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## Shadow Banking and Financial Stability under Limited Deposit Insurance

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# Shadow Banking and Financial Stability under Limited Deposit Insurance

Lukas Voellmy\*

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## Abstract

This paper studies the relation between shadow banking and financial stability in an economy in which deposit insurance is limited by a cap and in which self-fulfilling, systemic bank runs can occur. Due to the cap on deposit insurance, a certain part of short-term claims cannot be protected by deposit insurance. Referring to the present situation in the United States, this applies to institutional cash pools whose endowments are very large relative to the deposit insurance cap. I show that financial stability depends on the way how insured and uninsured deposits are distributed across financial institutions. The presence of a 'shadow banking sector' that issues only uninsured short-term claims may be desirable from a financial stability perspective. Shadow banks absorb uninsured (and uninsurable) deposits from the commercial banking sector, which limits the extent of systemic bank runs in the sense that systemic runs will be confined to the shadow banking sector rather than encompassing the entire financial system.

**JEL-Codes:** E44, G21, G28

**Keywords:** Shadow Banking, Deposit Insurance, Bank Runs, Financial Intermediation

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# 1 Introduction

The recent decades have witnessed the growth of a so-called shadow banking sector in the United States, which provides very short-term claims similar to bank deposits outside the traditional banking system (Poszar et al. 2010, Ricks 2012). Prominent examples of shadow bank claims are money market mutual fund shares, overnight asset backed commercial paper, or certain forms of repo. Since the financial crisis of 2007-08, and especially since the run on money market mutual funds in September 2008, the shadow banking sector is widely thought to pose a threat to financial stability.<sup>1</sup> This paper shows that the financial stability implications of the shadow banking sector should not be analyzed separately from the cap on deposit insurance. Shadow banks cater mostly to institutional investors managing large cash-balances, who have a preference for extremely safe, short-term assets (Poszar 2011). For instance, cash-pools of large non-financial corporations today commonly amount to several hundred million USD, a large part of which is held in the form of shadow bank assets rather than traditional bank deposits (Poszar 2011). Given the cap on deposit insurance, it is impossible or impracticable for these institutional cash-pools to hold their entire endowment in insured bank deposits. At the same time, supply of short-term government debt is limited. In this context of limited deposit insurance, shadow banks can have the effect of absorbing uninsured (and uninsurable) short-term claims from the commercial banking sector. This paper shows that this may be desired from a financial stability perspective in the sense that a flow of uninsured deposits back into the commercial banking sector can be detrimental to aggregate financial stability.

The deposit insurance scheme, and in particular the cap, are taken as exogenous in this paper. In this sense, the paper speaks to a regulator that cannot change the deposit insurance scheme. Furthermore, it is taken as given that there is a certain demand for short-term claims ("deposits" henceforward). The demand exceeds the cap on deposit insurance in the sense that some fraction of the deposits cannot be protected by deposit insurance. Banks are illiquid due to the short-term nature of deposits, which opens up the possibility of coordination failure in the form of self-fulfilling bank runs *à-la* Diamond and

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<sup>1</sup>Schmidt et al. (2016) provide a detailed description of the run on money market mutual funds in 2008. Episodes that can be characterized as bank runs were also observed in other segments of the shadow banking system such as the market for short-term asset backed commercial paper (Covitz et al. 2013, Kacperczyk and Schnabl 2010)

Dybvig (1983). Banks issue both insured and uninsured deposits. Whether an individual bank is susceptible to a run depends both on the share of uninsured deposits at the bank as well as on the liquidation price of assets, which in turn depends on how many other banks are hit by a run. The latter introduces a systemic element to bank runs. The model abstracts from fundamental risk, so that these self-fulfilling, systemic bank runs constitute the only source of risk in the economy.

I show that, given a certain total amount of insured and uninsured deposits outstanding in the economy, the potential magnitude of systemic runs depends on the way how insured and uninsured deposits are distributed across banks. In general, the magnitude of systemic runs is minimized if the financial system exhibits a two-tiered structure, with one sector that issues both insured and uninsured deposits (labelled commercial banking sector) and another sector that issues only uninsured deposits (labelled shadow banking sector). If the cap on deposit insurance is low, meaning that the share of uninsurable deposits in the economy is high, it is not feasible to avoid systemic runs altogether. However, the magnitude of systemic runs can be minimized by setting the shadow banking sector to the smallest possible size at which it absorbs enough of the uninsurable deposits from the commercial banking sector so as to keep the commercial banking sector shielded from systemic runs. While systemic runs may occur in the shadow banking sector, the presence of the shadow banking sector also implies that the share of insured deposits at commercial banks is relatively high, so that systemic runs do not encompass the commercial banking sector.

In a competitive allocation, investors may face conflicting incentives regarding the type of bank at which they hold deposits: At the one hand, the presence of insured depositors who do not participate in runs reduces expected losses caused by runs for uninsured depositors at commercial banks. This gives investors an incentive to hold uninsured deposits at commercial banks rather than shadow banks. At the other hand, if the deposit insurance agency charges a fee on deposits issued by commercial banks, investors have an incentive to move into shadow banks in order to avoid the fee.

Whether the shadow banking sector is larger or smaller in a competitive allocation compared to the optimal size depends on the level of the cap on deposit insurance. If the cap is high, so that the share of uninsurable deposits in the economy is low, a social planner that minimizes the magnitude of systemic runs would set the shadow banking sector to a

small size or might not set up a shadow banking sector at all. If a fee is charged on commercial bank deposits, the shadow banking sector will be larger than the optimal size in a competitive allocation, due to investors' private incentive to avoid the fee on commercial bank deposits. At the other hand, if the cap on deposit insurance is low, so that the share of uninsurable deposits is high, a social planner would set up a relatively large shadow banking sector prone to systemic runs. This situation does not constitute a competitive equilibrium since investors have a private incentive to move uninsured deposits from the unstable shadow banking sector into the stable commercial banking sector, causing the commercial banking sector to become unstable as well. As a result, the shadow banking sector will be smaller than the optimal size. The optimal size of the shadow banking sector can be implemented in a competitive allocation by first, imposing a tax on shadow bank deposits that mimics any fee charged on commercial bank deposits by the deposit insurance agency and second, limiting the issuance of uninsured deposits by commercial banks.

This paper remains silent on the optimal level of the cap on deposit insurance. Davila and Goldstein (2016) study the optimal level of the cap, including the case where runs have a systemic element as in the present paper. Increasing the cap has the benefit of reducing expected losses caused by runs but entails social costs such as deadweight losses of taxation. The present paper is complementary to Davila and Goldstein (2016) by showing that the trade-offs studied in Davila and Goldstein (2016) may be improved if, in addition to choosing the level of the cap, an appropriate distribution of insured and uninsured deposits across banks can be implemented. Another closely related paper is Luck and Schempp (2016) who study financial stability implications of the shadow banking sector in an economy in which commercial banks issue insured- and shadow banks uninsured short-term claims. The magnitude of systemic runs increases in the size of the shadow banking sector. The present paper shows that some of the conclusions reached in Luck and Schempp (2016) regarding shadow banking and financial stability may be reversed if deposit insurance is limited and commercial banks issue both insured and uninsured deposits.

More generally, this paper is related to a recent theoretical literature on shadow banking, in particular with regard to the financial stability implications of the shadow banking sector. In Gennaioli et al. (2013), shadow banks allow for a better diversification of idiosyncratic risk due to pooling and tranching of assets but make the financial system

more vulnerable to miscalculations of aggregate risk. Hanson et al. (2015) characterize shadow banking and commercial banking as two different ways to provide riskless claims. The occurrence of fire sales in the shadow banking sector is inherent to shadow banks' business model. In Gertler et al. (2016) shadow banks are modelled as wholesale banks that issue debt to other (retail) banks. Due to a relatively low degree of agency frictions compared to retail banking, shadow banking can reduce the financial accelerator in the aftermath of real shocks. However, high leverage in the shadow banking sector can also lead to instability in the form of bank runs. In Kim and Mangla (2017) and Ordonez (2017), shadow banking can increase investment in socially desirable risky projects given regulatory constraints imposed on commercial banks, but may also contribute to socially excessive risk-taking. Plantin (2015) and Gornicka (2016) focus on the interlinkages between commercial banks and shadow banks, in particular with respect to regulatory arbitrage by part of commercial banks. Martin et al. (2014) study run equilibria on various types of shadow banks, taking into account the specifics of the debt contracts used. Moreira and Savov (2017) characterize shadow banking as the provision of risky claims which are information-insensitive and therefore provide liquidity services. This leads to a socially desirable expansion of liquidity in normal times but makes the economy more vulnerable to changes in aggregate uncertainty. The present paper highlights that the presence or absence of a shadow banking sector has an effect on the way how uninsurable short-term claims are distributed across different financial institutions in the economy, with consequences for aggregate financial stability. This paper abstracts from many issues relevant to shadow banking and should be seen as complementary to the papers mentioned above.

## 2 The Environment

The economy lasts for two periods, indexed by  $t=0,1$ . Period 1 is further subdivided into middle of period and end of period. An infinitely divisible good is used for consumption and investment. Two types of agents populate the economy at  $t=0$ : A double continuum of households, indexed by  $h \in \mathcal{H} = [0, 1] \times [0, 1]$  and a continuum of banks, indexed by  $i \in \mathcal{I} = [0, 1]$ .<sup>2</sup> Each household receives an endowment of one unit of good at  $t=0$ . Households maximize expected utility  $E\{u(c_1)\}$ , where  $c_1$  equals total consumption during

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<sup>2</sup>There are infinitely many banks, but many more households than banks.

$t=1$ . Utility is strictly increasing, strictly concave and twice continuously differentiable. The only storage opportunity available at  $t=0$  is a riskless, constant returns to scale investment technology that returns one unit of good at the end of  $t=1$  per unit of good invested at  $t=0$ .

For reasons that are outside of the model, households only want to invest into demand deposits. Demand deposits can be issued by banks and they allow households to withdraw a fixed amount of good per unit of good invested at  $t=0$  at any time during  $t=1$  (middle or end of period) from the bank. Households are indifferent about when to consume during  $t=1$  and withdraw early (in the middle of  $t=1$ ) only if they have a strict incentive to do so. At  $t=0$ , banks can sell demand deposits to households and invest the proceeds into the investment technology. Banks do not receive an endowment of their own and maximize expected profits over both periods. Since the investment technology pays out only at the end of  $t=1$ , banks are illiquid in the middle of  $t=1$ . If households withdraw early, banks need to raise good by selling claims to the investment return on a secondary market. The secondary market is represented by a double continuum  $[0, 1] \times [0, 1]$  of outside investors that are born at the beginning of  $t=1$  with an endowment of  $\Lambda^S$  units of good each, where  $\Lambda^S \in (0, 1)$  represents the secondary market's capacity relative to households' endowment.

The model features an exogenous scheme of limited deposit insurance. After buying deposits at  $t=0$ , households can choose for which deposits to obtain deposit insurance. The total face value of insured deposits held at all banks together is limited to  $\theta \in [0, 1]$  per household, where  $\theta$  stands for the cap on deposit insurance. Different to real-world deposit insurance arrangements, the cap is modelled as a pure cap per person, without a specific limit on insured deposits held at a certain bank.<sup>3</sup> If a household obtains deposit insurance for a deposit, it is guaranteed to receive an amount of good equal to the face value of the deposit at the end of  $t=1$ . Whenever the bank is not able to pay out an amount of good corresponding to the face value of the deposit, the deposit insurance makes up for the difference. Deposit insurance payments are made at the end of  $t=1$  and are financed by levying a lump-sum tax on all households.<sup>4</sup> In the baseline version of

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<sup>3</sup>Modelling the cap as a cap per person and intermediary would call for a richer model that allows to pin down the number of banks offering insured deposits in equilibrium, for instance by introducing a fixed cost of opening a bank.

<sup>4</sup>Consumption  $c_1$  equals the total return received at  $t=1$  from a household's investment into deposits minus taxes to deposit insurance. Since deposit insurance payments represent transfers from households to themselves, all households can pay the lump-sum tax in a symmetric allocation. In a hypothetical

the model, households can obtain deposit insurance for deposits issued by all banks at no cost. In section 6, I study a version model of the model where a fee is charged on deposits issued by banks with access to deposit insurance. Figure 1 summarizes the timeline.

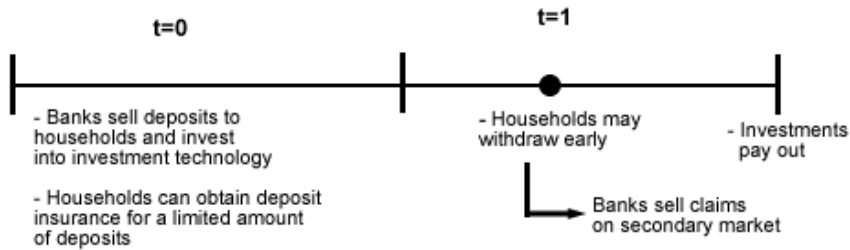


Figure 1: Timeline.

Perfect information and perfect commitment is assumed throughout in order to abstract from any issues related to moral hazard, both with regard to trades between households and banks and with regard to deposit insurance. Deposit insurance only insures deposits of banks whose deposits are actually backed by a corresponding amount of real investment at  $t=0$  and which commit to pay out insured depositors at  $t=1$  whenever they can, rather than "running away" with the investment return.

### 3 Runs

When faced with early withdrawals by depositors (households), banks sell claims to the investment return to outside investors in order to pay out the depositors. Depositors are served sequentially and paid out at face value, as in Diamond and Dybvig (1983). If many depositors withdraw early at  $t=1$ , the order of the line in which a bank processes the orders is determined randomly.<sup>5</sup> Let  $P$  denote the mid-period 1 price of an asset paying out one unit of good at the end of  $t=1$ . There is no uncertainty regarding the real investment returns and no discounting within  $t=1$ , which means that outside investors will buy assets at a price of  $P = 1$  (the fundamental value) as long as their endowment

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non-symmetric allocation in which some households' consumption level  $c_1$  would go to negative if they paid the entire tax, these households consume  $c_1 = 0$  and the tax will be increased accordingly for the remaining households.

<sup>5</sup>The order of the line is determined independently at each bank, which allows households to diversify away idiosyncratic risk regarding the order in the line by spreading their investment over many banks. See also the discussion in section 5 and footnote 13.



is sufficient to do so. Let  $\Lambda^D$  denote the total fundamental value of all assets sold in the middle of  $t=1$ . If  $\Lambda^D > \Lambda^S$ , then outside investors' endowment is not enough to buy all claims sold in the middle of  $t=1$  at their fundamental value and the market-clearing price is determined by cash-in-the-market pricing à-la Allen and Gale (1994):

$$\underbrace{P(\Lambda^D)}_{\text{liquidation price}} = \left\{ \frac{\overbrace{\Lambda^S}^{\text{secondary market capacity}}}{\underbrace{\Lambda^D}_{\text{assets sold}}}, 1 \right\} \quad (1)$$

From Diamond and Dybvig (1983) we know that the combination of payment-on-demand deposits and liquidation losses can lead to self-fulfilling run equilibria. Liquidation losses occur whenever assets trade below fundamental value ( $P < 1$ ). However, since insured depositors never have an incentive to withdraw early, susceptibility to runs depends also on the share of insured deposits among the deposits issued by a bank. Denote  $\vartheta(i)$  as the share of insured deposits among all deposits issued by bank  $i$ , in terms of the face value. To illustrate how susceptibility to runs depends both on  $\vartheta(i)$  and on  $P$ , consider a bank with  $\vartheta(i) = 0.5$  (that is, 50% of deposits are insured) and suppose the liquidation price equals  $P = 0.8$ . Then this bank could pay out all uninsured depositors if they all withdraw early, by selling 80% of the portfolio at the current market price of  $P = 0.8$ .<sup>6</sup> Since uninsured depositors know this, they will not run the bank in the first place. Suppose now the liquidation price  $P$  is below 0.5. Then the bank cannot pay out all uninsured depositors if they all withdraw early, even by liquidating the entire portfolio at the current market price. Hence the bank will be susceptible to self-fulfilling runs since nothing will be left in the bank for the last uninsured depositor that shows up at the bank, if all other uninsured depositors withdraw. It is then relatively easy to see that a bank will be susceptible to runs if and only if:

$$\underbrace{1 - \vartheta(i)}_{\text{share of uninsured deposits}} > \underbrace{P}_{\text{liquidation price}} \quad (2)$$

A bank with only insured depositors ( $\vartheta(i) = 1$ ) will never be susceptible to runs, independent of the liquidation price. A bank with no insured depositors ( $\vartheta(i) = 0$ ) will be susceptible to a run whenever assets trade below fundamental value ( $P < 1$ ). By (1), the

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<sup>6</sup>The important implicit assumption here is that the deposit insurance agency allows the bank to liquidate a large part of its portfolio in a run, thereby becoming insolvent.

liquidation price  $P$  depends itself on how many banks are hit by a run, which introduces a systemic element to the bank runs.

**Proposition 3.1.** *Let  $f(\vartheta) = 1 - P(\Lambda^D(\vartheta))$ . Then  $f(\vartheta)$  has a greatest fixed point, denoted by  $\vartheta^{SR}$ . A situation in which all banks whose share of insured deposits satisfies  $\vartheta(i) < \vartheta^{SR}$  are hit by a run constitutes the largest run that is possible, that is, the run encompassing the largest set of banks.*

*Proof.* By (1), the liquidation price is decreasing in the number of banks that liquidate their assets. Hence  $f(\vartheta)$  is an increasing function mapping  $[0, 1]$  into itself. By Tarski's fixed point theorem, the set of fixed points of  $f(\vartheta)$  is non-empty and has a greatest element, denoted by  $\vartheta^{SR}$ . In addition we have that  $\vartheta \geq f(\vartheta)$  for any  $\vartheta \geq \vartheta^{SR}$ . Next, I show that there exists a run equilibrium in which all banks with  $\vartheta(i) < \vartheta^{SR}$  are hit by a run. Suppose all banks with  $\vartheta(i) < \vartheta^{SR}$  liquidate their portfolios. Then, by (2), all banks with  $\vartheta(i) < 1 - P(\Lambda^D(\vartheta^{SR})) = f(\vartheta^{SR}) = \vartheta^{SR}$  are susceptible to a run. It remains to show that a run encompassing all banks with  $\vartheta(i) < \vartheta^{SR}$  is the largest run that is possible. Let  $\tilde{\vartheta} \geq \vartheta^{SR}$  and the set of banks with  $\vartheta(i) = \tilde{\vartheta}$  is non-empty. Suppose there is a run encompassing all banks with  $\vartheta \leq \tilde{\vartheta}$ . If banks with  $\vartheta(i) = \tilde{\vartheta}$  are hit by a run, then by (2), all banks with  $\vartheta(i) \leq \tilde{\vartheta}$  are susceptible to a run as well. We have that  $\tilde{\vartheta} \geq f(\tilde{\vartheta}) = 1 - P(\Lambda^D(\tilde{\vartheta}))$  and hence, by condition (2), banks with  $\vartheta(i) = \tilde{\vartheta}$  are not susceptible to a run, which leads to a contradiction. Hence there is no run encompassing all banks with  $\vartheta(i) \leq \tilde{\vartheta}$ , which completes the proof. ■

In what follows, a run encompassing all institutions with  $\vartheta(i) < \vartheta^{SR}$  will be called a *systemic run*. If  $\vartheta^{SR} = 0$  then systemic runs cannot occur in the economy. Equilibrium selection is driven by an exogenous sunspot shock that realizes at  $t=1$ . The realization of the sunspot variable is denoted by  $\xi \in \{0, 1\}$ . Households select the no-run equilibrium if  $\xi = 0$  occurs and they select the systemic run equilibrium if  $\xi = 1$  occurs (in case the economy exhibits a systemic run equilibrium).<sup>7</sup>  $\xi = 1$  occurs with some probability  $\pi^r > 0$ . The liquidation price in a systemic run is denoted by:

$$P^{SR} = \min \left\{ \frac{\Lambda^S}{\Lambda^D(\vartheta^{SR})}, 1 \right\} \quad (3)$$

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<sup>7</sup>It is assumed that households select either the no-run equilibrium or the systemic run equilibrium. In principle the economy may exhibit many different run equilibria in which smaller subsets of banks are hit by a run.

If systemic runs cannot occur in the economy, then  $P^{SR} = 1$ . I will now illustrate by way of an example why the magnitude of systemic runs in this economy depends on how insured and uninsured deposits are distributed across banks. Figure 2 shows three alternative structures of the financial system, with an identical total amount of insured deposits (in grey) and uninsured deposits outstanding. Secondary market capacity is given by  $\Lambda^S = 0.25$ .

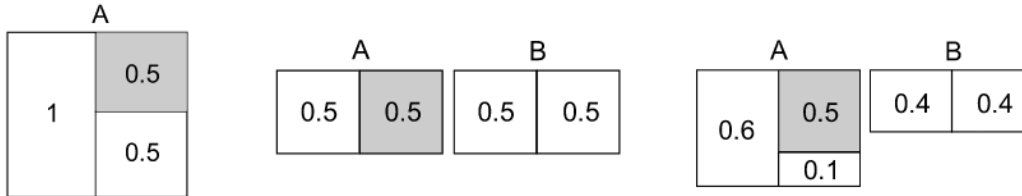


Figure 2: Alternative distributions of insured and uninsured deposits across banks.

The left-hand side of figure 2 shows a financial system in which insured and uninsured deposits are distributed uniformly across banks. There is one representative bank labelled A, which may stand for many identical banks, with 50% insured deposits. Suppose now all banks A liquidate their portfolios. The liquidation price falls to  $P = \frac{0.25}{1} = 0.25$ . If the liquidation price equals  $P = 0.25$  then all banks A are susceptible to runs (see 2). Hence the systemic run equilibrium in this economy encompasses the entire financial system.

The middle of figure 2 shows a two-tiered structure of the financial system in which deposits are distributed in such a way that all insured deposits are held in sector A of the financial system and all uninsured deposits are held in sector B. Banks A are never susceptible to runs. Consider now a hypothetical situation in which all banks B liquidate their portfolios. The liquidation price falls to  $P = \frac{0.25}{0.5} = 0.5$ . By condition (2), banks B are susceptible to runs at this liquidation price, which means that there is a systemic run equilibrium that encompasses the entire sector B of the financial system. Compared to the one-tiered structure depicted on the left-hand side of figure 2, systemic runs encompass only half of the financial system.

A social planner would choose the distribution of insured and uninsured deposits across banks such as to minimize losses caused by systemic runs for households (see section 4). The extreme distribution of insured deposits across banks that is depicted in the middle

of figure 2 is generally not optimal. To see this, suppose that, starting from the situation depicted in the middle of figure 2, a certain amount of uninsured deposits is moved from sector B into sector A, which leads to the situation depicted on the right-hand side of figure 2. The share of uninsured deposits in sector A now equals  $\frac{0.1}{0.6} < 0.25$ , which means that systemic runs still do not encompass sector A of the financial system. Sector B, which is still susceptible to systemic runs, is now relatively smaller, which means that the magnitude of systemic runs has decreased compared to the situation depicted in the middle of figure 2. In general, the magnitude of systemic runs will be minimized by setting sector B to the smallest possible size at which it is large enough to absorb enough of the uninsurable deposits from sector A so that sector A is not susceptible to systemic runs (see the section below).

## 4 The Planner's Allocation

Consider a social planner that maximizes expected utility of households, giving equal weight to all households. The planner takes as given that households want to hold demand-deposits with a total face value equal to their endowment,<sup>8</sup> and he takes as given the deposit insurance scheme in place, in particular the cap  $\theta$ . At  $t=0$ , the planner invests households' endowment into banks and distributes the demand deposits issued by the banks to households. The planner can choose the *structure* of the financial system by determining the distribution of insured and uninsured deposits across banks at  $t=0$ .<sup>9</sup>

Denote  $\delta(i)$  as the amount of endowment invested into bank  $i$  by the planner and  $\vartheta(i)$  as the share of insured deposits at bank  $i$ . The planner's allocation is given by two measurable functions  $\delta(i), \vartheta(i) : \mathcal{I} \rightarrow \mathbb{R}^+$ . The planner will not allocate any profits to banks so that the face value of deposits equals the amount invested into banks. Denote  $D(\mathcal{I}') \equiv \int_{\mathcal{I}'} \delta(i) di$  as total investment into some subset  $\mathcal{I}' \subseteq \mathcal{I}$  of banks, which equals the face value of deposits issued by these banks. We also denote  $D^I \equiv \int_{\mathcal{I}} \vartheta(i) \delta(i) di$  as the total face value of insured deposits in the economy.

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<sup>8</sup>This paper is concerned with the optimal distribution of short-term debt across banks and not with the optimal amount of short-term debt.

<sup>9</sup>The social planner can choose the allocation ex ante at  $t=0$  but he cannot intervene ex post at  $t=1$ . Without this restriction, the social planner problem as described above would not be sensible since the planner could simply emulate a deposit insurance scheme by redistributing good ex post at  $t=1$ .

Risk aversion implies that the planner will give each household an identical portfolio of deposits. We assume that some version of the law of large numbers holds in the continuum so that idiosyncratic risk regarding the order in the line at individual banks in case of a systemic run is eliminated by giving households deposits at many (formally a continuum) different banks. Hence all households will consume an identical amount  $c_1(\xi)$  in state of nature  $\xi \in \{0, 1\}$ . If no systemic run occurs at  $t=1$  ( $\xi = 0$ ), consumption of households equals the total face value of deposits. In case of a systemic run ( $\xi = 1$ ), consumption of households equals the face value of deposits minus losses caused by the run. Cash-in-the-market pricing of assets in a run means that total losses in a systemic run from the point of view of households are equal to the fundamental value of the assets sold to outside investors in the run minus the amount that outside investors pay for it. The latter simply equals outside investors' total endowment  $\Lambda^S$ . Note also that any payments made by deposit insurance at  $t=1$  represent transfers from households to themselves so that they cancel out in the aggregate. Consumption of households in case of no run and a systemic run respectively is hence given by:

$$\begin{aligned}
c(0) &= D(\mathcal{I}) \\
c(1) &= D(\mathcal{I}) - \max \left\{ \underbrace{D(i \in \mathcal{I} : \vartheta(i) < \vartheta^{SR})}_{\substack{\text{fundamental value of} \\ \text{assets sold in systemic run} \\ \text{total investment into banks} \\ \text{susceptible to run}}} - \underbrace{\Lambda^S}_{\substack{\text{secondary} \\ \text{market} \\ \text{capacity}}}, 0 \right\}
\end{aligned} \tag{4}$$

If systemic runs cannot occur in the economy, the set of banks with  $\vartheta(i) < \vartheta^{SR}$  is empty (see also section 3). The planner's problem is given by:

$$\max_{[\delta(i), \vartheta(i)]} E_{\xi} \{u(c_1(\xi))\} \quad \text{subject to} \quad D(\mathcal{I}) = 1 \quad \text{and} \quad D^I \leq \theta \tag{5}$$

Note that the planner's problem is equivalent to minimizing losses caused by systemic runs. We proceed with the following result:

**Lemma 4.1.** *The planner distributes insured deposits across banks such that  $D(i \in \mathcal{I} : \vartheta(i) = 0) + D(i \in \mathcal{I} : \vartheta(i) \geq \vartheta^{SR}) = 1$ .*

In words, lemma 4.1 says that the planner distributes insured and uninsured deposits across banks so that there are at most two types of banks: banks with no insured deposits ( $\vartheta(i) = 0$ ) and banks with enough insured deposits to prevent them from being susceptible

to runs ( $\vartheta(i) \geq \vartheta^{SR}$ ). Figure 3 provides a graphical illustration of the proof of lemma 4.1. Suppose the planner sets up a set of banks with  $\vartheta \in (0, \vartheta^{SR})$  and allocates a mass  $D > 0$  of endowment to these banks. The fundamental value of assets sold by these banks in a systemic run equals  $D$ . The deposits allocated to these banks can always be redistributed as depicted in figure 3, decreasing the mass of endowment allocated to banks that are susceptible to systemic runs to some  $D' < D$ . This means that losses caused by systemic runs decrease, and therefore the initial policy cannot be optimal.

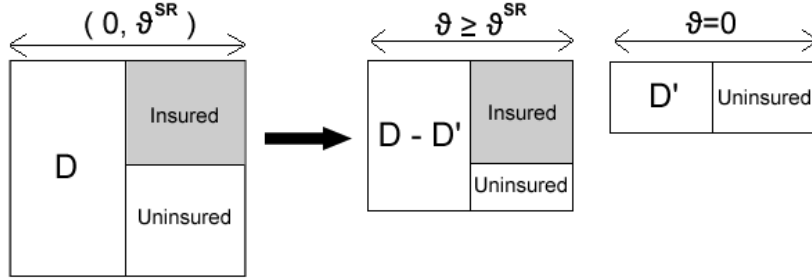


Figure 3: Illustration of proof of lemma 4.1

In what follows I will label banks with no insured deposits ( $\vartheta(i) = 0$ ) "shadow banks" and banks with insured deposits ( $\vartheta(i) > 0$ ) "commercial banks". The sets of commercial banks and shadow banks respectively set up by the planner are denoted by  $\mathcal{I}_{CB}, \mathcal{I}_{SB} \subseteq \mathcal{I}$ . The (relative) sizes of the two sectors are given by  $D(\mathcal{I}_{CB})$  and  $D(\mathcal{I}_{SB})$ , with  $D(\mathcal{I}_{CB}) + D(\mathcal{I}_{SB}) = 1$ . Without loss of generality, I assume the planner sets the share of insured deposits at all commercial banks to the same level  $\vartheta_{CB} \geq \vartheta^{SR}$ .<sup>10</sup> Since insuring more deposits entails no social cost, it is always optimal to insure the maximum possible amount of deposits, that is:

$$\underbrace{D(\mathcal{I}_{CB})}_{\substack{\text{total} \\ \text{deposits} \\ \text{at CBs}}} \vartheta_{CB} = \theta \quad (6)$$

total face value of insured deposits

By lemma 4.1, commercial banks are not susceptible to systemic runs in the planner's allocation. This implies that the share of insured deposits at commercial banks  $\vartheta_{CB}$  must be above a certain threshold:

**Lemma 4.2.**  $\vartheta_{CB} \geq \vartheta^{SR}$  is equivalent to  $\vartheta_{CB} \geq 1 - \Lambda^S$

<sup>10</sup>The continuum of commercial banks can also be regarded as one representative commercial bank.

*Proof.* First I show that  $\vartheta_{CB} \geq \vartheta^{SR} \Rightarrow \vartheta_{CB} \geq 1 - \Lambda^S$ . Suppose  $\vartheta_{CB} \geq \vartheta^{SR}$  and  $\vartheta_{CB} < 1 - \Lambda^S$ . In a hypothetical situation in which all banks liquidate their portfolios, we have that  $\Lambda^D = 1$  so that the liquidation price equals  $P(1) = \Lambda^S$  (see 1). By (2), commercial banks are susceptible to runs in such a situation if  $\vartheta_{CB} < 1 - \Lambda^S$ . Since shadow banks are always susceptible to runs if commercial banks are susceptible to runs (see 2) this implies that there is a systemic run equilibrium that affect all banks if  $\vartheta_{CB} < 1 - \Lambda^S$ , which is a contradiction to  $\vartheta_{CB} \geq \vartheta^{SR}$ . Next, since the liquidation price cannot fall below  $P(1) = \Lambda^S$ , it follows from (2) that commercial banks are never susceptible to systemic runs if  $\vartheta_{CB} \geq 1 - \Lambda^S$ . Hence  $\vartheta_{CB} \geq 1 - \Lambda^S \Rightarrow \vartheta_{CB} \geq \vartheta^{SR}$  which completes the proof. ■

Combining (6) with lemma 4.2 and substituting  $D(\mathcal{I}_{CB}) = 1 - D(\mathcal{I}_{SB})$  yields the following condition on the relative size of the shadow banking sector in the planner's allocation:

$$D(\mathcal{I}_{SB}) \geq \max \left\{ 1 - \frac{\theta}{1 - \Lambda^S}, 0 \right\} \equiv D_{SB}^{min}(\theta) \quad (7)$$

Condition (7) says that the shadow banking sector must be large enough to absorb enough of the uninsurable deposits from the commercial banking sector so as to keep the commercial banking sector shielded from systemic runs. By condition (2) and expression (3), shadow banks are susceptible to systemic runs iff  $P^{SR} < 1$ . Given that commercial banks are not susceptible to systemic runs, this is the case iff the size of the shadow banking sector satisfies  $D(\mathcal{I}_{SB}) > \Lambda^S = D_{SB}^{max}$ , where  $D_{SB}^{max}$  denotes the maximum relative size of the shadow banking sector at which the shadow banking sector is just not susceptible to systemic runs. We conclude this section with the following proposition:

**Proposition 4.1.** *The relative size of the shadow banking sector  $D(\mathcal{I}_{SB})$  in the planner's allocation depends on the cap  $\theta$  as follows:*

- i) For  $\theta \in [(1 - \Lambda^S), 1]$ , the planner sets  $D(\mathcal{I}_{SB}) \in [0, D_{SB}^{max}]$
- ii) For  $\theta \in [(1 - \Lambda^S)^2, (1 - \Lambda^S)]$ , the planner sets  $D(\mathcal{I}_{SB}) \in [D_{SB}^{min}(\theta), D_{SB}^{max}]$
- iii) For  $\theta \in [0, (1 - \Lambda^S)^2]$ , the planner sets  $D(\mathcal{I}_{SB}) = D_{SB}^{min}(\theta)$  is optimal.

The cap  $\theta$  can be divided into three regions: Consider first region i), in which the cap is at a relatively high level. From condition (7), we get that  $D_{SB}^{min}(\theta) = 0$ , meaning that commercial banks are not susceptible to systemic runs even if all uninsurable deposits

remain in the commercial banking sector. Systemic runs do not occur in the economy as long as the size of the shadow banking sector is within  $[0, D_{SB}^{max}]$ .<sup>11</sup>

Consider next the case where the cap is within the intermediate region ii). For  $\theta < (1 - \Lambda^S)$  we have that  $D_{SB}^{min}(\theta) = 1 - \frac{\theta}{1 - \Lambda^S} > 0$ , that is, the relative size of the shadow banking sector must be larger than zero in order to absorb enough uninsurable deposits from the commercial banking sector. At the other hand, we have that  $D_{SB}^{min}(\theta) \leq D_{SB}^{max}$ , with strict inequality for  $\theta > (1 - \Lambda^S)^2$ . This means that the planner can avoid systemic runs in both the commercial banking- and the shadow banking sector by setting the relative size of the shadow banking within  $[D_{SB}^{min}(\theta), D_{SB}^{max}]$ . Hence, if the cap is within region ii), the planner can avoid systemic runs in the economy by setting the shadow banking sector large enough to keep commercial banks shielded from systemic runs, but not too large, so that the shadow banking sector itself is not susceptible to systemic runs either.

Lastly, consider region iii), in which the cap is at a relatively low level. For  $\theta < (1 - \Lambda^S)^2$ , we have that  $D_{SB}^{min}(\theta) > D_{SB}^{max}$ . This means that the smallest size of the shadow banking sector at which the commercial banking sector is not susceptible to systemic runs is such that shadow banks are susceptible to systemic runs. At this level of the cap it is not feasible to avoid systemic runs altogether. The planner minimizes the scope of systemic runs by setting the shadow banking to the smallest size necessary to absorb enough uninsurable deposits from the commercial banking sector. To put it differently, the share of uninsured deposits in the commercial banking sector is set to the highest level at which commercial banks are just not susceptible to systemic runs.

The formal proof of proposition 4.1 is omitted. Figure 4 illustrates how the optimal size of the shadow banking sector (green line/area) depends on the cap  $\theta$ , for an economy with  $\Lambda^S = 0.25$ . The dotted line is the 45°-line. Note that if  $1 - \theta = 1$ , then the relative size of the shadow banking sector equals one by definition.

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<sup>11</sup>Within this region of the cap, we have that  $1 - \theta < D_{SB}^{max}$ , which means that the shadow banking sector is not susceptible to systemic runs even if it is larger than the share of uninsurable deposits. Hence constraint (6) may be slack in the planner's allocation within this region of  $\theta$ .



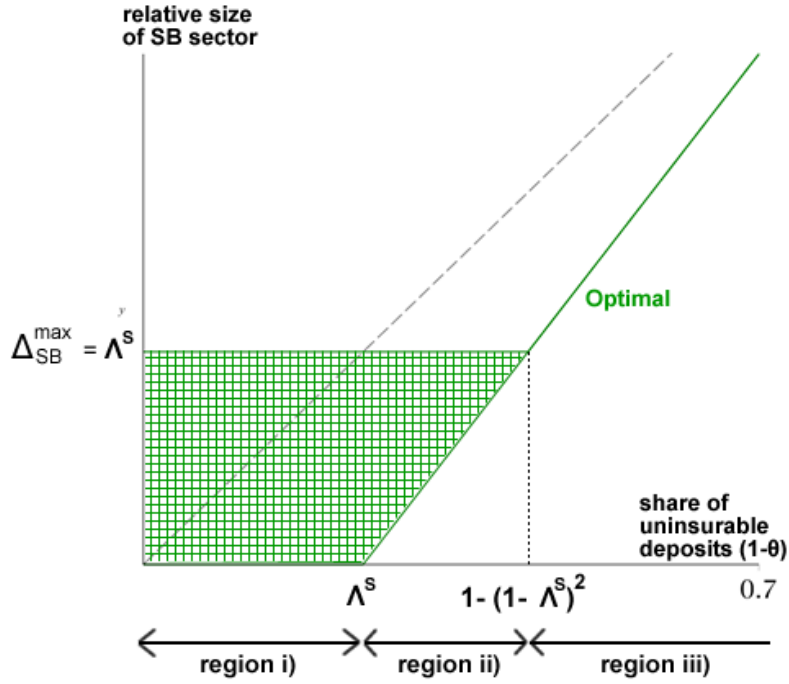


Figure 4: Optimal relative size of the shadow banking sector.

## 5 Competitive Equilibrium

In a competitive allocation, banks sell deposits to households at  $t=0$  and households choose for which deposits to obtain deposit insurance. Households take aggregate financial stability (captured by the liquidation price  $P^{SR}$ ), payments to deposit insurance, as well as the share of insured deposits  $\vartheta(i)$  at different banks as given when making their investment decision.<sup>12</sup> In the baseline version of the model, deposit insurance can be obtained for deposits issued by all banks and obtaining deposit insurance entails no fee for households. Each bank  $i$  offers deposit contracts allowing households to withdraw some fixed amount  $R(i)$  at  $t=1$  per unit of good invested into the bank. Insured deposits held at bank  $i$  yield

<sup>12</sup>The first two result endogenously from the fact that each household is a negligible part of the entire economy. While it is in principle possible that an individual bank serves only a finite number of households, there are many more households than banks. In any symmetric allocation, each bank serves a continuum of households so that each individual household makes up a negligible part of any bank's clientele.

a riskless return of  $R(i)$ . Define:

$$\varphi(\vartheta(i)) \equiv \min \left\{ \overbrace{\frac{P^{SR}}{1 - \vartheta(i)}}^{\text{liquidation price}}, 1 \right\} \quad (8)$$

share of uninsured deposits

If bank  $i$  is susceptible to runs, then  $\varphi(\vartheta(i)) < 1$  equals the share of uninsured deposits (in terms of the face value) that bank  $i$  can serve in a systemic run. The higher the share of insured deposits at the bank ( $\vartheta(i)$ ), the smaller is the share of deposits that are actually withdrawn in a run. Hence whenever a bank  $i$  is susceptible to systemic runs, the probability that an individual depositor can withdraw her uninsured deposits from the bank is increasing in  $\vartheta(i)$ . If bank  $i$  is not susceptible to runs, then  $\varphi(\vartheta(i)) = 1$  (see also condition 2).

Denote  $\delta^h(i)$  as the amount invested into bank  $i$  by household  $h$  and  $\mu^h(i) \in [0, 1]$  as the share of deposits bought from bank  $i$  for which household  $h$  obtains deposit insurance. Households' investment choice is given by two measurable functions  $\delta^h(i), \mu^h(i) : \mathcal{I} \rightarrow \mathbb{R}^+$ . In what follows, attention is restricted to symmetric equilibria in which all households make the same investment choice  $[\delta^h(i), \mu^h(i)] = [\delta(i), \mu(i)]$ . For future reference, and analogous to section 4, we denote  $D(\mathcal{I}') \equiv \int_{\mathcal{I}'} \delta(i) R(i) di$  as the face value of deposits held by each household at a given subset  $\mathcal{I}' \subseteq \mathcal{I}$  of banks.

Households' decision making at  $t=0$  is based on the assumption that some version of the law of large numbers holds in the continuum that allows households to diversify away any idiosyncratic risk regarding the order of the line at individual banks in case of a systemic run by investing in a continuum of different banks. Hence given that households invest in a continuum of banks, their portfolio choice is based on the expected return of the given portfolio of deposits in state of nature  $\xi \in \{0, 1\}$ .<sup>13</sup> Payments to deposit insurance

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<sup>13</sup> Intuitively, the return received from a portfolio of uninsured deposits in a systemic run by an individual household  $h$  should be given by  $\int_{\mathcal{I}} I(i)^h (1 - \mu(i)) \delta(i) R(i) di$  where  $I(i)^h$  is random and takes the value 1 if household  $h$  is early in the line at bank  $i$  and can withdraw her deposits and 0 if household  $h$  is late in the line at bank  $i$ . However, since the order of the line is random and independent at each bank, the sample path  $I(i)^h$  might not be measurable even if all banks are ex ante identical. I will not review the literature addressing the "measurability problem" of continuum models with idiosyncratic risk here. The approach taken here allows to keep internal consistency and is loosely related to Al-Najjar (1995).

in state  $\xi$  are denoted by  $T(\xi)$ . The expressions for consumption  $c_1(\xi)$  in state of nature  $\xi$  resulting from a given investment choice are then given by:

$$\begin{aligned}
\underbrace{c_1(0)}_{\text{consumption if no run}} &= \underbrace{\int_{\mathcal{I}} \delta(i) R(i) di}_{\text{total face value of deposits}} - \underbrace{T(0)}_{=0} \\
\underbrace{c_1(1)}_{\text{consumption if run}} &= \int_{\mathcal{I}} \left[ \underbrace{\mu(i)}_{\text{insured deposits at bank } i} + \underbrace{(1 - \mu(i)) \vartheta(i)}_{\text{uninsured deposits at bank } i \text{ that can be withdrawn in run}} \right] \delta(i) R(i) di - T(1)
\end{aligned} \tag{9}$$

A higher total face value of deposits increases consumption in every state of nature which implies that households' budget constraint at  $t=0$  will be binding. Households' utility maximization problem is then given by:

$$\max_{[\delta(i), \mu(i)]} E_{\xi} \{u(c_1(\xi))\} \quad \text{subject to} \quad \underbrace{\int_{\mathcal{I}} \delta(i) di = 1}_{\text{budget constraint}} \quad \text{and} \quad \underbrace{\int_{\mathcal{I}} \mu(i) \delta(i) R(i) di \leq \theta}_{\text{cap on deposit insurance / total face value of insured deposits}} \tag{10}$$

The equilibrium share of insured deposits at bank  $i$  is given by:<sup>14</sup>

$$\vartheta(i) = \mu(i) \tag{11}$$

A (symmetric) competitive allocation of the economy is an allocation in which: (i) households' portfolio choice  $[\delta(i), \mu(i)]$  is such that it solves households' utility maximization problem (10), (ii) banks maximize expected profits over both periods and (iii) the share of insured deposits at banks ( $\vartheta(i)$ ) and aggregate financial stability ( $P^{SR}$ ) are consistent with households' choices (expressions 3 and 11).

Note first that deposits with a higher face value  $R(i)$  first-order stochastically dominate deposits with a lower face value. Perfect competition then implies that the return offered by all banks equals the real return to the storage technology, that is, all banks set  $R(i) = 1$  and make zero profits in equilibrium. I assume that all banks participate in the market if equilibrium profits are zero. Note further that uninsured deposits at banks with a higher share of insured deposits first-order stochastically dominate uninsured deposits at

<sup>14</sup>A bank  $i$  with no depositors ( $\delta(i) = 0$ ) is treated by households as if  $\vartheta(i) = 0$ .

banks with lower share of insured deposits whenever systemic runs occur in the economy (whenever  $P^{SR} < 1$ ). The reason is that, if a systemic run occurs, the probability that uninsured deposits can be withdrawn from a bank is increasing in the share of insured deposits  $\vartheta(i)$  at the bank (expression 8). If systemic runs do not occur ( $P^{SR} = 1$ ), then all deposits pay the same (riskless) return, independent of  $\vartheta(i)$ . It follows that, when choosing how to invest the uninsurable part of their endowment, households always weakly prefer banks with a higher share of insured deposits. If systemic runs occur in the economy, banks with a higher  $\vartheta(i)$  are strictly preferred.<sup>15</sup> Households never have an incentive to invest into "shadow banks" with no insured depositors. This leads us to the following proposition that will be stated without separate proof:

**Proposition 5.1.** *In the competitive economy without fee for deposit insurance:*

- i) Either all banks are susceptible to systemic runs or none are.*
- ii) If systemic runs occur, then  $\vartheta(i) = \theta$  for all banks  $i \in \mathcal{I}$ .*

Since households never have a strict incentive to invest into shadow banks, it is a trivial result that the size of the shadow banking sector is (weakly) smaller than in the planner's allocation. Since it is not feasible to avoid systemic runs if  $\theta < (1 - \Lambda^S)^2$  (see section 4), it follows from proposition 5.1 that the shadow banking sector is strictly smaller than the optimal size if  $\theta < (1 - \Lambda^S)^2$ . At this level of the cap, the planner would set up shadow banking sector prone to systemic runs. This does not constitute a competitive equilibrium of the economy because households have a private incentive to move uninsured (and uninsurable) deposits from the unstable shadow banking sector into the stable commercial banking sector, causing the commercial banking to become susceptible to systemic runs as well.<sup>16</sup> In the next section I study a version of the model where the deposit insurance agency charges a fee on deposits issued by banks with access to deposit insurance. This

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<sup>15</sup>Note also that it is always (weakly) optimal for households to hold the maximum possible amount in insured deposits, given that deposit insurance entails no fee. If systemic runs occur, then uninsured deposits are risky and it is strictly optimal to hold the maximum amount  $\theta$  in insured deposits.

<sup>16</sup>If the cap is within the region  $\theta \in [(1 - \Lambda^S)^2, (1 - \Lambda^S)]$ , then systemic runs do not occur in the economy if the shadow banking sector is at the "right size" (see section 4). If systemic runs do not occur ( $P^{SR} = 1$ ), all banks pay identical returns, independent of  $\vartheta(i)$ . This implies that a situation in which the shadow banking sector is "accidentally" at the right size so that systemic runs do not occur, constitutes a competitive equilibrium of the economy if  $\theta \in [(1 - \Lambda^S)^2, (1 - \Lambda)]$ . Since households never have a strict incentive to invest into shadow banks, it is questionable how plausible this equilibrium is. In any case, there is also an equilibrium in which shadow banks do not exist and systemic runs affect the entire financial system if  $\theta \in [(1 - \Lambda^S)^2, (1 - \Lambda^S)]$ .

creates an incentive for households to invest into shadow banks which do not have access to deposit insurance and whose deposits are therefore not subject to the fee.

## 6 Fee on Commercial Bank Deposits

The setting is now modified in the following way: Before posting deposit contracts, banks decide whether or not to get access to deposit insurance. If a bank decides not to get access to deposit insurance, it will be labelled a shadow bank and households cannot obtain deposit insurance for the demand deposits issued by the bank in question. Banks that decide to get access to deposit insurance will be labelled commercial banks. Households can, but do not have to, obtain deposit insurance for deposits issued by commercial banks. Deposit insurance charges a fee on all deposits issued by commercial banks, insured or uninsured. This set up is motivated by real-world institutional features and, consistent with the approach taken in this paper, the fee on commercial bank deposits is taken as an exogenous parameter.

The fee equals a fraction  $\tau$  of the face value of deposits and is charged directly on households after they have withdrawn the deposits from the bank. The fee revenue is rebated to households in a lump-sum fashion. The fee is set up purposefully in such a way that it has no other effect besides the effect on household's private incentives how to invest their endowment. This implies that the results regarding the optimal size of the shadow banking sector derived in section 4 are not affected by the fee.<sup>17</sup>

The definition of the competitive equilibrium in the economy with a fee on commercial bank deposits is analogous to section 5 except for the explicit distinction between commercial banks and shadow banks. It is assumed that a positive mass of banks enters each sector whenever profits in the two sectors are the same. Analogous to section 4, the subsets of banks operating as commercial banks and shadow banks are denoted by  $\mathcal{I}_{CB}$  and  $\mathcal{I}_{SB}$  respectively and the relative sizes of the two sectors are given by  $D(\mathcal{I}_{CB})$  and

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<sup>17</sup>In particular, the fee is set up such that it does not affect the face value of outstanding deposits at  $t=1$ . If the fee were charged at  $t=0$ , or if it was charged on banks rather than directly on households, the aggregate fee revenue would affect the aggregate face value of outstanding deposits at  $t=1$ , even if by very little. Among other things this would imply that the severity and scope of systemic runs depends on the fee revenue. This effect would be quantitatively negligible for any reasonable parametrization of the fee and it does not seem to have any meaningful interpretation.

$D(\mathcal{I}_{SB})$ . Perfect competition again implies that all banks set  $R(i) = 1$ . Insured deposits at commercial banks now pay a riskless return of  $(1-\tau)R(i) = (1-\tau)$ . Uninsured deposits at commercial banks pay a return  $(1-\tau)$  if they can be withdrawn. In a systemic run, a given commercial bank  $i \in \mathcal{I}_{CB}$  can pay out a fraction  $\varphi(\vartheta(i))$  of the uninsured deposits (see 8). Shadow bank deposits pay a return  $R(i) = 1$  if they can be withdrawn from the bank. In a systemic run, any given shadow bank can pay out a fraction  $\varphi(0)$  of deposits. When choosing how to invest uninsured deposits, households trade off the fee  $\tau$  charged on commercial bank deposits against higher losses caused by run at shadow banks due to the fact that shadow banks do not have insured depositors among their creditors. We then immediately get the following result:

**Lemma 6.1.** *In the economy with a fee on commercial bank deposits, there is no equilibrium with stable shadow banking.*

*Proof.* Suppose there is an equilibrium with stable shadow banks, that is, with shadow banks that are not susceptible to systemic runs ( $\vartheta(0) = 1$ ). Since shadow bank deposits do not entail the fee  $\tau$ , they first-order stochastically dominate commercial bank deposits (both insured and uninsured). This means that households invest all endowment into shadow banks ( $D(\mathcal{I}_{SB}) = 1$ ). If all endowment is invested into shadow banks, shadow banks are susceptible to systemic runs, which follows from limited secondary market capacity ( $\Lambda^S < 1$ ). Hence we have a contradiction. ■

It follows from lemma 6.1 that we only need to consider the following two types of equilibria:

- i) Equilibria in which a stable commercial banking sector coexists with a shadow banking sector susceptible to systemic runs. (**Type A** equilibria).
- ii) Equilibria in which systemic runs affect all banks. (**Type B** equilibria).

Before proceeding, we add the following assumption about parameters:

**Assumption 6.1.**  $0 < \tau < (1 - \Lambda^S) \pi^r$

Assumption 6.1 puts an upper bound on the fee on commercial bank deposits. The upper bound on  $\tau$  is such that, in a situation of minimal financial stability in which systemic

runs affect the entire financial system (implying  $P^{SR} = P(1) = \Lambda^S$ ), the riskless return to insured commercial bank deposits is higher than the expected return to shadow bank deposits.<sup>18</sup> Assumption 6.1 hence implies that households prefer insured commercial bank deposits to shadow bank deposits if financial stability is at the lowest possible level.

### Type A equilibria

In a type A equilibrium only shadow banks are susceptible to systemic runs. Since commercial banks are not susceptible to systemic runs, commercial bank deposits (both insured and uninsured) pay a riskless return of  $R_{CB} = 1 - \tau$ . The amount of assets sold in a systemic run increases in the size of the shadow banking sector. The liquidation price of assets in a systemic run in a type A equilibrium equals  $P_A^{SR} = \frac{\Lambda^S}{D(\mathcal{I}_{SB})} < 1$  (see 1). Households' decision making is again based on the assumption that, by some version of the law of large numbers, idiosyncratic risk regarding the order in the line at individual banks in case of a systemic run is diversified away by investing in a continuum of shadow banks. Denote  $\tilde{R}_{SB,A}(D(\mathcal{I}_{SB}), \xi)$  as the effective return to a portfolio of shadow bank deposits in a type A equilibrium in state of nature  $\xi$ , given that the (relative) size of the shadow banking sector equals  $D(\mathcal{I}_{SB})$ . We have:

$$\tilde{R}_{SB,A}(D(\mathcal{I}_{SB}), \xi) = \begin{cases} 1 & \text{if } \xi = 0 \text{ (no run)} \\ \frac{\Lambda^S}{D(\mathcal{I}_{SB})} = P_A^{SR} & \text{if } \xi = 1 \text{ (run)} \end{cases} \quad (12)$$

Denote  $\alpha \in [0, 1]$  as the share of the endowment invested into the shadow banking sector by an individual household. Expected utility is continuous, as well as strictly concave in  $\alpha$  (see appendix C). By the maximum theorem, households' optimal choice, denoted  $\alpha_A^{opt}$ , is a continuous function of the size of the shadow banking sector  $D(\mathcal{I}_{SB})$ . We can also note that  $\alpha_A^{opt}$  must be decreasing in  $D(\mathcal{I}_{SB})$ . A larger size of the shadow banking sector makes shadow bank deposits less attractive in the sense that shadow bank deposits with a smaller shadow banking sector first-order stochastically dominate shadow bank deposits with a larger shadow banking sector. The only other effect of an increase in  $D(\mathcal{I}_{SB})$  from the point of view of an individual household is that the fee revenue rebated

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<sup>18</sup>In a systemic run, shadow banks can pay out a fraction  $\varphi(0) = \min\{P^{SR}, 1\}$  of deposits. The probability of a systemic run equals  $\pi^r$ . Hence the ex ante expected return to shadow bank deposits given that  $P^{SR} = \Lambda^S$  equals  $(1 - \pi^r) + \pi^r \Lambda^S$ . Assumption 6.1 can be rewritten as  $1 - \tau > (1 - \pi^r) + \pi^r \Lambda^S$ .

by the deposit insurance agency decreases as a result of the smaller size of the commercial banking sector, which affects consumption in all states identically.<sup>19</sup>

**Lemma 6.2.** *There exists a threshold  $\underline{D}_{SB} \geq \frac{\Lambda^S}{1-\tau}$  so that households' optimal choice is  $\alpha_A^{opt} = 1$  (all endowment invested into shadow banks) if and only if  $D(\mathcal{I}_{SB}) \leq \underline{D}_{SB}$ , and there exists a threshold  $\overline{D}_{SB} \leq 1$  so that households' optimal choice is  $\alpha_A^{opt} = 0$  (all endowment invested into commercial banks) if and only if  $D(\mathcal{I}_{SB}) \geq \overline{D}_{SB}$ .*

The fact that shadow banks are susceptible to systemic runs implies that  $D(\mathcal{I}_{SB}) > \Lambda^S$  in any type A equilibrium. However, as  $D(\mathcal{I}_{SB})$  approaches  $\Lambda^S$  from above, losses caused by runs on shadow banks go to zero (see 12). By the same reasoning as in lemma 6.1 this implies that it is optimal to invest only in shadow banks ( $\alpha_A^{opt} = 1$ ) if the relative size of the shadow banking sector is higher, but very close to,  $\Lambda^S$ . At the other hand, assumption 6.1 implies that it is optimal to invest only in commercial banks ( $\alpha_A^{opt} = 0$ ) as the relative size of the commercial banking sector approaches one. The full proof of lemma 6.2 is given in appendix A. To summarize, households' optimal choice, given that the economy is in a type A equilibrium, satisfies:

$$\alpha_A^{opt}(D(\mathcal{I}_{SB})) = \begin{cases} 1 & \text{if } D(\mathcal{I}_{SB}) \leq \underline{D}_{SB} \\ \in (0, 1) & \text{and decreasing in } D(\mathcal{I}_{SB}) \text{ if } D(\mathcal{I}_{SB}) \in [\underline{D}_{SB}, \overline{D}_{SB}] \\ 0 & \text{if } D(\mathcal{I}_{SB}) \geq \overline{D}_{SB} \end{cases} \quad (13)$$

Market clearing means that  $D(\mathcal{I}_{SB}) = \alpha_A^{opt}$  in equilibrium. Hence we can express  $\alpha_A^{opt}(\overbrace{D(\mathcal{I}_{SB})}^{\alpha})$  as a continuous and decreasing function mapping  $[0, 1]$  onto itself. It follows that there is a unique fixed point  $D_{SB,A}^* = \alpha_A^{opt}(D_{SB,A}^*)$  and  $D_{SB,A}^*$  is the unique candidate for a type A equilibrium. Intuitively, in a type A equilibrium, the size of the shadow banking sector is such that households are indifferent at the margin between investing into the shadow banking sector which is prone to systemic runs and paying the fee  $\tau$  on commercial bank deposits. Since  $\alpha_A^{opt}(0) = 1$  and  $\alpha_A^{opt}(1) = 0$ , corner solutions are ruled out and we have  $D_{SB,A}^* \in (\underline{D}_{SB}, \overline{D}_{SB})$ . It remains to check whether, at  $D(\mathcal{I}_{SB}) = D_{SB,A}^*$ , the economy is indeed in a type A equilibrium as presumed in (13), that is, shadow banks but not commercial banks are susceptible to systemic runs. It follows from lemma 4.2 that commercial

<sup>19</sup>Since systemic runs only encompass the shadow banking sector, deposit insurance never needs to make payments in a type A equilibrium.



banks are *not* susceptible to systemic runs if and only if the share of insured deposits at commercial banks satisfies  $\vartheta_{CB} \geq 1 - \Lambda^S$ . We have that:

$$\vartheta_{CB} \leq \frac{\theta}{D(\mathcal{I}_{CB})} = \frac{\text{total insurable deposits}}{\text{total endowment collected by commercial banks}} \quad (14)$$

Inserting  $D(\mathcal{I}_{CB}) = 1 - D(\mathcal{I}_{SB})$  into condition 14, we get that, given that the relative size of the shadow banking sector equals  $D(\mathcal{I}_{SB}) = D_{SB,A}^*$ , a stable commercial banking sector is feasible if and only if the cap on deposit insurance satisfies:

$$\theta \geq (1 - \Lambda^S)(1 - D_{SB,A}^*) = \underline{\theta}_A \quad (15)$$

It follows that  $D(\mathcal{I}_{SB}) = D_{SB,A}^*$  constitutes a type A equilibrium if and only if the cap satisfies  $\theta \geq \underline{\theta}_A$ . Intuitively, if  $\theta < \underline{\theta}_A$ , then the economy does not exhibit a type A equilibrium because the cap is too low to allow for a stable commercial banking sector. Since  $D_{SB,A}^* \in (\underline{D}_{SB}, \overline{D}_{SB})$  we have that  $\underline{\theta}_A \in \left(0, (1 - \Lambda^S) \left(1 - \frac{\Lambda^S}{1 - \tau}\right)\right)$ . The preceding discussion leads us to the following proposition which will be stated without separate proof:

**Proposition 6.1.** *The economy exhibits a (unique) type A equilibrium if and only if the cap on deposit insurance satisfies  $\theta \geq \underline{\theta}_A$ , with  $\underline{\theta}_A \in \left(0, (1 - \Lambda^S) \left(1 - \frac{\Lambda^S}{1 - \tau}\right)\right)$ .*

The comparative statics regarding the relative size of the shadow banking sector in the type A equilibrium ( $D_{SB,A}^*$ ) are rather straightforward: All else equal,  $D_{SB,A}^*$  increases in the fee on commercial bank deposits ( $\tau$ ) and secondary market capacity ( $\Lambda^S$ ) and decreases in the probability of systemic runs ( $\pi^r$ ) and the degree of households' risk aversion. I refer to appendix C for a derivation of the comparative statics results.

### Type B equilibria

In a type B equilibrium, all banks are susceptible to systemic runs. This means that the total fundamental value of assets sold in systemic runs equals  $\Lambda^D = 1$  and the liquidation price of assets in a systemic run equals  $P_B^{SR} = P(1) = \Lambda^S$  (see 1). Note that  $P_B^{SR} < P_A^{SR}$  since the extent of systemic runs is larger in a type B equilibrium compared to a type A equilibrium. Different to a type A equilibrium, the magnitude of systemic runs does not depend on the size of the shadow banking sector.

In a type B equilibrium, uninsured deposits at commercial banks with higher share of insured deposits  $\vartheta(i)$  first-order stochastically dominate uninsured deposits at commercial banks with lower  $\vartheta(i)$ . The reason is that uninsured deposits held at commercial banks with higher  $\vartheta(i)$  can be withdrawn with higher probability in case of a run while the returns are otherwise identical. It follows that all commercial banks have an identical share of insured deposits in a type B equilibrium, denoted by  $\vartheta_{CB}$ . Denote  $\tilde{R}_{CB,B}(\vartheta_{CB}, \xi)$  as the effective return to a portfolio of uninsured commercial bank deposits in a type B equilibrium in state of nature  $\xi$ , given that the share of insured deposits in the commercial banking sector equals  $\vartheta_{CB}$ . We have:

$$\tilde{R}_{CB,B}(\vartheta_{CB}, \xi) = \begin{cases} 1 - \tau & \text{if } \xi = 0 \text{ (no run)} \\ \underbrace{\frac{\Lambda^S}{1 - \vartheta_{CB}}}_{\varphi(\vartheta_{CB})} (1 - \tau) & \text{if } \xi = 1 \text{ (run)} \end{cases} \quad (16)$$

The effective return to a portfolio of shadow bank deposits in a type B equilibrium is given by:

$$\tilde{R}_{SB,B}(\xi) = \begin{cases} 1 & \text{if } \xi = 0 \text{ (no run)} \\ \underbrace{\Lambda^S}_{\varphi(0)} & \text{if } \xi = 1 \text{ (run)} \end{cases} \quad (17)$$

Insured commercial bank deposits pay a riskless return of  $1 - \tau$ . The upper bound on  $\tau$  (assumption 6.1) means that, in a situation in which systemic runs affect the entire financial system, insured commercial bank deposits are preferred to uninsured deposits at any bank. Hence, different to a type A equilibrium, in a type B equilibrium households will always hold the maximum possible amount ( $\theta$ ) in insured commercial bank deposits. This implies that condition 14 is binding in any type B equilibrium. The relevant choice of households is how to allocate the uninsurable part of their investment between commercial banks and shadow banks. Denote the share of a households' uninsurable endowment invested into shadow banks by  $\tilde{\alpha} \in [0, 1]$ . Households' expected utility is continuous, as well as strictly concave in  $\tilde{\alpha}$  (see appendix D). By the maximum theorem, households' optimal choice, denoted  $\tilde{\alpha}_B^{opt}$ , is a continuous function of  $\vartheta_{CB}$ .

If the share of insured deposits in the commercial banking sector ( $\vartheta_{CB}$ ) increases, uninsured commercial bank deposits become more attractive in the sense that uninsured commercial bank deposits under higher  $\vartheta_{CB}$  first-order stochastically dominate uninsured

commercial bank deposits under lower  $\vartheta_{CB}$ . It follows that households' optimal investment into the shadow banking sector ( $\tilde{\alpha}_B^{opt}$ ) is decreasing in  $\vartheta_{CB}$ .

**Lemma 6.3.** *There exists a  $\underline{\vartheta} \geq \tau$  so that households choose  $\tilde{\alpha}_B^{opt} = 1$  (all uninsured deposits held at shadow banks) if and only if  $\vartheta_{CB} \leq \underline{\vartheta}$ , and there exists an  $\bar{\vartheta} \leq 1 - \Lambda^S$  so that households choose  $\tilde{\alpha}_B^{opt} = 0$  (all uninsured deposits held at commercial banks) if and only if  $\vartheta_{CB} \geq \bar{\vartheta}$ .*

The fact that commercial banks are susceptible to systemic runs implies that  $\vartheta_{CB} < 1 - \Lambda^S$  in any type B equilibrium (lemma 4.2). However, as  $\vartheta_{CB}$  approaches  $1 - \Lambda^S$  from below, actual losses caused by runs for uninsured depositors at commercial banks go to zero (16). For  $\vartheta_{CB}$  below, but very close to  $1 - \Lambda^S$ , it is optimal to hold all uninsured deposits at commercial banks ( $\tilde{\alpha}_B^{opt} = 1$ ). At the other hand, in the limit as  $\vartheta_{CB}$  approaches zero, losses caused by runs at commercial banks are the same as on shadow banks, while commercial bank deposits entail the fee  $\tau$ . For  $\vartheta_{CB}$  close to zero, it is optimal to hold all uninsured deposits at shadow banks ( $\tilde{\alpha}_B^{opt} = 0$ ). The full proof of lemma 6.3 is given in appendix B. To summarize, households' optimal choice, given that the economy is in a type B equilibrium, satisfies:

$$\tilde{\alpha}_B^{opt}(\vartheta_{CB}) = \begin{cases} 1 & \text{if } \vartheta_{CB} \leq \underline{\vartheta} \\ \in (0, 1) \text{ and } \begin{matrix} \text{continuous} \\ \text{and decreasing} \\ \text{in } \vartheta_{CB} \end{matrix} & \text{if } \vartheta_{CB} \in [\underline{\vartheta}, \bar{\vartheta}] \\ 0 & \text{if } \vartheta_{CB} \geq \bar{\vartheta} \end{cases} \quad (18)$$

Market clearing means that  $D(\mathcal{I}_{SB}) = (1 - \theta)\tilde{\alpha}$  in equilibrium. By setting expression (14) to equality and inserting  $D(\mathcal{I}_{SB}) = 1 - D(\mathcal{I}_{CB})$ , we can express the share of insured deposits at commercial banks as an increasing function of households' investment into shadow banks:  $\vartheta_{CB}(\tilde{\alpha}) = \frac{\theta}{1 - (1 - \theta)\tilde{\alpha}}$ . Since shadow banks absorb uninsurable deposits from the commercial banking sector, the share of insured deposits at commercial banks is increasing in the size of the shadow banking sector. Hence a larger shadow banking sector implies lower losses caused by runs on commercial banks and therefore increases the relative attractiveness of uninsured commercial bank deposits compared to shadow bank deposits. Similar to the type A equilibrium, we can therefore conclude that optimal investment into shadow banks  $\tilde{\alpha}_B^{opt}$  must be decreasing in the size of the shadow banking

sector.<sup>20</sup> It follows that we can express  $\tilde{\alpha}_B^{opt}(\tilde{\alpha})$  as a continuous and decreasing function mapping  $[0, 1]$  into itself. This means that there is a unique fixed point  $\tilde{\alpha}_B^* = \tilde{\alpha}_B^{opt}(\tilde{\alpha}_B^*)$  which is the only candidate for a type B equilibrium. The size of the shadow banking sector in the unique candidate for a type B equilibrium is given by  $D_{SB,B}^* = (1 - \theta)\tilde{\alpha}_B^*$ . It then remains to check whether, at  $D(\mathcal{I}_{SB}) = D_{SB,B}^*$ , the economy is indeed in a type B equilibrium, that is, in a situation in which all banks are susceptible to systemic runs as has been presumed in (18). This is the case if and only if the share of insured deposits at commercial banks satisfies  $\vartheta_{CB}(\tilde{\alpha}_B^*) < 1 - \Lambda^S$  (see lemma 4.2). We get the following result:

**Proposition 6.2.** *The economy exhibits a (unique) type B equilibrium if and only if the cap satisfies  $\theta < 1 - \Lambda^S$ .*

*Proof.* As shown above, there is a unique candidate  $\tilde{\alpha}_B^*$  for a type B equilibrium for any given value of  $\theta$ . First I show that there cannot be a type B equilibrium if  $\theta \geq 1 - \Lambda^S$ . Suppose  $\theta \geq 1 - \Lambda^S$  and the economy is in a type B equilibrium. Then we have  $\vartheta_{CB}(\tilde{\alpha}) \geq \vartheta_{CB}(0) = \theta \geq 1 - \Lambda^S$ . By lemma 4.2, this implies that commercial banks are not susceptible to runs. Hence the economy is not in a type B equilibrium, which leads to a contradiction. Next, I show that there is a (unique) type B equilibrium if  $\theta < 1 - \Lambda^S$ . For this it needs to be shown that, if  $\theta < 1 - \Lambda^S$ , then  $\vartheta_{CB}(\tilde{\alpha}_B^*) < 1 - \Lambda^S$ . Suppose  $\theta < 1 - \Lambda^S$  and  $\vartheta_{CB}(\tilde{\alpha}_B^*) \geq 1 - \Lambda^S$ . Then, by (18), we have  $\tilde{\alpha}_B^{opt} = \tilde{\alpha}_B^* = 0$  and the share of insured deposits at commercial banks is given by  $\vartheta_{CB}(0) = \theta < 1 - \Lambda^S$  which leads to a contradiction. ■

If the cap on deposit insurance is relatively high ( $\theta \geq 1 - \Lambda^S$ ), then the aggregate share of uninsurable deposits is relatively low and there is no equilibrium in which systemic runs affect the entire financial system. Note the following: If all uninsured deposits are held at commercial banks ( $\tilde{\alpha} = 0$ ) then the share of insured deposits at commercial banks equals  $\vartheta_{CB}(0) = \theta$ . Now suppose we have  $\theta \in [\bar{\vartheta}, 1 - \Lambda^S)$ . Then it holds that  $\bar{\vartheta} \leq \vartheta_{CB}(0) < 1 - \Lambda^S$ . Hence if all endowment is invested into commercial banks, systemic runs affect the entire financial system. At the other hand, since  $\vartheta_{CB}(0) \geq \bar{\vartheta}$ , it is optimal for households to hold

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<sup>20</sup>As in the type A equilibrium, the only other effect of a larger shadow banking sector from the point of view of an individual household is that the fee revenue rebated from the deposit insurance agency decreases as a result of the smaller commercial banking sector. This affects consumption in all states identically. In case of a systemic run, the tax raised by deposit insurance equals the aggregate face value of insured deposits ( $\theta$ ), independent of the size of the shadow banking sector.

all deposits at commercial banks in this situation. It follows that the economy exhibits a type B equilibrium with only commercial banks and no shadow banks if  $\theta \in [\bar{\vartheta}, 1 - \Lambda^S)$ . Note that  $\bar{\vartheta}$  itself may change with  $\theta$ .

**Proposition 6.3.** *There exists a  $\hat{\theta}_B$ , with  $0 < \hat{\theta}_B < 1 - \Lambda^S$ , so that the economy exhibits a type B equilibrium with only commercial banks if and only if  $\theta \in [\hat{\theta}_B, 1 - \Lambda^S)$ .*

The proof of proposition 6.3 is given in appendix D.1. The interval  $[\hat{\theta}_B, 1 - \Lambda^S)$  seems to be quite large for most reasonable parametrizations. To understand the intuition behind proposition 6.3 it is useful to consider again the limit case as  $\theta$  approaches  $1 - \Lambda^S$  from below. In this case, if all endowment is invested into commercial banks, we have that  $\vartheta_{CB}(0) = \theta < 1 - \Lambda^S$ , which implies that systemic runs affect the entire financial system. However, private losses caused by runs for uninsured depositors at commercial banks become arbitrarily small as  $\theta$  (and therefore  $\vartheta_{CB}(0)$ ) approaches  $1 - \Lambda^S$  from below (16). Hence for  $\theta$  smaller but arbitrarily close to  $1 - \Lambda^S$  it is optimal for households to invest only in commercial banks, given that systemic runs affect the entire financial system. Intuitively, in a situation of low aggregate financial stability in which the entire financial system is prone to systemic runs, it can be privately optimal for households to hold all uninsured deposits at commercial banks rather than investing into even less stable shadow banks. This is only true if the cap on deposit insurance, and hence the share of insured deposits at commercial banks, is not all too low however. If  $\vartheta_{CB}(0) = \theta < \bar{\vartheta}$ , then the stability provided by the (now relatively low) share of insured deposits at commercial banks does not compensate for the fee on commercial banks anymore and households move part of their uninsurable deposits into shadow banks.

The model exhibits a discontinuity at  $\theta = 1 - \Lambda^S$ . By proposition 6.3 the economy exhibits a type B equilibrium with only commercial banks as  $\theta$  approaches  $1 - \Lambda^S$  from below. If  $\theta = 1 - \Lambda^S$  however, we have that  $\vartheta_{CB}(0) = 1 - \Lambda^S$ , which means that commercial banks are not susceptible to runs anymore if all endowment is invested into commercial banks (lemma 4.2). If systemic runs do not occur in the economy ( $P^{SR} = 1$ ) then shadow bank deposits first-order stochastically dominate commercial bank deposits (lemma 6.1), which implies that households invest part of their endowment into shadow banks, moving the economy to a type A equilibrium.

Note next that  $\underline{\theta}_A < 1 - \Lambda^S$ , which means that the economy exhibits both a type A and a type B equilibrium if the cap on deposit insurance is within  $\theta \in [\underline{\theta}_A, 1 - \Lambda^S)$ . Ex-

pected utility of households is higher in the type A equilibrium due to the smaller scope of systemic runs. Multiplicity of equilibria arises because households' optimal investment choice depends on aggregate financial stability (captured by the liquidation price  $P^{SR}$ ) which in turn depends on households' investment choices at  $t=0$ . In addition, changes in  $P^{SR}$  affect the effective return on shadow bank deposits differently than the effective return on uninsured commercial bank deposits. While changes in  $P^{SR}$  have a 1:1 effect on the return to shadow bank deposits in case of a systemic run, the effect on the return to uninsured commercial bank deposits is mitigated by the fact that insured depositors at commercial banks do not participate in runs. Roughly speaking, changes in aggregate financial stability  $P^{SR}$  affect riskiness of shadow bank deposits more strongly than riskiness of commercial bank deposits. To illustrate why the economy exhibits multiple equilibria for a certain range of the cap, suppose that  $\theta \in [\underline{\theta}_A, 1 - \Lambda^S)$  and all endowment is invested into commercial banks. Then  $\vartheta_{CB} = \theta < 1 - \Lambda^S$  and, by lemma 4.2, the commercial banking sector and therefore the entire financial system is prone to systemic runs. From proposition 6.3 we know that this situation may constitute a type B equilibrium of the economy.<sup>21</sup> Suppose now that, starting from an equilibrium with only commercial banks, a large part of the uninsured deposits is moved at once from the commercial banking sector into a newly created shadow banking sector. If enough uninsured deposits are moved into the shadow banking sector, the share of insured deposits at commercial banks ( $\vartheta_{CB}$ ) increases above  $1 - \Lambda^S$  so that the commercial banking sector is not prone to systemic runs anymore. This leads to a large increase in the liquidation price  $P^{SR}$ . The increase in aggregate financial stability lowers riskiness of both shadow bank deposits and uninsured commercial bank deposits. However, the effect is more pronounced for shadow bank deposits, which increases the relative attractiveness of shadow bank deposits compared to uninsured commercial bank deposits. Given that aggregate financial stability improved, it is now privately optimal for households to invest part of their endowment into shadow banks. Hence the new situation with a shadow banking sector and a smaller extent of systemic runs constitutes a competitive equilibrium of the economy as well.

Figure (5) illustrates how the equilibrium of the economy with a fee on commercial bank deposits depends on the deposit insurance cap  $\theta$ , according to propositions 6.1 and 6.2.

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<sup>21</sup> Whether this is true for the entire range  $[\underline{\theta}_A, 1 - \Lambda^S)$  depends on the position of  $\hat{\theta}_B$  relative to  $\underline{\theta}_A$ , which depends on parameters.

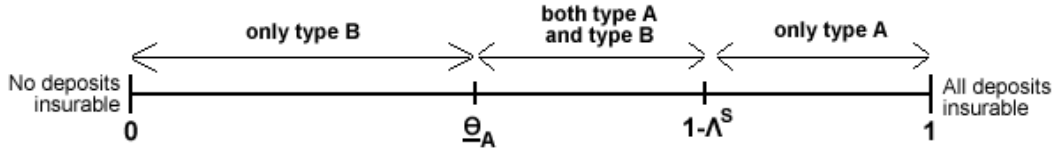


Figure 5: Equilibria of the economy depending on the cap  $\theta$ .

**Example 6.1.**  $\overbrace{u(c) = \ln(c)}^{\text{households' utility}}$ ,  $\overbrace{\Lambda^S = 0.25}^{\text{secondary market capacity}}$ ,  $\overbrace{\pi^r = 0.2}^{\text{probability of run}}$ ,  $\overbrace{\tau = 0.05}^{\text{fee on commercial bank deposits}}$ .<sup>22</sup>

Appendices C and D describe how to solve for the type A and type B equilibria. In example 6.1 we have that  $\underline{\theta}_A = 0.51$ , which means that the economy exhibits a type A equilibrium as long as the cap on deposit insurance satisfies  $\theta \geq 0.51$ . The relative size of the shadow banking sector in a type A equilibrium is given by  $D_{SB,A}^* = 0.33$ . For any  $\theta < 1 - \Lambda^S = 0.75$  the economy exhibits a type B equilibrium, which means that the economy exhibits multiple equilibria (type A and type B) if the cap is within  $\theta \in [0.51, 0.75)$ . We also get that  $\hat{\theta}_B = 0.21$  so that shadow banks do not exist in the type B equilibrium if  $\theta \in [0.21, 0.75)$ . Figure 6 shows the relative size of the shadow banking sector in the competitive equilibria compared to the the optimal size, for the economy of example 6.1. The dotted lines are the 45°-lines.

In general, if the cap on deposit insurance satisfies  $\theta \geq (1 - \Lambda^S)$ , then the type A equilibrium is the unique competitive equilibrium of the economy and the shadow banking sector is larger than the optimal size. If the cap is at such a high level, a social planner would set up a relatively small shadow banking sector, or no shadow banking sector at all (proposition 4.1). However, households have a private incentive to invest into shadow banks in order to avoid the fee imposed on commercial bank deposits. The shadow banking sector grows up to a size at which households are indifferent at the margin between investing in (stable) commercial banks and (unstable) shadow banks. We have the opposite situation if  $\theta < \underline{\theta}_A$ . If the cap is at such a low level, then the type B equilibrium is the unique equilibrium of the economy and the shadow banking sector is smaller than the optimal size. At this level of the cap, a social planner would set up a shadow banking sector that is larger than in a hypothetical type A equilibrium, with relatively large losses caused by

<sup>22</sup>I do not attempt to calibrate the model. One might argue that a more realistic parametrization would involve a lower probability of a run and a higher degree of risk aversion. This would lead to quantitatively similar results.

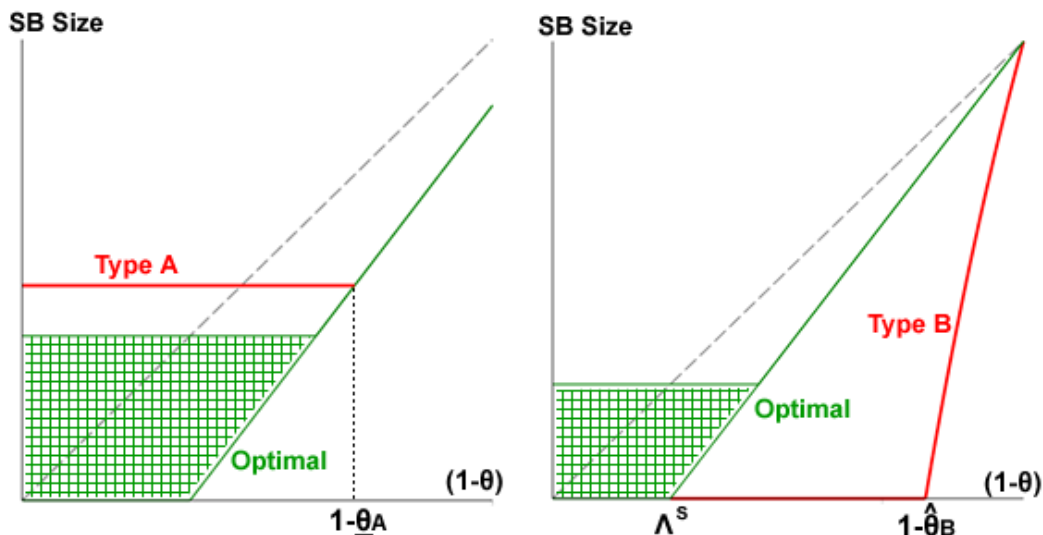


Figure 6: Size of shadow banking sector in competitive equilibrium vs. optimal size.

systemic runs in the shadow banking sector. The commercial banking sector is not prone to systemic runs in the planner's allocation. This allocation does not constitute a competitive equilibrium because households would have a private incentive to move uninsured deposits from the unstable shadow banking sector into the stable commercial banking sector. If the cap is within the intermediate region  $\theta \in [\underline{\theta}_A, 1 - \Lambda^S)$ , the shadow banking sector may be larger or smaller than the optimal size, depending on which equilibrium (type A or type B) is selected in the competitive allocation.<sup>23</sup>

In this section it has been assumed that the deposit insurance agency charges a fee on all commercial bank deposits, insured or uninsured, which seems consistent with current policy. Consider now an alternative situation where all intermediaries have access to deposit insurance and the fee is only charged on insured deposits. A result equivalent to lemma 6.1 would still hold, implying that systemic runs occur in every competitive equilibrium. The reason is that households only have an incentive to pay the fee on insured deposits if systemic runs make an investment into uninsured deposits risky. In addition, given that systemic runs occur, households have an incentive to hold uninsured deposits at these intermediaries with the highest share of insured deposits (see the discussion in section 5). Hence the only equilibrium of this economy is an equilibrium where all banks have an identical share of insured deposits and systemic runs affect the entire financial

<sup>23</sup>In the special case where  $\theta = \underline{\theta}_A$  and the economy is in a type A equilibrium, the size of the shadow banking sector corresponds to the optimal size.



system. The equilibrium of this economy essentially corresponds to item ii) in proposition 5.1 except that the share of insured deposits at the representative commercial bank is endogenous and is such that households are indifferent at the margin between paying the fee for deposit insurance and bearing losses caused by runs on the representative bank.

For any given level of the cap on deposit insurance, the optimal size of the shadow banking sector can be implemented in the competitive equilibrium with a two-pronged policy: First, the share of uninsured deposits issued by banks that issue insured deposits ("commercial banks") must be limited to  $1 - \Lambda^S$ , for instance by charging a marginal tax rate of plus infinity on uninsured deposits issued by any commercial bank whose share of uninsured deposits has reached  $1 - \Lambda^S$ . This prevents the sector of the financial system issuing insured deposits from becoming susceptible to systemic runs and ensures that the shadow banking sector is not too small relative to the optimal size. At the other hand, the incentive effects of fees levied on insured deposits by the deposit insurance agency should be offset, for instance by levying an equivalent tax on shadow bank deposits. (As well as on uninsured commercial bank deposits if this is not already done by the deposit insurance agency). This eliminates households' incentive to invest into uninsured (shadow bank-) deposits rather than insured deposits in order to avoid the fee on deposit insurance and prevents the shadow banking sector from growing too large relative to the optimal size.

## 7 Discussion

This paper shows that it is not only the total amount of outstanding short-term claims that matters for financial stability but also the way how insured and uninsured short-term claims are distributed across banks. If there is significant demand for short-term claims by investors with large endowments relative to the cap on deposit insurance, the presence of a shadow banking sector that issues short-term claims which are not protected by deposit insurance may be beneficial from a financial stability point of view. One of the main conclusions of this paper is that, in the context of limited deposit insurance, policies aimed at curtailing the shadow banking sector should be viewed with caution. This is especially true if such policies lead to a flow of uninsured deposits into the commercial banking sector. It should be noted that these results are derived in a highly simplified environment with no lender-of-last-resort, no implicit bail-out guarantees to commercial

banks and no sponsoring of shadow banks by commercial banks, to name just a few of the abstractions. In addition, the underlying causes for the demand for short-term claims by investors with large endowments are not addressed. While these limitations must be kept in mind, this paper does shed light on important aspects of shadow banking in the context of limited deposit insurance that have not been analyzed so far.

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# Appendix

## A Proof of lemma 6.2

If  $D(\mathcal{I}_{SB}) \leq \frac{\Lambda^S}{1-\tau}$ , then we have that  $\tilde{R}_{SB,A}(D(\mathcal{I}_{SB}), 1) \geq (1-\tau)$ . Hence commercial banks do not pay a higher effective return in case of a systemic run compared to shadow banks. Since shadow banks pay a higher return if no run takes place, shadow bank deposits first-order stochastically dominate commercial bank deposits, which implies that households' optimal choice is to invest only in shadow banks ( $\alpha_A^{opt} = 1$ ). Suppose next that  $D(\mathcal{I}_{SB}) = 1$ . The expected return to shadow bank deposits is then given by  $E_\xi\{\tilde{R}_{SB,A}(D(\mathcal{I}_{SB}), \xi)\} = \pi^r \Lambda^S + (1-\pi^r)$ . By assumption 6.1 we have  $\pi^r \Lambda^S + (1-\pi^r) < (1-\tau)$ , meaning that the expected return of (risky) shadow bank deposits is lower than the riskless return to commercial bank deposits. This implies that households' optimal choice is to invest only in commercial banks ( $\alpha_A^{opt} = 0$ ). The rest follows from the fact that  $\alpha_A^{opt}$  is continuous and decreasing in  $D(\mathcal{I}_{SB})$ . ■

## B Proof of lemma 6.3

If  $\vartheta_{CB} \leq \tau$  then we have that  $\tilde{R}_{CB,B}(\vartheta_{CB}, 1) \leq \Lambda^S$ , which means that uninsured commercial bank deposits do not pay a higher effective return in case of a systemic run than shadow bank deposits. Since shadow bank deposits pay a higher return if no run takes place, shadow bank deposits first-order stochastically dominate uninsured commercial bank deposits and households' optimal choice is to hold all uninsured deposits at shadow banks ( $\tilde{\alpha}_B^{opt} = 1$ ). At the other hand we have that  $\lim_{\vartheta_{CB} \nearrow (1-\Lambda^S)} \varphi(\vartheta_{CB}) = 1$  (see expression 8), which means that actual losses for uninsured depositors caused by runs on commercial banks go to zero in the limit as  $\vartheta_{CB}$  approaches  $1-\Lambda^S$  from below. Hence we have that  $\lim_{\vartheta_{CB} \nearrow (1-\Lambda^S)} \tilde{R}_{CB,B}(\vartheta_{CB}, 1) = 1-\tau$ . In the limit, uninsured commercial bank deposits are riskless and pay a higher expected return than risky shadow bank deposits, which implies that  $\lim_{\vartheta_{CB} \nearrow (1-\Lambda^S)} \tilde{\alpha}_B^{opt}(\vartheta_{CB}) = 0$ . The rest follows from the fact that  $\tilde{\alpha}_B^{opt}$  is continuous and decreasing in  $\vartheta_{CB}$ . ■

## C Solving for the type A equilibrium

The relevant choice variable in a type A equilibrium is  $\alpha$ , the share of households' endowment invested into the shadow banking sector. Denote  $c_1^A(\alpha, \xi)$  as consumption of a household at  $t=1$  if sunspot  $\xi$  realizes, given the economy is in a type A equilibrium. Denote  $T(\xi)$  as the tax levied by the deposit insurance agency, minus the fee revenue rebated. We have:

$$\underbrace{c_1^A(\alpha, 1)}_{\text{consumption if run}} = \alpha \underbrace{\frac{\Lambda^S}{D(\mathcal{I}_{SB})}}_{\text{return to SB deposits}} + (1 - \alpha) \underbrace{(1 - \tau)}_{\text{return to CB deposits}} - T(1) \quad (19)$$

$$\underbrace{c_1^A(\alpha, 0)}_{\text{consumption if no run}} = \alpha + (1 - \alpha)(1 - \tau) - T(0) \quad (20)$$

Households choose  $\alpha$  such as to maximize expected utility, given by:

$$E_\xi\{u(c_1^A(\alpha, \xi))\} = (1 - \pi^r) u(c_1^A(\alpha, 0)) + \pi^r u(c_1^A(\alpha, 1)) \quad (21)$$

We have that:

$$\frac{d E_\xi\{u(c_1^A(\alpha, \xi))\}}{d \alpha} = (1 - \pi^r) \tau u'(c_1^A(\alpha, 0)) + \pi^r \left[ \frac{\Lambda^S}{D(\mathcal{I}_{SB})} - (1 - \tau) \right] u'(c_1^A(\alpha, 1)) \quad (22)$$

And:

$$\frac{d^2 E_\xi\{u(c_1^A(\alpha, \xi))\}}{d \alpha^2} = \overbrace{(1 - \pi^r) \tau^2}^{>0} \overbrace{u''(c_1^A(\alpha, 0))}^{<0} + \overbrace{\pi^r \left[ \frac{\Lambda^S}{D(\mathcal{I}_{SB})} - (1 - \tau) \right]^2}^{>0} \overbrace{u''(c_1^A(\alpha, 1))}^{<0} < 0 \quad (23)$$

Since expected utility is continuous, and strictly concave in  $\alpha$ , the optimal choice of  $\alpha$  is unique, as well as continuous in all the arguments. From the discussion in the main text we know that we only need to consider interior solutions. To solve for the equilibrium  $\alpha_A^* = D_{SB,A}^*$  we insert the market clearing condition  $D(\mathcal{I}_{SB}) = \alpha$  into expression (22), cancel out the deposit insurance fee payments, set expression (22) to zero and solve it for

$\alpha$ . This yields the following condition:

$$\left[ (1 - \tau) - \frac{\Lambda^S}{\alpha} \right] \frac{\overbrace{u'((1 - \alpha) + \Lambda^S)}^{c_1^A(\alpha, 1)}}{\underbrace{u'(1)}_{c_1^A(\alpha, 0)}} = \frac{1 - \pi^r}{\pi^r} \tau \quad (24)$$

The left hand side (*LHS*) of expression (24) is continuous and strictly increasing in  $\alpha$ , while the right hand side (*RHS*) is a constant. For  $\alpha = \Lambda^S$  we have that  $LHS < RHS$  and for  $\alpha = 1$  we have that  $LHS > RHS$  (which follows from assumption 6.1). This confirms that there is unique  $\alpha = \alpha_A^* = D_{SB,A}^* \in (\Lambda^S, 1)$  solving equation (24). Note also that, all else equal, an increase in  $\pi^r$  shifts *RHS* downwards, an increase in  $\Lambda^S$  shifts *LHS* downwards, an increase in households' risk aversion shifts *LHS* upwards, and an increase in  $\tau$  shifts *RHS* upwards and *LHS* downwards. This leads to the comparative static results mentioned in the main text.

## D Solving for the type B equilibrium

In a type B equilibrium, the relevant choice variable of households is  $\tilde{\alpha}$ , the share of the uninsurable part of the endowment invested into the shadow banking sector. Denote  $c_1^B(\tilde{\alpha}, 0)$  and  $c_1^B(\tilde{\alpha}, 1)$  as consumption in case of no run and a systemic run respectively, given the economy is in a type B equilibrium. Note that, in a type B equilibrium we have that  $\vartheta_{CB} < 1 - \Lambda^S$ , since commercial banks would not be susceptible to runs otherwise. We have:

$$\begin{aligned} \overbrace{c_1^B(\tilde{\alpha}, 1)}^{\text{consumption if run}} &= \overbrace{\tilde{\alpha}(1 - \theta)\Lambda^S}^{\text{SB deposits}} + \left[ \overbrace{(1 - \tilde{\alpha})(1 - \theta)\frac{\Lambda^S}{1 - \vartheta_{CB}}}^{\text{uninsured CB deposits}} + \overbrace{\theta}^{\text{insured CB deposits}} \right] (1 - \tau) - T(1) \\ &= \theta(1 - \tau) + (1 - \theta) \left[ \tilde{\alpha} + (1 - \tilde{\alpha})\frac{1 - \tau}{1 - \vartheta_{CB}} \right] \Lambda^S - T(1) \\ \overbrace{c_1^B(\tilde{\alpha}, 0)}^{\text{consumption if no run}} &= \overbrace{\tilde{\alpha}(1 - \theta)}^{\text{SB deposits}} + \overbrace{[\theta + (1 - \tilde{\alpha})(1 - \theta)](1 - \tau)}^{\text{CB deposits}} - T(0) = (1 - \tau) + \tilde{\alpha}(1 - \theta)\tau - T(0) \end{aligned}$$

Households choose  $\tilde{\alpha}$  such as to maximize expected utility, given by:

$$E_{\xi}\{u(c_1^B(\alpha, \xi))\} = (1 - \pi^r) u(c_1^B(\alpha, 0)) + \pi^r u(c_1^B(\alpha, 1))$$

Derivation of expected utility with respect to  $\tilde{\alpha}$  yields:

$$\begin{aligned} \frac{d E_{\xi}\{u(c_1^B(\tilde{\alpha}, \xi))\}}{d \tilde{\alpha}} &= (1 - \pi^r) (1 - \theta) \tau u'(c_1^B(\tilde{\alpha}, 0)) \\ &\quad + \pi^r (1 - \theta) \Lambda^S \left( \frac{\tau - \vartheta_{CB}}{1 - \vartheta_{CB}} \right) u'(c_1^B(\tilde{\alpha}, 1)) \end{aligned} \quad (25)$$

As in section C of the appendix it is straightforward to show that  $\frac{d^2 E_{\xi}\{u(c_1^B(\tilde{\alpha}, \xi))\}}{d \tilde{\alpha}^2} < 0$ . Hence expected utility is continuous, as well as strictly concave in  $\tilde{\alpha}$ , so that the optimal choice  $\tilde{\alpha}_B^{opt}$  is unique and continuous in all the arguments.

### D.1 Proof of proposition 6.3

Since expected utility is strictly concave in  $\tilde{\alpha}$ , we have that  $\tilde{\alpha} = 0$  is the optimal choice for households if and only if:

$$\frac{d E_{\xi}\{u(c_1^B(\tilde{\alpha}, \xi))\}}{d \tilde{\alpha}} \leq 0 \text{ at } \tilde{\alpha} = 0 \quad (26)$$

Market clearing implies that  $\vartheta_{CB} = \frac{\theta}{1 - (1 - \theta)\tilde{\alpha}}$  (see section 6 in main text). Inserting  $\vartheta_{CB} = \theta$  (market clearing for  $\tilde{\alpha} = 0$ ) into condition 26, and cancelling out all transfers to and from deposit insurance, yields:

$$\frac{\theta - \tau}{1 - \theta} \geq \frac{1 - \pi^r}{\pi^r} \frac{\tau}{\Lambda^S} \frac{u'(\overbrace{1}^{c_1^B(0,0)})}{u'(\underbrace{\Lambda^S}_{c_1^B(0,1)})} \quad (27)$$

Whenever the cap  $\theta$  satisfies condition (27) and also satisfies  $\theta < 1 - \Lambda^S$ , then  $\tilde{\alpha}_B^* = D_{SB,B}^* = 0$  is the unique type B equilibrium of the economy. The left hand side (*LHS*) of condition (27) is increasing in  $\theta$  while the right hand side (*RHS*) does not change in  $\theta$ . For  $\theta = 0$  we have  $LHS < RHS$  and for  $\theta = 1 - \Lambda^S$  we have  $LHS > RHS$  (which follows



from assumption 6.1). Hence there is a  $\hat{\theta}_B$  with  $0 < \hat{\theta}_B < 1 - \Lambda^S$  so that condition (27) is fulfilled if and only if  $\theta \geq \hat{\theta}_B$ . ■

To solve for  $\hat{\theta}_B$  we solve (27) for  $\theta$ . Note next that corner solutions with  $\tilde{\alpha}_B^* = 1$  are not possible if  $\theta > 0$ . To see this, consider the following: If all uninsured deposits are held at shadow banks ( $\tilde{\alpha}_B^* = 1$ ) then all deposits at commercial banks are insured ( $\vartheta_{CB} = 1$ ). By (18) this means that households' optimal choice is to invest only in commercial banks  $\tilde{\alpha}_B^{opt} = 0$  which leads to a contradiction. Hence, whenever  $\theta < \hat{\theta}_B$ , then the economy exhibits a (unique) type B equilibrium with an interior solution  $\tilde{\alpha}_B^* \in (0, 1)$  and  $D_{SB,B}^* \in (0, 1 - \theta)$ . To solve for interior equilibria, we insert the market clearing condition  $\vartheta_{CB} = \frac{\theta}{1 - (1 - \theta)\tilde{\alpha}}$  into expression (25), cancel out all transfers to- and from deposit insurance, set the expression to zero and solve for  $\tilde{\alpha}$ . If  $\theta = 0$ , then only shadow banks exist by definition.