipositive and irreducible with main diagonal elements being normalized to unity: $\mathbf{G} \geq \mathbf{I}$.*

Remark 1. This normalization can be done w.l.o.g. so that we may set, more generally, $g_{ii}(\mathcal{N}) = c, \forall i \in \mathcal{N}$, for some nonnegative constant c , and our assumption will be replaced by $\mathbf{G} \geq c\mathbf{I}$; in particular, for $c = 0$, our assumption says that \mathbf{G} is semipositive. The next lemma provides a formal proof of this. In order to acknowledge the possibility of an arbitrary, nonnegative normalization of the diagonal elements, we will only require \mathbf{G} to be semipositive, *i.e.*, $\mathbf{G} \geq \mathbf{0}$, in the definition of the EVP (Definition 1), but will assume $\mathbf{G} \geq \mathbf{I}$ for expository purposes apart from that. \diamond

Lemma 1. *Given some quadratic matrix \mathbf{A} , define $\mathbf{B} := \mathbf{A} + \alpha\mathbf{I}$ with $\alpha \in \mathbb{R}$. Then, λ is an eigenvalue of \mathbf{A} and \mathbf{x} is the associated eigenvector if, and only if, $\mu = \lambda + \alpha$ is an eigenvalue of \mathbf{B} with the associated eigenvector \mathbf{x} .*

Proof. By the definition of an eigenvalue and its associated eigenvector, we have $\mathbf{0} = (\mathbf{A} - \lambda\mathbf{I}) \cdot \mathbf{x} = (\mathbf{A} + \alpha\mathbf{I} - (\lambda + \alpha)\mathbf{I}) \cdot \mathbf{x} = (\mathbf{B} - \mu\mathbf{I}) \cdot \mathbf{x}$ \square

According to Lemma 1, a translation of the diagonal elements yields a corresponding translation of the eigenvalues but leaves the eigenvectors unchanged. For this reason, such a shift leaves the Perron vector unchanged (provided that \mathbf{A} and $\mathbf{B} := \mathbf{A} - \alpha \mathbf{I}$ are both nonnegative). Consequently, we may choose any normalization of the diagonal elements of \mathbf{G} , i.e., $g_{ii} = c, \forall i \in \mathcal{N}$ for any $c \geq 0$, without affecting the values of EVP.

Remark 2. If $g_{ij} = 0 \forall i, j, i \neq j$, we obtain from Eq. (1) $g_{ii} = (1/\lambda)g_{ii} \forall i$, and thus $\lambda = 1$. But $\mathbf{G} = \mathbf{I}$ (or, more generally, $\mathbf{G} = c\mathbf{I}$ according to Remark 1) is an uninteresting case, of course, as in this case all (pairwise) team effects would be absent. Therefore, we assumed $\mathbf{G} \geq \mathbf{I}$ in Assumption 1. \diamond

4.2. Existence and uniqueness of the eigenvalue productivity. This section contains our first two main results.

Proposition 1 (Existence of the EVP). *Let the set of team members be $\mathcal{N} = \{1, \dots, n\}$ with $n \in \mathcal{N}$. For any semipositive $n \times n$ matrix of pairwise, directional production coefficients \mathbf{G} , there exists a semipositive vector of eigenvalue productivities $\mathbf{p} \geq \mathbf{0}$.*

Proof. The proof follows from the existence of a nonnegative Perron vector for nonnegative matrices, see Theorem 1 in Appendix C. Then, since $\mathbf{p} \neq \mathbf{0}$ by definition of an eigenvector, we conclude that \mathbf{p} is semipositive. \square

The next result establishes the uniqueness of the EVP under the prerequisite that \mathbf{G} is irreducible.

Proposition 2 (Uniqueness of EVP). *For any nonnegative, irreducible $n \times n$ matrix of pairwise, directional production coefficients \mathbf{G} , the EVP-vector $\mathbf{p} \geq \mathbf{0}$ is unique up to a scalar multiple.*

Proof. The proof follows from the uniqueness of the nonnegative Perron vector for irreducible matrices, see Theorem 2, in particular part 3, in Appendix C. \square

If we had not assumed in Proposition 2 that \mathbf{G} was irreducible, the root $\hat{\lambda}$ would not necessarily have been a simple root, and the EVP-vector would not necessarily be unique. That is, \mathbf{p} and \mathbf{q} with $\mathbf{p} \neq \theta \mathbf{q}$ for any $\theta \in \mathbb{R}$ can be eigenvectors associated with $\hat{\lambda}$.

By Assumption 1, it holds that $\mathbf{G} \geq \mathbf{I}$, and thus $\mathbf{G} \neq \mathbf{I}$, so the Perron root is larger than one.

Corollary 1. *Given the semipositive matrix \mathbf{G} and some normalization of the diagonal elements $g_{ii} = c \geq 0$, the Perron root of \mathbf{G} satisfies $\hat{\lambda} > c$. In particular, for the normalization $g_{ii} = 1$, we have $\hat{\lambda} > 1$.*

Proof. The proof follows from the fact that $g_{ii} = c \geq 0$ implies $\mathbf{G} \geq c\mathbf{I}$ and from Lemma 3 in Appendix C. \square

Finally, if \mathbf{G} is an irreducible nonnegative matrix, then \mathbf{G} is primitive if it has at least one nonzero diagonal element (see Lemma 4 in Appendix C). Since we normalized the diagonal elements to unity,¹⁶ any irreducible productivity matrix \mathbf{G} is also primitive, and consequently, we may apply the Perron–Frobenius theorem for primitive matrices, (see Theorem 3 in Appendix C).

4.3. Properties of eigenvalue productivity. We use the following notation. We write $\mathbf{x}_{(-i)} \equiv (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$, and similarly we write $\mathbf{A}_{(-i)}$ for the matrix generated from \mathbf{A} by removing column i and row i . Also, let $\mathbf{x}_{(-[i])} \equiv (x_{i+1}, \dots, x_n)$ denote the vector after removal of the first i components. For any two vectors \mathbf{x} and \mathbf{y} , we write $\langle \mathbf{x}, \mathbf{y} \rangle$ to denote their (standard) inner product. We write \mathbf{e}_i to denote the i th unit vector of length n ; and $\mathbf{1}$ to denote the all-ones vector $(1, \dots, 1)^\top$ of length n . Also, let π be a permutation of the team members (*i.e.*, a one-to-one function from \mathcal{N} to itself), and let $\pi(\mathbf{G})$ represent the correspondingly permuted matrix where the rows and the columns of \mathbf{G} are permuted as specified by π . In particular, π_{ij} denotes the minimal permutation where only the indices of players i and j are permuted, *i.e.*, $\pi_{ij} : \mathcal{N} \rightarrow \mathcal{N}$ with $\pi_{ij}(i) = j$, $\pi_{ij}(j) = i$ and $\pi_{ij}(k) = k$ for all $k \neq i, j$. This allows us to define the i – j *permutation equality* of vectors \mathbf{x} and \mathbf{y} by $\mathbf{x} \stackrel{\circ}{=}_{ij} \mathbf{y} :\Leftrightarrow \pi_{ij}(\mathbf{x}) = \mathbf{y} \Leftrightarrow \pi_{ij}(\mathbf{y}) = \mathbf{x}$.

Let $\mathbf{g}_{i\bullet}$ denote the i th row of matrix \mathbf{G} ; and $\mathbf{g}_{\bullet j}$, the j th column of \mathbf{G} . We say that players i and j *contribute equally in total terms*, if $\langle \mathbf{g}_{i\bullet}, \mathbf{1} \rangle = \langle \mathbf{g}_{j\bullet}, \mathbf{1} \rangle$. A player $i \in \mathcal{N}$ who is contributing nothing to the productivity of the other team members, *i.e.*, $\mathbf{g}_{i\bullet}^\top = \mathbf{e}_i$, is referred to as a *null player*. Let $M(i) \equiv \{j \mid \mathbf{g}_{j\bullet} \stackrel{\circ}{=}_{ij} \mathbf{g}_{i\bullet}, j \in \mathcal{N}\}$ denote the set of players with (after suitable permutations) identical pairwise productivities as player i , including player i . We refer to a player $j \in C(i) \equiv \{j \mid g_{ij} = g_{ji} = 0, j \in M(i) \setminus \{i\}\}$ as a *clone* of i , and to $C(i)$ as the set of clones of i .

Using this notation, we formalize the following properties for a productivity measure φ , and then show that EVP has those properties. We say that a productivity measure $\varphi : \mathbb{R}_+^{n \times n} \rightarrow \mathbb{R}_+^n$ satisfies

symmetry: if for two players $i, j \in \mathcal{N}$, $\mathbf{g}_{i\bullet} \stackrel{\circ}{=}_{ij} \mathbf{g}_{j\bullet}$, then $\varphi_i = \varphi_j$.

permutation covariance: for any permutation π : $\pi(\varphi(\mathbf{G})) = \varphi(\pi(\mathbf{G}))$.

null player property: if player $i \in \mathcal{N}$ is a null player, then $\varphi_i = 0$.

aggregate balance: if all players contribute equally in total terms, then $\varphi_i = \varphi_j$, $\forall i, j \in \mathcal{N}$.

differentiability: φ is differentiable with respect to the team coefficients g_{ij} .

The properties *symmetry* and *permutation covariance*¹⁷ represent quite natural properties of a productivity measure. In particular, *symmetry* requires that players contributing

¹⁶The same argument applies to any positive normalization of the diagonal elements $g_{ii} = c > 0$.

¹⁷Property *permutation covariance* can be equivalently expressed as follows. Suppose π is a permutation. Let $\mathbf{P} = [p_{ij}]$ be the permutation matrix defined by π , *i.e.*, $p_{\pi(j)j} = 1$, $j = 1, \dots, n$, and $p_{ij} = 0$, for all $i \neq \pi(j)$. Then $\mathbf{P}^\top \varphi(\mathbf{G}) = \varphi(\mathbf{P}^\top \mathbf{G} \mathbf{P})$.

equally to each team member should be treated identically by the productivity measure, while *permutation covariance* demands that a renumbering (or renaming) of the players should not affect their productivity measures. Thus, upon renumbering the players, the productivity measures change accordingly, and in this sense the productivity measure is covariant with that renumbering.

The *null player property* states the natural requirement that a player contributing nothing to a team should be assigned a productivity measure of zero. The property of *aggregate balance* postulates that if *all* players contribute equally in *total* terms, then the same productivity measure should be assigned to each team member irrespective of the distribution of their pairwise productivities. This property does not require, though, that two players contributing in equal total terms have the same productivity measure, unless all other $n - 2$ players also contribute the same total amounts (or unless, due to *symmetry*, the two players contribute equally to each player). *differentiability* is an economically reasonable and mathematically convenient property implying that small productivity changes do not bring about abrupt changes in the productivity measure: rather, small perturbations of the productivity matrix lead to small changes in the productivity measure.¹⁸

In addition, we define the following monotonicity properties. We say that a productivity measure $\varphi : \mathbb{R}_+^{n \times n} \rightarrow \mathbb{R}_+^n$ satisfies

relative monotonicity: if the i th row of \mathbf{G} is perturbed by $d\mathbf{g}_{i\bullet} \geq \mathbf{0}^\top$, then the relative productivity measure of player i , φ_i/φ_j , $\forall i, j$, $i \neq j$, does not decrease.

absolute monotonicity: normalize φ such that $\varphi_i = 1$, then a semipositive perturbation of the i th row of \mathbf{G} by $d\mathbf{g}_{i\bullet} \geq \mathbf{0}^\top$ does not increase the productivity of any player $j \in \mathcal{N}$, $j \neq i$, i.e., $\varphi(\mathbf{G}) \geq \varphi(\mathbf{G} + \mathbf{e}_i \mathbf{g}_{i\bullet})$.

duplication monotonicity: if player $j \in \mathcal{N}$ is a clone of player $i \in \mathcal{N}$, then $\varphi_i(\mathcal{N}) \leq \varphi_i(\mathcal{N} \setminus \{j\})$.

The properties *relative monotonicity* and *absolute monotonicity* characterize the behavior of the productivity measure if the pairwise production coefficients of player i (weakly) increase; that is, they characterize the effects of semipositive perturbations of the i th row of \mathbf{G} . Naturally, since a semipositive perturbation of the i th row of \mathbf{G} means that player i becomes more productive than at least one other player, we should expect the productivity measure to respect this increase in productivity of player i . In particular, *relative monotonicity* and *absolute monotonicity* say that, after suitable normalization, the (relative) productivity measure of player i should not decrease in response to an increase in the pairwise contributions of that player. For this reason, we refer to these properties as monotonicity.

Finally, the property *duplication monotonicity* states that if we add a clone of player i to the team \mathcal{N} , the EVP of player i , and thus of all players of type i , i.e., all players $j \in M(i)$, does not increase. Loosely speaking, enlarging a team by adding clones (weakly) decreases the EVP of all players whose characteristics are duplicated. Intuitively, the more clones of

¹⁸The results for the derivatives of the Perron vector with respect to the matrix elements can be found, for example, in Meyer and Stewart (1988) and Deutsch and Neumann (1985).

player i the team contains, the less crucial or the more dispensable this player (or type of a player) becomes.

Proposition 3 (Properties of EVP). *Let \mathbf{G} satisfy Assumption 1. Then, the productivity measure EVP, $\mathbf{p} \geq \mathbf{0}$, has the properties of symmetry, permutation covariance, null player property, aggregate balance, differentiability, relative monotonicity, absolute monotonicity, and duplication monotonicity.*

Proof. Proof of *symmetry*. Let two players have identical pairwise productivities. W.l.o.g. we permute the ordering of these players so that we have for players 1 and 2: $\mathbf{g}_{1\bullet} \stackrel{\circ}{=}_{12} \mathbf{g}_{2\bullet}$. Assume, on the contrary, $p_1 \neq p_2$. We obtain from the first two rows of Eq. (2) $\lambda p_1 = \langle \mathbf{g}_{1\bullet}, \mathbf{p} \rangle$ and $\lambda p_2 = \langle \mathbf{g}_{2\bullet}, \mathbf{p} \rangle$. Subtracting both equations and using $g_{12} = g_{21}$ yields

$$(1 - g_{12})p_1 + (g_{12} - 1)p_2 = \lambda(p_1 - p_2) \quad \Leftrightarrow \quad (\lambda - 1 + g_{12})(p_1 - p_2) = 0.$$

Since by Lemma 1 $\lambda > 1$, and $g_{12} \geq 0$, this is impossible, and thus $p_1 \neq p_2$ implies a contradiction.

The proof of *permutation covariance* follows from Lemma 2, see Appendix C, page 30.

Proof of *null player property*. W.l.o.g. we permute the ordering of the players such that the null player gets index 1. Then,

$$\mathbf{G} = \begin{pmatrix} 1 & \mathbf{0}^\top \\ \mathbf{b} & \mathbf{G}_{(-1)} \end{pmatrix}.$$

If \mathbf{p} is the eigenvector associated with $\hat{\lambda}$, we obtain from Eq. (2)

$$\begin{aligned} \mathbf{0} &= (\mathbf{G} - \hat{\lambda} \mathbf{I}_n) \cdot \mathbf{p} \\ &= \left(\begin{pmatrix} 1 & \mathbf{0}^\top \\ \mathbf{b} & \mathbf{G}_{(-1)} \end{pmatrix} - \hat{\lambda} \begin{pmatrix} 1 & \mathbf{0}^\top \\ \mathbf{0} & \mathbf{I}_{n-1} \end{pmatrix} \right) \cdot \begin{pmatrix} p_1 \\ \mathbf{p}_{(-1)} \end{pmatrix} \\ &= \begin{pmatrix} 1 - \hat{\lambda} & \mathbf{0}^\top \\ \mathbf{b} & \mathbf{G}_{(-1)} - \hat{\lambda} \mathbf{I}_{n-1} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ \mathbf{p}_{(-1)} \end{pmatrix} \\ \Leftrightarrow \quad p_1 &= 0 \wedge (\mathbf{G}_{(-1)} - \hat{\lambda} \mathbf{I}_{n-1}) \cdot \mathbf{p}_{(-1)} = \mathbf{0}, \end{aligned} \tag{4}$$

where we again used the fact that $\lambda > 1$ by Lemma 1.

Proof of *aggregate balance*. If \mathbf{G} has a constant row sum of μ , we have $\mathbf{G} \cdot \mathbf{1} = \mu \mathbf{1}$, implying that μ is an eigenvalue of \mathbf{G} and $\mathbf{1}$ is the associated eigenvector. Since \mathbf{G} is irreducible, μ equals the Perron root $\hat{\lambda}$ (alternatively, this follows from Lemma 3) and its associated eigenvector is unique (up to a scalar multiple).

Proof of *differentiability*. A proof of the differentiability of a simple eigenvector with respect to the entries of the matrix can be found in Wilkinson (1965, pp. 66-77).

Proof of *relative monotonicity*. See Theorem 2.1 and the proof thereof in Elsner et al. (1982). We reproduce that theorem in Appendix D, see Theorem 4 on page 32.

Proof of *absolute monotonicity*. See Theorem 3.1 and the proof thereof in Elsner et al. (1982). We reproduce that theorem in Appendix D, see Theorem 5 on page 32.

Proof of *duplication monotonicity*. By definition of a clone, we have $\mathbf{g}_{1\bullet} \stackrel{\circ}{=}_{ij} \mathbf{g}_{2\bullet}$ with the first two elements being 1 and 0 (and the same holds for the first two columns *i.e.* $\mathbf{g}_{\bullet 1} \stackrel{\circ}{=}_{ij} \mathbf{g}_{\bullet 2}$). Hence,

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & \mathbf{x}^\top \\ 0 & 1 & \mathbf{x}^\top \\ \mathbf{y} & \mathbf{y} & \mathbf{B} \end{pmatrix} \quad \text{and} \quad \mathbf{G}_{(-1)} = \begin{pmatrix} 1 & \mathbf{x}^\top \\ \mathbf{y} & \mathbf{B} \end{pmatrix},$$

with length $n-2$ vectors $\mathbf{x}, \mathbf{y} \geq \mathbf{0}$, and an $(n-2) \times (n-2)$ matrix \mathbf{B} satisfying Assumption 1. Let (μ, \mathbf{q}) be the leading eigenpair (*i.e.* the Perron root and the Perron eigenvector) of \mathbf{G} , then

$$\mathbf{0} = (\mathbf{G} - \mu \mathbf{I}) \cdot \mathbf{q} = \left(\begin{pmatrix} 1 & 0 & \mathbf{x}^\top \\ 0 & 1 & \mathbf{x}^\top \\ \mathbf{y} & \mathbf{y} & \mathbf{B} \end{pmatrix} - \mu \mathbf{I} \right) \cdot \begin{pmatrix} 1 \\ 1 \\ \mathbf{q}_{(-[2])} \end{pmatrix} = \begin{pmatrix} 1 - \mu + \mathbf{x}^\top \cdot \mathbf{q}_{(-[2])} \\ 1 - \mu + \mathbf{x}^\top \cdot \mathbf{q}_{(-[2])} \\ 2\mathbf{y} + (\mathbf{B} - \mu \mathbf{I})\mathbf{q}_{(-[2])} \end{pmatrix}.$$

Since the first two equations coincide, we may drop either of both, arriving at

$$\mathbf{0} = \left(\begin{pmatrix} 1 & \mathbf{x}^\top \\ 2\mathbf{y} & \mathbf{B} \end{pmatrix} - \mu \mathbf{I} \right) \cdot \begin{pmatrix} 1 \\ \mathbf{q}_{(-[2])} \end{pmatrix} = (\tilde{\mathbf{G}} - \mu \mathbf{I}) \cdot \mathbf{q}_{(-1)},$$

where

$$\tilde{\mathbf{G}} \equiv \begin{pmatrix} 1 & \mathbf{x}^\top \\ 2\mathbf{y} & \mathbf{B} \end{pmatrix} = \mathbf{G}_{(-1)} + \begin{pmatrix} 0 & \mathbf{0}^\top \\ \mathbf{y} & \mathbf{0} \end{pmatrix} = \mathbf{G}_{(-1)} + \mathbf{Y},$$

with $\mathbf{Y} \equiv \tilde{\mathbf{y}} \cdot \mathbf{e}_1^\top$ and $\tilde{\mathbf{y}} \equiv (0, \mathbf{y}^\top)^\top$. We decompose the perturbation

$$\mathbf{Y} = \sum_{i=2}^n \mathbf{Y}_i, \quad \text{with} \quad \mathbf{Y}_i \equiv \tilde{y}_i \mathbf{e}_i \cdot \mathbf{e}_1^\top.$$

into $n-1$ successive perturbations $\mathbf{Y}_i, i = 2, \dots, n$. Since perturbation \mathbf{Y}_i (weakly) increases the i th row of $\mathbf{G}_{(-1)}$ leaving all other rows unaffected, we know from Theorem 5 that this perturbation (weakly) increases the i th component of the (normalized) Perron vector of $\mathbf{G}_{(-1)}$. Successively repeating this argument, all $n-1$ components of the Perron vector are (weakly) increased relative to the first component. \square

EVP thus has a set of desirable properties. Since we explained them above, we may restrict ourselves here to some additional comments related to the specific structure of the EVP. It is a remarkable observation that while the Perron root of \mathbf{G} is a non-decreasing function of positive perturbations,¹⁹ the components of the corresponding (normalized) Perron vector are non-increasing. Since this translates into a relative increase in the EVP of the player whose row-entries are semipositively perturbed, *i.e.*, weakly increased, this is a

¹⁹As we are interested in the Perron vector, we do not explicitly discuss the properties of the Perron root. Among others, Seneta (1981), Berman and Plemmons (1994), Varga (2009), and Pinkus (2010) analyze how the eigenvalues of a matrix vary as functions of the elements of the matrix and show that the eigenvalues, and hence the Perron root, exhibit nice monotonicity properties.

plausible, and even natural feature of the EVP. In view of the property *permutation covariance*, the self-referential structure, and the linearity of the EVP, it is quite intuitive that the property *aggregate balance* holds: if all players contribute by the same total amount to the team, the same productivity measure will be assigned to all players. Notably, the reverse also holds, so that we infer:

Corollary 2. *The EVPs of all players coincide if, and only if, all players have equal total productivities.*

$$\forall i, j \in \mathcal{N} : \langle \mathbf{g}_{i\bullet}, \mathbf{1} \rangle = \langle \mathbf{g}_{j\bullet}, \mathbf{1} \rangle \quad \Leftrightarrow \quad \mathbf{p} = \alpha \mathbf{1}, \alpha > 0.$$

Proof. Since \mathbf{p} is an eigenvector and λ the associated eigenvalue of \mathbf{G} , we have by Eq. (1), $\mathbf{G} \cdot \alpha \mathbf{1} = \lambda \alpha \mathbf{1} \Leftrightarrow \mathbf{G} \cdot \mathbf{1} = \lambda \mathbf{1} \Leftrightarrow \langle \mathbf{g}_{i\bullet}, \mathbf{1} \rangle = \lambda, \forall i = 1, \dots, n.$ \square

While a sufficient condition for two players having the same productivity measure is that, by *symmetry*, both players equally contribute to the productivities of all players, equal pairwise contributions are not necessary for two players to have the same productivity measure.

Example 1. A simple example is the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

with eigenvalues $\{3, -1, 1, 1\}$ and an eigenvector associated with the (absolutely) largest eigenvalue, $(2, 1, 1, 1)^\top$. Hence, players 2, 3, and 4 all have the same EVP although their pairwise contributions do not coincide. Also, this example illustrates that the reverse implication of *aggregate balance* does not hold: equal total contributions are not necessary for two players to have equal productivity measures: players 3 and 4 have the same EVP although their total contributions do not coincide, as $\mathbf{A} \cdot \mathbf{1} = (5, 3, 3, 2)^\top$. \diamond

The property *null player property* not only ensures that such a player obtains an EVP equal to zero, but also that that we may add or remove null players from the team without affecting the EVPs of the other players (including the null players).

Corollary 3. *A null player can be removed from (or added to) a team without affecting the EVP of any player, that is, $\mathbf{p}_{(-i)}(\mathbf{G}) = \mathbf{p}(\mathbf{G}_{(-i)})$.*

Proof. This result follows from the proof of Proposition 3, see Eq. (4). \square

Comparing the EVPs of the teams \mathcal{N} and $\mathcal{N} \setminus \{i\}$ where i is a dummy player, we see from Corollary 3 that $\mathbf{p}(\mathbf{G})$ and $\mathbf{p}(\mathbf{G}_{(-i)})$ only differ by the element $p_i = 0$ (the EVP of the dummy player), which is contained in $\mathbf{p}(\mathbf{G})$ but not in $\mathbf{p}(\mathbf{G}_{(-i)})$. In this sense, the EVP vector of the team is unaffected by the removal (or the addition) of null players.

In this section we analyzed and discussed the properties of the EVP, presuming a pairwise productivity matrix \mathbf{G} . The following section demonstrates how \mathbf{G} can actually be obtained in a given situation.

5. CALCULATION OF EIGENVALUE PRODUCTIVITY

In this section we first show how to calculate the productivity matrix \mathbf{G} in general, and then we employ two examples to illustrate the applicability of our approach. The first example is a small artificial example, the second a real-world example where we consider a group of directors, actors and directors of photography who work together to produce different movies. (A third example of a team consisting of a group of researchers who collaborate on different research projects, co-authorships in publications, can be found in Appendix B.) In all cases, we have a situation where teams work in different compositions to realize a project, *e.g.*, movies, publications, and where a suitable measure of success is available. Variations in the team composition and in the success of the project are essential, as those allow us to calculate the matrix \mathbf{G} .

5.1. Calculation of the matrix \mathbf{G} in general. First, delete from the team, *i.e.*, from the set \mathcal{N} , those workers who have never been in action during the given period. Then, calculate the entries of \mathbf{G} as follows: For each given pair²⁰ of workers $\{i, j\}$, $i, j \in \mathcal{N}$, consider those projects in which i and j have worked together, that is, where both were included in the team composition. Calculate the ratio of the points²¹ achieved in these projects to the maximal number of points the teams including the pair $\{i, j\}$ could have potentially achieved. Let s_{ij} denote this point ratio, which measures the success (performance) of the pair $\{i, j\}$ over all team compositions. Next, consider those projects where worker i was a member of the team (worker j may or may not have been a member of the team), and calculate the points the respective teams have achieved in these projects divided by the maximal number of points those teams could have potentially achieved; denote this point ratio by $s_i \equiv s_{ii}$, which measures the success (performance) of worker i . Collecting all pair-success ratios gives the symmetric matrix $\mathbf{S} := (s_{ij})_{i,j \in \mathcal{N}}$.

Then define $g_{ij} := s_{ij}/s_j$, which represents the success of the pair $\{i, j\}$ relative to the success of worker j , an effect which is arguably associated with the presence of worker i . This allows us to define \mathbf{G} by $\mathbf{G} := (g_{ij})_{i,j \in \mathcal{N}}$. As a convention, for pairs of workers $\{i, j\}$ that have never been jointly included in some team composition during the period, we set $s_{ij} = \sqrt{s_i s_j}$ and thus $g_{ij} = \sqrt{s_i/s_j}$. Observe that \mathbf{G} is nonnegative, and (generically) not symmetric.

Next, in order to obtain the entries of \mathbf{G} , we relate the performance of a pair of workers to the performance of either of its two workers. More precisely, comparing the performance of the pair $\{i, j\}$, measured by s_{ij} , with the performance of worker j , s_j , we obtain the ratio

²⁰In this context, the word *pair* refers to an unordered pair; that is, to a set containing exactly two elements.

²¹For convenience, we use the term *points* to refer to the success measure.

TABLE 1. Results of the team for varying compositions

| no. | compositions | projects | points | max. pts. | ratio |
|-----|--------------|----------|--------|-----------|-----------------|
| 1 | <i>ACD</i> | 2 | 3 | 6 | $\frac{1}{2}$ |
| 2 | <i>ACE</i> | 4 | 12 | 12 | 1 |
| 3 | <i>ADE</i> | 3 | 1 | 9 | $\frac{1}{9}$ |
| 4 | <i>BCD</i> | 3 | 4 | 9 | $\frac{4}{9}$ |
| 5 | <i>BCE</i> | 2 | 6 | 6 | 1 |
| 6 | <i>BDE</i> | 3 | 0 | 9 | 0 |
| | sum | 17 | 26 | 51 | $\frac{26}{51}$ |

$g_{ij} := s_{ij}/s_j$ for all team members $i, j \in \mathcal{N}$. Then, g_{ij} measures the relative performance of the pair $\{i, j\}$ compared to the overall performance of worker j . In this way we are able to build the matrix \mathbf{G} , where, by construction, the main diagonal elements equal unity, *i.e.*, $g_{ii} = 1$ for all $i \in \mathcal{N}$. Then, the i th row of \mathbf{G} gives the performance of each pair $\{i, j\}$ (i fixed) normalized by the performance of worker j . Loosely speaking, the elements of the i th row capture the increase in the productivity of each worker due to the contribution of worker i . Inversely, the i th column of \mathbf{G} represents how each of the team members contributes to the performance of worker i . In this way, \mathbf{G} represents all relative normalized pairwise performance measures.

5.2. Example 1: A simple example. Consider a team consisting of six workers $N = \{A, B, C, D, E, F\}$. Assume that over a specific time period the team has put into effect 17 projects in six different compositions, and has realized 26 out of 51 possible (abstract) points of success. The detailed results for the specific team compositions are displayed in Table 1.

Apparently worker F has never been included in a team during the year, so that F 's team productivity can neither be reasonably defined nor measured. Accordingly, we completely disregard worker F henceforth. Moreover, observe that workers A and B were never included in the same team composition, a feature which may occur, for example, for workers with the same area of expertise, *e.g.*, IT specialists, or in team sports for players playing the same position, *e.g.*, goalies.

Before we proceed to calculate the EVP of each single worker, it is worthwhile to pause for a second and to inspect Table 1 for the individual contributions of the workers. Apparently, the success of the team has improved whenever C joined the team: compare the composition *ADE* with *ACD*, *ADE* with *ACE*, *BDE* with *BCD*, and *BDE* with *BCE*. In all of these comparisons, the ratio of achieved points to maximal points has gone up by replacing either D or E by C . It thus appears that the coworker productivity of C is relatively high. This should be reflected in our measure of individual productivity, the EVP. Also, it is easy to verify that the team performance has improved whenever B has been replaced by A ; and that the team performance has declined whenever D has been included. Accordingly, the EVP should assign a higher coworker productivity to A than to B , and it should assign

TABLE 2. Individual results for each worker

| worker | incl. in composition | points | max. pts. | s_i |
|--------|----------------------|--------|-----------|-----------------|
| A | $\{1, 2, 3\}$ | 16 | 27 | $\frac{16}{27}$ |
| B | $\{4, 5, 6\}$ | 10 | 24 | $\frac{5}{12}$ |
| C | $\{1, 2, 4, 5\}$ | 25 | 33 | $\frac{25}{33}$ |
| D | $\{1, 3, 4, 6\}$ | 8 | 33 | $\frac{8}{33}$ |
| E | $\{2, 3, 5, 6\}$ | 19 | 36 | $\frac{19}{36}$ |

worker D a particularly poor coworker productivity. At the end of our calculation of the EVPs, we shall see that this is exactly the case.

From Table 1 we calculate the results for each single worker i by disregarding those projects where i was not included in the composition. The individual results are shown in Table 2. In the next step we have to calculate the success for each pair of workers $\{i, j\}, i, j \in N$, which is done in Table 3.

TABLE 3. Pair results

| pair | incl. in composition | points | max. pts. | s_{ij} |
|------------|----------------------|--------|-----------|-----------------|
| $\{A, C\}$ | $\{1, 2\}$ | 15 | 18 | $\frac{5}{6}$ |
| $\{A, D\}$ | $\{1, 3\}$ | 4 | 15 | $\frac{4}{15}$ |
| $\{A, E\}$ | $\{2, 3\}$ | 13 | 21 | $\frac{13}{21}$ |
| $\{B, C\}$ | $\{4, 5\}$ | 10 | 15 | $\frac{2}{3}$ |
| $\{B, D\}$ | $\{4, 6\}$ | 4 | 18 | $\frac{2}{9}$ |
| $\{B, E\}$ | $\{5, 6\}$ | 6 | 15 | $\frac{2}{5}$ |
| $\{C, D\}$ | $\{1, 4\}$ | 7 | 15 | $\frac{7}{15}$ |
| $\{C, E\}$ | $\{2, 5\}$ | 18 | 18 | 1 |
| $\{D, E\}$ | $\{3, 6\}$ | 1 | 18 | $\frac{1}{18}$ |

Using that data we build the matrix \mathbf{S} , the matrix of pairwise success,²² displayed in Eq. 5. We then proceed to calculate the entries of \mathbf{G} (displayed also in Eq. 5) as described in the last subsection.

$$\mathbf{S} = \begin{pmatrix} \frac{16}{27} & \frac{2\sqrt{5}}{9} & \frac{5}{6} & \frac{4}{15} & \frac{13}{21} \\ \frac{2\sqrt{5}}{9} & \frac{5}{12} & \frac{2}{3} & \frac{2}{9} & \frac{2}{5} \\ \frac{5}{6} & \frac{2}{3} & \frac{25}{33} & \frac{7}{15} & 1 \\ \frac{4}{15} & \frac{2}{9} & \frac{7}{15} & \frac{8}{33} & \frac{1}{18} \\ \frac{13}{21} & \frac{2}{5} & 1 & \frac{1}{18} & \frac{36}{36} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} 1 & \frac{8}{3\sqrt{5}} & \frac{11}{10} & \frac{11}{10} & \frac{156}{133} \\ \frac{3\sqrt{5}}{8} & 1 & \frac{22}{25} & \frac{11}{12} & \frac{72}{95} \\ \frac{45}{32} & \frac{8}{5} & 1 & \frac{77}{40} & \frac{36}{19} \\ \frac{9}{20} & \frac{8}{15} & \frac{77}{125} & 1 & \frac{2}{19} \\ \frac{117}{112} & \frac{24}{25} & \frac{33}{25} & \frac{11}{48} & 1 \end{pmatrix} \quad (5)$$

Finally, we have to calculate the eigenvalues of \mathbf{G} , which are 4.97, 0.8665, -0.8585 , 0.1017, and -0.0797 . Since the first eigenvalue is the (absolutely) largest one, we obtain

²²Recall, that in building \mathbf{S} we have used the conventions $s_{ii} := s_i$ and $s_{ij} := \sqrt{s_i s_j}$ for pairs $\{i, j\}$ who have not been included in any team composition, which here applies only to $\{A, B\}$.

$\hat{\lambda} = 4.97$, and the associated eigenvector is (after normalizing by the first component) $\mathbf{p}(\hat{\lambda}) = (1, 0.7849, 1.3276, 0.4492, 0.9203)$, which is the vector we have been looking for: the between-EVP vector.²³

As appraised earlier when inspecting Table 1, the EVPs of worker A and C are high, while those of B and D are low. Apparently, the EVP reflects our intuitive notion of coworker productivity.

5.3. Example 2: Success of teams in movies. The movie industry offers suitable data to exemplify the applicability of EVP. Since movies are produced by varying teams of directors, actors, directors of photography, writers etc., team compositions change from one movie to another. Apparently, for any given genre of movies, some teams perform better or are more successful than others, implying that the team composition is crucial for the specific success of a movie. At the same time, success measures for movies are readily available: numbers of moviegoers, license fees, awards received, rankings in journals or on internet platforms may be used as indicators of success or quality.

In order to illustrate that our concept of EVP also functions well on this data, we consider a particular class of movies and the team of actors and directors of photography producing them, and compute the EVPs for all of those team members. To keep the example tractable, and thus the number of movies manageable, we consider a class of movies where the castings have significant overlaps, so that pairs of team members repeatedly take part in different movies of this class. To this end, we restrict the class of movies to a specific class: movies directed by the German director, movie maker and actor Rainer Werner Fassbinder.²⁴ Among other works, his oeuvre includes 40 feature-length movies that he completed between 1969 and 1982.²⁵ From those 40 movies we exclude *Theater in Trance*, because it is a documentary movie, and *Wie ein Vogel auf dem Draht*, because it is basically a one-woman TV show for and of Brigitte Mira. Disregarding those movies, our data set contains 38 movies with 38 different teams. By construction, as a producer Fassbinder is a member of all teams, and thus we remove him unless he (also) has a role as an actor (9 movies). Our data set contains the main actors, including those who play a ‘significant’ part, and the directors of photography for those 38 movies—together 23 team members. (The complete list of these 38 movies and 23 team members is included in Tables 4 and 9, respectively, the latter displayed in Appendix B. The data was collected on the Homepage of the Rainer Werner Fassbinder Foundation (Fassbinder, 2017).) We use the *IMDb*–user rating as an indicator of success of a movie (IMDb.com, 2017).

²³Note that we may normalize $\mathbf{p}(\hat{\lambda})$ in any way which appears to be convenient: thus, any multiple of $\mathbf{p}(\hat{\lambda})$: $\alpha\mathbf{p}(\hat{\lambda})$, $\alpha > 0$ may also serve as a vector of coworker productivities. In particular, we could have chosen to normalize so that $\sum \mathbf{p}_i(\hat{\lambda}) = n$ yielding $\mathbf{p}(\hat{\lambda}) = (1.1156, 0.8756, 1.481, 0.5011, 1.0267)$.

²⁴Fassbinder, born May 31, 1945 in Bad Wörishofen, Bavaria, Germany, died June 10, 1982 in Munich, Germany, is considered as one of the most important representatives of the so-called New German Cinema movement.

²⁵In addition, he produced three short movies (*This Night*, *Der Stadstreicher*, and *Das kleine Chaos*), an episode within the movie *Deutschland im Herbst*, and a TV-version of the cinema movie *Bolwieser*.

TABLE 4. EVP-ranking of the artist

| rank | artist | label | no. movies | <i>IMDb</i> | EVP |
|------|--------------------------|----------|------------|-------------|-------|
| 1 | Irm Herrmann | <i>F</i> | 12 | 0.768 | 1.048 |
| 2 | Lilo Pempeit | <i>K</i> | 11 | 0.773 | 1.043 |
| 3 | Karl Scheydt | <i>N</i> | 9 | 0.761 | 1.04 |
| 4 | Klaus Löwitsch | <i>I</i> | 6 | 0.763 | 1.036 |
| 5 | Xavier Schwarzenberger | <i>W</i> | 5 | 0.772 | 1.034 |
| 6 | Juliane Lorenz | <i>S</i> | 10 | 0.759 | 1.024 |
| 7 | Kurt Raab | <i>L</i> | 10 | 0.754 | 1.021 |
| 8 | Michael Ballhaus | <i>U</i> | 14 | 0.754 | 1.013 |
| 9 | Jürgen Jürgens | <i>V</i> | 4 | 0.752 | 1.01 |
| 10 | Volker Sprengler | <i>P</i> | 6 | 0.738 | 1.001 |
| 11 | Ulli Lommel | <i>J</i> | 10 | 0.734 | 1. |
| 12 | Rudolf Waldemar Brehm | <i>B</i> | 5 | 0.722 | 0.995 |
| 13 | Hans Hirschmüller | <i>G</i> | 5 | 0.732 | 0.995 |
| 14 | Margit Carstensen | <i>C</i> | 12 | 0.733 | 0.988 |
| 15 | Hannah Schygulla | <i>O</i> | 17 | 0.716 | 0.986 |
| 16 | Rainer Werner Fassbinder | <i>E</i> | 9 | 0.717 | 0.986 |
| 17 | Thea Eymesz | <i>R</i> | 13 | 0.719 | 0.985 |
| 18 | Ingrid Caven | <i>D</i> | 8 | 0.729 | 0.979 |
| 19 | Dietrich Lohman | <i>T</i> | 13 | 0.708 | 0.977 |
| 20 | Franz Walsch | <i>Q</i> | 14 | 0.721 | 0.969 |
| 21 | Harry Baer | <i>A</i> | 11 | 0.71 | 0.968 |
| 22 | Katrin Schaake | <i>M</i> | 4 | 0.693 | 0.96 |
| 23 | Günther Kaufmann | <i>H</i> | 9 | 0.692 | 0.943 |

Since we repeat the same steps as in Example 1, we only provide the basic results. First, we use the *IMDb*-ratings to compute *IMDb*-based scores for each artist, which are displayed in the fifth column of Table 4. Then, we compute the *IMDb*-based scores for each pair of artists, and use this data to compute the matrices \mathbf{S} and \mathbf{G} (both of which can be found in Appendix B, see page 30). Finally, we use matrix \mathbf{G} to compute the EVP-vector: We find that the (absolutely) largest eigenvalue is $\hat{\lambda} = 22.9679$, and the associated eigenvector is (after normalizing $\sum \mathbf{p}_i = n$)

$$\mathbf{p}(\hat{\lambda}) = (0.968, 0.995, 0.988, 0.979, 0.986, 1.048, 0.995, 0.943, 1.036, 1., 1.043, 1.021, 0.96, 1.04, 0.986, 1.001, 0.969, 0.985, 1.024, 0.977, 1.013, 1.01, 1.034) .$$

The ranking of the artists implied by $\mathbf{p}(\hat{\lambda})$ is provided in Table 4.²⁶

²⁶The coefficient of correlation between EVP and the *IMDb*-score is 0.9601.

6. CONCLUSION

In this paper, we treated a classic problem in labor and personnel economics: how can one measure individual contributions to a team or a group? Owing to the fact that a team's output is, by definition, produced jointly, and frequently simultaneously, coworker productivity, *i.e.*, individual contributions to the joint output, is both difficult to conceptualize and hard to measure. In this paper, we contribute to resolving both issues. We first derived a new concept with which to measure coworker productivity, which we referred to as 'eigenvalue productivity' (EVP), as it is built upon eigenvector centrality, a concept well established in network analysis. Then, we showed that under rather mild regularity conditions, such an EVP vector, measuring the contributions of all team members, exists and is unique; and we demonstrated that this vector has convenient economic properties, namely *symmetry*, *permutation covariance*, *null player property*, *aggregate balance*, *differentiability*, *relative monotonicity*, *absolute monotonicity*, and *duplication monotonicity*. Finally, using data of team success, we provided one artificial and two real-world examples to demonstrate how the productivity matrix can be generated, and then calculated the EVP vector for all cases.

With a procedure for obtaining the productivity matrix from the compositions of a given team and its performance over a period, we are equipped with an applicable method to calculate the individual productivity of each worker, as defined by the EVP. Since this or a similar type of data is frequently available, our concept can be widely used to calculate coworker productivities. Therefore, our concept can be easily employed in empirical work in labor economics (and team sports economics), as it can be suitably adapted to a specific data set. In sum, we showed that EVP is a properly defined concept, which has several economically desirable properties and is suitable for real-world data. We are therefore confident that EVP not only constitutes a significant theoretical contribution, but may also help calculate coworker productivities in applied empirical work.

APPENDIX A. EXAMPLE 3: A RESEARCH GROUP

We consider a team consisting of a group of researchers of various economics departments at European universities²⁷ and take their publication records as a measure of the success of cooperation within this team. As we are interested in the mutual productivity effects between the members of this group and their implied pairwise productivity coefficients, we disregard any stand-alone work as well as joint work with external, *i.e.* out of group researchers. Hence, we drop all external coauthors and—after removal of those—also disregard all single-authored publications. This procedure helps focus on the immediate cooperation of the members of the research group and their productivity effects; also, it avoids boosting the example excessively by including (a large number of) external researchers.

After removing those publications that are not coauthored by at least two members of the research group, the team consists of 14 members labeled by

$$\mathcal{N} = \{A, B, C, F, G, H, I, J, K, L, M, N, O, P\}.$$

The members of team \mathcal{N} have published 45 articles in peer-reviewed journals within the two-year period 2015–2016. They collaborated in 26 different teams of coauthors, publishing up to seven research articles a team. Each member of \mathcal{N} collaborated in up to 13 different teams of coauthors, publishing up to 15 research articles (team member C).

To measure the significance of a publication (success of a coauthorship) we use the ranking of economics journals established by the German economic newspaper *Handelsblatt* (*HB*) in 2015, (Handelsblatt, 2015). This list assigns each (established) journal to either of the categories: A+, A, B+, B, C+, C, and D, which are associated with the cardinal measures (points) 1, 0.6, 0.3, 0.2, 0.15, 0.1 and 0.05, respectively. We multiply these points with 100 such that the maximal possible points for a publication are 100 for a publication in a top-ranked journal (A+), and 5 for a mediocre-ranked journal (D).²⁸

Table 5 represents our database consisting of the publication data associated with the joint publications of the team members. The columns of this table display the (internal) coauthors, the number of publications of this team of coauthors (frequency with which the team of authors appears in our data set), the number of all coauthors (internal and external), the number of internal coauthors (members of \mathcal{N}) and the ranking score of the journal. The average points achieved by a paper amounts to 22.67, while the average points achieved by a team of coauthors amounts to 22.02, both of which is slightly better than a publication in a B-journal. Using the data set shown in Table 5, we compile the data for any particular team; this data is provided in Table 6.

From Table 5 we calculate the results for each single researcher $i \in \mathcal{N}$ by focusing on those projects where i was included in the team of authors. The individual results are shown in Table 7. The individual success ratio s_i represents the individual *HB*-score. While this

²⁷The data source is an existing research group, but to guarantee anonymity we do not use real names. The details of the publication records were collected from the researchers' personal and/or institutional websites.

²⁸Publications in journals that were not included in the ranking received 5 points.

TABLE 5. Results of the research group

| no. | team of coauthors | no. pub- lications | no. authors | no. authors internal | journal ranking |
|-----|----------------------|-----------------------|-------------|-------------------------|--------------------|
| 1 | <i>ACK</i> | 1 | 5 | 3 | 100 |
| 2 | <i>AO</i> | 7 | 7 | 2 | 10 |
| 3 | <i>BCK</i> | 1 | 4 | 3 | 20 |
| 4 | <i>CFK</i> | 1 | 4 | 3 | 10 |
| 5 | <i>AMO</i> | 3 | 4 | 3 | 60 |
| 6 | <i>AO</i> | 7 | 4 | 2 | 5 |
| 7 | <i>AMO</i> | 3 | 4 | 3 | 60 |
| 8 | <i>BM</i> | 1 | 2 | 2 | 5 |
| 9 | <i>BC</i> | 2 | 2 | 2 | 20 |
| 10 | <i>BC</i> | 2 | 3 | 2 | 5 |
| 11 | <i>CJMN</i> | 1 | 4 | 4 | 20 |
| 12 | <i>CH</i> | 1 | 3 | 2 | 20 |
| 13 | <i>CHIK</i> | 1 | 5 | 4 | 5 |
| 14 | <i>HO</i> | 3 | 5 | 2 | 10 |
| 15 | <i>HO</i> | 3 | 4 | 2 | 5 |
| 16 | <i>CK</i> | 1 | 2 | 2 | 60 |
| 17 | <i>BCFM</i> | 1 | 5 | 4 | 60 |
| 18 | <i>JK</i> | 1 | 5 | 2 | 5 |
| 19 | <i>JP</i> | 4 | 3 | 2 | 60 |
| 20 | <i>JLP</i> | 2 | 4 | 3 | 5 |
| 21 | <i>BJ</i> | 2 | 3 | 2 | 10 |
| 22 | <i>JP</i> | 4 | 4 | 2 | 30 |
| 23 | <i>JLP</i> | 2 | 3 | 3 | 5 |
| 24 | <i>KN</i> | 1 | 13 | 2 | 5 |
| 25 | <i>IK</i> | 3 | 3 | 2 | 5 |
| 26 | <i>IK</i> | 3 | 3 | 2 | 20 |
| 27 | <i>FK</i> | 1 | 3 | 2 | 10 |
| 28 | <i>CGK</i> | 1 | 4 | 3 | 5 |
| 29 | <i>CHKL</i> | 1 | 4 | 4 | 5 |
| 30 | <i>IK</i> | 3 | 2 | 2 | 20 |
| 31 | <i>AO</i> | 7 | 2 | 2 | 30 |
| 32 | <i>AO</i> | 7 | 3 | 2 | 20 |
| 33 | <i>AMO</i> | 3 | 3 | 3 | 60 |
| 34 | <i>AO</i> | 7 | 2 | 2 | 15 |
| 35 | <i>AO</i> | 7 | 5 | 2 | 30 |
| 36 | <i>BJ</i> | 2 | 3 | 2 | 5 |
| 37 | <i>CMN</i> | 2 | 3 | 3 | 20 |
| 38 | <i>CN</i> | 1 | 3 | 2 | 10 |
| 39 | <i>CMN</i> | 2 | 4 | 3 | 60 |
| 40 | <i>MN</i> | 1 | 2 | 2 | 10 |
| 41 | <i>HO</i> | 3 | 4 | 2 | 5 |
| 42 | <i>AO</i> | 7 | 4 | 2 | 10 |
| 43 | <i>JP</i> | 4 | 2 | 2 | 5 |
| 44 | <i>JP</i> | 4 | 3 | 2 | 60 |
| 45 | <i>KP</i> | 1 | 4 | 2 | 20 |

score measures the success of each member of the team accomplished in collaboration with different sets of coauthors, it disregards the specific team compositions and hence the coworker productivities. Acknowledging these coworker productivities, EVP will (generically) result in a measure of coworker productivity different from the HB -score; however, we would, of course, expect a strong (positive) correlation between both, the HB -score and the EVP. (We use the HB -scores to compare the EVP-scores with some established ranking method; yet, any other reasonable ranking method may equally serve to check the plausibility of EVP.)

In the next step, we calculate the success ratio for each pair of researchers $\{i, j\}$, $i, j \in \mathcal{N}$, which is done in a way analogous to that of calculating Table 7. For example, the pair $\{C, F\}$ collaborated in the teams $BCFM$ and CFK publishing two articles with scores 60 and 10; thus, their pair score equals $s_{CF} = 70/200 = 7/20$. Using that data, the matrix of pairwise success coefficients \mathbf{S} is given as in Eq. (6). In the next step we use the matrix \mathbf{S} and proceed to calculate the entries of \mathbf{G} as described above (see page 16). The result is displayed in Eq. (7).

TABLE 6. Publications of the different coauthorships

| coauthors | publications | no. papers | points | max points | success ratio | |
|-------------|----------------------------|------------|--------|------------|-----------------|-------|
| <i>ACK</i> | {1} | 1 | 100 | 100 | 1 | 1. |
| <i>AMO</i> | {5, 7, 33} | 3 | 180 | 300 | $\frac{3}{5}$ | 0.6 |
| <i>AO</i> | {2, 6, 31, 32, 34, 35, 42} | 7 | 120 | 700 | $\frac{6}{35}$ | 0.171 |
| <i>BC</i> | {9, 10} | 2 | 25 | 200 | $\frac{1}{8}$ | 0.125 |
| <i>BCFM</i> | {17} | 1 | 60 | 100 | $\frac{3}{5}$ | 0.6 |
| <i>BCK</i> | {3} | 1 | 20 | 100 | $\frac{1}{5}$ | 0.2 |
| <i>BJ</i> | {21, 36} | 2 | 15 | 200 | $\frac{3}{40}$ | 0.075 |
| <i>BM</i> | {8} | 1 | 5 | 100 | $\frac{1}{20}$ | 0.05 |
| <i>CFK</i> | {4} | 1 | 10 | 100 | $\frac{1}{10}$ | 0.1 |
| <i>CGK</i> | {28} | 1 | 5 | 100 | $\frac{1}{20}$ | 0.05 |
| <i>CH</i> | {12} | 1 | 20 | 100 | $\frac{1}{5}$ | 0.2 |
| <i>CHIK</i> | {13} | 1 | 5 | 100 | $\frac{1}{20}$ | 0.05 |
| <i>CHKL</i> | {29} | 1 | 5 | 100 | $\frac{1}{20}$ | 0.05 |
| <i>CJMN</i> | {11} | 1 | 20 | 100 | $\frac{1}{5}$ | 0.2 |
| <i>CK</i> | {16} | 1 | 60 | 100 | $\frac{3}{5}$ | 0.6 |
| <i>CMN</i> | {37, 39} | 2 | 80 | 200 | $\frac{2}{5}$ | 0.4 |
| <i>CN</i> | {38} | 1 | 10 | 100 | $\frac{1}{10}$ | 0.1 |
| <i>FK</i> | {27} | 1 | 10 | 100 | $\frac{1}{10}$ | 0.1 |
| <i>HO</i> | {14, 15, 41} | 3 | 20 | 300 | $\frac{1}{15}$ | 0.067 |
| <i>IK</i> | {25, 26, 30} | 3 | 45 | 300 | $\frac{3}{20}$ | 0.15 |
| <i>JK</i> | {18} | 1 | 5 | 100 | $\frac{1}{20}$ | 0.05 |
| <i>JLP</i> | {20, 23} | 2 | 10 | 200 | $\frac{1}{20}$ | 0.05 |
| <i>JP</i> | {19, 22, 43, 44} | 4 | 155 | 400 | $\frac{31}{80}$ | 0.388 |
| <i>KN</i> | {24} | 1 | 5 | 100 | $\frac{1}{20}$ | 0.05 |
| <i>KP</i> | {45} | 1 | 20 | 100 | $\frac{1}{5}$ | 0.2 |
| <i>MN</i> | {40} | 1 | 10 | 100 | $\frac{1}{10}$ | 0.1 |

TABLE 7. Publication success of the team members

| author | no. teams | no. publ. | points | success ratio s_i |
|----------|-----------|-----------|--------|------------------------|
| <i>A</i> | 3 | 11 | 400 | $\frac{4}{11}$ 0.364 |
| <i>B</i> | 5 | 7 | 125 | $\frac{5}{28}$ 0.179 |
| <i>C</i> | 13 | 15 | 420 | $\frac{7}{25}$ 0.28 |
| <i>F</i> | 3 | 3 | 80 | $\frac{4}{15}$ 0.267 |
| <i>G</i> | 1 | 1 | 5 | $\frac{1}{20}$ 0.05 |
| <i>H</i> | 4 | 6 | 50 | $\frac{1}{12}$ 0.083 |
| <i>I</i> | 2 | 4 | 50 | $\frac{1}{8}$ 0.125 |
| <i>J</i> | 5 | 10 | 205 | $\frac{41}{200}$ 0.205 |
| <i>K</i> | 12 | 14 | 290 | $\frac{29}{140}$ 0.207 |
| <i>L</i> | 2 | 3 | 15 | $\frac{1}{20}$ 0.05 |
| <i>M</i> | 6 | 9 | 355 | $\frac{71}{180}$ 0.394 |
| <i>N</i> | 5 | 6 | 125 | $\frac{5}{24}$ 0.208 |
| <i>O</i> | 3 | 13 | 320 | $\frac{16}{65}$ 0.246 |
| <i>P</i> | 3 | 7 | 185 | $\frac{37}{140}$ 0.264 |

$$\mathbf{S} = \begin{pmatrix} 0.3636 & 0.2548 & 1. & 0.3114 & 0.1348 & 0.1741 & 0.2132 & 0.273 & 1. & 0.1348 & 0.6 & 0.2752 & 0.3 & 0.31 \\ 0.2548 & 0.1786 & 0.2625 & 0.6 & 0.0945 & 0.122 & 0.1494 & 0.075 & 0.2 & 0.0945 & 0.325 & 0.1929 & 0.2097 & 0.2172 \\ 1. & 0.2625 & 0.28 & 0.35 & 0.05 & 0.1 & 0.05 & 0.2 & 0.2929 & 0.05 & 0.4 & 0.275 & 0.2625 & 0.272 \\ 0.3114 & 0.6 & 0.35 & 0.2667 & 0.1155 & 0.1491 & 0.1826 & 0.2338 & 0.1 & 0.1155 & 0.6 & 0.2357 & 0.2562 & 0.2655 \\ 0.1348 & 0.0945 & 0.05 & 0.1155 & 0.05 & 0.0645 & 0.0791 & 0.1012 & 0.05 & 0.05 & 0.1404 & 0.1021 & 0.1109 & 0.115 \\ 0.1741 & 0.122 & 0.1 & 0.1491 & 0.0645 & 0.0833 & 0.05 & 0.1307 & 0.05 & 0.05 & 0.1813 & 0.1318 & 0.0667 & 0.1484 \\ 0.2132 & 0.1494 & 0.05 & 0.1826 & 0.0791 & 0.05 & 0.125 & 0.1601 & 0.125 & 0.0791 & 0.222 & 0.1614 & 0.1754 & 0.1818 \\ 0.273 & 0.075 & 0.2 & 0.2338 & 0.1012 & 0.1307 & 0.1601 & 0.205 & 0.05 & 0.05 & 0.2 & 0.2 & 0.2246 & 0.275 \\ 1. & 0.2 & 0.2929 & 0.1 & 0.05 & 0.05 & 0.125 & 0.05 & 0.2071 & 0.05 & 0.2858 & 0.05 & 0.2258 & 0.2 \\ 0.1348 & 0.0945 & 0.05 & 0.1155 & 0.05 & 0.05 & 0.0791 & 0.05 & 0.05 & 0.05 & 0.1404 & 0.1021 & 0.1109 & 0.05 \\ 0.6 & 0.325 & 0.4 & 0.6 & 0.1404 & 0.1813 & 0.222 & 0.2 & 0.2858 & 0.1404 & 0.3944 & 0.275 & 0.6 & 0.3229 \\ 0.2752 & 0.1929 & 0.275 & 0.2357 & 0.1021 & 0.1318 & 0.1614 & 0.2 & 0.05 & 0.1021 & 0.275 & 0.2083 & 0.2265 & 0.2346 \\ 0.3 & 0.2097 & 0.2625 & 0.2562 & 0.1109 & 0.0667 & 0.1754 & 0.2246 & 0.2258 & 0.1109 & 0.6 & 0.2265 & 0.2462 & 0.2551 \\ 0.31 & 0.2172 & 0.272 & 0.2655 & 0.115 & 0.1484 & 0.1818 & 0.275 & 0.2 & 0.05 & 0.3229 & 0.2346 & 0.2551 & 0.2643 \end{pmatrix} \quad (6)$$

$$\mathbf{G} = \begin{pmatrix} 1. & 1.427 & 3.5714 & 1.1677 & 2.6968 & 2.0889 & 1.7056 & 1.3319 & 4.8276 & 2.6968 & 1.5211 & 1.3212 & 1.2188 & 1.173 \\ 0.7008 & 1. & 0.9375 & 2.25 & 1.8898 & 1.4639 & 1.1952 & 0.3659 & 0.9655 & 1.8898 & 0.8239 & 0.9258 & 0.8517 & 0.822 \\ 2.75 & 1.47 & 1. & 1.3125 & 1. & 1.2 & 0.4 & 0.9756 & 1.4138 & 1. & 1.0141 & 1.32 & 1.0665 & 1.0293 \\ 0.8563 & 3.36 & 1.25 & 1. & 2.3094 & 1.7889 & 1.4606 & 1.1405 & 0.4828 & 2.3094 & 1.5211 & 1.1314 & 1.0408 & 1.0045 \\ 0.3708 & 0.5292 & 0.1786 & 0.433 & 1. & 0.7746 & 0.6325 & 0.4939 & 0.2414 & 1. & 0.356 & 0.4899 & 0.4507 & 0.435 \\ 0.4787 & 0.6831 & 0.3571 & 0.559 & 1.291 & 1. & 0.4 & 0.6376 & 0.2414 & 1. & 0.4596 & 0.6325 & 0.2708 & 0.5615 \\ 0.5863 & 0.8367 & 0.1786 & 0.6847 & 1.5811 & 0.6 & 1. & 0.7809 & 0.6034 & 1.5811 & 0.5629 & 0.7746 & 0.7126 & 0.6877 \\ 0.7508 & 0.42 & 0.7143 & 0.8768 & 2.0248 & 1.5684 & 1.2806 & 1. & 0.2414 & 1. & 0.507 & 0.96 & 0.9126 & 1.0405 \\ 2.75 & 1.12 & 1.0459 & 0.375 & 1. & 0.6 & 1. & 0.2439 & 1. & 1. & 0.7247 & 0.24 & 0.9173 & 0.7568 \\ 0.3708 & 0.5292 & 0.1786 & 0.433 & 1. & 0.6 & 0.6325 & 0.2439 & 0.2414 & 1. & 0.356 & 0.4899 & 0.4507 & 0.1892 \\ 1.65 & 1.82 & 1.4286 & 2.25 & 2.8087 & 2.1756 & 1.7764 & 0.9756 & 1.3799 & 2.8087 & 1. & 1.32 & 2.4375 & 1.2217 \\ 0.7569 & 1.0801 & 0.9821 & 0.8839 & 2.0412 & 1.5811 & 1.291 & 0.9756 & 0.2414 & 2.0412 & 0.6972 & 1. & 0.92 & 0.8879 \\ 0.825 & 1.1741 & 0.9376 & 0.9608 & 2.2188 & 0.8 & 1.4033 & 1.0958 & 1.0901 & 2.2188 & 1.5211 & 1.087 & 1. & 0.9651 \\ 0.8525 & 1.2166 & 0.9715 & 0.9955 & 2.2991 & 1.7809 & 1.4541 & 1.3415 & 0.9655 & 1. & 0.8185 & 1.1263 & 1.0362 & 1. \end{pmatrix} \quad (7)$$

TABLE 8. EVP-ranking of the researchers

| rank | author | EVP | <i>HB</i> -score | no. publ. |
|------|----------|--------|------------------|-----------|
| 1 | <i>A</i> | 1.8082 | 0.3636 | 11 |
| 2 | <i>M</i> | 1.6208 | 0.3944 | 9 |
| 3 | <i>F</i> | 1.3146 | 0.2667 | 3 |
| 4 | <i>C</i> | 1.2889 | 0.28 | 15 |
| 5 | <i>O</i> | 1.1088 | 0.2462 | 13 |
| 6 | <i>P</i> | 1.0622 | 0.2643 | 7 |
| 7 | <i>B</i> | 1.033 | 0.1786 | 7 |
| 8 | <i>K</i> | 0.9855 | 0.2071 | 14 |
| 9 | <i>N</i> | 0.9282 | 0.2083 | 6 |
| 10 | <i>J</i> | 0.8132 | 0.205 | 10 |
| 11 | <i>I</i> | 0.6768 | 0.125 | 4 |
| 12 | <i>H</i> | 0.5212 | 0.0833 | 6 |
| 13 | <i>G</i> | 0.4386 | 0.05 | 1 |
| 14 | <i>L</i> | 0.4001 | 0.05 | 3 |

Finally, we have to calculate the eigenvalues of \mathbf{G} . Computing these, we find that the (absolutely) largest eigenvalue is $\hat{\lambda} = 14.4093$, and the associated eigenvector is (after normalizing $\sum \mathbf{p}_i = n$)

$$\mathbf{p}(\hat{\lambda}) = (1.8082, 1.033, 1.2889, 1.3146, 0.4386, 0.5212, 0.6768, 0.8132, \\ 0.9855, 0.4001, 1.6208, 0.9282, 1.1088, 1.0622) .$$

This is the EVP-vector we have been looking for.²⁹ The resulting ranking of the members of the research group \mathcal{N} is provided in Table 8. We infer that in view of the publications of the team members, the ranking according to EVP appears to be reasonable. In particular, the ranking according to EVP is similar to, yet different from, the naive method when we rank the members according to their individual *HB*-scores disregarding the effects of cooperation.³⁰

APPENDIX B. SUPPLEMENTARY MATERIAL OF EXAMPLE 2

Table 9 contains all full-length Fassbinder movies and their *IMDb* ratings and their years of production.

²⁹We may normalize $\mathbf{p}(\hat{\lambda})$ in any way which appears to be convenient, and thus any multiple of $\mathbf{p}(\hat{\lambda})$, *i.e.*, $\alpha \mathbf{p}(\hat{\lambda})$, $\alpha > 0$ may also serve as an EVP-vector.

³⁰The coefficient of correlation between EVP and the *HB*-score is 0.9675.

TABLE 9. Full-length Fassbinder movies: teams and *IMDb*-user ratings

| no. | team | <i>IMDb</i> | German title | English title | year |
|-----|-----------------|-------------|---|---|------|
| 4. | <i>EGJOQT</i> | 6.7 | <i>Liebe ist kälter als der Tod</i> | <i>Love is Colder than Death</i> | 1969 |
| 5. | <i>ADHOQRT</i> | 6.5 | <i>Götter der Pest</i> | <i>Gods of the Plague</i> | 1969 |
| 6. | <i>ABEFGOQT</i> | 7.2 | <i>Katzelmacher</i> | <i>Katzelmacher</i> | 1969 |
| 7. | <i>AKLQT</i> | 7.7 | <i>Warum läuft Herr R. Amok?</i> | <i>Why does Herr R. run Amok?</i> | 1969 |
| 8. | <i>HMORT</i> | 6.5 | <i>Rio das Mortes</i> | <i>Rio das Mortes</i> | 1979 |
| 9. | <i>ABCDGHLO</i> | 6.6 | <i>Das Kaffeehaus</i> | <i>Das Kaffeehaus</i> | 1970 |
| 10. | <i>AHJMOQRU</i> | 6.6 | <i>Whity</i> | <i>Whity</i> | 1970 |
| 11. | <i>CEOQT</i> | 6.1 | <i>Niklashauser Fahrt</i> | <i>The Niklashausen Journey</i> | 1970 |
| 12. | <i>EMNRT</i> | 6.8 | <i>Der amerikanische Soldat</i> | <i>The American Soldier</i> | 1970 |
| 13. | <i>DEJLNOQU</i> | 7.0 | <i>Warnung vor einer heiligen Nutte</i> | <i>Beware of a Holy Whore</i> | 1970 |
| 14. | <i>ABFHORT</i> | 6.6 | <i>Pioniere in Ingolstadt</i> | <i>Pioneers in Ingolstadt</i> | 1970 |
| 15. | <i>FGINRT</i> | 7.6 | <i>Händler der vier Jahreszeiten</i> | <i>The Merchant of Four Seasons</i> | 1971 |
| 16. | <i>CFMORU</i> | 7.8 | <i>Die bitteren Tränen der Petra von Kant</i> | <i>The Bitter Tears of Petra von Kant</i> | 1971 |
| 17. | <i>ABRT</i> | 7.2 | <i>Wildwechsel</i> | <i>Jailbait</i> | 1972 |
| 18. | <i>BFGJLNOT</i> | 8.5 | <i>Acht Stunden sind kein Tag</i> | <i>Eight Hours are not a Day</i> | 1972 |
| 19. | <i>ACEJOT</i> | 7.6 | <i>Bremer Freiheit</i> | <i>Bremen Freedom</i> | 1972 |
| 20. | <i>CDIJKLNU</i> | 7.9 | <i>Welt am Draht</i> | <i>World on a Wire</i> | 1973 |
| 21. | <i>CFIJK</i> | 7.2 | <i>Nora Helmer</i> | <i>Nora Helmer</i> | 1973 |
| 22. | <i>HIU</i> | 7.8 | <i>Martha</i> | <i>Martha</i> | 1973 |
| 23. | <i>EFKNRV</i> | 8.1 | <i>Angst essen Seele auf</i> | <i>Fear Eats the Soul</i> | 1973 |
| 24. | <i>FJKNORTV</i> | 7.1 | <i>Fontane Effi Briest</i> | <i>Effi Briest</i> | 1974 |
| 25. | <i>AEFLNRU</i> | 7.8 | <i>Faustrecht der Freiheit</i> | <i>Fox and his Friends</i> | 1974 |
| 27. | <i>CDFKLRLU</i> | 7.7 | <i>Mutter Küsters' Fahrt zum Himmel</i> | <i>Mother Küster Goes to Heaven</i> | 1975 |
| 28. | <i>CDFKLVL</i> | 7.7 | <i>Angst vor der Angst</i> | <i>Fear of Fear</i> | 1975 |
| 29. | <i>U</i> | 8.0 | <i>Ich will doch nur, dass ihr mich liebt</i> | <i>I Only Want You to Love Me</i> | 1975 |
| 30. | <i>CDJLPRUV</i> | 7.2 | <i>Satansbraten</i> | <i>Satan's Brew</i> | 1975 |
| 31. | <i>CJPSU</i> | 7.6 | <i>Chinesisches Roulette</i> | <i>Chinese Roulette</i> | 1976 |
| 33. | <i>LPQSU</i> | 7.3 | <i>Bolwieser (Kino)</i> | <i>The Stationmaster's Wife</i> | 1976 |
| 34. | <i>CU</i> | 7.5 | <i>Frauen in New York</i> | <i>Women in New York</i> | 1977 |
| 35. | <i>IPQSU</i> | 7.4 | <i>Despair — Eine Reise ins Licht</i> | <i>Despair</i> | 1977 |
| 37. | <i>IKOQSU</i> | 7.9 | <i>Die Ehe der Maria Braun</i> | <i>The Marriage of Maria Braun</i> | 1978 |
| 38. | <i>DHKNP</i> | 7.7 | <i>In einem Jahr mit 13 Monden</i> | <i>I a Year of 13 Moons</i> | 1978 |
| 39. | <i>ACHOPS</i> | 7.1 | <i>Die dritte Generation</i> | <i>The Third Generation</i> | 1978 |
| 40. | <i>FKOQSW</i> | 8.8 | <i>Berlin Alexanderplatz</i> | <i>Berlin Alexanderplatz</i> | 1979 |
| 41. | <i>AEKOQSW</i> | 7.2 | <i>Lili Marleen</i> | <i>Lili Marleen</i> | 1980 |
| 42. | <i>QSW</i> | 7.7 | <i>Lola</i> | <i>Lola</i> | 1981 |
| 44. | <i>SW</i> | 8.0 | <i>Die Sehnsucht der Veronika Voss</i> | <i>Veronika Voss</i> | 1981 |
| 45. | <i>HQSW</i> | 6.9 | <i>Querelle</i> | <i>Querelle</i> | 1981 |

$$\begin{aligned}
 \mathbf{S} = & \begin{pmatrix}
 0.71 & 0.69 & 0.71 & 0.655 & 0.745 & 0.72 & 0.69 & 0.668 & 0.736 & 0.71 & 0.745 & 0.737 & 0.66 & 0.78 & 0.692 & 0.71 & 0.704 & 0.694 & 0.715 & 0.713 & 0.72 & 0.731 & 0.72 \\
 0.69 & 0.722 & 0.66 & 0.66 & 0.72 & 0.743 & 0.743 & 0.66 & 0.742 & 0.85 & 0.747 & 0.755 & 0.707 & 0.85 & 0.722 & 0.73 & 0.72 & 0.69 & 0.74 & 0.737 & 0.738 & 0.737 & 0.747 \\
 0.71 & 0.66 & 0.733 & 0.742 & 0.685 & 0.76 & 0.66 & 0.685 & 0.755 & 0.75 & 0.762 & 0.742 & 0.78 & 0.79 & 0.704 & 0.73 & 0.61 & 0.757 & 0.735 & 0.685 & 0.762 & 0.745 & 0.752 \\
 0.655 & 0.66 & 0.742 & 0.729 & 0.7 & 0.77 & 0.66 & 0.693 & 0.79 & 0.737 & 0.775 & 0.735 & 0.71 & 0.753 & 0.67 & 0.745 & 0.675 & 0.713 & 0.744 & 0.65 & 0.745 & 0.745 & 0.75 \\
 0.745 & 0.72 & 0.685 & 0.7 & 0.717 & 0.77 & 0.695 & 0.704 & 0.74 & 0.71 & 0.765 & 0.74 & 0.68 & 0.742 & 0.697 & 0.727 & 0.684 & 0.757 & 0.72 & 0.688 & 0.74 & 0.81 & 0.72 \\
 0.72 & 0.743 & 0.76 & 0.77 & 0.77 & 0.768 & 0.777 & 0.66 & 0.74 & 0.76 & 0.777 & 0.793 & 0.78 & 0.782 & 0.767 & 0.753 & 0.8 & 0.753 & 0.88 & 0.74 & 0.777 & 0.763 & 0.88 \\
 0.69 & 0.743 & 0.66 & 0.66 & 0.695 & 0.777 & 0.732 & 0.66 & 0.76 & 0.76 & 0.752 & 0.755 & 0.712 & 0.805 & 0.725 & 0.735 & 0.695 & 0.76 & 0.745 & 0.75 & 0.743 & 0.742 & 0.752 \\
 0.668 & 0.66 & 0.685 & 0.693 & 0.704 & 0.66 & 0.66 & 0.692 & 0.78 & 0.66 & 0.77 & 0.66 & 0.655 & 0.77 & 0.665 & 0.74 & 0.667 & 0.655 & 0.7 & 0.653 & 0.72 & 0.722 & 0.69 \\
 0.736 & 0.742 & 0.755 & 0.79 & 0.74 & 0.74 & 0.76 & 0.78 & 0.763 & 0.755 & 0.767 & 0.79 & 0.727 & 0.775 & 0.79 & 0.74 & 0.765 & 0.76 & 0.765 & 0.76 & 0.775 & 0.758 & 0.768 \\
 0.71 & 0.85 & 0.75 & 0.737 & 0.71 & 0.76 & 0.76 & 0.66 & 0.755 & 0.734 & 0.74 & 0.765 & 0.66 & 0.762 & 0.725 & 0.74 & 0.677 & 0.697 & 0.76 & 0.748 & 0.726 & 0.715 & 0.753 \\
 0.745 & 0.747 & 0.762 & 0.775 & 0.765 & 0.777 & 0.752 & 0.77 & 0.767 & 0.74 & 0.773 & 0.775 & 0.732 & 0.77 & 0.775 & 0.77 & 0.79 & 0.763 & 0.797 & 0.74 & 0.783 & 0.763 & 0.8 \\
 0.737 & 0.755 & 0.742 & 0.735 & 0.74 & 0.793 & 0.755 & 0.66 & 0.79 & 0.765 & 0.775 & 0.754 & 0.723 & 0.78 & 0.737 & 0.725 & 0.733 & 0.757 & 0.73 & 0.81 & 0.748 & 0.745 & 0.763 \\
 0.66 & 0.707 & 0.78 & 0.71 & 0.68 & 0.78 & 0.712 & 0.655 & 0.727 & 0.66 & 0.732 & 0.723 & 0.693 & 0.68 & 0.697 & 0.715 & 0.66 & 0.692 & 0.725 & 0.665 & 0.72 & 0.722 & 0.731 \\
 0.78 & 0.85 & 0.79 & 0.753 & 0.742 & 0.782 & 0.805 & 0.77 & 0.775 & 0.762 & 0.77 & 0.78 & 0.68 & 0.761 & 0.753 & 0.77 & 0.7 & 0.748 & 0.76 & 0.75 & 0.757 & 0.76 & 0.767 \\
 0.692 & 0.722 & 0.704 & 0.67 & 0.697 & 0.767 & 0.725 & 0.665 & 0.79 & 0.725 & 0.775 & 0.737 & 0.697 & 0.753 & 0.716 & 0.71 & 0.711 & 0.685 & 0.775 & 0.698 & 0.732 & 0.71 & 0.8 \\
 0.71 & 0.73 & 0.73 & 0.745 & 0.727 & 0.753 & 0.735 & 0.74 & 0.74 & 0.74 & 0.77 & 0.725 & 0.715 & 0.77 & 0.71 & 0.738 & 0.735 & 0.72 & 0.735 & 0.723 & 0.738 & 0.72 & 0.755 \\
 0.704 & 0.72 & 0.61 & 0.675 & 0.684 & 0.8 & 0.695 & 0.667 & 0.765 & 0.677 & 0.79 & 0.733 & 0.66 & 0.7 & 0.711 & 0.735 & 0.721 & 0.655 & 0.76 & 0.684 & 0.724 & 0.737 & 0.765 \\
 0.694 & 0.69 & 0.757 & 0.713 & 0.757 & 0.753 & 0.76 & 0.655 & 0.76 & 0.697 & 0.763 & 0.757 & 0.692 & 0.748 & 0.685 & 0.72 & 0.655 & 0.719 & 0.739 & 0.69 & 0.742 & 0.747 & 0.745 \\
 0.715 & 0.74 & 0.735 & 0.744 & 0.72 & 0.88 & 0.745 & 0.7 & 0.765 & 0.76 & 0.797 & 0.73 & 0.725 & 0.76 & 0.775 & 0.735 & 0.76 & 0.739 & 0.759 & 0.733 & 0.755 & 0.756 & 0.772 \\
 0.713 & 0.737 & 0.685 & 0.65 & 0.688 & 0.74 & 0.75 & 0.653 & 0.76 & 0.748 & 0.74 & 0.81 & 0.665 & 0.75 & 0.698 & 0.723 & 0.684 & 0.69 & 0.733 & 0.708 & 0.731 & 0.71 & 0.74 \\
 0.72 & 0.738 & 0.762 & 0.745 & 0.74 & 0.777 & 0.743 & 0.72 & 0.775 & 0.726 & 0.783 & 0.748 & 0.72 & 0.757 & 0.732 & 0.738 & 0.724 & 0.742 & 0.755 & 0.731 & 0.754 & 0.72 & 0.763 \\
 0.731 & 0.737 & 0.745 & 0.745 & 0.81 & 0.763 & 0.742 & 0.722 & 0.758 & 0.715 & 0.763 & 0.745 & 0.722 & 0.76 & 0.71 & 0.72 & 0.737 & 0.747 & 0.756 & 0.71 & 0.72 & 0.752 & 0.762 \\
 0.72 & 0.747 & 0.752 & 0.75 & 0.72 & 0.88 & 0.752 & 0.69 & 0.768 & 0.753 & 0.8 & 0.763 & 0.731 & 0.767 & 0.8 & 0.755 & 0.765 & 0.745 & 0.772 & 0.74 & 0.763 & 0.762 & 0.772
 \end{pmatrix} \\
 \\
 \mathbf{G} = & \begin{pmatrix}
 1. & 0.956 & 0.968 & 0.899 & 1.04 & 0.938 & 0.943 & 0.965 & 0.964 & 0.967 & 0.964 & 0.977 & 0.953 & 1.025 & 0.967 & 0.962 & 0.976 & 0.965 & 0.942 & 1.007 & 0.955 & 0.971 & 0.933 \\
 0.972 & 1. & 0.9 & 0.906 & 1.005 & 0.969 & 1.015 & 0.953 & 0.973 & 1.158 & 0.967 & 1.001 & 1.021 & 1.117 & 1.008 & 0.989 & 0.998 & 0.959 & 0.975 & 1.041 & 0.979 & 0.98 & 0.967 \\
 1. & 0.914 & 1. & 1.018 & 0.956 & 0.99 & 0.902 & 0.99 & 0.989 & 1.022 & 0.987 & 0.984 & 1.126 & 1.038 & 0.983 & 0.989 & 0.846 & 1.052 & 0.968 & 0.967 & 1.011 & 0.99 & 0.975 \\
 0.923 & 0.914 & 1.012 & 1. & 0.977 & 1.003 & 0.902 & 1.002 & 1.035 & 1.004 & 1.003 & 0.975 & 1.026 & 0.99 & 0.935 & 1.009 & 0.936 & 0.992 & 0.98 & 0.917 & 0.989 & 0.99 & 0.972 \\
 1.049 & 0.997 & 0.934 & 0.961 & 1. & 1.003 & 0.949 & 1.018 & 0.969 & 0.967 & 0.99 & 0.981 & 0.982 & 0.976 & 0.972 & 0.985 & 0.948 & 1.052 & 0.949 & 0.971 & 0.982 & 1.076 & 0.933 \\
 1.014 & 1.03 & 1.036 & 1.057 & 1.074 & 1. & 1.061 & 0.953 & 0.969 & 1.035 & 1.005 & 1.051 & 1.126 & 1.027 & 1.07 & 1.02 & 1.109 & 1.047 & 1.159 & 1.045 & 1.031 & 1.014 & 1.14 \\
 0.972 & 1.03 & 0.9 & 0.906 & 0.97 & 1.012 & 1. & 0.953 & 0.996 & 1.035 & 0.973 & 1.001 & 1.028 & 1.058 & 1.012 & 0.996 & 0.963 & 1.057 & 0.982 & 1.059 & 0.986 & 0.986 & 0.974 \\
 0.941 & 0.914 & 0.934 & 0.951 & 0.983 & 0.86 & 0.902 & 1. & 1.022 & 0.899 & 0.996 & 0.875 & 0.946 & 1.012 & 0.928 & 1.002 & 0.924 & 0.911 & 0.922 & 0.922 & 0.955 & 0.959 & 0.894 \\
 1.037 & 1.028 & 1.03 & 1.084 & 1.032 & 0.964 & 1.038 & 1.127 & 1. & 1.029 & 0.992 & 1.048 & 1.05 & 1.018 & 1.103 & 1.002 & 1.06 & 1.057 & 1.008 & 1.073 & 1.028 & 1.007 & 0.994 \\
 1. & 1.177 & 1.023 & 1.011 & 0.991 & 0.99 & 1.038 & 0.953 & 0.989 & 1. & 0.958 & 1.015 & 0.953 & 1.002 & 1.012 & 1.002 & 0.938 & 0.969 & 1.001 & 1.055 & 0.963 & 0.95 & 0.975 \\
 1.049 & 1.035 & 1.04 & 1.063 & 1.067 & 1.012 & 1.027 & 1.112 & 1.004 & 1.008 & 1. & 1.028 & 1.056 & 1.012 & 1.082 & 1.043 & 1.095 & 1.061 & 1.05 & 1.045 & 1.039 & 1.014 & 1.036 \\
 1.038 & 1.046 & 1.012 & 1.009 & 1.033 & 1.033 & 1.031 & 0.953 & 1.035 & 1.042 & 1.003 & 1. & 1.043 & 1.025 & 1.028 & 0.982 & 1.017 & 1.052 & 0.962 & 1.143 & 0.993 & 0.99 & 0.988 \\
 0.93 & 0.979 & 1.064 & 0.975 & 0.949 & 1.016 & 0.973 & 0.946 & 0.952 & 0.899 & 0.947 & 0.958 & 1. & 0.893 & 0.972 & 0.968 & 0.915 & 0.963 & 0.955 & 0.939 & 0.955 & 0.959 & 0.947 \\
 1.099 & 1.177 & 1.077 & 1.034 & 1.036 & 1.019 & 1.1 & 1.112 & 1.015 & 1.039 & 0.996 & 1.034 & 0.982 & 1. & 1.051 & 1.043 & 0.97 & 1.04 & 1.001 & 1.059 & 1.004 & 1.01 & 0.993 \\
 0.975 & 1.001 & 0.96 & 0.919 & 0.972 & 0.999 & 0.99 & 0.961 & 1.035 & 0.988 & 1.003 & 0.977 & 1.006 & 0.99 & 1. & 0.962 & 0.986 & 0.952 & 1.021 & 0.985 & 0.972 & 0.944 & 1.036 \\
 1. & 1.011 & 0.995 & 1.022 & 1.015 & 0.981 & 1.004 & 1.069 & 0.969 & 1.008 & 0.996 & 0.962 & 1.033 & 1.012 & 0.991 & 1. & 1.019 & 1.001 & 0.968 & 1.021 & 0.979 & 0.957 & 0.978 \\
 0.992 & 0.997 & 0.832 & 0.926 & 0.954 & 1.042 & 0.949 & 0.963 & 1.002 & 0.922 & 1.022 & 0.973 & 0.953 & 0.92 & 0.993 & 0.995 & 1. & 0.911 & 1.001 & 0.965 & 0.961 & 0.979 & 0.991 \\
 0.977 & 0.956 & 1.032 & 0.979 & 1.056 & 0.981 & 1.038 & 0.946 & 0.996 & 0.949 & 0.988 & 1.004 & 1. & 0.983 & 0.956 & 0.975 & 0.908 & 1. & 0.973 & 0.974 & 0.985 & 0.992 & 0.965 \\
 1.007 & 1.025 & 1.002 & 1.021 & 1.005 & 1.147 & 1.018 & 1.011 & 1.002 & 1.035 & 1.031 & 0.968 & 1.047 & 0.999 & 1.082 & 0.995 & 1.053 & 1.027 & 1. & 1.035 & 1.002 & 1.004 & 1. \\
 1.005 & 1.021 & 0.934 & 0.892 & 0.96 & 0.964 & 1.025 & 0.944 & 0.996 & 1.018 & 0.958 & 1.074 & 0.96 & 0.985 & 0.974 & 0.98 & 0.948 & 0.959 & 0.966 & 1. & 0.97 & 0.944 & 0.958 \\
 1.014 & 1.022 & 1.039 & 1.022 & 1.033 & 1.012 & 1.015 & 1.04 & 1.015 & 0.989 & 1.014 & 0.992 & 1.04 & 0.994 & 1.022 & 0.999 & 1.004 & 1.032 & 0.995 & 1.031 & 1. & 0.957 & 0.988 \\
 1.029 & 1.021 & 1.016 & 1.022 & 1.13 & 0.995 & 1.014 & 1.043 & 0.993 & 0.974 & 0.988 & 0.988 & 1.042 & 0.999 & 0.991 & 0.975 & 1.021 & 1.038 & 0.996 & 1.002 & 0.955 & 1. & 0.987 \\
 1.014 & 1.034 & 1.026 & 1.029 & 1.005 & 1.147 & 1.027 & 0.997 & 1.006 & 1.026 & 1.035 & 1.012 & 1.056 & 1.007 & 1.117 & 1.023 & 1.06 & 1.036 & 1.017 & 1.044 & 1.012 & 1.013 & 1.
 \end{pmatrix}
 \end{aligned}$$

APPENDIX C. PERRON–FROBENIUS THEOREMS

The following results are collected from Seneta (1981), Takayama (1985, ch. 4.B), Graham (1987), and Berman and Plemmons (1994).

Let \mathbf{P} be an $n \times n$ permutation matrix, *i.e.*, a matrix with all elements but one in each row and in each column being equal to zero.³¹ An $n \times n$ matrix \mathbf{A} is called *reducible* if there exists a permutation matrix \mathbf{P} such that

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{A}_{22} \end{pmatrix},$$

where \mathbf{A}_{11} and \mathbf{A}_{22} are square submatrices. If this is impossible, *i.e.*, if such a permutation matrix \mathbf{P} does not exist, \mathbf{A} is called *irreducible*. (Note that in this definition, the off-diagonal matrix \mathbf{A}_{12} need not be square.) Moreover, if a permutation matrix \mathbf{P} exists such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ is block-diagonal, *i.e.*, $\mathbf{A}_{12} = \mathbf{0}$, \mathbf{A} is called *completely reducible*.

Lemma 2 (Graham, 1987, p. 50). *Let \mathbf{A} be an $n \times n$ square matrix and \mathbf{P} be an $n \times n$ permutation matrix. Then the matrices \mathbf{PAP}^T and \mathbf{A} have the same eigenvalues and, except for permutation of their elements, the same eigenvectors.*

As a direct consequence of Lemma 2, a permutation, that is, a renumbering of the workers, does not affect their productivity measures. Consequently, eigenvalue productivity is covariant with any permutation of the team members.

Lemma 3. *Let $A = (a_{ij})$ be an $n \times n$ irreducible, nonnegative matrix. Then the Perron root satisfies the inequalities*

$$\min_i \sum_j a_{ij} \leq \hat{\lambda} \leq \max_i \sum_j a_{ij},$$

with equality on either side implying equality throughout (i.e., $\hat{\lambda}$ can only be equal to the maximal or minimal row sum if all row sums are equal).

Proof. See Seneta (1981) Theorem 1.5 in conjunction with Corollary 1 therein. □

The first version of the Perron–Frobenius Theorem is concerned with nonnegative matrices.

Theorem 1 (Perron–Frobenius I). *Let \mathbf{A} be a nonnegative $n \times n$ matrix. Then,*

- (1) \mathbf{A} has a nonnegative real eigenvalue $\hat{\lambda}$.
- (2) A nonnegative eigenvector $\hat{\mathbf{x}} \geq \mathbf{0}$ can be associated with $\hat{\lambda}$.
- (3) If $\mathbf{Ax} \geq \mu\mathbf{x}$ for some μ and $\mathbf{x} \geq \mathbf{0}$, then $\mu \leq \hat{\lambda}$.
- (4) For any eigenvalue λ of \mathbf{A} , $|\lambda| \leq \hat{\lambda}$.
- (5) If $\mathbf{A} \geq \mathbf{B} \geq \mathbf{0}$, then $\hat{\lambda}_{\mathbf{A}} \geq \hat{\lambda}_{\mathbf{B}}$.

A stronger result can be obtained for irreducible nonnegative matrices.

³¹For any permutation matrix \mathbf{P} we have $\mathbf{P}^T = \mathbf{P}^{-1}$.

Theorem 2 (Perron–Frobenius II). *Let \mathbf{A} be a nonnegative irreducible $n \times n$ matrix. Then,*

- (1) \mathbf{A} has a positive real eigenvalue $\hat{\lambda}$ equal to its spectral radius $\rho(\mathbf{A})$.
- (2) The eigenvector $\hat{\mathbf{x}}$ associated with $\hat{\lambda}$ is positive.
- (3) The eigenvector $\hat{\mathbf{x}}$ is unique up to a scalar multiple.
- (4) If $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ for some $\lambda \geq 0$ and $\mathbf{x} \geq \mathbf{0}$, then $\lambda = \hat{\lambda}$.
- (5) For any eigenvalue λ of \mathbf{A} , $|\lambda| \leq \hat{\lambda}$.
- (6) $\hat{\lambda}$ increases when any entry of \mathbf{A} increases.
- (7) $\hat{\lambda}$ is a simple eigenvalue of \mathbf{A} .

Since a positive matrix is irreducible (Graham, 1987), Theorem 2 equally applies to positive matrices.

Observe that if \mathbf{A} is not irreducible, then

- (1) The root $\hat{\lambda}$ can be zero.
(In economic applications this case is rather uninteresting.)
- (2) Some, but not all, elements of $\hat{\mathbf{x}}$ can be zero.
- (3) Both $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ with $\hat{\mathbf{y}} \neq \theta\hat{\mathbf{x}}$ for any $\theta \in \mathbb{R}$ can be eigenvectors associated with $\hat{\lambda}$.
- (4) The root $\hat{\lambda}$ is not necessarily a simple root.

We now impose an additional restriction on irreducible nonnegative matrices. The class of irreducible matrices can be partitioned into primitive (or acyclic) and imprimitive (or cyclic) matrices.

A square nonnegative matrix \mathbf{A} is called *primitive* if there exists a positive integer k such that $\mathbf{A}^k > \mathbf{0}$. Whether a matrix is primitive or not can easily be verified by means of the following result. Let $\tilde{\mathbf{A}}$ be the incidence matrix of \mathbf{A} , then \mathbf{A} is primitive if, and only if, $\tilde{\mathbf{A}}$ is primitive. (A matrix $\tilde{\mathbf{A}}$ is called the *incidence matrix* corresponding to a nonnegative matrix \mathbf{A} if all positive entries of \mathbf{A} are replaced by ones.)

The following result gives a sufficient condition for \mathbf{A} to be primitive, providing an easy method to test whether or not a matrix is primitive.

Lemma 4. *A nonnegative irreducible square matrix \mathbf{A} is primitive if it has at least one diagonal element that is positive.*

Since a primitive matrix \mathbf{A} is irreducible (see Graham, 1987, p. 136), Theorem 2 also applies to \mathbf{A} :

Theorem 3 (Perron–Frobenius III). *Let \mathbf{A} be a nonnegative primitive $n \times n$ matrix. Then, conditions (1) to (4) of Theorem 1 hold. Furthermore,*

- (5) The eigenvector $\hat{\mathbf{x}}$ is unique up to a scalar multiple.
- (6) $\hat{\lambda}$ increases when any entry of \mathbf{A} increases: if $\mathbf{0} < \mathbf{B} \leq \mathbf{A}$ and μ is an eigenvalue of \mathbf{B} , then $|\mu| \leq \hat{\lambda}$. Moreover, if $|\mu| = \hat{\lambda}$, then $\mathbf{B} = \mathbf{A}$.
- (7) $\hat{\lambda}$ is a simple eigenvalue of \mathbf{A} .

APPENDIX D. MONOTONICITY RESULTS FOR THE PERRON VECTOR

The following results are borrowed from Elsner et al. (1982).

Theorem 4 (Elsner, Johnson, and Neumann, 1982). *Let \mathbf{A} be an $n \times n$ nonnegative, irreducible matrix. Then for any nonnegative n -vector $\mathbf{v} \neq \mathbf{0}$,*

$$\frac{y_i}{x_i} > \frac{y_k}{x_k}, \quad k \neq i, \quad 1 \leq k \leq n,$$

where $\mathbf{x} = (x_1, \dots, x_n)^\top$ and $\mathbf{y} = (y_1, \dots, y_n)^\top$ are Perron vectors of \mathbf{A} and $\mathbf{A} + \mathbf{e}_i \mathbf{v}^\top$, respectively.

Note that the largest relative change in the component of a Perron vector corresponding to the row in \mathbf{A} which has been positively perturbed does not imply the largest absolute change for that component; that is, it does not follow from Theorem 4 that

$$y_i - y_k > x_i - x_k, \quad k \neq i, \quad 1 \leq k \leq n.$$

Yet, if both Perron vectors \mathbf{x} and \mathbf{y} are normalized so that $x_i = y_i = 1$, then Theorem 4 implies that

$$y_k < x_k, \quad k \neq i, \quad k = 1, \dots, n.$$

If in Theorem 4 the assumption that \mathbf{A} is irreducible is replaced by the assumption that the Perron roots of \mathbf{A} and $\mathbf{A} + \mathbf{e}_i \mathbf{v}^\top$ are simple, then the strict inequalities have to be weakened to accommodate the possibility that no greater relative change occurs in the i th component over all other components. Moreover, since some of the entries in the Perron vectors may well be zero, these inequalities have to be re-arranged to meet such an eventuality.

Elsner et al. (1982) discuss perturbations of the matrix \mathbf{A} and the consequences for the Perron vector in more detail.³²

Theorem 5. *Let \mathbf{A} be an $n \times n$ nonnegative matrix whose Perron root is simple with a non-zero first entry. Assume that $\mathbf{v} \neq \mathbf{0}$ is a nonnegative vector. Then, there exists a positive number ε_0 (possibly dependent on \mathbf{v}), such that*

$$\frac{d}{d\varepsilon} z_k(\varepsilon) \leq 0$$

for each $\varepsilon \in [0, \varepsilon_0]$ and for each $1 \leq k \leq n$, where for each $\varepsilon \in [0, \varepsilon_0]$, $\mathbf{z}(\varepsilon) = (z_1(\varepsilon), \dots, z_n(\varepsilon))^\top$ denotes the Perron vector of

$$\mathbf{A}_\varepsilon := \mathbf{A} + \varepsilon \mathbf{e}_1 \mathbf{v}^\top.$$

³²For an analysis of marginal changes of the matrix elements and the corresponding changes of the Perron vector, see Franklin (1968, Sec. 6.12).

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