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**Preprint**

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*Suggested Citation:* Vasilev, Aleksandar (2017) : Insurance-markets Equilibrium with Sequential Non-convex Market-Sector and Divisible Informal-Sector Labor Supply, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

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# Insurance-markets Equilibrium with Sequential Non-convex Market-Sector and Divisible Informal-Sector Labor Supply

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September 5, 2017

## Abstract

This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex market-sector employment and informal sector work. In contrast to Vasilev (2016a), the discrete-continuous labor supply decision in this paper is a sequential one, and instead of home production, we focus on informal activity. This still requires a single insurance market to operate - in particular, one for market-sector employment. In addition, given that the labor choice for market- and informal-sector hours is made in succession, the insurance market for market employment needs to close before the labor supply choice in the grey economy is made. This segmentation is reminiscent of the results obtained in Vasilev (2015) and also a direct consequence of the sequential nature of the discrete-continuous sectoral labor supply decision.

**Keywords:** Indivisible labour, Lotteries, Discrete-continuous mix, Informal economy, Insurance

*JEL Classification Codes:* E1, J22, J46

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# 1 Introduction and Motivation

The purpose of this paper is to explore the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions. The novelty relative to those studies is that the earlier setups (Vasilev 2016a,b) were dealing with private-public sector and full-time work vs. overtime, respectively, while here the focus is on the market vs unofficial work margin. The particular focus in this paper is on the lottery- and insurance-market equilibrium in an economy with non-convex market-sector employment and informal sector work. In contrast to Vasilev (2016a), the discrete-continuous labor supply decision in this paper is a sequential one, and instead of home production, we focus on informal activity. This still requires a single insurance market to operate - in particular, one for market-sector employment. In addition, given that the labor choice for market- and informal-sector hours is made in succession, the insurance market for market employment needs to close before the labor supply choice in the grey economy is made. This segmentation is reminiscent of the results obtained in Vasilev (2015) and also a direct consequence of the sequential nature of the discrete-continuous sectoral labor supply decision.

## 2 Model Setup

The model follows to a great extent Vasilev (2017a). The economy is static, there is no physical capital, and agents face a sequential convex decision, followed by a continuous labor supply decision in a two-sector economy. Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by  $i$  and distributed uniformly on the  $[0, 1]$  interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index  $i$  to save on notation.

### 2.1 Description of the model

Each household maximizes the following utility function:

$$U(c, l) = \ln c + \alpha \ln l, \tag{1}$$

where  $c$  denotes consumption of market output,  $l$  is the leisure enjoyed by each individual household, and  $\alpha > 0$  is the relative weight attached to utility of leisure. Each household is endowed with a time endowment of unity, which can be split between hours worked in either the official sector,  $h_m$ , hours worked in the informal economy ("black market"),  $h_b$ , and leisure  $l$ , so that

$$h_m + h_b + l = 1 \quad (2)$$

The households make a sequential labor supply choice: The first is whether to work full-time in the market sector, or not at all. In other words,  $h_m \in \{0; \bar{h}\}$ . Conditional on not working in the market sector, a household may decide to go and work in the grey sector, where it can supply any number of hours, i.e.  $h_b \in [0, 1]$ . That is, the first labor choice is indivisible, while the second is divisible. Also, it will be assumed that  $h_b = 0$  whenever  $h_m = \bar{h}$ , or a household employed full-time in the market sector would choose not to supply any hours in the grey economy. This assumption is put in place to guarantee that each worker can only participate in one of the production sectors. Next, the hourly wage rate in the official ("market") sector and the implicit rate in the informal economy ("black market") sectors are denoted by  $w^m$  and  $w^b$ , respectively. Finally, the households own the firm in the market economy, and are entitled an equal share of the profit ( $\pi$ ).

The problem faced by a household that decides to work full-time in the market sector is then to set  $h_m = \bar{h}$  and enjoy

$$U^m = \ln(w^m \bar{h} + \pi) + \alpha \ln(1 - \bar{h}), \quad (3)$$

while a household that decides not to work in the market sector chooses  $h_b \in [0, 1]$  to maximize its utility function

$$\max_{h_b} U^b = \ln(w^b h_b + \pi) + \alpha \ln(1 - h_b) \quad (4)$$

The optimal labor choice in the grey economy is then characterized by the following first-order condition:

$$\frac{w^b}{w^b h_b + \pi} = \frac{\alpha}{1 - h_b}, \quad (5)$$

or

$$h_b = \frac{w^b - \alpha\pi}{(1 + \alpha)w^b} \quad (6)$$

That is, optimal choice of hours worked in the informal economy is a function of both the wage and profit rate in the official sector, which the household takes as given.

## 2.2 Stand-in firm: market sector

There is a representative firm in the model economy, which operates in the market sector. It produces a homogeneous final product using a production function that requires labor  $H_m$  as the only input. For simplicity, output price will be normalized to unity. The production function  $f(H_m)$  features decreasing returns to scale and satisfies Inada conditions:  $f'(H_m) > 0$ ,  $f''(H_m) < 0$ ,  $f'(0) = \infty$ ,  $f'(\bar{h}) = 0$ . The representative firm acts competitively by taking the wage rate  $w^m$  as given, and chooses  $H_m$  to maximize profit:

$$\pi = f(H_m) - w^m H_m \quad \text{s.t.} \quad 0 \leq H_m \leq \bar{h}. \quad (7)$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

## 2.3 Stand-in firm: unofficial sector

Each worker in the unofficial sector has access to an individual concave production function ("backyard technology") that uses only labor,  $g(h_b)$ , where  $g'(h_b) > 0$ ,  $g''(h_b) < 0$ ,  $g'(0) = \infty$ ,  $g'(1) = 0$ . Each firm in the unofficial sector will then hire labor  $h_b$  in every period to maximize static profit

$$\max_{h_b} g(h_b) - w^b h_b \quad \text{s.t.} \quad 0 \leq h_b \leq 1. \quad (8)$$

With free entry, there are zero profits in the sector, hence the implicit wage  $w^b$  in the unofficial sector equals the average product of labor, i.e.

$$w^b = \frac{g(h^b)}{h_b}. \quad (9)$$

### 3 Insurance Markets

Alternatively, we could regard the labor selection arrangement as follows: the workers are participating in a lottery with the proportions representing the probability of being selected for work. Conditional on the discrete labor choice in the first stage, a household would receive the same income (in expected terms). Lastly, we can introduce insurance markets, and allow households to buy insurance, which would allow them to equalize the actual income received independent of the employment status in the market sector. Given the difference in market and informal sector wages, insurance markets needed in order to provide actuarially fair insurance.

There is one representative insurance company for market sector employment. At the beginning of the period, the households decide if and how much insurance to buy against the probability of being chosen for market-sector work. Then, the company closes, and the households not selected for official-sector work, choose how many hours to supply in the informal sector. Insurance costs  $q^m$  per unit, and provides one unit of income if the household is not employed in the market sector. We can think of insurance as bonds that pay out only in case the household is not chosen for work in the official economy. Thus, household will also choose the quantity of insurance to purchase  $b^m$ . Without the segmented and sequential nature of the insurance market described above, insurance will not be actuarially fair, one of the groups will face better odds versus price, the company will not be able to break even, and/or at least one type of household will not be able to buy full insurance, which would completely smooth consumption across employment states, given the non-convexity constraint of labor supply.

#### 3.1 Market-sector Insurance Company

The insurance company for market-sector employment maximizes sprofit. The company services all households. It receives revenue if a household is working in the market sector and makes payment if it is not. More specifically, the proportion of people working in the market sector contribute towards the unemployment benefits pool, which are then distributed in the form of benefits to the unemployed. The amount of insurance sold is a solution to the

following problem:

Taking  $q^m(i)$  as given,  $b^m(i)$  solves

$$\max_{b^m(i)} \lambda^m(i)q^m(i)b^m(i) - [1 - \lambda^m(i)]b^m(i) \quad (10)$$

With free entry profits are zero, hence

$$\max_{b^m(i)} \lambda^m(i)q^m(i)b^m(i) - [1 - \lambda^m(i)]b^m(i) = 0. \quad (11)$$

This condition implicitly clears the insurance market for each household.

## 4 Decentralized Competitive Equilibrium (DCE) with lotteries

### 4.1 Definition of the DCE with lotteries

A competitive Equilibrium **with sequential Lotteries** for this economy is a list

$$(c^m(i), c^b(i), \lambda^m(i), h_b(i), p, w^m, w^b, \pi) \quad (12)$$

such that the following conditions are fulfilled.

1. **Consumers maximization condition.** Taking prices  $w^m, w^b, \pi$  as given, for each  $i$ , the sequence

$$\sigma = (c^m(i), c^b(i), \lambda^m(i), h_b(i)) \quad (13)$$

solves the maximization problem

$$\max_{\sigma \in \Sigma} \{ \lambda^m(i) \ln c^m + (1 - \lambda^m(i)) \ln c^b + \lambda^m(i) \alpha \ln(1 - \bar{h}) + (1 - \lambda^m(i)) \alpha \ln(1 - h_b) \} \quad (14)$$

s.t

$$\lambda^m(i)c^m + (1 - \lambda^m(i))c^b = \lambda^m(i)w^m\bar{h} + (1 - \lambda^m(i))w^bh_b + \pi \quad (15)$$

$$c_m \geq 0, c^b \geq 0, 0 < \lambda^m(i) < 1 \quad (16)$$

where  $\Sigma$  is the constraint defined by the above relations.

2. **Market-sector firm maximization condition.** Taking prices  $w^m, w^b, \pi$  as given,

$$\max_{\bar{H}^m \geq 0} F(\bar{H}^m) - w^m \bar{H}^m. \quad (17)$$

3. **Informal-sector firm maximization condition.** Taking prices  $w^m, w^b, \pi$  as given,

$$\max_{h^b \geq 0} g(h^b) - w^b h^b \quad s.t. \quad g(h^b) - w^b h^b = 0. \quad (18)$$

4. **Market-clearing condition:** We have

$$\int_i \lambda^m(i) \bar{h} di = \bar{H}^m. \quad (19)$$

$$\int_i (1 - \lambda^m(i)) h_b(i) di = H_b. \quad (20)$$

$$\int_i \{\lambda^m(i) c^m + (1 - \lambda^m(i)) c^b\} di = F(\bar{H}^m) + \int_i (1 - \lambda^m(i)) g(h_b(i)) di. \quad (21)$$

The last equation describes clearing in the goods market. Note that in line with national income accounting, output from informal activity also counts towards total gross domestic product.

## 4.2 Characterizing the DCE

The household's problem is as follows:

$$\begin{aligned} \mathcal{L} = \max_{\sigma \in \Sigma} & \lambda^m(i) \ln c^m + (1 - \lambda^m(i)) \ln c^b + \lambda^m(i) \alpha \ln(1 - \bar{h}) + (1 - \lambda^m(i)) \alpha \ln(1 - h_b) \\ & - \mu [\lambda^m(i) c^m + (1 - \lambda^m(i)) c^b - \lambda^m(i) w^m \bar{h} - (1 - \lambda^m(i)) w^b h_b - \pi], \end{aligned} \quad (22)$$

where  $\mu$  is the Lagrangian multiplier in front of the households' budget constraint. The first-order optimality conditions are as follows:

$$c^m : \frac{1}{c^m} = \mu, \quad (23)$$

$$c^b : \frac{1}{c^b} = \mu. \quad (24)$$



It follows that

$$c = c^m = c^b = 1/\mu. \quad (25)$$

We simplify the Lagrangian by suppressing all consumption superscripts and  $i$  notation in the derivations to follow

$$h_b : \frac{\alpha}{1 - h_b} = \mu w^b \quad (26)$$

$$\lambda^m : \alpha[\ln(1 - \bar{h}) - \ln(1 - h_b)] = \mu[w^b h_b - w^m \bar{h}] \quad (27)$$

The former condition states that the marginal rate of substitution between labor in the informal sector and consumption equals the implicit sector in the black market. The latter says that the difference from enjoying leisure (given that consumption utility has been equalized across states equals the difference in the labor income, multiplied by the shadow price of consumption (i.e. expressed in consumption utility terms). This implicitly characterizes optimal market sector participation rate  $\lambda^m$ . Note that it is optimal from the benevolent planner/government point of view to choose randomly  $\lambda^m$  and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness is achieved. A household is exposed to risk: first, it can be chosen to work with some probability; second, conditional on not being chosen to work in the market sector, it can choose to work any number of hours in the informal sector. Given the risk in the economic environment, it would be optimal to have insurance. The government sells employment lotteries, and individuals will buy insurance to cover any risk exposure. With insurance, the employer pays market wage to individuals only if they work. Now we extend the commodity space to include insurance markets explicitly.

## 5 Decentralized Competitive Equilibrium (DCE) with insurance markets

### 5.1 Definition of the DCE with insurance markets

A competitive Equilibrium **with sequential Lotteries and insurance markets** for this economy is a list

$$(c^m(i), c^b(i), \lambda^m(i), b^m(i), q^m(i), h_b(i), p, w^m, w^b, \pi) \quad (28)$$

such that the following conditions are fulfilled.

1. **Consumers maximization condition.** Taking prices  $p, w^m, w^b, \pi$  as given, for each  $i$ , the sequence

$$\sigma = (c^m(i), c^b(i), \lambda^m(i), h_b(i)) \quad (29)$$

solves the maximization problem

$$\max_{\sigma \in \Sigma} \lambda^m(i) \{ \ln c^m + \alpha \ln(1 - \bar{h}) + (1 - \lambda^m(i)) [\ln c^b + \alpha \ln(1 - h_b)] \} \quad (30)$$

s.t

$$pc^m + b^m q^m = w^m \bar{h} + \pi \quad (31)$$

$$pc^b = b^m + w^b h_b + \pi \quad (32)$$

$$c^m \geq 0, c^b \geq 0, 0 < \lambda^m(i) < 1 \quad (33)$$

or

$$pc^m + pq^m c^b = w^m \bar{h} + q^m w^b h_b + (1 + \pi)q^m \quad (34)$$

where  $\Sigma$  is the constraint defined by the above relations.

2. **Market-sector firm maximization condition.** Taking prices  $w^m, w^b, \pi$  as given,

$$\max_{\bar{H}^m \geq 0} F(\bar{H}^m) - w^m \bar{H}^m. \quad (35)$$

3. **Informal-sector firm maximization condition.** Taking prices  $w^m, w^b, \pi$  as given,

$$\max_{h^b \geq 0} g(h^b) - w^b h^b \quad s.t \quad g(h^b) - w^b h^b = 0. \quad (36)$$

4. **Insurance-company** Taking  $g^m(i)$  as given,  $b^m(i)$  solves

$$\max_{b^m} \lambda^m(i) q^m(i) b^m - (1 - \lambda^m(i)) b^m \quad (37)$$

With free entry profits are zero, hence

$$\lambda^m(i)q^m(i)b^m - (1 - \lambda^m(i))b^m = 0. \quad (38)$$

This implicitly clears the insurance market for each individual in the market sector.

**5. Market-clearing condition:** We have

$$\int_i \lambda^m(i)\bar{h}di = \bar{H}^m. \quad (39)$$

$$\int_i (1 - \lambda^m(i))h_b(i)di = H_b. \quad (40)$$

$$\int_i \{\lambda^m(i)c^m + (1 - \lambda^m(i))c^b\}di = F(\bar{H}^m) + \int_i (1 - \lambda^m(i))g(h_b(i))di. \quad (41)$$

## 5.2 Characterization of the DCE with insurance markets

$$\mathcal{L} = \max_{\sigma \in \Sigma} \lambda^m(i) \{ \ln c^m + \alpha \ln(1 - \bar{h}) + (1 - \lambda^m(i)) [\ln c^b + \alpha \ln(1 - h_b)] \} \quad (42)$$

$$- \mu [p c^m + p q^m c^b - w^m \bar{h} - q^m w^b h^b - (1 + \pi) q^m] \quad (43)$$

Normalize  $p = 1$ .

$$c^m : \frac{\lambda^m(i)}{c^m} = p\mu \quad (44)$$

$$c^b : \frac{(1 - \lambda^m(i))}{c^b} = p q^m \mu \quad (45)$$

Optimal  $\lambda^m$  ( $\lambda^m(i) = \lambda^m, \forall i$ ) is implicitly characterized by the zero-profit condition from the market-sector insurance company:

$$\frac{\lambda^m}{1 - \lambda^m} = \frac{1}{q^m} \quad (46)$$

The price of insurance depends on probability of the event you are insuring against. We cannot force  $q^m(i) = q^m$  although ex post that would indeed be the case. For the insurance firms, profits are linear in  $q^m$ . This implies that profits cannot be positive or negative in equilibrium. Zero profits in the private sector insurance market then mean  $q^m = \frac{1 - \lambda^m}{\lambda^m}$ . A common interpretation for the market-sector insurance companies is that this price of the insurance is the odds ratio, or the ratio of probabilities of the two events. Combining then with the FOCs for state-contingent consumption, we obtain that  $c^m = c^b, \forall i$ . That is, households buy full insurance to equalize consumption in all states. Since labor income is

stochastic, i.e., it is uncertain whether the individual will be employed in the market sector, we need an institution that could offer insurance. More specifically, lotteries can then be introduced to achieve market completeness.

## 6 Conclusions

This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex market-sector employment and informal sector work. In contrast to Vasilev (2016a), the discrete-continuous labor supply decision is a sequential one, and instead of home (non-market) production, here we focus on informal activity. This still requires one insurance market to operate - for market-sector employment. In addition, given that the labor choice for market- and informal-sector hours is made in succession, the insurance market for official-sector employment needs to close before the labor supply choice in the informal sector is made. This segmentation is reminiscent of Vasilev (2015) and also a direct consequence of the sequential nature of the discrete-continuous sectoral labor supply decision.

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