Hinz, Holger

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A Transshipment - Type Model for the Optimal Allocation of Foreign Currencies in Multinational Corporations

Holger Hinz

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Diplom-Kaufmann Holger Hinz
Institut fuer Betriebswirtschaftslehre
Lehrstuhl fuer Finanzwirtschaft
Director: Prof. Dr. Reinhart Schmidt

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OPTIMAL ALLOCATION OF FOREIGN CURRENCIES
IN MULTINATIONAL CORPORATIONS

1. Introduction

Starting in the late 1970's the attempt to rationalize multinational corporations' (MNCs') short-term liquid funds management led to the development and implementation of so-called "Netting Systems": In order to reduce the costs associated with transferring inter-affiliate payments, the idea behind these systems is to net out payables and receivables so that only this netted amount needs be transferred. Depending upon the MNCs' internal payments structure there are two possible ways of determining these fund flows: In case affiliates just sell back and forth to each other, a bilateral netting approach will suffice. Generally, though, more complex interdependencies among MNCs' subsidiaries call for a multilateral netting system to be presented here.

While scientific publications on international cash management problems have brought up applications of a great variety of methodological approaches (1), in my opinion the problem outlined above has proven to be highly suggestive to mathematical programming techniques. Starting out I will therefore describe a classical transportation - type approach ( cha-

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Chapter 2), then develop a modified linear program (LP) in chapter 3 along with a numerical example (chapter 4), and finally draw some conclusions in chapter 5.

2. A Classical Transportation - Type Model

As an example, let us consider the transportation - type model suggested by SHAPIRO (2):

\[
\begin{align*}
\text{minimize} & \quad \sum_{i,j} C_{ij} X_{ij} \\
\text{subject to} & \quad \sum_{j} X_{ij} = O_i ; \quad i = 1, \ldots, m \\
& \quad \sum_{i} X_{ij} = O_j ; \quad j = m+1, \ldots, n \\
& \quad X_{ij} \geq 0 ; \quad i,j = 1, \ldots, n
\end{align*}
\]

The potential savings of such a system clearly depend upon the nature of the costs associated with funds transfer, i.e. the structure of the cost matrix and the magnitude of its elements (the C_{ij}'s). Typically, these costs are a result of

(1) SPREAD, i.e. the difference between bid and ask rates for a currency and

(2) FLOAT, i.e. the opportunity cost of idle money-in-transit.

If we inspect the SHAPIRO-model delineated above in some detail, it becomes obvious that the algorithm cannot generate an optimal solution with respect to the conversion costs (SPREAD). For one thing, it is assumed that the amounts due between affiliates are given in the MNC's home currency units, while any information on the underlying subsidiary's currency portfolio is lost.(3) Also, since the cost coefficients are an amalgam of FLOAT- and SPREAD-related particles, the choice among alternative ways of transferring funds is biased: Fund flows between any two connected nodes of the MNC's network are evaluated as if currency conversion necessarily has to take place. On the other hand, the model cannot account for real-world FLOAT-costs either, since the opportunity (interest-) cost involved has to be considered a function of the fund-in-transit's underlying currency.

SHAPIRO's model has been introduced despite these deficiencies, since it is well suited as a starting point for a more general problem formulation to be presented in the next section, whose aim will be to suggest optimal decisions with respect to the uses and sources of short-term liquid funds in MNCs.

(3) The same applies to:
Planning short-term cashflows in MNCs typically involves a two-step-procedure: First, subsidiaries have to independently determine the amounts, the timing and the currency denomination of cash inflows and -requirements respectively. In order to be able to exploit potential economies of scale which may exist from an MNC-wide perspective, headquarter's treasury management now has to centrally decide upon the intra-corporate financial transactions to be taken.

Since in the short-run it can be assumed that the MNCs' cash requirements will not exceed its credit lines there is no need to formulate a liquidity constraint on the company as a whole.

The reason why a multiperiod approach proves advantageous to modeling the problem at hand may best be explained considering the following situation: Suppose, a subsidiary's cash requirements include a currency not available in any of the affiliates. The solution to be generated must account for the fact that the subsidiary's choice is between either converting currencies or taking a currency loan. In case a loan is considered favorable, this will have an impact on future cashflows. Therefore, a meaningful evaluation of conversion costs can only be achieved in a multiperiod setting.
To sum up the arguments put forward thus far, Exhibit #1 displays the variables affecting currency-balances which are considered relevant in our model. For expository convenience we restricted this presentation to an MNC consisting of

--three units (FRG,USA,GB)

--financial headquarters ('bank')

dealing in at most three different currencies each (DM, $, FF). Each node is characterized by the name of the subsidiary (s), the planning-period (t), and the currency (c) involved.

Exhibit #1

Variables affecting the US-subsidiary's first-period Dollar-balance
Breaking down Shapiro's cost coefficients into its main components then yields the following problem formulation:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i,j,t,l} C_{ijtl} X_{ijtl} + \sum_{i,j,m,n} W_{itmn} Y_{itmn} \\
& \quad + \sum_{j,t,l} B_{jtl} X_{0jtl} - \sum_{j,t,l} L_{jtl} X_{j0tl} \\
\text{subject to} & \quad \sum_{j,t} X_{ijtl} - \sum_{j,m} Y_{itml} + \sum_{m} Y_{itlm} \\
& \quad + X_{i0tl} - X_{0itl} \\
& \quad = Z_{itl} + X_{i0(t-1)l} - X_{0i(t-1)l} \\
& \quad (\text{for all } i,t,l) \\
\sum_{t,l} X_{ijtl} & \leq \text{TRANSMAX}_{ij} \\
\sum_{t,l} X_{jitl} & \leq \text{TRANSMAX}_{ji} \\
(\text{for all } i,j) \\
\sum_{i} X_{i0tl} & \leq \text{BORROWMAX}_{i} \\
\sum_{i} X_{i0tl} & \leq \text{LENDMAX}_{i} \\
(\text{for all } t,l)
\end{align*}
\]

The objective function (1) depicts the cost/benefit structure involved in allocating the MNC's foreign currency resources. It is important to note that the $C_{ij}$'s here should be interpreted as a subset of Shapiro's coefficients, denoting the (linear) costs of transferring funds, including float, wire- and cable charges etc. The second summation

<table>
<thead>
<tr>
<th>Variables</th>
<th></th>
<th>Coefficients</th>
<th>Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>Amount transferred (decision)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>Amount converted (decision)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>Cash Flow (exogenous)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>Transfer costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>Conversion costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>Borrowing rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Lending rate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0 MNC's central treasury
i,j Subsidiaries
t Period
l,m,n Currency Denomination
accounts for conversion costs between currencies ("Conversion from currency m to currency n") and allows for the possibility of shipping cash without currency conversion necessarily having to take place. The remainder of the objective function relates to the subsidiary's interest income. Constraints (2) account for the financial plans of the subsidiaries, where we assume that the loans are short-term in nature and to be payed back after each period. The RHS - variables "X" are exogenous for the first period only, denoting the subsidiaries' beginning cash balances. In addition to these liquidity constraints, one can think of a wide variety of institutional and management-policy-type influences on the allocation process: Constraints (3) may be interpreted as one possible form of capital transfer restrictions, while a simple, currency-based credit policy is represented by constraints (4).

4. Numerical Example

For a better understanding of how the model works, consider the following numerical example for a fictitious MNC consisting of headquarters and two subsidiaries with cashflows in two foreign currencies and a planning horizon of two periods.
minimize

\[
\begin{align*}
&0.11 \times X_{1211} + 0.11 \times X_{1212} + 0.11 \times X_{1221} + 0.11 \times X_{1222} \\
&+ 0.33 \times X_{2111} + 0.33 \times X_{2112} + 0.33 \times X_{2121} + 0.33 \times X_{2122} \\
&+ 0.51 \times Y_{1112} + 0.51 \times Y_{1121} + 0.61 \times Y_{1212} + 0.61 \times Y_{1221} \\
&- 0.79 \times X_{1011} - 0.79 \times X_{1012} + 1.15 \times X_{0112} \\
&- 0.59 \times X_{1021} + 1.11 \times X_{1022} + 1.30 \times X_{0122} \\
&- 1.16 \times X_{2011} + 1.12 \times X_{2012} + 1.50 \times X_{2022} \\
&- 0.55 \times X_{2021} + 1.14 \times X_{2022} + 1.12 \times X_{2022}
\end{align*}
\]

subject to

\[
\begin{align*}
X_{1211} - X_{2111} + Y_{1112} - Y_{1121} + X_{1011} - X_{0111} &= -74 \\
X_{1212} - X_{2112} - Y_{1112} + Y_{1121} + X_{1012} - X_{0112} &= 64 \\
X_{1221} - X_{2121} + Y_{1212} - Y_{1221} - X_{1011} + X_{0111} + X_{1021} - X_{0121} &= -29 \\
X_{1222} - X_{2122} - Y_{1212} + Y_{1221} - X_{1012} + X_{0112} + X_{1022} - X_{0122} &= 110 \\
- X_{1211} + X_{2111} + Y_{2112} - Y_{2121} - X_{2011} + X_{2021} - X_{0211} &= -63 \\
- X_{1212} + X_{2112} - Y_{2112} + Y_{2121} + X_{2012} - X_{2022} &= -120 \\
- X_{1221} + X_{2121} + Y_{2212} - Y_{2221} - X_{2011} + X_{2021} + X_{2021} - X_{0221} &= -16 \\
- X_{1222} + X_{2122} - Y_{2212} + Y_{2221} - X_{2012} + X_{2022} + X_{2022} - X_{0222} &= 92 \\
X_{1211} + X_{1212} + X_{1221} + X_{1222} &\leq 50 \\
X_{2111} + X_{2112} + X_{2121} + X_{2122} &\leq 50
\end{align*}
\]

Since the data do not affect the model's applicability, the RHS - values and the coefficients have been chosen arbitrarily, i.e. Bitl > Litl > Wtmn > Cijtl. The first constraint then shows the need for 74 currency units (type 1) by subsidiary one in period one.
Exhibit #2 displays the optimal MNC-wide strategy:

<table>
<thead>
<tr>
<th>Indices</th>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>Currency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Transfer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conversion</td>
<td>60.00</td>
<td>50.00</td>
</tr>
<tr>
<td>Borrowing</td>
<td>14.00</td>
<td>14.00</td>
</tr>
<tr>
<td>Lending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transfer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conversion</td>
<td>63.00</td>
<td>70.00</td>
</tr>
<tr>
<td>Borrowing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lending</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each cell of the solution matrix displays the model-based financial actions suggested, where "Transfer" denotes the amount to be sent to the receiving unit and "Conversion" stands for the amount to be changed into the complimentary currency. In our example, subsidiary one will take a loan (60) and convert 14 to satisfy its demand for 74 currency units. Here, the advantage to the joint usage of the MNC's financial resources mainly stems from the ability to circumvent the banks' borrowing-/lending-rate spread, since parts of the loans now can be viewed as "homemade". To give the reader a first numerical impression of the possible savings of such a system, we compared the above example's solution to a situation, where the "TRANSMAXij's" were set to zero and observed an increase in allocation costs from 36.58 to 39.40 home-currency units (objective function values).

5. Conclusion

Due to the complex interdependencies involved in allocating foreign currency in MNCs an LP-based model structure and a corresponding numerical example have been presented. The advantages of the modeling
approach taken are manifold: First, the clear-cut definition of different costs involved in the allocation process and the model's multiperiod nature resolve suboptimalities encountered in former research. Secondly, the model can be viewed as the basis to developing an integrated international cash management system: According to Teck(4) the management of an MNC's economic exposure consists of sequentially deciding upon 1) internal actions (e.g. netting), and 2) the use of external hedging instruments (e.g. forward cover).

In our example, the MNC's overall currency position after the allocation process has taken place (internal measures) can be monitored by inspecting the affiliates' payables/receivables - accounts with headquarters. Decisions involving external exposure management activities will have to be based on these data.

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Literature


