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Evolutionary game approach on fixed-mobile market

Nariaki Ikematsu

Keywords: Evolutionary game, Lotka-Volterra equations, fixed-mobile market

Abstract

Lotka-Volterra equations which equivalent to the replicator equation in evolutionary game theory under mathematical dynamic ecosystem model will provide the new basic behavior of network services. The FTTH-DSL market in Japan shifted from the co-existence model of horizontal market to the Winner-Takes-All model of FTTH. This model also shows that the fixed-mobile market in Thai and Japan have the model of the bi-stable model. This report also submits some data and materials to start discussing the new future infrastructure.

The parameters of Lotka-Volterra equations are also dynamic. If parameter is constant the market does not have any innovation. It means that **the survival-of-the-fittest world does not have any innovation.**

Introduction

Call for Papers of 4th ITS Asia-Pacific Conference on June, 2017, Kyoto, Japan states that the fundamental ecosystem of ICT consists of the three layers, infrastructure, platform, and applications. The new platform and application layer will replace the traditional systems by the new infrastructure, because the big or rich data needs the bigger capacity of infrastructure such as the new photonics with high-speed and precise light-wave control.

So, this report begins applying the evolutionary game theory to the typical replacement or substitution of FTTH-DSL market in Japan and fixed line mobile market in existing network ecosystem infrastructure to understand the nature of this model before we simulate the replacement of the new infrastructure ecosystem by this model. The ecosystem in this report is not familiar among the researchers but the purpose of this report does not include the definitions of ICT ecosystem. However, this the replicator equation, the evolutionary game mathematical model which is equivalent to Lotka-Volterra-type equations (hereinafter called LV equations) is the basic dynamic mathematical model to study the ecosystem among species in nature. This model has been also widely studied among the researchers in social science academy. This definition of ICT ecosystem in this report is simply the market described by the numbers of subscribers derived from this model because the advantage of this dynamic model is to explain the

changes of the subscribers of the ecosystem or market.

This report also submits some data and materials to start discussing the new future infrastructure. Even if this report fails to reach the goal, the evolutionary game theory have a long and huge knowledges to explain competition, corporation and so forth. If this model in this report works well, we may probably apply this model to develop the policy or strategy to replace the traditional systems by the new infrastructure.

1. Previous study in theory (1)

1.1 Early studies

Before applying the LV equations to the existing network, we introduce the previous study in theory of LV equations in networks.

Francisco Javier Sanchez Casado, Raquel Nunez Lopez and Cecilia Sanchez, Sanchez [1996] explains the substitution effect of mobile telephones on fixed telephony. The model proposed in this paper is LV equations. Hayashi [2005] discusses independently applied modified LV equations for the competitions between two network facilities services and Hayashi [2008] also applied the modified LV equations for predator-prey systems to the network operator (NO) and vertical network operator (VNO).

1.2 Recent Study

Ikematsu[2017] goes back to the following simple original LV equations and discuss the nature of these equations in order to apply the linear regression analysis to the changes in the numbers of subscriber among FTTH-DSL market in Japan.

$$\begin{aligned} \frac{1}{x_1} \frac{dx_1}{dt} &= \varepsilon_1 + \mu_{11}x_1 + \mu_{12}x_2 \\ \frac{1}{x_2} \frac{dx_2}{dt} &= \varepsilon_2 + \mu_{21}x_1 + \mu_{22}x_2 \end{aligned} \quad (1)$$

where i is the type of service, x is the numbers (populations) of a service, t is time, ε is the intrinsic growth rate and μ is intensity rate of interaction. We call the parameter μ the intensity rate of self-interaction at $i=j$ and the intensity rate of interaction at $i \neq j$. The LV equation is equivalent to replicator equation in evolutionally game theory. Under ecosystem where services survive, evolutionally game theory shows intensity rate of self-interaction is intensity of self-adaptation to environment and intensity rate of interaction is intensity of strategy to competitors' in environment. The equation is equivalent to Rolf's model at $i=1$.

Ikematsu and shoji [2017] goes back to the original knowledge of biological dynamic ecosystem model and re-defined and re-categorize these equations by the signs of parameters as Table 1:

TABLE1 The LV model for two services in the network market

Mathematical Mode		Competition System	Predator-Prey System
Service 1	ε_1 intrinsic growth rate of service 1	+	+
	μ_{11} intensity rate of self-interaction for service 1	-	-
	μ_{12} intensity rate of interaction of service 2 against 1	-	-
Service2	ε_2 intrinsic growth rate of service 2	+	-
	μ_{21} intensity rate of interaction of service 1 against 2	-	+
	μ_{22} intensity rate of self-interaction for service 2	-	-
Expected Target market		Horizontal market	Vertical market NO -VNO

(1) Competition System

Suppose the absence of one of the services is 0, the network service falls into network effect by Rolfs and the sign of intrinsic growth rate of service 1 or 2 becomes positive.

Under competitive system in horizontal market, when two players provide the similar service in the same target, it competes each other. A player attempts to decrease the subscriber of the competitor in same market and the sign of intensity rate of interactions becomes negative.

(2) Predator-prey system

Under predator-prey system in vertical market, Hayashi [2008] assumes the LV model to the NO-VNO market. VNO, the service 2 as the predator cannot survive without NO as the prey and the sign of ε shall be negative. Note the sign of intensity rate of interaction is positive, because the population of service 2 increases when service 2 connects to service 1.

2 Previous study in practice (1)

Ikematsu and Shoji [2017] also apply the LV equations to FTTH-DSL market from 2006 to 2015 (data sources are TCA) and then determines the FTTH-DSL market as the replacement market falls into the competition model, because the sing of each ε at both FTTH and DSL are positive, and the sign of each μ is negative as shown Table 2.

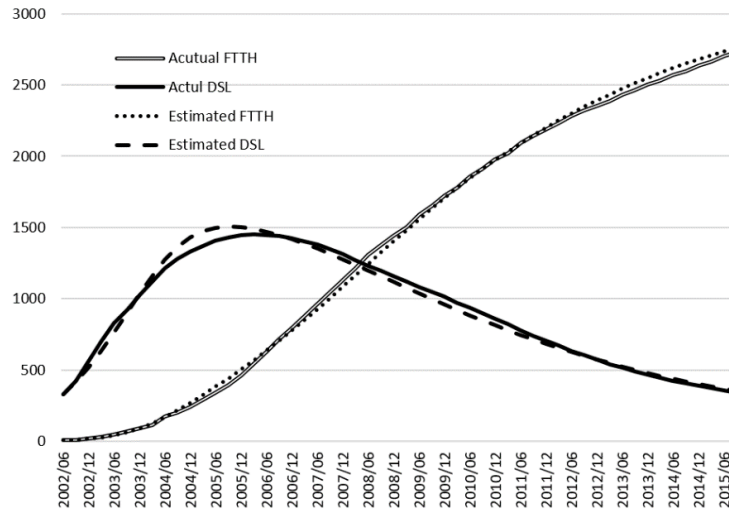
Table 2 THE Parameters of LV model in the FTTH-DSL Market (June 2002~December 2015)

Parameters	FTTH		DSL	
	ε_1	7.523×10^{-1}	ε_2	3.226×10^{-1}
Intensity rate of self-interaction	μ_{11}	-2.257×10^{-4}	μ_{22}	-1.835×10^{-4}
Intensity rate of interaction	μ_{12}	-3.396×10^{-4}	μ_{21}	-1.091×10^{-4}

The positive sign of each intrinsic growth rate ε shows broadband market of FTTH and

DSL has been growing. The negative sign of each of intensity rate of interaction shows that a player replaces the competitor's subscriber. In another word, the FTTH and DSL services have competed.

Chart 1 Trends of actual and estimated numbers of subscribers in FTTH-DSL market



The estimated data of FTTH and DSL calculated by the parameters on the Table 2 are fitted with the actual data of FTTH and DSL (Chart 1). Statistical analysis also supported the LV equations works in the FTTH-DSL market except the D/W value of DSL.

3. Previous study in theory (2) – Winner-Takes- All, diversity, and bi-stable model

Ikematsu and Shoji (2017) theoretically continue analyzing the LV model to understand this dynamic nature of this market based on the modified theory of mathematical ecosystem. However, we go back the original theory of mathematical ecosystem approach. First, we introduce new parameters K and γ . We substitute the equation (3) to equation (1) and we obtain equation (3).

$$\begin{aligned}
 K_1 &= \frac{\varepsilon_1}{-\mu_{11}} > 0, & K_2 &= \frac{\varepsilon_2}{-\mu_{22}} > 0, & \frac{d}{dt} x_1 &= \varepsilon_1 x_1 \left(1 - \frac{x_1 + \gamma_{12} x_2}{K_1}\right) \\
 \gamma_{12} &= \frac{-\mu_{12}}{-\mu_{11}} > 0, & \gamma_{21} &= \frac{-\mu_{21}}{-\mu_{22}} > 0 & \frac{d}{dt} x_2 &= \varepsilon_2 x_2 \left(1 - \frac{\gamma_{21} x_1 + x_2}{K_2}\right)
 \end{aligned} \tag{2} \tag{3}$$

The equilibrium points which stop changing the numbers of subscribers in market is given by (3) = 0. Then we obtain

$$\frac{x_1}{K_1} + \frac{x_2}{K_1/\gamma_{12}} = 1, \frac{x_1}{K_2/\gamma_{21}} + \frac{x_2}{K_2} = 1 \quad (4)$$

K_1 is the matured market size of service 1, assuming only service 1 survives at $x_2=0$. Also, K_2 is the matured market size of service 2. We call γ_{12} and γ_{21} intensity of strategy of service 1 and service 2, respectively.

The LV model for 2 services competition can be classified 4 models as shown Table 3. We can see these 4 models can be arranged into three type of model. We call these three models; the Winner Takes All (WTA) model and the diversity model, the bi-stable model. The WTA model shows only one service takes all market shares. The diversity model shows two services survive jointly or co-exist in market. The bi-stable model is a WTA model and two service have the unstable equitable points where two services co-exist. These patterns are categorized by the conditions shown at Table 3.

TABLE3 WTA, diversity and bi-stable model

	$K_1 > K_2/\gamma_{21}$	$K_1 < K_2/\gamma_{21}$
$K_2 < K_1/\gamma_{12}$	Diversity Model Equilibrium point $P_3 = \left[\frac{K_1 - \gamma_{12}K_2}{1 - \gamma_{12}\gamma_{21}}, \frac{K_2 - \gamma_{21}K_1}{1 - \gamma_{12}\gamma_{21}} \right]$ Service 1 and 2 are globally stable at P_3 .	WTA Model Equilibrium point $P_1 = (K_1, 0)$ Service 1 is globally stable at P_1 .
$K_2 > K_1/\gamma_{12}$	WTA Model Equilibrium point $P_2 = (0, K_2)$ Service 2 is globally stable at P_2	Bi-stable Service 1 and service 2 is globally stable at P_1 or P_2 , respectively by the initial population of service 1 and 2. P_3 is local unstable equilibrium point.

When the ICT policy need the standard or ICT strategy needs the monopoly, we must select the WTA model. When the ICT policy or strategy needs competitive market, we must select the WTA model.

4. Previous study in practice (2)

Ikematsu and Shoji (2017) apply this theory to the FTTH-DSL market (data sources are TCA), and the FTTH-DSL market between June 2012 and December 2015 falls into the WTA market and FTTH will survive.

Table 4: WTA model: FTTH-DSL market from June 2012 to December 2015

FTTH (x_1 :service 1)		DSL (x_2 :service 2)	
$K_1 = \varepsilon_1 / -\mu_{11}$	33,330	$K_2 = \varepsilon_1 / -\mu_{11}$	17,580
$\gamma_{12} = -\mu_{12} / -\mu_{11}$	1.505	$\gamma_{21} = -\mu_{21} / -\mu_{22}$	0.595
K_2 / γ_{21}	29,560	K_1 / γ_{12}	22,150
$K_1 > K_2 / \gamma_{21}$		$K_2 < K_1 / \gamma_{12}$	

For your information, the FTTH-DSL market between June 2002 and December 2007 falls into the diversity model (See Table 4).

Table 4: The diversity model: FTTH-DSL market from June 2012 to December 2015

FTTH (x_1 :service 1)		DSL (x_2 :service 2)	
$K_1 = \varepsilon_1 / -\mu_{11}$	119,510	$K_2 = \varepsilon_1 / -\mu_{11}$	14,320
K_2 / γ_{21}	394,240	K_1 / γ_{12}	18,000
$K_1 < K_2 / \gamma_{21}$		$K_2 < K_1 / \gamma_{12}$	

The FTTH-DSL market shifted the diversity model to the WTA model which only FTTH will survive. This conclusion means that the policy and strategy in FTTH-DSL market works well. I discussed this issue further at Information Processing Society of Japan soon. In this report, I would like to discuss the fixed-mobile market in ASEAN countries, the main theme of this report. We have provided basic information from the pre-studies to proceed this issue.

5. Fixed-Mobile market

Now, we apply the LV equations for the regression analysis to the fixed-mobile market in ASEAN countries and Japan from 2000 to 2014 and from 2000 to 2010. Data sources are ITU for ASEAN countries and TCA for Japan. Japanese fixed market does not include cable, IP-phone and ISDN. When we use the terminology from 2000 to 2014, we use the data from 2000 to 2015, but the rate of changes in these data for regression analysis from 2000 to 2014.

5.1 The signs of the parameters

We apply the LV equations for the regression analysis the fixed-mobile market in ASEAN countries and Japan from 2000 to 2014 and from 2000 to 2010. For the conclusion, the fixed-mobile market in Thai follows the horizontal market of LV equations for both the period both from 2000 to 2014 and from 2000 to 2010, because the

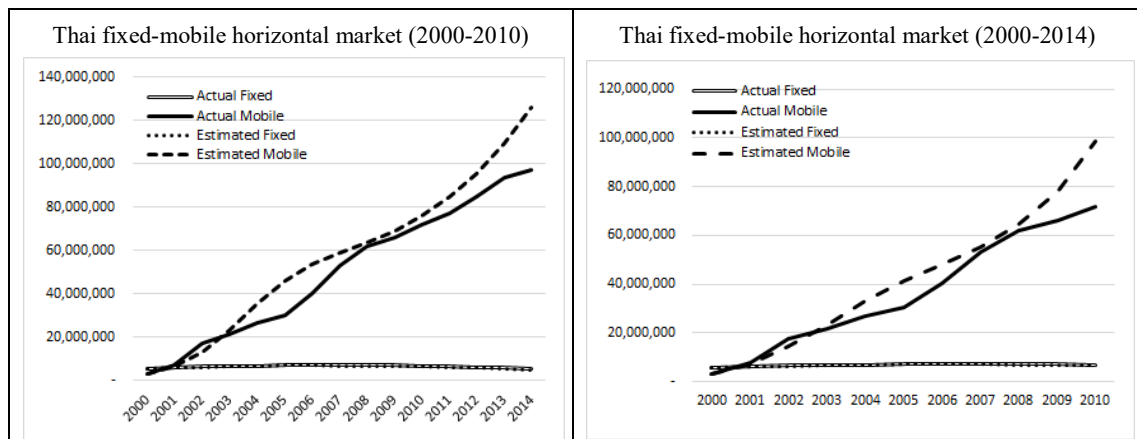
signs of all μ are negative and the signs of all ε are positive. Philippines follows the horizontal market for the period from 2000 to 2014. Brunei and Japan follows the horizontal market for the period from 2000 to 2010. See Table 5.

The other data for ASEAN countries were provided at the appendix in this report.

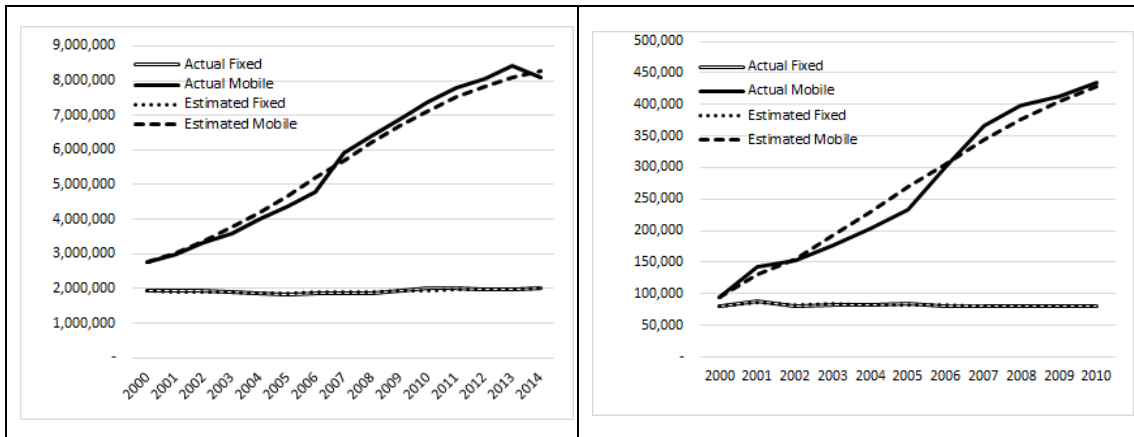
Table 5

Countries	2014				2010			
	ε_1		ε_2		ε_1		ε_2	
Thai	ε_1	1.4233389947E-1	ε_2	3.5263518152	ε_1	3.0062589381E-1	ε_2	5.8984982202E+00
	μ_{11}	▲1.1209913054E-8	μ_{21}	▲4.1232594320E-7	μ_{11}	▲3.7788865386E-8	μ_{21}	▲8.0947922987E-07
	μ_{12}	▲1.4071241312E-9	μ_{22}	▲9.7545992130E-9	μ_{12}	▲7.9243778783E-10	μ_{22}	▲9.4795000334E-10
Philippines	ε_1	3.5982944065E-1	ε_2	1.1252001910	ε_1	5.2056122597E-1	ε_2	1.1109338372
	μ_{11}	▲9.1657825795E-8	μ_{21}	▲1.8206545566E-7	μ_{11}	▲1.4750896251E-7	μ_{21}	▲1.7694253261E-7
	μ_{12}	▲6.0493462659E-10	μ_{22}	▲4.5362660758E-9	μ_{12}	4.3663575867E-10	μ_{22}	▲4.6580896065E-9
Brunei	ε_1	▲2.9161689503E-1	ε_2	4.1565559991E-1	ε_1	1.5036179365	ε_2	2.7377446476
	μ_{11}	4.0855426800E-6	μ_{21}	▲6.3472329664E-7	μ_{11}	▲1.7567096997E-5	μ_{21}	▲2.8120740861E-5
	μ_{12}	▲2.1305161505E-7	μ_{22}	▲7.8319933466E-7	μ_{12}	▲2.4889919683E-7	μ_{22}	▲1.0524966199E-6
Japan	ε_1	▲2.9161689503E-1	ε_2	4.1565559991E-1	ε_1	2.8712786425E-1	ε_2	4.7384813041E-1
	μ_{11}	4.0855426800E-6	μ_{21}	▲6.3472329664E-7	μ_{11}	▲1.0519753222E-9	μ_{21}	▲4.7489417355E-9
	μ_{12}	▲2.1305161505E-7	μ_{22}	▲7.8319933466E-7	μ_{12}	▲2.8879819317E-9	μ_{22}	▲1.9736299190E-9

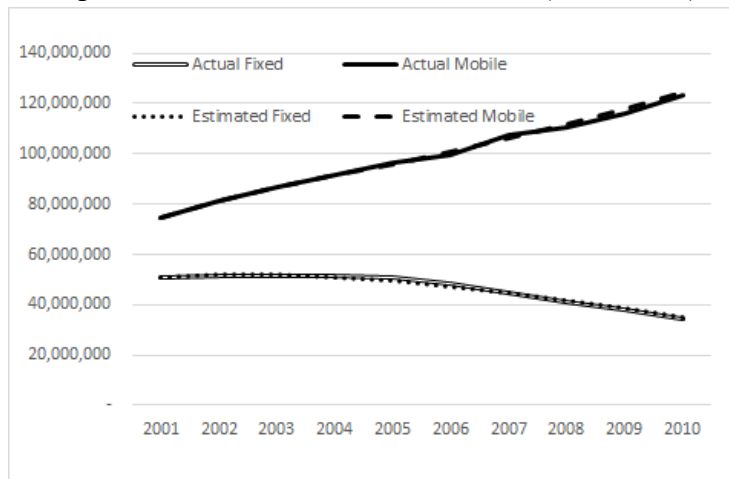
Chart 2



Philippines fixed-mobile horizontal market (2000-2014)	Brunei fixed-mobile horizontal market (2000-2010)
--	---



Japan fixed-mobile horizontal market (2001-2010)



5.2 The WTA model and bi-stable model.

Next, we determine the model type by Table 3 for Thai, Philippines, Brunei and Japan horizontal market as following Table 6.

Table 6 The model in horizontal market for fixed-mobile market

Countries	Period	Model	Winner if any
Thai	2000-2010 2000-2014	Bi-stable	Mobile
Philippines	2000-2014	Diversity	
Brunei	2000-2010	Diversity	
Japan	2000-2010	Bi-stable	Mobile

The supporting data are shown as the following tables.

Table 7 Bi-stable market in Thai fixed-mobile market (2000 to 2014)

$K_1=\varepsilon_1/\mu_{11}$	12,697,146	$K_2=\varepsilon_2/\mu_{22}$	361,506,581
$\gamma_{12}=\mu_{12}/\mu_{11}$	1.2552498172E-1	$\gamma_{21}=\mu_{21}/\mu_{22}$	4.2269901018E+1
K_2/γ_{21}	8,552,340	K_1/γ_{12}	101,152,341
$K_1 > K_2/\gamma_{21}$		$K_2 > K_1/\gamma_{12}$	

Table 8 Bi-stable market in Thai fixed-mobile market (2000 to 2014)

$K_1=\varepsilon_1/\mu_{11}$	7,955,409	$K_2=\varepsilon_2/\mu_{22}$	6,222,372,698
$\gamma_{12}=\mu_{12}/\mu_{11}$	2.0970139742E-2	$\gamma_{21}=\mu_{21}/\mu_{22}$	8.5392607945E+2
K_2/γ_{21}	7,286,781	K_1/γ_{12}	379,368,448
$K_1 > K_2/\gamma_{21}$		$K_2 > K_1/\gamma_{12}$	

Table 9 The diversity model in Philippines fixed-mobile market (2000 to 2014)

Fixed (x_1 :service 1)		Mobile (x_2 :service 2)	
$K_1=\varepsilon_1/\mu_{11}$	3,925,791	$K_2=\varepsilon_2/\mu_{22}$	248,045,457
$\gamma_{12}=\mu_{12}/\mu_{11}$	6.5999233709E-3	$\gamma_{21}=\mu_{21}/\mu_{22}$	4.0135532753E+1
K_2/γ_{21}	6,180,196	K_1/γ_{12}	594,823,680
$K_1 < K_2/\gamma_{21}$		$K_2 > K_1/\gamma_{12}$	

Table 10 The diversity model in Brunei fixed-mobile market (2000 to 2010)

$K_1=\varepsilon_1/\mu_{11}$	85,593	$K_2=\varepsilon_2/\mu_{22}$	2,601,191
$\gamma_{12}=\mu_{12}/\mu_{11}$	1.4168487649E-2	$\gamma_{21}=\mu_{21}/\mu_{22}$	2.6718129378E+1
K_2/γ_{21}	97,357	K_1/γ_{12}	6,041,072
$K_1 < K_2/\gamma_{21}$		$K_2 > K_1/\gamma_{12}$	

Table 11 The Bi-stable model in Japanese fixed-mobile market (2001 to 2010)

$K_1=\varepsilon_1/\mu_{11}$	272,941,635	$K_2=\varepsilon_2/\mu_{22}$	240,089,657
$\gamma_{12}=\mu_{12}/\mu_{11}$	2.7452943720	$\gamma_{21}=\mu_{21}/\mu_{22}$	2.4061966683
K_2/γ_{21}	99,779,731	K_1/γ_{12}	99,421,628
$K_1 > K_2/\gamma_{21}$		$K_2 > K_1/\gamma_{12}$	

6 Discussion

My purpose of writing this paper is simple. How to market Radio-over-Fiber (RoF) market in the world. I am not sure but I believe that RoF will be one of the next access network infrastructure. I would like to find the best countries to start this plan. Japan has

started the collaborative research and development works in RoF in ASEAN countries and so, I studied these countries. Either fixed line operator or mobile operator may start the RoF. So, I study the fixed-mobile market. In another word which operator has key role to develop IoT access network.

In the bi-stable market, if the mobile operator will win the market, the fixed line operator must start co-existence strategy. However, the equitable point is unstable and this model is the most difficult market to re-built the market. Line (4) are rotating and these lines must rotate 180 degrees to be diversity model. There is another option. The fixed line operator select NO strategy for mobile operator as a VNO. To verify this strategy, we need to verify the vertical market on Table 1. I am not sure but bi-stable model markets are not reasonable countries to start this replacement. More than 30 years ago the several ASEAN countries and the middle east countries started installing the ISDN networks before the advanced countries started installing ISDN. In another word, several ASEAN countries are the advance countries in telecommunication network. So, I believe that ASEAN countries will become first advance countries for IoT and RoF.

I am not able to contribute the policy making knowledge in this report, but I provided several discussion materials and data in this report.

7 Future study

7.1 LV equation for interaction among n kinds of network services

Hayashi (2005) requests that we need the the multi-layers and multi-services LV equations because in the introduction the fundamental ecosystem of ICT consists of the three layers, infrastructure, platform, and applications. Yes, when we apply LV equations to these services and layers, we need the multi-layers and multi-services LV equations. The LV model for n kinds of network services under multi-layer system is given by the following equation

$$(1) \frac{d}{dt} x_i = x_i (\varepsilon_i + \sum_j^n \mu_{ij} x_j) \quad (i = 1, 2, \dots, n)$$

where i is the type of service, x is the numbers (populations) of a service, t is time, ε is the intrinsic growth rate and μ is intensity rate of interaction. Again, we call the parameter μ the intensity rate of self-interaction at $i=j$ and the intensity rate of interaction at $i \neq j$. Please recall the LV equation is equivalent to replicator equation in evolutionally game theory. Under ecosystem where services survive, evolutionally game theory shows intensity rate of self-interaction is intensity of self-adaptation to

environment and intensity rate of interaction is intensity of strategy to competitors' in environment. We have this model and need the further studies.

7.2 Dynamics

The original concepts of LV equations assume the parameters is constant value. However, we have already known that the parameters of LV equation are also valuable. In this report, LV model does not have concept of price, but the parameters has the dynamics, It means that the behavior of parameters probably include the price dynamics.

However, the changes in parameters probably have more valuable secret. Martin [2008] explains that LV equation assume non-evolution world because their parameters are constant value. Also, Fusimi [2007] says that the survival-of-the-fittest works only in the world without evolution when he explains the similar equations of LV equations called Eigen equations. In another word, **the survival-of-the-fittest world does not have any innovation. The static world has no innovation.** When we find the dynamic parameters model to explain the actual world, we will be able to go next step. This is the core work of the evolutionary games for long years. We have many pre-studies in social science.

7.3 Life Cycle

The LV equations parameters has been changed and model of market is shifted and this shift probably provide us the new mechanism of life cycle of services. If we obtain some knowledge from this cycle, this knowledge support our ICT policy or strategy. We need further study of this issue. I will announce this approach of FTTH-DSL soon. We also need same approach to the fixed and mobile market. As we showed at Appendix in this paper, we have different type of model from LV equation. For example, the models in, Malaysia, Lao, Cambodia, Australia and New Zealand are the typical corporation model or so-called mutualism model in biology. It means both fixed and mobile service support to increase the other service subscribers. We have the other type of models. We need to understand the nature of all of models by theoretical point views.

8. Conclusion

This report introduced the evolutionary game theory to the typical replacement or substitution of FTTH-DSL market in Japan as the pre-studies and applied the fixed line mobile market in excising network ecosystem infrastructure in ASEAN countries and some markets fall into the diversity or bi-stable market.

This report explained the very basic idea of LV equations only. I hope many researchers studies these models. The evolutionary game theory will probably support to plan the

policy or corporate strategy for n kinds of network services under multi-layer system over the IoT network.

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Acknowledgement

This paper has benefited greatly from late Dr. Toshohiko HAYASHI's work. I would like to express the great appreciation to him. We lost a big hero of the ICT society.

APPENDIX

<i>Brune</i>	ϵ_1	▲2.9161689503E-01	ϵ_2	4.156559991E-01	ϵ_1	1.5036179365E+00	ϵ_2	2.7377446476E+00
	μ_{11}	4.0855426800E-06	μ_{21}	▲6.3472329664E-07	μ_{11}	▲1.7567096997E-05	μ_{21}	▲2.8120740861E-05
	μ_{12}	▲2.1305161505E-07	μ_{22}	▲7.8319933466E-07	μ_{12}	▲2.4889919683E-07	μ_{22}	▲1.0524966199E-06
<i>Cambodia</i>	ϵ_1	5.7699566740E-01	ϵ_2	5.8437394797E-01	ϵ_1	▲1.7272435228E-01	ϵ_2	5.5001798007E-01
	<i>Mutu</i>	▲4.6735129096E-06	μ_{21}	8.0106173015E-07	μ_{11}	▲1.4917721386E-05	μ_{21}	1.3077977324E-06
	μ_{12}	9.7966076707E-08	μ_{22}	▲4.4791386269E-08	μ_{12}	7.4277996979E-07	μ_{22}	▲4.1035544654E-08
<i>Malaysia</i>	ϵ_1	3.2592684248E-01	ϵ_2	▲1.5975483357E-01	ϵ_1	2.4550660908E-01	ϵ_2	1.4159191158E-01
	<i>Mutu</i>	▲7.2551992540E-08	μ_{21}	1.1254574180E-07	μ_{11}	▲5.5527475298E-08	μ_{21}	5.1700586330E-08
	μ_{12}	4.5487542718E-12	μ_{22}	▲.6643329327E-09	μ_{12}	1.9822833501E-10	μ_{22}	▲9.4597024391E-09

<i>Myanmar</i>	ϵ_1	3.7078084193E-01	ϵ_2	▲3.5943882211E-02	ϵ_1	3.9268452685E-01	ϵ_2	1.2109066679E+00
	μ_{11}	▲7.0724737079E-07	μ_{21}	1.9144097406E-06	μ_{11}	▲7.8531884404E-07	μ_{21}	▲1.8359029118E-06
	μ_{12}	▲6.3255786755E-10	μ_{22}	▲9.1047651390E-10	μ_{12}	4.5582051914E-08	μ_{22}	5.1931591582E-07
<i>Indonesia</i>	ϵ_1	2.0145302357E-01	ϵ_2	6.9797233760E-01	ϵ_1	7.6304868481E-01	ϵ_2	6.9899845304E-01
	μ_{11}	4.7655263242E-09	μ_{21}	▲5.9581521510E-09	μ_{11}	▲1.0429874402E-07	μ_{21}	▲4.8413789915E-09
	μ_{12}	▲1.5611741222E-09	μ_{22}	▲.4757737323E-09	μ_{12}	1.7544773776E-08	μ_{22}	▲1.8272706386E-09
<i>Lao</i> <i>Mutu</i>	ϵ_1	3.3909666955E-03	ϵ_2	1.0977533268E+00	ϵ_1	4.9126559651E-01	ϵ_2	1.4158677430E+00
	μ_{11}	▲1.7343473750E-06	μ_{21}	▲1.3786788485E-07	μ_{11}	▲5.0247247763E-06	μ_{21}	▲4.7680288345E-06
	μ_{12}	3.3078074214E-07	μ_{22}	▲2.3655249546E-07	μ_{12}	2.2546997949E-08	μ_{22}	▲1.8682269656E-07
<i>Philippines</i> <i>WTA</i>	ϵ_1	3.5982944065E-01	ϵ_2	1.1252001910E+00	ϵ_1	5.2056122597E-01	ϵ_2	1.1109338372E+00
	μ_{11}	▲9.1657825795E-08	μ_{21}	▲1.8206545566E-07	μ_{11}	▲1.4750896251E-07	μ_{21}	▲1.7694253261E-07
	μ_{12}	▲6.0493462659E-10	μ_{22}	▲4.5362660758E-09	μ_{12}	4.3663575867E-10	μ_{22}	▲4.6580896065E-09
<i>Singapore</i>	ϵ_1	2.3968261677E-01	ϵ_2	9.4774786933E-01	ϵ_1	1.7339481259E-02	ϵ_2	9.1337801786E-01
	μ_{11}	▲1.3947768014E-07	μ_{21}	▲4.1815306761E-07	μ_{11}	▲2.8637332846E-08	μ_{21}	▲4.1079221923E-07
	μ_{12}	5.5751058226E-09	μ_{22}	▲1.1446920491E-08	μ_{12}	8.5214329936E-09	μ_{22}	▲6.5936124409E-09
<i>Thai</i> <i>Bi-stable</i>	ϵ_1	1.4233389947E-01	ϵ_2	3.5263518152E+00	ϵ_1	3.0062589381E-01	ϵ_2	5.8984982202E+00
	μ_{11}	▲1.1209913054E-08	μ_{21}	▲4.1232594320E-07	μ_{11}	▲3.7788865386E-08	μ_{21}	▲8.0947922987E-07
	μ_{12}	▲1.4071241312E-09	μ_{22}	▲9.7545992130E-09	μ_{12}	▲7.9243778783E-10	μ_{22}	▲9.4795000334E-10
<i>Viet Nam</i>	ϵ_1	4.7493773858E-01	ϵ_2	6.9074748888E-01	ϵ_1	5.6437190161E-01	ϵ_2	4.7373714542E-01
	μ_{11}	▲2.1674424229E-08	μ_{21}	2.1442919950E-08	μ_{11}	▲3.9762191925E-08	μ_{21}	6.4368164767E-08
	μ_{12}	▲2.9840329489E-09	μ_{22}	▲7.1108464452E-09	μ_{12}	▲4.2546359312E-10	μ_{22}	▲1.2796933900E-08

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