

Guo, Zi-Yi

Article — Manuscript Version (Preprint)

Martingale Regressions for a Continuous Time Model of Exchange Rates

Global Conference on Business and Finance Proceedings

Suggested Citation: Guo, Zi-Yi (2017) : Martingale Regressions for a Continuous Time Model of Exchange Rates, Global Conference on Business and Finance Proceedings, ISSN 1941-9589, Institute for Business and Finance Research, Hilo, HI, USA, Vol. 12, Iss. 2, pp. 40-45

This Version is available at:

<https://hdl.handle.net/10419/168350>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Martingale Regressions for a Continuous Time Model of Exchange Rates

Zi-Yi Guo^{*}

This version: September 2017

Abstract: One of the daunting problems in international finance is the weak explanatory power of existing theories of the nominal exchange rates, the so-called “foreign exchange rate determination puzzle”. We propose a continuous-time model to study the impact of order flow on foreign exchange rates. The model is estimated by a newly developed econometric tool based on a time-change sampling from calendar to volatility time. The estimation results indicate that the effect of order flow on exchange rates is more than doubled compared with the traditional econometric estimations. The normality tests of the distribution of regression residuals confirm our application of the new econometric tool.

Keywords: High frequency data; Time change; Brownian motion.

JEL classification:

^{*} Zi-Yi Guo, Corporate Model Risk Management Group, Wells Fargo Bank, N.A., 301 S College St, Charlotte, NC 28202, zachguo0824@gmail.com. *The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.*

Introduction

In international finance, the weak power of existing theories in explaining nominal exchange rate fluctuations is one of the daunting problems. Lyons (2001) named this weak explanatory power as the "exchange rate determination puzzle." Following the pioneer work by Evans and Lyons (2002), recent empirical evidence from market microstructure approach has shown that most short-run exchange rate volatility can be explained by a new variable - order flow (see Vitale (2007) for a survey). The order flow is defined as the net of buyer-initiated and seller-initiated orders for the asset trading in the market, a measure of net buying pressure. The reason why order flow drives the nominal exchange rates is because order flow conveys heterogeneous information, either of the future macroeconomic fundamentals or of unobserved liquidity demands, hedging demands, or speculative demands and so on. In the traditional macro approach, with homogeneous information the mapping from the information to equilibrium exchange rates is immediate, so order flow does not convey any information about the market clearing prices. While within the market microstructure framework individuals possess private information, the private information is conveyed by order flow during the trading process, which in turn affects market-clearing prices.

Most empirical estimations about the nominal exchange rates with order flow in the literature have relied on high-frequency data, such as daily, hourly and five-minute (e.g. Evans and Lyons, 2002, 2008, Evans, 2002, Rime, Sarno and Sojli, 2010, Guo, 2017a, Guo, 2017b, Guo, 2017c, and Guo and Luo, 2017), and traditional econometric tools (such as OLS). However, it is well known that the direct use of high frequency data in traditional econometric estimations has several drawbacks. First, the conditional mean processes of interest in many economic and financial models are dominated by the error processes, since the later one has a higher magnitude; second, the distributions of errors in many models are changing overtime and far from the normal distributions, such as the peakedness and fat-tails phenomena in financial data; finally, the variables in conditional mean part might correlate with the errors if the orthogonality condition is approximated by the Euler scheme, which creates severe identification issues. In this paper, we empirically study the impact of order flow on the nominal exchange rates by using high-frequency data in a continuous time framework. We take an advantage of a newly developed econometric methodology by Park (2010). The methodology relies on random

sampling using a time change from calendar to volatility time instead of a fixed-interval sampling. The sampling chronometer runs at a rate inversely proportional to the volatility. After using this chronometer, the error processes become a standard Brownian motion and samples could be treated as being *i.i.d.* normal¹. With this new methodology, our estimation reflects that the impact of order flow on exchange rate increases significantly compared with the traditional econometric estimation in Evans and Lyons (2002). The normality tests of regression residuals confirm the validity of the new methodology and after time change regression residuals are normally distributed.

A model of exchange rate determination in continuous time

We build our model in a continuous-time setting. We assume the individual demand b_{Ft}^i for foreign currency is linearly determined by three different components: the public information I_t^p , the private information I_t^i and the exchange rates (asset prices) s_t . Bacchetta and van Wincoop (2006) demonstrate this assumption in a discrete model (see equation (12) in their paper). The assumption can be rewritten as

$$db_{Ft}^i = \alpha_1 dI_t^p + \alpha_2 ds_t + \alpha_3 dI_t^i. \quad (1)$$

The demand b_{Ft}^i consists of two components: market orders (order flow) and limit orders. In the paper, we treat the foreign exchange market as an explicit auction market². In the market, limit orders are on the passive side, and provide liquidity to the market. Market orders are defined as the initiator orders and be confronted with the passive outstanding limit orders. We assume limit orders only depend on public information, and market orders exclusively depend on private information. Since market orders only depend on private information, we have

$$dx_t^i = \alpha_3 dI_t^i - E(\alpha_3 dI_t^i | I_t^p), \quad (2)$$

where x_t^i denotes individual i 's market orders until time t . We do not include ds_t in the public information set, because market orders are placed before the exchange rate is revealed. Further,

¹ This idea has also been used by Yu and Phillips (2001), Jeong, Kim and Park (2015), Chang, Choi, Kim and Park (2016), and so on.

² Bacchetta and van Wincoop (2006) also make this assumption. Lyons (2001) has a detailed discussion of the nature of the foreign exchange market.

we assume $E(\alpha_3 dI_t^i | dI_t^p) = \alpha_4 dI_t^p$ and define order flow as $x_t = \int x_t^i di$. With the market clearing condition in equilibrium $\int b_{Ft}^i di = 0$, the foreign exchange rates are determined as follows

$$ds_t = \eta_1 dI_t^p + \eta_2 dx_t, \quad (3)$$

where $\eta_1 = -(\alpha_1 + \alpha_4)/\alpha_2$ and $\eta_2 = -1/\alpha_2$. This equation indicates that the foreign exchange rate is jointly determined by a public information component and a private information component. The later one is summarized by order flow x_t . We use the interest rate differential dr_t to approximate the public information component I_t^p . The equation eventually can be written as

$$ds_t = \eta_1 dr_t + \eta_2 dx_t + \sigma_t dw_t, \quad (4)$$

where w_t is a standard Brownian motion, and the volatility term $\sigma_t dw_t$ is assumed to summarize all the measurement errors. Evans and Lyons (2002) build a portfolio shifts model and prove the change of the foreign exchange rates is a linear function of the public-information increment and the unobservable portfolio shift, which is essentially a discrete version of equation.

Estimation methodology

We adopt a newly developed estimator by Park (2010) to estimate equation. Instead of a fixed-interval sampling which might have a time-varying and higher-magnitude volatility term, we use a random sampling by using a time change from calendar to volatility time. The key idea of Park (2010) is that all continuous martingales become Brownian motion if their sample paths are read using a clock running at the speed set inversely to the rate of increase in their quadratic variations. Since we assume all the volatilities are summarized by the term $\sigma_t dw_t$ and $\int_0^t \eta_1 dr_t + \eta_2 dx_t$ is of bounded variation, the stop time then can be calculated as

$$T_t = \inf_{s>0} \{ [\int_0^s \sigma_t dw_t]_s > t \} = \inf_s \{ [s]_s > t \}. \quad (5)$$

After time change, equation (4) can be written as

$$ds_{T_t} = \eta_1 dr_{T_t} + \eta_2 dx_{T_t} + dV_t, \quad (6)$$

where V_t is a Brownian motion.

Assuming $\Delta > 0$, equation (6) can be rewritten as

$$z_i = \Delta^{-1/2} [s_{T_{i\Delta}} - s_{T_{(i-1)\Delta}} - \eta_1(r_{T_{i\Delta}} - r_{T_{(i-1)\Delta}}) - \eta_2(x_{T_{i\Delta}} - x_{T_{(i-1)\Delta}})],$$

for $i = 1, \dots, n$. $\{z_i\}_{i=1, \dots, n}$ are distributed as *i.i.d.* standard normal. $T_{i\Delta}$ is defined as

$$T_{i\Delta} = \arg \min_{t_k > T_{(i-1)\Delta}} |\sum_{j=T_{(i-1)\Delta}+1}^k (s_{t_j} - s_{t_{j-1}})^2 - \Delta|.$$

Denote $\theta = (\eta_1, \eta_2)$, let (z_i) be the observed values of $z_i(\theta)$ arranged in the ascending order, i.e., $\{z_{(1)} < \dots < z_{(N)}\}$, and suppose $w_i = \Phi(z_{(i)})$, where Φ is the standard normal distribution function.

The 1-dimensional ($d=1$) Martingale Estimator (MGE) of θ can be introduced as

$$\theta_N = \operatorname{argmin}_{\theta \in N} \left\{ \frac{1}{N} \sum_{i=1}^N \left(\frac{z_{(i)}}{2N} - \omega_i \right)^2 + \frac{1}{12N^2} \right\}. \quad (7)$$

Park (2010) proves its asymptotic consistency and provides other MGEs with high dimensions. For simplicity, we only consider the 1-dimensional MGE.

Empirical results

The dataset of order flow is the same as in Evans (2002). The original dataset contains time-stamped, tick-by-tick observations of actual transactions on the Reuters D2000-1 system for the largest spot market (DM/\$) over a four-month period, May 1 - August 31, 1996. According to Reuters, over 90 percent of the world's direct interdealer transactions took place through the system. Although trading can be made on the system 24 hours a day, 7 days a week, the dataset excludes weekends (too few observations in the weekends) and a feed interruption caused by a power failure, and has 79 full trading days in the sample with 255,497 trades. That interdealer order flow is positive (negative) is defined as a dealer initiating a bilateral conversation purchases (sells) foreign exchange at the ask (bid) quote. The dataset does not have the information of the size of individual transactions and we use the number of transactions as a proxy variable (see Evans (2002) for a detailed discussion). The order flow (in thousands) is accumulated in every five-minute interval. We use the mean of the last purchase price and the last sale price in that time interval as the exchange rate. The dataset covers 13,434 observations. We can only collect the daily data of overnight interest rates for the dollar and the Deutsche

mark from Datastream. We assume the interest rates do not change during each day and use the daily interest rates to approximate the instantaneous interest rates within that day.

The theory of the estimator does not have any requirement for Δ . However, in practice we choose the optimal Δ based on two considerations. First, Δ should not be too small, and otherwise we do not have enough samples to effectively estimate the quadratic variations and the estimation of time change might have serious bias. On the other hand, if we choose Δ in a large value, after time change sampling we could not have enough observations for the martingale estimation. Based on these two considerations, we choose $\Delta=0.4188$, which gives us $N=80$ observations after time change sampling. We change the value of Δ as a robust check.

Table 1: Main Results of Estimation

	N=80		N=40	
	Fixed-time	Time-change	Fixed-time	Time-change
η_1	0.224	0.258	0.127	0.080
η_2	1.9498**	5.470**	1.704**	5.325**

** indicates statistical significance at 5% level.

As we can see in Table 1, when $\Delta=0.4188$ and $N=80$, with fixed-interval sampling, the coefficient of 1.949 in the estimation equation implies that 1,000 more dollar purchase than sales increases the deutsche mark price of a dollar by 1.949 percent, which is consistent with the results obtained in Evans and Lyons (2002). However, with time-change random sampling, the coefficient increases to 5.470., which not only supports the view that order flow conveys information and correlates with foreign exchange rates, but also improves the prediction by a large amount. Given an average trade size in our sample of \$3.9 million, our MGE indicates \$1 billion of net dollar purchases increases the deutsche mark price by 1.403 percent ($=5.470/3.9$) instead of by 0.500 percent ($=1.949/3.9$) with the traditional econometric estimation. Our estimation shows very similar results with different values of Δ and N .

The reason why it is better to use time-change sampling than fixed-interval sampling is because the nature of high-frequency data. Figure 1 presents the distribution of the estimation errors. The solid line is the true distribution of our estimation errors, and the dotted line is the normal distribution which has the same mean and variance as the true distribution. Panel A

denotes the estimated errors sampled at fixed-time intervals, and Panel B denotes the estimated errors sampled at time change. The figure shows that if we sample at fixed-time intervals, the estimation errors are far away from being normal, while if we sample at time change, the estimated errors are very close to be normal. Table 2 shows the p-value of the normality tests for the case of $N=80$, which supports our interpretation. With time-change sampling, we cannot reject the null hypothesis at 1%, 5%, or 10% level.

Figure 1: Regression residuals with fixed-time sampling and time-change sampling

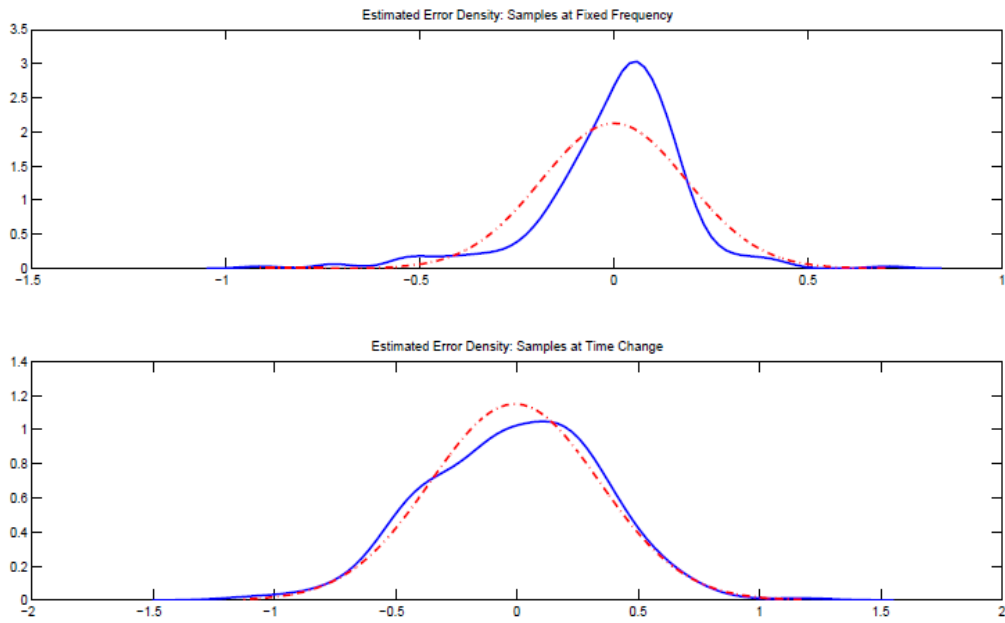


Table 2: Normality Test for the Estimated Residuals

	Kolmogorov-Smirnov	Shapiro-Wilk	Cramer-von Mises	Anderson-Darling
Fixed-time	0.009	0.000	0.006	0.003
Time-change	0.434	0.125	0.354	0.398

Conclusion

This paper makes use of a new econometric methodology to estimate the correlation of exchange rates and order flow, a key variable to explain the short-run exchange rate fluctuations. We collect our samples with a time change instead of the traditional fixed-interval approach. Our empirical results support the argument that order flow has a strong impact on foreign exchange rates, and improve the existing estimation results by traditional econometric approaches. The normality tests of the distribution of regression residuals confirm our application of the new econometric methodology.

References

- Bacchetta, P. and E. van Wincoop, 2006. Can information heterogeneity explain the exchange rate determination puzzle? *American Economic Review* 96, 552--576.
- Chang, Y., Y. Choi, H. Kim and J. Park, 2016. Evaluating factor pricing models using high frequency panels. *Quantitative Economics* 7, 889-933.
- Evans, M., 2002. FX trading and exchange rate dynamics. *Journal of Finance* 57, 2405--2447.
- Evans, M. and R. Lyons, 2002. Order flow and exchange rate dynamics. *Journal of Political Economy* 110, 170--180.
- Evans, M. and R. Lyons, 2008. How is macro news transmitted to exchange rates 88, 26-50.
- Jeong, D., H. Kim and J. Park, 2015. Does ambiguity matter? estimating asset pricing models with a multiple-priors recursive utility. *Journal of Financial Economics* 115, 361-382.
- Guo, Z., 2017a. Empirical performance of GARCH models with heavy-tailed innovations. Working paper.
- Guo, Z., 2017b. Models with short-term variations and long-term dynamics in risk management of commodity derivatives. Working paper.
- Guo, Z., 2017c. Comparison of error-correction models and first-difference models in CCAR deposits modeling. *Global Journal of Management and Business Research*, 17, 13-32.
- Guo, Z. and Y. Luo, 2017. The effect of sales and evidence from micro data. *Research in World Economy*, 8, 37-44.
- Lyons, R., 2001. *The microstructure approach to exchange rates*. Cambridge MA: MIT Press.

Park, J., 2010. Martingale regressions on conditional mean models in continuous time. mimeo.

Rime, D., L. Sarno and E. Sojli, 2010. Exchange rate forecasting, order flow and macroeconomic information. *Journal of International Economics* 80, 72-88.

Vitale, P., 2007. A guided tour of the market microstructure approach to exchange rate determination. *Journal of Economic Surveys* 21, 903--934.

Yu, J. and P. Phillips, 2001. A Gaussian approach for continuous time models of the short-term interest rates. *Econometrics Journal* 2, 210-224.