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# Aggregation with non-convex labor supply, unobservable effort, and efficiency wages of the no-shirking type

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## Abstract

The purpose of this note is to explore the problem of non-convex labor supply decision in an economy with imperfect observability of work effort, and the need to use efficiency wages to prevent shirking as in Shapiro and Stiglitz (1984). In addition, the paper and explicitly performs the aggregation presented in Vasilev (2017) without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply increases from unity to infinity.

**Keywords:** Aggregation, Indivisible labor, Non-convexities, Shirking, Efficiency Wages

*JEL Classification Codes:* E1, J22, J41, J46

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# 1 Introduction and Motivation

The purpose of this note is to explore the problem of non-convex labor supply decision in an economy with imperfect observability of work effort, and the need to use efficiency wages to prevent shirking as in Shapiro and Stiglitz (1984). In addition, the paper and explicitly performs the aggregation presented in Vasilev (2017) without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply increases from unity to infinity.

## 2 Model setup

The theoretical setup follows to a great extent Vasilev (2017). There is a unit mass of households, indexed by  $i$  and distributed uniformly on the  $[0, 1]$  interval, as well as a representative firm. In the exposition below, we will use small case letters to denote individual variables and suppress the index  $i$  to save on notation. To simplify the analysis, the model economy is static, without physical capital, and agents will face a non-convex labor supply decision. The firm produces output using labor and capital, but cannot observe the effort exerted by workers. Given that effort is not directly contractible (due to its unobservability on the firm's side), the firm sets a reservation wage to induce an optimal level of effort.

### 2.1 Description of the model

Each household maximizes the following utility function:

$$U = \ln c + \eta \ln(1 - eh - \xi), \tag{1}$$

where  $\eta > 0$  is the weight attached to leisure, and  $\xi > 0$  represents some fixed cost of working. Variable  $c$  denotes household  $i$ 's consumption,  $h$  denotes a fixed number of hours worked, and  $e$  is the amount of effort exerted. The time available to each worker is normalized to unity. In addition, we assume that that worker's effort will be imperfectly observable by firms.

All households have equal share in the firm's profit. Total profit is pooled together (within the "family" of households), and then distributed equally among all households. In this way, households can partially insure one another against unfavorable outcomes in the labor market, e.g. not being selected for work. The common consumption can be represented as

$$c^h = \pi, \quad (2)$$

$\pi = \Pi$ , or the sum of individual profit income equals firm's total profit. The other type of income is the labor income, and households would differ in each period depending on their employment status. In addition, given that the level of effort is not directly observable, some of the employed workers will work and exert the required effort level,  $e$ , stipulated in the contract, while others may decide to shirk. If caught, which happens with probability  $d$  due to the imperfect technology of detection, the individual is fired and receives a fraction  $0 < s < 1$  of the wage. Therefore, the labor contract that the firms then needs to offer is to be one that induces workers not to cheat in equilibrium. The contract would specify a wage rate, an effort level, and an implementable rule that a worker caught cheating on the job will be fired and paid only a fraction  $s$  of the wage,  $0 < s < 1$ . All workers know this in advance, and take the terms of the contract and the labor demand as given.

In addition, each employed transfers/contributes  $T$  units of income to the unemployment pool, where the proceeds are used to payout to the unemployed. The consumption of an employed worker who does not decide to shirk then equals:

$$c = c^h + wh - T, \quad (3)$$

where  $w$  is the hourly wage rate. In contrast, a worker who is employed, decides to cheat, and is caught, receives

$$c^s = c^h + swh - T. \quad (4)$$

The consumption of an individual that does not work,  $c^u$ , is then

$$c^u = c^h + \frac{n}{1-n}T, \quad (5)$$

The corresponding instantaneous utility levels are in turn:

$$u(c^u, e^u = 0, h^u = 0) = \ln c^u + \eta \ln 1 = \ln c^u, \quad (6)$$

if unemployed,

$$u(c, e, h) = \ln c + \eta \ln(1 - eh - \xi), \quad (7)$$

if employed and the worker does not shirk,

$$u(c, e, h) = \ln c + \eta \ln(1) = \ln c, \quad (8)$$

if the person shirks, but is not caught, and

$$u(c^s, e^s = 0, h^s = 0) = \ln c^s + \eta \ln(1) = \ln c^s, \quad (9)$$

if the person shirks, and is caught.

## 2.2 Firm

There is a representative firm that produces output via the following production function

$$y_t = (nhe)^{1-\alpha}, \quad (10)$$

where  $0 < \alpha < 1$  is a parameter characterizing the concavity of the production function. The firm chooses both the employment rate and the wage rate (to induce a certain optimal effort level, i.e.  $e = e(w)$ ) to maximize

$$(nhe)^{1-\alpha} - wnh \quad (11)$$

s.t. "no shirking condition" (the ICC):

$$\ln c + \eta \ln(1 - \xi - h) \geq (1 - d) \ln c + d \ln c^s. \quad (12)$$

## 2.3 Decentralized competitive equilibrium (DCE): Definition

A DCE is defined by allocations  $\{c, c^u, c^s, e, H\}$ , wage rate  $\{w\}$ , and aggregate profit ( $\Pi = \pi$ ) s.t. (1) all households maximize utility; (2) the stand-in firm maximizes profit; (3) all markets clear.

### 3 Characterization the DCE and derivation of the aggregate utility function

It will be shown that in the DCE, if it exists, only some of the households will be employed and work full-time, while the rest will be unemployed. Following the arguments in Rogerson (1988) and Hansen (1985), it can be easily shown that polar cases in which each household either, or a case in which nobody works, cannot not be equilibrium outcomes. Therefore, it must be the case that a only proportion of the agents in the economy are working. Denote this mass of employed by  $n$ . Workers who do not shirk will receive consumption  $c$ , workers who shirk but are not caught also get  $c$ ; those who shirk and are caught receive  $c^s$ , and those not selected for work will consume  $c^u$ . Note that if a household is selected for work and rejects the job offer, there will be no unemployment insurance, or it would receive just the common consumption  $c^h$ . Therefore, no household selected for work would have an incentive to reject the offer.

Lastly, a worker caught shirking will be optimally fined to bring his/her utility down to the utility of an unemployed person. However, given the detection probability of shirking  $d$ , some workers may still decide to shirk. Let  $n^s$  be the proportion of shirkers and given a detection probability  $d$  of a shirker being caught, this implies  $dn^s$  would be the proportion of shirkers being caught, and  $(1 - d)n^s$  are the shirkers not being caught. In turn,  $n - n^s$  are the employed individuals who decide not to shirk. There will be a unique level of intra-household transfer  $T$  that equalizes  $c^s = c^u$  and would uniquely determine  $n^s \in (0, 1)$ .

Firm's profit is then

$$\pi = \Pi = (nhe)^{1-\alpha} - wnh > 0, \quad (13)$$

which follows from the decreasing returns to scale featured by the production function. Note that the firm will set the efficiency wage in order to prevent shirking. Next, to show that the DCE actually exists, it is sufficient to show the existence of a fixed point  $n \in (0, 1)$  by analyzing a non-linear equation. In particular, it is trivial to show that everyone working in the market sector ( $n = 1$ ) is not an equilibrium, since then  $w = 0$ . Similarly, nobody working ( $n = 0$ ) is not an equilibrium outcome either, since the firm would then offer a very

high wage for the first unit of labor, and by taking a full-time job a marginal worker could increase his/her utility a lot.

Thus, if there is a DCE, then it must be that not all households would receive the same consumption bundle. If  $n \in (0, 1)$  is an equilibrium, then total utility for households that work should equal to the utility of households that do not supply any hours. This equation is monotone in  $n$ , as the utility function is a sum of monotone functions. Thus we can explore the behavior of that function (the difference between the utility of working and not working) as we let  $n$  vary in the  $(0, 1)$  interval. As  $n \rightarrow 0$ , the left-hand-side dominates (utility of working is higher), while when  $n \rightarrow 1$  the right-hand-side dominates (utility of not working is higher), where the results follow from the concavity of the utility functions and the production technology. In addition, from the continuity of those functions,  $\exists n \in (0, 1)$ , which is consistent with equilibrium. The unique value of  $n$  follows from the monotonicity of the utility and production functions. Let  $c$  and  $c^u$  denote equilibrium consumption allocations of individuals selected for work, and those who are unemployed. Therefore, aggregate utility becomes:

$$(n - n^s)[\ln c + \eta \ln(1 - \xi - eh)] + n^s[(1 - d) \ln c + d \ln c^s] + (1 - n) \ln c^s, \quad (14)$$

which is the representation used in Vasilev (2017). On the aggregate, when each household faces an indivisible labor choices, the representative agent obtained from the aggregation features different preferences of work: as in Hansen (1985), the disutility of work in the market sector is now linear in employment.

## 4 Conclusions

The purpose of this note is to explore the problem of non-convex labor supply decision in an economy with imperfect observability of work effort, and the need to use efficiency wages to prevent shirking as in Shapiro and Stiglitz (1984). In addition, the paper and explicitly performs the aggregation presented in Vasilev (2017) without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to

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