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# Bank Capital Regulation in a Model of Modern Banking Crises \*

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## Abstract

We study the macroeconomic effects of retail bank capital regulation in an economy with a retail and a shadow banking sector. The financial instability in our paper takes the form of systemic bank runs on the shadow banking sector. Such runs are results of self-fulfilling beliefs and coordination failures of the retail banks. Retail bank capital regulation reduces the frequency of systemic bank runs by mitigating the drops in capital price during fire sales. However, the aggregate capital stock decreases as a result of capital misallocation. In our calibrated model, retail bank capital regulation is effective in reducing systemic risk: increasing the bank capital requirement from 8 to 10 percent, the frequency of bank runs decreases from 3 to 1.2 runs per 100 years. Meanwhile, the steady state capital stock decreases by approximately 1 percent. The cost of bank capital requirement outweighs its benefit of fewer bank runs.

**Keywords:** Bank capital regulation, shadow banking, systemic bank runs.

**JEL Classification:** E440; G240; G280.

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# 1 Introduction

The 07-09 global financial crisis revealed the fragility of the financial sector as well as the weakness of the existing financial regulatory system. Indeed, stringent bank regulations have been introduced globally subsequent to the financial crisis: In 2010, Basel III was released by the Basel Committee on Banking Supervision to strengthen the regulation, supervision and risk management of the banking sector of its member countries. In the same year, the Dodd-Frank Wall Street Reform and Consumer Protection Act was signed into federal law in the United States. In 2011, the European Commission adopted a legislative package called Capital Requirements Directive (CRD) IV, which reflects the Basel II and Basel III rules on capital measurement and capital standards.

Nonetheless, there is little consensus among policymakers, bankers and researchers regarding the optimal level of bank capital regulation. On the one hand, advocates of regulation argue that higher capital requirements increase financial stability. On the other hand, opponents of regulation weigh in that higher capital requirements reduce financial intermediation. This leads to our research question: what is the optimal level of bank capital requirement?

In this paper, we study the macroeconomic effects of bank capital regulation in a general equilibrium framework with two banking sectors, retail and shadow banking. Taking shadow banking into consideration is crucial: for one thing, it has grown tremendously over the last decades into an essential part of the modern financial system<sup>1</sup>; for another, it was the collapse of the shadow banking sector that led to the financial turmoil which eventually turned into a global financial crisis. We build our model on the canonical framework of banking and financial crises developed by [Gertler, Kiyotaki, and Prestipino \(2016\)](#). We add capital accumulation and bank capital requirement on the retail banking sector in order to analyze the general equilibrium effects of bank capital regulation. The [Gertler, Kiyotaki, and Prestipino \(2016\)](#) framework features financial crises in the form of systemic bank runs on the shadow banking sector, which enables us to link the capital structure of banks to the outbreak of financial crises and to analyze the effect of bank capital regulation on financial stability. We calibrate our model using data from the Euro area. With the calibrated model, we investigate the effects of different capital requirements on financial stability and welfare.

Our model captures the following trade-off of bank capital requirements. On the positive side, a higher capital requirement on retail banks has a positive spillover effect on the capital ratio of shadow banks thanks to the structure of the wholesale funding market linking the two banking sectors. Meanwhile, the size of the shadow banking sector shrinks as retail bank regulation tightens. As a result, the drops in capital price during fire-sales are mitigated and systemic runs on the shadow banking sector becomes less frequent. Therefore the financial stability of the economy is enhanced. On the negative side, a higher capital requirement pushes up the cost of financing for

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<sup>1</sup>According to the Global Shadow Banking Monitoring Report 2015 by the Financial Stability Board, in 2014, shadow banking accounts for 12% of the total financial system, and the shadow banking to GDP ratio is around 60%.

banks, which is then passed on to the production sector, resulting in lower investment, a lower capital stock, and eventually lower output of the economy. In our calibrated model, we find that increasing the bank capital requirement from 8 to 10 percent leads to a decrease in the bank run frequency from 3 to 1.2 runs per 100 years. However, the steady state capital stock decreases by approximately 1 percent under this capital requirement policy. In terms of welfare, households and shadow bankers lose with a higher bank capital requirement, whereas retail bankers benefit from it. The optimal capital requirement level depends on the welfare weights assigned to different agents in the economy.

There are two forms of financial frictions in our model, which are key for the positive effect of retail bank regulation in reducing financial crises. First, there is a missing market between households and the shadow banking sector. In other words, shadow banks cannot borrow directly from households. The motivation for this assumption lies in the observation that shadow banks rely heavily on short-term credit from other financial institutions and not individual investors. Second, as in [Gersbach, Rochet, and Scheffel \(2015\)](#), shadow banks cannot pledge the entire amount of repayments from capital investment to retail banks<sup>2</sup> - their creditors. The non-pledgeability of shadow banks' investment revenue implies an endogenous leverage constraint for the shadow banks, where shadow banks' leverage ratio is linked to their profitability. As the capital requirement tightens, the retail banks' financing cost increases, which is passed on to the shadow banking sector, pushing down its profit margin and leverage ratio, and the size of the shadow banking sector shrinks.

Another important assumption of our model is the ranking of agents' investment skills: households are the least skilled investors and shadow banks are the most skilled investors. The investment skills of agents are captured by a capital holding cost, which corresponds to screening and monitoring expenses in capital investment. With a higher bank capital requirement, capital reallocates from shadow banks, the most efficient investors, to households and the retail banks, the less efficient investors. As a result, the total investment efficiency in the economy declines.

Bank runs happen in equilibrium and are unanticipated<sup>3</sup>. We focus on the case that retail banks run on shadow banks and not retail bank runs<sup>4</sup>. A bank run equilibrium exists when the liquidation value of shadow banks' assets is too low to cover their liabilities to retail banks, even though shadow banks are solvent if there is no run. But a run only actually happens when there is a coordination failure among retail banks; therefore bank runs are results of self-fulfilling beliefs of retail banks. When a bank run happens, the entire shadow banking sector is wiped out.

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<sup>2</sup>The non-pledgeability of shadow banks' revenue can occur in several forms of financial frictions, such as moral hazard problem (e.g. [Holmstrom and Tirole, 1997](#)) and asset diversion (e.g. [Gertler and Karadi, 2011](#)) etc.

<sup>3</sup>One interpretation for this assumption is that bank runs are infrequent events and people tend to forget that these events could happen again. Another explanation lies in the "this time is different" thinking.

<sup>4</sup>During the 07-09 financial crisis, many shadow banks experienced a run from their creditors and experienced insolvency as a consequence, the most well-known cases being the collapse of Bear Stearns Companies and the failure of Lehman Brothers. The traditional banking sector, on the other hand, was largely shielded from the financial catastrophe by government interventions.

Households and retail banks hold all the capital goods of the economy and make the investments. This is exactly the reason for the low fire-sale price of capital: during a crisis, capital becomes unproductive, as investments are made by the less efficient investors in the economy.

In this paper we define shadow banks as financial institutions that (i) are outside the regulatory framework of banks, (ii) borrow from other financial institutions using money market instruments, and (iii) are more efficient than traditional banks in making investments. Our notion of shadow banks corresponds to the external shadow banking sector in contrast to the off-balance-sheet shadow banking activities carried out by traditional banks. We consider the internal shadow banking as part of the traditional banking as bank capital regulations (such as Basel III) are, or at least supposed to be, implemented on a fully consolidated basis. In contrast to internal shadow banking, external shadow banking is a result of gains from specialization and vertical integration rather than a result of regulatory arbitrage as is the internal shadow banking ([Adrian and Ashcraft, 2012](#)). Examples of shadow banks by our definition include finance companies, hedge funds, investment banks, private equity firms, stand-alone broker-dealers, independent wealth management firms, and nonbank affiliated SIVs etc.

## Literature Review

This paper is closely related with two strands of literature. The first literature studies the role of financial frictions as a driving force of financial crisis. The early wave of this literature is rooted in the financial accelerator and credit cycle framework developed by [Bernanke and Gertler \(1989\)](#) and [Kiyotaki and Moore \(1997\)](#). In these early works, financial frictions are embedded in the financial structure of firms rather than financial intermediaries. After the 07-09 financial crisis, there is an emerging literature that links financial frictions in the financial intermediation to the outbreak of the worst global financial crisis since the Great Depression<sup>5</sup>. In [Gertler, Kiyotaki, and Prestipino \(2016\)](#), the authors develop a canonical macroeconomic framework of financial crisis in the form of bank runs, which extends the current literature by featuring a role of the fast growing shadow banking sector, which played an important part in the onset of the 07-09 financial crisis. In this paper, we build on their framework and modify it by adding capital accumulation and bank capital regulation in the retail banking sector to analyze the welfare and financial stabilization effect of bank capital regulation.

The second related literature studies optimal bank capital regulation in various frameworks which consider different trade-offs of the regulation policy. The positive effect of bank capital regulation captured by this literature is less risk-taking by banks (see [Nguyen, 2014](#) and [Begenau, 2015](#)) and limiting moral hazard problems of banks ([Van den Heuvel, 2008](#)). On the downside of bank capital regulation, [Van den Heuvel \(2008\)](#) argues that a tighter capital regulation reduces banks' ability to create liquidity. Similarly, [Nguyen \(2014\)](#) emphasizes that capital regulations

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<sup>5</sup>E.g. [Brunnermeier and Sannikov, 2014](#); [Gertler and Kiyotaki, 2015](#); [Boissay, Collard, and Smets, 2016](#); [Gertler, Kiyotaki, and Prestipino, 2016](#).

lead to less bank lending, which in turn causes lower growth of the economy. [Begenau \(2015\)](#) proposes the opposite effect in bank lending as in [Van den Heuvel \(2008\)](#) and [Nguyen \(2014\)](#). She argues that the safe and liquid bank deposit is a desirable asset for households, and as banks face a higher capital requirement, the supply of bank debt decreases, resulting in a lower financing cost for the bank; therefore, bank lending increases as a result of bank capital regulation rather than decreases.

This paper departs from the existing literature on optimal bank capital regulation in two ways: First, by modeling endogenous banking crises directly, we consider the impact of bank capital regulation on the stability of the financial system, rather than on the risk-taking behavior of financial institutions. Second, we explicitly model shadow banking as a part of the financial sector and study the implications of bank capital regulation in this setup, which has not been done in these studies.

Recently, there is a growing literature that studies the role of shadow banking system in evaluating the bank regulation policy, such as [Plantin \(2014\)](#) and [Huang \(2015\)](#). The way they model shadow banking corresponds to the off-balance sheet shadow banking activities conducted by traditional banks. In contrast, we consider shadow banking to be an independent banking sector that conducts financial intermediation outside the regulatory framework of banks. Our paper is closely related with [Begenau and Landvoigt \(2016\)](#), in which they also model an unregulated external shadow banking sector and capital accumulates endogenously. The key difference to their framework is how resources flow in our economy. In our model, households have direct access to capital markets but no access to shadow banking finance, and there is a wholesale funding market that links the retail and the shadow banking sector. In their model, households can lend and invest directly at shadow banks but have no access to the capital market, and there is no interbank market between the two banking sectors. These differences lead to our opposite implications of retail bank capital regulation on the the shadow banking sector.

## 2 Model

The core framework is the general equilibrium model with endogenous financial crisis in the form of bank runs developed by [Gertler, Kiyotaki, and Prestipino \(2016\)](#). A key feature of this framework is that the financial intermediary sector is subdivided into a retail banking sector and a wholesale banking sector. In their framework, aggregate capital supply is fixed and banks are unregulated. To study the welfare implications of bank capital regulation, we add capital accumulation and bank capital requirement on retail banks to this framework.

We consider a closed economy populated with five types of agents: households, retail bankers, shadow bankers, final goods producers and capital producers. [Figure 1](#) shows an overview of the model.

Retail banks and shadow banks are balance sheet-constrained: Retail banks face a Basel-III

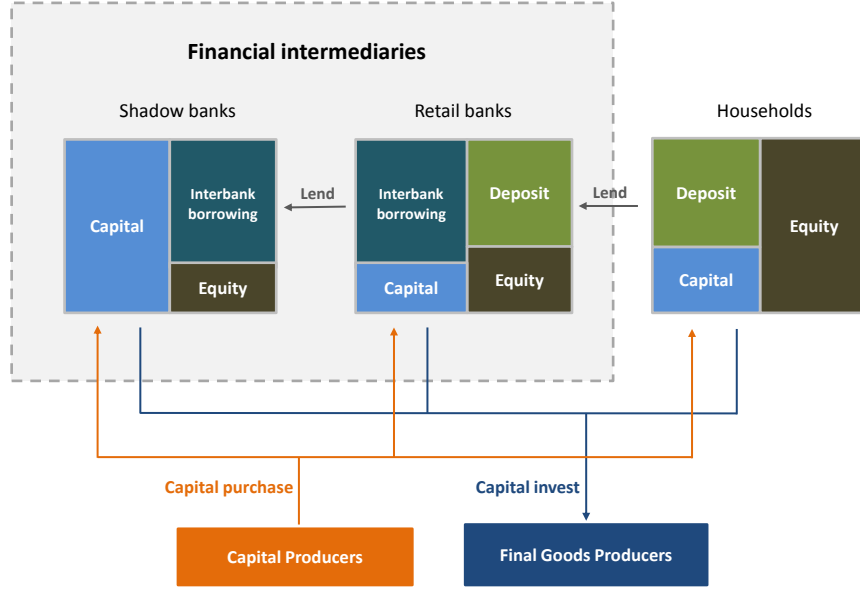


Figure 1: Model Overview

type capital requirement and shadow banks face a borrowing constraint for not able to pledge to repay a fraction of their assets.

## 2.1 Households

Households consume, invest in capital, save through deposits at retail banks and supply labor inelastically. The utility function of the representative household is given by

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} (\beta^H)^t \ln(C_t^H) \right], \quad (2.1)$$

where  $\beta^H$  is the discount factor of the household.  $C_t^H$  denotes household consumption in period  $t$ .

In each period, the household decides how much to consume and save. It supplies one unit of labor inelastically to final goods producers at wage  $W_t$ . The household can save in two assets: deposits at retail banks  $D_{t+1}^H$ , and capital  $K_{t+1}^H$ . Deposits yield a safe gross return  $R_{t+1}^D$  in the subsequent period. Capital can be purchased at price  $Q_t$  and yields a stochastic net return  $r_{t+1}^K$  in the subsequent period. Capital depreciates at rate  $\delta$ . The remaining fraction of the capital stock in the next period is valued at the next period price of capital,  $Q_{t+1}$ . The cash flow the household receives in  $t + 1$  for investing 1 unit of capital in  $t$  is therefore

$$R_{t+1}^K = r_{t+1}^K + (1 - \delta)Q_{t+1}. \quad (2.2)$$

The household obtains labor income from the final goods producers, capital income from its retail banks deposits and capital holdings. In addition, the household owns the capital producers, which yield a profit of  $\Pi_t^Q$ . The net worth of the household at the beginning of period  $t$  is given by

$$N_t^H = R_t^K K_t^H + R_t^D D_t^H + W_t + \Pi_t^Q. \quad (2.3)$$

The household pays a quadratic holding cost for holding capital. In reality, households delegate credit supply to banks, because banks can perform maturity transformation and have a cost advantage at monitoring non-financial firms. To capture this efficiency advantage of banks in a simplified way, we follow [Gertler, Kiyotaki, and Prestipino \(2016\)](#) and introduce a quadratic holding cost for new capital. This capital holding cost takes the form

$$\frac{\eta^H}{2} (K_{t+1}^H)^2.$$

This capital holding cost represents the cost of screening and monitoring investment projects.

The optimization problem of the household can be summarized as

$$\max_{\{C_t^H, K_{t+1}^H, D_{t+1}^H\}_{t=0}^\infty} \mathbb{E}_0 \left[ \sum_{t=0}^\infty (\beta^H)^t \ln(C_t^H) \right], \quad (2.4)$$

s.t.

$$\begin{aligned} C_t^H &= N_t^H - Q_t K_{t+1}^H - D_{t+1}^H - \frac{\eta^H}{2} (K_{t+1}^H)^2, \\ K_{t+1}^H, D_{t+1}^H &\geq 0, \end{aligned} \quad (2.5)$$

with  $N_t^H$  given by Equation 2.3.

The first-order conditions of the households' problem with respect to capital holding  $K_{t+1}^H$  and deposit  $D_{t+1}^H$  are given by:

$$FOC(K_{t+1}^H) : \quad \frac{1}{C_t^H} (Q_t + \eta^H K_{t+1}^H) = \beta^H \mathbb{E}_t \left( \frac{1}{C_{t+1}^H} R_{t+1}^K \right) \quad (2.6)$$

$$FOC(D_{t+1}^H) : \quad \frac{1}{C_t^H} = \beta^H \mathbb{E}_t \left( \frac{1}{C_{t+1}^H} R_{t+1}^D \right) \quad (2.7)$$

The interpretation of these first-order conditions is standard. In the first expression, the left-hand side and the right-hand side are the marginal cost and marginal benefit of capital holding, respectively. The marginal cost of capital holding has two components. One is the price the households have to pay for purchasing the capital goods, and the second is the capital holding cost due to households' low investment skills.



## 2.2 Retail Bankers

Retail bankers are agents who own and operate retail banks. Retail bankers consume, invest in capital, take deposits from households and lend money to shadow banks on the repo market. Their utility function is given by

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} (\beta^R)^t \ln(C_t^R) \right], \quad (2.8)$$

where  $\beta^R$  is the discount factor of the retail banker and  $C_t^R$  is its consumption in period  $t$ . We assume  $\beta_R < \beta_H$ , which means retail bankers are less patient than households. This assumption allows us to motivate retail bankers taking on debt in the form of household deposits in a simple and stylized way.

In period  $t$ , retail bankers consume  $C_t^R$ , invest  $K_{t+1}^R$  units of capital<sup>6</sup>, borrow  $D_{t+1}^R$  from households and lend  $B_{t+1}^R$  to shadow banks. Similar to households, retail bankers have to pay a quadratic capital holding cost for the next period capital stock. This capital holding cost takes the form

$$\frac{\eta^R}{2} (K_{t+1}^R)^2.$$

We assume  $\eta^R < \eta^H$ , which means retail bankers are better at screening and monitoring investment projects than households.<sup>7</sup>

In period  $t$ , retail bankers obtain a gross return on capital,  $R_t^K K_t^R$ , and a gross return from lending to shadow banks,  $R_t^B B_t^R$ . In addition, they receive an exogenous endowment  $W^R$  each period. They return  $R_t^D D_t^R$  to households for their deposits. In summary, the retail banker's net worth in period  $t$  is given by

$$N_t^R = R_t^K K_t^R + R_t^B B_t^R + W^R - R_t^D D_t^R. \quad (2.9)$$

The regulator imposes a Basel-III type capital requirement on retail banks, which stipulates that the bank's equity cannot be less than a fraction  $\Gamma^R$  of its risk-weighted assets.  $\omega^B$  and  $\omega^K$  are the weights on shadow bank lending and capital holding, respectively, for the calculation of risk-weighted assets. Accordingly, the capital requirement can be formulated as

$$E_{t+1}^R \geq \Gamma^R (\omega^B B_{t+1}^R + \omega^K K_{t+1}^R), \quad (2.10)$$

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<sup>6</sup>In practice, banks' lending to the non-financial sector is largely in the form of debt rather than equity. In the context of our model, banks' investment in the non-financial sector takes the form of equity investment rather than debt. This is a common assumption in the literature with financial intermediation for simplicity - otherwise another layer of liability of the non-financial sector has to be added. Under the current assumption, default on bank loans can be related to bankruptcy of the non-financial firms.

<sup>7</sup>It might be possible to actually show conditions under which a population in which people differ by ability select into each type of agent.

where equity of the retail bank is equal to the fraction of assets that is not funded by deposits:

$$E_{t+1}^R = Q_t K_{t+1}^R + B_{t+1}^R - D_{t+1}^R. \quad (2.11)$$

The problem of the retail banker can be summarized as

$$\max_{\{C_t^R, K_{t+1}^R, D_{t+1}^R, B_{t+1}^R\}_{t=0}^\infty} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} (\beta^R)^t \ln(C_t^R) \right], \quad (2.12)$$

s.t.

$$C_t^R = N_t^R + D_{t+1}^R - Q_t K_{t+1}^R - B_{t+1}^R - \frac{\eta^R}{2} (K_{t+1}^R)^2, \quad (2.13)$$

$$Q_t K_{t+1}^R + B_{t+1}^R - D_{t+1}^R \geq \Gamma^R (\omega^B B_{t+1}^R + \omega^K Q_t K_{t+1}^R), \quad (2.14)$$

$$K_{t+1}^R, D_{t+1}^R, B_{t+1}^R \geq 0,$$

with  $N_t^R$  given by Equation 2.9.

The capital requirement 2.14 can be rewritten as:

$$D_{t+1}^R \leq (1 - \Gamma^R \omega^K) Q_t K_{t+1}^R + (1 - \Gamma^R \omega^B) B_{t+1}^R. \quad (2.15)$$

Recall that there are two classes of assets on the retail bank's balance sheet: retail bank's capital holding,  $Q_t K_{t+1}^R$ , and its wholesale lending,  $B_{t+1}^R$ . The interpretation of 2.15 is that the retail bank can finance at most  $(1 - \Gamma^R \omega^K)$  share of its capital holding by households' deposit and at most  $(1 - \Gamma^R \omega^B)$  share of its wholesale lending by households' deposit. In other words, at least  $\Gamma^R \omega^K$  share of capital holding and  $\Gamma^R \omega^B$  share of wholesale lending has to be financed by retail banks' own equity.

Assuming that the retail bank capital requirement is always binding, the first-order conditions of the retail bankers' problem with respect to capital holding  $K_{t+1}^R$  and wholesale lending  $B_{t+1}^R$  are given by:

$$FOC(K_{t+1}^R): \quad \frac{1}{C_t^R} (Q_t \Gamma^R \omega^K + \eta^R K_{t+1}^R) = \beta^R \mathbb{E}_t \left\{ \frac{1}{C_{t+1}^R} [R_{t+1}^K - R_{t+1}^D (1 - \Gamma^R \omega^K) Q_t] \right\} \quad (2.16)$$

$$FOC(B_{t+1}^R): \quad \frac{1}{C_t^R} \Gamma^R \omega^B = \beta^R \mathbb{E}_t \left\{ \frac{1}{C_{t+1}^R} [R_{t+1}^B - R_{t+1}^D (1 - \Gamma^R \omega^B)] \right\} \quad (2.17)$$

Again, the left-hand side of the first-order conditions are the marginal cost of the retail banks' investments (capital investment and wholesale lending), and the right-hand side are the marginal benefit of the investments. One thing worth noticing is, given that the capital requirement is binding, the cost of purchasing one additional unit of capital goods is not  $Q_t$  but  $Q_t \Gamma^R \omega^K$ . This is because retail banks will finance the additional unit of capital with  $Q_t \Gamma^R \omega^K$  units of equity and

$Q_t(1 - \Gamma^R \omega^K)$  units of households' deposit. Subsequently, in the marginal benefit of capital holding on the right-hand side, the share  $Q_t(1 - \Gamma^R \omega^K)$  that is borrowed from the households has to be paid back to households with a gross return of  $R_{t+1}^D$ . Similarly,  $1 - \Gamma^R \omega^B$  units of the retail banks' wholesale lending is financed by households deposits and has to be paid back to the households with a gross return on  $R_{t+1}^D$  in the subsequent period.

## 2.3 Shadow Bankers

Shadow bankers are agents who own and operate shadow banks. Shadow bankers consume, borrow funds from retail banks on the repo market and invest in capital. Their utility function is given by

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} (\beta^S)^t \ln(C_t^S) \right], \quad (2.18)$$

where  $\beta^S$  is the discount factor of the shadow banker. In period  $t$ , shadow bankers consume  $C_t^S$ , invest  $K_{t+1}^S$  units of capital, and borrow  $B_{t+1}^S$  from retail banks. They receive  $R_t^K K_t^S$  from capital holding from last period and an exogenous endowment  $W^S$ , and they have to repay  $R_t^B B_t^S$  to retail banks. The shadow banker's net worth in period  $t$  is therefore given by

$$N_t^S = R_t^K K_t^S + W^S - R_t^B B_t^S. \quad (2.19)$$

In contrast to retail banks, shadow banks are unregulated and cannot be regulated in our model. While the former results from regulatory oversight, the latter might be a consequence of shadow banks operating from outside the regulators' reach. Instead, we assume that shadow banks face a moral hazard problem: they cannot pledge to repay more than a fraction  $\phi$  of their worst-scenario gross return on capital holdings. As a consequence, shadow banks face the following borrowing constraint <sup>8</sup>:

$$R_{t+1}^B B_{t+1}^S \leq (1 - \phi) \min_{Z_{t+1}} (R_{t+1}^K) K_{t+1}^S. \quad (2.20)$$

This constraint also guarantees that the lending from retail banks to shadow banks on the repo market is risk-free as long as bank runs do not occur. We can rewrite 2.20 as:

$$B_{t+1}^S \leq (1 - \phi) \frac{\min_{Z_{t+1}} (R_{t+1}^K)}{R_{t+1}^B} K_{t+1}^S. \quad (2.21)$$

The interpretation of 2.21 is that at most  $(1 - \phi) \frac{\min_{Z_{t+1}} (R_{t+1}^K)}{R_{t+1}^B}$  share of shadow bank's capital holding  $K_{t+1}^S$  can be financed by wholesale funding.

We assume shadow banks have a lower discount factor than retail banks:  $\beta^S < \beta^R$ . This is a

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<sup>8</sup>Another interpretation of this borrowing constraint: In the worst productivity outcome, after  $\phi$  share of gross return on capital investment is diverted, the rest of the investment revenue is still large enough to pay off shadow bank's liability to retail banks.

technical assumption that ensures that shadow bankers always prefer to use borrowed money than their own equity. In other words, with this assumption the borrowing constraint for shadow banks is always binding.

The problem of a shadow bank can be summarized as

$$\max_{\{C_t^S, B_{t+1}^S, K_{t+1}^S\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} (\beta^S)^t \ln(C_t^S) \right], \quad (2.22)$$

s.t.

$$C_t^S = N_t^S - Q_t K_{t+1}^S + B_{t+1}^S \quad (2.23)$$

$$R_{t+1}^B B_{t+1}^S \leq (1 - \phi) \min_{Z_{t+1}} (R_{t+1}^K) K_{t+1}^S, \quad (2.24)$$

$$K_{t+1}^S, B_{t+1}^S \geq 0,$$

with  $N_t^S$  as in equation 2.19.

Assuming the borrowing constraint is binding, the first-order condition of the shadow bankers' problem with respect to  $K_{t+1}^S$  is given by:

$$FOC(K_{t+1}^S) : \quad \frac{1}{C_t^S} \left[ Q_t - (1 - \phi) \frac{\min_{Z_{t+1}} (R_{t+1}^K)}{R_{t+1}^B} \right] = \beta^S \mathbb{E}_t \left\{ \frac{1}{C_{t+1}^S} \left[ R_{t+1}^K - (1 - \phi) \min_{Z_{t+1}} (R_{t+1}^K) \right] \right\} \quad (2.25)$$

The left-hand side of the first-order condition is the marginal cost of capital holding of shadow banks which includes two components: first is the price,  $Q_t$ , shadow banks pay for purchasing an additional unit of capital, the second component is the share of this one unit of capital that can be financed through wholesale funding from retail banks. Accordingly, the marginal benefit of capital holding, the right-hand side of the equation, also has two components: the return on the additional unit of capital holding subtract what shadow banks have to repay retail banks,  $(1 - \phi) \min_{Z_{t+1}} (R_{t+1}^K)$ .

## 2.4 Final Goods Producers

Final goods producers use the aggregate capital stock and labor from households to produce a homogenous final consumption good. The price of the consumption good is normalized to one. Aggregate uncertainty in the model takes the form of a productivity shock to the production function of the final goods, where  $Y_t$  units of final goods are produced using a Cobb-Douglas production technology that takes labor and capital as input:

$$Y_t = F(K_t, L_t; Z_t) = Z_t K_t^\alpha L_t^{1-\alpha}. \quad (2.26)$$

The final good producers maximize profits taking the aggregate wage  $W_t$  and the rental rate of capital  $r_t^K$  as given. The first order conditions of the final goods producers' problem yield two standard equations which determine the wage and rental rate of capital in equilibrium:

$$W_t = (1 - \alpha)Z_t K_t^\alpha, \quad (2.27)$$

$$r_t^K = \alpha Z_t K_t^{\alpha-1}. \quad (2.28)$$

We assume the total factor productivity of the final goods producer in period  $t$ ,  $Z_t$ , follows an exogenous AR(1) process in the following form:

$$\log(Z_t) = \rho_Z \log(Z_{t-1}) + \epsilon_t, \quad (2.29)$$

where  $|\rho_Z| < 1$  and  $\epsilon_t \sim N(0, \sigma_Z)$ .

## 2.5 Capital Producers

Capital producers adopt a technology which transforms one unit of consumption goods into one unit of capital goods:

$$Y_t^K = F^K(I_t) = I_t, \quad (2.30)$$

where  $Y_t^K$  is the amount of capital produced in period  $t$  and  $I_t$  is the amount of consumption goods used for the production. Adjustment to the production of capital goods is costly. Following the literature, the capital adjustment cost takes the form

$$\frac{\theta}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \quad (2.31)$$

where  $\delta$  is the depreciation rate of capital. The form of capital adjustment cost implies whenever the investment rate differs from depreciation rate, a positive proportional adjustment cost is incurred. This adjustment cost implies an endogenous relative price of capital goods.

As the capital adjustment cost depends on the capital stock from last period, the profit of capital producers are not necessarily zero outside steady state. We assume the capital producers are owned by households and any profits or losses are subsequently transferred to the households each period.

The capital producers' problem can be summarized as following:

$$\max_{\{I_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ Q_t Y_t^K - I_t - \frac{\theta}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right]. \quad (2.32)$$

## 2.6 No Run Equilibrium

In the absence of bank runs, we use a standard sequential equilibrium definition: Taking the bank capital regulation policy  $\Gamma^R, \omega^K, \omega^B$  as given, an equilibrium is a sequence of prices,  $\{Q_t, r_t^K, r_t^D, r_t^B, W_t\}_{t=0}^\infty$  and quantities  $\{K_t, L_t, C_t^H, C_t^R, C_t^S, K_t^H, K_t^R, K_t^S, D_t^H, D_t^R, B_t^S, B_t^R, I_t\}_{t=0}^\infty$  such that

- Households, retail banks, shadow banks, final goods producers, and capital producers solve their respective optimization problems as defined above.
- Markets clearing for

$$\text{i. Capital sale: } K_{t+1}^H + K_{t+1}^R + K_{t+1}^S = Y_t^K + (1 - \delta)(K_t^H + K_t^R + K_t^S), \quad (2.33)$$

$$\text{ii. Capital rental: } K_t = K_t^H + K_t^R + K_t^S, \quad (2.34)$$

$$\text{iii. Deposits: } D_t^R = D_t^H, \quad (2.35)$$

$$\text{iv. Money market funds: } B_t^S = B_t^R, \quad (2.36)$$

$$\text{v. Labor: } L_t = 1. \quad (2.37)$$

## 2.7 Bank Runs

As in the model of [Gertler et al. \(2016\)](#), retail bankers can run on shadow bankers. If such a run happens, the assets of the shadow banks are liquidated at the liquidation price  $Q_t^*$ . The retail bankers recover the assets of the shadow bankers instead of their lending. The recovery rate of retail banks is given by

$$(1 - \phi) \frac{[r_t^K + (1 - \delta)Q_t^*] K_t}{(1 + r_t^B)B_t} \quad (2.38)$$

Shadow bankers get excluded from financial markets and hence can neither invest nor borrow. Therefore, they have to consume their endowment and the divertible share of their assets. In the period subsequent to the bank run, shadow bankers stay excluded from financial markets with probability  $\pi$ . With probability  $1 - \pi$ , they reenter the financial markets.

Bank runs are self-fulfilling in our model. In that case, the market price of capital deteriorates in anticipation of a bank run. This weakens balance sheets of shadow banks so much that they cannot repay their liabilities. As a consequence, it is optimal for the retail banks to run on shadow banks. Such a self-fulfilling run can occur with probability  $\nu$  every period. However, it will only occur if the assets of the shadow banks, valued at the liquidation price of capital, are insufficient to cover the liabilities of shadow banks, that is, if

$$(1 - \phi) \frac{[r_t^K + (1 - \delta)Q_t^*] K_t}{(1 + r_t^B)B_t} < 1. \quad (2.39)$$

Finally, bank runs are unanticipated. That is, agents do not expect runs to happen. Even after

bank runs have actually happened, agents do not expect runs to happen again.

## 2.8 The Liquidation Price of Capital

The FOC from the problem of the capital producers 2.32 yields an expression for the price of capital:

$$Q_t = 1 + \theta \left( \frac{I_t}{K_t} - \delta \right) = 1 + \theta \frac{K_{t+1} - K_t}{K_t}. \quad (2.40)$$

If the economy invests, the price of capital exceeds 1, if it disinvests, the price of capital is below 1.

During a bank run, the desired capital holdings of the economy decrease, since shadow banks no longer can invest and their capital stock is liquidated. Denote the optimal capital stock choice during a bank run by  $K_{t+1}^*$  and the optimal dis-investment by  $I_t^*$ . The liquidation price of capital is then given by

$$Q_t^* = 1 + \theta \left( \frac{I_t^*}{K_t} - \delta \right) = 1 + \theta \frac{K_{t+1}^* - K_t}{K_t}. \quad (2.41)$$

## 3 Steady State Implications of Bank Capital Regulation

We explore the steady state<sup>9</sup> dynamics of the model in this section. We characterize a non-stochastic steady state equilibrium of the model (See Appendix A) absent of bank runs in which productivity  $Z_t$  is constant. There is no capital adjustment cost for the capital producers when depreciation rate equals investment rate, therefore the price of capital is equal to 1 in steady state,  $Q_{SS} = 1$ .

We provide a comparative statics analysis of the impact of strengthening bank capital regulation on retail banks, i.e. the consequence of an increase in the policy parameter  $\Gamma^R$  while keeping other parameters constant.

### Proposition 3.1. (*The Price Effects of A Higher Capital Requirement*)

*An increase in the capital requirement on retail banks leads to:*

1. *higher return on capital,  $R_{SS}^K$ ,*
2. *higher return on wholesale funding,  $R_{SS}^B$ , and*
3. *lower spread between return on capital and return on wholesale lending,  $R_{SS}^K - R_{SS}^B$ .*

Retail bankers are more impatient than households by assumption. Therefore, it is more costly for retail bankers to use own equity than to use borrowed money from the households. As capital

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<sup>9</sup>The steady state we consider here is under normal economic conditions as opposed to the bank run scenarios.

requirement tightens, retail banks have to increase the share of equity used for the same asset position. Their cost of funding increases accordingly. Retail banks pass this increased cost to their debtors by demanding a higher return on their capital investment on the final good producers,  $R_{SS}^K$ , and a higher return on wholesale lending to the shadow banks,  $R_{SS}^B$ . The increased borrowing cost of shadow banks further pushes up the required return on capital. As a result, the cost of investment for the production sector increases. Meanwhile, the spread between the return on capital and the cost of wholesale funding decreases as capital requirement decreases, which means shadow banks' profit margin shrinks as a result of a higher capital requirement.

**Proposition 3.2. (*The Reallocation Effects of A Higher Capital Requirement*)**

*With a higher capital requirement on retail banks:*

1. *households' capital holding increases,*
2. *retail banks' capital holding increases, when the risk weight on retail banks' capital holding is sufficiently small, and*
3. *shadow banks' capital holding decreases.*

As discussed in Proposition 3.1, a higher capital requirement drives up the return on capital investment, making capital holdings more attractive for investors. As a result, households' capital holding increases.

For the retail banks, however, there are two forces driving their capital investment decisions. On the one hand, retail banks become more constrained in their ability to borrow due to the higher capital requirement and have to use more equity to finance their investments. Because equity is more expensive than borrowing from households, this means capital investments become more costly for retail banks. On the other hand, retail banks want to hold more capital, as return on capital investment increases. The first is a partial equilibrium effect. The second is a general equilibrium effect, which affects retail banks' capital holding through the equilibrium prices. When the risk weight on capital holding,  $\omega^K$ , is sufficiently small, i.e. retail banks do not have to raise so much equity for capital investments, then the general equilibrium effect dominates the partial equilibrium effect, and retail banks increase their capital holding as the capital requirement increases.

Similarly, a higher capital requirement also has two opposite effects on shadow banks' capital holding. For one thing, it is more costly for shadow banks to finance capital holdings as retail banks charge a higher return on wholesale lending. For another, capital investment becomes more profitable as it pays a higher return. As we have shown in the last point in Proposition 3.1, the spread between return on capital and return on wholesale lending becomes smaller as capital



requirement increases, meaning the profit margin of shadow banks shrinks. Therefore, the first effect dominates the second effect, and shadow banks reduce their capital holding as a result of higher capital requirements.

In summary, a result of a higher capital requirement on retail banks is capital reallocating from shadow banks (the more efficient investors) to retail banks and households (the less efficient investors).

**Proposition 3.3. (*The Spillover Effect of A Higher Capital Requirement*)**

*A higher capital requirement on retail banks leads to:*

1. *less lending from retail banks to shadow banks,  $B_{SS}$ , and*
2. *a lower leverage ratio of shadow banks,  $B_{SS}/K_{SS}^S$ .*

In steady state all banks are borrowing constrained, i.e. they borrow up to their borrowing limit. The shadow banks' borrowing constraint 2.21 can be rewritten as:

$$B_{SS} = (1 - \phi) \frac{R_{SS}^K}{R_{SS}^B} K_{SS}^S. \quad (3.1)$$

The interpretation of this borrowing constraint is that the amount shadow banks are allowed to borrow,  $B_{SS}^S$ , is positively correlated with their profit margin,  $R_{SS}^K/R_{SS}^B$ , and the amount of capital holding of shadow banks,  $K_{SS}^S$ . That is, the more profitable the shadow banking sector is and the more assets shadow banks hold, the more they can borrow. As we discussed in Propositions 3.1 and 3.2, a higher capital requirement of the retail banks depresses the profit margin of the shadow banking sector and reduces the capital holding of shadow banks. As a result, retail banks lend less to shadow banks.

Rewrite the shadow banks' borrowing constraint in steady state 3.1 we get:

$$\frac{B_{SS}}{K_{SS}^S} = (1 - \phi) \frac{R_{SS}^K}{R_{SS}^B}. \quad (3.2)$$

The left-hand side of the expression is the leverage ratio of the shadow banks. This expression means that shadow banks' leverage ratio is endogenously determined by their profitability. The higher their profit margin is, the larger leverage ratio shadow banks can have. Therefore, when a higher capital requirement is imposed on retail banks, shadow banks' profit margin shrinks, and their leverage ratio decreases. As leverage ratio is negatively correlated with capital ratio, this means a higher capital requirement on retail banks not only leads to a higher capital ratio of retail banks but the shadow banking sector also increases its capital ratio. We call this the positive spillover effect of retail bank capital requirement.

**Proposition 3.4. (*The Aggregate Output Effect of A Higher Capital Requirement*)**

*With a higher capital requirement on retail banks:*

1. *aggregate capital stock decreases, and*
2. *aggregate output of the economy decreases.*

From the first-order condition of the final goods producers with respect to capital 2.28, the aggregate capital stock in steady state is given by:

$$K_{SS} = \left( \frac{r_{SS}^K}{\alpha Z} \right)^{\frac{1}{\alpha-1}}, \quad (3.3)$$

where  $r_{SS}^K = R_{SS}^K - 1 + \delta$ . Under a higher capital requirement, the supply of capital investment decreases, driving up the required return on capital,  $r_{SS}^K$ . The demand for capital by the final goods producers decreases, and aggregate capital stock decreases. Accordingly, aggregate output of the economy also decreases.

## 4 Numerical Results

In this section we discuss our calibration strategy and examine how costly bank runs are in terms of welfare. We calibrate the model using data from the Euro area at quarterly frequency from q1 1999 to q4 2015.

### 4.1 Calibration

We divide all parameters into three groups. Group 1 parameters are fairly standard and therefore are taken directly from the literature. Group 2 parameters are the policy instruments for bank capital regulation. Group 3 parameters are calibrated to match certain data counterparts.

Table 1: Parameters from the literature

Name	Role	Value	Source / Target
$\alpha$	Capital share	0.36	Christoffel, Coenen, and Warne, 2008
$\delta$	Depreciation rate	0.025	Christoffel, Coenen, and Warne, 2008
$\sigma$	Risk aversion	1	Log utility
$W^R, W^S$	Bank endowments	2.5	

Parameters in table 1 are set following the literature. The capital share of final good production and quarterly depreciation rate of capital are set to be 0.36 and 0.025, respectively, following

Christoffel, Coenen, and Warne (2008). We assume the utility function of the agents to take the form of log utility, implying a risk aversion level of 1 for all agents.

Table 2: Policy parameters

Name	Role	Value	Source / Target
$\Gamma^R$	Capital requirement	0.08	Basel III
$\omega^K$	Risk weight, capital	0.5	Basel III
$\omega^B$	Risk weight, interbank loans	1	Basel III

Parameters in table 2 are the three policy parameters: minimum capital requirement on retail banks,  $\Gamma^R$ ; risk weight for capital holding of retail banks,  $\omega^K$ ; risk weight for wholesale funding,  $\omega^B$ . We refer to the current capital requirement as in Basel III as our benchmark in the policy experiments that we conduct later on. At the time this paper is written, the minimum total capital plus conservation buffer is 8%. The risk weights of capital holding and wholesale funding are taken from Basel II <sup>10</sup>, as there are no new changes in risk weighing in Basel III from Basel II.

Table 3: Calibrated parameters

Name	Role	Value	Source / Target
$\beta^H$	HH discount factor	0.9921	$R^D = 0.8\%$ (Euribor)
$\beta^R$	RB discount factor	0.9763	$R^B - R^D = 0.13\%$ (Repo Spread)
$\beta^S$	SB discount factor	0.9369	$R^K = 1.35\%$ (Euro Stoxx 600)
$\eta^H$	HH capital holding cost	7.8721e-4	$K^H/K = 0.29$
$\eta^R$	RB capital holding cost	4.5356e-4	$K^R/K = 0.45$
$\phi$	SB Diversion share	0.1051	SB leverage of 90%
$\pi$	Persistence of bank runs	7/8	2 year crisis length (Laeven and Valencia, 2013)
$\nu$	Probability of sunspot	0.025	Crisis probability of 0.627 % per quarter
$\rho_Z$	TFP shock persistence	0.8	Output autocorrelation of 0.8773
$\sigma_Z$	TFP shock volatility	0.01	Output standard deviation of 0.0115
$\theta$	Capital adjustment cost	0.25	Investment standard deviation of 0.0277

Parameters in table 3 are calibrated using data of the Euro area. These parameters are further divided into two subgroups. In the first subgroup, parameters are calibrated jointly such that the steady state of the model matches its data counterpart. The variables in steady state we target at are return on deposit, spread between wholesale funding and deposit, return on capital, capital holding share of households, capital holding share of retail banks and leverage ratio of shadow

<sup>10</sup>International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version, in part 2, the first pillar - Minimum Capital Requirements. <http://www.bis.org/publ/bcbs128b.pdf>

banks. We use the average quarterly Euribor rates as a proxy for interest rate on deposit and Repo spread as a proxy for the spread between wholesale funding and deposit. Return on capital investment is captured by the average quarterly return on Euro Stoxx 600. Households hold 29 percent share of the capital stock of the economy and retail banks hold 45 percent. The rest 26 per cent is held by shadow banks. Average leverage ratio of shadow banks is 90 percent <sup>11</sup>.

In the second subgroup in Table 3, the parameters are calibrated such that the simulated model moments match their data counterparts. The persistence of financial crises is chosen such that the average length of a financial crisis is 2 years. The probability of sunspot shock is calibrated to match a quarterly probability of bank run of 0.627 percent. TFP shock persistence and volatility together with capital adjustment cost are calibrated jointly to match the autocorrelation and standard deviation of output and investment of the Euro area.

## 4.2 How Costly Are Bank Runs

Before we study how efficient bank capital requirement is for reducing bank runs in the economy, we want to know how costly are the bank runs in our calibrated model. In this subsection, we conduct two experiments to measure the cost of bank runs. First, we compare two impulse response functions (IRFs) of a negative two standard deviations productivity shock. In the first IRF, the productivity shock triggers a normal recession only. In the second IRF, the recession is accompanied by a run on the shadow banking sector. We compare the two IRFs to get an idea how severe are shadow bank runs in comparison to a normal recession. Second, we look at how much the agents are willing to give up in terms of permanent consumption to avoid bank runs. For this purpose, we first simulate the full model to get the permanent consumption equivalent for each type of agents living in this economy. Next, we shut down the bank run channel by setting the probability of coordination failure to 0 to have an economy where bank runs never happen. We simulate the modified model and calculate the permanent consumption equivalent for each type of agents living in this economy without bank runs. The difference between the two consumption equivalents is therefore the amount each agent is willing to pay to avoid bank runs.

As shown in figure 2, a recession triggered by a negative productivity shock is much more severe when accompanied by a shadow bank run. In a normal recession, capital stock drops by merely 1.8% from steady state, while in a recession with bank run, capital stock drops by more than 3%, and the recovery also takes longer. Similarly, total output drops more from steady state and recovers more slowly in a recession with bank run comparing to a normal recession. The fire sale of capital goods depresses the price of capital when the bank run happens and leads to a 0.5% drop in price of capital from steady state. In the case of bank run, capital holding shifts from shadow banks to households and retail banks, as shadow banks are liquidated and exit the economy during the period of bank run. Consequently, investment and borrowing of shadow banks ( $K^S$

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<sup>11</sup>Here the leverage ratio is defined as the ratio of debt over total assets

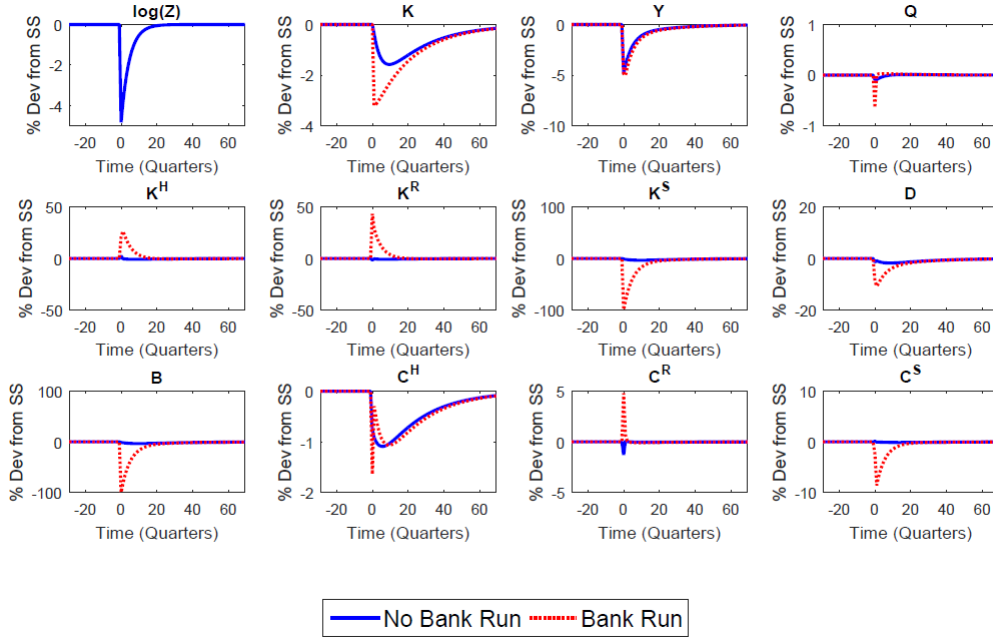


Figure 2: Normal recession versus financial crisis in the form of bank run

Table 4: Agents' willingness to pay to avoid bank runs

	Households	Retail Bankers	Shadow Bankers	Average
$\Delta$ Perm Cons Equiv (%)	0.74	-0.42	1.94	0.75

and  $B$  respectively) decrease 100% to 0. In terms of consumption, households and shadow bankers experience a larger consumption loss in a bank run recession compare to a normal recession. On the contrary, retail bankers experience some kind of "consumption boom" in the period of bank run. The interpretation for this observation is that, when shadow banks get liquidated in a bank run, retail banks can no longer lend to shadow banks but only invest in capital. However retail banks are inefficient in making direct investments. Therefore, they optimally choose to consume more in the period of bank run.

Table 4 shows how much households, retail bankers and shadow bankers are willing to pay to stay in a world without bank runs, which can also be understood as the welfare cost of bank runs for different agents. It appears that shadow bankers dislike bank runs the most and would be willing to pay almost 2% of their permanent consumption to stay out of bank runs. Households also find bank runs costly. They are willing to pay 0.74% of their permanent consumption to avoid bank runs. On the other hand, bank runs are somewhat beneficial for retail bankers, who would pay 0.42% of their permanent consumption to stay in a world *with* bank runs. This is due to the consumption boom for retail bankers we observe in the period of bank run.

## 5 Policy Experiments

In this section, we conduct two policy experiments: one is varying the minimum capital requirement  $\Gamma^R$  and the other is varying the risk weight on wholesale funding  $\omega^B$ . By increasing the minimum capital requirement  $\Gamma^R$ , retail banks are required to employ more equity to fund both asset classes, i.e. capital investment and wholesale funding. In contrast, by increasing the risk weight only on wholesale funding, retail banks are only required to increase the equity holding for wholesale funding and not for capital investment. The latter, therefore, becomes relatively less costly to invest than wholesale funding.

For each policy experiment, we answer the following four questions:

1. What are the steady state implication of the policy?
2. How effective is the policy in reducing bank runs?
3. What is the welfare effect of the policy?
4. What is the optimal level of the policy?

### 5.1 Minimum Capital Requirement

In this subsection, we discuss the first policy tool: minimum capital requirement for the retail banks.

#### 5.1.1 The Cost of Minimum Capital Requirement

If the policymakers impose a tighter bank capital requirement, retail banks have to raise relatively more costly equity to finance their assets. Retail banks' cost of funding increases. As a result, the required rate of return on capital investment increases. Accordingly, the production sector demands less capital. The aggregate capital stock decreases, and output decreases as well. The increased cost of funding also results in a higher required return on wholesale funding. For shadow banks, this means funding from the interbank market becomes more costly compare to equity funding, therefore demand for interbank funds decreases. For households, return on capital investment increases relative to the return on deposits<sup>12</sup>. Therefore, households' reduce their deposits and invest more in capital. In sum, the steady state effect of a higher capital requirement can be summarized as: lower capital intermediation through the financial sector, investment becomes less efficient, the aggregate capital stock decreases, and aggregate output decreases.

Figure 3 shows the comparative statics for varying the minimum capital requirement between 2% to 20%. We can see that the steady state aggregate capital stock decreases by around 10% from 32 to 29, and the steady state aggregate output decreases by around 2% from 8.5 to 8.35. Capital

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<sup>12</sup>The required return on deposit does not change, as households' time preference for resources does not change

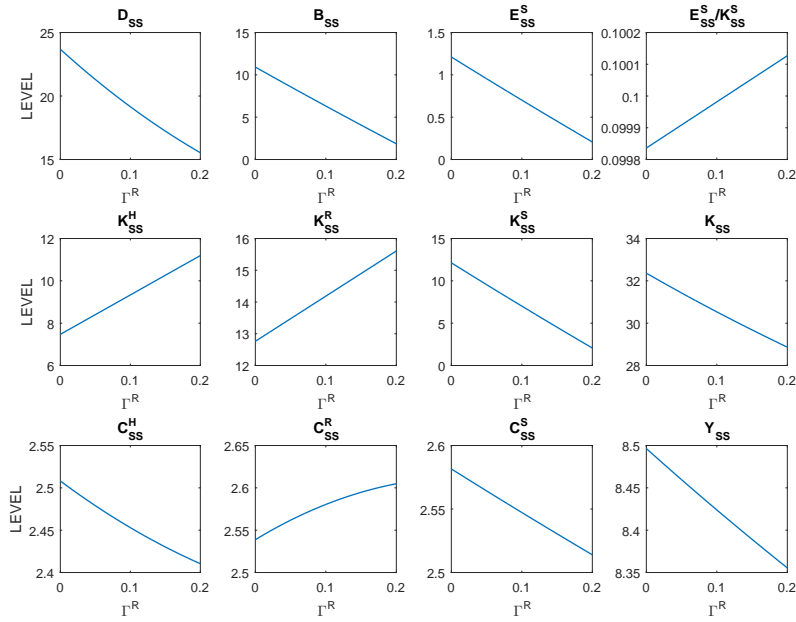


Figure 3: Steady State Effect of Bank Capital Requirement

reallocates from shadow bankers, the most efficient investors, to households and retail bankers, the less efficient investors.

It is interesting to notice that increasing the retail bank capital requirement has a positive "spillover" effect on the capital ratio of the shadow banks. As shown in the top-right subplot of Figure 3, shadow banks' capital to asset ratio increases as the capital requirement on retail banks increases. This is because retail banks demand a higher return on wholesale funding as the bank capital requirement tightens, while the shadow banks' cost of equity remains unchanged. This means, everything else being equal, equity becomes cheaper relative to wholesale funding. Therefore, shadow banks increase their capital ratio endogenously.

In terms of consumption, households and shadow banks consume less in a regime with a higher capital requirement. For households this is because they save less through deposits and make more inefficient direct investment. For shadow banks this is due to the higher cost of funding and reduced investment. Retail banks, on the other hand, enjoy higher consumption under a higher capital requirement. This results seems surprising at first sight. Why do regulated banks consume more the more regulated they are? The answer is, retail bankers are impatient by definition and as a result, their savings rate is sub-optimally low. By imposing a higher capital requirement on them, they are forced to increase their savings in the form of equity. This is why they can consume more while being more tightly constrained.

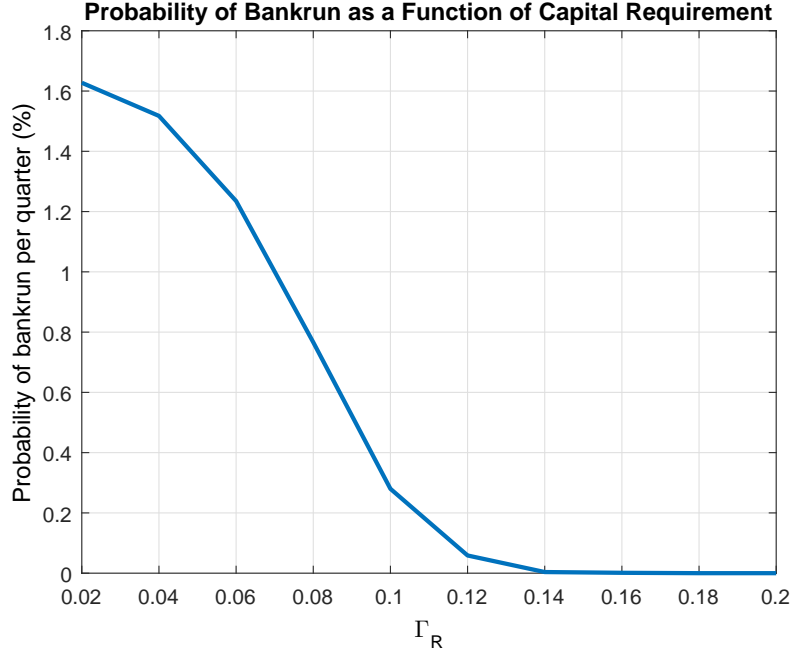


Figure 4: Probability of Bank Runs as a Function of Minimum Capital Requirement

### 5.1.2 The Impact of Minimum Capital Requirement on Bank Runs

Now, we look at the effectiveness of minimum capital requirement in reducing bank runs. To do this, for each given minimum capital requirement level, we simulate the model to get 100 time series each containing 1000 periods. The 100 time series represent 100 different economies. The 100 economies differ by three types of shocks that hit the economy: an AR(1) productivity shock, a random sunspot shock on the coordination of retail bankers when the bank run equilibrium exist, and a random shock on the continuation of the shadow bank run. We drop the first 200 periods of each economy as burn-in. For each of the 100 economies, we count the number of bank runs that happened over the remaining 800 periods, and calculate the frequency of bank runs of each economy over the simulated periods. Then we take the average of the bank run frequencies of the 100 economies to get the expected possibility of bank run each period for a given minimum capital requirement.

Figure 4 shows how probability of bank runs per quarter varies as the minimum capital requirement increases. As the minimum capital requirement increases from 8% to 14%, the annual probability of bank run decreases from 3% to 0. The intuition lies in the steady state effect of the bank capital requirement which we detailed in the last subsection. As discussed before, under a higher minimum capital requirement, capital shifts from shadow banks to households and retail banks. That means, the size of the shadow banking sector shrinks as the capital requirement tightens. When bank runs happen, the price of capital would not decrease as much as in the case of a large shadow banking sector, because smaller shadow banking sector means that the amount of capital liquidated during a run is smaller. Therefore, the difference between normal and fire



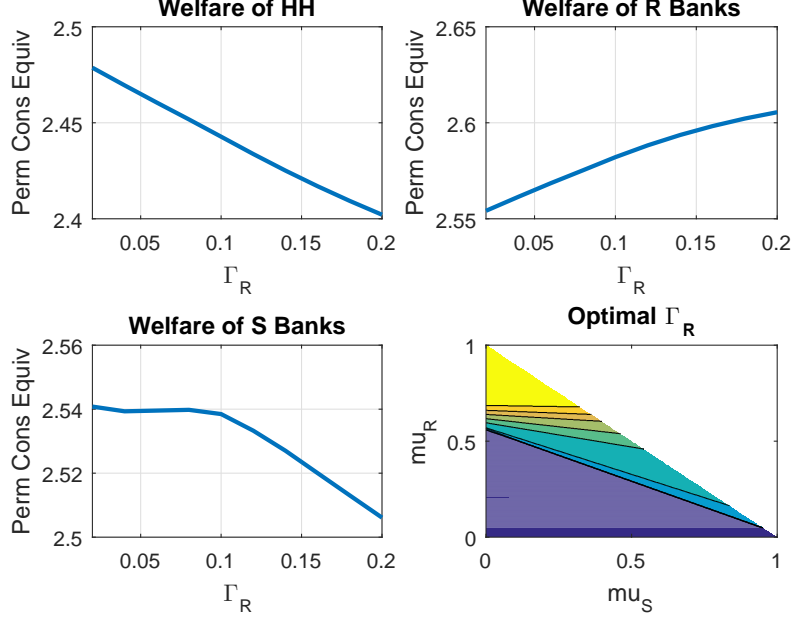


Figure 5: Welfare as a Function of Minimum Capital Requirement

sale price of capital is smaller, and bank runs are less likely to occur.

### 5.1.3 Welfare Effect and the Optimal Level of Minimum Capital Requirement

We discussed the cost and benefit of the minimum capital requirement policy in the last two subsections respectively. In this subsection, we will discuss the welfare measure of minimum capital requirement that combines the two effects of the policy and find the optimal policy using this welfare measure. We define the optimal policy as following:

$$\max_{\Gamma^R} \{ (1 - \mu^R - \mu^S) U^H + \mu^R U^R + \mu^S U^S \}, \quad (5.1)$$

where  $\mu_R$  and  $\mu_S$  represent the welfare weights assigned to retail banks and shadow banks respectively. Accordingly, the welfare weight of the household is equal to  $1 - \mu_R - \mu_S$ .  $U^H$ ,  $U^R$  and  $U^S$  are welfare of households, retail bankers and shadow bankers measured by the permanent consumption of each type of agents respectively.

To calculate the agent's welfare for each given minimum capital requirement level, we use the same simulation method as we used to calculate the probability of bank run given a certain capital requirement. For each minimum capital requirement, we simulate 100 economies each contains 1000 periods and drop the first 200 periods. We calculate the permanent consumption equivalent for each economy over the 800 periods and take the average of the 100 economies to get a welfare measure for each minimum capital requirement level.

Figure 5 shows the agents' welfare as functions of minimum capital requirement and the optimal

minimum capital requirement policy in the last sub-plot. As we increase the minimum capital requirement, households and shadow bankers are worse off, and retail bankers are better off. Comparing Figure 5 and Figure 3 we can easily see that the shape of the welfare functions is to a great extent in line with the shape of the steady state consumption as a function of minimum capital requirement. This observation suggests that the "steady state effect" is more dominant than the "bank run elimination effect", which means the costs of minimum capital requirement outweighs its benefit.

The last sub-plot in Figure 5 shows the optimal minimum capital requirement using the definition of optimal policy defined above. The warmer the color is, the higher the optimal minimum capital requirement is. Therefore, in the top left region, where we care the most about retail bankers, the optimal capital requirement is very high as retail bankers benefit from high capital requirements. At the origin and in the bottom right, we care about households and shadow bankers respectively. In these regions, the optimal capital requirement is the lowest, as households and shadow bankers prefer lower capital requirements. In the middle region, where we care about all three types of agents, an intermediate capital requirement would be optimal.

## 6 Conclusion

We study the macroeconomic effects of retail bank capital regulation in a quantitative model with regulated retail banks and unregulated shadow banks. In our model, financial crises occur in the form of runs on shadow banks. There is a role for regulation in the model, because banks do not internalize that their decisions affect the likelihood of financial crises.

From the perspective of a policymaker, the trade-off which determines the optimal capital requirement is the following: On the one hand, higher capital requirements reduce the leverage ratio and the size of the shadow banking sector. This reduces financial instability. On the other hand, raising capital requirements reduces the steady state capital stock and output due to capital misallocation. The main assumptions behind this trade-off are the financial frictions and investment inefficiency of retail banks relative to shadow banks. Financial frictions take the form of an endogenous leverage constraint for shadow banks and a missing market between shadow banks and households.

We conclude that capital requirements can impose substantial costs on the economy, if capital accumulation is endogenous. Therefore, the optimal capital requirement in a model with endogenous capital accumulation should be substantially lower than the optimal capital requirement in a model without endogenous capital accumulation.

An interesting extension of our model would be to include aggregate demand. A bank run could then result in a downward spiral: The initial effects of the run depress aggregate demand, which worsens the effect of the run and so on. In this case, bank runs would also affect how monetary policy should be conducted.

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## A Steady State Equilibrium

A steady state equilibrium is a special case of the competitive equilibrium and is characterized by constant prices and allocations of resource over time.

Steady state prices are determined by the first order conditions with respect to  $D_{t+1}^H$ ,  $B_{t+1}^R$ , and  $K_{t+1}^S$ :

$$R_{SS}^D = \frac{1}{\beta^H} \quad (\text{A.1})$$

$$R_{SS}^B = \Gamma^R \frac{\omega^B}{\beta^R} + (1 - \Gamma^R \omega^B) \frac{1}{\beta^H} \quad (\text{A.2})$$

$$R_{SS}^K = \frac{R_{SS}^B}{\beta^S \phi R_{SS}^B + 1 - \phi} \quad (\text{A.3})$$

Given the prices, capital holdings of households and retail bank owners can be characterized by first order conditions with respect to  $K_{t+1}^H$  and  $K_{t+1}^R$ :

$$K_{SS}^H = \frac{\beta^H R_{SS}^K - 1}{\alpha^H} \quad (\text{A.4})$$

$$K_{SS}^R = \frac{\beta^R [R_{SS}^K - R_{SS}^D (1 - \Gamma^R \omega^K)] - \Gamma^R \omega^K}{\alpha^R} \quad (\text{A.5})$$

Total capital stock and wage is determined by the first order conditions of the firms:

$$K_{SS} = \left( \frac{r_{SS}^K}{\alpha Z} \right)^{\frac{1}{\alpha-1}} \quad (\text{A.6})$$

$$W_{SS} = (1 - \alpha) Z K_{SS}^\alpha \quad (\text{A.7})$$

where  $r_{SS}^K = R_{SS}^K - 1 + \delta$ .

Total output is given by:

$$Y_{SS} = Z K_{SS}^\alpha \quad (\text{A.8})$$

Capital holding of shadow banks is given by the market clearing condition of capital goods:

$$K_{SS}^S = K_{SS} - K_{SS}^H - K_{SS}^R \quad (\text{A.9})$$

Shadow banks' borrowing can then be determined by the market imposed capital requirement:

$$B_{SS}^S = \frac{(1 - \phi) R_{SS}^K K_{SS}^S}{R_{SS}^B} \quad (\text{A.10})$$

Retail banks' equity ( $E_{SS}^R$ ) can be characterized by the regulatory capital requirement:

$$E_{SS}^R = \Gamma^R(\omega^K K_{SS}^R + \omega^B B_{SS}^R) \quad (\text{A.11})$$

Capital of shadow banks ( $E_{SS}^S$ ) and retail banks' borrowing from households ( $D_{SS}^R$ ) are characterized by the balance sheet constraints of the two banking sectors:

$$E_{SS}^S = K_{SS}^S - B_{SS}^S \quad (\text{A.12})$$

$$D_{SS}^R = K_{SS}^R + B_{SS}^R - E_{SS}^R \quad (\text{A.13})$$

Market clearing for wholesale and retail funding markets implies wholesale lending from retail banks ( $B_{SS}^R$ ) and households' deposit at retail banks ( $D_{SS}^H$ ):

$$B_{SS}^R = B_{SS}^S \quad (\text{A.14})$$

$$D_{SS}^H = D_{SS}^R \quad (\text{A.15})$$

Finally, consumption is characterized by the budget constraints of the agents:

$$C_{SS}^H = W_{SS} + (R_{SS}^D - 1)D_{SS}^H + (R_{SS}^K - 1)K_{SS}^H - \frac{\alpha^H}{2}(K_{SS}^H)^2 \quad (\text{A.16})$$

$$C_{SS}^R = W_{SS} + (R_{SS}^B - 1)B_{SS}^R + (R_{SS}^K - 1)K_{SS}^R - \frac{\alpha^R}{2}(K_{SS}^R)^2 - (R_{SS}^D - 1)D_{SS}^R \quad (\text{A.17})$$

$$C_{SS}^S = W_{SS} + (R_{SS}^K - 1)K_{SS}^S - (R_{SS}^B - 1)B_{SS}^S \quad (\text{A.18})$$

## B Proofs for Propositions

### B.1 Proof of Proposition 3.1

*Proof.* From the steady state expressions A.1 - A.3 we can show:

$$\begin{aligned} \frac{\partial R_{SS}^B}{\partial \Gamma^R} &= \omega^B \left( \frac{1}{\beta^R} - \frac{1}{\beta^S} \right) &> 0, \\ \frac{\partial R_{SS}^K}{\partial \Gamma^R} &= \frac{1 - \phi}{(\phi\beta^S R_{SS}^B + (1 - \phi))^2} \frac{\partial R_{SS}^B}{\partial \Gamma^R} &> 0, \\ \frac{\partial (R_{SS}^K/R_{SS}^B)}{\partial \Gamma^R} &= -\frac{\beta^S \phi}{(\phi\beta^S R_{SS}^B + (1 - \phi))^2} &< 0. \end{aligned}$$

Therefore,  $R_{SS}^K$  and  $R_{SS}^B$  are increasing in  $\Gamma^R$  and  $R_{SS}^K/R_{SS}^B$  is decreasing in  $\Gamma^R$ .  $\square$

## B.2 Proof of Proposition 3.2

*Proof.* From the steady state expressions A.4 and A.5 we have:

$$\begin{aligned}\frac{\partial K_{SS}^H}{\partial \Gamma^R} &= \frac{\beta^H}{\eta^H} \frac{\partial R_{SS}^K}{\partial \Gamma^R} > 0, \\ \frac{\partial K_{SS}^R}{\partial \Gamma^R} &= \frac{1}{\eta^R} \left[ \underbrace{\beta^R \frac{\partial R_{SS}^K}{\partial \Gamma^R}}_{\text{General Equilibrium Effect}} + \underbrace{(R_{SS}^D \beta^R - 1) \omega^K}_{\text{Partial Equilibrium Effect}} \right] \\ &= \frac{1}{\eta^R} \frac{\beta^H - \beta^R}{\beta^H} \left[ \frac{1 - \phi}{(\phi \beta^S R_{SS}^B + 1 - \phi)^2} \omega^B - \omega^K \right].\end{aligned}$$

$\partial K_{SS}^R / \partial \Gamma^R > 0$  if  $\omega^K < \omega^B(1 - \phi) / (\phi \beta^S R_{SS}^B + 1 - \phi)^2$ , where  $R_{SS}^B$  is given by A.2.

$K_{SS}^S$  is characterized by the residual capital holding:  $K_{SS}^S = K_{SS} - K_{SS}^H - K_{SS}^R$ . We have proved that  $K^H$  and  $K^R$  are increasing in  $\Gamma^R$  when  $\omega^K < \omega^B(1 - \phi) / (\phi \beta^S R_{SS}^B + 1 - \phi)^2$ , the proof the  $K_{SS}$  is decreasing in  $\Gamma^R$  can be found in the Proof of Proposition 3.4. Therefore,  $K_{SS}^S$  is decreasing as  $\Gamma^R$  increases.  $\square$

## B.3 Proof of Proposition 3.3

*Proof.* From the steady state expression A.10 one can show that:

$$\begin{aligned}B_{SS} &= (1 - \phi) \frac{R_{SS}^K}{R_{SS}^B} K_{SS}^S, \\ \frac{B_{SS}}{K_{SS}^S} &= (1 - \phi) \frac{R_{SS}^K}{R_{SS}^B}.\end{aligned}$$

As we have proved in Proposition 3.1 and 3.2,  $\frac{\partial(R_{SS}^K/R_{SS}^B)}{\partial \Gamma^R} < 0$  and  $\frac{\partial K_{SS}^S}{\partial \Gamma^R} < 0$ . Therefore,  $B_{SS}$  and  $\frac{B_{SS}}{K_{SS}^S}$  are decreasing in  $\Gamma^R$ .  $\square$

## B.4 Proof of Proposition 3.4

*Proof.* From the steady state sexpressions A.6 - A.8 we have:

$$\begin{aligned}\frac{\partial K_{SS}}{\partial \Gamma^R} &= \frac{1}{\alpha - 1} \frac{1}{\alpha Z} \left( \frac{r_{SS}^K}{\alpha Z} \right)^{\frac{\alpha}{1 - \alpha}} \frac{\partial r_{SS}^K}{\partial \Gamma^R} < 0 \\ \frac{\partial Y_{SS}}{\partial \Gamma^R} &= \alpha Z K_{SS}^{\alpha - 1} \frac{\partial K_{SS}}{\partial \Gamma^R} < 0\end{aligned}$$

Therefore,  $K_{SS}$  and  $Y_{SS}$  are decreasing in  $\Gamma^R$ .  $\square$

## C Computation

We solve the model nonlinearly using a time iteration algorithm. Solving the model nonlinearly is important, because bank runs can lead to large deviations from steady state, where perturbation algorithms are inaccurate.

The state space of the model is  $\mathcal{S} = (N^H, N^R, N^S, K, Z)$  in the "*no bank run*" equilibrium and  $\mathcal{S}^* = (N^{H,*}, N^{R,*}, K, Z)$  in the "*bank run*" equilibrium. We approximate the consumption policy functions  $C^H(\mathcal{S}), C^R(\mathcal{S}), C^S(\mathcal{S}), C^{H,*}(\mathcal{S}^*)$  and  $C^{R,*}(\mathcal{S}^*)$  and the capital prices  $Q(\mathcal{S})$  and  $Q^*(\mathcal{S}^*)$  using second order polynomials. We compute the polynomial coefficients by imposing that the polynomial approximations must be equal to the original functions on the grid. Specifically, denoting the polynomial coefficients by  $\alpha$  and the polynomials by  $\Pi(\mathcal{S})$ , we impose for example for the consumption of households

$$\Pi(\mathcal{S}_i)\alpha_{C^H} = C^H(\mathcal{S}_i) \quad i = 1, \dots, N. \quad (\text{C.1})$$

for all  $N$  grid points. We use a second order Smolyak grid computed using the toolbox by [Judd, Maliar, Maliar, and Valero \(2014\)](#).

One slight complication of the model is that the future net worth values,  $N_H, N_R$  and  $N_S$ , depends on  $Q(\mathcal{S})$ . This implies that, for example, the household net worth for a given function  $Q(\cdot)$  must be computed as a solution to the nonlinear function<sup>13</sup>

$$N^H = (r^K + (1 - \delta)Q(N^H, N^R, N^S, K, Z))K^H + R^D D^H + W + \Pi^Q. \quad (\text{C.2})$$

With this in mind, we will now outline our solution algorithm for the "*no bank run*" equilibrium. Suppose we are in iteration  $k$  and have initial guesses for the no run consumption policy functions  $C_{(k)}^H(\mathcal{S}), C_{(k)}^R(\mathcal{S})$ , and  $C_{(k)}^S(\mathcal{S})$  and the capital price function  $Q_{(k)}(\mathcal{S})$  as well as values for the future net worth  $N_{(k)}^{H'}, N_{(k)}^{R'}$  and  $N_{(k)}^{S'}$ .

1. Compute the new values for  $(K^{H'}, K^{R'}, K^{S'}, D', B', R^{D'}, R^{B'}, Q)$  for all grid points  $i = 1, \dots, N$  using the first order conditions [2.6](#), [2.7](#), [2.16](#), [2.17](#), [2.25](#), [2.40](#) and the leverage constraints [2.14](#) and [2.20](#). Compute the future net worth where necessary according to

$$\tilde{N}_{(k+1)}^{H'} = (r^{K'} + (1 - \delta)Q_{(k)}(N_{(k)}^{H'}, N_{(k)}^{R'}, N_{(k)}^{S'}, K', Z'))K^{H'} + R^{D'} D^{H'} + W' + \Pi^{Q'}. \quad (\text{C.3})$$

We compute expectations using Gauss-Hermite quadrature. Note that for each quadrature node, a different value of  $\tilde{N}_{(k+1)}^{H'}$  must be computed.

2. Using the new policies and prices, update the consumption functions using the budget constraints [2.5](#), [2.13](#) and [2.23](#).

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<sup>13</sup>In principle,  $\Pi^Q$  is also a function of the states. We ignore this here for the sake of exposition. We do however account for this correctly in the code.



3. Update the next period net worth values using C.3, with some attenuation:  $N_{(k+1)}^{H'} = (1 - \iota)N_{(k)}^{H'} + \iota\tilde{N}_{(k+1)}^{H'}$ , with  $\iota = 0.25$ .
4. Repeat until the errors in the consumption, capital price and net worth values on the grid are small. We iterate until the maximum error in consumption is smaller than 1e-4 and the maximum error in the net worth is smaller than 1e-2.

Since bank runs are unanticipated, we can first solve for the "*no bank run*" equilibrium and then afterwards for the "*bank run*" equilibrium. Importantly, expectations during a bank run are taken over the future "*bank run*" and "*no bank run*" states. It is therefore necessary to keep track of two sets of net worth values,  $N_{(k)}^{H'}, N_{(k)}^{R'}$  and  $N_{(k)}^{H',*}, N_{(k)}^{R',*}$ . Otherwise, the algorithm works in the same way as for the "*no bank run*" equilibrium.