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# Unilateral climate Policy and the Green Paradox: Extraction Costs matter

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**Abstract**

To analyze the effect of unilaterally tightened climate policies, we augment the two country model of Hoel (2011) with fossil fuel extraction costs that convexly increase in current extraction and with a decreasing stock. We find that the introduction of extraction costs give rise to more optimistic results. It turns out that a tighter climate policy of the country with the initially stricter policy causes neither an increase of early fossil fuel extraction (weak green paradox) nor an increase of pollution costs (strong green paradox) if the fossil fuel stock is sufficiently small. In case of a tighter climate policy in the country with the initially laxer policy, a weak green paradox depends on the price-elasticity of energy demand and the strength of flow- and stock-dependence of extraction costs. Furthermore, a weak green paradox does not directly give rise to a strong one, as the stock-dependence connotes a policy driven reduction of total fossil fuel extraction. We find that total extraction reacts the stronger to policy a tighter climate policy the higher the initial fossil fuel tax rate or backstop subsidy, respectively.

*Keywords:* Climate change, green paradox, exhaustible resources, renewable energy

JEL classification: Q41; Q42; Q54; Q58

## 1. Introduction

Natural non-renewable resources have been the subject of economic research at least since the seminal work of Hotelling (1931). While Hotelling, Stiglitz (1974), Dasgupta and Heal (1974), Barbier (1999), and Tsur and Zemel (2005) focus on the problem of exhaustibility, later studies such as Farzin (1996), Hoel and Kverndokk (1996), Tahvonen

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(1997), Chakravorty et al. (2008), Chakravorty et al. (2012), Henriet (2012), Van der Ploeg and Withagen (2012), and Kollenbach (2015b) emphasize pollution caused by non-renewable resources. In this respect, fossil fuels have attracted most attention, as they are responsible for about 75% of greenhouse gas emissions and therefore the main driving force of global warming.[1] As shown by the non-global adoption of the Kyoto Protocol and the Paris Agreement, there is an international consensus about the goal of climate policy - the 2° target - but hardly a globally coordinated policy. Rather, different countries or regions, such as the European Union, act independently by introducing climate policies that affect mainly the demand side.[2] However, Sinn (2008a) and Sinn (2008b) have shown that ill-designed demand side climate policies may increase early $CO_2$ emissions instead of reducing them. This phenomenon, called the "green paradox", occurs because fossil fuel owners accelerate resource extraction as a response to a rapidly tightening climate policy to avoid selling their resources at heavily depressed prices in the future.

The green paradox concept has been studied in more detail by Gerlagh (2011), Grafton et al. (2012), Van der Ploeg and Withagen (2012) and Hoel (2013). This literature strand assumes a single economy, which can be interpreted as the whole world. Therefore, the analyzed climate policies are implicitly assumed to be the same across the world. Recently, a two-country, two-period model has been developed by Eichner and Pethig (2011). Ritter and Schopf (2014) extend this model by stock-dependent extraction costs. However, both studies do not consider a backstop technology. A two-country model with continuous time and a backstop is used by Hoel (2011) and Ryszka and Withagen (2014) to analyze the effects of climate policy changes. Hoel assumes that countries do not grow and are identical in all respects but climate policy. The respective instruments, a fossil fuel tax and a subsidy for a clean backstop, are exogenously given and time invariant. Since the utilization of fossil fuels causes pollution which is ignored by anyone but the governments, climate policy can be welfare enhancing. It is assumed that one (high-tax/high-subsidy) country levies a higher tax and/or grants a higher subsidy than the other (low-tax/low-subsidy) one. Hoel (2011) shows under which conditions a green paradox is caused by an increase of common or unilateral taxes and subsidies. Furthermore, he determines

[1] Cf. Hoel (2011) and Van der Ploeg and Withagen (2012).

[2] Some countries do not follow a specific climate policy at all. Others, like the EU and some US states are using a wide array of climate policy instruments such as carbon taxes, renewable energy subsidies, or emission trading. Cf. also Hoel (2011) for examples.

the welfare effect of the policy changes. According to Hoel, the increase of the backstop subsidy by the high-subsidy country at least increases early emissions. Furthermore, a tighter climate policy of the low-tax/low-subsidy country worsens the pollution problem, i.e the climate costs increase.

While Ryszka and Withagen (2014) assume constant marginal fossil fuel extraction costs, Hoel (2011) completely abstains from them. Consequently, Hoel states in his conclusion that he is "using an extremely simple model" and acknowledges that a stock independent and perfectly elastic fossil fuel supply function is a restrictive assumption. According to Farzin (1992), the knowledge about geology and extraction technology suggests that extraction costs are not only stock-dependent but also increasing in the current resource flow. Both stock- and flow-dependent extraction costs are applied in numerous studies.[3] The present paper augments Hoel's (2011) model by both.

It turns out that extraction costs may give rise to more optimistic results. On the one hand, we find that a tighter climate policy of the high-tax/high-subsidy country does not cause a green paradox if the fossil fuel stock is sufficiently small, so that the country uses both energy sources in the moment of the policy change. On the other hand, both a tighter climate policy of the low-tax/low-subsidy country and an increase of a common subsidy increase the stock left in situ. Depending on the initial tax or subsidy rate, respectively, the relative reduction of total extraction is large enough to guarantee decreasing climate costs.

The outline of the paper is as follows. In section 2 we describe the model. The equilibrium on the fossil fuel market is discussed in section 3, while section 4 presents the fossil fuel extraction path. The effects of climate policy changes are analyzed in section 5. Subsequently, in section 6 we discuss the policy implications of our results. Section 7 concludes.

## 2. Model

Our analysis is based on the model of Hoel (2011), which we augment with fossil fuel extraction costs.[4] In the following the assumptions of the model are briefly discussed. Consider an economy consisting of the two countries 1 and 2. Energy generation rests on

[3] Cf. Farzin (1992), Farzin (1995), Farzin (1996), Farzin and Tahvonen (1996), Grafton et al. (2012), Tsur and Zemel (2003), Tsur and Zemel (2005), and Kollenbach (2015a).

[4] To increase the comparability, we adopt the notation of Hoel (2011).

the utilization of exhaustible fossil fuels (oil) $x$ and a perfect renewable substitute referred to as backstop (solar power) $y$. The latter is available in both countries at all points of time. Both energy sources are supplied competitively on a global energy market.

### 2.1. Energy supply

The representative backstop firm exhibits the supply cost function $f(y) = by$, which is linear in backstop supply. Therefore, every desired backstop amount will be supplied in period $t$ if the energy producer price $p(t)$ equals or exceeds the backstop unit costs $b > 0$. In case of fossil fuels, we assume that $n$ identical fossil fuel firms sell the resource on the world market. Each firm is endowed with the stock $g_0$, so that the whole economy has a fossil fuel endowment of $G_0 = ng_0$. The stocks decrease in extraction according to[5]

$$\dot{g} = -\chi, \tag{1}$$

$$\dot{G} = -x, \tag{2}$$

with $\chi$ denoting fossil fuel extraction of the representative firm and $x = n\chi$ representing extraction in the whole economy. The costs of extracting fossil fuels are given by the twice differentiable function $M(g(t), \chi(t))$.[6] Thus, extraction costs of the representative firm depend on both the remaining stock $g(t)$ and the current extraction $\chi(t)$. As we abstain from fixed costs, $M(g(t)), 0) = 0$ holds. The cost function convexly increases in current extraction, i.e. $M_\chi > 0$ and $M_{\chi\chi} \geq 0$. Furthermore, $M_g < 0$ and $M_{gg} \geq 0$, so that the costs convexly increase with a declining stock. Finally, we assume that the marginal extraction costs are the higher the lower the remaining stock, i.e. $M_{\chi g} = M_{g\chi} < 0$, but $M_{jkl} = 0$, $j, k, l = g, \chi$.[7] The intertemporal optimization problem of the representative fossil firm is given by

$$\max_{\chi(t)} \int_0^\infty e^{-rt}[p(t)\chi(t) - M(g(t), \chi(t))]dt \quad \text{s.t.} \quad \int_0^\infty \chi(t)dt \leq g_0 \quad \text{and} \quad g(t), \chi(t) \geq 0, \tag{3}$$

with $r$ denoting the time preference rate. The corresponding current-value Lagrangian reads

$$L = p(t)\chi(t) - M(g(t), \chi(t)) - \tau(t)\chi(t) + \zeta_\chi(t)\chi(t) + \zeta_g(t)g(t), \tag{4}$$

[5]The notation $\dot{z}$ is used to describe the change of the arbitrary variable $z$ in time, i.e. $\dot{z} = \frac{dz}{dt}$. Analogously, $\ddot{z} = \frac{d^2z}{dt^2}$.

[6] For the sake of simplicity, we omit the time index $t$ if not necessary for understanding.

[7]The extraction cost function properties assumed here are those originally used in Farzin (1992, 1995, 1996). With respect to the flow-dependence, see also Tsur and Zemel (2003) and Tsur and Zemel (2005) and with respect to the stock-dependence, Krautkraemer (1998), Van der Ploeg and Withagen (2012), Tahvonen (1997), and Tahvonen and Salo (2001).

with $\tau$ as the shadow price (costate variable) of the fossil fuel stock and $\zeta_i,\ i = \chi, g$ as the Lagrange-multiplier of the non-negativity conditions $\chi(t) \geq 0$ and $g(t) \geq 0$. The necessary conditions for an inner solution connote[8]

$$p(t) = \tau(t) + M_\chi(g(t), \chi(t)), \tag{5}$$

$$\dot{\tau}(t) = r\tau(t) + M_g(g(t), \chi(t)). \tag{6}$$

At every point in time, (5) implicitly determines the fossil fuel supply function

$$\chi^s(t) = \chi^s(p(t), \tau(t), g(t)). \tag{7}$$

According to (5), the sum of scarcity rent and marginal extraction costs equals the fossil fuel producer price. If the price determined by the fossil fuel market falls short of this level, the representative producer will cease extraction. (6) states the modified Hotelling-rule.

The transversality conditions, which determine the point in time $0 < T \leq \infty$ at which fossil fuel supply vanishes forever, read[9]

$$p(T)\chi_T - M(g_T, \chi_T) - \tau(T)\chi_T \left\{ \begin{array}{c} = \\ \geq \end{array} \right\} \tau(T)\chi_T \text{ if } \begin{cases} 0 < T < \infty \\ T = \infty \end{cases}, \tag{8}$$

$$\tau(T) \geq 0,\ \tau(T)g_T = 0, \tag{9}$$

with $g_T$ as the remaining fossil fuel stock and $\chi_T$ as fossil fuel extraction at time $T$. Following Farzin (1992), we focus on the economical but not on the physical exhaustion of the fossil fuel stock, i.e. $g_T > 0$. In this case, (9) connotes $\tau(T) = 0$, i.e. the non-extracted fossil fuel stock has no value. Consequently, (8) can be written as

$$p(T)\chi_T - M(g_T, \chi_T) \left\{ \begin{array}{c} = \\ \geq \end{array} \right\} 0 \text{ if } \begin{cases} 0 < T < \infty \\ T = \infty \end{cases}. \tag{10}$$

### 2.2. Energy demand and climate costs

Total energy demand equals the sum of demand of country 1 and country 2. Both countries are inhabited by a large number of identical individuals. The representative

[8] The complementary slackness conditions are $\zeta_\chi(t) \geq 0,\ \zeta_\chi(t)\chi(t) = 0$ and $\zeta_g(t) \geq 0,\ \zeta_g(t)g(t) = 0$.

[9] Cf. Feichtinger and Hartl (1986), Satz 7.6. See also Seierstad and Sydsaeter (1987), page 337, theorem 4. $T = 0$ is ruled out by assumption.

individual of country $i = 1, 2$ exhibits the well-behaved utility function $u_i(x_i + y_i)$, which depends on the sum of fossil fuel $x_i$ and backstop use $y_i$ in country $i$, with $u_i' > 0$ and $u_i'' < 0$. The corresponding energy demand function is denoted by $D_i(Q_i)$, satisfying $u_i'(D_i(Q_i)) = Q_i$, where $Q_i$ is the consumer price of energy in country $i$. Due to $u_i'' < 0$, the demand function decreases in the price, i.e. $D_i'(Q_i) < 0$.

Burning fossil fuels unleashes $CO_2$ emissions, which accumulate in the atmosphere. $S(t)$ denotes the stock of emissions that exceeds the pre-industrial level at time $t$. On the one hand, the stock increases in current fossil fuel burning. On the other hand, it decreases with the rate $\gamma$ due to natural regeneration. Thus, from $\omega$ units emitted at time $t$ only $\omega e^{-\gamma(\mu-t)}$ remain in the atmosphere at time $\mu > t$. The corresponding climate cost function is denoted by $C(S(t))$. Following Hoel (2011), we assume $C_S(S) > 0$ and $C_{SS}(S) = 0$, i.e. a linear increasing climate cost function. Hoel shows that the social costs of carbon $v(t)$, i.e. the social costs of one additional carbon unit emitted at time $t$, read $v = \frac{1}{r+\gamma} C_S$. Therefore, the total climate costs of fossil fuel extraction are

$$\Omega = v \int_0^\infty e^{-rt} x(t) dt. \tag{11}$$

$\Omega$ may give rise to climate policy of the two countries. Each country applies its own policy by taxing fossil fuels and subsidizing the backstop. The finite fossil fuel unit tax of country $i = 1, 2$ is denoted by $q_i \geq 0$ and the finite backstop unit subsidy by $\sigma_i \geq 0$. Following Hoel (2011), we assume that the taxes and subsidies are exogenously given and constant in time.[10] The taxes and subsidies connote the following relations between the consumer and producer prices for country $i = 1, 2$,

$$Q_i^x(t) = p(t) + q_i, \tag{12}$$

$$Q_i^y(t) = b - \sigma_i, \tag{13}$$

where $Q_i^x(t)$ and $Q_i^y(t)$ refer to the consumer prices of fossil fuel and the backstop, respectively.[11] To ensure positive fossil fuel extraction at early points in time, we assume

[10] Neither the reason for nor the optimal level of climate policy is subject of this paper. According to Hoel (2011), the socially optimal tax for all countries equals $v$. Suppose there are $m$ countries, that are identical in all aspects. If there is no internationally coordinated climate policy, the optimal individual tax rate of each country reads $\frac{v}{m}$. However, there are many political reason, e.g. lobbying, high information costs or rent-seeking behavior, why tax rates differ among countries and are not equal to the optimal level.

[11] Recall that fossil fuel and the backstop are perfect substitutes.

$M_\chi(g_0, 0) = 0 < b - q_i - \sigma_i$, i.e. the first marginal fossil fuel unit is cheaper than the backstop in both countries.

Climate costs are only one component of a country's welfare. According to Hoel (2011), welfare of country $i = 1, 2$ is described by

$$W_i = \int_0^\infty e^{-rt} \Big[ u_i(x_i(t) + y_i(t)) - b y_i(t) + p(t) \left( n_i \chi(t) - x_i(t) \right) - n_i M(g(t), \chi(t)) \Big] dt - \beta_i \Omega, \tag{14}$$

where $n_i$ denotes the number of fossil fuel firms located in country $i$ and $\beta_i$ is the country's share of climate costs.[12] Thus, $\sum_i n_i = n$ and $\sum_i \beta_i = 1$. The first two terms in the square brackets indicate utility from energy net of backstop supply costs. The third term is the trade balance of fossil fuels. By defining $\alpha_i := \frac{n_i}{n}$ as the share of fossil fuel firms in country $i$, the term can be rewritten as $p(t)(\alpha_i x(t) - x_i(t))$. The fourth term gives the share of fossil fuel extraction costs. $\beta_i \Omega$ denotes the climate costs of country $i$.

### 2.3. Objects of investigation

By disregarding extraction costs, i.e. by using the additional assumption $M(g, \chi) = 0$, Hoel (2011) analyzes the effects of exogenously changed climate policies (higher fossil fuel taxes or backstop subsidies) and backstop costs. To present his results, we adopt the definition of a weak and a strong green paradox made by Gerlagh (2011).

**Definition 1** *Suppose the climate policy of at least one country is intensified.*

*a) A weak green paradox is defined as an increase of fossil fuel extraction in early periods.*

*b) A strong green paradox is defined as an increase of total climate costs $\Omega$.*

Among others, Hoel's results read as follows[13]

(i) Suppose the carbon tax is increased in the country that initially has the lowest tax. Then, a strong green paradox occurs if the energy demand function is sufficiently price-inelastic.

[12] A different impact of climate costs on welfare and an uneven distribution of fossil fuel firms among countries are possible explanations for disparate climate policies. A country facing a high $\beta_i$ but a low $n_i$ might be more willing to tax fossil fuels than a country with low climate costs and much fossil fuel reserves.

[13] Cf. Hoel (2011) proposition 1, 3, 4, 7, 9, and page 861. We focus on results that are changed by our consideration of extraction costs. Thus, the propositions 2 and 10, which are robust to extraction costs, are omitted. Since they refer to an exogenous decrease of backstop costs $b$, which we do not analyze, the propositions 5 and 6 are also omitted.

(ii) Suppose the carbon tax is increased in the country that initially has the lowest tax. Then, total welfare for both countries declines if the social costs of carbon $v$ are sufficiently high and the energy demand function is sufficiently price-inelastic.

(iii) Suppose a common backstop subsidy is increased. If the difference in carbon taxes is small, a strong green paradox occurs.

(iv) If fossil fuel taxes and backstop subsidies are equal among countries and the common tax does not exceed the Pigovian rate $v$, an increase of the common subsidy reduces welfare in both countries.

(v) A strong (weak) green paradox occurs if the subsidy in the country with the initially lowest (highest) subsidy is increased.

In the following we investigate how the consideration of extraction costs $M(g, \chi)$ alters the results (i) - (v). For this purpose, we analyze how unilateral changes of climate policies affect the fossil fuel extraction path. Consequently, the equilibrium on the fossil fuel market at every point in time and the resulting fossil fuel extraction path are presented in the following before we turn to the effect of a tighter climate policy.

## 3. The energy and fossil fuel markets

At every point in time, the energy and fossil fuel markets incorporate the energy demand functions $D_i(Q_i(t))$ of both countries, the aggregated fossil fuel supply function

$$x^s(p(t), \tau(t), g(t)) = n\chi^s(p(t), \tau(t), g(t)), \tag{15}$$

and the backstop supply function. As shown in section 2, every desired backstop amount is supplied if the energy producer price $p(t)$ equals or exceeds marginal backstop costs $b$. To determine and illustrate the equilibria on the fossil fuel and energy market, we substitute $Q_i$ by (12) and take notice of (13). Thus, fossil fuel demand $B_i$ in country $i = 1, 2$ reads

$$B_i(p) = \begin{cases} D_i(p + q_i) & \text{if } p < b - q_i - \sigma_i, \\ B_i \in [0, D_i(b - \sigma_i)] & \text{if } p = b - q_i - \sigma_i, \\ 0 & \text{if } p > b - q_i - \sigma_i. \end{cases} \tag{16}$$

If the producer price of fossil fuel $p$ falls short of the fossil fuel reservation price $b - q_i - \sigma_i$, energy demand $D_i(p+q_i)$ in country $i$ is solely satisfied by fossil fuels. However, if $p$ equals

the reservation price, the representative individual is indifferent between fossil fuels and the backstop. Thus, fossil fuel demand $B_i$ takes on any value of the interval $[0, D_i(b-\sigma_i)]$, such that the energy market is cleared.[14] In other words, energy demand can be satisfied by a mix of both fossil fuel and the backstop. Finally, fossil fuel demand of country $i$ vanishes if the producer price exceeds the reservation price.

To state the aggregated fossil fuel demand function, we follow Hoel (2011) by assuming that the climate policy of one country, say country 1, is stricter than the policy of the other country, i.e. $q_1 + \sigma_1 > q_2 + \sigma_2$. Therefore, country 1 (2) is called the *high-tax* (*low-tax*) country. The aggregated fossil fuel demand function is given by

$$A^x(p) = \begin{cases} D_1(p+q_1) + D_2(p+q_2) & \text{if} \quad p < b - q_1 - \sigma_1, \qquad (I) \\ B_1 + D_2(p+q_2) & \text{if} \quad p = b - q_1 - \sigma_1, \qquad (II) \\ D_2(p+q_2) & \text{if} \quad \begin{cases} p > b - q_1 - \sigma_1 \\ p < b - q_2 - \sigma_2 \end{cases}, \ (III) \\ B_2 & \text{if} \quad p = b - q_2 - \sigma_2, \qquad (IV) \\ 0 & \text{if} \quad p > b - q_2 - \sigma_2, \end{cases} \tag{17}$$

whereas the aggregated energy demand function reads

$$A(p) = \begin{cases} D_1(p+q_1) + D_2(p+q_2) & \text{if} \quad p < b - q_1 - \sigma_1, \qquad (I) \\ D_1(b-\sigma_1) + D_2(p+q_2) & \text{if} \quad p = b - q_1 - \sigma_1, \qquad (II) \\ D_1(b-\sigma_1) + D_2(p+q_2) & \text{if} \quad \begin{cases} p > b - q_1 - \sigma_1 \\ p < b - q_2 - \sigma_2 \end{cases}, \ (III) \\ D_1(b-\sigma_1) + D_2(b-\sigma_2) & \text{if} \quad p \geq b - q_2 - \sigma_2. \qquad (IV) \end{cases} \tag{18}$$

The equilibrium on both the fossil fuel and the energy market at any point in time $t$ can be illustrated by using (17) and the fossil fuel supply function (15), as shown in Fig. 1.[15] In the depicted case, the fossil fuel supply function $x^s(p(t), \tau(t), g(t))$ intersects part (II) of the aggregated fossil fuel demand function, so that the producer price of fossil fuel equals

[14] Note that $B_i$ is a function of time, i.e. $B_i = B_i(p(t), t)$, as explained below. To avoid an inconvenient notation, the time argument has been suppressed. Cf. footnote 6.

[15] Graphically, one gets the aggregated supply function by adding up the single functions described by (5). Note that the $p$-intercept is the same for the aggregated function and the single functions, as the extraction costs of the first marginal fossil fuel unit $M_\chi(g(t), 0)$ are the same for all firms.

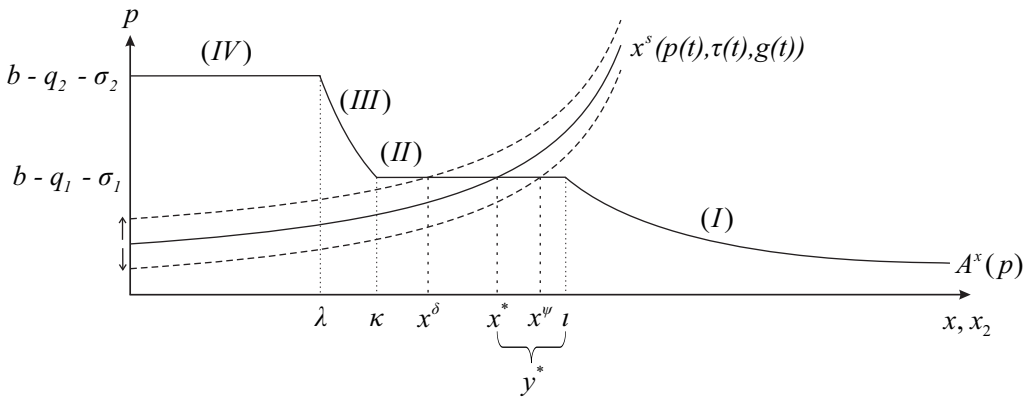



**Figure 1:** Energy market equilibrium at time $t$

the reservation price in the high-tax country $b - q_1 - \sigma_1$. Therefore, total energy demand $A(b - q_1 - \sigma_1)$ is given by $\iota$. The energy demand of the low-tax country is solely satisfied by fossil fuels, i.e. $D_2(b - q_1 - \sigma_1 + q_2) = B_2 = \kappa$. In contrast, energy demand of the high-tax country $D_1(b - \sigma_1) = \iota - \kappa$ is satisfied by a mix of fossil fuels and the backstop. We assume that as much fossil fuel is used as could be supplied at marginally lower costs (including the scarcity rent) than the backstop. Thus, we get that fossil fuel demand of the high-tax country is given by $B_1 = x^* - \kappa$, while the remaining energy demand of this country $D_1(b - \sigma_1) - (x^* - \kappa)$ is satisfied by backstop utilization $y^* = \iota - x^*$.

If the fossil fuel supply function intersects part (I) of the aggregated fossil fuel demand function, the fossil fuel producer price would fall short of the reservation price in the high-tax country. Consequently, only fossil fuel is used in both countries, i.e. $A(p) = A^x(p) = D_1(p + q_1) + D_2(p + q_2)$.

In case that the fossil fuel supply function intersects part (III), the fossil fuel producer price exceeds the reservation price in the high-tax country but falls short of the reservation price in the low-country. Therefore, energy demand of the high-tax country is solely satisfied by the backstop, i.e. $D_1(b - \sigma_1) = y_1 = \iota - \kappa$. In contrast, the low-tax country only uses fossil fuels, so that we get $D_2(p - q_2) = B_2$.

Finally, an intersection at part (IV) of the aggregated fossil fuel demand function connotes the sole utilization of backstop in the high-tax country, i.e. $D_1(b - \sigma_1) = y_1 = \iota - \kappa$. The energy demand of the low-tax country is satisfied by a mix of fossil fuels and backstop. Thus, $D_2(b - \sigma_2) = B_2 + y_2 = \lambda$, where $B_2 \in [0, D_2(b - \sigma_2)]$. The results are summarized in Table 1.

If climate policy does not change in time, the aggregated fossil fuel demand function

| | (I) | (II) | (III) | (IV) |
|---|---|---|---|---|
| $B_1$ | $D_1(p+q_1)$ | $\in [0, D_1(b-\sigma_1)]$ | 0 | 0 |
| $B_2$ | $D_2(p+q_2)$ | $D_2(b-q_1-\sigma_1+q_2)$ | $D_2(p+q_2)$ | $\in [0, D_2(b-q_2)]$ |
| $y_1$ | 0 | $D_1(b-\sigma_1)-B_1$ | $D_1(b-\sigma_1)$ | $D_1(b-\sigma_1)$ |
| $y_2$ | 0 | 0 | 0 | $D_2(b-\sigma_2)-B_2$ |

**Table 1:** Fossil fuel and backstop utilization for the high-tax and low-tax country given an equilibrium on part (I), (II), (III) or (IV) of the aggregated fossil fuel demand function

is time invariant. In contrast, aggregated fossil fuel supply is a function of time, as (15) depends not only on the price $p$ but also on the scarcity rent $\tau(t)$ and the remaining fossil fuel stock $g(t)$. The scarcity rent evolves in time in accordance with (6), while the fossil fuel stock continuously decreases. Thus, at every point in time we get a new fossil fuel market equilibrium until fossil fuel extraction vanishes at time $T$. This set of fossil fuel market equilibria determines the fossil fuel extraction path.

To illustrate the evolution of the supply function in time we differentiate (5) with respect to $t$ for a given extraction rate $\chi$ and take account of (1) and (6)

$$\frac{dp}{dt} = r\tau(t) + M_g(g(t), \chi(t)) - M_{\chi g}(g(t), \chi(t)) \cdot \chi(t). \tag{19}$$

While the first and third term on the right hand side are positive, the second one is negative, so that the sign of the price change in time is unknown. Graphically, an inferior (dominant) second effect connotes an upward (downward) shift of the supply function in Fig. 1. An upward shifted supply function may also intersect part $(II)$ of the aggregated demand function, with $x^\delta$ as equilibrium fossil fuel extraction. In this case, fossil fuel use of the high-tax country $B_1 = x^\delta - \kappa$ falls short of its equivalent in the former equilibrium at $x^*$, while backstop utilization increases to $y^\delta = \iota - x^\delta$. If the second effect is dominant, the aggregated supply function is shifted downwards in the $(x, p)$ space. Consequently, the equilibrium fossil fuel quantity $x^\psi$ exceeds $x^*$, so that fossil fuel use of the high-tax country is increased to $B_1 = x^\psi - \kappa$, while backstop use is reduced to $y^\psi = \iota - x^\psi$. Thus, both discussed variants show that there can be time periods characterized by the simultaneous use of both energy sources by one country.[16] In both cases, energy demand of the low-tax country is still solely satisfied by fossil fuels, i.e. $D(b - q_1 - \sigma_1 + q_2) = B_2 = \kappa$.

[16]For an early analysis of the possibility of simultaneous use of an exhaustible resource and its substitute, see Farzin (1986). Chakravorty et al. (2006) finds that a ceiling on the stock of pollution can give

A similar argument holds with respect to the fossil fuel use of the low-tax country if the supply function $x^s(p(t), \tau(t), g(t))$ intersects part $(IV)$ of the aggregated demand function. However, if the supply function intersects part $(I)$ or $(III)$ of the demand function, a simultaneous use of both energy sources by one country is not possible.

## 4. The fossil fuel extraction path

To explicitly determine the extraction path and to analyze the effects of climate policy changes, we use the specific energy demand and fossil fuel extraction cost functions of Tab. 2. Fig. 2 illustrates an exemplary fossil fuel extraction path. Similar to the

| | Function |
|---|---|
| Energy demand country 1 | $D_1(Q_1(t)) = a_1 - c_1 Q_1(t)$ |
| Energy demand country 2 | $D_2(Q_2(t)) = a_2 - c_2 Q_2(t)$ |
| Fussil fuel extraction costs | $M(\chi(t), g(t)) = \theta\chi(t)^2 + \eta\chi(t)[g_0 - g(t)]$ |

**Table 2:** Energy demand and fossil fuel extraction cost functions

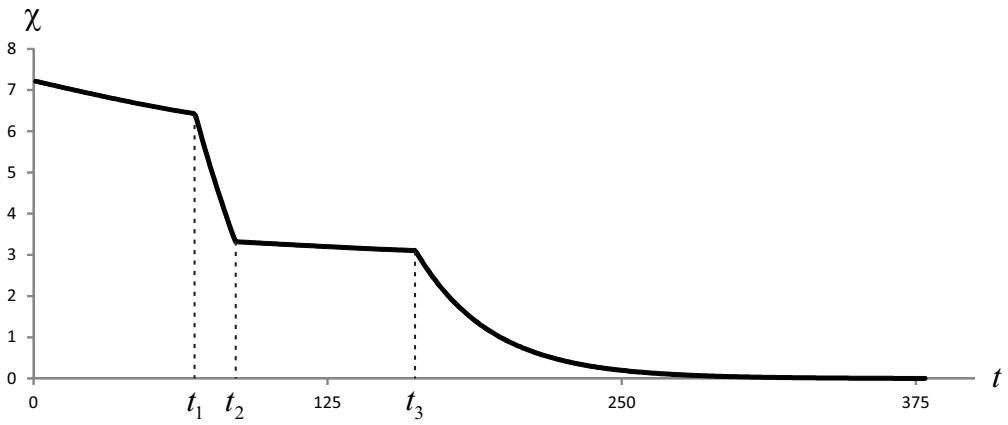



**Figure 2:** Fossil fuel extraction path for $g_0 = 1000$, $r = 0.05$, $b = 0.9$, $q_1 = 0.22$, $q_2 = \sigma_1 = \sigma_2 = 0.005$, $a_1 = a_2 = 4$, $c_1 = c_2 = 1$, $\theta = 0.01$, $\eta = 0.001$

aggregated fossil fuel demand function, the extraction path consists of four parts. The junction points are denoted by $t_1$, $t_2$ and $t_3$, so that, in terms of the notation of Fig. 1, $n\chi(t_1) = \iota$, $n\chi(t_2) = \kappa$ and $n\chi(t_3) = \lambda$. Therefore, we use the index $j = I, ..., IV$ to refer

rise to the simultaneous use of both energy sources without a flow-dependent fossil fuel extraction cost function.

to the different parts of the extraction path. Using the fossil fuel extraction cost function, $\dot{g}_j = -\chi_j$ and $\ddot{g}_j = -\dot{\chi}_j$, we can combine (5) and (6) to get[17]

$$\dot{p}_j = r[p_j + 2\theta\dot{g}_j - \eta(g_0 - g_j)] - 2\theta\ddot{g}_j, \;\; j = I, ..., IV. \tag{20}$$

According to (17), the energy producer price and fossil fuel demand depend on which part of the aggregated fossil fuel demand function $A^x(p)$ the market equilibrium is located. Substituting the energy demand functions of Tab. 2, taking note of $x_i(t) = n\chi_i(t)$ and rearranging gives

$$p = \begin{cases} \frac{a_1+a_2}{c_1+c_2} - \frac{c_1}{c_1+c_2}q_1 - \frac{c_2}{c_1+c_2}q_2 - \frac{1}{c_1+c_2}n\chi & \text{if } \iota \leq n\chi, \qquad (I) \\ b - q_1 - \sigma_1 & \text{if } \kappa \leq n\chi \leq \iota, \qquad (II) \\ \frac{a_2}{c_2} - q_2 - \frac{1}{c_2}n\chi & \text{if } \lambda \leq n\chi \leq \kappa, \quad (III) \\ b - q_2 - \sigma_2 & \text{if } n\chi \leq \lambda. \qquad (IV) \end{cases} \tag{21}$$

By substituting (21) and its derivative with respect to time into (20), we find that the evolution of the fossil fuel stock along part $(I), ..., (IV)$ of the aggregated demand function is described by the following second order linear inhomogeneous differential equations

$$\begin{aligned} &\ddot{g}_I - r\dot{g}_I - \frac{B}{A_I}g_I = \frac{C_I}{A_I}, \\ &\ddot{g}_{II} - r\dot{g}_{II} - \frac{B}{A_{II}}g_{II} = \frac{C_{II}}{A_{II}}, \\ &\ddot{g}_{III} - r\dot{g}_{III} - \frac{B}{A_{III}}g_{III} = \frac{C_{III}}{A_{III}}, \\ &\ddot{g}_{IV} - r\dot{g}_{IV} - \frac{B}{A_{IV}}g_{IV} = \frac{C_{IV}}{A_{IV}}, \end{aligned} \tag{22}$$

with

$$(a)\; A_I = \frac{n}{c_1 + c_2} + 2\theta, \qquad (b)\; C_I = r\left[\frac{a_1 + a_2}{c_1 + c_2} - \frac{c_1}{c_1 + c_2}q_1 - \frac{c_2}{c_1 + c_2}q_2 - \eta g_0\right], \tag{23}$$

$$(a)\; A_{II} = A_{IV} = 2\theta, \qquad (b)\; C_{II} = r[b - q_1 - \sigma_1 - \eta g_0], \tag{24}$$

$$(a)\; A_{III} = \frac{n}{c_2} + 2\theta, \qquad (b)\; C_{III} = r\left[\frac{a_2}{c_2} - q_2 - \eta g_0\right], \tag{25}$$

$$(a)\; B = r\eta, \qquad (b)\; C_{IV} = r[b - q_2 - \sigma_2 - \eta g_0]. \tag{26}$$

[17]Recall that $\ddot{z} = \frac{d^2z}{dt^2}$.

Solving the differential equations gives

$$g_I(t) = K_I^1 e^{\lambda_I^1 t} + K_I^2 e^{\lambda_I^2 t} - \frac{C_I}{B}, \tag{27}$$

$$g_{II}(t) = K_{II}^1 e^{\lambda_{II}^1 t} + K_{II}^2 e^{\lambda_{II}^2 t} - \frac{C_{II}}{B}, \tag{28}$$

$$g_{III}(t) = K_{III}^1 e^{\lambda_{III}^1 t} + K_{III}^2 e^{\lambda_{III}^2 t} - \frac{C_{III}}{B}, \tag{29}$$

$$g_{IV}(t) = K_{IV}^1 e^{\lambda_{IV}^1 t} + K_{IV}^2 e^{\lambda_{IV}^2 t} - \frac{C_{IV}}{B}, \tag{30}$$

where $K_j^1$ and $K_j^2$ are constants and where for $j = I, ..., IV$ and

$$(a)\ \lambda_i^1 = \frac{r}{2} + \left[\left(\frac{r}{2}\right)^2 + \frac{B}{A_i}\right]^{0.5} > 0, \qquad (b)\ \lambda_i^2 = \frac{r}{2} - \left[\left(\frac{r}{2}\right)^2 + \frac{B}{A_i}\right]^{0.5} < 0. \tag{31}$$

(27) - (30) determine the fossil fuel stock development along part $(I), ..., (IV)$ of the aggregated demand function and, therefore, the fossil fuel extraction path. At the junction points $t_1$, $t_2$ and $t_3$, the evolution smoothly switches from one part to the next, i.e. from $g_I(t)$ to $g_{II}(t)$ at $t = t_1$, from $g_{II}(t)$ to $g_{III}(t)$ at $t = t_2$ and from $g_{III}(t)$ to $g_{IV}(t)$ at $t = t_3$.[18]

## 5. Climate policy changes

Based on the results of section 4, we are going to analyze the effects of a tighter climate policy by assuming that one instrument, i.e. $q_1$, $q_2$, $\sigma_1$ or $\sigma_2$, is increased unilaterally.[19] At first, we turn to the case of a large fossil fuel endowment such that the initial supply function $x^s(p(0), \tau(0), g_0)$ intersects $(I)$ of the aggregated demand function is intersected. Consequently, both countries use only fossil fuel at time $t = 0$. Subsequently, we consider a fossil fuel endowment that is sufficiently small to guarantee an initial fossil market equilibrium along part $(II)$ of the demand function. In this case, the high-tax country uses a mix of both fossil fuel and the backstop.

### 5.1. Large fossil fuel endowment

Consider the case of a sufficiently large fossil fuel endowment, such that the initial fossil fuel market equilibrium is located on part $(I)$ of the aggregated demand function,

[18] Note that the first junction point only exists if the fossil fuel endowment is sufficiently large. Otherwise, fossil fuel extraction is initially described by $g_{II}(t)$, i.e. $g_0 = g_{II}(0)$. If the fossil fuel endowment is too small, country 1 only uses the backstop at all points in time. A sharper climate policy of country 1 has then no effect. Therefore, this case is neglected in the following.

[19] Following Hoel (2011), we abstain from discussing the reasons for policy changes.

i.e. $\chi(0) = \chi_I(0) = -\dot{g}_I(0)$ and

$$g_I(t_1) = g_{II}(t_1), \tag{32}$$

$$g_{II}(t_2) = g_{III}(t_2), \tag{33}$$

$$g_{III}(t_3) = g_{IV}(t_3) \tag{34}$$

hold at the junction points. Furthermore, using $p(t_1) = p(t_2) = b - q_1 - \sigma_1$, $p(t_3) = b - q_2 - \sigma_2$ and (21), we find that fossil fuel extraction at the junction points is described by

$$\chi_I(t_1) = \chi_{II}(t_1) = \frac{c_1 + c_2}{n}\frac{1}{r}(C_I - C_{II}), \tag{35}$$

$$\chi_{II}(t_2) = \chi_{III}(t_2) = \frac{c_2}{n}\frac{1}{r}(C_{III} - C_{II}), \tag{36}$$

$$\chi_{III}(t_3) = \chi_{IV}(t_3) = \frac{c_2}{n}\frac{1}{r}(C_{III} - C_{IV}). \tag{37}$$

From (27) and (30), we get that the initial and final fossil fuel stock and extraction can be written as

$$g_0 = K_I^1 + K_I^2 - \frac{C_I}{B}, \tag{38}$$

$$\chi_0 = -\lambda_I^1 K_I^1 - \lambda_I^2 K_I^2, \tag{39}$$

$$g_T = K_{IV}^1 e^{\lambda_{IV}^1 T} + K_{IV}^2 e^{\lambda_{IV}^2 T} - \frac{C_{IV}}{B}, \tag{40}$$

$$\chi_T = -\lambda_{IV}^1 K_{IV}^1 e^{\lambda_{IV}^1 T} - \lambda_{IV}^2 K_{IV}^2 e^{\lambda_{IV}^2 T}. \tag{41}$$

Finally, using $\tau(T) = 0$, (21) and the fossil fuel extraction cost function from Tab. 2, we can rewrite the transversality condition (10) as

$$(b - q_2 - \sigma_2)\chi_T - \theta\chi_T^2 - \eta\chi_T(g_0 - g_T) \left\{ \begin{array}{c} = \\ \geq \end{array} \right\} 0 \text{ if } \begin{cases} 0 < T < \infty \\ T = \infty \end{cases} \tag{42}$$

and (5) as

$$b - q_2 - \sigma_2 = 2\theta\chi_T + \eta(g_0 - g_T). \tag{43}$$

As shown in Appendix A.1, (30) and (40) - (43) connote that fossil fuel extraction lasts forever, as $T \to \infty$, but that extraction decreases in the long-run, since $\lim_{T\to\infty} \chi_T = 0$. Furthermore, we find $K_{IV}^1 = 0$ and

$$g_T = -\frac{C_{IV}}{B} = g_0 - \frac{b - q_2 - \sigma_2}{\eta}. \tag{44}$$

Thus, the stock left in situ depends on the backstop unit costs, the fossil fuel tax rate $q_2$ and the backstop subsidy $\sigma_2$ of the low-tax country and the stock-dependence of extraction costs covered by $\eta$.

The remaining system of equations (32) - (39) and (44) depends on the variables $K_I^1$, $K_I^2$, $K_{II}^1$, $K_{II}^2$, $K_{III}^1$, $K_{III}^2$, $K_{IV}^2$, $t_1$, $t_2$, $t_3$, and $\chi_0$. To identify the effect of a sharper climate policy, we differentiate the system of equations with respect to the tax and subsidy rates $q_1$, $q_2$, $\sigma_1$ and $\sigma_2$.

### 5.1.1. Tax increase of the high-tax country

At first, consider the case of a tax increase in the high-tax country, i.e. $dq_1 > 0$. By differentiating (32) - (37) and (44) with respect to $q_1$ and solving for $\frac{\partial g_T}{\partial q_1}$, $\frac{\partial \chi_0}{\partial q_1}$ and $\frac{\partial K_I^1}{\partial q_1}$, we find[20]

$$\frac{\partial g_T}{\partial q_1} = 0, \tag{45}$$

$$\frac{\partial \chi_0}{\partial q_1} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial q_1} + \lambda_I^2 \frac{1}{\eta}\frac{c_1}{c_1 + c_2}, \tag{46}$$

$$\frac{\partial K_I^1}{\partial q_1} = \frac{\left(\frac{1}{\frac{2\theta}{c_2}(c_1+n)+2\theta} - z_1\right) z_4 + \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\left[\frac{\Psi_\lambda}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_3)}\left(\frac{1}{2\theta + \frac{n}{c_2}}\Lambda^a + \frac{1}{\eta}\Lambda^b\right) + z_2 z_3\right]}{|\Phi_I|\Theta}, \tag{47}$$

where $z_1, z_3, \Phi_I, \Lambda^a, \Theta < 0$, and $z_2, z_4, \Psi_\lambda, \Lambda^b > 0$ are defined in Tab. 3.

According to (45), the higher tax does not affect total fossil fuel extraction, so that a sufficiently strong increase of early fossil fuel extraction will give rise to a strong green paradox. (46) shows that the increase of $q_1$ does not cause a weak green paradox if $\frac{\partial K_I^1}{\partial q_1} > 0$. However, as revealed by (47), the sign of $\frac{\partial K_I^1}{\partial q_1}$ is ambiguous, since the first term of the numerator and $\Lambda^b$ are positive, while $\Lambda^a$ and the last term of the numerator are negative. Hoel (2011) highlights the importance of a low price-elasticity of energy demand as a requirement for a green paradox in several cases.[21] However, the ambiguity is neither eliminated by a high nor by a low price-elasticity of demand, i.e. by high or low values of $c_1$ or $c_2$.

[20] See Appendix A.2 for the proof. Note that the $\dot{\chi}_i(j)$ denote the evolution of fossil fuel extraction along part $i = I, II, III, IV$ of the aggregated fossil fuel demand function at the junction points $j = t_1, t_2, t_3$. By definition, fossil fuel extraction decreases at these points, so that $\dot{\chi}_i(j) < 0$.

[21] Hoel (2011) refers to a price-inelastic fossil fuel demand function in his propositions 1, 3, 4, 6 and 8. The importance of demand elasticity is also stressed by Eichner and Pethig (2011) and Ritter and Schopf (2014).

$$z_1 = \lambda_I^2 \frac{1}{\eta} \frac{c_1 e^{\lambda_I^2 t_1}}{c_1 + c_2} < 0 \qquad z_2 = \frac{1}{\eta} \frac{c_1 e^{\lambda_I^2 t_1} + c_2}{c_1 + c_2} > 0$$

$$z_3 = \lambda_{II}^1 \lambda_{II}^2 \Phi_{II} \Lambda^a - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)} \Psi_{II} \Lambda^b < 0 \qquad z_4 = \Psi_{II}^a \Lambda^a - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)} \Phi_{II} \Lambda^b > 0$$

$$\Psi_\lambda = (\lambda_{II}^1 - \lambda_{II}^2)^2 e^{-(\lambda_{II}^1 + \lambda_{II}^2)(t_2 - t_1)} > 0 \qquad \Phi_I = e^{\lambda_I^2 t_1} - e^{\lambda_I^1 t_1} < 0$$

$$\Lambda^b = \frac{\lambda_{III}^1 \lambda_{III}^2}{\lambda_{IV}^2} \Phi_{III} \dot{\chi}_{IV}(t_3) - \Psi_{III} \dot{\chi}_{III}(t_3) > 0 \qquad \Lambda^a = \frac{\Psi_{III}^a}{\lambda_{IV}^2} \dot{\chi}_{IV}(t_3) - \Phi_{III} \dot{\chi}_{III}(t_3) < 0$$

$$\Theta = \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)} z_3 + \frac{\Psi_I}{\Phi_I} z_4 < 0$$

with

$$\Psi_I = \lambda_I^1 e^{\lambda_I^1 t_1} - \lambda_I^2 e^{\lambda_I^2 t_1} > 0$$

$$\Psi_{II} = \lambda_{II}^1 e^{-\lambda_{II}^1 (t_2 - t_1)} - \lambda_{II}^2 e^{-\lambda_{II}^2 (t_2 - t_1)} > 0 \qquad \Phi_{II} = e^{-\lambda_{II}^1 (t_2 - t_1)} - e^{-\lambda_{II}^2 (t_2 - t_1)} < 0$$

$$\Psi_{III} = \lambda_{III}^1 e^{-\lambda_{III}^1 (t_3 - t_2)} - \lambda_{III}^2 e^{-\lambda_{III}^2 (t_3 - t_2)} > 0 \qquad \Phi_{III} = e^{-\lambda_{III}^1 (t_3 - t_2)} - e^{-\lambda_{III}^2 (t_3 - t_2)} < 0$$

$$\Psi_{III}^a = \lambda_{III}^2 e^{-\lambda_{III}^1 (t_3 - t_2)} - \lambda_{III}^1 e^{-\lambda_{III}^2 (t_3 - t_2)} < 0 \qquad \Psi_{II}^a = \lambda_{II}^2 e^{-\lambda_{II}^1 (t_2 - t_1)} - \lambda_{II}^1 e^{-\lambda_{II}^2 (t_2 - t_1)} < 0$$

**Table 3:** Definitions of variables

### 5.1.2. Subsidy increase of the high-tax country

Consider now the case of a subsidy increase in the high-tax country, i.e. $d\sigma_1 > 0$. The differentiated system of equations reveals[22]

$$\frac{\partial g_T}{\partial \sigma_1} = 0, \tag{48}$$

$$\frac{\partial \chi_0}{\partial \sigma_1} = -(\lambda_I^1 - \lambda_I^2) \frac{\partial K_I^1}{\partial \sigma_1}, \tag{49}$$

$$\frac{\partial K_I^1}{\partial \sigma_1} = \frac{\frac{\dot{\chi}_I(t_1)}{\chi_{II}(t_1)} \frac{1}{\eta} z_3 + \frac{\Psi_\lambda}{\lambda_{II}^1 + \lambda_{II}^2} \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)} \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)} \left( \frac{1}{2\theta + \frac{n}{c_2}} \Lambda^a + \frac{1}{\eta} \Lambda^b \right) + \frac{1}{2\theta + \frac{n}{c_1 + c_2}} z_4}{|\Phi_I| \Theta}, \tag{50}$$

with $z_3, \Lambda^a, \Phi_I, \Theta < 0$ and $z_4, \Psi_\lambda, \Lambda^b > 0$, from Tab. 3.

Similar to the case of an increased tax $q_1$, (48) shows that total fossil fuel extraction is not affected. According to (49), no weak green paradox occurs if $\frac{\partial K_I^1}{\partial \sigma_1} > 0$. However, (50) reveals that the sign of $\frac{\partial K_I^1}{\partial \sigma_1}$ is ambiguous, as both the first and the last term of the numerator and the terms in parentheses are of opposite sign.

According to Hoel (2011), an increased subsidy of the high-tax country causes a weak green

[22] Cf. Appendix A.3 for the proof.

paradox if there are no extraction costs. The simple graphical argument of Appendix A.4 shows that this result also holds for purely flow-dependent extraction costs. Therefore, the stock-dependence of extraction costs counters a weak green paradox, as indicated by $\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\frac{1}{\eta}z_3 + \frac{\Psi_\lambda}{\lambda^1_{II}+\lambda^2_{II}}\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\frac{1}{2\theta+\frac{n}{c_2}}\Lambda^a$. If these terms outweigh $\frac{\Psi_\lambda}{\lambda^1_{II}+\lambda^2_{II}}\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\frac{1}{\eta}\Lambda^b + \frac{1}{2\theta+\frac{n}{c_1+c_2}}z_4$ the subsidy hike will not cause a weak green paradox.

**Proposition 1** *Suppose that the fossil fuel endowment is large and that the high-tax country unilaterally increases its backstop subsidy. If fossil fuel extraction costs do not depend on the fossil fuel stock, a weak green paradox arises. Otherwise, the weak green paradox effect is mitigated due to the stock-dependence.*

The mechanism caused by the stock-dependence is clarified by (5) and (6), which can be rewritten as

$$p(t) = \tau(t) + 2\theta\chi(t) + \eta[g_0 - g(t)], \tag{51}$$

$$\dot{\tau}(t) = r\tau(t) - \eta\chi(t). \tag{52}$$

Note that $\tau(t) + \eta[g_0 - g(t)]$ equals the $p$-intercept of the fossil fuel supply function $x^s(p(t), \tau(t), g(t))$ at time $t$. Consider now a subsidy hike induced weak green paradox, i.e. an increase of $\chi(t)$ for early points in time.[23] Ceteris paribus, the scarcity rent growth rate $\dot{\tau}(t)$ is depressed due to the stock-dependence of extraction costs. Consequently, more fossil fuel extraction lowers the position of the supply function $x^s(p(t), \tau(t), g(t))$ in the $(\chi, p)$-space. Given the time invariant fossil fuel demand function, fossil fuel extraction is boosted. In other words, fossil fuel extraction is shifted towards early periods. Note that this extraction acceleration effect is the stronger the higher the stock-dependence, i.e. the higher $\eta$.

However, a lower fossil fuel stock $g(t)$ boosts the third term of (51), so that the $p$-intercept of the supply function $x^s(p(t), \tau(t), g(t))$ is increased ceteris paribus. The higher the position of the supply function in the $(\chi, p)$-space the lower fossil fuel extraction. Similar to the acceleration effect, this retardation effect is the stronger the higher $\eta$. By weakening both effects the mechanism guarantees a balance in the sense that $\tau(T) + \eta[g_0 - g_T]$ equals the reservation price $b - q_2 - \sigma_2$ in the moment the fossil fuel stock becomes economically exhausted.

[23] In the following we refer to a tax/subsidy hike induced weak/strong green paradox as a tax/subsidy induced weak/strong green paradox.

### 5.1.3. Tax increase of the low-tax country

After having discussed a sharper climate policy of the high-tax country, we now turn to the low-tax country. At first, suppose that the tax $q_2$ is increased. Solving the differentiated system of equations gives[24]

$$\frac{\partial g_T}{\partial q_2} = \frac{1}{\eta} > 0, \tag{53}$$

$$\frac{\partial \chi_0}{\partial q_2} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial q_2} + \lambda_I^2 \frac{1}{\eta}\frac{c_2}{c_1 + c_2}, \tag{54}$$

$$\begin{aligned}\frac{\partial K_I^1}{\partial q_2} = {} & \frac{1}{\Phi_I \Theta}\left[\frac{\Psi_\lambda}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\dot\chi_I(t_1)}{\dot\chi_{II}(t_1)}\frac{\dot\chi_{II}(t_2)}{\dot\chi_{III}(t_2)}\left(\frac{1}{2\theta + \frac{n}{c_2}}\Lambda^a + \frac{1}{\eta}\Lambda^b\right)\right] \\ & + \frac{1}{\Phi_I \Theta}\left[\frac{\dot\chi_I(t_1)}{\dot\chi_{II}(t_1)}\frac{c_2(1 - e^{\lambda_I^2 t_1})}{c_1 + c_2}\frac{1}{\eta}z_3 + \left(\frac{c_2}{2\theta(c_1 + c_2) + n} + \frac{c_2}{c_1}z_1\right)z_4\right],\end{aligned} \tag{55}$$

where $z_1, z_3, \Phi_I, \Theta, \Lambda^a < 0$ and $z_4, \Psi_\lambda, \Lambda^b > 0$ as defined in Tab. 3.

Given $\frac{\partial K_I^1}{\partial q_2} > 0$, (54) connotes a reduction of early fossil fuel extraction. However, the sign of (55) is ambiguous, since the terms in parentheses in both lines are of opposite sign. Suppose that the flow-dependence of extraction costs $\theta$ is large and energy demand in country 1 is price-elastic, i.e. $c_1$ is high. In this case, the terms in the second line and $\frac{1}{2\theta + \frac{n}{c_2}}\Lambda^a$ vanish, so that $\frac{\partial K_I^1}{\partial q_2} > 0$. Consequently, no weak green paradox occurs.

**Proposition 2** *Suppose that the fossil fuel endowment is large and that the low-tax country unilaterally increases its fossil fuel tax rate. If the flow-dependence of extraction costs is sufficiently strong and energy demand in country* 1 *is sufficiently price-elastic, no weak green paradox occurs.*

This result confirms the findings of Hoel (2011), Eichner and Pethig (2011) and Ritter and Schopf (2014), who stress the importance of the price-elasticity of energy demand for a green paradox. However, in our case a high price-elasticity needs to be combined with a strong flow-dependence of extraction costs to rule out a weak green paradox. The role of the latter becomes clear by inspecting the fossil fuel supply function $x^s(p(t), \tau(t), g(t))$ in a $(x, p)$-diagram, such as Fig. 1. The higher the flow-dependence of extraction costs the steeper the supply function at every point in time. Ceteris paribus, the effect of a scarcity rent reduction on fossil fuel extraction is weak if the supply function steeply increases. Consequently, a strong flow-dependence of extraction costs reduces the risk of a weak green paradox.

[24] Cf. Appendix A.5 for the proof.

(53) shows that total extraction $g_0 - g_T$ is reduced by the tax increase. As we focus on the economical exhaustion of fossil fuels, this is an intuitive result. Furthermore, the effect of a higher tax $q_2$ on total extraction is the weaker the stronger the stock-dependence of extraction costs $\eta$, since, according to (44), a stronger stock-dependence connotes less fossil fuel extraction. However, the (positively defined) elasticity of total fossil fuel extraction with respect to the tax rate $q_2$

$$\epsilon_{g_0-g_T,q_2} = -\frac{\partial(g_0 - g_T)}{\partial q_2}\frac{q_2}{g_0 - g_T} = \frac{q_2}{b - q_2 - \sigma_2} \tag{56}$$

shows that the relative reduction of fossil fuel use is the larger the higher the tax rate. As less fossil fuel extraction counters the effect of a potential weak green paradox with respect to climate costs, we conclude as follows.

**Proposition 3** *Suppose that the fossil fuel endowment is large and that the low-tax country unilaterally increases its fossil fuel tax rate. If the initial tax rate is sufficiently high, no strong green paradox occurs.*

Proposition 3 contrasts with result (i) of Hoel (2011), which postulates a tax induced strong green paradox given a sufficiently price-inelastic demand.[25] As result (ii) is closely related to (i), we scrutinize the welfare effect of the tax increase for country $i = 1, 2$, which is given by

$$\begin{aligned}\frac{dW_i}{dq_2} =& q_i \int_0^\infty e^{-rt}\frac{dx_i(t)}{dq_2}dt - \sigma_i \int_0^\infty e^{-rt}\frac{dy_i(t)}{dq_2}dt + \int_0^\infty e^{-rt}(\alpha_i x(t) - x_i(t))\frac{dp(t)}{dq_2}dt \\ & - \beta_i \frac{d\Omega}{dq_2} + \alpha_i \int_0^\infty e^{-rt}\left[\tau(t)\frac{dx(t)}{dq_2} + |M_g(g(t), \chi(t))|\frac{dG(t)}{dq_2}\right]dt.\end{aligned} \tag{57}$$

The first two terms reflect the distortions in the economy caused by the tax and the subsidy if climate effects are ignored. The third term is a pure terms-of-trade effect. If we consider total welfare $W_1 + W_2$, the terms-of-trade terms sum up to zero, since $\sum_i \alpha_i = 1$. Therefore, we follow Hoel (2011) and ignore these terms subsequently. The fourth term represents the change of climate costs of country $i$. The last term reflects the stock-dependence of extraction costs. Its sign is ambiguous. On the one hand, less total fossil fuel extraction decreases the present value of used resources. On the other hand, less total extraction in combination with a flattened extraction path reduces the value of

[25] Cf. page 8.

the associated costs. Ignoring the terms-of-trade term and using (11), we can rewrite (57) for $i = 1, 2$ as

$$\begin{aligned}\frac{dW_i}{dq_2} =& q_i \int_0^\infty e^{-rt} \frac{dx_i(t)}{dq_2} dt - \sigma_i \int_0^\infty e^{-rt} \frac{dy_i(t)}{dq_2} dt + \alpha_i \int_0^\infty e^{-rt} |M_g(g(t), \chi(t))| \frac{dG(t)}{dq_2} dt \\ &+ \alpha_i \int_0^\infty e^{-rt} \tau(t) \frac{dx(t)}{dq_2} dt - \beta_i v \int_0^\infty e^{-rt} \frac{dx(t)}{dq_2}.\end{aligned} \tag{58}$$

As stated above, the tax induced reduction of the fossil fuel reservation price in country 2 implies less total extraction. Consequently, if proposition 3 holds, the last term in (58) is positive. In contrast, the sign of the fourth term is negative, as the present value of extracted fossil fuel reserves decreases. Without a strong carbon leakage effect, a reduction of total extraction connotes a negative first term in both countries, while the sign of the third term depends on the effect of the tax increase on fossil fuel stock evolution. However, if the social costs of carbon $v$ are sufficiently high, the last term dominates all other effects. In particular, the gaps between the tax rates $q_i$ and the social cost of carbon $v$ need to be sufficiently large.[26] In this case, welfare in both countries increases, so that the effect on total welfare $W_1 + W_2$ is positive.

**Proposition 4** *Suppose that the fossil fuel endowment is large and that the low-tax country unilaterally increases its fossil fuel tax. If the initial tax rate and the social costs of carbon are sufficiently high, welfare of both countries and, therefore, total welfare increases.*

Propositions 3 and 4 reverse Hoel's (2011) results presented by (i) and (ii) on page 8. The reason is that without stock-dependent extraction costs, total fossil fuel extraction is not altered. In contrast, with the stock-dependence and an only economically exhausted fossil fuel stock a lower reservation price of the low-tax country connotes a decrease of total fossil fuel extraction. As less total extraction has a climate costs reducing effect, the tax increase of the low-tax country can boost total welfare. For a closed economy, Gerlagh (2011) and Van der Ploeg and Withagen (2012) derive similar results.

[26] Our reference to the gap between the tax rates and the social costs of carbon is based on the following consideration. Suppose $\beta_i$ is such that $\beta_i \int_0^\infty e^{-rt} \frac{dx(t)}{dq_2} dt \approx \int_0^\infty e^{-rt} \frac{dx_i(t)}{dq_2} dt$. If $q_i > v$, a negative integral $\int_0^\infty e^{-rt} \frac{dx(t)}{dq_2} dt$ reduces welfare.

### 5.1.4. Subsidy increase of the low-tax country

In this section we focus on the last remaining case with a large fossil fuel endowment, that is the increase of the subsidy in the low-tax country. The differentiated system of equations connotes[27]

$$\frac{\partial g_T}{\partial \sigma_2} = \frac{1}{\eta} > 0, \tag{59}$$

$$\frac{\partial \chi_0}{\partial \sigma_2} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial \sigma_2}, \tag{60}$$

$$\frac{\partial K_I^1}{\partial \sigma_2} = \frac{\frac{\Psi_\lambda}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\Psi_\lambda^a}{\lambda_{III}^1 - \lambda_{III}^2}\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_3)}\left(\frac{1}{\lambda_{IV}^2}\frac{1}{2\theta + \frac{\eta}{c_2}}\dot{\chi}_{IV}(t_3) + \frac{1}{\eta}\dot{\chi}_{III}(t_3)\right)}{|\Phi_I|\Theta}, \tag{61}$$

with $\Psi_\lambda > 0$ and $\Phi_I, \Theta < 0$ from Tab. 3 and

$$\Psi_\lambda^a = (\lambda_{III}^1 - \lambda_{III}^2)^2 e^{-(\lambda_{III}^1 + \lambda_{III}^2)(t_3 - t_2)} > 0. \tag{62}$$

According to (60), a weak green paradox is avoided if $\frac{\partial K_I^1}{\partial \sigma_2} > 0$. (61) shows that $K_I^1$ increases in $\sigma_2$ if the second term in parentheses outweighs the first one. In particular, this is the case if the flow-dependence of extraction costs $\theta$ is strong, the price-elasticity in the low-tax country $c_2$ is small or the stock-dependence of extraction costs $\eta$ is weak. In the first two cases the term $\frac{1}{2\theta + \frac{\eta}{c_2}}$ vanishes, while in the latter one $\frac{1}{\eta}$ is large.

**Proposition 5** *Suppose that the fossil fuel endowment is large and that the low-tax country unilaterally increases its backstop subsidy. If the flow-dependence of extraction costs is sufficiently strong, the stock-dependence sufficiently is weak and/or the price-elasticity of energy demand in the low-tax country is sufficiently small, no weak green paradox occurs.*

The intuition for this result is as follows. As explained in section 5.1.3, a strong flow-dependence increases the slope of the fossil fuel supply function and, therefore, reduces the risk of a weak green paradox. With respect to the stock-dependence, it seems counterintuitive that a low $\eta$ is beneficial. However, the weaker the stock-dependence the more fossil fuel is used and, therefore, the higher the subsidy induced reduction of total extraction in absolute numbers. As a sharp decrease in total extraction counters the subsidy induced necessity to reduce the initial scarcity rent, the risk of a weak green paradox is reduced. Finally, a small price-elasticity of the low-tax country works in a similar way. Consider Fig. 1. By increasing its subsidy the low-tax country lowers the position of part $(IV)$ of

[27] Cf. Appendix A.6 for the proof.

the aggregated demand function. This connotes a demand shortfall that is the smaller the steeper part $(III)$, i.e. the less price-elastic demand in the low-tax country. For a given evolution of the fossil fuel supply function in time, a small demand shortfall connotes a weaker necessity to reduce the initial scarcity rent.

Similar to the case of an increased $q_2$, a higher subsidy of the low-tax country reduces total extraction, as indicated by (59). While the effect decreases in the stock-dependence, the relative reduction is given by

$$\epsilon_{g_0-g_T,\sigma_2} = -\frac{\partial(g_0 - g_T)}{\partial\sigma_2}\frac{\sigma_2}{g_0 - g_T} = \frac{\sigma_2}{b - q_2 - \sigma_2}. \tag{63}$$

Thus, the higher the initial subsidy rate the larger the relative reduction of total extraction. Consequently, for a sufficiently high initial subsidy rate, the increase of $g_T$ outweighs every potential weak green paradox effect. Similarly to proposition 3, we conclude as follows.

**Proposition 6** *Suppose that the fossil fuel endowment is large and that the low-tax country unilaterally increases its backstop subsidy. If the initial subsidy rate is sufficiently high, no strong green paradox occurs.*

Proposition 6 contrasts with the finding of Hoel (2011) presented by result (v) on page 8, which postulates a strong green paradox. Similar to proposition 4 the result is driven by the stock-dependence of extraction costs, which give rise to a reduction of total extraction not covered by Hoel (2011). As the argumentation with respect to welfare of section 5.1.3 can be applied in a similar manner, proposition 7 holds.

**Proposition 7** *Suppose that the fossil fuel endowment is large and that the low-tax country unilaterally increases its backstop subsidy. If the initial subsidy rate and the social costs of carbon are sufficiently high, welfare of both countries and, therefore, total welfare increases.*

It is noteworthy that a stock-dependence is not necessary for a higher unilateral backstop subsidy to decrease climate costs. According to Ryszka and Withagen (2014), constant but internationally different marginal extraction costs may alter the extraction periods of the respective fossil fuel reserves such that a weak green paradox effect of a higher subsidy is compensated, i.e. total climate costs decrease.

### 5.2. Small fossil fuel endowment

Recall that the previous section focuses on the case of a large fossil fuel endowment, so that the initial fossil fuel market equilibrium is located on part $(I)$ of the aggregated

demand function. In this section we turn to a fossil fuel endowment that is sufficiently small to guarantee an initial fossil fuel market equilibrium located in part $(II)$ of the demand function. That is $\chi(0) = \chi_{II}(0) = -\dot{g}_{II}(0)$. Consequently, the junction point $t_1$ does not exist and the system of equations (32) - (39) and (44) reduces to

$$g_{II}(t_2) = g_{III}(t_2), \tag{64}$$

$$g_{III}(t_3) = g_{IV}(t_3), \tag{65}$$

$$\chi_{II}(t_2) = \chi_{III}(t_2) = \frac{c_2}{n}\frac{1}{r}(C_{III} - C_{II}), \tag{66}$$

$$\chi_{III}(t_3) = \chi_{IV}(t_3) = \frac{c_2}{n}\frac{1}{r}(C_{III} - C_{IV}), \tag{67}$$

$$g_0 = K^1_{II} + K^2_{II} - \frac{C_I}{B}, \tag{68}$$

$$\chi_0 = -\lambda^1_{II}K^1_{II} - \lambda^2_{II}K^2_{II}, \tag{69}$$

$$g_T = -\frac{C_{IV}}{B} = g_0 - \frac{b - q_2 - \sigma_2}{\eta}. \tag{70}$$

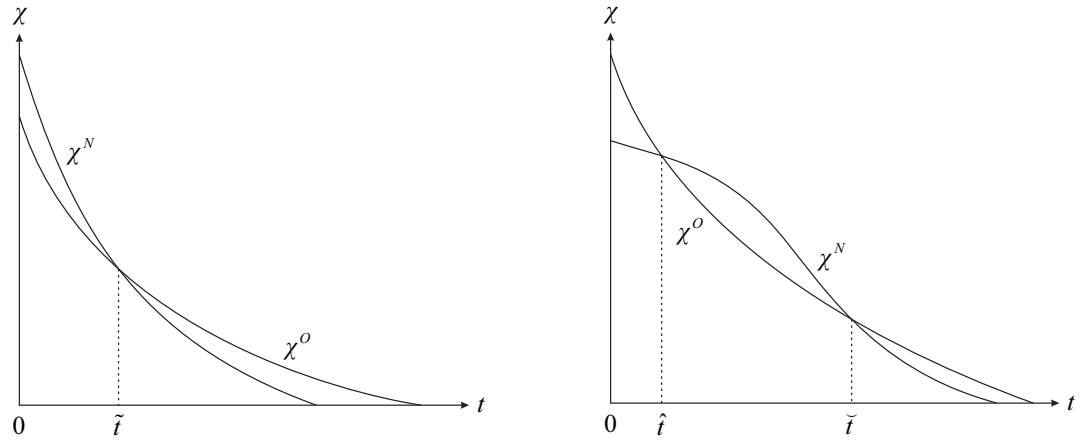
To identify the effects of a sharper climate policy, we differentiate this system, which depends on $K^1_{II}$, $K^2_{II}$, $K^1_{III}$, $K^2_{III}$, $K^2_{IV}$, $t_2$, $t_3$, and $\chi_0$, with respect to the tax and subsidy rates $q_1$, $q_2$, $\sigma_1$ and $\sigma_2$.

### 5.2.1. Tighter climate policy of the high-tax country

Similar to our previous procedure, we turn at first to a tighter climate policy of the high-tax country. Appendix A.7 shows that the results given by the differentiation of (64) - (70) with respect to the tax rate $q_1$ and the subsidy $\sigma_1$ are ambiguous. However, an alternative method provides more insight. To apply the method we make only use of the more general assumptions of section 2, that is we abstain from the function defined in Tab. 2. The method rests on the comparison of the fossil fuel extraction path valid without any climate policy change, i.e. with $dq_1 = d\sigma_1 = 0$, and the fossil fuel extraction path valid for $dq_1 > 0$ or $d\sigma_1 > 0$, respectively. In the following, we refer to the former path as the *old* and to the latter one as the *new* extraction path. Without loss of generality, suppose that the policy change occurs at $t = 0$.[28] A weak green paradox requires the new path $\chi^N(t)$ to be located above the old one $\chi^O(t)$ for early periods of time, as illustrated

[28]Suppose the time the policy change is announced and implemented is $\omega$. Then, we can split the optimization problem of the representative fossil fuel owner into one for the time period $[0, \omega[$ and one for the time period $[\omega, \infty[$. According to Bellman's principle of optimality the solutions to both problems need to be optimal on their own.

in Fig. 3(a).[29] Since the reservation price $b - q_2 - \sigma_2$ is not altered by a tighter climate




(a) Increase of early extraction (b) Increase of mid-term extraction

**Figure 3:** New and old extraction path with green paradoxes

policy of the high-tax country, total fossil fuel extraction is not affected. Thus, the areas below the two paths are equal, i.e. $\int_0^\infty \chi^O(t)dt = \int_0^\infty \chi^N(t)dt$. Consequently, in case of a weak green paradox, the new extraction path has to intersect the old one from above. In Fig. 3(a), the respective moment is denoted by $\tilde{t}$.

However, a weak green paradox is not a requirement for a strong one. If extraction is sufficiently shifted to the mid-term, total climate costs increase also without a weak green paradox. This case is illustrated in Fig. 3(b). Due to the discount rate in (11), mid-term extraction during the time period $\left[\, \hat{t}, \breve{t} \,\right]$ needs to increase to a higher degree than early extraction decreases. A pure shift of early to mid-term extraction, i.e. a flatten of the extraction path, would reduce climate costs. Thus, for a strong green paradox to occur, also late extraction must decline. Therefore, there is a second intersection of the old and new extraction path at $t = \breve{t}$.

Both discussed intersections, that is at $\tilde{t}$ and $\breve{t}$, are characterized by a new extraction path that intersects the old one from above. As shown in Appendix A.8 the conditions

$$\frac{d\tau^N(t)}{dt} < \frac{d\tau^O(t)}{dt}, \tag{71}$$

$$\chi^O(t) = \chi^N(t), \tag{72}$$

$$\frac{d\chi^O(t)}{dt} > \frac{d\chi^N(t)}{dt} \tag{73}$$

[29] In the following, the superscripts $N$ and $O$ are used to refer to variables associated with the new and the old extraction path, respectively.

need to hold in both cases. Recall that at every point in time fossil fuel extraction is determined by the equilibrium on the fossil fuel market, which is given by equating (15) and (17). Thus, by checking for (71) - (73) to hold, we compare two fossil fuel market equilibria, one on the old and one on the new aggregated demand function. Due to (72), these equilibria have to be located on one vertical line in a $(x, p)$-diagram, such as Fig. 1. In other words, we get equilibria combinations. These are written in the manner $[(O), (N)]$, with the first (second) element referring to the position of the equilibrium on the old (new) aggregated demand function.

Consider at first a tax increase of the high-tax country. The effect of a higher tax rate $q_1$ on the aggregated fossil fuel demand function is illustrated in Fig. 4. The tax increase

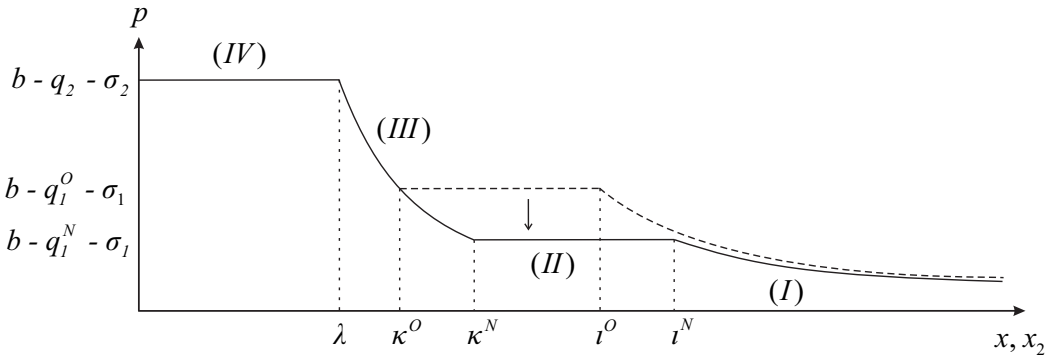



**Figure 4:** Aggregated fossil fuel demand before (dashed line) and after (solid line) the tax increase of the high-tax country

lowers the fossil fuel reservation price of the high-tax country $b - q_1 - \sigma_1$, so that part $(II)$ of the aggregated demand is shifted downwards in the $(x, p)$ space. Since an initial equilibrium on part $(I)$ of the aggregated demand function is ruled out by the assumption of a small fossil fuel endowment, the equilibria combinations where (71) - (73) may hold are $[(II), (II)]$, $[(II), (III)]$, $[(III), (III)]$ and $[(IV), (IV)]$. However, Appendix A.9 proves that in all four cases (73) connotes a contradiction to (71). Consequently, neither a weak nor a strong green paradox can occur.

A similar argument holds with respect to an increase of the subsidy rate of the high-tax country $\sigma_1$. As illustrated by Fig. 5, the reduction of the reservation price $b - q_1 - \sigma_1$ lowers the position of part $(II)$ of the aggregated demand function, while the position of the remaining part $(I)$ is not affected. However, similar to the case of a tax hike, the equilibria combinations that may allow (71) - (73) to hold are $[(II), (II)]$, $[(II), (III)]$, $[(III), (III)]$ and $[(IV), (IV)]$, so that Appendix A.9 can be applied. Consequently, both

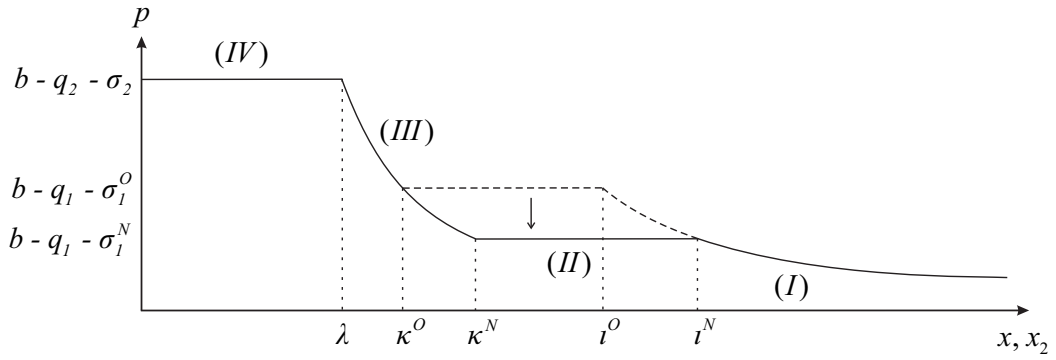



**Figure 5:** Aggregated fossil fuel demand function before (dashed line) and after (solid line) the subsidy increase of the high-tax country

a weak and a strong green paradox are ruled out.

**Proposition 8** *Suppose the high-tax country unilaterally increases either its fossil fuel tax rate or its backstop subsidy. If the fossil fuel endowment is small, so that the high-tax country initially uses a mix of fossil fuels and the backstop, neither a weak nor a strong green paradox occurs.*

Proposition 8 contrasts with the results of Hoel (2011) and section 5.1, which are either ambiguous or suggest a weak green paradox. To understand the result, recall that the price-elasticity of energy demand is mentioned by Hoel (2011), Eichner and Pethig (2011) and Ritter and Schopf (2014).[30] Along part $(II)$ of the aggregated demand function fossil fuel demand is perfectly price-elastic. Compared with a less price-elastic demand, e.g. part $(I)$, any shift of the supply function will cause a larger change of fossil fuel extraction. Consequently, the set of fossil fuel market equilibria given by the intersections of the supply function (15) with part $(II)$ result in an extraction path that is too steep to be intersected from above.

### 5.2.2. Tighter climate policy of the low-tax country

Analogous to section 5.1, we finally turn to climate policy changes of the low-tax country. At first, consider the case of an increase of the fossil fuel tax rate $q_2$. Differentiating

[30]Hoel refers to a price-inelastic fossil fuel demand function in the propositions 1, 3, 4, 6 and 8.

(64) - (70) with respect to $q_2$ and solving gives[31]

$$\frac{\partial g_T}{\partial q_2} = \frac{1}{\eta} > 0, \tag{74}$$

$$\frac{\partial \chi_0}{\partial q_2} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_{II}^1}{\partial q_2}, \tag{75}$$

$$\frac{\partial K_{II}^1}{\partial q_2} = \frac{1}{|\Phi_{II}^a|\Theta^a}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\left[\frac{1}{2\theta + \frac{n}{c_2}}\Lambda^a + \frac{1}{\eta}\Lambda^b\right], \tag{76}$$

where $\Lambda^a < 0$, $\Lambda^b > 0$ from Tab. 3 and

$$(a)\ \Theta^a = \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b + \frac{\Psi_{II}^b}{\Phi_{II}^a}\Lambda^a > 0, \qquad (b)\ \Phi_{II}^a = e^{\lambda_{II}^2 t_2} - e^{\lambda_{II}^1 t_2} < 0, \tag{77}$$

$$\Psi_{II}^b = \lambda_{II}^1 e^{\lambda_{II}^1 t_2} - \lambda_{II}^2 e^{\lambda_{II}^2 t_2} > 0. \tag{78}$$

In a similar manner, the differentiation with respect to $\sigma_2$ yields[32]

$$\frac{\partial g_T}{\partial \sigma_2} = \frac{1}{\eta} > 0, \tag{79}$$

$$\frac{\partial \chi_0}{\partial \sigma_2} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_{II}^1}{\partial q_2}, \tag{80}$$

$$\frac{\partial K_{II}^1}{\partial \sigma_2} = \frac{1}{\Phi_{II}^a\Theta^a}\frac{\Psi_\lambda^a}{\lambda_{III}^1 - \lambda_{III}^2}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\left[\frac{1}{\lambda_{IV}^2}\frac{1}{2\theta + \frac{n}{c_2}}\dot{\chi}_{IV}(t_3) + \frac{1}{\eta}\dot{\chi}_{III}(t_3)\right], \tag{81}$$

where $\Psi_\lambda^a > 0$ from (62).

According to (75) and (80), early fossil fuel extraction is reduced if $\frac{\partial K_{II}^1}{\partial q_2}, \frac{\partial K_{II}^1}{\partial \sigma_2} > 0$. However, the signs of both (76) and (81) are ambiguous. In both cases, $K_{II}^1$ increases in the policy instrument if the second term in squared brackets dominates the first one. This is the case for a strong flow-dependence of extraction costs $\theta$, a low price-elasticity of energy demand in the low-tax country $c_2$ and/or for a small stock-dependence of extraction costs $\eta$. In the first two cases the term $\frac{1}{2\theta + \frac{n}{c_2}}$ vanishes, in the latter one $\frac{1}{\eta}$ is sufficiently large. Therefore, we conclude similar to proposition 5.

**Proposition 9** *Suppose that the fossil fuel endowment is small and that the low-tax country unilaterally increases either its fossil fuel tax rate or its backstop subsidy. If the flow-dependence of extraction costs is sufficiently strong, the stock-dependence is sufficiently weak and/or the price-elasticity of energy demand in the low-tax country is sufficiently small, no weak green paradox occurs.*

Recall from the discussion of proposition 5 that a strong flow-dependence of extraction costs weakens the green paradox effect of a scarcity rent reduction, while both a weak

[31] Cf. Appendix A.10 for the proof.
[32] Cf. Appendix A.11 for the proof.

stock-dependence of extraction costs and a price-inelastic demand in the low-tax country alleviates the subsidy or tax induced necessity to lower the initial scarcity rent.
Finally, (74) and (79) imply a reduction of total extraction $g_0 - g_T$. By applying (56) and (63), it is revealed that the relative reduction of total extraction is the larger the higher the tax rate or the backstop subsidy, respectively. Consequently, a strong green paradox is avoided if the initial tax rate or the initial backstop subsidy is sufficiently high. Note that we can apply the argumentation of section 5.1.3 with respect to welfare in a similar manner.

**Proposition 10** *Suppose that the fossil fuel endowment is small and that the low-tax country unilaterally increases its fossil fuel tax rate or backstop subsidy. If the initial tax rate or backstop subsidy is sufficiently high, no strong green paradox occurs. In this case, welfare of both countries is increased if the social costs of carbon are sufficiently high.*

With respect to the backstop subsidy, proposition 10 resembles the result for a large fossil fuel endowment stated in proposition 6. Both results contrast with the finding of Hoel (2011) presented by result (v) on page 8, which postulates a strong green paradox. The driving force behind our result is the stock-dependence of extraction costs, as it gives rise to a reduction of total extraction not covered by Hoel (2011).

Evidently, the argumentation with respect to the stock-dependence also applies to the case of a common backstop subsidy. Consequently, an increase of such a common subsidy will not cause a strong green paradox if its initial rate is sufficiently high. Rather, the subsidy hike increases welfare in both countries if the social costs of carbon are sufficiently high. These findings contrast with Hoel's (2011) results presented by (iii) and (iv) on page 8.

## 6. Policy implications

Based on our analysis, we can derive policy implications for countries with a strict or lax climate policy. At first, consider the stricter ones. It is reasonable to regard several industrialized nations, in particular the members of the European Union, as high-tax countries.[33] According to sections 5.1.1 and 5.1.2, the effects of a tighter climate policy of these countries with respect to climate costs and welfare are ambiguous. However, this result substantially changes if we consider a small fossil fuel endowment, indicated by

[33] According to OECD (2013), the average carbon tax level in the majority of EU countries exceeds €40, while the federal levels in the USA and Canada are €4.8 and €7.7, respectively.

the early utilization of the backstop by the high-tax country, as in section 5.2.1. In this case, both a weak and a strong green paradox are ruled out. Indeed, it seems that this setting is the more realistic one, since all EU members are already using both renewable energies and fossil fuels.[34] Thus, our analysis suggests that a sharper climate policy of these countries will not harm the environment but lower total climate costs. As shown above, a reduction of total climate costs will increase welfare if the social costs of carbon are sufficiently high. The corresponding estimates range from low double-digit numbers to over $200 per ton of carbon dioxide.[35]

Furthermore, this result also shows that the timing of the policy change can be important. Even if the fossil fuel endowment is large, it is only a matter of time until the stock decreases to a level such that proposition 8 holds. Consequently, if the governments of high-tax countries want to avoid the risk of a green paradox, they may postpone the policy change until both fossil fuels and renewable energies are used.

In case of low-tax countries, proposition 3, 4, 6, 7 and 10 provide support for attempts to tighten climate policy in countries with a currently rather lax policy, such as the USA, Canada, Mexico, or India.[36] While a sharper climate policy of the low-tax countries may cause a weak green paradox, it decreases total fossil fuel extraction. If total extraction is sufficiently reduced, a more ambitious climate policy reduces total climate costs and increases welfare. According to our results, this outcome depends on the current fossil fuel tax or backstop subsidy, respectively, and the social costs of carbon. Real world data and estimates about both display a large variation. On the one hand, carbon taxes considerably vary between countries and sectors. According to OECD (2013), the average effective tax rates on $CO_2$ in OECD countries range from €2.8 (Mexico) to €107.3 (Switzerland).[37] On the other hand, as stated above, estimates regarding social costs of carbon lie between low double-digit numbers and over $200 per ton of carbon dioxide. In case that the rather high estimates with respect to the social costs of carbon are correct,

[34] Renewable energy accounted for 15% of gross final energy consumption of the EU28 in 2013.

[35] A comprehensive review of estimates including a meta-analysis is given by Tol (2005). He finds that the social costs of carbon are very likely below $50 per ton of $CO_2$. In contrast, Moore and Diaz (2015) estimate $220 per ton.

[36] According to OECD (2013), the average effective carbon tax rates of the former three countries are €4.8 (federal level), €7.7 (federal level) and €2.8, respectively. India levies a coal tax of approximately $3.1.

[37] See also OECD (2015) page 47 for a comparison of effective $CO_2$ taxes in transport use and heating and process use.

our results may very well hold. However, a unilateral intensification of climate policy in only one low-tax country has no effect on total extraction if other low-tax countries are inactive. Thus, our results suggest a coordinated climate policy action of, at least, the most important low-tax countries in terms of energy demand.

## 7. Conclusion

To analyze the effects of unilateral climate policy changes in a more general framework, we augment the two country model of Hoel (2011) with fossil fuel extraction costs. It turns out that the consideration of extraction costs give rise to more optimistic results than obtained by Hoel (2011). At first, consider countries which already apply a strict climate policy in terms of a fossil fuel tax or backstop subsidy. If the fossil fuel stock is sufficiently small, an intensification of these policies causes neither a weak nor a strong green paradox. It is reasonable to assign the European Union and its member states, such as Germany, to this set of countries. Furthermore, all these nations are already using both fossil fuels and renewable energies, which serves as an indicator for a small fossil fuel stock. Thus, our result suggests that concerns regarding a negative climate effect of a tighter EU climate policy are not justified. Even if we consider a large fossil fuel stock, our results show that a green paradox can be avoided by postponing the intensification of climate policy till the stock has decreased sufficiently.

With respect to countries, which are characterized by a lax climate policy, we show that the occurrence of a weak green paradox depends on the price-elasticity of energy demand and the strength of flow- and stock-dependence of extraction costs. In particular, if energy demand in the lax country is price-inelastic or the flow-dependence of extraction costs high, a subsidy hike will not cause a weak green paradox. Furthermore, given stock-dependent extraction costs a weak green paradox does not give directly rise to a strong green paradox, as a tighter climate policy of the lax countries reduces total fossil fuel extraction. Obviously, this has a favorable reducing effect on climate costs. The relative reduction of total extraction depends on the initial tax rate or backstop subsidy, respectively. The higher these instruments the more elastic total extraction. Thus, if the current fossil fuel tax or backstop subsidy are sufficiently high, a tax or subsidy hike of the lax countries will reduce total climate costs. In case of sufficiently large social costs of carbon, this reduction gives rise to an increase of welfare. This argument also holds with respect to an increase of a common subsidy if climate policies are equal among countries.

So far, we have focused on the effect of a higher fossil fuel tax or backstop subsidy with respect to a green paradox and an increase or a decrease of welfare. However, it should be noted that the effects concerning energy source utilization at specific points in time and, therefore, on the welfare level differ between the two policy instruments.

While we assume a quite general extraction cost function, we adopt other simplifying assumptions made by Hoel (2011). In particular, these are linear backstop costs, the perfect substitutability of fossil fuels and the backstop, and only one uniform fossil fuel type. In particular, non-linear backstop costs, as considered by Farzin (1986), seem to be promising for future research.

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## A. Appendix

### A.1. Fossil fuel exhaustion time $T$

Suppose that $T$ is finite, so that (42) holds with equality. Then, (42) holds for $\chi_T = 0$ and for

$$(b - q_2 - \sigma_2) - \theta\chi_T - \eta(g_0 - g_T) = 0 \tag{A.1}$$

$$\Leftrightarrow \chi_T = \frac{1}{\theta}[b - q_2 - \sigma_2 - \eta(g_0 - g_T)]. \tag{A.2}$$

Substituting into (43) gives $g_T = g_0 - \frac{b-q_2-\sigma_2}{\eta}$ and, therefore, $\chi_T = 0$. As the final fossil fuel market equilibrium is located on part $(IV)$ of the aggregated demand function,

$$\chi_T = -\lambda_{IV}^1 K_{IV}^1 e^{\lambda_{IV}^1 T} - \lambda_{IV}^2 K_{IV}^2 e^{\lambda_{IV}^2 T} = 0. \tag{A.3}$$

Ruling out the degenerate solution $K_{IV}^1 = K_{IV}^2 = 0$, (A.3) can only hold for $T \to \infty$ and $K_{IV}^1 = 0$, as $\lambda_{IV}^1 > 0$ and $\lambda_{IV}^2 < 0$. However, $T \to \infty$ contradicts the assumption of a finite $T$.

Consequently, fossil fuel is used at all points in time and $T \to \infty$. Since $\lambda_{IV}^1 > 0$ and $\lambda_{IV}^2 < 0$, $\lim_{T\to\infty} x_T = \pm\infty$ if $K_{IV}^1 \gtrless 0$. As both cases are not possible, $K_{IV}^1 = 0$, so that (30) reads

$$g_{IV}(t) = K_{IV}^2 e^{\lambda_{IV}^2 t} - \frac{C_{IV}}{B}, \tag{A.4}$$

which implies

$$\chi_{IV}(t) = -\lambda_{IV}^2 K_{IV}^2 e^{\lambda_{IV}^2 t} \geq 0. \tag{A.5}$$

Consequently, $K_{IV}^2 > 0$, so that (40) and (41) read $\lim_{T\to\infty} g_T = -\frac{C_{IV}}{B} > 0$ and $\lim_{T\to\infty} x_T = 0$.

### A.2. Tax increase of the high-tax country

In the subsequent discussion we will make use if the following notation

$$\Lambda^c = \frac{\Psi^a_{III}}{\lambda^2_{IV}} - \Phi_{III} > 0; \qquad \Lambda^d = \frac{\lambda^1_{III}\lambda^2_{III}}{\lambda^2_{IV}}\Phi_{III} - \Psi_{III} < 0;$$

$$z_5 = \frac{X_1}{\dot\chi_{II}(t_1)}\frac{c_2}{n} > 0; \qquad z_6 = \lambda^1_{II}\lambda^2_{II}\Phi_{II}\frac{1}{\eta} + \frac{X_2}{\dot\chi_{III}(t_2)}\Psi_{II}\frac{c_2}{n} > 0;$$

$$z_7 = \Psi^a_{II}\frac{1}{\eta} + \frac{X_2}{\dot\chi_{III}(t_2)}\Phi_{II}\frac{c_2}{n} < 0; \qquad z_8 = \Psi^a_{III}\frac{1}{\eta} - \Lambda^c\frac{c_2}{n} < 0;$$

$$z_9 = \lambda^1_{III}\lambda^2_{III}\Phi_{III}\frac{1}{\eta} - \Lambda^d\frac{c_2}{n} > 0;$$

$$X_1 = \dot\chi_{II}(t_1) - \dot\chi_I(t_1) < 0; \qquad X_2 = \dot\chi_{II}(t_2) - \dot\chi_{III}(t_2) < 0.$$

Differentiating the equation system (32) - (37), (44) with respect to $i = q_1, q_2, \sigma_1, \sigma_2$ gives

$$e^{\lambda^1_I t_1}\frac{\partial K^1_I}{\partial i} + e^{\lambda^2_I t_1}\frac{\partial K^2_I}{\partial i} - e^{\lambda^1_{II} t_1}\frac{\partial K^1_{II}}{\partial i} - e^{\lambda^2_{II} t_1}\frac{\partial K^2_{II}}{\partial i} = \frac{1}{B}\left(\frac{\partial C_I}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \tag{A.6}$$

$$\dot\chi_I(t_1)\frac{\partial t_1}{\partial i} - \lambda^1_I e^{\lambda^1_I t_1}\frac{\partial K^1_I}{\partial i} - \lambda^2_I e^{\lambda^2_I t_1}\frac{\partial K^2_I}{\partial i} = \frac{c_1 + c_2}{n}\frac{1}{r}\left(\frac{\partial C_I}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \tag{A.7}$$

$$\dot\chi_{II}(t_1)\frac{\partial t_1}{\partial i} - \lambda^1_{II} e^{\lambda^1_{II} t_1}\frac{\partial K^1_{II}}{\partial i} - \lambda^2_{II} e^{\lambda^2_{II} t_1}\frac{\partial K^2_{II}}{\partial i} = \frac{c_1 + c_2}{n}\frac{1}{r}\left(\frac{\partial C_I}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \tag{A.8}$$

$$e^{\lambda^1_{II} t_2}\frac{\partial K^1_{II}}{\partial i} + e^{\lambda^2_{II} t_2}\frac{\partial K^2_{II}}{\partial i} - e^{\lambda^1_{III} t_2}\frac{\partial K^1_{III}}{\partial i} - e^{\lambda^2_{III} t_2}\frac{\partial K^2_{III}}{\partial i} = \frac{1}{B}\left(\frac{\partial C_{II}}{\partial i} - \frac{\partial C_{III}}{\partial i}\right), \tag{A.9}$$

$$\dot\chi_{II}(t_2)\frac{\partial t_2}{\partial i} - \lambda^1_{II} e^{\lambda^1_{II} t_2}\frac{\partial K^1_{II}}{\partial i} - \lambda^2_{II} e^{\lambda^2_{II} t_2}\frac{\partial K^2_{II}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \tag{A.10}$$

$$\dot\chi_{III}(t_2)\frac{\partial t_2}{\partial i} - \lambda^1_{III} e^{\lambda^1_{III} t_2}\frac{\partial K^1_{III}}{\partial i} - \lambda^2_{III} e^{\lambda^2_{III} t_2}\frac{\partial K^2_{III}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \tag{A.11}$$

$$e^{\lambda^1_{III} t_3}\frac{\partial K^1_{III}}{\partial i} + e^{\lambda^2_{III} t_3}\frac{\partial K^2_{III}}{\partial i} - e^{\lambda^2_{IV} t_3}\frac{\partial K^2_{IV}}{\partial i} = \frac{1}{B}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{IV}}{\partial i}\right), \tag{A.12}$$

$$\dot\chi_{III}(t_3)\frac{\partial t_3}{\partial i} - \lambda^1_{III} e^{\lambda^1_{III} t_3}\frac{\partial K^1_{III}}{\partial i} - \lambda^2_{III} e^{\lambda^2_{III} t_3}\frac{\partial K^2_{III}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{IV}}{\partial i}\right), \tag{A.13}$$

$$\dot\chi_{IV}(t_3)\frac{\partial t_3}{\partial i} - \lambda^2_{IV} e^{\lambda^2_{IV} t_3}\frac{\partial K^2_{IV}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{IV}}{\partial i}\right), \tag{A.14}$$

$$\frac{\partial K^1_I}{\partial i} + \frac{\partial K^2_I}{\partial i} = \frac{1}{B}\frac{\partial C_I}{\partial i}, \tag{A.15}$$

$$\frac{\partial \chi_0}{\partial i} = -\lambda^1_I\frac{\partial K^1_I}{\partial i} - \lambda^2_I\frac{\partial K^2_I}{\partial i}, \tag{A.16}$$

$$\frac{\partial g_T}{\partial i} = -\frac{1}{B}\frac{\partial C_{IV}}{\partial i}. \tag{A.17}$$

With $i = q_1$ and making use of Tab. A.4 we find

| i | $\frac{\partial C_I}{\partial i}$ | $\frac{\partial C_{II}}{\partial i}$ | $\frac{\partial C_{III}}{\partial i}$ | $\frac{\partial C_{IV}}{\partial i}$ |
|---|---|---|---|---|
| $q_1$ | $-r\frac{c_1}{c_1+c_2}$ | $-r$ | 0 | 0 |
| $q_2$ | $-r\frac{c_2}{c_1+c_2}$ | 0 | $-r$ | $-r$ |
| $\sigma_1$ | 0 | $-r$ | 0 | 0 |
| $\sigma_2$ | 0 | 0 | 0 | $-r$ |

**Table A.4:** Derivations of $C_I$, $C_{II}$, $C_{III}$ and $C_{IV}$ with respect to $q_1$, $q_2$, $\sigma_1$ and $\sigma_2$

$$\frac{\partial \chi_0}{\partial q_1} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial q_1} + \lambda_I^2 \frac{r}{B}\frac{c_1}{c_1+c_2}, \tag{A.18}$$

$$e^{\lambda_{IV}^2 t_3}\frac{\partial K_{IV}^2}{\partial q_1} = \frac{1}{\lambda_{IV}^2}\dot{\chi}_{IV}(t_3)\frac{\partial t_3}{\partial q_1}, \tag{A.19}$$

$$e^{\lambda_{III}^1 t_3}\frac{\partial K_{III}^1}{\partial q_1} = \frac{\lambda_{III}^2}{\lambda_{III}^1 - \lambda_{III}^2}\frac{1}{\lambda_{IV}^2}\left[\frac{\lambda_{IV}^2}{\lambda_{III}^2}\dot{\chi}_{III}(t_3) - \dot{\chi}_{IV}(t_3)\right]\frac{\partial t_3}{\partial q_1}, \tag{A.20}$$

$$e^{\lambda_{III}^2 t_3}\frac{\partial K_{III}^2}{\partial q_1} = \frac{\lambda_{III}^1}{\lambda_{III}^1 - \lambda_{III}^2}\frac{1}{\lambda_{IV}^2}\left[\dot{\chi}_{IV}(t_3) - \frac{\lambda_{IV}^2}{\lambda_{III}^1}\dot{\chi}_{III}(t_3)\right]\frac{\partial t_3}{\partial q_1}, \tag{A.21}$$

$$\frac{\partial t_2}{\partial q_1} = \frac{1}{\dot{\chi}_{III}(t_2)}\left[\frac{c_2}{n} - \frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\Lambda^b\frac{\partial t_3}{\partial q_1}\right], \tag{A.22}$$

$$e^{\lambda_{II}^1 t_2}\frac{\partial K_{II}^1}{\partial q_1} = \frac{\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left(\lambda_{II}^2\Lambda^a - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b\right)\frac{\partial t_3}{\partial q_1} + \lambda_{II}^2\frac{r}{B} + \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right]}{\lambda_{II}^1 - \lambda_{II}^2}, \tag{A.23}$$

$$e^{\lambda_{II}^2 t_2}\frac{\partial K_{II}^2}{\partial q_1} = \frac{\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left(\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b - \lambda_{II}^1\Lambda^a\right)\frac{\partial t_3}{\partial q_1} - \lambda_{II}^1\frac{r}{B} - \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right]}{\lambda_{II}^1 - \lambda_{II}^2}, \tag{A.24}$$

$$\frac{\partial t_1}{\partial q_1} = \frac{1}{\dot{\chi}_{II}(t_1)}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_3\frac{\partial t_3}{\partial q_1} + z_6\right) + \frac{c_2}{n}\right], \tag{A.25}$$

$$\frac{\partial K_I^1}{\partial q_1} = \frac{1}{|\Phi_I|}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_4\frac{\partial t_3}{\partial q_1} + z_7\right) + \frac{r}{B}\frac{c_1 e^{\lambda_I^2 t_1} + c_2}{c_1 + c_2}\right], \tag{A.26}$$

$$\frac{\partial K_I^1}{\partial q_1} = \frac{1}{\Psi_I}\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_3\frac{\partial t_3}{\partial q_1} + z_6\right] - \frac{1}{\Psi_I}z_5 + \frac{1}{\Psi_I}z_1, \tag{A.27}$$

$$\frac{\partial t_3}{\partial q_1} = \frac{(\lambda_{II}^1 - \lambda_{II}^2)(z_5 - z_1) - \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_2)}z_6 - \frac{\Psi_I}{\Phi_I}z_7 - \frac{\Psi_I}{\Phi_I}(\lambda_{II}^1 - \lambda_{II}^2)z_2}{\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left[\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}z_3 + \frac{\Psi_I}{\Phi_I}z_4\right]}, \tag{A.28}$$

where $X_1 = \dot{\chi}_{II}(t_1) - \dot{\chi}_I(t_1)$ and $X_2 = \dot{\chi}_{II}(t_2) - \dot{\chi}_{III}(t_2)$. Using (21), (22) and the identities $g_I(t_1) = g_{II}(t_1)$, $\chi_I(t_1) = \chi_{II}(t_1)$, $g_{II}(t_2) = g_{III}(t_2)$ and $\chi_{II}(t_2) = \chi_{III}(t_2)$ we can rewrite $X_1$ and $X_2$ as

$$X_1 = \frac{1}{1 + 2\theta\frac{c_1+c_2}{n}}\dot{\chi}_{II}(t_1) < 0, \tag{A.29}$$

$$X_2 = \frac{1}{1 + 2\theta\frac{c_2}{n}}\dot{\chi}_{II}(t_2) < 0. \tag{A.30}$$

Substituting (A.28) into (A.26) or (A.27) and using (A.29) and (A.30) gives (47).

### A.3. Subsidy increase of the high-tax country

By using (A.6) - (A.17) with $i = \sigma_1$ and Tab. A.4 we find

$$\frac{\partial \chi_0}{\partial \sigma_1} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial \sigma_1}, \tag{A.31}$$

$$e^{\lambda_{IV}^2 t_3}\frac{\partial K_{IV}^2}{\partial \sigma_1} = \frac{1}{\lambda_{IV}^2}\dot{\chi}_{IV}(t_3)\frac{\partial t_3}{\partial \sigma_1}, \tag{A.32}$$

$$e^{\lambda_{III}^1 t_3}\frac{\partial K_{III}^1}{\partial \sigma_1} = \frac{\lambda_{III}^2}{\lambda_{III}^1 - \lambda_{III}^2}\frac{1}{\lambda_{IV}^2}\left[\frac{\lambda_{IV}^2}{\lambda_{III}^2}\dot{\chi}_{III}(t_3) - \dot{\chi}_{IV}(t_3)\right]\frac{\partial t_3}{\partial \sigma_1}, \tag{A.33}$$

$$e^{\lambda_{III}^2 t_3}\frac{\partial K_{III}^2}{\partial \sigma_1} = \frac{\lambda_{III}^1}{\lambda_{III}^1 - \lambda_{III}^2}\frac{1}{\lambda_{IV}^2}\left[\dot{\chi}_{IV}(t_3) - \frac{\lambda_{IV}^2}{\lambda_{III}^1}\dot{\chi}_{III}(t_3)\right]\frac{\partial t_3}{\partial \sigma_1}, \tag{A.34}$$

$$\frac{\partial t_2}{\partial \sigma_1} = \frac{1}{\dot{\chi}_{III}(t_2)}\left[\frac{c_2}{n} - \frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\Lambda^b\frac{\partial t_3}{\partial \sigma_1}\right], \tag{A.35}$$

$$e^{\lambda_{II}^1 t_2}\frac{\partial K_{II}^1}{\partial \sigma_1} = \frac{\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left(\lambda_{II}^2\Lambda^a - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b\right)\frac{\partial t_3}{\partial \sigma_1} + \lambda_{II}^2\frac{r}{B} + \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right]}{\lambda_{II}^1 - \lambda_{II}^2}, \tag{A.36}$$

$$e^{\lambda_{II}^2 t_2}\frac{\partial K_{II}^2}{\partial \sigma_1} = \frac{\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left(\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b - \lambda_{II}^1\Lambda^a\right)\frac{\partial t_3}{\partial \sigma_1} - \lambda_{II}^1\frac{r}{B} - \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right]}{\lambda_{II}^1 - \lambda_{II}^2}, \tag{A.37}$$

$$\frac{\partial t_1}{\partial \sigma_1} = \frac{1}{\dot{\chi}_{II}(t_1)}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_3\frac{\partial t_3}{\partial \sigma_1} + z_6\right) + \frac{c_1 + c_2}{n}\right], \tag{A.38}$$

$$\frac{\partial K_I^1}{\partial \sigma_1} = \frac{1}{|\Phi_I|}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_4\frac{\partial t_3}{\partial \sigma_1} + z_7\right) + \frac{r}{B}\right], \tag{A.39}$$

$$\frac{\partial K_I^1}{\partial \sigma_1} = \frac{1}{\Psi_I}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_3\frac{\partial t_3}{\partial \sigma_1} + z_6\right) - \frac{X_1}{\dot{\chi}_{II}(t_1)}\frac{c_1 + c_2}{n}\right], \tag{A.40}$$

$$\frac{\partial t_3}{\partial \sigma_1} = \frac{(\lambda_{II}^1 - \lambda_{II}^2)\frac{X_1}{\dot{\chi}_{II}(t_1)}\frac{c_1+c_2}{n} - (\lambda_{II}^1 - \lambda_{II}^2)\frac{\Psi_I}{\Phi_I}\frac{r}{B} - \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_2)}z_6 - \frac{\Psi_I}{\Phi_I}z_7}{\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left[\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}z_3 + \frac{\Psi_I}{\Phi_I}z_4\right]} \tag{A.41}$$

Substituting (A.41) into (A.39) or (A.40) and using (A.29) and (A.30) gives (50).

### A.4. Weak green paradox caused by $d\sigma_1 > 0$ in case of purely flow-dependent extraction costs

If the extraction costs are only flow-dependent, (5) reads

$$p(t) = \tau(t) + M_\chi(\chi(t)),$$

so that the fossil fuel supply function (7) is given by

$$\chi^s(t) = \chi^s(p(t), \tau(t))$$

at every point in time. Furthermore, the Hotelling-rule reads

$$\dot{\tau}(t) = r\tau(t),$$

so that, depending only on $\tau(0)$, the position of the supply function $x^s(t) = n\chi^s(t)$ at every point in time in a $(x, p)$-diagram, such as Fig. 1, is determined. The increasing scarcity rent $\tau(t)$ can be illustrated by an upward-shift of the supply function, where the $p$-intercept equals $\tau(t)$. For $\tau(0)$ to be optimal the $p$-intercept, i.e. $\tau(T)$, has to become equal to $b - q_2 - \sigma_2$ in the moment $T$ the fossil fuel stock becomes exhausted.

Suppose $\tau(0)$ is such that the initial equilibrium on the fossil fuel market is located on part $(I)$ of the aggregated demand function $A^x(p)$, i.e. $\chi_0 > \iota$. A marginal increase of the subsidy $\sigma_I$ lowers the position of part $(II)$ of $A^x(p)$. Suppose $\tau(0)$ is not altered by the subsidy hike, so that the position of the supply function $x^s(t)$ at every point remains the same. Due to the lowered position of part $(II)$ of $A^x(p)$ the ensuing fossil fuel extraction will be lower for all fossil fuel market equilibria along part $(II)$. Consequently, $\tau(t)$ will equal $b - q_2 - \sigma_2$ at a moment, the fossil fuel stock is not exhausted. Since $M_\chi(0) = 0$, this is not optimal. Rather, the optimal $\tau(0)$ has to decrease to compensate for lowered demand. Since the old initial equilibrium is located on part $(I)$ of $A^x(p)$, whose position in the $(x, p)$-diagram is not altered by the subsidy hike, the new initial equilibrium is located to the right of the old one. In other words, a weak green paradox arises.

### A.5. Tax increase of the low-tax country

By using (A.6) - (A.17) with $i = q_2$ and Tab. A.4 we find

$$\frac{\partial \chi_0}{\partial q_2} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial q_2} + \lambda_I^2 \frac{r}{B}\frac{c_2}{c_1 + c_2}, \tag{A.42}$$

$$e^{\lambda_{IV}^2 t_3}\frac{\partial K_{IV}^2}{\partial q_2} = \frac{1}{\lambda_{IV}^2}\dot{\chi}_{IV}(t_3)\frac{\partial t_3}{\partial q_2}, \tag{A.43}$$

$$e^{\lambda_{III}^1 t_3}\frac{\partial K_{III}^1}{\partial q_2} = \frac{\lambda_{III}^2}{\lambda_{III}^1 - \lambda_{III}^2}\frac{1}{\lambda_{IV}^2}\left[\frac{\lambda_{IV}^2}{\lambda_{III}^2}\dot{\chi}_{III}(t_3) - \dot{\chi}_{IV}(t_3)\right]\frac{\partial t_3}{\partial q_2}, \tag{A.44}$$

$$e^{\lambda_{III}^2 t_3}\frac{\partial K_{III}^2}{\partial q_2} = \frac{\lambda_{III}^1}{\lambda_{III}^1 - \lambda_{III}^2}\frac{1}{\lambda_{IV}^2}\left[\dot{\chi}_{IV}(t_3) - \frac{\lambda_{IV}^2}{\lambda_{III}^1}\dot{\chi}_{III}(t_3)\right]\frac{\partial t_3}{\partial q_2}, \tag{A.45}$$

$$\frac{\partial t_2}{\partial q_2} = -\frac{1}{\dot{\chi}_{III}(t_2)}\left[\frac{c_2}{n} + \frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\Lambda^b \frac{\partial t_3}{\partial q_2}\right], \tag{A.46}$$

$$e^{\lambda_{II}^1 t_2}\frac{\partial K_{II}^1}{\partial q_2} = \frac{\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left(\lambda_{II}^2\Lambda^a - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b\right)\frac{\partial t_3}{\partial q_2} - \lambda_{II}^2\frac{r}{B} - \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right]}{\lambda_{II}^1 - \lambda_{II}^2}, \tag{A.47}$$

$$e^{\lambda_{II}^2 t_2}\frac{\partial K_{II}^2}{\partial q_2} = \frac{\left[\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left(\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b - \lambda_{II}^1\Lambda^a\right)\frac{\partial t_3}{\partial q_2} + \lambda_{II}^1\frac{r}{B} + \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right]}{\lambda_{II}^1 - \lambda_{II}^2}, \tag{A.48}$$

$$\frac{\partial t_1}{\partial q_2} = \frac{1}{\dot{\chi}_{II}(t_1)}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_3\frac{\partial t_3}{\partial q_2} - z_6\right) - \frac{c_2}{n}\right], \tag{A.49}$$

$$\frac{\partial K_I^1}{\partial q_2} = \frac{1}{|\Phi_I|}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_4\frac{\partial t_3}{\partial q_2} - z_7\right) - \frac{r}{B}\frac{c_2(1 - e^{\lambda_I^2 t_1})}{c_1 + c_2}\right], \tag{A.50}$$

$$\frac{\partial K_I^1}{\partial q_2} = \frac{1}{\Psi_I}\left[\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\left(\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}z_3\frac{\partial t_3}{\partial q_2} - z_6\right) + z_5 + \frac{c_2}{c_1}z_1\right], \tag{A.51}$$

$$\frac{\partial t_3}{\partial q_2} = \frac{\frac{\Psi_I}{\Phi_I}z_7 + \frac{\Psi}{\Phi}(\lambda_{II}^1 - \lambda_{II}^2)\frac{r}{B}\frac{c_2(1 - e^{\lambda_I^2 t_1})}{c_1 + c_2} + \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_2)}z_6 - (\lambda_{II}^1 - \lambda_{II}^2)(z_5 + \frac{c_2}{c_1}z_1)}{\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left[\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}z_3 + \frac{\Psi_I}{\Phi_I}z_4\right]}, \tag{A.52}$$

Substituting (A.52) into (A.50) or (A.51) and using (A.29) and (A.30) gives (55).

### A.6. Subsidy increase of the low-tax country

Define $X_3$ as $X_3 := \dot{\chi}_{IV}(t_3) - \dot{\chi}_{III}(t_3)$. By making use of (21), (22) and the identities $g_{III}(t_3) = g_{IV}(t_3)$ and $\chi_{III}(t_3) = \chi_{IV}(t_3)$ we can rewrite $X_3$ as

$$X_3 = \frac{1}{2\theta + \frac{n}{c_2}}\dot{\chi}_{IV}(t_3). \tag{A.53}$$

By using (A.6) - (A.17) with $i = \sigma_2$ and Tab. A.4 we find

$$\frac{\partial \chi_0}{\partial \sigma_2} = -(\lambda_I^1 - \lambda_I^2)\frac{\partial K_I^1}{\partial \sigma_2}, \tag{A.54}$$

$$e^{\lambda_{IV}^2 t_3}\frac{\partial K_{IV}^2}{\partial \sigma_2} = \frac{1}{\lambda_{IV}^2}\left[\dot{\chi}_{IV}(t_3)\frac{\partial t_3}{\partial \sigma_2} - \frac{c_2}{n}\right], \tag{A.55}$$

$$e^{\lambda_{III}^1 t_3}\frac{\partial K_{III}^1}{\partial \sigma_2} = \frac{\frac{\lambda_{III}^2}{\lambda_{IV}^2}\left[\frac{\lambda_{IV}^2}{\lambda_{III}^2}\dot{\chi}_{III}(t_3) - \dot{\chi}_{IV}(t_3)\right]\frac{\partial t_3}{\partial \sigma_2} - \lambda_{III}^2\frac{r}{B} + \frac{\lambda_{III}^2 - \lambda_{IV}^2}{\lambda_{IV}^2}\frac{c_2}{n}}{\lambda_{III}^1 - \lambda_{III}^2}, \tag{A.56}$$

$$e^{\lambda_{III}^2 t_3}\frac{\partial K_{III}^2}{\partial \sigma_2} = \frac{\frac{\lambda_{III}^1}{\lambda_{IV}^2}\left[\dot{\chi}_{IV}(t_3) - \frac{\lambda_{IV}^2}{\lambda_{III}^1}\dot{\chi}_{III}(t_3)\right]\frac{\partial t_3}{\partial \sigma_2} + \lambda_{III}^1\frac{r}{B} - \frac{\lambda_{III}^1 - \lambda_{IV}^2}{\lambda_{IV}^2}\frac{c_2}{n}}{\lambda_{III}^1 - \lambda_{III}^2}, \tag{A.57}$$

$$\frac{\partial t_2}{\partial \sigma_2} = -\frac{1}{\dot{\chi}_{III}(t_2)}\frac{1}{\lambda_{III}^1 - \lambda_{III}^2}\left[\Lambda^b\frac{\partial t_3}{\partial \sigma_2} + \lambda_{III}^1\lambda_{III}^2\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}\right], \tag{A.58}$$

$$\begin{aligned} e^{\lambda_{II}^1 t_2}\frac{\partial K_{II}^1}{\partial \sigma_2} =& \frac{\lambda_{II}^2}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\Lambda^a\frac{\partial t_3}{\partial \sigma_2} + \Psi_{III}^a\frac{r}{B} - \Lambda^c\frac{c_2}{n}}{\lambda_{III}^1 - \lambda_{III}^2} \\ & - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\Lambda^b\frac{\partial t_3}{\partial \sigma_2} + \lambda_{III}^1\lambda_{III}^2\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}}{\lambda_{III}^1 - \lambda_{III}^2}, \end{aligned} \tag{A.59}$$

$$\begin{aligned} e^{\lambda_{II}^2 t_2}\frac{\partial K_{II}^2}{\partial \sigma_2} =& \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\frac{1}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\Lambda^b\frac{\partial t_3}{\partial \sigma_2} + \lambda_{III}^1\lambda_{III}^2\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}}{\lambda_{III}^1 - \lambda_{III}^2} \\ & - \frac{\lambda_{II}^1}{\lambda_{II}^1 - \lambda_{II}^2}\frac{\Lambda^a\frac{\partial t_3}{\partial \sigma_2} + \Psi_{III}^a\frac{r}{B} - \Lambda^c\frac{c_2}{n}}{\lambda_{III}^1 - \lambda_{III}^2}, \end{aligned} \tag{A.60}$$

$$\begin{aligned} \frac{\partial t_1}{\partial \sigma_2} =& \frac{z_3\frac{\partial t_3}{\partial \sigma_2} + \lambda_{II}^1\lambda_{II}^2\Phi_{II}\left(\Psi_{III}^a\frac{r}{B} - \Lambda^c\frac{c_2}{n}\right)}{\dot{\chi}_{II}(t_1)(\lambda_{II}^1 - \lambda_{II}^2)(\lambda_{III}^1 - \lambda_{III}^2)} \\ & - \frac{\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Psi_{II}\left(\lambda_{III}^1\lambda_{III}^2\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}\right)}{\dot{\chi}_{II}(t_1)(\lambda_{II}^1 - \lambda_{II}^2)(\lambda_{III}^1 - \lambda_{III}^2)}, \end{aligned} \tag{A.61}$$

$$\begin{aligned} \frac{\partial K_I^1}{\partial \sigma_2} =& \frac{z_4\frac{\partial t_3}{\partial \sigma_2} + \Psi_{II}^a\left(\Psi_{III}^a\frac{r}{B} - \Lambda^c\frac{c_2}{n}\right)}{|\Phi_I|(\lambda_{II}^1 - \lambda_{II}^2)(\lambda_{III}^1 - \lambda_{III}^2)} \\ & - \frac{\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Phi_{II}\left(\lambda_{III}^1\lambda_{III}^2\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}\right)}{|\Phi_I|(\lambda_{II}^1 - \lambda_{II}^2)(\lambda_{III}^1 - \lambda_{III}^2)}, \end{aligned} \tag{A.62}$$

$$\begin{aligned} \frac{\partial K_I^1}{\partial \sigma_2} =& \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\frac{z_3\frac{\partial t_3}{\partial \sigma_2} + \lambda_{II}^1\lambda_{II}^2\Phi_{II}\left(\Psi_{III}^a\frac{r}{B} - \Lambda^c\frac{c_2}{n}\right)}{\Psi_I(\lambda_{II}^1 - \lambda_{II}^2)(\lambda_{III}^1 - \lambda_{III}^2)} \\ & - \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\frac{\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Psi_{II}\left(\lambda_{III}^1\lambda_{III}^2\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}\right)}{\Psi_I(\lambda_{II}^1 - \lambda_{II}^2)(\lambda_{III}^1 - \lambda_{III}^2)}, \end{aligned} \tag{A.63}$$

$$\frac{\partial t_3}{\partial \sigma_2} = \frac{z_8\left(\frac{\Psi_I}{|\Phi_I|}\Psi_{II}^a - \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\lambda_{II}^1\lambda_{II}^2\Phi_{II}\right) - \frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}z_9\left(\frac{\Psi_I}{|\Phi_I|}\Phi_{II} - \frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}\Psi_{II}\right)}{\frac{\dot{\chi}_I(t_1)}{\dot{\chi}_{II}(t_1)}z_3 + \frac{\Psi_I}{\Phi_I}z_4}, \tag{A.64}$$

Substituting (A.64) into (A.62) or (A.63) and using (A.53) gives (61).

## A.7. Tax and subsidy increase of the high-tax country in case of a small fossil fuel endowment

Differentiating the equation system (64) - (70) with respect to $i = q_1, q_2, \sigma_1, \sigma_2$ gives

$$e^{\lambda^1_{II}t_2}\frac{\partial K^1_{II}}{\partial i} + e^{\lambda^2_{II}t_2}\frac{\partial K^2_{II}}{\partial i} - e^{\lambda^1_{III}t_2}\frac{\partial K^1_{III}}{\partial i} - e^{\lambda^2_{III}t_2}\frac{\partial K^2_{III}}{\partial i} = \frac{1}{B}\left(\frac{\partial C_{II}}{\partial i} - \frac{\partial C_{III}}{\partial i}\right), \quad \text{(A.65)}$$

$$\dot\chi_{II}(t_2)\frac{\partial t_2}{\partial i} - \lambda^1_{II}e^{\lambda^1_{II}t_2}\frac{\partial K^1_{II}}{\partial i} - \lambda^2_{II}e^{\lambda^2_{II}t_2}\frac{\partial K^2_{II}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \quad \text{(A.66)}$$

$$\dot\chi_{III}(t_2)\frac{\partial t_2}{\partial i} - \lambda^1_{III}e^{\lambda^1_{III}t_2}\frac{\partial K^1_{III}}{\partial i} - \lambda^2_{III}e^{\lambda^2_{III}t_2}\frac{\partial K^2_{III}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{II}}{\partial i}\right), \quad \text{(A.67)}$$

$$e^{\lambda^1_{III}t_3}\frac{\partial K^1_{III}}{\partial i} + e^{\lambda^2_{III}t_3}\frac{\partial K^2_{III}}{\partial i} - e^{\lambda^2_{IV}t_3}\frac{\partial K^2_{IV}}{\partial i} = \frac{1}{B}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{IV}}{\partial i}\right), \quad \text{(A.68)}$$

$$\dot\chi_{III}(t_3)\frac{\partial t_3}{\partial i} - \lambda^1_{III}e^{\lambda^1_{III}t_3}\frac{\partial K^1_{III}}{\partial i} - \lambda^2_{III}e^{\lambda^2_{III}t_3}\frac{\partial K^2_{III}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{IV}}{\partial i}\right), \quad \text{(A.69)}$$

$$\dot\chi_{IV}(t_3)\frac{\partial t_3}{\partial i} - \lambda^2_{IV}e^{\lambda^2_{IV}t_3}\frac{\partial K^2_{IV}}{\partial i} = \frac{c_2}{n}\frac{1}{r}\left(\frac{\partial C_{III}}{\partial i} - \frac{\partial C_{IV}}{\partial i}\right), \quad \text{(A.70)}$$

$$\frac{\partial K^1_{II}}{\partial i} + \frac{\partial K^2_{II}}{\partial i} = \frac{1}{B}\frac{\partial C_{II}}{\partial i}, \quad \text{(A.71)}$$

$$\frac{\partial \chi_0}{\partial i} = -\lambda^1_{II}\frac{\partial K^1_{II}}{\partial i} - \lambda^2_{II}\frac{\partial K^2_{II}}{\partial i}, \quad \text{(A.72)}$$

$$\frac{\partial g_T}{\partial i} = -\frac{1}{B}\frac{\partial C_{IV}}{\partial i}. \quad \text{(A.73)}$$

By setting $j = q_1 = \sigma_1$ and using Tab. A.4 we find

$$\frac{\partial \chi_0}{\partial j} = -(\lambda^1_{II} - \lambda^2_{II})\frac{\partial K^1_{II}}{\partial j} - \frac{r}{B}, \quad \text{(A.74)}$$

$$e^{\lambda^2_{IV}t_3}\frac{\partial K^2_{IV}}{\partial j} = \frac{1}{\lambda^2_{IV}}\dot\chi_{IV}(t_3)\frac{\partial t_3}{\partial j}, \quad \text{(A.75)}$$

$$e^{\lambda^1_{III}t_3}\frac{\partial K^1_{III}}{\partial j} = \frac{\lambda^2_{III}}{\lambda^1_{III} - \lambda^2_{III}}\frac{1}{\lambda^2_{IV}}\left[\frac{\lambda^2_{IV}}{\lambda^2_{III}}\dot\chi_{III}(t_3) - \dot\chi_{IV}(t_3)\right]\frac{\partial t_3}{\partial j}, \quad \text{(A.76)}$$

$$e^{\lambda^2_{III}t_3}\frac{\partial K^2_{III}}{\partial j} = \frac{\lambda^1_{III}}{\lambda^1_{III} - \lambda^2_{III}}\frac{1}{\lambda^2_{IV}}\left[\dot\chi_{IV}(t_3) - \frac{\lambda^2_{IV}}{\lambda^1_{III}}\dot\chi_{III}(t_3)\right]\frac{\partial t_3}{\partial j}, \quad \text{(A.77)}$$

$$\frac{\partial t_2}{\partial j} = \frac{1}{\dot\chi_{III}(t_2)}\left[\frac{c_2}{n} - \frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Lambda^b\frac{\partial t_3}{\partial j}\right], \quad \text{(A.78)}$$

$$\frac{\partial K^1_{II}}{\partial j} = \frac{1}{\Phi^a_{II}}\left[\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Lambda^a\frac{\partial t_3}{\partial j} + \frac{r}{B}(1 - e^{\lambda^2_{II}t_2})\right], \quad \text{(A.79)}$$

$$\frac{\partial K^1_{II}}{\partial j} = \frac{1}{\Psi^b_{II}}\left[\frac{X_2}{\dot\chi_{III}(t_2)}\frac{c_2}{n} - \frac{\dot\chi_{II}(t_2)}{\dot\chi_{III}(t_2)}\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Lambda^b\frac{\partial t_3}{\partial j} + \lambda^2_{II}e^{\lambda^2_{II}t_2}\frac{r}{B}\right], \quad \text{(A.80)}$$

$$\frac{\partial t_3}{\partial j} = \frac{\frac{X_2}{\dot\chi_{III}(t_2)}\frac{c_2}{n} + \lambda^2_{II}e^{\lambda^2_{II}t_2}\frac{r}{B} + \frac{\Psi^b_{II}}{|\Phi^a_{II}|}\frac{r}{B}(1 - e^{\lambda^2_{II}t_2})}{\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Theta^a} \quad \text{(A.81)}$$

Substituting (A.81) into (A.79) or (A.80) gives

$$\frac{\partial K^1_{II}}{\partial j} = \frac{1}{\Phi^a_{II}\Theta^a}\left[\frac{X_2}{\dot\chi_{III}(t_2)}\frac{c_2}{n}\Lambda^a + \lambda^2_{II}e^{\lambda^2_{II}t_2}\frac{r}{B}\Lambda^a + (1 - e^{\lambda^2_{II}t_2})\frac{\dot\chi_{II}(t_2)}{\dot\chi_{III}(t_2)}\frac{r}{B}\Lambda^b\right] \quad \text{(A.82)}$$

As the first term brackets in negative, while the two others are positive, the sign of (A.82) is ambiguous.

**A.8.** (71) **-** (73)

Recall that we have $n$ identical fossil fuel firms. The following discussion makes, therefore, use of a representative firm.

Every tightening of climate policies alters fossil fuel demand by lowering the position of at least one part of the aggregated demand function, so that demand is lower for a specific producer price range. Therefore, a weak green paradox requires a lower initial scarcity rent $\tau^N(0) < \tau^O(0)$, so that the initial supply function intersects the new demand function to the right of the old initial equilibrium. Furthermore, a weak green paradox connotes $\chi^N(t) > \chi^O(t)$ for all $t < \tilde{t}$, so that $g^N(t) < g^O(t)$. Due to $M_{gg} > 0$, (5)(b) implies that (71) holds for all $t < \tilde{t}$ and, in particular, at $t = \tilde{t}$. The two other requirements are directly evident from Fig. 3. Firstly, the extraction paths intersect at $\tilde{t}$ or $\breve{t}$, respectively, so that (72) holds. Secondly, the new path intersects the old from above, which implies (73).

It remains to be shown that (71) is also a requirement for the intersection at $t = \breve{t}$. Note that a strong green paradox requires $g^N(\breve{t}) < g^O(\breve{t})$. Assume that $\frac{d\tau^N(\breve{t})}{dt} \geq \frac{d\tau^O(\breve{t})}{dt}$. According to (5)(b), this is possible if either $\tau^N(t) \geq \tau^O(t)$ or $|M_g(g^N(t), \chi^N(t))| \leq |M_g(g^O(t), \chi^O(t))|$. However, $\tau^N(t) \geq \tau^O(t)$, (72) and $g^N(t) < g^O(t)$ imply a higher position and a steeper slope of the new fossil fuel supply function. Together with an unchanged or lowered position of the parts of the aggregated demand function this contradicts (72).

**A.9. Alternative method for tax and subsidy increase of the high-tax country in case of a small fossil fuel endowment**

We can distinguish between four energy market equilibria types. For $j = O, N$ the corresponding market clearing conditions read

$$p^s(x) = \begin{cases} H^j(x^j(t)) & (I) \\ b - q_1^j - \sigma_1^j & (II) \\ u_x^2(x^j(t)) - q_2^j & (III) \\ b - q_2^j - \sigma_2^j, & (IV) \end{cases} \tag{A.83}$$

with $p^s(x)$ as the inverse of the aggregated fossil fuel supply function and $H(D(p+q_1) + D(p+q_2)) = p$. For a given fossil fuel value the right-hand side of (A.83) gives a unique

price value. As the representative fossil fuel firm has no market power, it takes the price as given. Therefore, also

$$\tau^j(t) + M_\chi(g^j(t), \chi^j(t)) = \begin{cases} H^j(x^j(t)) & (I) \\ b - q_1^j - \sigma_1^j & (II) \\ u_2'(x^j(t)) - q_2^j & (III) \\ b - q_2^j - \sigma_2^j, & (IV) \end{cases} \tag{A.84}$$

has to hold. Differentiating (A.84) with respect to time, taking notice of $\frac{dH^j(D_1(p^j+q_1^j)+D_2(p^j+q_2^j))}{dp^j} = \frac{dp^j}{dp^j} \Leftrightarrow H_x^j(x^j) = \frac{1}{D_1'(p^j+q_1^j)+D_2'(p^j+q_2^j)}$, and rearranging gives

$$\frac{d\chi^j}{dt} = \begin{cases} \left[\frac{n}{D_1'(p^j+q_1^j)+D_2'(p^j+q_2^j)} - M_{\chi\chi}\right]^{-1} \left[\frac{d\tau^j}{dt} - M_{\chi g}\chi^j(t)\right] & (I) \\ -\frac{1}{M_{\chi\chi}} \left[\frac{d\tau^j}{dt} - M_{\chi g}\chi^j(t)\right] & (II),\ (IV) \\ \left[\frac{n}{D_2'(p^j+q_2^j)} - M_{\chi\chi}\right]^{-1} \left[\frac{d\tau^j}{dt} - M_{\chi g}\chi^j(t)\right]. & (III) \end{cases} \tag{A.85}$$

The fossil fuel market equilibria combinations that may allow for (71) - (73) to hold are $[(II),(III)]$, $[(II),(II)]$, $[(III),(III)]$ and $[(IV),(IV)]$.[38] By using (A.85), (72) and (73) we get Table A.5. All inequalities contradict (71). In the case of $[(II),(III)]$ because

| O | N | $\frac{dx^O(t)}{dt} > \frac{dx^N(t)}{dt} \Leftrightarrow$ |
|---|---|---|
| (II) | (III) | $\frac{d\tau^N(t)}{dt} > \frac{M_{\chi\chi} - \frac{n}{D_2'(Q_2^{xN}(t))}}{M_{\chi\chi}} \frac{d\tau^O(t)}{dt} + \left[\frac{M_{\chi\chi} - \frac{n}{D_2'(Q_2^{xN}(t))}}{M_{\chi\chi}} - 1\right] \lvert M_{\chi g}\rvert \chi(t)$ |
| (II) | (II) | $\frac{d\tau^N(t)}{dt} > \frac{d\tau^O(t)}{dt}$ |
| $(III)$ | $(III)$ | $\frac{d\tau^N(t)}{dt} > \frac{d\tau^O(t)}{dt}$ |
| $(IV)$ | $(IV)$ | $\frac{d\tau^N(t)}{dt} > \frac{d\tau^O(t)}{dt}$ |

**Table A.5:** Conditions for a weak and/or green paradox caused by a tax increase of the high-tax country

$M_{\chi\chi} > 0$ and $D_2' < 0$. Consequently, if the high-tax country initially uses a mix of fossil

[38] Recall that we assume here a small fossil fuel endowment, so that the initial fossil fuel market equilibrium is located on part $(II)$ of the aggregated demand function.

fuel and backstop without the tax or the subsidy increase, the conditions for a weak green paradox are not met. As a strong green paradox that is not caused by a weak one also requires (71) - (73) to hold, both a weak and a strong green paradox are ruled out.

### A.10. Tax increase of the low-tax country in case of a small fossil fuel endowment

By using (A.65) - (A.73) with $i = q_2$ and Tab. A.4 we find

$$\frac{\partial \chi_0}{\partial q_2} = -(\lambda^1_{II} - \lambda^2_{II})\frac{\partial K^1_{II}}{\partial q_2}, \tag{A.86}$$

$$e^{\lambda^2_{IV} t_3}\frac{\partial K^2_{IV}}{\partial q_2} = \frac{1}{\lambda^2_{IV}}\dot{\chi}_{IV}(t_3)\frac{\partial t_3}{\partial q_2}, \tag{A.87}$$

$$e^{\lambda^1_{III} t_3}\frac{\partial K^1_{III}}{\partial q_2} = \frac{\lambda^2_{III}}{\lambda^1_{III} - \lambda^2_{III}}\frac{1}{\lambda^2_{IV}}\left[\frac{\lambda^2_{IV}}{\lambda^2_{III}}\dot{\chi}_{III}(t_3) - \dot{\chi}_{IV}(t_3)\right]\frac{\partial t_3}{\partial q_2}, \tag{A.88}$$

$$e^{\lambda^2_{III} t_3}\frac{\partial K^2_{III}}{\partial q_2} = \frac{\lambda^1_{III}}{\lambda^1_{III} - \lambda^2_{III}}\frac{1}{\lambda^2_{IV}}\left[\dot{\chi}_{IV}(t_3) - \frac{\lambda^2_{IV}}{\lambda^1_{III}}\dot{\chi}_{III}(t_3)\right]\frac{\partial t_3}{\partial q_2}, \tag{A.89}$$

$$\frac{\partial t_2}{\partial q_2} = -\frac{1}{\dot{\chi}_{III}(t_2)}\left[\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Lambda^b\frac{\partial t_3}{\partial q_2} + \frac{c_2}{n}\right], \tag{A.90}$$

$$\frac{\partial K^1_{II}}{\partial q_2} = \frac{1}{\Phi^a_{II}}\left[\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Lambda^a\frac{\partial t_3}{\partial q_2} - \frac{r}{B}\right], \tag{A.91}$$

$$\frac{\partial K^1_{II}}{\partial q_2} = -\frac{1}{\Psi^b_{II}}\left[\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\Lambda^b\frac{\partial t_3}{\partial q_2} + \frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n}\right], \tag{A.92}$$

$$\frac{\partial t_3}{\partial q_2} = -\frac{\frac{X_2}{\dot{\chi}_{III}(t_2)}\frac{c_2}{n} + \frac{\Psi^b_{II}}{|\Phi^a_{II}|}\frac{r}{B}}{\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\Theta^a} < 0. \tag{A.93}$$

Substituting (A.93) into (A.91) or (A.92) gives (76).

### A.11. Subsidy increase of the low-tax country in case of a small fossil fuel endowment

By using (A.65) - (A.73) with $i = \sigma_2$ and Tab. A.4 we find

$$\frac{\partial \chi_0}{\partial \sigma_2} = -\left(\lambda^1_{II} - \lambda^2_{II}\right)\frac{\partial K^1_{II}}{\partial \sigma_2}, \tag{A.94}$$

$$e^{\lambda^2_{IV} t_3}\frac{\partial K^2_{IV}}{\partial \sigma_2} = \frac{1}{\lambda^2_{IV}}\dot{\chi}_{IV}(t_3)\frac{\partial t_3}{\partial \sigma_2} - \frac{1}{\lambda^2_{IV}}\frac{c_2}{n}, \tag{A.95}$$

$$e^{\lambda^1_{III} t_3}\frac{\partial K^1_{III}}{\partial \sigma_2} = \frac{\frac{\lambda^2_{III}}{\lambda^2_{IV}}\left[\frac{\lambda^2_{IV}}{\lambda^2_{III}}\dot{\chi}_{III}(t_3) - \dot{\chi}_{IV}(t_3)\right]\frac{\partial t_3}{\partial \sigma_2} - \lambda^2_{III}\frac{r}{B} + \frac{\lambda^2_{III} - \lambda^2_{IV}}{\lambda^2_{IV}}\frac{c_2}{n}}{\lambda^1_{III} - \lambda^2_{III}}, \tag{A.96}$$

$$e^{\lambda^2_{III} t_3}\frac{\partial K^2_{III}}{\partial \sigma_2} = \frac{\frac{\lambda^1_{III}}{\lambda^2_{IV}}\left[\dot{\chi}_{IV}(t_3) - \frac{\lambda^2_{IV}}{\lambda^1_{III}}\dot{\chi}_{III}(t_3)\right]\frac{\partial t_3}{\partial \sigma_2} + \lambda^1_{III}\frac{r}{B} - \frac{\lambda^1_{III} - \lambda^2_{IV}}{\lambda^2_{IV}}\frac{c_2}{n}}{\lambda^1_{III} - \lambda^2_{III}}, \tag{A.97}$$

$$\frac{\partial t_2}{\partial \sigma_2} = \frac{1}{\dot{\chi}_{III}(t_2)}\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\left[\Lambda^d\frac{c_2}{n} - \Lambda^b\frac{\partial t_3}{\partial \sigma_2} - \lambda^1_{III}\lambda^2_{III}\Phi_{III}\frac{r}{B}\right], \tag{A.98}$$

$$\frac{\partial K^1_{II}}{\partial \sigma_2} = \frac{1}{|\Phi^a_{II}|}\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\left[\Lambda^c\frac{c_2}{n} - \Lambda^a\frac{\partial t_3}{\partial \sigma_2} - \Psi^a_{III}\frac{r}{B}\right], \tag{A.99}$$

$$\frac{\partial K^1_{II}}{\partial \sigma_2} = \frac{1}{\Psi^b_{II}}\frac{1}{\lambda^1_{III} - \lambda^2_{III}}\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\left[\Lambda^d\frac{c_2}{n} - \Lambda^b\frac{\partial t_3}{\partial \sigma_2} - \lambda^1_{III}\lambda^2_{III}\Phi_{III}\frac{r}{B}\right], \tag{A.100}$$

$$\frac{\partial t_3}{\partial \sigma_2} = -\frac{\frac{\dot{\chi}_{II}(t_2)}{\dot{\chi}_{III}(t_2)}\left[\lambda^1_{III}\lambda^2_{III}\Phi_{III}\frac{r}{B} - \Lambda^d\frac{c_2}{n}\right] + \frac{\Psi^b_{II}}{\Phi^a_{II}}\left[\Psi^a_{III}\frac{r}{B} - \Lambda^c\frac{c_2}{n}\right]}{\Theta^a} < 0. \tag{A.101}$$

Substituting (A.101) into (A.99) or (A.100) and using (A.53) gives (81).