

Mass, Helene; Fugger, Nicolas; Gretschno, Vitali; Wambach, Achim

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# Imitation perfection – a simple rule to prevent discrimination in procurement\*

Helene Mass <sup>†a</sup>, Nicolas Fugger<sup>b</sup>,  
Vitali Gretschko<sup>b</sup>, and Achim Wambach<sup>b</sup>

<sup>a</sup>University of Cologne

<sup>b</sup>ZEW Mannheim

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## Abstract

Procurement regulation aimed at curbing discrimination requires equal treatment of sellers. However, [Deb and Pai \(2016\)](#) show that such regulation poses virtually no restriction on the ability to discriminate. We propose a simple rule – imitation perfection – that restricts discrimination significantly. With homogeneous bidders, imitation perfection implies that all bidders earn the same surplus in every equilibrium conditional on their valuation. Revenue and social-surplus-optimal auctions exist that are consistent with imitation perfection. Imitation perfection also limits discrimination amongst heterogeneous bidders. It is incompatible, however, with revenue and social-surplus optimization. Thus, a trade-off between non-discrimination and optimality exists.

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<sup>†</sup>Corresponding author, [mass@wiso.uni-koeln.de](mailto:mass@wiso.uni-koeln.de)

# 1 Introduction

Regulators go to great lengths to prevent discrimination in procurement. In its rule on public procurement, for example, the WTO (World Trade Organization) demands that governments comply with “non-discrimination, equality of treatment, transparency and mutual recognition”, furthermore the WTO seeks “to avoid introducing or continuing discriminatory measures that distort open procurement.”<sup>1</sup> The European commission requires public buyers to reach their decision “in full accordance with the principles of equal treatment, non-discrimination and transparency.”<sup>2</sup> These regulations imply that the *rules and procedures* of a procurement process should treat suppliers equally. That is, the rules of a procurement process should not depend on the identity of the suppliers. However, [Deb and Pai \(2016\)](#) show that regulation requiring equal treatment of suppliers poses virtually no restriction on the ability to discriminate. In particular, such *symmetric auctions* allow for *perfect discrimination*. That is, there exists a symmetric auction and an equilibrium of this auction, in which the project is allocated with probability one at the most favorable price to a particular bidder. Hence, an auctioneer can favor a particular bidder in the harshest way possible without violating existing legal hurdles. This in turn, indicates that existing legal hurdles are not sufficient to prevent discrimination and that regulators should not focus on discrimination-free *rules* but rather discrimination-free *outcomes*.

This article is complementary to [Deb and Pai \(2016\)](#) and provides an answer to the question as to what rules are sufficient in order to achieve discrimination-free outcomes. We propose a simple rule – *imitation perfection*. *Imitation perfection* requires that for any realization of bids and the resulting allocation and payments, every bidder should have had the opportunity to imitate the allocation and payment of any other bidder that outbid him. We show that imposing imitation perfection prevents *perfect*

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<sup>1</sup>See the General Agreement on Tariffs and Trade (GATT) (Article 1), General Agreement on Trade in Services (GATS) (Article 2), and Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) (Article 4) and [World Trade Organization \(2012\)](#)

<sup>2</sup>See Directive 2004/18/EC of the European Parliament and of the Council of 31 March 2004 on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts.

*discrimination*. Moreover, in case of symmetric bidders, *imitation perfection* leads to discrimination-free outcomes in the following sense: in every equilibrium of an *imitation-perfect* and symmetric auction every bidder earns the same expected surplus conditional on her valuation.<sup>3</sup> This is due to the fact that with *imitation perfection* every bidder could have won the auction with the *same payment* as the winning bidder by changing her bid ex-post to a slightly higher bid than the winning bid. That is, an *imitation-perfect* auction gives each bidder the opportunity to come arbitrarily close to the ex-post surplus of every outbidding bidder with the same valuation.

With symmetric bidders it follows that bidders with the same valuation adopt identical strategies and have the same expected surplus. If bidders are ex-ante heterogeneous, the definition of a discrimination-free outcome is not straightforward. We demonstrate that *imitation perfection* still prevents perfectly discriminatory outcomes. Moreover, we show that if asymmetry amongst bidders is not too high, all imitation-perfect auctions lead to a similar expected surplus to all bidders conditional on their valuation. That is, the change in worst-case discrimination in equilibrium is linear and continuous if the setting changes from a symmetric to an asymmetric one.

Usually, the beneficiary of a procurement organization (the people of a country, the CPO of a company, or its shareholders) is responsible for thousands of different procurement projects with thousands of different bidders. According to the European Commission, there are over 250,000 public authorities involved in procurement in the EU. Delegating the specific procurement project to a (potentially large) group of agents is therefore unavoidable. Most of these agents will have the buyer's best interest in mind and will use the optimal procedures. There may, however, be some agents who are corrupt and/or favor certain bidders.<sup>4</sup> For the buyer, it is impossible to oversee each of the procurement transactions and check whether the implemented procedures were optimal. Thus, there is a need to set gen-

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<sup>3</sup>The process of a procurement auction is the same as the process of a sales auction, the only difference being that the lowest bid is awarded the contract. The bidders do not have valuations for the good but costs for fulfilling the contract. Due to the existence of the correspondence between selling auctions and procurement auctions, the formal framework will be set up for selling auctions and we will use the term auctions from now on. This has the advantage that most readers are more familiar with this notation.

<sup>4</sup>See [Mironov and Zhuravskaya](#) ([Mironov and Zhuravskaya](#)) for some recent empirical evidence.

eral procurement rules. The set of procurement regulations should have the following properties. Firstly, it should be easy to check whether those regulations have been followed. In particular, this should not require knowledge of unobservable qualities such as personal beliefs, or the use of complicated calculations such as equilibrium analyses. Secondly, the regulation should restrict corrupt agents in a meaningful way. Finally, honest agents should maintain enough freedom in order that they are able to implement optimal procedures. *Imitation perfection* has all of these desirable properties. Firstly, it is easy to check from the rules of the particular auction whether the procurement process satisfies *imitation perfection*. This is due to the fact that *imitation perfection* is an ex-post requirement on the payment rule and does not depend on knowledge of beliefs or equilibrium calculations. Secondly, *imitation perfection* prevents corrupt agents from implementing perfectly discriminatory outcomes and in case of symmetric bidders, restricts them to outcomes in which bidders earn the same expected surplus conditional on their valuation. Finally, in the case of symmetric bidders, *imitation perfection* allows the honest agents to implement the efficient auction as well as the buyer-surplus-optimal one. In this respect, ensuring that the procurement mechanism is *imitation perfect* comes at no cost if bidders are symmetric.

However, with heterogeneous bidders, *imitation perfection* is neither compatible with social-surplus maximization nor with revenue maximization. This result should not come as a surprise, as *imitation perfection* ensures that bidders achieve discrimination-free outcomes and with heterogeneous bidders, both the efficient and the revenue optimal outcome are inherently discriminatory in the sense that different bidders with the same valuation do not expect the same pay-off. Intuitively, *imitation perfection* implies that the payment of the winning bidder does not depend on the bids of the losing bidders. It can then be shown that bidders with the same valuation expect the same payment in equilibrium if the allocation is efficient. This, however, implies that if bidders have asymmetric beliefs, one of the bidders is not behaving optimally. Thus, there is a trade-off between non-discrimination and optimality.

## Relation to the literature

Only few papers deal with the question as to how general procurement rules must be designed in order to achieve the goals of procurement organizations. [Deb and Pai \(2016\)](#) analyze the common desideratum of "non-discrimination". However, they show that even equal and anonymous treatment of all bidders does not prevent discrimination. [Gretschko and Wambach \(2016\)](#) analyze how far public scrutiny can help to prevent corruption and discrimination. They consider a setting in which the agent is privately informed about the preferences of the buyer over the specifications of the horizontally differentiated sellers. The agent colludes with one exogenously chosen seller. They show that in the optimal mechanism the agent should have no discretion with respect to the winning probability of the favorite seller, which in turn induces the agent to truthfully report the preference of the buyer whenever his favorite seller fails to win. Moreover, they demonstrate that intransparent negotiations have this feature of the optimal mechanism and thus may outperform transparent auctions. Even though we do not explicitly model an agent of the buyer, our model could easily be extended by the introduction of an agent who in exchange for a bribe, would bend the rules of the mechanism in the most favorable way that is consistent with the procurement regulations. Contrary to [Gretschko and Wambach \(2016\)](#), we do not focus on the ability of the agent to manipulate the quality assessment or preferences of the buyer but rather on the ability of an agent to directly manipulate the rules of the mechanism. To the best of our knowledge, our article is therefore the first to investigate the design of procurement regulations in the presence of corruption and manipulation of the rules of the mechanism.<sup>5</sup>

In the majority of work on corruption in auctions, the ability of the agent to manipulate is defined in respect to the particular mechanism. Either the agent is able to favor one of the sellers within the rules of a particular mechanism (typically, bid-rigging in first-price auctions) or the agent is able to manipulate the quality assessment of the sellers for a particular

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<sup>5</sup>Previous work on mechanism design with corruption focused on the ability of the agent to manipulate the quality assessment and the principals optimal reaction to this. In particular, the mechanism designed by the principal is tailored to the situation at hand and does not imply general procurement regulations. See, [Celentani and Ganuza \(2002\)](#) and [Burguet \(2014\)](#) for details.

mechanism. Examples of the first strand of literature include [Arozamena and Weinschelbaum \(2009\)](#), [Burguet and Perry \(2007\)](#), [Burguet and Perry \(2009\)](#), [Cai et al. \(2013\)](#), [Compte et al. \(2005\)](#), [Menezes and Monteiro \(2006\)](#), and [Lengwiler and Wolfstetter \(2010\)](#). Examples of the second strand include [Laffont and Tirole \(1991\)](#), [Burguet and Che \(2004\)](#), and [Koessler and Lambert-Mogiliansky \(2013\)](#).

Finally, our article is related to the literature on mechanism design with fairness concerns. As pointed out by [Bolton et al. \(2005\)](#) and [Saito \(2013\)](#) (among others), market participants care about whether the rules governing a particular market are procedurally fair. Thus, *imitation perfection* can be seen not only as a device to prevent favoritism and corruption, but also as a possible way of ensuring that all equilibria of a particular mechanism yield fair (discrimination-free) outcomes. Previous approaches to mechanism design with fairness concerns in auctions and other settings include [Budish \(2011\)](#), [Bierbrauer et al. \(2015\)](#), [Bierbrauer and Netzer \(2016\)](#), [Englmaier and Wambach \(2010\)](#), and [Rasch et al. \(2012\)](#).

## 2 Model

### Environment

Let  $N = \{1, \dots, n\}$  denote a set of risk-neutral bidders. Bidders compete for one indivisible item. Bidder  $i$ 's valuation  $v_i$  for the item is drawn independently from the interval  $V_i = [0, \bar{v}_i]$  according to a distribution function  $F_i$  and is private knowledge to this bidder.  $F_i$  is common knowledge among the competitors. Denote by  $\mathbf{v}_{-i} \in \prod_{j \neq i} V_j$  the vector containing all competitors' valuations.

### Symmetric auctions

We are interested in the effectiveness of current procurement regulation. Thus, we focus on auction mechanisms in which all participants submit bids  $b_i \in \mathbb{R}^+$  and the auction mechanism assigns the item based on these bids. Let  $\mathbf{b} = (b_1, \dots, b_n)$  be the tuple of bids.<sup>6</sup> An *auction mechanism* is a

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<sup>6</sup>We will use  $\mathbf{b}_{-i}$  to denote the vector of all bids except the bid of bidder  $i$ .

double  $(x, p)$  of an allocation function  $x$  and a payment function  $p$ . The allocation function  $x : \mathbf{b} \rightarrow (x_1, \dots, x_n)$  determines the probability of each participant receiving the item and the payment function  $p : \mathbf{b} \rightarrow (p_1, \dots, p_n)$  each participant's payment. That is,  $x_i \in [0, 1]$ ,  $\sum x_i \leq 1$ , and  $p_i \in \mathbb{R}$ . Current public procurement regulation aimed at curbing discrimination imposes restrictions on the class of auction mechanisms that can be used by the auctioneer. In particular, regulators require that the rules of an auction have to treat all bidders equally.<sup>7</sup> The requirement that all bidders are treated equally by the auction mechanism was analyzed by [Deb and Pai \(2016\)](#), who provide the following definition.

**Definition** (Symmetric auction). *In a symmetric auction the allocation is given by*

$$x_i(b_i, \mathbf{b}_{-i}) = \begin{cases} \frac{1}{\#\{j \in N : b_j = b_i\}} & \text{if } b_i \geq \max\{\mathbf{b}_{-i}, r\} \\ 0 & \text{otherwise,} \end{cases}$$

where  $r$  is a reservation bid.

Let  $\pi_n$  be a permutation of the elements  $1, \dots, n$ . In a symmetric auction, it holds true that for all  $\mathbf{b} = (b_1, \dots, b_n) \in B$  that

$$p_i(b_i, \mathbf{b}_{-i}) = p_i(b_i, b_{\pi_n^{-1}(1)}, \dots, b_{\pi_n^{-1}(i-1)}, b_{\pi_n^{-1}(i+1)}, \dots, b_{\pi_n^{-1}(n)})$$

and

$$p_i(b_{\pi_n(i)}, b_{\pi_n(1)}, \dots, b_{\pi_n(i-1)}, b_{\pi_n(i+1)}, \dots, b_{\pi_n(n)}) = p_{\pi_n(i)}(b_i, b_{-i}).$$

In a *symmetric auction*, the highest bidder wins and the payment function is anonymous. That is, payment by a bidder depends only on her bid, and not on her identity. Hence, a bidder's payment is not affected if competitors permute their bids. Moreover, a permutation of all bids would lead to the same permutation of payments and allocations.

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<sup>7</sup>The *Directive 2004/18/EC of the European Parliament and of the Council of 31 March 2004 on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts* requires the buyers to post in advance all decision criteria including their weightings and reach their decision based on "two award criteria only: the lowest price and the most economically advantageous tender [...] in full accordance with the principles of equal treatment, non-discrimination and transparency."



A *bidding strategy* is a mapping  $\beta_i : V_i \rightarrow \mathbb{R}^+$ . A tuple  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$  constitutes an *equilibrium* of a mechanism if for all  $i$  and for all  $v_i \in V_i$  the bid  $\beta_i(v_i)$  maximizes participant  $i$ 's expected payoff

$$U_i^\beta(v_i) = \int_{V_{-i}} [v_i \cdot x_i(b, \beta_{-i}^*(\mathbf{v}_{-i})) - p_i(b, \beta_{-i}^*(\mathbf{v}_{-i}))] \cdot f_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}.$$

We will refer to  $(U_1^\beta, \dots, U_n^\beta)$  generated by a particular equilibrium  $(\beta_1, \dots, \beta_n)$  as the *outcome* of the auction.

## Perfect discrimination via symmetric auctions

The main insight of [Deb and Pai \(2016\)](#) is that even though the *rules* of a symmetric auction treat all bidders equally, mechanisms with discriminating *outcomes* can still be implemented. In particular, they demonstrate that almost every reasonable mechanism has an implementation as a symmetric auction. Thus, requiring the implementation of a symmetric auction is not an effective anti-discrimination measure. To illustrate the amount of potential discrimination in symmetric auctions, consider the following example.

**Example 1.** *An agency is in charge of running an auction among  $n$  bidders with valuations in  $[0, 1]$ . One of the bidders, say bidder 1, has close ties to the agency. Thus, the aim of the agency is not to maximize overall surplus but only the surplus of bidder 1. In this case, the agency can implement the following symmetric auction. The highest bidder wins the object. If only one bidder bids a strictly positive amount, all payments are zero. If more than one bidder bids a strictly positive amount, all bidders who bid a strictly positive amount pay (a penalty of) one. This auction has a Bayes-Nash equilibrium in undominated strategies in which bidder 1, irrespective of her valuation, bids some strictly positive amount  $b_1 > 0$ . All other bidders bid zero, irrespective of their valuations. In this case, bidder 1 receives the object with certainty and pays zero which constitutes the optimal outcome for bidder 1.*

We call an equilibrium a *perfect discrimination equilibrium* if one bidder wins the auction with certainty independently of her valuation and pays

nothing. All other bidders receive the object with zero probability and do not receive any payments from the auctioneer.

**Definition** (Perfect-discrimination equilibrium). *An equilibrium  $\beta_1, \dots, \beta_n$  of an auction mechanism  $(x, p)$  is called perfect-discrimination equilibrium if there exists a bidder  $i$  such that for any vector of valuations  $(v_1, \dots, v_n)$  it holds that:*

$$\begin{aligned} x_i(\beta_1(v_1), \dots, \beta_n(v_n)) &= 1 \\ p_i(\beta_1(v_1), \dots, \beta_n(v_n)) &= 0 \\ p_j(\beta_1(v_1), \dots, \beta_n(v_n)) &\geq 0 \quad \text{for } i \neq j \end{aligned}$$

*The corresponding outcome is called a perfect-discrimination outcome.*

Given that symmetric auctions do not prevent perfect-discrimination equilibria, the aim of this article is to provide a simple adjustment of symmetric auctions that restricts discrimination in a meaningful way.

Note that [Deb and Pai \(2016\)](#) propose adjustments of symmetric auctions that may restrict the class of implementable mechanisms. In particular, they consider auction mechanisms with inactive losers, continuous payment rules, monotonic payment rules and ex-post individual rationality. However, it is easy to see that none of these adjustments taken in isolation prevents the existence of perfect-discrimination equilibria. This is due to the fact that any of these adjustments allows for the implementation of the second-price auction. The second-price auction has  $n$  perfectly-discriminating equilibria in which one of the bidders bids  $\max\{\bar{v}_1, \dots, \bar{v}_n\}$  and all other bidders bid zero.

### 3 Imitation Perfection

In what follows we introduce a simple additional restriction that can be placed on symmetric auctions. We call this restriction *imitation perfection* and show that all mechanisms that comply with imitation perfection do not have perfect-discrimination equilibria. Imitation perfection requires that the rules of the auction are such that for any realization of bids and the resulting allocation and payment, every bidder would have achieved

the same allocation and payment as any other bidder who placed a higher bid by bidding slightly higher than the other bidder.

**Definition** (Imitation-perfection). *A symmetric auction  $(x, p)$  is imitation-perfect if  $p$  is non-decreasing in every component and for all bidder  $i$ , all bids  $b_i$  and all  $\epsilon > 0$  there exists a bid  $b' > b_i$  such that for all vectors of bids  $(b_1, \dots, b_i, \dots, b_j, \dots, b_n)$  it holds for all  $j \in \{1, \dots, n\}$  with  $b_j < b_i$  that*

$$|p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_j(b_1, \dots, b_i, \dots, b_{j-1}, b', b_{j+1}, \dots, b_n)| < \epsilon.^8$$

Under imitation perfection every bidder could have imitated the allocation and payment of the winning bidder ex-post. In other words, similar bids from different bidders should lead to similar outcomes *irrespective* of the behavior of the other bidders.<sup>9</sup> It is easy to verify whether an auction is imitation perfect since it can be verified ex-post and thus is independent of the beliefs of the bidders or of the selection of a particular equilibrium.

In the following we show that the outcome of an imitation perfect auction in terms of expected surplus has desired properties although the outcome depends on the particular equilibrium.

In a first step, we show that imitation perfection ensures that a symmetric auction cannot have perfect-discrimination equilibria. To clarify, suppose there exists a symmetric auction that is imitation perfect and has a perfect discrimination equilibrium. This would imply that in such an equilibrium there is one bidder who receives the object with probability one and pays zero. This cannot constitute an equilibrium in an imitation-perfect auction as any other bidder could outbid the winning bidder and receive the object at a payment arbitrary close to zero.

**Proposition 1.** *An imitation-perfect symmetric auction does not have a perfect-discrimination equilibrium.*

<sup>8</sup>For the definition of imitation perfection it is sufficient to consider only the payment function, because in a symmetric auction the allocation rule is fixed.

<sup>9</sup>To see how imitation perfection may fail, consider the second-price auction with two bidders. If bidder 1 is bidding  $b_1 = 0.5$  and bidder 2 is bidding  $b_2 = 0$ , bidder 1 will receive the object and pay a price of zero. Bidder 2 cannot imitate this outcome. By bidding slightly above one half, bidder 2 would win the object but her payment would be one half – the bid of bidder 1.

*Proof.* Let  $(x, p)$  be an imitation-perfect symmetric auction mechanism and  $(\beta_1, \dots, \beta_n)$  a perfect-discrimination equilibrium. In a perfect-discrimination equilibrium exists a bidder  $i$  who wins with certainty and has to pay zero whereas all other bidders have at most a payoff of zero. Consider an arbitrary bidder  $j \neq i$  with valuation  $\bar{v}_j$ . This bidder could bid  $b' > \beta_i(\bar{v}_i)$  such that for all  $\epsilon > 0$  :

$$|p_i(\beta_1(v_1), \dots, \beta_i(\bar{v}_i), \dots, \beta_j(\bar{v}_j), \dots, \beta_n(v_n)) -$$

$$p_j(\beta_1(v_1), \dots, \beta_i(\bar{v}), \dots, \beta_{j-1}(v_{j-1}), b', \beta_{j+1}(v_{j+1}), \dots, \beta_n(v_n))| < \epsilon$$

for all  $v_1, \dots, v_{j-1}, v_{j+1}, \dots, v_n$ . Because bidder  $i$  pays zero in the perfect-discrimination equilibrium and  $p$  is non-decreasing in each component, it follows that:

$$p_j(\beta_1(v_1), \dots, \beta_i(v_i), \dots, \beta_{j-1}(v_{j-1}), b', \beta_{j+1}(v_{j+1}), \dots, \beta_n(v_n)) < \epsilon$$

for all  $v_1, \dots, v_i, \dots, v_{j-1}, v_{j+1}, \dots, v_n$ . Since bidder  $i$  wins with probability 1 in equilibrium and  $b' > \beta_i(\bar{v})$ , bidder  $j$  would outbid every bidder with probability 1 and would pay an amount smaller than  $\epsilon$ . Bidder  $j$  therefore has an incentive to deviate to  $b'$  and therefore a perfect-discrimination equilibrium cannot exist in an imitation-perfect symmetric auction.  $\square$

In the following, we provide a formal definition of discrimination-free outcomes and explore the implications of imitation perfection for auction outcomes and whether imitation perfection is at odds with other potential goals of the seller such as revenue and social-surplus maximization.

### 3.1 Imitation perfection with symmetric bidders

We start our analysis by considering the case of symmetric bidders. That is, for this section we assume that  $F_i = F$  and  $\bar{v}_i = \bar{v}$  for all  $i \in N$ . So far, we have shown that imitation-perfect symmetric auctions do not allow for perfect discrimination. However, if bidders are symmetric, a stronger notion of “non-discrimination” can be applied:

**Definition** (Discrimination-free auction). *An equilibrium  $(\beta_1, \dots, \beta_n)$  of a*

*symmetric-auction is called non-discriminatory if it holds that:*

$$U_i^\beta(v) = \int_{V_{-i}} [v \cdot x_i(\beta_i(v), \beta_{-i}(\mathbf{v}_{-i})) - p_i(\beta_i(v), \beta_{-i}(\mathbf{v}_{-i}))] \cdot f(\mathbf{v}_{-i}) d\mathbf{v}_{-i} = \\ \int_{V_{-j}} [v \cdot x_j(\beta_j(v), \beta_{-j}(\mathbf{v}_{-j})) - p_j(\beta_j(v), \beta_{-j}(\mathbf{v}_{-j}))] \cdot f(\mathbf{v}_{-j}) d\mathbf{v}_{-j} = U_j^\beta(v)$$

for all  $i, j \in \{1, \dots, n\}$  and  $v \in [0, \bar{v}]$ . A symmetric auction is called discrimination-free if all equilibria of this auction are non-discriminatory.

In a discrimination-free equilibrium all bidders earn the same surplus conditional on their realized valuation. With symmetric bidders this is a reasonable requirement, which seems to be in line with the aim of procurement regulators to achieve equal treatment of bidders. For a given symmetric auction to be considered discrimination-free, all equilibria of this auction must be discrimination-free. The following proposition establishes that imitation-perfect symmetric auctions only allow for discrimination-free equilibria.

**Proposition 2.** *An imitation-perfect symmetric auction with reserve price  $r$  such that for every  $i$  the payment function  $p_i(b_1, \dots, b_n)$  of a winning bidder is strictly increasing in  $b_i$  for  $b_i \geq r$  is discrimination-free.*

*Proof.* The proof is relegated to the appendix.  $\square$

Intuitively, the proof of Proposition 2 builds on the fundamental idea of imitation perfection that bidders can imitate the allocation and payment of the other bidders that have outbid them. Formally, we prove that bidders implement identical strategies. This ensures that bidders with the same valuation have the same expected surplus. In order to do so, we adapt a technique of Chawla and Hartline (2013). They show that for a given auction, if some interval  $[\underline{z}, \bar{z}]$  satisfies *utility crossing*, that is, if for some bidders  $i$  and  $j$  it holds that  $U_i^\beta(\bar{z}) \geq U_j^\beta(\bar{z})$  and  $U_j^\beta(\underline{z}) \geq U_i^\beta(\underline{z})$  and  $\beta_j(v) \geq \beta_i(v)$  for all  $v \in [\underline{z}, \bar{z}]$ , then the strategies of bidder  $i$  and bidder  $j$  must be identical on this interval. If there is an interval of valuations of positive measure such that the equilibrium prescribes that one bidder strictly outbids the other, we apply imitation perfection at the upper endpoint of this interval in order to demonstrate that this interval satisfies

utility crossing. Due to imitation perfection, a deviating bid for bidder  $i$  exists, such that bidder  $i$  can achieve the same expected surplus as bidder  $j$ . Bidder  $i$ 's surplus in equilibrium cannot, therefore, be lower than bidder  $j$ 's surplus as bidder  $i$  would otherwise have an incentive to deviate.

One important class of imitation-perfect auctions are pay-your-bid auctions. Pay-your-bid auctions, in the same way as first-price and the all-pay auction, provide a simple and standard way to implement imitation-perfect symmetric auctions.

**Definition** (Pay-your-bid auction). *A symmetric auction satisfies the pay-your-bid rule if the payment of every bidder depends only on whether or not she wins and on her bid, i.e. for every bidder  $i$  her payment can be written as:*

$$p_i(b_1, \dots, b_i, \dots, b_n) = W_i * p_i^{win}(b_i) + [1 - W_i] * p_i^{lose}(b_i)$$

where  $W_i$  is equal to one if  $i$  is the winning bidder, and is equal to zero if she is not the winning bidder.

This definition leads directly to the following proposition.

**Proposition 3.** *A symmetric auction with a reserve price that satisfies the pay-your-bid rule such that  $p_i^{win}$  is continuous and strictly increasing and  $p_i^{lose}$  is continuous and non-decreasing, is imitation perfect.*

*Proof.* We have to show that for every bidder  $i$ , every bid  $b_i$  and every  $\epsilon > 0$  there exists a  $b' > b_i$  such that for every bid vector  $(b_1, \dots, b_i, \dots, b_n)$  it holds that:

$$|p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_j(b_1, \dots, b_i, \dots, \dots, b_{j-1}, b', b_{j+1}, \dots, b_n)| < \epsilon.$$

If  $b_i$  is the highest bid, due to the symmetry of the mechanism and the pay-your-bid rule, it holds for any  $b' > b_i$  that:

$$\begin{aligned} & |p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - \\ & p_j(b_1, \dots, b_i, \dots, \dots, b_{j-1}, b', b_{j+1}, \dots, b_n)| \\ & = |p_i^{win}(b_i) - p_i^{win}(b')|. \end{aligned}$$

As  $p_i^{win}$  is continuous, one can find a  $b'$  such that  $|p_i^{win}(b_i) - p_i^{win}(b')|$  is smaller than  $\epsilon$ . If  $b_i$  is not the highest bid, by applying the same argument

one can find a  $b'_i > b_i$  with  $|p_i^{lose}(b_i) - p_i^{lose}(b')| < \epsilon$  and a  $b''_i > b_i$  such that  $b''_i$  is still not the highest bid. The appropriate  $b'$  will be  $b' = \min\{b'_i, b''_i\}$ .  $\square$

Note that imitation perfection does not imply the pay-your-bid rule. To illustrate this, consider any mechanism where the payment of a bidder depends only on her bid and on the bids of higher ranked bidders. Such a mechanism is imitation-perfect but does not satisfy the pay-your-bid rule.

We have now established that all pay-your-bid auctions are imitation perfect. Thus, we can conclude that all pay-your-bid auctions are discrimination-free. In a setting with symmetric bidders, the revenue-optimal auction can be implemented as a first-price auction with a reservation price and the social-surplus optimal auction can be implemented with a first-price auction without reservation price, we can state the following corollary.

**Corollary 1.** *The following holds true:*

- (i) *There exists a symmetric and discrimination-free auction that is revenue optimal among all incentive compatible mechanisms*
- (ii) *There exists a symmetric and discrimination-free auction that is welfare maximizing among all incentive compatible mechanisms.*

It generally holds that if bidders are symmetric, there is no conflict between a discrimination-free implementation and the aim of revenue or welfare maximization.

### 3.2 Imitation perfection with asymmetric bidders

We now turn our attention to the case in which bidders are ex-ante asymmetric. That is, we assume that there are at least two bidders, say bidder  $i$  and bidder  $j$ , such that  $F_i \neq F_j$ . If bidders are asymmetric, the notion of a discrimination-free equilibrium as it was used in the previous section is no longer reasonable. This is due to the fact that asymmetry implies that different bidders with the same valuation expect different degrees of competition and thus have different expected surpluses even if identical strategies are implemented. As a consequence, it is not obvious that these bidders should earn the same surplus. Thus, to further extend the idea of

a discrimination-free equilibrium, we will show that if the asymmetry between bidders is not too pronounced, the difference between the expected payoffs for a bidder with a given valuation in various imitation-free mechanisms, will also not vary greatly. We follow [Fibich et al. \(2004\)](#) in order to provide a precise and tractable definition of what exactly we mean in saying that asymmetry is not too pronounced. We thereby assume that  $F_1, \dots, F_n$  are defined on an  $[\underline{v}, \bar{v}]$  such that:

$$F_i(v) = F(v) + \epsilon H_i(v) \quad (1)$$

with,  $\epsilon > 0$ ,  $F(\underline{v}) = 0$ ,  $F(\bar{v}) = 1$ ,  $H_i(\underline{v}) = H_i(\bar{v}) = 0$  and  $|H_i(v)| \leq 1$  for all  $v$  and all  $i$ .

We start our analysis by introducing a slightly stronger version of imitation perfection.

**Definition** (Strong imitation perfection). *A symmetric auction  $(x, p)$  is strongly imitation-perfect if  $p$  is non-decreasing in every component and if for all bidders  $i$ , for all bids  $b_i$  and for all  $\epsilon > 0$  there exists a bid  $b' > b_i$ , such that for all vectors of bids  $(b_1, \dots, b_i, \dots, b_j, \dots, b_n)$  it holds for all  $j \in \{1, \dots, n\}$  that*

$$|p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_j(b_1, \dots, b_i, \dots, \dots, b_{j-1}, b', b_{j+1}, \dots, b_n)| < \epsilon.$$

In contrast to simple imitation perfection, the strong version requires that a bidder cannot only imitate bidders that have outbid him, but rather all bidders. While conceptually the definitions do not greatly differ, the strong version of imitation perfection greatly simplifies the proof of the following proposition.

**Proposition 4.** *Let bidders' values be distributed as in (1). If  $A$  and  $B$  are strongly imitation-perfect auctions with the same reservation bid, then for every bidder  $i$  with a given valuation  $v$  the difference between the expected payoffs in any equilibrium of  $A$  and  $B$  is given by at most*

$$\max \{2((v - \underline{v})\epsilon + \epsilon), 4\epsilon\}.$$

*In particular, this implies that for a given strongly imitation-perfect*



auction  $A$  and for every bidder  $i$  with a given valuation  $v$ , the difference between the expected payoffs in potentially different equilibria of  $A$  is given by at most

$$\max \{2((v - \underline{v})\epsilon + \epsilon), 4\epsilon\}.$$

*Proof.* The proof is relegated to the Appendix.  $\square$

In case of asymmetric bidders, a bidder with a given valuation and a given distribution of the competitor's values does not earn the same expected payoff in every imitation perfect auction. An auctioneer who knows the distributions of the bidders and the valuation of her favorite bidder is still able influence her favorite bidder's expected payoff by choosing among imitation perfect auctions. However, proposition 4 demonstrates that if the asymmetry between bidders is not too pronounced, this possibility is limited.

We now turn our attention to the question as to whether imitation perfection is compatible with efficiency and revenue maximization in case of asymmetric bidders.

**Proposition 5.** *Let  $\bar{v}_i = \bar{v}$  for all  $i$  and let there exist at least one pair of bidders  $j, k$  such that  $\int_0^{\bar{v}} F_j(z)dz \neq \int_0^{\bar{v}} F_k(z)dz$ . In this case, all equilibria of a symmetric and imitation perfect auction yield inefficient outcomes. That is, the object is not always allocated to the bidder with the highest valuation.*

*Proof.* The proof is relegated to the appendix.  $\square$

Proposition 5 states an impossibility result. If the rules of an auction allow bidders to mimic each others' strategies and thus surplus, then the asymmetry of the bidders will always lead to an inefficient allocation. The intuition for this result is the following. Imitation perfection implies that the payment of the winning bidder does not depend on the bids of the losing bidders. Given that the auction is symmetric, it can be shown that two bidders with different beliefs but the same valuation expect the same payment if the allocation is efficient. However, due to the fact that the bidders have different beliefs, the winning probabilities of those bidders would differ. This implies that one of the bidders is not behaving optimally and that

the efficient allocation therefore cannot constitute an equilibrium. Applying similar reasoning indicates that imitation perfection is not compatible with revenue maximization in the case of an asymmetric auction.

**Proposition 6.** *Let  $\bar{v}_i = \bar{v}$  for all  $i$  and let there be at least one pair of bidders  $j, k$  such that  $\int_0^{\bar{v}} F_j(z) dz \neq \int_0^{\bar{v}} F_k(z) dz$ . In this case, all equilibria of a symmetric and imitation perfect auction yield non-optimal outcomes. That is, the object is not always allocated to the bidder with the highest virtual valuation.*

*Proof.* The proof is relegated to the appendix. □

## 4 Conclusion

This article demonstrates that the existing rules imposed to prevent discrimination in procurement, which require equal treatment of bidders, are not sufficient to prevent even perfect discrimination of bidders. We provide a simple extension to the existing rules which guarantees discrimination-free outcomes and which is easily verified. If bidders are symmetric both the welfare optimal and the revenue maximizing procurement mechanism can be implemented in a discrimination-free way. If bidders are asymmetric, imitation perfection still ensures that discrimination is limited in the following sense: the change in worst-case discrimination in equilibrium is linear and continuous if the setting changes from a symmetric to an asymmetric one.

# Appendices

## Definition of a direct mechanism

In a *direct mechanism* bidders report their valuations. Given a direct mechanism  $(x, p)$  the functions  $X_i$  and  $P_i$  are called *interim allocations* and *interim payments* for bidder  $i$  and are given by

$$X_i(v_i) = \int_{V_{-i}} x_i(v_i, \mathbf{v}_{-i}) \cdot f_{-i}(\mathbf{v}_{-i}) d(\mathbf{v}_{-i})$$

and

$$P_i(v_i) = \int_{V_{-i}} p_i(v_i, \mathbf{v}_{-i}) \cdot f_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}$$

Here  $x_i(v_i, \mathbf{v}_{-i})$  and  $p_i(v_i, \mathbf{v}_{-i})$  denote the allocation and payment when all bidders submit their true valuation.

Interim allocations and payments can also be defined for an equilibrium of an arbitrary mechanism. If  $(\beta_1, \dots, \beta_n)$  is an equilibrium of an arbitrary mechanism  $(x, p)$ , interim allocations and payments are defined by

$$X_i(\beta_i(v_i)) = \int_{V_{-i}} x_i(\beta_i(v_i), \beta_{-i}(\mathbf{v}_{-i})) f_{-i}(\mathbf{v}_{-i}) d(\mathbf{v}_{-i})$$

and by

$$P_i(\beta_i(v_i)) = \int_{V_{-i}} p_i(\beta_i(v_i), \beta_{-i}(\mathbf{v}_{-i})) f_{-i}(\mathbf{v}_{-i}) d(\mathbf{v}_{-i})$$

for all  $i$  and all  $v_i \in V_i$ . Note that interim allocations and interim payments for bidder  $i$  depend not only on her strategy  $\beta_i$  but on the whole strategy profile  $(\beta_1, \dots, \beta_n)$ .

We will make use of Lemma 1 in the following proofs. It shows that the payment of a bidder  $i$  in an imitation perfect auction cannot depend on bids which are lower than the bid  $b_i$ . In particular, the payment of the winning bidder depends only on her bid.

**Lemma 1.** *In an imitation perfect mechanism it holds for every bidder  $i$  and every two vectors of bids  $(b_1, \dots, b_i, \dots, b_j, \dots, b_n)$ ,  $(b_1, \dots, b_i, \dots, b'_j, \dots, b_n)$  with  $b_j < b_i$  and  $b'_j < b_i$  that*

$$p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) = p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n).$$

*Proof.* Suppose that

$$p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) \neq p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n).$$

Then, there exists some  $\alpha > 0$  such that

$$|p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n)| > \alpha.$$

Due to imitation perfection there is a  $b' > b_i$  such that for every  $\epsilon = \frac{\alpha}{2}$  it holds that

$$\begin{aligned} |p_j(b_1, \dots, b_i, \dots, b', \dots, b_n) - p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n)| &< \epsilon, \\ |p_j(b_1, \dots, b_i, \dots, b', \dots, b_n) - p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n)| &< \epsilon. \end{aligned}$$

However, the triangle inequality implies that

$$\begin{aligned} &|p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n)| \\ &\leq |p_i(b_1, \dots, b_i, \dots, b_j, \dots, b_n) - p_j(b_1, \dots, b_i, \dots, b', \dots, b_n)| \\ &\quad + |p_j(b_1, \dots, b_i, \dots, b', \dots, b_n) - p_i(b_1, \dots, b_i, \dots, b'_j, \dots, b_n)| \\ &< \frac{\alpha}{2} + \frac{\alpha}{2} = \alpha, \end{aligned}$$

which leads to a contradiction.  $\square$

## Proof of Proposition 2

*Proof.* To show that a mechanism is discrimination-free, it is sufficient to show that all bidders adopt identical strategies. If there is a reservation bid, it is sufficient to show that strategies are identical above the value of the reservation bid, as bidders bidding below the value of the reservation bid have the same expected payoff of zero. Let  $\beta = (\beta_1, \dots, \beta_n)$  be an equilibrium. We consider two arbitrary bidders, w.l.o.g. these are bidder 1 and bidder 2. Let  $r$  be the reservation bid. We denote the endpoints of an interval of values over which  $\beta_i(v) = b$  by  $\underline{v}_i(b)$  and  $\bar{v}_i(b)$ . For an arbitrary

valuation  $v$  and bids  $b_1 = \beta_1(v), b_2 = \beta_2(v)$  with  $b_2 > b_1 > r$  it holds that

$$\begin{aligned}
X_1(b_1) &= F(\underline{v}_2(b_1)) \cdot F(\underline{v}_3(b_1)) \cdot \dots \cdot F(\underline{v}_n(b_1)) + \\
&\quad \sum_{k=1}^{n-2} \frac{1}{k} \cdot Pr(k \text{ bidders have bid } b_1) \\
&\leq F(\underline{v}_2(b_1)) \cdot F(\underline{v}_3(b_1)) \cdot \dots \cdot F(\underline{v}_n(b_1)) + \\
&\quad \sum_{k=1}^{n-2} Pr(k \text{ bidders have bid } b_1) \\
&\leq F(\underline{v}_1(b_2)) \cdot F(\underline{v}_3(b_2)) \cdot \dots \cdot F(\underline{v}_n(b_2)) \\
&\leq X_2(b_2).
\end{aligned}$$

Because  $p^{win}$  is strictly increasing, it cannot hold that  $X_1(b_1) = X_2(b_2)$ . Otherwise, bidding  $b_2$  would not be a best response for bidder 2. Therefore,  $b_2 > b_1$  implies

$$X_1(b_1) < X_2(b_2). \quad (2)$$

We now require the following definition.

**Definition** (Utility crossing). *An interval  $[\underline{z}, \bar{z}]$ , with  $\underline{z} > r$ , satisfies utility crossing if  $\beta_2 \geq \beta_1$  for all  $v \in (\underline{z}, \bar{z})$  and  $U_2(\underline{z}) \geq U_1(\underline{z})$  and  $U_1(\bar{z}) \geq U_2(\bar{z})$ .*

We will show that  $\beta_1(v) = \beta_2(v)$  then holds for all  $v \in [\underline{z}, \bar{z}]$ .

Assume that  $\beta_2 > \beta_1$  over some measurable interval of valuations. It then follows from (2) that  $X_2(\beta_2(v)) > X_1(\beta_1(v))$  for all  $v$  with  $\beta_2(v) > \beta_1(v)$ . According to Myerson, it holds for every  $i$  and every  $v_i$  that

$$U_i^\beta(v_i) = U_i^\beta(\underline{v}) + \int_{\underline{v}}^{v_i} X_i(\beta_i(z)) dz.$$

Using this equation for  $\bar{z}$  and rearranging it accordingly gives

$$\begin{aligned}
U_1^\beta(\bar{z}) - U_1^\beta(\underline{z}) &= \int_{\underline{z}}^{\bar{z}} X_1(\beta_1(z)) dz, \\
U_2^\beta(\bar{z}) - U_2^\beta(\underline{z}) &= \int_{\underline{z}}^{\bar{z}} X_2(\beta_2(z)) dz \\
\Rightarrow U_2^\beta(\bar{z}) - U_2^\beta(\underline{z}) &> U_1^\beta(\bar{z}) - U_1^\beta(\underline{z}),
\end{aligned}$$

which contradicts utility crossing. It therefore holds that  $\beta_1(v) = \beta_2(v)$  for all  $v$  in  $[\underline{z}, \bar{z}]$ .

Now assume that the strategies of bidder 1 and bidder 2 differ over some measurable interval. We will show that the interval of values over which the strategies differ lies in an interval satisfying utility crossing. Hence, their strategies cannot differ.

Suppose that strategies are equal at  $\bar{v}$ . A mass point in the bid distribution of bidder 2 at  $\bar{v}$  is not possible in equilibrium. Otherwise, bidder 1 could increase her allocation probability by some  $\delta > 0$  by raising her bid marginally above  $\beta_2(\bar{v})$ . Due to imitation perfection, there exists some  $b' > \beta_2(\bar{v})$  such that for every vector of bids  $(\beta_1(\bar{v}), \beta_2(\bar{v}), b_3, \dots, b_n)$  it holds that

$$|p_2(\beta_1(\bar{v}), \beta_2(\bar{v}), b_3, \dots, b_n) - p_1(b', \beta_2(\bar{v}), b_3, \dots, b_n)| < \delta \bar{v}.$$

From Lemma 1 we know that the payment of the winning bidder depends only on her bid. Hence, the expected payment of bidder 1 and bidder 2 can be written as  $p^{win}(\beta_1(\bar{v}))$  and  $p^{win}(\beta_2(\bar{v}))$  for an appropriate function  $p^{win}$ . It holds that

$$\begin{aligned} & |p_2(\beta_1(\bar{v}), \beta_1(\bar{v}), b_3, \dots, b_n) - p_1(b', \beta_2(\bar{v}), b_3, \dots, b_n)| \\ &= |p^{win}(\beta_2(\bar{v})) - p^{win}(b')| < \delta \bar{v}. \end{aligned}$$

Therefore, bidder 1 would have an incentive to deviate. Since there is no mass point at  $\bar{v}$ , the allocation probabilities of the bidders are equal. Furthermore,  $\beta_1(\bar{v}) = \beta_2(\bar{v})$  will be the highest bid in every bid vector. Hence, the expected payment of bidder 1 and bidder 2 can be written as  $p^{win}(\beta_1(\bar{v})) = p^{win}(\beta_2(\bar{v}))$  for an appropriate function  $p^{win}$ . We conclude that the expected utilities of the bidders are equal.

Assume that strategies differ at  $\bar{v}$ . W.l.o.g. bidder 2's bids are higher than bidder 1's bids around  $\bar{v}$ . Due to imitation perfection, for every  $\epsilon > 0$  there exists a bid  $b' > \beta_2(\bar{v})$  such that for every bid vector

$(\beta_1(\bar{v}), \beta_2(\bar{v}), b_3, \dots, b_n)$  it holds that

$$|p_2(\beta_1(\bar{v}), \beta_2(\bar{v}), b_3, \dots, b_n) - p_1(b', \beta_2(\bar{v}), b_3, \dots, b_n)| < \epsilon.$$

If bidder 1 deviates to  $b'$ , (2) implies that her winning probability would be higher than the probability of bidder 2 winning. The bid  $b'$  would be the highest bid in every bid vector. Since the payment of the highest bidder depends only on the highest bid, it holds that

$$\begin{aligned} & |p_2(\beta_1(\bar{v}), \beta_1(\bar{v}), b_3, \dots, b_n) - p_1(b', \beta_2(\bar{v}), b_3, \dots, b_n)| \\ & = p^{win}(\beta_2(\bar{v})) - p^{win}(b') < \epsilon. \end{aligned}$$

Hence, the expected payment of bidder 1 exceeds the expected payment of bidder 2 by at most  $\epsilon$ . For every  $\epsilon > 0$  we have found a deviation bid for bidder 1 such that bidder 2's expected utility exceeds bidder 1's expected utility by at most  $\epsilon$ . We conclude that  $U_1^\beta(\bar{v}) \geq U_2^\beta(\bar{v})$ .

Let  $\bar{z}$  be the highest value at which the strategies differ, i.e. it holds that

$$\bar{z} = \sup \{v | \beta_2(v) > \beta_1(v)\}.$$

If we go from  $\bar{z}$  to  $\underline{v}$ , letting  $\underline{z}$  be the first value at which the strategies of bidder 1 and bidder 2 imply equal bids, i.e.

$$\underline{z} = \sup \{v < \bar{z} | \beta_1(v) = \beta_2(v)\}.$$

To prove that the interval  $[\underline{v}, \bar{v}]$  satisfies utility crossing, we must show that  $U_2(\underline{z}) \geq U_1(\underline{z})$ . If the strategy of bidder 1 implies never bidding higher than bidder 2 for the same valuation, then this follows from [Myerson \(1981\)](#). If the strategy of bidder 1 implies higher bids than the strategy of bidder 2 over some measurable interval, however, then either the bidding strategies cross in  $\underline{z}$ , or they are equal over some interval. In the latter case, we redefine  $\underline{z}$  to be some point in this interval. In both cases it holds that  $\beta_1(\underline{z}) = \beta_2(\underline{z})$ .

The expected payment of bidder 1 is given by

$$P_1(\underline{z}) = \int_{v_2 \in [\underline{v}, \bar{z}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_1(\beta_1(\underline{z}), \beta_2(v_2), \dots, \beta_n(v_n)) f_{-1}(\mathbf{v}_{-1}) d\mathbf{v}_{-1} \\ + \int_{v_2 \in [\bar{z}, \bar{v}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_1(\beta_1(\underline{z}), \beta_2(v_2), \dots, \beta_n(v_n)) f_{-1}(\mathbf{v}_{-1}) d\mathbf{v}_{-1}.$$

Because the payment does not depend on lower bids and the mechanism is symmetric, this is equal to

$$\int_{v_1 \in [\underline{v}, \bar{z}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_2(\beta_1(v_1), \beta_2(\underline{z}), \dots, \beta_n(v_n)) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \\ + \int_{v_2 \in [\bar{z}, \bar{v}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_1(\beta_1(\underline{z}), \beta_2(v_2), \dots, \beta_n(v_n)) f_{-1}(\mathbf{v}_{-1}) d\mathbf{v}_{-1}.$$

Since the payment function is non-decreasing in every component and bidder 2's bids are higher than, or equal to those of bidder 1, above  $\underline{z}$ , this is smaller or equal than

$$\int_{v_1 \in [\underline{v}, \bar{z}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_2(\beta_1(v_1), \beta_2(\underline{z}), \dots, \beta_n(v_n)) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \\ + \int_{v_2 \in [\bar{z}, \bar{v}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_1(\beta_2(\underline{z}), \beta_1(v_2), \dots, \beta_n(v_n)) f_{-1}(\mathbf{v}_{-1}) d\mathbf{v}_{-1}.$$

Due to the symmetry of the mechanism, this is equal to

$$\int_{v_1 \in [\underline{v}, \bar{z}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_2(\beta_1(v_1), \beta_2(\underline{z}), \dots, \beta_n(v_n)) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \\ + \int_{v_1 \in [\bar{z}, \bar{v}], v_3, \dots, v_n \in [\underline{v}, \bar{v}]} p_2(\beta_1(v_1), \beta_2(\underline{z}), \dots, \beta_n(v_n)) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2}.$$

Suppose that at  $\underline{z}$  bidder 2's allocation probability is lower than that of bidder 1. There is then a mass point in the bid distribution of bidder 2 at  $\underline{z}$  and it holds that  $\underline{v}_2(\beta_1(\underline{z})) < \underline{v}_1(\beta_1(\underline{z})) < \underline{z}$ . For a sufficiently small  $\delta$ , it therefore holds that  $\beta_1(\underline{v}_1[\beta_1(\underline{z}) - \delta]) > \beta_2[\underline{v}_2(\beta_1(\underline{z}) - \delta)] = \beta_2(\underline{z}) = \beta_1(\underline{z} - \delta)$ . This results in a contradiction as bidding strategies cannot be decreasing in equilibrium. Since bidder 2 has at least the same allocation probability, and at most the same expected payment as bidder 1 at  $\underline{z}$ , it follows that  $U_2^\beta(\underline{z}) \geq U_1^\beta(\underline{z})$  and therefore, the strategies of bidder 1 and



bidder 2 are equal on  $[z, \bar{z}]$ .

□

## Proof of Proposition 4

*Proof.* To prove the proposition, we first state two lemmas.

**Lemma 2.** *Let bidders' values be distributed as in Proposition 4. In a strongly imitation perfect auction it holds for every valuation  $v$  and every pair of bidders  $i$  and  $j$  that*

$$\int_{\underline{v}}^v |X_i(z) - X_j(z)| dz \leq \epsilon.$$

*Proof.* We adapt the proof of Proposition 2. We fix an equilibrium  $\beta = (\beta_1, \dots, \beta_n)$ . Let  $v$  be an arbitrary valuation and  $b_1 = \beta_1(v)$  and  $b_2 = \beta_2(v)$  with  $b_2 > b_1$ . It holds that

$$X_1(\beta_1(b_1)) \leq X_2(\beta_2(b_2)) + \epsilon. \quad (3)$$

Similarly to the proof of Proposition 2 we have:

$$\begin{aligned} X_1(\beta_1(b_1)) &= F_2(v_2(b_1)) \cdot F_3(v_3(b_1)) \cdot \dots \cdot F_n(v_n(b_1)) + \\ &\quad \sum_{k=1}^{n-2} \frac{1}{k} \cdot Pr(k \text{ bidders have bid } b_1) \\ &\leq F_2(v_2(b_1)) \cdot F_3(v_3(b_1)) \cdot \dots \cdot F_n(v_n(b_1)) + \\ &\quad \sum_{k=1}^{n-2} Pr(k \text{ bidders have bid } b_1) \\ &\leq F_2(v_1(b_2)) \cdot F_3(v_3(b_2)) \cdot \dots \cdot F_n(v_n(b_2)) \\ &\leq F_1(v_1(b_2) + \epsilon) \cdot F_3(v_3(b_2)) \cdot \dots \cdot F_n(v_n(b_2)) \\ &\leq X_2(\beta_2(b_2)) + \epsilon. \end{aligned}$$

Let  $v$  be an arbitrary valuation and  $b_2 = \beta_2(v)$ . Using the same steps as in Lemma 1, we can show that under the stronger version of imitation perfection the payment of a bidder depends only on her bid and whether or not she wins. There exists therefore a bid  $b'_1 > b_2$  with which bidder 1

could achieve an expected payment arbitrarily close to bidder 2's expected payment. Due to (3) it holds that  $X_2(b_2) \leq X_1(b'_1) + \epsilon$ . Therefore, there exists a deviation bid for bidder 1 with which she could achieve the same expected payoff as bidder 2. Hence,  $U_2^\beta(v) \leq U_1^\beta(v) + \epsilon$ .

$$\begin{aligned} U_1^\beta(v) &= U_1^\beta(\underline{v}) + \int_{\underline{v}}^v X_1(\beta_1(z))dz \\ U_2^\beta(v) &= U_2^\beta(\underline{v}) + \int_{\underline{v}}^v X_2(\beta_2(z))dz \\ \Rightarrow \int_{\underline{v}}^v X_2(\beta_2(z))dz - \int_{\underline{v}}^v X_1(\beta_1(z))dz &= U_2^\beta(v) - U_1^\beta(v) \leq \epsilon. \end{aligned}$$

With the same reasoning we can deduce that

$$\int_{\underline{v}}^v X_1(\beta_1(z))dz - \int_{\underline{v}}^v X_2(\beta_2(z))dz \leq \epsilon.$$

□

**Lemma 3.** *Let bidders' values be distributed as in Proposition 4 and let  $A$  and  $B$  be two mechanisms with reservation bid  $r$ . Let  $X_i^k(v)$  denote the expected winning probability of bidder  $i$  with valuation  $v$  in mechanism  $k$  for  $k \in \{A, B\}$ . For every bidder  $i$  with valuation  $v \geq r$  it holds that*

$$\int_r^v |X_i^A(z) - X_i^B(z)| dz \leq \max \{2((v - \underline{v})\epsilon + \epsilon), 4\epsilon\}.$$

*Proof.* The idea of the proof is to find a constant for mechanism  $A$  and  $B$  which lies between the winning probability of the lowest and the highest bidder at  $v$ . Due to Lemma 2, the difference between these winning probabilities is limited by  $\epsilon$  and therefore, the difference between the constant and the lowest (or the highest) winning probability is limited by  $\epsilon$ . Lemma 2 also implies that the difference between the winning probability of the lowest bidder and bidder  $i$  is limited by  $\epsilon$  and therefore, the winning probability between bidder  $i$  and the constant is limited by  $2\epsilon$ . Since the constant is the same for both mechanisms, the result follows from integration and the triangle inequality.

Let  $\beta = (\beta_1, \dots, \beta_n)$  be an equilibrium of mechanism  $A$  and  $\beta' = (\beta'_1, \dots, \beta'_n)$  be an equilibrium of mechanism  $B$ . W.l.o.g. let  $\beta_1(v) =$

$\min_{i=1,\dots,n} \{\beta_i(v)\}$  and  $\beta_n(v) = \max_{i=1,\dots,n} \{\beta_i(v)\}$ . It subsequently holds that

$$\beta_2^{-1}(\beta_1(v)) \cdot \dots \cdot \beta_n^{-1}(\beta_1(v)) \leq v \cdot \dots \cdot v \leq \beta_1^{-1}(\beta_n(v)) \cdot \dots \cdot \beta_{n-1}^{-1}(\beta_n(v)).$$

It follows that

$$\begin{aligned} & F_2(\beta_2^{-1}(\beta_1(v))) \cdot \dots \cdot F_n(\beta_n^{-1}(\beta_1(v))) \leq F_2(v) \cdot \dots \cdot F_n(v) \\ & \leq F_1(\beta_1^{-1}(\beta_n(v))) \cdot \dots \cdot F_{n-1}(\beta_{n-1}^{-1}(\beta_n(v))) + \epsilon. \end{aligned}$$

Let  $X_i^k(v, F_1, \dots, F_n)$  denote the expected winning probability of bidder  $i$  in mechanism  $k$  for  $k \in \{A, B\}$  for value distributions  $F_1, \dots, F_n$ . It then holds for all  $v \geq r$  that

$$X_1^A(v, F_1, \dots, F_n) \leq F_2(v) \cdot \dots \cdot F_n(v) \leq X_n^A(v, F_1, \dots, F_n) + \epsilon.$$

Integration gives

$$\begin{aligned} & \int_r^v X_1^A(z, F_1, \dots, F_n) dz \leq \int_r^v F_2(v) \cdot \dots \cdot F_n(v) dz \\ & \leq \int_r^v X_n^A(z, F_1, \dots, F_n) dz + (v - \underline{v})\epsilon \leq \int_r^v X_1^A(z, F_1, \dots, F_n) dz + (v - \underline{v})\epsilon + \epsilon \end{aligned}$$

where the last inequality follows from Lemma 2. Therefore, it holds that

$$\int_r^v |X_1^A(z, F_1, \dots, F_n) - F_2(v) \cdot \dots \cdot F_n(v)| dz \leq (v - \underline{v})\epsilon + \epsilon.$$

We know from Lemma 2 that

$$\begin{aligned} \int_r^v X_1^A(z, F_1, \dots, F_n) dz & \leq \int_r^v X_i^A(z, F_1, \dots, F_n) dz + \epsilon \\ & \leq \int_r^v X_1^A(z, F_1, \dots, F_n) dz + 2\epsilon \end{aligned}$$

and therefore, it holds that

$$\int_r^v |X_i^A(z, F_1, \dots, F_n) - F_2(v) \cdot \dots \cdot F_n(v)| dz \leq \max\{((v - \underline{v})\epsilon + \epsilon), 2\epsilon\}.$$

The analogue result holds for mechanism  $B$ . Finally, our result follows

from the triangle inequality.  $\square$

Proposition 4 follows from the application of Lemma 2.

For bidder  $i$  with valuation  $v$  the difference in expected payoffs between the two mechanisms is given by

$$\int_r^v |X_i^A(z) - X_i^B(z)| dz.$$

It follows from Lemma 3 that

$$\int_r^v |X_i^A(z) - X_i^B(z)| dz \leq \max \{2((v - \underline{v})\epsilon + \epsilon), 4\epsilon\}.$$

$\square$

## Proof of Propositions 5 and 6

*Proof.* It is w.l.o.g. to assume that the pair of bidders with different distribution functions are bidder 1 and 2 and it holds that  $\int_0^{\bar{v}} F_1(z) dz < \int_0^{\bar{v}} F_2(z) dz$ . Suppose there exists an efficient equilibrium  $(\beta_1, \dots, \beta_n)$ . Let  $(x^d, p^d)$  be the corresponding direct mechanism, i.e.

$$\begin{aligned} x_i^d(v_1, \dots, v_n) &= x_i(\beta_1(v_1), \dots, \beta_n(v_n)) \\ p_i^d(v_1, \dots, v_n) &= p_i(\beta_1(v_1), \dots, \beta_n(v_n)) \end{aligned}$$

According to Myerson (1981) and using the fact that  $x^d$  is efficient it holds for every  $v \in [0, \bar{v}]$  that

$$\begin{aligned}
P_2^d(v) &= vX_2^d(v) - \int_0^v X_2^d(z)dz + P_2^d(0) \\
&= v \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}]} x_2^d(v, \mathbf{v}_{-2}) f_{-2}(v_{-2}) d\mathbf{v}_{-2} \\
&\quad - \int_0^v \left[ \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}]} x_2^d(z, \mathbf{v}_{-2}) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \right] dz + P_2^d(0) \\
&= v \int_{[0, v] \times \dots \times [0, v]} 1 \cdot f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \\
&\quad - \int_0^v \left[ \int_{[0, z] \times \dots \times [0, z]} 1 \cdot f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \right] dz + P_2^d(0) \\
&= F_{-2}(v)v - \int_0^v F_{-2}(z)dz + P_2^d(0), \tag{4}
\end{aligned}$$

where  $F_{-2}(z)$  denotes  $F_1(z) \cdots F_3(z) \cdots F_n(z)$ .

It follows from Lemma 1 that the payment of a winning bidder with valuation  $v$  can be written as a function  $pw^d(v)$  that does not depend on the losing bidders' reported values. Using the definition of interim payments we can conclude

$$\begin{aligned}
P_2^d(v) &= \int_{[0, v] \times \dots \times [0, v]} pw_2^d(v) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \\
&\quad + \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}] \setminus [0, v] \times \dots \times [0, v]} p_2^d(v, \mathbf{v}_{-2}) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \\
&= F_{-2}(v)pw_2^d(v) + \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}] \setminus [0, v] \times \dots \times [0, v]} p_2^d(v, \mathbf{v}_{-2}) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2}. \tag{5}
\end{aligned}$$

Equating (4) and (5) yields

$$\begin{aligned}
pw_2^d(v) &= (1/F_{-2}(v)) \left( F_{-2}(v)v - \int_0^v F_{-2}(z)dz + P_2^d(0) \right. \\
&\quad \left. - \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}] \setminus [0, v] \times \dots \times [0, v]} p_2^d(v, \mathbf{v}_{-2}) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \right). \tag{6}
\end{aligned}$$

Since in a symmetric auction a permutation of bids yields to an analogue permutation of outcomes, it follows for any vectors of valuations

$(\hat{v}, v_2, \dots, v_n), (\hat{v}, v'_2, \dots, v'_n)$  where  $\hat{v}$  is the highest value that  $pw_1^d(\hat{v}) = pw_1^d(\hat{v}, v_2, \dots, v_n) = pw_2^d(v_2, \hat{v}, \dots, v_n) = pw_2^d(\hat{v})$  and similarly  $pw_1^d(\hat{v}) = pw_1^d(\hat{v}, v'_2, \dots, v'_n) = pw_2^d(v'_2, \hat{v}, \dots, v'_n) = pw_2^d(\hat{v})$ . In other words, the payment of a winning bidder depends only on her bid and not on the bids of other bidders, nor on the identity of the other bidders. It follows from [Myerson \(1981\)](#) that in a direct mechanism the expected utility of bidder 1 with value  $v$  is

$$U_1(v) = \int_0^v X_i^d(z) dz + U_1(0) = \int_0^v F_{-1}(z) dz - P_1^d(0). \quad (7)$$

By definition, the interim utility for bidder 1 with value  $v$  is

$$\begin{aligned} U_1(v) &= - \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}] \setminus [0, v] \times \dots \times [0, v]} p_1^d(v, \mathbf{v}_{-2}) f_{-1}(\mathbf{v}_{-1}) d\mathbf{v}_{-1} \\ &\quad + F_{-1}(v)v - F_{-1}(v)pw_1^d(v) \\ &\stackrel{(6)}{=} - \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}] \setminus [0, v] \times \dots \times [0, v]} p_1^d(v, \mathbf{v}_{-1}) f_{-1}(\mathbf{v}_{-1}) d\mathbf{v}_{-1} + F_{-1}(v)v \\ &\quad (-F_{-1}(v)/F_{-2}(v)) \left( F_{-2}(v)v - \int_0^v F_{-2}(z) dz + P_2^d(0) \right. \\ &\quad \left. - \int_{[0, \bar{v}] \times \dots \times [0, \bar{v}] \setminus [0, v] \times \dots \times [0, v]} p_2^d(v, \mathbf{v}_{-2}) f_{-2}(\mathbf{v}_{-2}) d\mathbf{v}_{-2} \right). \end{aligned}$$

It holds that  $F_1(\bar{v}) = F_2(\bar{v})$  and therefore  $F_{-1}(\bar{v}) = F_{-2}(\bar{v})$  and the expression for the expected utility simplifies to

$$U_1(v) = \int_0^{\bar{v}} F_{-2}(z) dz - P_2^d(0) < \int_0^{\bar{v}} F_{-1}(z) dz - P_1^d(0) \stackrel{7}{=} U_1(v)$$

The strict inequality is due to  $\int_0^{\bar{v}} F_1(z) dz < \int_0^{\bar{v}} F_2(z) dz$ . This constitutes a contradiction. The proof of Proposition 6 works in the same way with the only difference being that the distributions are replaced with the corresponding virtual valuations.  $\square$

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