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Working Paper

A unified view of systemic risk: detecting SIFIs and forecasting the financial cycle via EWSs

Working Paper, No. 36

Provided in Cooperation with:
Università Cattolica del Sacro Cuore, Dipartimento di Economia e Finanza (DISCE)

Suggested Citation: Spelta, Alessandro (2016) : A unified view of systemic risk: detecting SIFIs and forecasting the financial cycle via EWSs, Working Paper, No. 36, Università Cattolica del Sacro Cuore, Dipartimento di Economia e Finanza (DISCE), Milano

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A unified view of systemic risk: detecting SIFIs and forecasting the financial cycle via EWSs

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January 11, 2016

Abstract

Following the definition of systemic risk by the Financial Stability Board, the International Monetary Fund and the Bank for International Settlements, this paper proposes a method able to simultaneously address the two dimensions in which this risk materializes: namely the cross-sectional and the time dimension. The method is based on the W-TOPHITS algorithm, that exploits the connectivity information of an evolving network, and decomposes its tensor representation as the outer product of three vectors: borrowing, lending and time scores. These vectors can be interpreted as indices of the systemic importance of borrowing and lending associated with each financial institution and of the systemic importance associated with each period, coherently with the realization of the whole network in that period. The time score, being able to simultaneously consider the temporal distribution of the whole traded volume over time as well as the spatial distribution of the transactions between players in each period, turns out to be a useful Early Warning Signal of the financial crisis. The W-TOPHITS is tested on the e-MID interbank market dataset and on the BIS consolidated banking statistics with the aim of discovering Systemically Important Financial Institutions and to show how the time score is able to signal a change in the bipartite network of borrowers and lenders that heralds the fall of the traded volume that occurred during the 2007/2009 financial crisis.

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†I am indebted to the European Commission for their financial support through the EU FP7 RASTANEWS project – research grant No. 320278.
**Keywords:** Systemic Risk, Tensor, Early Warning Signals, Evolving Networks.

**JEL Codes:** G01, G17, C63, C53.
1 The two dimensions of systemic risk and their interactions: a unified framework

Usually, in Economics and Finance, systemic risk refers to the risk of collapse of an entire financial system or an entire market rather than to the risk associated with an individual entity of the system [1]. Generally speaking, it concerns the risk posed by balance sheet relationships and interdependencies among players, where the default of a single entity can cause a cascading failure, which could potentially bring down the entire market. This definition, despite being correct is, however, incomplete. It considers only one side of the coin of systemic risk. Indeed, in the notes on macroprudential policy framework, three of the main supervisory authorities, namely, the Financial Stability Board (FSB), the International Monetary Fund (IMF) and the Bank for International Settlements (BIS) pointed out that systemic risk regards two related dimensions: the cross-sectional and the time dimension (see [2], [3], [4]).

As already mentioned, the cross-sectional dimension concerns linkages and interdependencies across institutions in a financial system. In this dimension, systemic risk arises because the distress of an institution can affect (directly or indirectly) other parts of the system via domino and cascade effects. But this is not the end of the story. Systemic risk materializes also in the time dimension. This dimension is related to the financial cycle, where systemic risk is due to the build-up of financial imbalances over time and to the procyclicality of the financial system. The temporal dimension of systemic risk stems from the existence of information asymmetries between borrowers and lenders and is related to the financial accelerator narrative\(^1\) ([5], [6]) and to the inappropriate responses of financial market participants to changes in risk over time [7].

On one hand, addressing systemic risk in the cross-sectional dimension requires the measurement of the interconnections among institutions with so as to evaluate the impact of their bilateral exposures on other participants within the system. Interconnectedness is thus related to the detection of the most central players in a network. In the context of financial networks, these players are called Systemically Important Financial Institutions (SIFIs) and researchers in

\(^1\)During crisis periods, even borrowers with profitable projects find some difficulties in obtaining funds because of information asymmetries and this depresses economic conditions even more. During booms, on the other hand, collateral values rise and these players are able to obtain easy credit via external finance and this adds to the economic stimulus.
complex systems have extensively dealt with this topic (see [8], [9], [10], [11], [12] among others). On the other hand, to address systemic risk in the time dimension it is necessary to forecast the turning point of the financial cycle via Early Warning Signals (EWSs) capable of indicating in advance the end of a period of excessive credit growth, in the run-up to a financial crisis. Therefore, roughly speaking, a financial EWSs can be seen as a change in a measurable property of the financial network that typically occurs prior to a financial crisis. At an early stage, the identification of financial EWSs was mostly based on cross-country OLS regressions aimed at discovering the causes of the crisis faced by various countries [13]. However this approach was not very successful in finding a set of variables able to predict the harshness of the crunch [14] probably because these covariates ignore that links between agents can help predict the intensity of the crisis ([15], [16]).

Moreover, the two dimensions of systemic risk are also closely intertwined and reinforce each other. Indeed, an increase in the number and/or in the size of transactions among players, feeds the increasing trend of the financial cycle. On the other hand, strong asset growth and credit expansion bring more connections into the system, as financial institutions increase leverage and non-core funding exposures with other agents [7].

The scope of this work is to render the view expressed by the FSB, the IMF and the BIS operational by translating the observations underlined by these institutions into a practical tool. Given the systemic nature of the financial crisis and the recognition that interconnectedness among financial intermediaries has played a crucial role in spreading the crisis, this article proposes a new network-based measure able to address systemic risk in a unified framework in which both the time and the cross-sectional dimension, together with their feedback mechanisms, are taken into account simultaneously.

In a nutshell, from the ground up, in this way a financial market is perceived as an evolving weighted directed network where nodes are financial institutions and arrows are loans from lenders to borrowers. The evolving network can be described by means of a time-ordered sequence of weighted adjacency matrices, each one characterizing the state of the financial system at a given point in time. The adjacency matrices are thus combined into a single mathematical object: a three-way tensor (the mathematical definition is provided in Subsection 2.1). The W-TOPHITS algorithm, the core of the proposed technique, exploits the connectivity information
of the evolving network and decomposes its tensor representation as the outer product of three vectors, the borrowing score, the lending score and the time score (see below and Subsection 2.2 for the definition of the algorithm). These vectors can be interpreted as indices of the systemic importance of the borrowing and the lending associated with each financial institution and of the systemic importance associated with the realizations of the network in each period. The time score, being a function of both the size of traded volume and of its distribution among players with different systemic importance, is able to perceive any change in the institutions’ trading behavior that heralds the fall of the traded volume which occurred during the crisis (see Subsections 3.3 and 3.4). Indeed, results indicate that prior to a crisis, the time score plummets, even for a still increasing traded volume, as a result of modifications in the topology of the bipartite network of lenders and borrowers with respect to "business as usual" periods. Prior to a crisis, the time score decreases because the transaction volume between institutions holding a high borrowing/lending score shrink and become smaller than the ones of less important institutions, which, in turn, increase their systemic importance only during the pre-crisis phase. Thus, the time score turns out to be an empirical financial EWSs to forecast the dynamic of the traded volume; used as a proxy of the financial cycle.

Figure 1 shows graphically how an evolving network can be expressed as a three dimensional tensor and how this tensor decomposes as the outer product of three vectors, each of which is associated with a particular dimension of the tensor. The first dimension regards the borrowing activities of entities that compose the network, the second dimension relates to the lending component and the third represents the evolution over time of the whole system.
The evolution of the network (a) can be captured by stacking the adjacency matrices (b), representing the flows of funds between entities, into a single mathematical object called tensor (c). Notice that, passing from a matrix to a tensorial representation of the financial system, the temporal component is captured by the third subscript (in purple) associated with each transaction. Then the tensor is decomposed as the outer product of three vectors (d) via the W-TOPHIS algorithm presented below in the paper. The borrowing and the lending score represent the systemic importance of each player as a lender and as a borrower. The time score ranks the systemic importance of each time-period coherently with the transactions executed by institutions with different importance in the cross-sectional dimension in each period.

As suggested by Figure 1, a financial market can be described by means of a three-way tensor $\mathcal{E}$ where the generic element $e_{ijt}$ represents the amount lent by institution $i$ to institution $j$ at time $t$. Assume that $I$ denotes the number of nodes and $T$ the number of periods, moreover, assume also that $i \rightarrow e_j$ denotes that player $i$ lends and amount $e$ to player $j$ at time $t$ and that $\tau$ represents the iteration step. Then the W-TOPHITS algorithm computes the borrowing, the lending and the time score iteratively in the context of weighted evolving networks as follow: starting from three random vectors representing the borrowing, the lending and the time score, three steps are computed until all convergence criteria are satisfied (see Subsection 2.2 for more details).

$$l_{si}^{(\tau+1)} = \sum_{i:j \rightarrow e_c} e_{ijt} b_{is}^{(\tau)} l_{ti}^{(\tau)} \quad i = 1, ..., I$$

(1)
In words, the lending score ($ls_i$) of institution $i$ is the weighted sum of the borrowing scores ($bs_j$) of the players that $i$ points to. The weight associated with each borrower is the product of the size of the transaction between $i$ and the borrower ($e_{ijt}$) times and the time score ($ts_t$) of the period in which the transaction is performed. The borrowing score ($bs_j$) of institution $j$ is the weighted sum of the lending scores ($ls_i$) of the players that point to $j$. The weight associated with each lender is the product of the size of the loans received from that lender ($e_{ijt}$) times the time score ($ts_t$) of the period in which the transaction is executed. Finally, the time score of period $t$ is the sum over all pairs of institutions $(i, j)$ involved in a transaction at time $t$, of the product, of the lending score ($ls_i$) of the creditor $i$ multiplied by the borrowing score ($bs_j$) of the debtor $j$ and by the size of the transaction ($e_{ijt}$) between $i$ and $j$. In other words, the time score is a weighted sum of the transactions executed in each period, where each transaction is weighted by the joint systemic importance of the pair of institutions performing that transaction ($ls_i bs_j$). Figure 2 provides a simple network example to show how the algorithm works.

$$bs_j^{(\tau+1)} = \sum_{i: j \rightarrow i} e_{ijt} ls_i^{(\tau+1)} ts_t^{(\tau)} j = 1, ..., I$$

$$ls_i^{(\tau+1)} = \sum_{t: i \rightarrow j} e_{ijt} bs_j^{(\tau+1)} ts_t^{(\tau)} t = 1, ..., T$$

Figure 2: Small network example. The network evolves through two periods $t1$ and $t2$. In $t1$ institution B4 lends an amount $e_{411}$ to institution B1 that, in turn, lends an amount $e_{131}$ to B3. In $t2$ institution B5 lends an amount $e_{512}$ to institution B1 that, in turn, lends an amount $e_{122}$ to B2. Consider node B1 first, for instance. The borrowing score of that node is the sum of the lending score of B4 and B5, each of them weighted by the strengths $e_{411}, e_{512}$ of the respective transaction, and by the values of time scores $ts_{t1}$ and $ts_{t2}$ of the periods in which these transactions occur. Mathematically:
bs_{B1} = e_{411}ls_{B4}ts_{t1} + e_{512}ls_{B5}ts_{t2}. Similarly the lending score of B1 is the sum of the borrowing score of B3 and B2, that borrow from B1, weighted by the respective size of the transaction e_{131}, e_{122} and by the time score’s values ts_{t1} and ts_{t2}. Mathematically: ls_{B1} = e_{131}bs_{B3}ts_{t1} + e_{122}bs_{B2}ts_{t2}.

Now consider period t1. The time score of t1 is given by the sum, for each pair of connected banks (B4 → B1 and B1 → B3) of the product of the borrowing score, lending score and transaction size regarding period t1. Mathematically: ts_{t1} = e_{411}ls_{B4}bs_{B1} + e_{131}ls_{B1}bs_{B3}.

Some remarks have to be made in order to disclose the main properties of the method. First, contrary to standard centrality measures that have to be computed every time that a new realization of the network is observed, the algorithm produces only one borrowing and one lending score value for each financial institution, using the information embedded in all the temporal observations (see Figure 1 and Figure 2). This feature is useful to evaluate ex-post, how transactions of different amounts are distributed in time between entities with different systemic importance (see Subsection 3.3). Given the total traded volume, the time score will be higher (lower), the greater (smaller) the bias of the transactions of the highest amount in the direction of the most systemically important institutions.

Second, the W-TOPHITS algorithm is specifically targeted to weighted directed networks, therefore it is able to distinguish between Systemically Important Borrowers (SIB) and Systemically Important Lenders (SIL). The proposed technique is thus consistent with the Basel III method that looks at both the lending and borrowing sides of banks’ balance sheets to evaluate their systemic importance \cite{basel}. SIBs are institutions more vulnerable to liquidity shocks, i.e. shocks that hit the liabilities of financial institutions because of a massive withdrawal of deposits, or a refusal of the counterpart to roll over the debt. SILs are institutions more vulnerable to devaluation/default shocks, i.e. shocks that hit the assets side of the balance sheet because of the default of some borrowers or due to a fall of the market value of assets in the institutions’ portfolio. The W-TOPHITS exploits the interplay between devaluation and liquidity shocks to calculate the borrowing and lending score of each financial intermediary. In other words, if a bor-

\footnote{The Basel Committee on Banking Supervision (BCBS) proposed the application of an indicator-based measure, based on five categories that should reflect the systemic importance of individual financial intermediaries, in order to identify Global Systemically Important Banks (G-SIBs). One of these categories, interconnectedness, is given by the weighted sum of intra-financial system assets, intra-financial system liabilities and securities outstanding. These correspond, for instance, to the in- and out-strength of every bank respectively.}
rowing institution is not able to repay its debt, the lender, suffering from a devaluation shock, might reduce his supply of funds in the market causing a liquidity shock to its borrowers. If an institution stops lending to a borrower, the borrower, suffering from a liquidity shock, might not be able to roll-over his obligations causing a devaluation shock to his lenders. Results can be easily interpreted in terms of hubs and authorities ([18], [21]) but in presence of a weighted directed evolving network.

Third, from equations (1-3) it clearly emerges that borrowing and lending scores (representing the cross-sectional dimension of systemic risk) are influenced by the time score that provides additional weights, depending on whether institutions execute transactions in a period of low or high systemic importance in the temporal dimension. Moreover, by definition, the time score is influenced by the cross-sectional dimension of systemic risk via the borrowing and lending scores. As advocated by the FSB, the IMF and the BIS the two dimensions in which systemic risk materializes and their mutual reinforcement are therefore taken into account by the algorithm.

2 Method

2.1 Tensor notation

Before presenting the details of the W-TOPHITS some basic notations are needed. A tensor is a multidimensional array. Formally, a $N$-way or $N$-th order tensor is a subset of the tensor product of $N$ vector spaces, each of which has its own coordinate system. The order of a tensor is the number of dimensions, also known as ways or modes. Vectors are tensors of order one and will be denoted by boldface lowercase letters, e.g., $a$. Matrices are tensors of order two and will be denoted by Euler script letters, e.g., $A$. Higher-order tensors (order three or higher) will be denoted by Euler script letters, e.g., $A$. Scalars are denoted by lowercase letters, e.g., $a$. The $i$-th entry of a vector $a$ is denoted by $a_i$. By analogy the element $(i,j)$ of a matrix $A$ is denoted by $a_{ij}$, and the element $(i,j,k)$ of a third-order tensor $A$ is denoted by $a_{ijk}$.

The 2-norm of a $N$-th dimensional tensor $A \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is the square root of the sum of
the squares of all its elements,

\[ \| A \|_2 = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_N=1}^{I_N} a_{i_1,i_2,...,i_N}^2} \] (4)

A $N$-way tensor $A \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is of rank-one if it can be written as the outer product of $N$ vectors, i.e.,

\[ A = a^{(1)} \odot a^{(2)} \odot \cdots \odot a^{(N)} \] (5)

where the symbol “$\odot$” represents the vector outer product.

Given matrices $A \in \mathbb{R}^{I \times J}$ and $B \in \mathbb{R}^{K \times L}$, their Khatri–Rao product is denoted by $A \odot B$. The result is a matrix of size $(IJ) \times K$. The Hadamard product is the elementwise matrix product, and is denoted by the symbol $\star$.

Matricization, also known as unfolding or flattening, is the process of reordering the elements of an $N$-way array into a matrix. The mode-$n$ matricization of a tensor $A \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is denoted by $A^{(n)}$ and arranges the mode-$n$ fibers to be the columns of the resulting matrix. Tensor elements $(i_1, i_2, ..., i_N)$ maps to matrix element $(i_n, j)$, where

\[ j = 1 + \sum_{k=1, k \neq n}^{N} (i_k - 1) J_k \quad \text{with} \quad J_k = \prod_{m=1, m \neq n}^{k-1} I_m \]

### 2.2 The W-TOPHITS and the Candecomp-Parafact decomposition

The W-TOPHITS algorithm is the core of the proposed method. The algorithm computes the borrowing, the lending and the time score iteratively similarly to the TOPHITS developed in [19] and [20] but in the context of weighted networks. As shown in [21], for economic networks that are intrinsically weighed, algorithms that also consider the weights are more appropriate for describing the main features of the system than methods based on its binary representation.

Equations (1-3) can be written, in tensor format, as:

\[ l^{(\tau+1)} = \mathcal{E} \times_2 b^{(\tau)} \times_3 t^{(\tau)} \] (6)

\[ b^{(\tau+1)} = \mathcal{E} \times_1 l^{(\tau+1)} \times_3 t^{(\tau)} \] (7)
where $\mathcal{E} \times_i x$ indicates that tensor $\mathcal{E}$ should be multiplied by the vector $x$ in dimension $i$.

Under appropriate conditions \[22\], the algorithm converges to the best rank-one approximation of $\mathcal{E}$. The best rank-one approximation of the tensor can be computed using a higher-order method called Candecomp-Parafact decomposition (see \[23\], \[24\]) similar to the singular value decomposition but in a multidimensional framework. The three way Candecomp-Parafact decomposition, yields a rank-$R$ approximation of $\mathcal{E}$ of the form:

$$\mathcal{E} \approx [\sigma; \mathbf{U}, \mathbf{V}, \mathbf{W}] = \sum_{r=1}^{R} \sigma_r \mathbf{u}_r \circ \mathbf{v}_r \circ \mathbf{w}_r$$

For $R = 1$ this decomposition produces the best rank-one approximation of $\mathcal{E}$, meaning that it seeks to find three vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$, such that the $2$-norm of the difference between the original tensor $\mathcal{E}$ and the approximate tensor $\hat{\mathcal{E}}$ is minimal.

$$\min_{\mathcal{E}} \left\| \mathcal{E} - \hat{\mathcal{E}} \right\|_2 \text{ with } \hat{\mathcal{E}} = \sigma \mathbf{u} \circ \mathbf{v} \circ \mathbf{w}$$

where the symbol "$\circ$" stands for the outer product and $\mathbf{u} \in \mathbb{R}^I$, $\mathbf{v} \in \mathbb{R}^I$, $\mathbf{w} \in \mathbb{R}^T$ and $\sigma = \|\mathbf{u}\| \|\mathbf{v}\| \|\mathbf{w}\|$.

Since bilateral financial transactions are always positive, non-negative tensor decomposition is employed to solve (10). This is customarily used to simplify the interpretation of the resulting decomposition; see \[25\]. The problem (10) is generally non-convex but convex in each block of variables, thus I employ the generalized block coordinate descent method proposed by \[26\] to solve iteratively:

$$\min_{\mathbf{u}} Y_{vw} \mathbf{u}^T - \mathbf{E}_{(1)}$$

where $Y_{vw} = \mathbf{v} \odot \mathbf{w}$ and $\mathbf{E}_{(1)}$ is the $(IT) \times I$ unfolded matrix of $\mathcal{E}$ and

$$\min_{\mathbf{v}} Y_{uw} \mathbf{v}^T - \mathbf{E}_{(2)}$$

where $Y_{uw} = \mathbf{u} \odot \mathbf{w}$ and $\mathbf{E}_{(2)}$ is the $(IT) \times I$ unfolded matrix of $\mathcal{E}$ and

$$\min_{\mathbf{w}} Y_{uw} \mathbf{w}^T - \mathbf{E}_{(2)}$$
where $Y_{uv} = u \odot w$ and $E_{(2)}$ is the $(IT) \times I$ unfolded matrix of $E$ and  

$$
\min_{w \geq 0} \| Y_{uv}w^T - E_{(3)} \|_2
$$

(13)

where $Y_{uv} = u \odot v$ and $E_{(3)}$ is the $(II) \times T$ unfolded matrix of $E$.

Once the minimization has been worked out, the $u$ and $v$ vectors contain the lending and the borrowing score values for each player, and the vector $w$ contains the time score value for each period:

$$
ls^{(r)} \rightarrow ls^* = u, \; bs^{(r)} \rightarrow bs^* = v, \; ts^{(r)} \rightarrow ts^* = w
$$

3 Application of the W-TOPHITS to banking data

3.1 Dataset

The W-TOPHITS is tested on the e-MID interbank market dataset and on the consolidated banking statistics collected by the BIS. The aim is to identify the systemic importance of each institution (as a borrower and as a lender) within the two networks and to analyze the systemic importance of each period in order to forecast the time of the financial crisis.

The e-MID is an electronic market based in Milan and developed for uncollateralized interbank transactions. It is available for interbank transactions with any bank operating in the European interbank market. Contracts are settled at different maturities ranging from overnight up to one year, with the largest bulk of the transactions settled overnight. The overnight (O/N) and the overnight long (ONL)\(^3\) segments encompass more than 90% of all e-MID transactions as the interbank market is mainly a market for short-term trades. For the purposes of this study I have used monthly aggregated data of the overnight transactions segment. I have considered a set of 354 banks each of them represented by the amount of its exposure vis-a-vis the rest of the reporting banks. The e-Mid interbank market is thus described as a third order interbank tensor

\(^3\)ONL are contracts encompassing more than one day between two consecutive business days.
composed of 156 matrices $E^{ib} \in \mathbb{R}^{354 \times 354}$ representing monthly transactions between European banks from January 2000 to December 2012.

The BIS dataset reports statistics on international banking activity. The International Banking Statistics (IBS) is comprised of the consolidated banking statistics (CBS), which measure worldwide consolidated claims of banks headquartered in reporting countries, including claims of their own foreign affiliates but excluding interoffice positions. The CBS include quarterly data on off-balance sheet exposures, such as risk transfers, guarantees and credit commitments. In this application I employ the consolidated banking statistics on ultimate risk basis (CBS/UR). The CBS/UR provide information about banking systems’ risk exposures, in particular country risk. I have considered a set of 31 countries each of which is represented by the amount of its foreign claims vis-a-vis the others, from the first quarter of 2000 to the last quarter of 2012. Starting from this dataset, I have built the 3-way ultimate risk tensor $E^{ur} \in \mathbb{R}^{31 \times 31 \times 52}$. The ultimate risk tensor is composed of 52 slices $E^{ur} \in \mathbb{R}^{31 \times 31}$ each of which represents financial transactions between countries in a quarter\(^4\).

### 3.2 The cross-sectional dimension

I will first focus on the cross-sectional dimension of systemic risk. Results about the systemic importance of each player are shown in Figure 3. Following the Basel Committee on Banking Supervision [17], the indices are normalized so that an overall score is given in basis points\(^5\). In particular, Figure 3 (a) displays the lending (upper) and the borrowing (lower) score values associated with each bank operating in the e-MID interbank market. As clearly emerges from the figure, some Italian banks turn out to be the main Systemically Important Borrowers and Lenders. Moreover, other Italian banks hold very low scores. These banks exploited the e-MID interbank market just for few periods at the beginning of the data sample\(^6\). Some of the foreign banks, having high centralities, are also systemically important; especially English, French and German credit institutions for the lending side and German banks for the borrowing side\(^7\) (the

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\(^4\)Since in the dataset there are only 16 reporting countries and more than 200 non reporting countries I have restricted the analysis to the 31 main advanced economies. These are the countries that received loans for at least 5\(^5\) billion dollar in all the quarters from 2000 to 2012 (Table 1 in Appendix A lists the countries).

\(^5\)The scope of the paper is not to provide a threshold for selecting SIB or SIL but to explore the systemic importance of each institution according to the risk posed on the whole system.

\(^6\)For the sake of brevity, the figures demonstrating this result are not shown but available upon request.

\(^7\)Despite the fact that institutions are anonymous the country of origin can be disclosed.
first two columns of Table 1 in Appendix A show the ID of the first 31 Systemically Important Lenders and Borrowers respectively, sorted in decreasing order. Figure 2 (b) presents the lending (upper) and the borrowing (lower) score for countries’ banking sectors reported in the ultimate risk tensor. From the figure it is clear that the United States and Great Britain play the role of the main SIBs, whereas Germany, France and Japan are the main SILs\(^8\). (Table 1 in Appendix A shows the ranking of the countries accordingly to their lending and borrowing score values in columns three and four respectively. This is also the list of the all countries analyzed).

\[\text{(a)}\]

\[\text{(b)}\]

Figure 3: Lending and borrowing score values for banks operating in the e-MID interbank market (a) and for countries’ banking sectors in the BIS dataset (b). The upper left panel shows the lending score value (red) of each bank in the e-MID dataset. The lower left panel shows the borrowing score value (blue) of each bank. The upper right panel displays the lending score value (red) of each country in the BIS dataset. The lower right panel encompasses the borrowing score (blue) value of each country. In each sub-figure the entities with the highest borrowing or lending score have been emphasized with arrows and names (Table 1 in Appendix A provides the ID of the 31 main SILs and SIBs for both datasets in decreasing order, for the BIS dataset this list coincides with all the countries that belong to the ultimate risk tensor).

\(^8\)A clarification. Note that the United States and Great Britain, despite being the main SIBs, have a lending score equal to zero. This is due to the lack of data about their claims because these countries are non-reporting ones (see Footnote 4) rather than to the fact that they are really not important in terms of lending activities to other economies.
3.3 The time dimension

This subsection is devoted to the results obtained by the inspection of the temporal dimension of systemic risk. Figure 4 shows the normalized\(^9\) time score (red) along with the entire normalized traded volume (blue) in each period for the e-MID dataset (a) and for the BIS dataset (b). The pre-crisis phase (from 2006-Q2 to 2007-Q2) is marked with a blue background and the financial crisis period is divided into two sub-phases (from 2007-Q3 to 2008-Q2 and from 2008-Q3 to 2009-Q2) highlighted with red and gray backgrounds respectively.

The traded volume of both series increases until the peak of 2007, and subsequently decreases, with a more pronounced fall for the e-MID dataset. Regarding the interbank tensor, time score increases faster than the traded volume up to 2004. This dynamic is explained by the entry of foreign banks in a market that was limited to Italian institutions prior to 2000\(^{10}\). Subsequently it reaches its maximum before the pre-crisis phase and stays approximately at the same level in the first half of the pre-crisis period, starting an abrupt fall in 2006. After the financial crisis, when most banks stop trading, the market collapse is identified by the low values of both the traded volume and the time score. The time score, computed using the ultimate risk tensor, follows more closely the traded volume dynamic. However, it is interesting to note that it becomes flatter than the traded volume during the end of the pre-crisis phase and that abruptly falls when the traded volume is still high or increasing in two specific episodes, i.e., in the middle of the first wave of the crisis and during the end of the second phase of the crisis\(^{11}\). This behavior is also emphasized by the enlargement of Figure 4 (b).

\(^9\)Normalization is obtained by dividing each series by its maximum value over all the sampling periods. This allows a better comparison between the dynamics of the time score and of the traded volume.

\(^{10}\)For sake of brevity, the figures demonstrating this result are not shown but available upon request.

\(^{11}\)The drop of the time score and of the traded volume that occurred in 2010-Q3 is due to the fact that reporting countries increased their transactions against credit aggregates like "the rest of Europe" or "off-shore areas" that are non reporting entities and therefore not included in the analysis rather than because of the financial crisis.
Figure 4: Normalized traded volume and time score for the e-MID interbank market dataset (a) and for countries’ banking sector claims reported in the BIS consolidated banking statistics (b). The total transaction volume is drawn in blue while the time score in red in both sub-figures. In all the panels, the pre-crisis phase (from 2006-Q2 to 2007-Q2) is marked with a blue background and the financial crisis period is divided into two sub-phases (from 2007-Q3 to 2008-Q2 and from 2008-Q3 to 2009-Q2) highlighted with a red and a gray background respectively. The enlargement in the panel (b) underlines periods near the crisis in which the time score increases less that the traded volume or in which its fall is followed by an abrupt decrease of the traded volume in the BIS dataset.

As anticipated in the introduction, the fall in the time score is due to a change in the business activities of institutions with high borrowing/lending score values that first lower their transaction volume with respect to other entities which, in turn, reduce loans/investments only when the financial crisis unfolds. To appreciate this feature I have computed the cumulative sum of the normalized\(^{12}\) weighted links for the e-MID dataset and for the BIS dataset during different periods. The weighted links have been previously arranged using the order produced by the decreasing sort of the elements that compose the joint systemic importance matrices of the interbank and of the ultimate risk tensors \(S_{ib,ur}^{th}\). The elements of these matrices indicate

\(^{12}\)The normalization of the weighed links is obtained by dividing each transaction by the traded volume of the period the links belong to. This allows a better comparison of the shape of their cumulative sum disregarding the difference in the magnitude of traded volume between different periods.
the systemic importance of all pairs of institutions connected by possible links; i.e. $s^{ib,ur}_{i,j} = l_i s_j^{ib,ur} b_j^{ib,ur}$, $\forall i, j$. Note that these matrices do not vary in time but that, period by period, the couples of players activated by the creation of links can be different. The shape of the cumulative sum of the links’ weights indicates whether the transactions of the highest amount are associated with couples of players with a high joint systemic importance or whether they are executed between normal entities. The closer the curve produced by the cumulative sum follows the left-hand border and then the top border of the plotting space, the more the weights of the links connecting systemically important players are high with respect to the others. Figure 5 displays the results for the e-MID dataset (a) and for the BIS dataset (b)-(e).
Figure 5: Cumulative sum of the normalized weighted links sorted according to the joint systemic importance of the nodes connected by all possible transactions, for the e-MID dataset (a), and for the BIS dataset (b)-(e). The small box in panel (a) shows the enlargement for the links belonging to the most systemically important pairs of banks in the e-MID dataset. In order to produce a better visualization, for the BIS dataset the episodes in which the time score increases less than the traded volume and in which its fall is followed by an abrupt decrease of the traded volume are shown separately in four sub-figures (b)-(e).

Regarding the e-MID interbank market, it clearly emerges from Figure 5 (a) that up to the pre-crisis phase, transactions involving the highest amounts connect pairs of banks with an increasing joint systemic importance. During the pre-crisis period, this trend reverts and less systemically important institutions connect with links having higher weights. For the BIS dataset the same behavior is visible in four different episodes which occurred near the crisis period as reported in the lower panels of Figure 5. During the end of the pre-crisis phase (b), from the second to the third quarter of 2007 the time score became flatter than the traded volume, while from 2007-Q4 to 2008-Q1 (c), from 2009-Q1 to 2009-Q2 (d) and from 2010-Q2 to 2010-Q3 (e) it decreased more steeply than the volume. In all these episodes, the change in the time score pattern is due to a repositioning of the business behavior of SIFIs that decrease their transaction volume with respect to other entities.

Note however that the conclusions expressed above regard only an ex-post valuation of the mechanism leading to the fall of the time score in the pre-crisis phase. Or, in other words, they are valid only when the borrowing and the lending score are computed using all the temporal information available from 2000 up to 2012.

A deeper investigation indeed clarifies that, prior to a crisis, the topology of the bipartite network of lenders and borrowers changes significantly with respect to "business as usual" periods. Recomputing the scores whenever new data are available, by adding each time the new adjacency matrix, sheds some light on the behavior of the players in different periods.
Figure 6: Borrowing score and lending score values calculated by adding during each period the adjacency matrix representing new observations. Panels (a)-(b) represent the borrowing and lending score values respectively, of all the banks active in the e-MID interbank market. Panels (c)-(d) show the borrowing and lending score values respectively, of the countries in the ultimate risk tensor. Near the crisis phase the rankings produced by the two scores change. This change is expressed by a decreases of the time score.

Figure 6 shows that the borrowing and lending score of different entities show remarkable changes in the pre-crisis phase. Some institutions lose importance and other players increase their systemic impact. In particular, players that have a high score at the end of the sample, show a burst in their systemic importance during the pre-crisis phase, as they are replaced by other institutions that became systemically more important only during this period. This means that the time score starts to fall in a pre-crisis phase, even for a still increasing traded volume, because of a change in the topological structure of the bipartite network of borrowers and lenders.

The considerations produced by the two types of analysis are, however, not mutually exclusive. In fact the borrowing and lending scores, computed at the end of the sample period, contain all the information about the market transactions executed up to 2012. This is a peculiarity of the tensor approach to systemic risk.

Furthermore, notice that all these features of the system could not have emerged by looking only at transactions executed period by period, meaning that a tensorial approach, that combines the cross-sectional and the time dimension, is needed to discover prior changes in SIFIs’ behavior.

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that anticipate a market crash.

3.4 Evaluating the performance of the time score as an EWS

The analysis conducted so far only demonstrates graphically the predictive power of the time score on the basis that the indicator is able to correctly recognize crisis events when these events have already been occurred. The time score, in other words, appears to have a good in-sample predictive power. The next step is to perform an out-of-sample analysis to evaluate whether this measure can be useful for predicting downturns of the traded volume even when data about crisis periods are not yet available. This exercise is fundamental in order to evaluate the practical implementation of the measure, given that, to be really exploitable, the time score must anticipate an unknown market fall.

Therefore, we assume that we are in the first period of the pre-crisis phase, unaware of what will happen in the future. Now the question is whether the computation of the time score would have been helpful to forecast the market crash of 2007. To answer this question, starting from the pre-crisis phase, the transaction volume and the time score obtained from the tensor build stacking the data up to that point, are collected (Appendix B shows the behavior of the time score computed using the out-of-sample approach together with the traded volume for both datasets). Then the Granger causality test [27] between the traded volume and the time score is computed whenever new data are available, using each time the new values of the time score and of the traded volume.

After having standardized the two series, so that they have zero mean and unit standard deviation, the Granger causality is defined as:

\[ ts_t = \sum_{z=1}^{Z} a_z ts_{t-z} + \sum_{z=1}^{Z} b_z tv_{t-z} + \epsilon_t \]

where \( \epsilon_t \) is an i.i.d. processes, \( Z \) is the maximum lag considered and \( a_z, b_z \) are the model coefficients. The causality test is based on the F-test where the null hypothesis is that the coefficient \( b_z \) is zero. By definition the time score (\( ts \)) is said to be Granger-caused by the traded volume (\( tv \)) if it can be better predicted using the histories of both \( ts \) and \( tv \) than it can using the history of \( ts \) alone. Periods in which this is no longer true could be intended as alarm bells of
looming financial crisis because in these periods strong changes in the cross-sectional dimension of the systemic risk occur. In fact, assessing whether the traded volume Granger-caused the time score means assessing whether the dynamic of the time score is given by that of the traded volume or whether it is mainly due to a change in the distribution of the transactions among players. As reported above, during pre-crisis phases systemically important institutions lower their transaction volume with respect to other entities that take over as new SIFIs, causing a strong change in the distribution of the traded volume. This change translates into a falling Granger-causality between $ts$ and $tv$ in pre-crisis phases.

Results are shown in Figures 7 and 8 for the e-MID and the BIS dataset respectively, from one lag up to four. In both figures it clearly emerges that the fall in the Granger causality between the two series anticipates the fall of the traded volume. For both the e-MID and the BIS dataset this feature is strong for two and three lags.

This means that, for the e-MID dataset, the time score predicts quite well the behavior of the traded volume three months in advance. While for the BIS dataset, the time score anticipates the volume two quarters in advance. The out-of-sample analysis thus reinforces the idea that the time score could be employed as an EWS to identify future market collapse.

Figure 7: Out-of-sample analysis of the e-MID interbank market. The subplots report, for different numbers of lags (from one month up to four), the difference between the F-test and the critical value of
the F-distribution at the 95% confidence level (blue line) together with the traded volume (green line).
The black arrows emphasize that a decrease of the Granger causality is followed by a decrease of the traded volume.

![Graphs showing F-distribution and traded volume over time with arrows indicating decrease in Granger causality leading to decrease in traded volume.]

Figure 8: Out-of-sample analysis of the BIS dataset. The subplots report, for different numbers of lags (from one quarter up to four), the difference between the F-test and the critical value of the F-distribution at the 95% confidence level (blue line) together with the traded volume (green line). Notice that a decrease of the Granger causality is followed by a decrease of the traded volume (emphasized by the black arrows).

4 Conclusions

In the notes on macroprudential policy framework, institutions devoted to safeguarding the financial system pointed out the presence of two sources of systemic risk for financial markets which, although related, affect the soundness of the system in two different dimensions: the cross-sectional dimension and the time dimension. The two sources of instability are intertwined and cannot be addressed separately. In fact a rise in the interconnectivity among institutions promotes the financial cycle but, at the same time, a strong asset growth leads to greater interconnectedness within the system. The algorithm proposed in this work explicitly addresses...
systemic risk in a unified framework in which both the time and the cross-sectional dimensions are taken into account simultaneously.

The W-TOPHITS is applied to the e-MID interbank market dataset and to the consolidated banking statistics collected by the Bank for International Settlement. The findings have non negligible implications both for modeling and policy making. Indeed, the time score is shown to be a useful early warning indicator of the financial cycle because it anticipates the fall of the traded volume that occurred during the financial crisis. The time score decreased in the pre-crisis phase, as before the market crash of 2007/2009, because of a change in the network structure. SIFIs, during that phase, lower their volume of transactions, making other institutions more systemically important. This paper points out that the time-analysis of the functioning of systemically important entities plays a fundamental role in predicting a market downturn because these players drive the conduct of the rest of the intermediaries.

The observed topological alteration could have been originated by a shift of the institutions’ confidence in their risk profile of their peers and could be interpreted as a precursor of the downturn of the financial cycle. This is because it signals a change in the relationships of trust among financial intermediaries that possibly led to the subsequent market collapse. In fact, in the presence of phenomena like information asymmetries and moral hazard, institutions have a strong incentive to monitor their partners; [28]. Because peer monitoring is costly, trust in partners’ soundness plays a significant role especially in markets for uncollateralized loans. Indeed financial institutions base their decisions to engage or not in a transaction on the level of trust in their peers. Thus the change in the network structure that occurred during the pre-crisis phase could imply a change in the institutions’ confidence in the risk profile of their peers, possibly indicating an increasingly riskier situation for the markets that materialized with the financial crisis.
References


## Appendices

### A Additional table

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Table 1: The first 31 most systemically important banks in the e-MID interbank market sorted in descending order by their lending score values (first column), and by their borrowing score values (second column). The 31 countries in the BIS dataset sorted in descending order by their lending score values (third column) and by their borrowing score values (fourth column).

B Out-of-sample analysis of the time score

The following figures represent the dynamic of the time score whenever it is computed starting from tensors of different length that represent all the available information up to each period. The time score begins to fall before the burst of 2007 even for the out-of-sample analysis. Inserting new data little affects the past dynamic of the time score, meaning that this variable could be useful to anticipate unknown financial crisis.

Figure 9: Out-of-sample analysis of the time score for the e-MID interbank tensor. The figure represents in blue the time score computed using tensors of different length in the third dimension. Each tensor represents the available data up to a certain period. The length of the time score therefore increases in time and its shape adjusts to take into account the new set of transactions and its feature.

In red traded volume.
Figure 9: Out-of-sample analysis of the time score for the BIS ultimate risk tensor. The figure represents in blue the time score computed using tensors of different length in the third dimension. Each tensor represents the available data up to a certain period. The length of the time score therefore increases in time and its shape adjusts to take into account the new set of transactions and its feature.

In red traded volume.