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# Capacity constrained price competition with transportation costs

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*Preliminary*

## **Abstract**

We characterize mixed-strategy equilibria in case of capacity constrained price competition, transportation costs and customer-specific pricing. The equilibrium prices weakly increase in the distance between supplier and customer. Despite prices above costs and excess capacities, the competing firms exclusively serve their home markets in equilibrium. Competition yields volatile market shares and an inefficient allocation of more distant customers to suppliers. Even ex-post cross supplies may restore efficiency only partly. Firms sometimes do not cross supply each other as this can intensify competition by relaxing the receiver's capacity constraint. We use our findings to discuss recent competition policy cases and provide hints for a more refined coordinated-effects analysis.

**JEL classification:** L11, L41, L61

**Keywords:** Bertrand-Edgeworth, hotelling, capacity constraints, mixed equilibrium, price differentiation, inefficient supply.

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# 1 Introduction

There are many markets with largely homogeneous products for which the costs of shipping from the production site to the customer are significant. Prominent examples include basic materials such as cement, concrete, chemicals and steel. In addition to their general economic relevance, these products also feature prominently in many competition policy cases. Recent examples include merger control procedures of the European Commission in relation to Holcim / Cemex West, Holcim / Lafarge, and Outokumpu / Inoxum. To assess the merger effects on competition, in several of these cases the authorities make use of arguments based on price competition with limited capacities.<sup>1</sup> However, the arguments are based on models without customer specific transport costs and pricing, although the markets at stake have these characteristics. With this article we fill this gap in the literature by analyzing such a model.

We first investigate the case of two symmetric firms that are only differentiated by their location and thus transport costs to the customers, which are located on a line in between them. Each firm has capacity to serve more than half of the market, but not the whole market. We allow the firms to charge location-specific prices. Interestingly, for moderate to low transportation costs we find an equilibrium with uniform prices. However, if transport costs are sufficiently high, firms prefer to charge location specific prices. These prices have the feature that they increase in the distance between each firm and the respective customer. This yields the outcome that firms always serve their nearest customers (“home markets”), and that at prices above the costs of the competitor. Instead, further away customers are served either by the one or the other firm, as firms play mixed strategies in prices. This yields both volatile market shares and a productive inefficiency, as not always the firm with the lowest cost serves each customer.

We extend our analysis in several ways. First, we endogenize the capacity in case of volatile demand and demonstrate that the analyzed case of over-capacity may well result in equilibrium when firms optimally choose their capacities. In addition, we analyze how subcontracting between the firms affects the market outcome in this setting. There is a rationale for a subcontract when a firm makes the most competitive offer to a customer, while another firm can serve that customer at lower costs. We show that subcontracts can lead to an efficient production in certain situations, but not in all. In particular, firms refrain from cross-supplies when this frees up capacity of a constrained firm that set too low prices – as the additional capacity can increase competition on (otherwise) residual demand segments of the market. We also show that cross supplies can increase customer prices when the cross supplier has a weak bargaining position towards the receiver.

The remainder of the article is structured as follows. We discuss the related literature in the next section, introduce the model in Section 3, solve the model and derive the main results in Section 4, endogenize the capacity choice in Section 5, discuss cross supplies in Section 6, and then conclude in Section 7.

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<sup>1</sup>See Section 7 for a more detailed discussion and references.

## 2 Related literature

This article contributes to several strands of the existing literature. There is a well-known literature based on Bertrand (1883) – Edgeworth (1925) that analyzes price competition in case of capacity constraints – and does so mostly for homogeneous products (a recent example is Acemoglu et al. (2009)). We contribute at a methodological level with a model where customers are differentiated by location and thus transportation costs of the firms. We allow the firms to price discriminate by charging customer specific prices. This leads to mixed price strategies which may consist of different prices for different customers. Customer specific prices are – to our knowledge – knew to this strand of literature.

There are a few articles and working papers which introduce differentiation in this context, notably Canoy (1996); Sinitsyn (2007); Somogyi (2016); Boccard and Wauthy (2016). Canoy investigates the case of increasing marginal costs in a framework with differentiated products. However, he does not allow for customer specific costs and customer specific prices. Somogyi considers Bertrand-Edgeworth competition in case of substantial horizontal product differentiation in a standard Hotelling setup. Boccard and Wauthy focus on less strong product differentiation in a similar Hotelling setup. Somogyi uses a logit demand specification and shows that pure-strategy equilibria exist for small and large over-capacities, but only mixed-strategy equilibria for intermediate capacity levels. For some of these models equilibria with mixed-price strategies over a finite support exist (Boccard and Wauthy (2016); Sinitsyn (2007); Somogyi (2016)). This appears to be due to the combination of uniform prices and demand functions which, given the specified form of customer heterogeneity, have interior local optima as best responses.<sup>2</sup> Overall, these contributions appear to be mostly methodological and partly still preliminary.

This article is also related to the literature on subcontracting relationships between competitors (also referred to as cross supplies). With a subcontract, a firm can essentially use the production capabilities of a competitor. Efficiencies can for instance arise when a firm with decreasing returns to scale has won a large contract, so that subcontracting part of the production to an identical firm reduces costs (Kamien et al. (1989)). Similarly, if there are increasing returns to scale, pooling the production can reduce costs (Baake et al. (1999)). More indirectly, if firms with asymmetric costs compete, the resulting allocation of demand to each firm may not exactly minimize costs, such that again subcontracting increases efficiency (Spiegel (1993)).

The above literature has essentially pointed out two competitive effects which depend on the distribution of the efficiency rent from subcontracting between the firm that receives the cross supply and the one that delivers it. If the receiver obtains the efficiency rent, its effective costs are lower as it uses the partly more efficient production technology of its competitor at costs. This tends to increase competition. Instead, if the cross supplier obtains the efficiency rent, it foregoes a profit when competing potential subcontracting business away from its

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<sup>2</sup>In a related vein, some work has considered heterogenous consumers in a model with different consumer segments where mixed-strategy price equilibria result Sinitsyn (2008, 2009), in line with Varian (1980) model of sales.

competitor. This tends to soften competition.<sup>3</sup>

We contribute to this literature on subcontracting in several ways. First, we point out that horizontal subcontracting may also occur when firms are symmetric and there are no generic reasons for subcontracting (if there was a symmetric equilibrium in pure price strategies, there would be no need for subcontracting). The only reason for subcontracting is that price competition with capacity constraints can be so chaotic that the equilibrium prices and allocation of customers to firms is not reflecting the differences in productions costs, although customer specific pricing is feasible. We show that subcontracting can increase or decrease consumer surplus, depending on the distribution of the efficiency gains among the subcontracting competitors. Second, we show that there is a – to our knowledge – new reason for why firms may not engage in welfare-improving subcontracting, which relates to capacity constraints. When a firm that produces nearly at capacity asks an unconstrained firm for a cross supply to a customer which that firm can supply more efficiently, the unconstrained firm may deny this. The reason is that such a supply would leave the demanding firm with additional capacity, which can intensify competition for other customers.

## 3 Model

### 3.1 Setup

There are two symmetric firms. Firm  $L$  is located at the left end of a line, and firm  $R$  at the right end of this line. Four customers are located on the line, named 1, 2, 3 and 4 (from left to right). The firms produce homogeneous goods. Each customer has unit demand and a willingness to pay of  $v$ . Firm  $L$  incurs transport costs of  $1c$ ,  $2c$ ,  $3c$ , and  $4c$  to reach the customers from left to right. For firm  $R$ , there are transport costs of  $4c$ ,  $3c$ ,  $2c$ ,  $1c$  to reach the same customers. There are no other costs of supply. We assume that the willingness to pay is higher than the transport costs even for the most distant customers:  $v > 4c$ , so that each customer is contestable. See Figure 1 for an illustration.

Each firm  $j \in \{L, R\}$  can price discriminate between customers by charging a separate price  $p_i^j$  to each customer  $i \in \{1, 2, 3, 4\}$ . A pure prices strategy for firm  $i$  is a vector  $(p_i^1, p_i^2, p_i^3, p_i^4) \in R^4$ . We solve the game from the perspective of firm  $L$  and apply symmetry. If we suppress superscript  $j$ ,  $p_i$  belongs to firm  $L$ .

The game has the following timing:

1. Firms  $L$  and  $R$  simultaneously set the eight prices  $p_i^L$  and  $p_i^R$ .
2. The customers buy at the lowest price as long as the price is not above  $v$ .

In our primary case each firm has a capacity to serve up to 3 of the 4 customers. As a consequence, a single firm cannot serve the whole market, whereas overall there is 50% overcapacity.

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<sup>3</sup>Marion (2015) finds that in California highway construction auctions the winning bid is uncorrelated with horizontal subcontracting and attributes this to an efficiency motive for cross-supplies. See also Huff (2012) for a similar work.

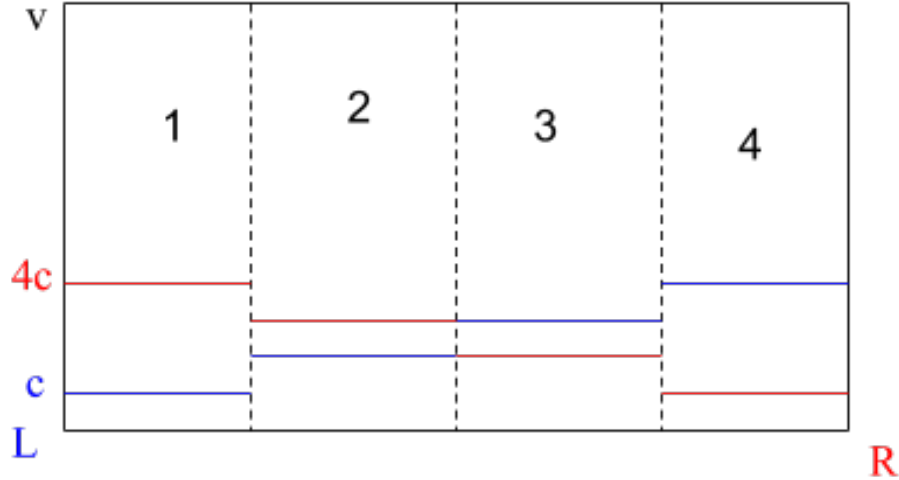


Figure 1: Setting: Customers 1 to 4 with unit demand are located between suppliers L and R; transport costs increase in distance.

## Rationing

We employ efficient rationing, in particular, we use the following rationing rule:

1. In case of equal prices charged by both firms to the same customer, the customer buys where the transport costs are lowest.
2. If one firm charges lower prices to all customers than the other firm and does not have capacity to serve all customers, we assume that the customers are allocated to firms so that consumer surplus<sup>4</sup> is maximized. In other words the consumer with the best outside option is rationed.

While this is not the only rationing rule possible, we consider this rule appropriate for the following reasons:

- The rationing rule corresponds to the usual efficient rationing (as, for instance, used by Kreps and Scheinkman (1983)) in that the customers with the highest willingness to pay are served first. A difference is, however, that the willingness to pay is endogenous in that it depends on the (higher) prices charged by the other supplier. These may differ across customers, and so does the surplus for a customer to purchase from the low-price supplier.
- The rationing rule gears at achieving efficiencies, in particular for equilibria in which the supplier's prices weakly increase in the costs of serving each customer. Our results of inefficiencies in the competitive equilibrium are thus particularly robust.

<sup>4</sup>By consumer surplus we mean the surplus of the four customers.

- At least for the case of uniform prices ( $p_1^j = p_2^j = p_3^j = p_4^j$ ) other rationing rules yield the same outcome. In particular, the supplier with lower prices would also choose to serve the closest three customers, as this minimizes the transportation costs.
- The rationing rule is the natural outcome if the customers can coordinate their purchases: They will reject the offer of the low-price firm that yields the lowest customer surplus. This occurs, for instance, if interim-contracts with side payments among the customers are allowed. It would also occur if there is only one customer.
- Similarly, if a supplier has to compensate a customer to which he made an offer that he cannot fulfill, this might also incentivize a supplier to ration according to the customers' net utility from this contract. More generally, in a repeated game suppliers may at least partially internalize the customers' willingness to pay, which again supports the employed rationing rule.

### **3.2 Reference case with capacity constraints of 2 units each: Equilibrium in pure strategies**

Suppose that each firm has capacity to serve only 2 out of the 4 customers. As a consequence, it is an equilibrium in pure strategies that each firm sets the price for each customer equal to the willingness to pay of  $v$ , and that each customer buys the good from the closest firm. This is efficient and thus maximizes total surplus as all customers are served by the firm with the lowest transport costs. Each firm obtains the highest profit that is feasible with two units of capacity, which equals  $2v - 3c$ . Customer surplus is given by  $4v - 4v = 0$ .

### **3.3 Reference case without capacity constraints: Equilibrium in pure strategies**

Suppose that each firm has capacity to serve all the 4 customers. As a consequence, for each customer the two firms face Bertrand competition with asymmetric costs. It is thus an equilibrium in pure strategies that each firm sets the price for each customer equal to the highest marginal costs of the two firms for serving that customer, and that the customer buys the good from the firm with the lower marginal costs. This is again efficient (for given capacities) in that all customers are served by the closest firm with the lowest transport costs. Each firm makes a margin of  $4c - c = 3c$  from selling to the closest customer, and  $3c - 2c = c$  from selling to the second closest customer. The equilibrium profit of a firm is thus  $4c$ . Consumer surplus is given by  $4v - 2 \cdot 4c - 2 \cdot 3c = 4v - 14c$ .

### **3.4 Non-existence of a pure strategy equilibrium when each firm can serve three quarter of the market**

Suppose each firm can only serve at most 3 customers and both firms set prices as if there were no capacity constraints, as discussed in the previous subsection. Is this an equilibrium?

For each firm, the prices of the two most distant customers already equal its incremental costs of supplying each of these customers ( $3c$  and  $4c$ ). There is thus still no incentive to undercut these prices. Similarly, there is still no incentive to undercut the prices for the two closest customers as these are already buying from the firm.

In view of the other firm's capacity constraint, the now potentially profitable deviation is to set the highest possible price of  $v$  for each customer. All customers prefer to buy from the other firm at the lower prices which range between  $3c$  and  $4c$ . However, as the other firm only has capacity to serve 3 customers, one customer ends up buying from the deviating firm at a price of  $v$ . Given the rationing rules, this is the closest customer as this maximizes consumer surplus in line with our rationing rules, because the price of the other firm is largest for that customer. The profit of the deviating firm is thus  $v - c$ . This is larger than the pure strategy candidate profit of  $4c$  if  $v > 5c$ .

The above condition for a profitable deviation is stricter by one  $c$  than the contestability assumption  $v > 4c$ . Stated differently, for a profitable deviation the willingness to pay must be 25% higher than the highest transport costs. We assume that this is the case throughout the rest of the paper. If instead  $5c > v > 4c$ , there is an equilibrium in pure strategies also when each firm can serve only  $3/4$  of total demand.<sup>5</sup>

## 4 Construction of the equilibrium with (in distance) increasing prices

We further analyze the case that each of the two firms has 3 units of capacity and thus can serve  $3/4$  of the total demand. The last subsection establishes that there is a pure strategy equilibrium if  $v \leq 5c$ , while for  $v > 5c$  the pure strategy equilibrium does not exist. In this section we construct a mixed strategy equilibrium for the latter case. A mixed strategy equilibrium consists of a joint distribution over the four prices for each firm. In what follows we use the term uniform prices. By this we mean that the four prices of one supplier are perfectly correlated and thus the same in each draw ( $p_1^j = p_2^j = p_3^j = p_4^j$ ).

We proceed by first postulating properties of the equilibria we search for. We then verify that such equilibria exist and describe them in detail.

**Characteristics of the equilibria we search for.** Both firms play mixed price strategies that are symmetric with prices that are weakly increasing in distance:  $p_1^L \leq p_2^L \leq p_3^L \leq p_4^L$  and  $p_4^R \leq p_3^R \leq p_2^R \leq p_1^R$ . This means that every individual realization of each player's price vector in the mixed strategy equilibrium has this price order. Moreover, each individual price is mixed over the same support  $[\underline{p}, v]$  and there are no mass points in the marginal distributions of the prices.

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<sup>5</sup>Note that the pure strategy equilibrium without capacity constraints is the only reasonable candidate equilibrium also with a constraint of 3 per firm. Any price above the marginal costs of the competitor could be profitably undercut. There are however, potential candidate equilibria with even lower prices, in which the player who loses a segment prices below its costs. We exclude those equilibria as is usual in the literature on asymmetric Bertrand competition.



We now first provide some base results that hold for all equilibria with the above defined characteristics. We start with a property for the sales allocation in equilibrium, which is derived from the characteristic that firms play only weakly increasing price vectors.

**Lemma 1.** *Given both firms play weakly increasing prices, there is zero probability that any firm serves the most distant customer, while the closest customer is served with probability 1.*

*Proof.* There are two cases to distinguish, either the capacity constraint of a firm is binding, or not binding:

1. All the prices of one firm lie below the prices of the other firm: the firm with the lowest prices serves its closest three customers up to the capacity limit; the most distant customer is served by the firm with the high prices. This minimizes the prices that are charged and thus maximizes customer surplus, in line with the rationing rule.
2. Instead, if prices intersect (each firm has the lowest price for at least one customer), the customer closest to each firm must be won by that firm.

□

Next we establish that a price vector with  $p_1^L < p_2^L$  cannot be a best response to a distribution of weakly increasing price vectors played by the other firm.

**Lemma 2.** *In any symmetric equilibrium with weakly increasing prices, the prices for the two closest customers are identical:  $p_1^L = p_2^L$ , and by analogy  $p_4^R = p_3^R$ .*

*Proof.* There are two cases to distinguish:

First, suppose that  $p_2^L \leq p_2^R$ . Given a weakly increasing price order of the right firm, the left firm wins the first customer with a price of  $p_1^L = p_2^L$  as  $p_1^R \geq p_2^L$ . This follows from increasing prices of the right firm. Consequently,  $p_1^L \leq p_1^R$ . Hence, there is no incentive to charge a price  $p_1^L < p_2^L$ , as  $p_1^L = p_2^L$  ensures a higher margin without losing demand.

Second, suppose that  $p_2^L > p_2^R$ . Given weakly increasing price orders, this means that the right firm also has the lowest prices for customers 3 and 4. In this case the left firm will serve customer 1 even if it has a higher price than the right firm, as the other firm can and will – given the rationing rule – serve its three closest customers, so that customer 1 only has the option to buy from the left firm or not at all. In this case setting  $p_1^L$  below  $p_2^L$  is dominated by setting  $p_1^L = p_2^L$ .

In both cases the price relation  $p_1^L < p_2^L$  – and by analogy  $p_4^R < p_3^R$  – is dominated by equal prices for the two closest customers, which establishes the lemma. □

In the equilibria we search for, the upper bound  $v$  is played with positive density and without mass points for all customers. There is thus a profit of  $\pi(v) = v - c$  that a firm can realize with probability one by choosing a price of  $v$  for all customers. As all price vectors

played in equilibrium must yield identical expected profits, the expected profit must equal  $v - c$ . This also implies  $\pi(\underline{p}) = \pi(v)$ , which yields  $3(\underline{p} - 2c) = v - c \implies$

$$\underline{p} = \frac{1}{3}v + \frac{5}{3}c. \quad (1)$$

Given weakly increasing prices, this implies that if  $p_1 = v$ , also the other three prices of firm  $L$  must equal  $v$ , so that  $p_1 = p_2 = p_3 = p_4 = v$  is played at the top of the distribution (recall that when suppressing the superscript we mean prices of firm  $L$ ). Similarly, the lower bound  $\underline{p}$  is played with positive density, which implies that  $p_4 = \underline{p}$  is played in equilibrium. Again, if  $p_4 = \underline{p}$  is played, weakly increasing prices imply  $p_1 = p_2 = p_3 = p_4 = \underline{p}$ . Thus uniform prices are played with positive density, at least at the boundaries of the price support. This does not exclude that strictly increasing prices of the form  $p_1 = p_2 < p_3 \leq p_4$  are also played in equilibrium. The lowest price

As an intermediate step, the next lemma characterizes the equilibrium when firms can only set uniform prices (so that always  $p_1 = p_2 = p_3 = p_4$ ). We later show that for some parameters uniform prices constitute an equilibrium also when it is possible to choose different prices.

**Lemma 3.** *It is an equilibrium that firms mix uniform prices according to the distribution function*

$$F(p) = \frac{3p - 5c - v}{2p - 5c} \quad (2)$$

on the support  $[\underline{p}, v]$  – at least when only uniform prices are feasible. The equilibrium profit equals  $v - c$  and there is an expected cost inefficiency of  $c$  compared to the first best market allocation.

*Proof.* If firms always play uniform prices, all prices have the same marginal distribution function  $F_1^j = F_2^j = F_3^j = F_4^j \equiv F$  for  $j \in \{L, R\}$  by construction. The expected profit, using the results of lemma 1, can be expressed as

$$\pi^L = (p_1 - c) + [1 - F_2^R(p_2)](p_2 - 2c) + [1 - F_3^R(p_3)](p_3 - 3c)$$

and under uniform  $p$  becomes

$$\pi^L = (p - c) + [1 - F(p)](p - 2c) + [1 - F(p)](p - 3c).$$

Let us characterize the distribution function  $F$  by using that  $L$  must be indifferent between all prices that it plays with positive density. As any marginal price change in the support cannot yield higher profits in equilibrium,  $F$  is defined by the following differential equation, where  $f(p)$  is the corresponding density:

$$\begin{aligned} \frac{\partial \pi^L}{\partial p} &= 1 - f(p)(p - 2c) + [1 - F(p)] - f(p)(p - 3c) + [1 - F(p)] = 0 \\ \implies &3 - f(p)(2p - 5c) - 2F(p) = 0 \end{aligned}$$

$$\implies 3 - 2F(p) = f(p)(2p - 5c)$$

$$\implies \frac{f(p)}{3 - 2F(p)} = \frac{1}{2p - 5c}.$$

The set of solutions to this differential equation is

$$F(p) = \frac{k - 3p}{5c - 2p}, \quad (3)$$

with  $k$  being a parameter. Given the support  $[p, v]$  and that there are no mass points at prices of  $v$ , we can use that

$$F(p = v) = 1 = \frac{k - 3v}{5c - 2v} \implies k - 3v = 5c - 2v \implies k = 5c + v.$$

Inserting for  $k$  in (3) yields (2).

Inserting  $\underline{p}$  for  $p$  in the above confirms that there is zero mass at the bottom of the distribution:

$$F(\underline{p}) = \frac{5c + v - 3(\frac{1}{3}v + \frac{5}{3}c)}{5c - 2(\frac{1}{3}v + \frac{5}{3}c)} = \frac{5c + v - v - 5c}{5c - \frac{2}{3}v - \frac{10}{3}c} = 0.$$

If both firms mix uniform prices according to the distribution function (2) on the support  $[p, v]$ , any uniform price vector on this support is a best response – at least when only uniform prices are feasible. The equilibrium profit is given by the residual demand profit at  $p = v$  and equals  $v - c$ . As almost certainly the prices of the firms are not equal, there is an inefficiency of  $c$  as almost certainly one of the two intermediate customers is served by the firm with transport costs that are higher by  $c$  than of the efficient firm, which in these cases has set higher prices and still has unused capacity.  $\square$

Checking whether uniform prices are best responses to the above distribution of uniform prices when price differentiation across customers is feasible yields

**Lemma 4.** *If  $v \geq 7c$ , i.e., the willingness to pay is at least 75% above the transport costs to the most distant customer, there is no incentive to respond to the uniform price distribution  $F$  with a price vector that has  $p_3 > p_2 = p_1$ . Instead, for a lower willingness to pay in the interval  $7c > v > 5c$ , there is an incentive to raise the price for the third closest customer above the level of  $p_1 = p_2$  in an interval with the lower bound of  $\underline{p}$*

*Proof.* As established in Lemma (2), a best response to weakly increasing prices using weakly increasing prices is of the form  $p_1 = p_2 \leq p_3 \leq p_4$ . As the most distant customer is never served, we restrict our search for best responses to price vectors with  $p_4$  equal to  $p_3$  (as  $p_4 = p_3$  is always a best response). This leaves only one critical price step in the best responses:  $p_3 > p_2$ . We first check whether there is an incentive to deviate with the price for one customer individually while maintaining the order of weakly increasing prices and

afterwards verify that only weakly increasing prices are best responses to uniform prices. Note that when price changes are considered that maintain the weakly increasing price order, changing a price for one customer does not affect the expected profits of that firm with other customers.

There might be an incentive for a firm to charge higher prices to more distant customers as these are more costly to serve. To see this, note that the expected profit for firm  $L$  from serving one customer  $i \in \{1, 2, 3, 4\}$  with the lowest price (i.e. without residual demand profits and in the absence of capacity constraints) is given by  $[1 - F(p_i)](p_i - i \cdot c)$ . Differentiating with respect to  $p_i$  yields

$$[1 - F(p_i)] - f(p_i)p_i + f(p_i) \cdot i \cdot c. \quad (4)$$

The marginal profit for firm  $L$  increases in the distance  $i$ . There is thus a natural incentive to set higher prices for more distant customers. We can evaluate the marginal profit (4) for customer  $i = 3$  by substituting for  $i$ , and for  $f$  and  $F$  from (2) and evaluate whether, and possibly for which parameters, the marginal profit is negative, such that a marginal price increase of only  $p_3$  is not profitable:

$$\begin{aligned} & \left[ 1 - \frac{3p - 5c - v}{2p - 5c} \right] - \frac{2v - 5c}{(2p - 5c)^2} (p - 3c) < 0 \\ \implies & (2p - 5c)(v - p) - (2v - 5c)(p - 3c) < 0. \end{aligned} \quad (5)$$

For  $p = \underline{p}$  the marginal profit condition reduces to  $-\frac{1}{3}v + \frac{7}{3}c < 0$ . As  $v = 5c + x$  with  $x > 0$ , we get  $-x + 2c < 0$ , which holds if  $x > 2c$ . This implies  $v > 7c$ .

Moreover, we show that the second derivative of the profit for customer 3 is negative in the relevant range. The second derivative is given by

$$2(v - p) - (2p - 5c) - (2v - 5c) = 2v - 2p - 2p + 5c - 2v + 5c = -4p + 10c.$$

This second derivative is already negative at the lower bound price of  $\underline{p}$  for the lowest possible value for  $v$  of  $5c$ , above which there are mixed strategy equilibria. It is also negative for all larger prices up to  $v$ . The profit function is thus strictly concave in the relevant range. This implies that whenever the marginal profit (4) is negative at  $\underline{p}$ , it is negative for the prices above  $\underline{p}$ .  $\square$

Intuitively, if the differences in transport costs across customers are sufficiently small in relation to the willingness to pay (although they may still be economically substantial), there is no incentive to price differentiate. Showing in addition that a firm cannot profitably deviate from uniform prices with prices that are not weakly increasing, given the other firms only plays uniform prices, establishes

**Proposition 1.** *If the willingness to pay is at least 75% above the transport costs to the most distant customer ( $v > 7c$ ), there exists a symmetric equilibrium in which the firms play*

uniform prices with distribution  $F$  in the support  $[p, v]$ . The expected profit of a firm is  $v - c$ . There is an allocative inefficiency of  $c$ .

*Proof.* Suppose that firm  $L$  chooses prices that are not weakly increasing in response to uniform price distribution played by firm  $R$ . In that case there is a pair of prices of  $L$  such that the price for the further away customer – with higher costs – is strictly below a price for a closer customer. In all such cases it is at least as profitable to switch the prices between the two customers. There are three possible situations: the uniform price of the competitor is above, below, or in between the price pair. If the uniform prices of firm  $R$  are above both prices, switching the prices weakly increases profits. In particular, it is profit neutral if the capacity constraint is not binding, and strictly profit increasing if the customer with the higher prices is rationed, as this reduces the costs for the customers served without affecting the average price level of the customers that are served. If the competitor's price is below both prices, switching the prices may affect which customer is served by firm  $L$  as residual demand (as it would only be the one with the lowest price) and is thus weakly profitable because the closer customer has lower costs. If the competitor's price is in between the price pair, the capacity constraint is not binding for either firm. In this case switching the prices to a weakly increasing price order is always profitable as it changes the customer that is served by firm  $L$  to a less costly one without changing the prices that are realized.  $\square$

### Candidate equilibrium with strictly increasing prices

Let us now investigate for which parameter range profitable deviations from uniform prices exist. Lemma 4 states that for  $7c > v > 5c$  a firm best respond to the uniform price distribution with strictly increasing prices in the sense of  $p_3 > p_2$  in an interval starting at  $\underline{p}$ . We thus search for a price distribution which allows for strictly increasing prices at the bottom of the support. At the top of the price support, it must still be the case that the price of  $v$  is played with positive density for all customers (recall that we search for equilibria with full support in  $[\underline{p}, v]$ ). Given weakly increasing prices, this implies that the combination  $p_1 = p_2 = p_3 = p_4 = v$  is played with positive density. Analogously, also the lower bound of the support is played with positive density in all segments. This still defines the lower bound price  $\underline{p}$ .

The (expected) profit of firm  $L$  can be written as

$$\pi_L = (p_1^L - c) + [1 - F_2^R(p_2^L)](p_2^L - 2c) + [1 - F_3^R(p_3^L)](p_3^L - 3c).$$

We have established that weakly increasing price vectors are of the form  $p_1 = p_2 \leq p_3 \leq p_4$  (Lemma 2). With weakly increasing prices, a firm never serves the most distant customer. For marginal price deviations which maintain the weakly increasing price order, a firm is thus not capacity constrained with respect to the three closest customers. Moreover, a firm always serves its closest customer. Denoting  $p_{12} \equiv p_1 = p_2$  and  $F_2 = F_2^L = F_3^R$  and  $F_3 = F_3^L = F_2^R$ ,

the profit becomes

$$p_{12} - c + (1 - F_3(p_{12})) (p_{12} - 2c) + (1 - F_2(p_3)) (p_3 - 3c). \quad (6)$$

The fact that a marginal deviation with  $p_1$  and  $p_2$  must not be profitable defines  $F_3$  through the condition that the firm must be indifferent at the margin for any price that is played with positive density:

$$1 + 1 - F_3(p_{12}) - f_3(p_{12})(p_{12} - 2c) = 0.$$

The solution to this differential equation is

$$F_3(p) = \frac{2p - k_3}{p - 2c}.$$

At the lower bound price  $\underline{p}$ , it must be  $F_3(\underline{p}) = 0$ . This implies  $k_3 = 2\underline{p} = \frac{2}{3}v + \frac{10}{3}c$  and thus

$$F_3(p) = \frac{2p - \frac{2}{3}v - \frac{10}{3}c}{p - 2c}.$$

To obtain  $F_2$ , we differentiate the profit in (6) with respect to  $p_3$  to obtain the marginal condition for that price:

$$1 - F_2(p_3) - f_2(p_3)(p_3 - 3c) = 0.$$

The solution to this differential equation is

$$F_2(p) = \frac{p - k_2}{p - 3c}.$$

Also this distribution function must be 0 at  $p = v$ , which implies  $k_2 = \frac{1}{3}v + \frac{5}{3}c$  and thus

$$F_2(p) = \frac{3p - v - 5c}{3p - 9c}.$$

Recall that  $F_2$  and  $F_3$  have been constructed so that if firm  $R$  plays increasing price vectors with  $p_1^R = p_2^R$  as well as  $p_3^R = p_4^R$  and in particular mixes the prices of its two closest customers with  $F_2$  and the prices of the two more distant segments with  $F_3$ , firm  $L$  is indifferent between playing uniform prices and increasing prices with  $p_2 < p_3$  in an interval starting at the lower bound  $\underline{p}$ .

At the top of the price support, it must be that firm  $L$  sets  $p_3 = p_4 = v$  only if  $p_1 = p_2 = v$ . In other words, firms play only uniform prices at the upper bound of the price support. The interval in which uniform prices are not a best response to uniform prices played with the distribution function  $F$  as defined in (2) is given by  $[\underline{p}, p^I]$ , where the upper bound is obtained by setting the marginal profit in (5) equal to zero and solving for  $p$ . This yields  $p^I = \frac{1}{2} (5c + \sqrt{2cv - 5c^2})$ . For the complementary upper part of the price support, that is  $[p^I, v]$ , uniform prices drawn from  $F$  could be mutually best responses. Indeed, the functions  $F_2$  and  $F_3$  do not generally attain a value of 1 at  $p = v$ , such that they can only describe

part of the price distribution. However, the functions  $F_2$  and  $F_3$  both intersect with  $F$  at  $p = 4c$ . Note that the intersection point  $4c$  is above the level  $p^J$  above which uniform prices are feasible for the parameter range  $7c > v > 5c$ . This means that from  $4c$  to  $v$  uniform prices are mutually best responses. In the symmetric candidate equilibrium the prices are therefore played from either  $F_2$  or  $F_3$  on the lower part of the support, and from  $F$  in the upper part. In detail, for the prices  $p_1^L, p_2^L, p_3^R, p_4^R$  of the two closest customers the distribution is

$$F_c(p_i^j) \equiv \begin{cases} \frac{3p_i^j - 5c - v}{2p_i^j - 5c} & , 4c < p_i^j \leq v \\ \frac{p_i^j - \frac{1}{3}v - \frac{5}{3}c}{p_i^j - 3c} & , \underline{p} \leq p_i^j < 4c \end{cases} \quad (7)$$

and for the prices  $p_3^L, p_4^L, p_1^R, p_2^R$  of two most distant customers it is

$$F_d(p_i^j) \equiv \begin{cases} \frac{3p_i^j - 5c - v}{2p_i^j - 5c} & , 4c < p_i^j \leq v \\ \frac{2p_i^j - \frac{2}{3}v - \frac{10}{3}c}{p_i^j - 2c} & , \underline{p} \leq p_i^j < 4c \end{cases} . \quad (8)$$

Note that for the corner case of  $v = 7c$ , the lower bound  $\underline{p}$  equals  $4c$  and the functions  $F_c$  and  $F_d$  coincide and equal  $F$ . This is consistent with our previous finding in Proposition 1 that for  $v > 7c$  there is an equilibrium in which uniform prices are played with the distribution  $F$  on the whole support  $[\underline{p}, v]$ .

Observe that the expected profit on the support is still  $v - c$ : It is  $v - c$  for uniform prices of  $v$  and – by construction of  $F$  – marginal price decreases do not change the expected profit in the upper part. The profit also equals  $v - c$  at the lower bound  $\underline{p}$ , and again  $F_2$  and  $F_3$  are constructed such that marginal price changes are not profitable on the lower part of the support.

This means that although firms are in their best response indifferent between uniform and strictly increasing prices, in this part of the price support firms they set on average lower prices for the two closer customers and higher prices for the two distant customers. Stated differently, the prices can be ordered in the sense of first order stochastic dominance. Note that in the relevant range it is the case that  $F_c \geq F_d$ , with a strict inequality in the range  $(\underline{p}, 4c)$  such that the prices of the closest two customers are first order stochastically dominated by the prices of the two further away customer.

There is an inefficiency of  $c$  when firm  $R$  sets a lower price for customer 2 than firm  $L$  because  $R$  then serves the customer with its higher transport costs (and the other way around for customer 3). The probability of firm  $R$  setting a lower price for customer 2 is given by

$$Pr(p_2^R < p_2^L) = \int_{\underline{p}}^v \int_{\underline{p}}^{p_2^L} f_d(s) f_c(t) ds dt = \int_{\underline{p}}^v F_d(t) f_c(t) dt. \quad (9)$$

Note that  $p_2^R$  and  $p_2^L$  are drawn independently as they belong to different players. This implies that the joint density is the product of densities as shown in the intermediate step of 9. The probability is 50% in case of uniform prices; this still holds in the interval  $[4c, v]$  where firms set uniform prices. For the interval  $[\underline{p}, 4c]$  firms set on average lower prices to closer

customers in line with the lower transportation costs. As a consequence, the probability of an inefficient supply is lower here, and thus also on average for the full support when compared to the case that firms are restricted to charge uniform prices. Price discrimination thus increases efficiency, but cannot fully achieve it as misallocations still occur with positive probability.

Showing that in response to weakly increasing prices it is not profitable for a firm to set prices that are not weakly increasing establishes

**Proposition 2.** *If  $5c < v < 7c$ , there exists a symmetric mixed strategy equilibrium with weakly increasing prices. Each firm draws the prices for its closest two customers from the price distribution  $F_c$  and for the two most distant customers from  $F_d$ , as defined in (7) and (8). All marginal price distributions are atomless with support  $[p, v]$ . The expected firm profit is  $v - c$  and firms play strictly increasing prices with positive probability.*

*Proof.* It is left to verify that there is no profitable drastic deviation that overturns weakly increasing prices. Consider firm  $L$  for the argument. Given firm  $R$  plays weakly increasing prices with  $p_1^R = p_2^R$  and  $p_3^R = p_4^R$  according to the equilibrium distributions  $F_c$  and  $F_d$  we establish that every best response to that strategy has weakly increasing prices.

Suppose to the contrary that  $L$  plays prices that are not weakly increasing. In that case, there is a pair of prices such that a segment further away has lower prices. Let us investigate the case  $p_4^L > p_3^L$ . By the the same logic as for uniform prices (proof to Proposition 1), it is profitable to switch prices, as only the same three cases can occur because  $R$  sets identical prices for its two closest customers. Moreover, the same logic holds for  $p_1^L$  and  $p_2^L$ . Consequently, the price order is such that  $p_1^L \leq p_2^L$  and  $p_3^L \leq p_4^L$ .

It is left to establish that  $p_2^L \leq p_3^L$ . Let us first show that  $p_1^L \leq p_2^L \leq p_4^L$ . Note that customers 2 and 4 cannot be the residual customer for  $L$  given the rationing rule according to customer surplus and costs. Only one of the customers 1 and 3 can become the residual customer as the difference of the price of  $R$  and  $L$  is weakler larger for those customers and the costs for  $L$  of serving ese customers 1 and 3 strictly lower than of serving the other customers 2 and 4 respectively..

Suppose  $p_4^L < p_2^L$  and that the capacity constraint is binding for  $L$  (this occurs when  $L$  has lower prices than  $R$  for all customers). In that case either customer 4 or 2 is rationed (given  $p_3^L \leq p_4^L$  and  $p_1^L \leq p_2^L$ ). If customer 2 is rationed (which implies that  $L$  serves customer 4, but not 2), then it is profitable to increase  $p_4^L$  to  $p_2^L$  as this ensures that a higher price  $p_2^L$  is realized at the lower costs for customer 2. If instead customer 4 is rationed, increasing  $p_4^L$  has no effect on profits, whereas a lower  $p_4^L$  reduces profits strictly if it results in customer 2 being rationed (recall that  $p_2^L > p_4^L$  and lower costs for customer 2). In summary, there is a strict incentive to increase  $p_4^L$  as long as it is not certain that customer 4 is rationed; once this is certain there is still a weak incentive.

Suppose  $p_4^L < p_2^L$  and that the capacity constraint of  $L$  is not binding. In that case increasing  $p_4^L$  increases the expected profits of  $L$ . This follows from the marginal profit of  $L$  with respect to  $p_4^L$  : As  $L$  faces equilibrium strategies of  $R$  which all have the property  $p_4^R = p_3^R$ , and  $F_3^R = F_4^R$  is designed such that  $L$  is indifferent over  $p_3^L$  (the marginal profit



(4) for  $i = 3$  is zero),  $L$  has a strict incentive to increase  $p_4^L$  for which it has larger costs (the marginal profit (4) for  $i = 4$  is positive). Thus, there is an incentive to increase  $p_4^L$  if it is below  $p_2^L$  up to the point where it is certain that customer 4 is rationed in a situation where the prices of  $R$  are all above the prices of  $L$ .

However, if it is certain that customer 4 is the one that is rationed in all situations where  $L$  is capacity constrained, the capacity constraint never binds for the first three customers. In that case  $p_3^L$  is chosen according to the marginal condition. This condition ensures that  $L$  is indifferent over  $p_3^L$  (the marginal profit (4) for  $i = 3$  is zero), such that it is always a best response to increase  $p_3^L$  up to  $p_2^L$ . This established that it is not profitable for a firm to set prices that are not weakly increasing in response to the price distributions of the candidate equilibrium.  $\square$

*Remark 1.* There are no equilibria with  $p_4 = v$  as a pure strategy. In those equilibria each firm would not be capacity constrained in any price realization. As a consequence, the firms would find themselves in the Bertrand logic, with only prices equal to a lower bound being played in equilibrium. This is not a stable situation.

## 5 Endogenous capacities

In this section we investigate which capacity levels firms choose before competing in prices. For this we first determine the profits obtained in case of asymmetric capacity levels, in particular when one firm has three and the other firm two units of capacity. We then introduce volatile demand and finally discuss the capacity choices of a social planner and cartelizing firms.

### 5.1 Asymmetric capacities: Firm L has 3 units, firm R 2 units

We first characterize an asymmetric pure strategy equilibrium.

Consider that firm  $L$  charges prices of  $p_1^L = p_2^L = v$  and  $p_3^L = p_4^L = p < v$ , while firm  $R$  charges prices of  $p_1^R = p_2^R = p_3^R = p_4^R = p$ . According to the rationing rules, firm  $R$  serves customers 3 and 4 as the firms charge equal prices for these customers and firm  $L$  has the lowest transport costs. Firm  $R$  could deviate by charging prices of  $v$  to customers 3 and 4 (or making no offers), and prices just below  $v$  to customers 1 and 2. It would make a profit of approximately  $2v - 7c$  from serving customers 1 and 2. This is not profitable if  $2v - 7c < 2p - 3c \implies p > v - 2c$ . Moreover, firm  $R$  could deviate by making an offer only to customer 4 at a price of  $v$  to obtain a profit of  $v - c$ . This is not profitable if  $2p - 3c > v - c \implies p > \frac{1}{2}v + c$ . Taken together this implies  $p > \max(v - 2c, \frac{1}{2}v + c)$ . The two expressions on the right hand side are equal at  $v = 6c$ .

Firm  $L$  serves customers 1 and 2 at a profit of  $2v - 3c$ . It could deviate by charging prices of just below  $p$  to all customers and would serve customers 1, 2 and 3 at a profit of  $3p - 6c$ . This deviation is not profitable if  $3p - 6c < 2v - 3c \implies p < \frac{2}{3}v + c$ . Together with the other inequality, this implies  $\frac{2}{3} \cdot v + c > p > \max(v - 2c, \frac{1}{2}v + c) \implies 9c > v$ . For

not too large differences between costs  $c$  and willingness to pay  $v$  we thus have pure strategy equilibria with a profit of  $2v - 3c$  for firm L with 3 units of capacity and profits in the range of  $\max(2v - 7c, v - c)$  to  $\frac{4}{3} \cdot v - c$  for firm R with 2 units. The maximum profit of  $R$  is smaller than the profit of  $2v - 3c$  which a firm obtains when each firm has 2 units of capacity. The profit of firm  $L$  with 3 units of capacity is still  $2v - 3c$ , which is the same profit as when each firm has two capacity units.

Consider now the incentives of firm  $R$  to increase its capacity so that each firm has 3 units. In this case the profit of each firm is  $v - c$ , which is the lower bound of the profit firm  $R$  obtains with 2 units when firm L has 3. In a game where firms simultaneously choose between 2 or 3 units of capacity and the third unit of capacity is costless, there can be an equilibrium in which both firms have 3 units of capacity each. However, this equilibrium would be in weakly dominated strategies. It is thus more plausible that each firm has two units of capacity when satisfying total demand requires 4 units.

**Lemma 5.** *Consider that each firm has 2 units of capacity, total demand is 4 units and  $v < 9c$ . It is not strictly profitable for a firm to increase its capacity to 3 units if firms play the equilibrium characterized above, even if capacity is costless.*

## 5.2 Capacity choices of competing firms when demand is volatile

So far we considered the case that demand is constant at 4 units. Let us now consider the case that demand is volatile: With probability  $\alpha$  demand is at 1 unit per customer, and with probability  $1 - \alpha$  demand is at 1.5 units per customer. We assume that in the latter case serving each of the 4 customers requires 1.5 units of capacity, with both the willingness to pay and transport costs being scaled up by 50%. With 2 units of capacity a firm can thus only serve 1 customer and would need one more unit to serve a second customer.

Consider for a moment that total demand is always at 6 units ( $\alpha = 0$ ). As increasing capacity until the total market is served does not decrease the price level (as argued in Subsection 3.2), a capacity level of 3 units per firm would thus be the likely outcome if capacity is costless or not too expensive.

For  $0 < \alpha < 1$ , a firm deciding between 2 and 3 units of capacity – given the other firm has 3 units – faces the following trade-off: In low states of demand (4 units of total demand), 3 units of capacity reduce profits from (up to)  $\frac{4}{3} \cdot v - c$  to  $4 - c$ , so by up to  $\frac{1}{3}v$  (as argued in Subsection 5.1 above). Instead, in high states of demand (6 units), an additional unit of capacity allows to serve the second closest customer at a profit of  $\frac{3}{2}(v - 2c)$ . If capacity is costless, it is thus profitable to add a third capacity unit if  $-\alpha \frac{1}{3}v + (1 - \alpha) \left(\frac{3}{2}v - 3c\right) > 0 \implies \alpha < \frac{9(v-2c)}{11v-18c}$ . For the lower bound of  $v = 5c$ , this yields  $\alpha < \frac{27}{37}$ , and  $\alpha < \frac{7}{9}$  at the upper bound  $v = 9c$ .<sup>6</sup> The high state of demand in which the capacity is fully utilized can thus be a rather seldom event, and overcapacity in a low state of demand a comparably frequent event. It is thus sufficient that the high state of demand is a rather seldom event for the

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<sup>6</sup>Recall that  $v = 9c$  is the upper bound for which we have established the 2 : 3 capacity equilibrium characterized in Lemma 5.

firms to each have 3 instead of 2 units of capacity – if a third unit of capacity is not too expensive to obtain.

Adding additional capacity beyond 3 units (weakly) decreases profits in both states of demand, even if capacity is costless.

**Conjecture 1.** *The case of overcapacity (modeled as a total demand of 4 units and a capacity of 3 units per firm) appears to be a plausible and possibly typical market situation when demand is sometimes higher than usually, and our results for this case thus relevant.*

## 6 Subcontracting between the two firms

In the mixed strategy equilibria described above the transportation costs are inefficiently high. With positive probability a firm that has free capacity and is closest to a specific customer does not serve that customer. There is thus scope to subcontract the delivery to the firm which has the lowest costs for this customer. The firm that initially has won the contract would still charge the customer the agreed price and compensate the efficient firm for delivering the product to that customer. The resulting efficiency rent can be shared among the two firms, yielding a Pareto-improvement.

In what follows we first set up a game with subcontracting where the above intuition fully carries through because the allocation of the customers to suppliers according to the rationing rules takes place before subcontracting. Afterward, we set up an alternative game where subcontracting takes place before rationing to show that firms sometimes prefer to not engage in subcontracting to relax competition.

### 6.1 Subcontracting after rationing

Consider the game as set up so far (subsection 3.1) and add an additional stage after customers are allocated to the suppliers according to the rationing rules. In the new sequence

- [1] suppliers set customer prices,
- [2] customers are allocated to suppliers – according to prices and capacity constraints,
- [3] subcontracting among suppliers may take place.

In stage [3] firms can agree to subcontract the supply of a customer to the other firm, which then serves the customer from its location and incurs the associated transportation costs. Subcontracting reduces total costs by the cost difference between the two firms for each subcontracted customer. Depending on the expected payments between the firms, subcontracting changes the perceived cost when competing for a specific customer. We follow Kamien et al. (1989) in assuming that the firms make take-it-or-leave-it offers in two different versions: the contract is either offered by

- (a) the firm that has won the customer, or

- (b) the firm that has lost the customer but has lower costs for that customer.

Let us solve this new game for SPNE using backward induction. First consider the third stage. As customers are allocated to suppliers in stage [2], in stage [3] firms know who has won which customer. Firms thus know all relevant information in stage [3]. The only possible effect of a subcontract in this stage is that it reduces production costs. This yields

**Lemma 6.** *If a firm has the lowest transport costs for a customer, has not won the customer in the second stage and has free capacity, subcontracting of that customer will always take place in equilibrium as it yields a Pareto-improvement for both firms.*

The behavior of customers in stage [2] is not affected subsequent subcontracting: they optimally accept the offers with the lowest prices and are rationed according to the rationing rules as described in subsection 3.1. When firms set prices to customer in stage [1], they anticipate two things:

- if a firm wins a customer, the costs of serving that customer may be lower than its own transport costs, due to subcontracting (case (a)), and
- if a firm loses a customer due to a higher price, it may nevertheless receive revenues from supplying this customer through a subcontract (case (b)).

#### (a) Winner of the price game determines the subcontract

Consider now case (a) that the winner of a customer decides on the terms of the contract. The winner can make an offer that extracts the additional rent due to saving in transport costs. As a consequence, the perceived costs become  $2c$  and  $c$  instead of  $3c$  and  $4c$  for the two most distant customers, which can be efficiently supplied by the other firm through a subcontract. The (perceived) costs of serving the two closest customers are still  $c$  and  $2c$ . Consequently, the effective costs for the first and the fourth customer are now the same, as well as the costs for the second and the third customer.

If a firm does not win one of its closest two customers as it charged a higher price than the competitor, it could nevertheless serve these through a subcontract. However, being such a cross supplier does not yield any additional rent when the winner offers the contract.

Let us consider the case of uniform prices. Each firm plays a uniform price schedule with identical prices for all four customers. Denote the uniform price of firm  $i$  by  $p^i$ . Three types of outcomes can arise: Both firms have the same prices for all customers, one firm has lower prices, and one firm has higher prices for all customers ( $p^L = p^R$ ,  $p^L < p^R$  and  $p^L > p^R$ ). If one firm has the lowest price for all costumers, it will serve its three closest customers according to the rationing rule. Recall that in case of identical prices our rationing rule chooses according to the costs of the firm. For the same reason a firm serves its closest customer as residual demand if it has higher prices than its competitor.

Let us describe the payoffs of each firm in the different outcomes. If the two firms have identical prices, each firm realizes a profit of  $2p^i - 3c$  by winning the closest two customers. If

$p^i < p^j$ , for  $j \neq i$ , firm  $i$  serves its closest three customers with a profit of  $p^i - c + 2(p^i - 2c)$ , while its competitor  $j$  makes a profit of  $p^j - c$ .

A pure strategy equilibrium in uniform prices does not exist. The profit from unilaterally undercutting any price  $p^j$  of the competitor is strictly positive if  $p^i - 2c > 0$ . Hence the only candidate of a pure strategy equilibrium is  $p^L = p^R = 2c$ , which yields a per firm profit of  $4c - 3c = c$ . However, it is always more profitable to deviate and charge a price of  $v$  as this ensures a profit of  $v - c$  from serving the residual customer. Recall that  $v > 4c$  by the assumption that all customers can be potentially served by each firm. Similarly, also an asymmetric pure strategy equilibrium cannot exist. This excludes the existence of pure strategy equilibria.

In the next step we characterize the symmetric mixed strategy equilibrium of the game. Denote the price distribution played by firm  $R$  (and also firm  $L$  in equilibrium) by  $G(p)$  as cdf and  $g(p)$  as pdf of  $p^R$ . In turn, the expected profit of firm  $L$  can be expressed as a function of the uniform price  $p$  and is

$$\begin{aligned}\pi^L(p) &= (1 - G(p)) [(p - c) + 2(p - 2c)] + G(p)(p - c) \\ &= (3p - 5c) - G(p)(2p - 4c).\end{aligned}$$

Suppose that the equilibrium price distribution  $G$  is atomless over the support  $[\underline{q}, v]$ . In the symmetric equilibrium firm  $L$  must be indifferent over all prices in the support. This yields the marginal condition

$$\frac{\partial \pi^L}{\partial p} = 3 - g(p)(2p - 4c) - 2G(p) = 0$$

which has to hold for all  $p \in [\underline{q}, v]$ . The ordinary differential equation can be solved for  $G(p)$  using that  $G(v) = 1$  and  $G(\underline{q}) = 0$ . This yields

$$G(p) = \frac{3p - v - 4c}{2p - 4c}. \quad (10)$$

The lowest price that is played in equilibrium is

$$\underline{q} = \frac{1}{3}v + \frac{4}{3}c.$$

As both firms play a price of  $v$  with positive density but with zero mass, the profit of each firm in the symmetric mixed strategy equilibrium is equal to the profit that a firm makes by serving only the residual customer at a price of  $v$ . This implies expected profits of  $\pi^L = \pi^R = v - c$ . The expected profits are as in the uniform price equilibrium without subcontracting. However, with subcontracting there is no inefficiency as there is always an ex post agreement that the firm with the lower costs serves each customer. The total costs are  $2(c + 2c)$ , whereas without subcontracting and uniform prices the expected total costs are  $2(c + 0.5 \cdot 2c + 0.5 \cdot 3c)$ , which is larger by  $c$ . Thus total surplus increases by  $c$  through subcontracting. As firm profits are the same, customer surplus – the difference between total

surplus and firm profits – also increases by  $c$ .

**Proposition 3.** *If we restrict strategies to uniform prices and the winner chooses the terms of the contract (case (a)), there is no pure strategy equilibrium when firms subcontract after rationing. In the unique symmetric mixed strategy equilibrium each firm has an expected profit of  $v - c$ . There is no inefficiency, and total surplus as well as expected consumer surplus is larger by  $c$  than in the case without subcontracting.*

If the firm that has won a customer by charging a lower price than its competitor is able to subcontract at its terms, it can extract all surplus and has a stronger incentive to win. Firms charge lower prices in expectation and all the additional surplus from subcontracting and realizing lower transportation costs are passed onto customers. Compared to no subcontracting, lower prices are sometimes played in equilibrium (the price support now starts at a lower value:  $q < p$ ).

Note that we have imposed the assumption that firms charge uniform prices for all customers. In the next step we derive some results that shed light on the robustness of this result if firms can price differentiate between the different customers.

*Remark 2.* If the winner chooses the terms of the contract and firms can charge different prices, there is no pure strategy equilibrium. In a symmetric mixed strategy equilibrium firms charge non-uniform prices.

*Proof.* The only candidate equilibrium with pure strategies is such that each firm sets prices equal to its perceived marginal costs. Note that the two firms now have the same perceived costs for each customer. Consider the strategy profile that each firm sets a price that is equal to its perceived marginal costs. This means that both firms set a price of  $c$  to customer 1 and 4 and a price of  $2c$  to customers 2 and 3. Undercutting would imply serving the market below costs, which is less profitable, while increasing all prices to  $v$  ensures a profit of  $v - c$  and is always profitable.

Uniform prices are not stable as increasing prices for the second or third closest customer ensures that that firm serves the customers with the lowest perceived costs in case of rationing.  $\square$

### **(b) Loser of the price game determines the subcontract**

When the supplier which has set the higher price for a customer offers the subcontract (version (b)), the residual profit is  $v$ . Note that a firm that charges a price of  $v$  to all customers does not only win a residual customer as the other supplier is capacity constrained, but in any case serves its two closest customers, either directly or through a subcontract. The cross supplier can now extract the efficiency rent of a subcontract, which is the cost difference of the two suppliers for the subcontracted customer. The equilibrium profit in any mixed strategy equilibrium must thus be at least as large as the residual demand profit and the efficiency rent. In case of uniform prices, a supplier can ensure a profit of  $v - c$  from serving the closest customer as residual demand, and an efficiency rent of  $c$  from cross supplying

the second closest customer. Consequently, the expected equilibrium profit is larger when the loser chooses the terms of the contract. Customer surplus is reduced compared to no subcontracting as total surplus increases by  $c$ , but profits of both firms together by  $2c$ .

**Proposition 4.** *If the loser chooses the terms of the contract, there is no pure strategy equilibrium (both for uniform and customer-specific prices). If we restrict strategies to uniform prices, in any mixed strategy equilibrium the expected profit is at least  $v$ . In such an equilibrium customer surplus is lower than without subcontracting. Such a mixed strategy equilibrium exists.*

*Proof.* Pure strategy equilibria with individual prices do not exist as reasoned in Subsection 3.4; the undercutting profit is the same and the deviation profit even higher by  $c$ ; hence there is no pure strategy equilibrium for  $v > 4c$ . Undercutting a uniform price with a slightly lower uniform price is profitable as long as the price is above the marginal costs  $3c$  of serving the third closest customer. The profit at symmetric prices of  $3c$  is  $6c - 3c = 3c$ . The profit that can be assured unilaterally by each firm is  $v$ . The strategy that ensures this is to set a price of  $v$  to all customers. By the rationing rule, in case of uniform prices, the competitor wins its closest three customers. Consequently, the supplier with a price of  $v$  serves its closest customer at a profit of  $v - c$  and gets a subcontract for the second closest customer. For that customer it can extract the complete cost difference of  $c$ . The deviation profit of  $v$  is larger than the profit  $3c$  in the pure candidate equilibrium given the assumption that all customers are contestable ( $v > 4c$ ). Customer surplus must be lower as total surplus increases by  $c$  through cost efficiencies and producer surplus increases by  $2c$ , which yields a reduction in customer surplus of  $c$ .

In the next step we characterize the symmetric mixed strategy equilibrium of the game. Denote the price distribution played by firm  $R$  (and also firm  $L$  in equilibrium) by  $G(p)$  as cdf and  $g(p)$  as pdf of  $p^R$ . In turn, the expected profit of firm  $L$  can be expressed as a function of the uniform price  $p$  is

$$\begin{aligned}\pi^L(p) &= (1 - G(p)) [(p - c) + 2(p - 2c)] + G(p)(p - c + c) \\ &= (3p - 5c) - G(p)(2p - 5c).\end{aligned}$$

Suppose that the equilibrium price distribution  $G$  is atomless over the support  $[\underline{q}, v]$ . In the symmetric equilibrium firm  $L$  must be indifferent over all prices in the support. This yields the marginal condition

$$\frac{\partial \pi^L}{\partial p} = 3 - g(p)(2p - 5c) - 2G(p) = 0$$

which has to hold for all  $p \in [\underline{q}, v]$ . The ordinary differential equation can be solved for  $G(p)$  using that  $G(v) = 1$  and  $G(\underline{q}) = 0$ . This yields

$$G(p) = \frac{3p - (v - 5c)}{2p - 5c}. \tag{11}$$

The lowest price that is played in equilibrium is  $\underline{q} = \frac{1}{3}v + \frac{5}{3}c$ . □

## 6.2 Subcontracting before rationing

So far we considered the sequence

- [1] suppliers set prices for customers,
- [2] allocation of customers to suppliers – according to prices and capacity constraints,
- [3] subcontracting among suppliers.

In this sequence, subcontracting is always a Pareto-improvement for the two suppliers if a supplier has won a customer for which it has the higher transportation costs and the other supplier has free production capacity, given the previous allocation of customers.

Let us now consider the sequence that

- (i) suppliers set prices for customers – as before,
- (ii) subcontracting among suppliers – in anticipation of customer allocation,
- (iii) allocation of customers to suppliers – according to prices and capacity constraints.

In that sequence, subcontracting still yields an efficiency rent for the suppliers if one has won a customer for which it has the higher transportation costs and the other supplier has free production capacity. However, for the cross-supplier there is now the potential disadvantage that a competitor receiving a cross supply has one more unit of capacity, which can be used to supply another customer. Consider that supplier  $R$  sets a price of  $v$  for all customers, and supplier  $L$  a strictly lower price  $v - x$  to all customers. Without subcontracting,  $L$  will simply supply customers 1, 2 and 3 up to its capacity limit, and supplier  $R$  serves the residual customer 4. Consider that supplier  $R$  agrees to supply customer 3 by means of a subcontract with  $L$ . Now supplier  $L$  has one more unit of capacity. As a consequence, customer 4 will be allocated to supplier  $L$  in stage (iii) as  $L$  charges a strictly lower price and – due to the subcontract – still has an unused unit of capacity. This implies that supplier  $R$  would forego its residual demand profit of  $v - c$ . Indeed, in anticipation of this, the suppliers could still agree to also subcontract for customer 4 to save the cost difference  $4c - c$ . This cost saving is, however, only a side-effect of the cross supply for customer 3, as otherwise firm  $L$  would be capacity constrained and firm  $R$  would serve customer 4 anyway – at its low transportation costs. The only effective cost saving is thus that of  $3c - 2c = c$  for customer 3. This needs to be traded off against the lost revenues when in turn firm  $L$  serves customer 4 at a price of  $p_L = v - x$  instead of firm  $R$  serving that customer at the higher price  $p_R = v$ . Taken together, a cross supply can only yield a Pareto-improvement for the suppliers when the cost saving on customer 3 is higher than the lost revenue on customer 4:  $c > p_R - p_L$ .

We proceed again by analyzing the two cases of either the cross supplier or the receiving supplier obtaining the full surplus of the cross supply and characterize the resulting equilibria.



**(a) Winner of the price game determines the subcontract**

With uniform prices played by the other firm according to the cdf  $F(p)$ , the expected profit of a supplier is given by

$$\begin{aligned} \pi(p) = & p - c + (1 - F(p))(p - 2c) + (1 - F(p))(p - 3c) \\ & + (F(p + c) - F(p)) \left( c - \int_p^{p+c} x \frac{f(x)}{(F(p + c) - F(p))} dx + p \right). \end{aligned}$$

The last term is new here compared to the case without cross supplies. It states that with a probability of  $(F(p + c) - F(p))$  the other firm sets a price in between  $p$  and  $p + c$ . In this case the cost saving of  $c$  on the third closest customer is larger than the foregone revenue of the cross supplier on the most distant customer. The expected lost revenue for this case is difference of the average price in the range  $p$  to  $p + c$  and  $p$ .

**Proposition 5.** *If subcontracting takes place before rationing, and if the winner determines the contract, and only uniform prices can be played, there is a symmetric mixed strategy equilibrium with an atomless price distribution with an upper bound of  $v$ . The expected profit in this symmetric equilibrium is  $v - c$ , customers benefit from subcontracting, but not all efficient cross supplies are realized.*

*Proof.* The existence of a symmetric mixed strategy equilibrium of the pricing game follows from Theorem 6 in Dasgupta and Maskin (1986). The conditions are met as the action space is a compact and non-empty subset of the real numbers, the sum of the suppliers' pay-offs is continuous, and individual pay-offs are bounded, and weakly lower semi-continuous, with a strict inequality at the point of symmetry. The Theorem 6 implies that all possible points of discontinuity are atomless. As every price can be a point of discontinuity, the whole price distribution must be atomless.

Note that the sum of the suppliers' pay-offs is  $3 \min(p_L, p_R) + \max(p_L, p_R) - 7c$  without subcontracting and  $4 \min(p_L, p_R) - 6c$  in case of subcontracting. These profits are equal if players are indifferent between subcontracting and no subcontracting. Individual profits are weakly lower semi-continuous as they do not jump downwards within the price support, except where the prices of both suppliers are equal.

Price  $v$  is played with positive density. Suppose to the contrary that there was an upper boundary  $\bar{p} < v$  in the symmetric equilibrium without mass points. The profit of such a price is  $\bar{p} - c < v - c$ , such that it is profitable to move density to set a price of  $v$  instead of  $\bar{p}$ . The profit at a price of  $v$  is the residual demand profit  $v - c$ , as the receiver of a subcontract obtains the associated rent. As the profit must be the same on the whole support, the equilibrium profit is  $v - c$ .

Not all efficient subcontracting takes place as the range of the support is larger than  $c$ . Thus price differences larger than  $c$  occur with positive probability. In these cases subcontracting does not take place as the firms lose more in revenues than they gain in cost reductions. Note that the smallest possible price is  $\underline{p}$  without subcontracting as defined in

(??). The smallest possible price with subcontracting is even lower as the profit at the upper bound is still  $v - c$ , and the profit at the lower bound is higher with subcontracting. The price difference  $v - \underline{p}$  is already larger than  $c$  given  $v \geq 4c$  as  $v - \underline{p} = \frac{1}{3}(2v - 5c)$ .  $\square$

In this setting customers gain from subcontracting through two channels:

1. Each subcontract reduces the price paid by the customer for which the firm with the lower price is then no longer capacity constrained.
2. Suppliers set lower prices as they have effectively lower costs of serving the third closest customer.

The second point is true because in equilibrium firms pass all efficiency gains from subcontracting on to their customers, as their expected profits remain at  $v - c$ , whereas total surplus is higher. As firms only engage in subcontracting when their joint surplus increases, they must on average set lower prices for their profit to remain constant in spite of lower costs.

### (b) Loser of the price game determines the subcontract

With uniform prices played by the other firm according to the cdf  $F(p)$ , the expected profit of a supplier is given by

$$\begin{aligned} \pi(p) = & p - c + (1 - F(p))(p - 2c) + (1 - F(p))(p - 3c) \\ & + (F(p) - F(p - c)) \left( c + \int_{p-c}^p x \frac{f(x)}{(F(p) - F(p - c))} dx - p \right). \end{aligned}$$

The last term is new here compared to the case without cross suppliers. It states that with a probability of  $(F(p) - F(p - c))$  the other firm sets a price in between  $p - c$  and  $p$ . In this case the cost saving of  $c$  on the second closest customer is larger than the foregone revenue on the closest customer (which is otherwise served by the loser as residual demand). The expected lost revenue for this case is the difference of  $p$  and the average price in the range  $p - c$  to  $p$ .

The expected profit of a firm choosing a price of  $v$  is

$$\pi(v) = v - c + (1 - F(v - c)) \left( c + \int_{v-c}^v x \frac{f(x)}{(F(v) - F(v - c))} dx - v \right),$$

which defines the expected equilibrium profit. Note that the second term is the efficiency gain minus the positive externality on customers, due to the price reduction for the closest customer. The size of that term increases in the density of prices close to  $v$ .

**Proposition 6.** *If subcontracting takes place before rationing, and if the loser proposes the subcontract, and only uniform prices can be played, there is a symmetric mixed strategy equilibrium with an atomless price distribution that includes  $v$  in the support of the price distribution. The expected profit in this symmetric equilibrium is between  $v$  and  $v - c$ . If  $v > 4.5c$ , not all efficient subcontracts are realized.*

*Proof.* The first part of the proof is virtually identical to that of Proposition 5. The highest price that is played with positive density is  $v$ . The expected profit at this price is bounded from above by  $v - c + c$ , where the first part is the residual demand profit and the second part the rent from subcontracting in the limit when the price of the competitor converges to  $v$ . The lower bound can be found if the subcontracting profit, which yields additional rents to the firm with higher prices, is excluded. That profits is  $v - c$ . The lowest price  $p'$  that is played must yield the same pay-off as the largest price. To obtain the minimum range of the support, the maximum value of  $p'$  is obtained by equalizing the associated profit with the upper bound profit at a price of  $v$ , that is  $v = 3p' - 6c \Leftrightarrow p' = v/3 + 2c$ . Hence, the range  $v - p' = \frac{2}{3}v - 2c$  is at least  $c$  if  $v \geq 4.5c$ . For this case clearly not all efficient subcontracting takes place as the range of the support is larger  $c$ .  $\square$

The expected profit increases by the net gain of subcontracts for a price of  $v$ , at the top of the price distribution:

$$\pi(p) = v - c + (1 - F(v - c)) \left( c - \left( \int_{v-c}^v x f(x) dx \right) + p \right).$$

## 7 Discussion

### Inefficient competition and efficient cartels?

We have shown that certain pricing pattern emerge in equilibrium. In particular, prices weakly increase in the costs of serving the different customers. This, together with limited over-capacities, yields the outcome that a firm typically serves its closest customers. However, customers in between competing firms are often served by a firm with high costs, although there is a firm with lower costs and free capacities. Importantly, this occurs although location and customer specific pricing is feasible and firms do price differentiate in equilibrium. The reason is that competition results in unstable prices. As the one competitor does not know which prices the other competitor will ultimately charge, there is strategic uncertainty and different prices are charged each time.

Price competition with limited overcapacities can thus lead to an inefficient customer allocation. One might therefore wonder whether a cartel among the suppliers would lead to a more efficient market outcome. Indeed, suppliers could benefit from a market sharing agreement such that each supplier only serves the customers for which it has to lowest transport costs. However, there are at least two potential disadvantages of such a cartel. First, the suppliers could also agree on higher customer prices. Although this might not be detrimental to total welfare if demand is inelastic as in our model, it might well do so if demand is elastic, and would in any case hurt customers. Second, the cartelists might have excessive incentives to invest in capacity. For example, consider again the case that total demand is 4 units with certainty. The plausible equilibrium is that each firm has two units of capacity, even it is costless, as a third unit of capacity would not increase profits (Lemma 5). However, if

cartelists bargain over the distribution of the cartel profits, how much each cartelist gains in comparison to the situation of no cartel might well matter for the outcome (for instance in case of Nash-bargaining). By investing in a third unit of capacity, a cartelist does not reduce its own competitive profit, but does reduce the competitive profit of the other firm (see section 5). As a consequence, the cartelist with the higher capacity will obtain more of the cartel profits and a firm has incentives to acquire a third unit of capacity, even it is costly (but not too costly). These costs clearly reduce welfare as they do not increase output or efficiency, but only redistribute profits among the cartelists. Similar observations have been made by for instance Davidson and Deneckere (1990); Fershtman and Gandal (1994). Moreover, empirical findings indicate that cartels are associated with excessive capacities (see for instance Normann and Tan (2014)).

### **Cross supplies**

As argued above, a cartel can reduce transport costs and improve allocative efficiency, but has potentially severe detrimental effects. A potentially better solution to increase allocative efficiency are cross supplies among the suppliers. However, also cross supplies are a two-sided sword. On the one hand, they can clearly increase efficiency. On the other hand, they can also dampen competition as a firm that anticipates to become a cross-supplier has less incentives to aggressively compete for that customer in the first place. We find that cross suppliers do not harm customers through higher prices if the cross-supplier earns no (or a low) margin on its cross-supplies. As a consequence, cross-supply arrangements where the cross supplier will sell to a competitor at marginal costs tend to be pro-competitive, while arrangements which foresee that the cross-supplier earns a significant profit from carrying out another cross supply might restrict competition. What matters here is the profit obtained from an additional cross supply. A cross-supplier may well be remunerated when signing a framework agreement that foresees cross-supplies at costs in certain situations. However, cross supplies may not always take place and thus inefficiencies may persist just because firms fear additional competition. In particular, when an almost capacity constrained firm asks the unconstrained firm for a cross supply to a customer which that firm can supply more efficiently, the unconstrained firm may deny this as such a supply would leave the demanding firm with additional capacity. This again can intensify competition for other customers.

### **Bertrand-Edgeworth arguments in competition policy**

Various competition policy cases deal with homogeneous products that have significant transport costs and where location or customer based price discrimination is common.<sup>7</sup> Some cases make explicit references to Bertrand-Edgeworth models, but without taking geographic differentiation and customer specific pricing into account. For instance, in relation to the merger OUTOKUMPU / INOXUM in 2012 the European Commission (Commission) noted that “one of the main criticisms of the Notifying Party of this [Bertrand-Edgeworth] model

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<sup>7</sup>For instance the major merger case M.7252 HOLCIM / LAFARGE of the European Commission dealt with cement, concrete and other basic materials.

of aggressive competition is that it tends to predict more competitive prices pre-merger than the observed pre-merger price”. The Commission acknowledged that there may be frictions which have not been accounted for, such as that “customers may have other preferences for a specific supplier (e.g. geographic proximity, preferences for a specific suppliers products based on quality concerns or experiences; or on-going business relationships or contracts etc)”.<sup>8</sup> Our model allows for both customer specific transport costs as well as prices and thus can aid the analysis in future cases.

Similarly, our model can help in assessing whether firms in a market compete with each other or coordinate their sales activities. For instance, in M.7009 HOLCIM / CEMEX WEST the Commission considered “that the most likely focal point for coordination in the cement markets under investigation would be customer allocation whereby competitors refrain from approaching rivals’ customers with low prices. Under such a coordination scenario, the sizable transport costs for cement would lead to a general allocation of customers based on proximity to a given plant. The Commission has thus investigated the hypothesis that cement competitors might face limited incentives to enter significantly into competitors’ geographic strongholds...”.<sup>9</sup> The Commission concludes that “given the low level of differentiation across suppliers and the existing overcapacities, it is difficult to explain the observed level of gross margins as being the result of competitive interaction between cement suppliers.”<sup>10</sup> As a supporting argument, the Commission refers to a Bertrand Edgeworth model with constant marginal costs and uniform pricing.<sup>11</sup>

Our model makes several predictions which can be related to the above reasoning. In particular, even with overcapacities of 50% we find that firms always serve their nearest customers (home market), and that at prices above the costs of the closest competitor. Moreover, firms set higher prices in the home markets of rival firms, although a unilateral undercutting there seems rational in view of their over-capacities. Such a pattern is difficult to reconcile with previous models. In case of a model without capacity constraints, one would expect that prices equal the marginal costs of the firm with the second lowest costs (asymmetric Bertrand competition). Instead, the typical Bertrand-Edgeworth model with capacity constraints and uniform pricing does not explain why more distant customers are charged higher prices. One might thus be tempted to conclude that if one observes such a pattern of less aggressive prices in the firms’ home markets and no significant switching of suppliers in spite of significant overcapacities, firms must at least tacitly coordinate their sales activities in that market. With our model we can instead explain such a pattern among competing firms. Of course, such a pattern may also be observed in case of collusion. To answer the question whether firms are indeed colluding or competing, a model based on more realistic assumptions about the underlying market, such as geographic differentiation and location specific pricing, could therefore improve the reliability of the analysis. In particular, we have characterized equi-

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<sup>8</sup>See recitals (725) and (407) in the Commission’s decision in M.6471 OUTOKUMPU / INOXUM, and more generally to Annex IV for the Bertrand-Edgeworth modelling.

<sup>9</sup>See recitals (167) and (168).

<sup>10</sup>Recital 178

<sup>11</sup>See the European Commission decision M.7009 HOLCIM / CEMEX WEST, fn. 195.

librium price distribution which make it possible to compute competitive price-cost margins for the various geographic market segments.

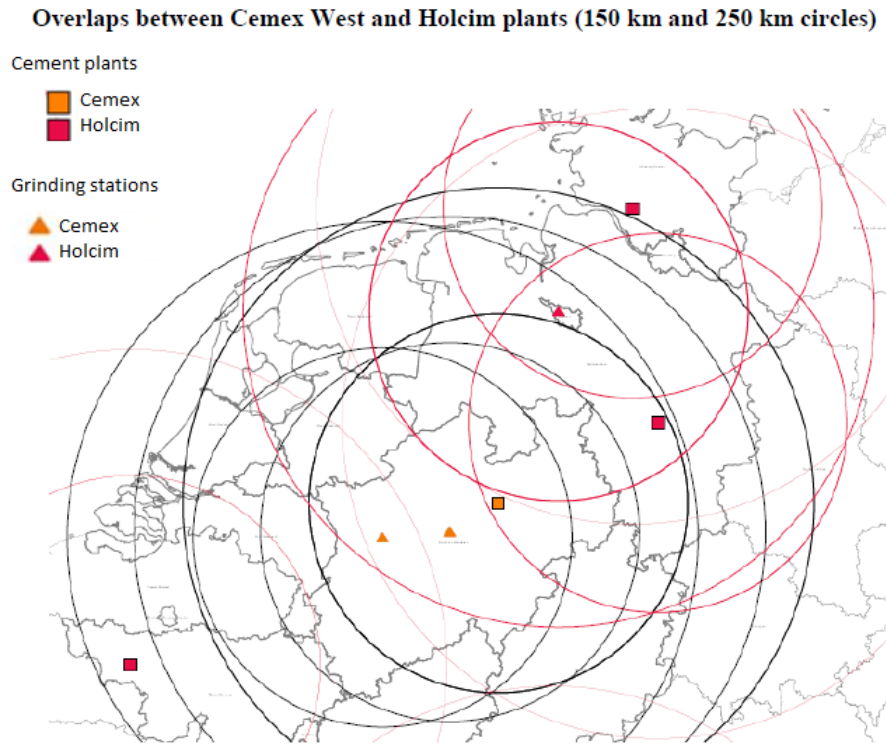


Figure 2: Overlap map – taken from European Commission decision M.7009 Holcim / Cemex West.

## 8 Conclusion

We have characterized mixed-strategy equilibria in case of capacity constrained price competition, transportation costs and customer-specific pricing. We have analyzed the case of two symmetric firms that are only differentiated by their location and thus transport costs to the customers, which are located on a line in between them.

Although we allow for price discrimination, uniform prices for different customers for which firms have different transportation costs can be sustained in equilibrium when the transport costs are not too high. If transport costs are sufficiently high, firms indeed prefer to charge location specific prices. These prices have the feature that they increase in the distance between each firm and the respective customer. This yields the outcome that firms always serve their nearest customers (“home markets”), and that at prices above the costs of the competitor. Instead, further away customers are served either by the one or the other firm, as firms play mixed strategies in prices. This yields both volatile market shares and a productive inefficiency, as not always the firm with the lowest cost serves each customer.

Furthermore, we discuss ex-post subcontracting between the firms, which is attractive if a firm with excess capacity does not serve all the customers for which it has the lowest costs.

However, even ex-post subcontracts may restore efficiency only partly. Firms sometimes do not cross supply each other as this can intensify competition by relaxing the receiver's capacity constraint. Additionally, subcontracting lowers or increases consumer prices depending on whether the firm with the lower price, or the firm with the lower costs obtains the efficiency rent from subcontracting.

## Annex: Explicit price strategy for strictly increasing prices

In this section we present an example of an equilibrium price strategy for the case of strictly increasing prices ( $5c < v < 7c$ ). In particular, we illustrate that a firm can draw prices from a joint distribution such that the resulting price vectors are always weakly increasing with  $p_1 = p_2 \leq p_3 = p_4$ , with marginal distributions  $F_c$  for the two closest and  $F_d$  for the two most distant prices, as defined in (7) and (8).

Suppose that firm  $L$  initially draws a price  $p_1 \in [\underline{p}, v]$  from the distribution function  $F_c$ . It then sets  $p_2 = p_1$  as also  $p_2$  must be played according to the marginal distribution  $F_c$  in equilibrium and  $p_1 = p_2$  is a requirement of the equilibrium strategies.

Recall that  $F_c(p)$  equals  $F_d(p)$  for  $p \in [4c, v]$ , which is only consistent with uniform price vectors. Firm  $L$  thus sets also the other prices  $p_3$  and  $p_4$  equal to  $p_1$  in this interval.

For  $p_1 \in [\underline{p}, 4c)$ , firm  $L$  faces the problem that only playing uniform prices is not consistent with the marginal distributions. In particular,  $F_d$  first-order stochastically dominates  $F_c$  in that range. In particular, it is the case that there is a price  $\tilde{p} \in [\underline{p}, 4c]$ , such that  $f_c(\tilde{p}) = f_d(\tilde{p})$  and  $f_c(p) > f_d(p)$  for  $[\underline{p}, \tilde{p})$ , and reversely  $f_c(p) < f_d(p)$  for  $(\tilde{p}, 4c]$ : small prices of close customers are played more often than small prices of distant customers. What firm  $L$  now can do is the following:

1. For each realized price  $p_1 \in [\underline{p}, \tilde{p})$ ,
  - (a) with probability  $\alpha \equiv f_d(p_1)/f_c(p_1)$  set uniform prices  $p_1 = p_2 = p_3 = p_4$ .
  - (b) with probability  $1 - \alpha$  set prices  $p_3$  and  $p_4$  in the interval  $[\tilde{p}, 4c]$  according to the density function  $f_{relocate} = \frac{f_d(p) - f_c(p)}{F_d(4c) - F_c(4c) - (F_d(\tilde{p}) - F_c(\tilde{p}))}$ .
2. For each realized price  $p_1 \in [\tilde{p}, 4c]$ ,
  - (a) with probability 1 set uniform prices  $p_1 = p_2 = p_3 = p_4$ .

In case of low prices below  $\tilde{p}$ , the firm draws a higher price from  $f_{relocate}$  with probability  $(1 - \alpha)$  according to step 1 (b). This density function is constructed in a way that density for distant prices is allocated from any point in the lower interval  $[\underline{p}, \tilde{p})$  to the upper interval  $[\tilde{p}, 4c)$  in proportion to the density  $f_d(p) - f_c(p)$ . This is the "missing" density when only uniform prices are played in response to realizations of  $p_1 \in [\tilde{p}, 4c]$  according to step 2 (a) of the above rule. As a consequence, the distant prices occur according to the marginal distribution function  $F_d$ . Note that the firm only plays strictly increasing prices in step 1 (b),

which happens with probability  $\int_{\underline{p}}^{\tilde{p}} f_1(p)(1 - \alpha)dp = F_1(\tilde{p}) - F_3(\tilde{p})$ . In summary, the joint distribution is characterized as follows: Only uniform prices are drawn in the upper part of the interval starting at  $4c$ . Uniform prices are also often played in the lower part of the interval. In order to ensure that the different marginal densities  $f_c$  and  $f_d$  in the lower part of the interval result, strictly increasing prices are drawn as described by the above rule.

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