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Consumption Insurance, Welfare, and Optimal Progressive Taxation*

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Abstract

We study consumption insurance and optimal progressive taxation in a heterogeneous-agent life-cycle model of consumption and labor supply. Partial insurance of consumption against wage shocks is achieved through (i) progressive taxation, (ii) labor supply adjustment, and (iii) precautionary wealth accumulation. Based on US data, the estimated model, fitting well with age profiles of wealth and labor supply distributions, suggests that wealth plays an important role in reducing the consumption response to wage shocks. The optimal degree of progressivity depends on preference and initial wealth conditions. Households that are more patient, more willing to work, and less wealthy prefer more progressivity. Using the same model estimated from German data, we show that the optimal degree of progressivity is similar in Germany to that in the United States, although the wealth channel in Germany has a greater impact on achieving consumption insurance.

Keywords consumption insurance · optimal taxation · wage risk · labor supply · wealth

JEL Classification D31 · E21 · H21 · J22

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1 Introduction

Measurements of risk-sharing mechanisms are important when designing optimal taxation and social welfare programs. This study quantifies those channels through which consumption insurance against idiosyncratic wage shocks is achieved by US and German households, and studies the optimal degree of progressivity within a class of progressive tax functions.

To this end, we use an estimated incomplete-markets heterogeneous-agent model, where households choose their consumption and labor supply over the life cycle. In this model, there are two types of idiosyncratic wage shocks, namely permanent and transitory. Households differ in their initial wealth, preferences, and wage shocks. Partial consumption insurance is achieved through three channels: progressive taxation, labor supply adjustment, and wealth accumulation.

Most of the existing literature on consumption insurance focuses on the comovement of income and consumption (Blundell et al. (2008), Jappelli and Pistaferri (2010), etc) and, more recently, on labor supply (see e.g., Heathcote et al. (2014)). Very few studies explicitly consider the role of wealth.¹ However, wealth can be important, for at least two reasons. First, it reduces the precautionary saving motive, thereby affecting consumption and labor supply responses to wage shocks. For instance, in response to the same windfall income, households vary in terms of whether to consume more or work less, depending on their wealth level. Second, wealth is a component of budget constraints. In the absence of high-quality data on consumption, data on wealth can be useful to discipline consumption in a structural model.

¹Exceptions include the work of Kaplan and Violante (2010), where the only insurance mechanism is wealth, and Krueger and Perri (2011), who find that consumption insurance depends crucially on wealth holdings.

In this study, we explicitly consider the role of wealth in our model. Specifically, we consider wealth and labor supply quantiles per age, and use the method of simulated moments to estimate the structural parameters of the model by matching the distributions of wealth and the labor supply over the life cycle.

A prerequisite for quantifying consumption insurance is to measure idiosyncratic risks in wage. We extend the literature on estimating wage and income processes by developing a new method that places both long-term and short-term restrictions on wage growth rates. An important observation is that the variance of income increases over the life cycle as a result of permanent shocks. This can be used to identify the variance of permanent shocks (Carroll (1992)). Essentially, this pattern restricts long-term income growth. Another observation is that income growth rates are autocorrelated owing to transitory shocks. The covariance between consecutive periods of income growth can be used to identify the variance of transitory shocks (Meghir and Pistaferri (2004)). This places a short-term restriction on income growth. We combine these observations to distinguish clearly between permanent and transitory shocks, making full use of available data.

To quantify how consumption insurance is achieved, we decompose the consumption response to wage shocks. We show that in the absence of wealth accumulation, the model has a closed-form solution. Consumption insurance can be decomposed exactly into a progressive taxation channel and a labor supply adjustment channel. In the presence of wealth, we can decompose consumption insurance into three channels by acknowledging that wealth changes the transmission of wage shocks to consumption and labor. Since the model no longer has a close-form solution, we resort to our numerically estimated model to quantify the strength of each channel.

Our main result can be summarized as follows. Over the life cycle, US households insure, on average, against 43% of permanent wage shocks, of which 11% is by progressive taxation, 7% by wealth, and 25% by labor supply adjustment. In contrast,

90% of transitory wage shocks are insured against, of which 11% is by progressive taxation, 54% by wealth, and 25% by labor supply adjustment. Wealth is more important in insurance against transitory shocks than it is for permanent shocks. Obviously, the role of wealth is not constant; as households accumulate wealth over the life cycle, the insurance mechanism through wealth strengthens.

Using the estimated model, we further analyze the optimal degree of progressive taxation within a class of progressive taxation, following [Bénabou \(2002\)](#). Abstracting from public goods, we adopt a simple framework where net tax revenue over the life cycle is fixed at its current level as we vary the degree of progressivity. We measure welfare as the expected discounted utility over the life cycle. Welfare trade-offs arise because progressivity changes the incentive to work and, hence, consumption levels, and because it changes the strengths of risk sharing mechanisms.

Based on our model, we find that US progressive taxation is a little more progressive than the average optimal degree of progressivity indicates. However, the optimal level for the economy as a whole is not necessarily optimal for heterogeneous households. Our model suggests that households that are more patient, more willing to work, and less wealthy prefer more progressivity.

Finally, we re-estimate our model using German data in order to compare our findings from the United States to the results from Germany. There are two reasons for the comparison. First, it serves as a robustness check of whether the model is capable of fitting reasonably well with similar data from other countries. Second, and more importantly, Germany has a more progressive tax system than the United States does. Thus, it is interesting to compare the consumption insurance and the optimal degree of progressivity in the two countries.

Compared with the United States, we find that idiosyncratic risks are lower in Germany. The extent to which consumption is insured is similar, although the wealth channel plays a more important role in Germany. Interestingly, the optimal degree

of progressivity is almost the same in both countries, despite differences in their preferences and distributions of wealth and labor.

Our analysis uses the concept of partial insurance developed in [Blundell et al. \(2008\)](#), and relates to studies quantifying partial insurance. [Heathcote et al. \(2014\)](#) derive a closed-form solution for consumption and labor supply in a general equilibrium model. We show that this solution is analytically similar to that obtained for hand-to-mouth households in our model. However, we emphasize the empirical relevance of our model with respect to the wealth distribution. [Kaplan and Violante \(2010\)](#) use a consumption-saving life-cycle model to study partial insurance, where wealth is the only insurance mechanism. In our model, progressive taxation, wealth, and labor supply adjustment insure households against idiosyncratic wage shocks.

The social insurance effect of progressive taxation has been investigated by [Mirrlees \(1974\)](#) and [Varian \(1980\)](#), but not with an explicit focus on precautionary saving. The literature on precautionary saving (e.g., [Carroll and Samwick, 1998](#)) typically assumes exogenous labor income, and abstracts from modeling taxation and transfers explicitly. At the same time, older studies show that precautionary behavior is influenced strongly by transfers ([Hubbard et al., 1995](#); [Gruber and Yelowitz, 1999](#); [Engen and Gruber, 2001](#)). We connect these findings and provide a comprehensive examination of the interdependencies between taxes, labor supply, and precautionary saving.

There is a significant body of literature on optimal taxation.² In terms of the

²For example, using simulations, [Fehr et al. \(2013\)](#) find that the optimal system involves higher progressivity, because insurance benefits over-compensate for additional labor market distortions. [Conesa et al. \(2009\)](#) ([Conesa and Krueger \(2006\)](#)) study the trade-off between efficiency and insurance under progressive taxation, and find that a proportional income tax with a constant marginal tax rate of 23 (17.2) percent and a deduction of roughly 7,200 (9,400) USD is optimal for the United States. [Floden and Lindé \(2001\)](#) examine idiosyncratic risks and the effects of government redistribution in the United States and Sweden in the context of proportional taxation, finding substantial welfare benefits of redistribution. [Low and Maldoom \(2004\)](#) find that optimal tax progressivity depends on the ratio of prudence to risk aversion.

class of progressive tax functions, our study is most closely related to the work of [Heathcote et al. \(forthcoming\)](#), who analyze the optimal degree of progressivity for the United States in a general equilibrium environment, where consumption, labor supply, human capital, and public goods shape welfare trade-offs. Interestingly, our estimate is similar to theirs, though we emphasize the match of wealth and labor supply distributions over the life cycle in a partial equilibrium framework.

This study is also related to the literature on the estimation of the income process and life-cycle models. The estimation of the wage process extends the methods developed by [Carroll \(1992\)](#), [Meghir and Pistaferri \(2004\)](#), [Blundell et al. \(2008\)](#), and [Hryshko \(2012\)](#). The key innovation here is that we exploit both long-term and short-term restrictions on wage growth to distinguish between permanent and transitory wage shocks. The estimation of the structural parameters of our life-cycle model is similar to those in [Gourinchas and Parker \(2002\)](#) and [Cagetti \(2003\)](#). We use our model to match a few quantiles of wealth and labor supply distributions over the life cycle. The empirical relevance to distributions of the wealth and labor supply is crucial to our quantitative results.

The rest of the paper is structured as follows. In [Section 2](#), we introduce a standard incomplete markets model with taxes and transfers, and discuss the insurance mechanisms and welfare implications of progressive taxation. [Section 3](#) briefly describes our data, the estimation procedure, and results of key parameters and variables. Our main results are reported in [Section 4](#). Lastly, [Section 5](#) concludes the paper. In [Appendix A](#), we describe our data in more detail, along with the solution to ([Appendix C](#)) and the estimation of the proposed model ([Appendix B](#)). [Appendix D](#) lists our parameter choices. Finally, we compare the results for the United States and Germany in [Appendix E](#).

2 Model

This section first introduces a standard incomplete-markets model of household consumption and labor supply over the life cycle, and then analyzes the insurance effects and welfare implications of progressive taxation in the context of this model. We discuss three channels through which consumption is partially insured under progressive taxation, and two types of welfare trade-offs under progressive taxation.

2.1 A Life-Cycle Model

The economy consists of a finite number of households, indexed by i . We assume that households start their economic life in period 0 and live for T years. Households work for T^w years in their life, and then enter the stage of retirement. During their working life, households derive utility from annual consumption C_{it} and disutility from labor H_{it} ; after retirement, households only enjoy utility from consumption. We assume that households face a positive probability of death after retirement, and all households die at time T with certainty. Let p_t^S be the conditional probability of survival at time t , and M_{it} be the market resources households leave as a bequest. Households maximize their expected discounted utility, as follows:

$$E_0 \left\{ \sum_{t=0}^{T^w-1} \beta_i^t u(C_{it}, H_{it}) + \sum_{t=T^w}^{T-1} \beta_i^t [p_t^S u^R(C_{it}) + (1 - p_t^S) u^B(M_{it})] \right\}, \quad (1)$$

where β_i is the discount factor for household i . We allow β to be heterogeneous among households to account for the empirical wealth distribution.

Period utility during working age follows

$$u(C_{it}, H_{it}) = \frac{C_{it}^{1-\rho}}{1-\rho} - e^{\phi_i} \frac{H_{it}^{1+\sigma}}{1+\sigma}.$$

The parameter ρ governs the intertemporal elasticity of substitution for consumption and σ governs the Frisch-elasticity of labor supply. $1/\phi_i$ captures the extent of disutility from the labor supply in terms of consumption goods. We allow ϕ to differ across households to account for the empirical distribution of the labor supply.

The period utility after retirement follows

$$u^R(C_{it}) = \frac{C_{it}^{1-\rho}}{1-\rho}.$$

We assume the utility from a bequest follows a simple functional form,

$$u^B(M_{it}) = e^\varphi \frac{M_{it}^{1-\rho}}{1-\rho},$$

where the parameter φ captures the degree of the bequest motive.³

Budget Constraint

Let A_{it} be the amount of financial assets household i holds at the beginning of period t , W_{it} be the wage in year t , and H_{it} be the labor supply in year t . Then, the budget constraint follows

$$A_{i,t+1} = A_{i,t}(1 + r(1 - \tau_A)) + W_{it}H_{it} - TX(W_{it}H_{it}) - (1 + \tau_C)C_{it}. \quad (2)$$

Here, r is the risk-free interest, τ_A and τ_C are the proportional tax rates on capital income and consumption, respectively, and $TX(\cdot)$ is the progressive tax function.

Progressive Taxation

³In principle, we could allow φ to differ across households, but we find that the heterogeneity in φ only makes the model marginally better in fitting the data. In light of the extra degree(s) of freedom introduced by this heterogeneity, we decide to drop it.

We specify a parsimonious tax function taken from the public finance literature (see [Feldstein \(1969\)](#)), following [Bénabou \(2002\)](#) and [Heathcote et al. \(2014\)](#). Taxes or transfers are given by

$$TX(Y_{it}) = Y_{it} - \lambda Y_{it}^{1-\tau}. \quad (3)$$

With this tax function, disposable (post-government) income \tilde{Y}_{it} is a function of pre-government income Y_{it} and the parameters τ and λ .

$$\tilde{Y}_{it} = \lambda Y_{it}^{1-\tau}. \quad (4)$$

The parameter τ determines the degree of progressivity of the tax system.⁴ For $\tau = 0$, the tax and transfer function becomes a proportional tax system with tax rate $1 - \lambda$; for $\tau = 1$, the tax and transfer function becomes so progressive that the after-government income is equal to λ , irrespective of the pre-government income. The parameter λ is related to tax revenue. Holding tax revenue constant, λ increases as the degree of progressivity τ increases.

Wage Process

We assume that the wage follows a permanent–transitory type of process:

$$\log Z_{it} = \gamma_t + \log Z_{it-1} + \eta_{it}, \quad (5)$$

$$\log W_{it} = \log Z_{it} + \epsilon_{it}, \quad (6)$$

where Z_{it} is the permanent component of the wage process, γ_t is the deterministic

⁴The standard definition of a progressive tax-transfer function in public economics is that the marginal tax rate is larger than the average tax rate for every level of pre-government income: $TX'(Y) > TX(Y)/Y$. Applying this definition to our specific tax function implies $1 - \lambda(1-\tau)Y^{-\tau} > 1 - \lambda Y^{-\tau}$, which is true when $\tau > 0$.

growth rate common to all households, and η_{it} and ϵ_{it} are normally distributed permanent and transitory shocks to wages, respectively,

$$\begin{aligned}\eta_{it} &\sim N\left(-\frac{\sigma_{\eta,t}^2}{2}, \sigma_{\eta,t}^2\right), \\ \epsilon_{it} &\sim N\left(-\frac{\sigma_{\epsilon,t}^2}{2}, \sigma_{\epsilon,t}^2\right).\end{aligned}$$

We allow the variances of the permanent and transitory shocks to the wage to vary by age because this feature is present in the data and is important for the rate of wealth accumulation over the life cycle.

Borrowing Constraint

The borrowing constraint can have a large impact on the consumption and labor supply choices of households with a low wage or wealth. We assume that the maximum amount that households can borrow depends on their permanent component of wage. In particular,

$$A_{i,t+1} \geq -\underline{a}Z_{it}. \tag{7}$$

This constraint can be interpreted as banks' decisions to lend on the basis of households' annual income: banks assess the average wage of households, calculate their annual income if they work full time, and then decide what fraction of that income can be lent.

2.2 Partial Insurance under Progressive Taxation

The key question we want to address is how much of a transitory or a permanent shock to wages transmits to a household's consumption response under progressive taxation. In other words, we analyze how shocks are insured against under progres-

sive taxation. To this end, it is useful to examine the optimality conditions of a household's problem.

The intratemporal optimal condition (in log form) is

$$(1 - \tau) \log W_{it} = (\sigma + \tau) \log H_{it} + \rho \log C_{it} + \phi_i \log \left(\frac{(1 + \tau_c)}{\lambda(1 - \tau)} \right).$$

Combined with the wage process in equations (5) and (6), we have

$$(1 - \tau)(\gamma_t + \log Z_{it-1} + \eta_{it} + \epsilon_{it}) = (\sigma + \tau) \log H_{it} + \rho \log C_{it} + \phi_i \log \left(\frac{(1 + \tau_c)}{\lambda(1 - \tau)} \right), \quad (8)$$

where γ_t and $\log Z_{it-1}$ are known at time t , and the last term on the right side of the equation is a constant. Permanent and transitory shocks (η_{it} and ϵ_{it} , respectively), after being adjusted by progressive taxation, pass on to labor and consumption.

Equation (8) makes it clear that there are three channels through which consumption is partially insured against idiosyncratic shocks to wages under progressive taxation.

The first channel is the reduction of wage shocks through progressive taxation, which appears on the left side of equation (8) as $(1 - \tau)$. Any shock to wage growth is reduced by a fraction τ . If tax is proportional ($\tau = 0$), households bear the full impact of wage shocks; on the other extreme, if tax is fully progressive ($\tau = 1$), no shocks will be passed on to households. We call this the *direct* effect of progressive taxation. Intuitively, because the marginal tax rate is increasing under progressive taxation, households will pay a higher (lower) average tax rate if there are positive (negative) changes to their income. As a result, post-government wages have a smaller variance than pre-government wages do,⁵ and part of the shock to pre-government wages is

⁵More precisely, the post-government wage refers to the *log* level of the post-government wage. Proportional tax also reduces the variance of the level of the post-government wage, but it does not reduce the log level of the wage.

insured against.

The second channel is labor supply adjustment, which appears as the first term on the right side of equation (8). This absorbs part of the post-government wage shock, partially insuring consumption against the shock. Progressive taxation distorts labor supply and, hence, affects its response to idiosyncratic shocks. The more progressive taxation is, the less elastic labor becomes.⁶

The third channel is precautionary wealth accumulation. Although this does not appear directly in equation (8), it changes the relationship between $\log W_{it}$ and $\log H_{it}$, thereby affecting how much of a wage shock transmits to consumption. To see this, we examine the intertemporal optimality condition,

$$C_{it}^{-\rho} = \beta(1 + r(1 - \tau_A))E_t [C_{it+1}^{-\rho}] + (1 + \tau_c)\mu_t, \quad (9)$$

where μ_t is the Lagrange multiplier on the borrowing constraint (7). If wealth is sufficiently high, the expectation operator E_t in equation (9) disappears and there is no precautionary saving motive. Consumption is deterministic and wage shocks are fully absorbed by progressive taxation and labor adjustment. On the other hand, if wealth is low, there is a strong precautionary saving motive and consumption can be quite responsive to wage shocks. Because progressive taxation reduces the uncertainty of current and future post-government wages, it affects the strength of the precautionary motive.

We call the latter two channels the *indirect* effects of progressive taxation.

⁶Here, $1/\sigma$ is the elasticity of labor with respect to pre-government wages under proportional taxation ($\tau = 0$). Under progressive taxation, the elasticity is $\frac{1-\tau}{\sigma+\tau}$.

2.3 Decomposition of Partial Insurance

To further disentangle insurance through labor supply and wealth accumulation, we start with a special case where there is no wealth accumulation and households are living hand-to-mouth.⁷ The intratemporal optimality condition (8) still holds, but the borrowing constraint is binding

$$C_{it} = \frac{1}{1 + \tau_C} (W_{it}H_{it} - TX(W_{it}H_{it})) = \frac{\lambda}{1 + \tau_C} (W_{it}H_{it})^{1-\tau}. \quad (10)$$

Equations (8) and (10) together fully characterize households' consumption and labor choices. In particular,

$$\log C_{it} = (1 - \tau) \frac{\sigma + 1}{\rho + \sigma + (1 - \rho)\tau} \log W_{it} + \tilde{C}, \quad (11)$$

$$\log H_{it} = (1 - \tau) \frac{1 - \rho}{\rho + \sigma + (1 - \rho)\tau} \log W_{it} + \tilde{H}, \quad (12)$$

where

$$\begin{aligned} \tilde{C} &= \frac{\sigma + 1}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\lambda}{1 + \tau_C} - \frac{1 - \tau}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\phi_i}{1 - \tau}, \\ \tilde{H} &= \frac{1 - \rho}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\lambda}{1 + \tau_C} - \frac{1}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\phi_i}{1 - \tau}. \end{aligned}$$

The coefficient before the wage term in equation (11) captures the consumption insurance under progressive taxation in the absence of wealth. Then, $(1 - \tau)$ is the direct effect of progressive taxation, and $\frac{\sigma+1}{\rho+\sigma+(1-\rho)\tau}$ is insurance through labor supply adjustment.⁸ The extent to which the labor supply responds to a wage shock under

⁷Alternatively, we can think of households as facing no transitory shocks and consuming their permanent income in each period.

⁸This is the same pass-through coefficient for permanent shocks as in [Heathcote et al. \(2014\)](#) (cf. Proposition 1) in a general equilibrium setting.

progressive taxation is captured by the coefficient before the wage term in equation (12). The parameter ρ determines the sign of the labor supply change in response to an increase in wages: if $\rho > 1$, the income effect dominates the substitution effect and labor supply decreases.

We define the consumption and labor responses to a particular type of wage shock

$$\begin{aligned}\xi_C &= \frac{\text{cov}(\log C_{it}, \psi_{it})}{\text{var}(\psi_{it})}, \\ \xi_H &= \frac{\text{cov}(\log H_{it}, \psi_{it})}{\text{var}(\psi_{it})},\end{aligned}$$

respectively, where ψ can be permanent or transitory.

Taking the covariance of both sides of equation (8) with ψ and rearranging, we arrive at an expression describing how wage shocks are absorbed,

$$\begin{aligned}1 - \tau &= (\sigma + \tau) \frac{\text{cov}(\log H_{it}, \psi_{it})}{\text{var}(\psi_{it})} + \rho \frac{\text{cov}(\log C_{it}, \psi_{it})}{\text{var}(\psi_{it})} \\ &= (\sigma + \tau)\xi_H + \rho\xi_C.\end{aligned}\tag{13}$$

A fraction τ is absorbed by the direct effect of progressive taxation. In the absence of wealth, equation (12) implies that $\xi_H = (1 - \tau) \frac{1 - \rho}{\rho + \sigma + (1 - \rho)\tau}$. If we define

$$\chi = \frac{(1 - \rho)(\sigma + \tau)}{\rho + \sigma + (1 - \rho)\tau},\tag{14}$$

then a fraction χ of the remaining impact of a wage shock is absorbed by the labor supply and $1 - \chi$ by consumption. That is,

$$\{\tau, (1 - \tau)(1 - \chi), (1 - \tau)\chi\}$$

characterizes the allocation of wage shocks in the absence of wealth. The labor

supply response is given by $\xi_H = (1 - \tau)\chi\frac{1}{\sigma+\tau}$ and the consumption response is $\xi_C = (1 - \tau)(1 - \chi)\frac{1}{\rho}$. To see how the labor supply adjustment affects consumption insurance, we need to compare the consumption response to wage shocks when labor is elastic and when it is inelastic. Let $\sigma \rightarrow \infty$ in equation (14) so that labor is completely inelastic. Then, we have $\chi_\infty = 1 - \rho$. Thus, the effect of the labor supply is $(\chi - \chi_\infty)/\rho$.

In the presence of wealth, because wealth changes the allocation of a wage shock between consumption and labor supply, but not the total amount of the shock to be absorbed by them,⁹ the allocation of a wage shock can be written as follows:

$$\{\tau, (1 - \tau)(1 - \chi - \chi'), (1 - \tau)(\chi + \chi')\},$$

where χ' denotes the effect of wealth on the allocation of a wage shock to consumption.¹⁰ Then, in the presence of wealth, $\xi_C = (1 - \tau)(1 - \chi - \chi')\frac{1}{\rho}$. After a little algebraic manipulation, we have

$$\xi_C = (1 - \tau)(1 - \chi - \chi')\frac{1}{\rho} = (1 - \tau) \left(1 - \frac{\chi'}{\rho} - \frac{\chi - \chi_\infty}{\rho} \right). \quad (15)$$

This makes it clear that consumption insurance is obtained through the direct effect of progressive taxation (τ), and the indirect effects of labor supply ($\frac{\chi - \chi_\infty}{\rho}$) and wealth ($\frac{\chi'}{\rho}$). Without progressive taxation, wealth accumulation, or elastic labor, consumption would respond on a one-to-one basis with a wage shock. Uncertainty in wages makes χ' intractable. Therefore, we resort to numerical methods for the estimation.

⁹Equation (8) holds with or without wealth.

¹⁰Note that χ' is an average, because the impact of wealth depends on the level of wealth, which differs across households and changes over time for the same household.

2.4 Welfare and the Optimal Progressive Taxation

We define the welfare of household i , $\widetilde{\mathcal{W}}(\lambda, \tau, A_{i,0})$, as the value of the maximized expected discounted utility over the life cycle:

$$\widetilde{\mathcal{W}}(\lambda, \tau, A_{i,0}) = E_0 \left\{ \sum_{t=0}^{T^w-1} \beta_i^t u(C_{it}, H_{it}) + \sum_{t=T^w}^{T-1} \beta_i^t [p_t^S u^R(C_{it}) + (1 - p_t^S) u^B(M_{it})] \right\}.$$

Welfare depends on the tax system (λ and τ) and the initial market resources ($A_{i,0}$). To facilitate a comparison of welfare under different degrees of progressivity, we use the following risk-adjusted transformation,

$$\mathcal{W} = \left((1 - \rho) \widetilde{\mathcal{W}} \right)^{\frac{1}{1-\rho}}, \quad (16)$$

where \mathcal{W} is given in units of consumption.

It is important to keep (expected) tax revenue constant in the discussion of the welfare implications of progressive taxation. One way to rationalize this is that the government redistributes the net tax revenue to households in the form of public goods. Once we assume that the utility derived from public goods is additive to our current specification of utility, then this redistribution does not change the ranking of welfare, as long as tax revenue is constant.

There are two types of welfare trade-offs under progressive taxation. The first type is intratemporal. As taxation becomes more progressive (an increase in τ), there are broadly two ways of keeping labor income tax revenue fixed. One is to cut the average tax rate to induce households to work more, which results in more consumption and more labor supply. The other is to raise the average tax rate to make households work less, which results in less consumption and less labor supply. Whether welfare improves depends on the balance between the contribution to wel-

fare of extra consumption and the loss of welfare from extra labor. In particular, welfare is not necessarily monotonic in the degree of progressivity.

The second type of welfare trade-off is intertemporal. A typical household's income increases as she progresses over the working life. As taxation turns more progressive, this implies a shift of taxation from young age to old age. In the presence of a borrowing constraint, this shift tends to increase household current consumption. In addition, because progressive taxation reduces future post-government wage uncertainty, it gives a further boost to current consumption. As such, welfare is affected by the time preference rate of households in relation to interest rates.

In light of the trade-offs, it is natural to ask what degree of progressive taxation is optimal within the class of tax functions in equation (3). This question is not trivial because we require the net tax revenue to remain constant. At one extreme, taxation can be so progressive that households barely want to work. Almost all of their income is taxed away to reach the revenue target, so consumption is close to zero and utility approaches negative infinity. In this case, welfare is undoubtedly less than that under proportional taxation.

Because there is no closed-form solution to household welfare, we again resort to numerical methods to determine the relationship between welfare and progressivity under fixed tax revenue.

3 Estimation

In this section, we estimate the parameters of our model. Throughout, we use the Panel Study of Income Dynamics (PSID). In the appendix, we re-estimate our model using comparable household survey data from the German Socio-Economic Panel (SOEP). It is interesting to see how consumption insurance differs in two advanced

economies when they differ in terms of their degree of progressivity, initial wealth distribution, and preferences.

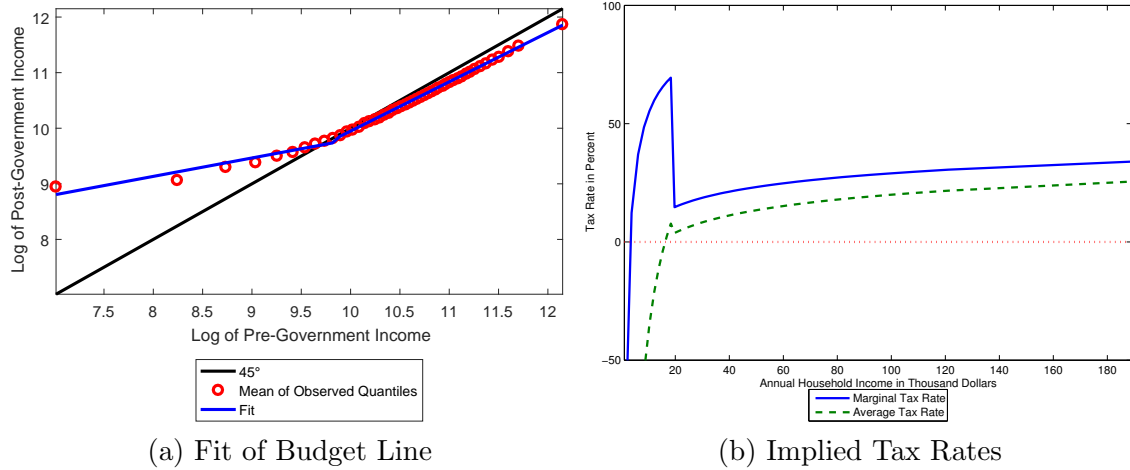
The key components of the model include progressive taxation, a wage process, and household preferences. We estimate the progressive tax function directly from information on pre-government and post-government income in the data. To estimate the wage process, we extend the literature on the income process and use the variance–covariance restriction of all possible residual wage growth rates to identify the variances of permanent and transitory shocks. Finally, we use the method of simulated moments to estimate the parameters related to household preferences by matching the empirical wealth and the labor supply distributions over the life cycle.

3.1 Progressive Taxation

The tax function given in equation (3) is parsimonious in its parametrization, but gives a remarkably good representation of the actual tax and transfer system in the United States. To estimate the two parameters, we use data on households’ pre-government and post-government labor income from the PSID. Pre-government household income includes labor earnings, private transfers (e.g., alimony), pension incomes, and income from interest, dividends, and rent. Post-government income is equal to pre-government income less income taxes (including income and payroll taxes, etc.), plus public transfers (unemployment benefits, social assistance etc.).

The left graph in Figure 1 compares our estimated tax function with the data. We construct the tax schedule implied in the data as follows. First, we collapse all waves of our data into 50 quantiles by pre-government income, and associate the mean of each quantile with the mean post-government income in the same quantile. These points are shown as circles in the figure. Then, we estimate equation (4) in logs using an ordinary least squares regression. Because transfers have a different slope and intercept, we interact $\log Y_{it}$ with a dummy indicating whether post-government

Figure 1: Estimation of Tax and Transfer Function and Implied Tax Rates



income exceeds pre-government income. This simple model fits the data quite well, with $R^2 = 0.996$. The point estimates are $\tau = 0.11$ and $\lambda = 2.95$ for the region where no transfers are paid.¹¹ This is similar to the estimate of $\tau^{US} = 0.15$ that [Heathcote et al. \(forthcoming\)](#) establishes for the United States. The 45 degree line is where pre- and post-government income are identical. As shown in the left graph, the predicted post-government income (the solid kinked line) aligns well with the data.

The right graph in Figure 1 displays the implied marginal and average tax rates, which we obtain from our estimates of τ and λ . The solid line shows the marginal tax rate, while the dashed line is the average tax rate. Marginal tax rates are high for transfer recipients and increase to about 70 percent because of transfer withdrawal. Once a household no longer receives transfers, the marginal tax rate drops to about 15 percent. For high-income households, the marginal tax rate increases to 35 percent. The red dashed line displays the average tax rate.

¹¹In the region where transfers are paid, $\tau = 0.67$ and $\lambda = 655$.

3.2 Wage Process

We use all waves available to estimate the wage process given in equation (6). A key aspect of estimating the wage process is to identify the potentially age-varying variances of transitory and permanent shocks, because they affect the strength of the precautionary motive and the path of wealth accumulation. To this end, we develop an estimation method that combines two related ideas in the literature. The first, as in [Carroll \(1992\)](#), is based on the fact that the variances of log wage (level) increase over the life cycle. This long-term increase in variances helps to identify the variance of permanent shocks. The second, as in [Blundell et al. \(2008\)](#), uses the variance–covariance restriction of the first difference in residual log wages. This tight restriction on short-term wage growth identifies the variances of permanent and transitory shocks.

We consider both short-term and long-term restrictions on residual wage growth. Specifically, we conduct a regression of log wages on year dummies, a fourth-order polynomial in age, household characteristics, an education indicator, and interactions between the education indicator and all other explanatory variables. The residuals from this regression, namely the residual log wages, are then used to estimate the variances of permanent and transitory shocks to wages. We use the generalized method of moments (GMM) estimators where moments include variance–covariance restrictions on all orders of the difference in residual log wages. Let $\Delta^k w_t = \log W_t - \log W_{t-k}$ denote the wage growth rate over k periods at time t , where $k = 1, 2, \dots$ up to the maximum difference in ages available in the sample, and $\Delta^p w_s = \log W_s - \log W_{s-p}$ denotes the wage growth rate over p periods at time s . Then, our moments include all possible covariances $\text{cov}(\Delta^k w_t, \Delta^p w_s)$. The inclusion of long-term ($k > 1$) wage growth ensures that we do not overestimate the variances. In other words, it makes sure that permanent shocks indeed have an impact on wages over the long

Figure 2: Growth Rate of Wages by Age and Variance of Log Wages



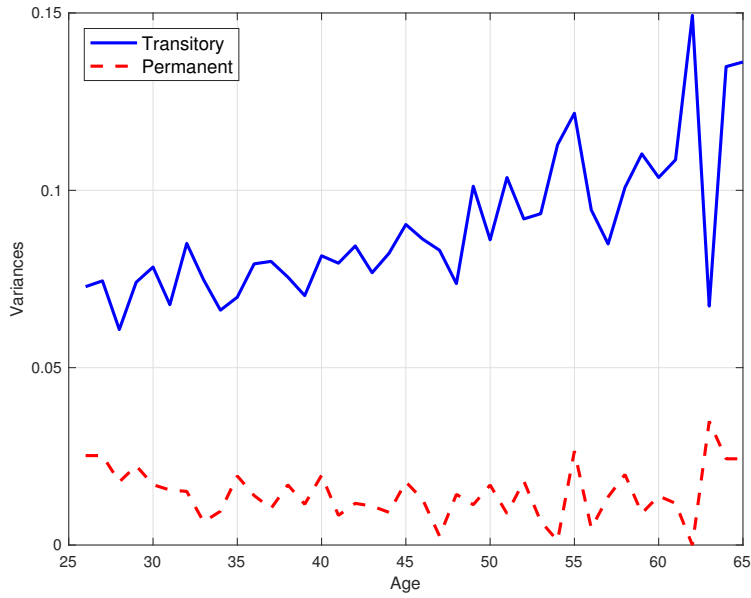
term, while transitory shocks do not. Appendix B describes the estimation process in more detail.

Figure 2 presents the average growth rates of wages γ_t in the left graph and the variance of log wages in the right graph. The measure of wage growth is the linear prediction from the regression mentioned above. Wage growth decreases over the life cycle. After age 50, wages start to decline. Wage inequality, measured by the variance of log wages, increases steadily up to age 60 and at a faster rate close to retirement.

Figure 3 illustrates the age-varying variances of permanent and transitory shocks over the life cycle. The variances of permanent shocks, with an average of 0.014 between ages 26 and 65, are roughly constant. The variances of transitory shocks display an increasing pattern: households face increasingly big transitory shocks as they age. The average of the variances of transitory shocks is 0.089.¹²

¹²Compared to the literature, such as Carroll (1992) and Hryshko (2012), our estimates of the variances of permanent shocks are at the lower end, and transitory shocks are at the upper end. In the Appendix, we provide estimates for Germany using SOEP data, which are of similar magnitudes.

Figure 3: Permanent and Transitory Variances by Age



3.3 Preferences

As mentioned in Section 2.2, consumption partial insurance is achieved under progressive taxation through a direct channel of shock reduction, and indirect channels of labor supply adjustment and wealth accumulation. Thus, it is natural to use our model to match the empirical wealth and labor supply distributions when estimating parameters related to household preferences.

Before we employ the method of simulated moments to estimate household preferences, there are a number of parameters that we have to account for, including life span, interest rate, and the initial distribution of wealth and wages, among others. Appendix D explains the details of our choices of these parameters.

We choose the 25th, 50th, and 75th percentiles of wealth and labor supply at each age from 26 to 60 as our targets. That is, we estimate the parameters such that the model-generated profiles of wealth and labor supply over the life cycle can be as close to the data as possible.

Formally, let $\theta = (\rho, \beta, \phi, \sigma, \underline{a}, \Delta\beta, \Delta\phi)$ be the parameters to be estimated.¹³¹⁴ Let $\pi_{A,t}^i(\theta), i = 1, 2, 3$ be the 25th, 50th, and 75th percentiles of wealth at age t , respectively, in the simulated data from the model, and $\pi_{H,t}^i(\theta), i = 1, 2, 3$ be the percentiles of labor supply. The estimator of θ solves the following problem:

$$\min \sum_{t=26}^{60} \sum_{i=1}^3 (L_A(\pi_{A,t}^i(\theta)) + L_H(\pi_{H,t}^i(\theta))) .$$

Here, $L_A(\pi_{A,t}^i(\theta))$ and $L_H(\pi_{H,t}^i(\theta))$ are the loss functions of wealth and labor supply profiles,

$$\begin{aligned} L_A(\pi_{A,t}^i(\theta)) &= E \{ [A_{j,t} - \pi_{A,t}^i(\theta)] [\pi^i - \mathbf{1}(A_{j,t} < \pi_{A,t}^i(\theta))] \}, \\ L_H(\pi_{H,t}^i(\theta)) &= E \{ [H_{j,t} - \pi_{H,t}^i(\theta)] [\pi^i - \mathbf{1}(H_{j,t} < \pi_{H,t}^i(\theta))] \}, \end{aligned}$$

where $\pi^1 = 0.25$, $\pi^2 = 0.5$, and $\pi^3 = 0.75$.

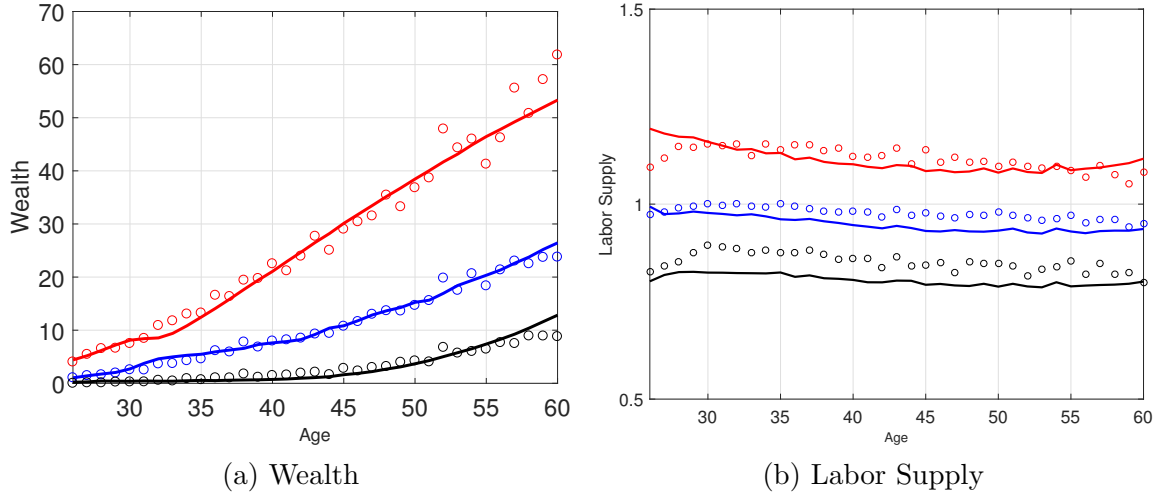
Figure 4 shows that our model is able to capture wealth and labor supply distributions over the life cycle reasonably well. Table 1 displays the parameter estimates of the model. Most of the estimates are in line with estimates from the previous literature.¹⁵ The heterogeneity in the discount factor (or equivalently, time preference rate) and in the disutility of labor is the key to the model's empirical success.

¹³To deal with heterogeneity, we assume for simplicity that half the households are patient ($\beta + \Delta\beta$) and half are impatient ($\beta - \Delta\beta$); half the households have high disutility $\phi + \Delta\phi$ and half have low disutility $\phi - \Delta\phi$. In other words, we assume that the discount factor and disutility in labor are independent, each following a binomial distribution with equal probability of outcomes. Then, $\Delta\beta$ and $\Delta\phi$ can be interpreted as the standard deviations of the distributions of the discount factor and the disutility of labor, respectively.

¹⁴We also include the borrowing constraint \underline{a} despite it not being a parameter related to preferences, because it cannot be estimated directly from the data.

¹⁵For example, our estimate of relative risk aversion falls within the conventional range of zero to five. Our estimate of the inverse of Frisch elasticity of substitution is similar to the micro estimates surveyed by Keane and Rogerson (2012) and Chetty et al. (2011).

Figure 4: Profiles Over the Life Cycle in the United States: Model vs. Data



Notes: For each age, red, blue, and black circles are the 75th, 50th, and 25th percentiles of data, respectively, while solid lines of the same color are the corresponding percentiles of the simulated data from the model.

Table 1: Parameter Estimates of the Model: United States

	Parameter	Estimates	Bootstrap std*
Preference			
Discount factor mean	$\bar{\beta}$	0.991	(0.001)
Discount factor std	$\text{std}(\beta)$	0.027	(0.003)
Coefficient of relative risk aversion	ρ	2.47	(0.309)
Inverse of Frisch elasticity	σ	2.41	(0.829)
Disutility of labor mean	$\bar{\phi}$	-0.617	(0.389)
Disutility of labor std	$\text{std}(\phi)$	0.695	(0.150)
Borrowing constraint	\underline{a}	0.0136	(0.003)
Bequest	φ	1.31	(0.539)

*Standard errors are based on 100 sample bootstraps.

Without these two types of heterogeneity, it is impossible for the model to generate the wide dispersion of wealth and labor supply in the data. The standard deviation of the distribution of the discount factor is quite small, suggesting that even very small differences in the degree of impatience can result in a big difference in wealth accumulation. The standard deviation of the distribution of disutility of labor, on the other hand, is rather large compared to the mean, suggesting that households have very distinct preferences for the trade-off between consumption and labor.

4 Results

In this section, we use our estimated structural model to answer the questions raised earlier. In particular, we address the relative strengths of different channels through which partial insurance of consumption is achieved under progressive taxation, and the optimal progressive taxation within the class of functions in equation (3).

4.1 Partial Insurance under Progressive Taxation

Direct effect

The degree of progressivity, τ , measures the direct effect of progressive taxation. Section 3.1 shows that the point estimate of τ is 0.11. Thus progressive taxation reduces shocks to post-government wages by 11%.

Indirect effect

The indirect effects of progressive taxation cannot be measured directly in the data. However, with the help of our estimated structural model, we are able to quantitatively evaluate their magnitude. Section 2.3 has shown that consumption

response to wage shocks is the result of both direct and indirect effects:

$$\xi_C = \frac{cov(\log C_{it}, \psi_{it})}{var(\psi_{it})} = (1 - \tau)(1 - \chi - \chi')\frac{1}{\rho},$$

where

$$\chi = \frac{(1 - \rho)(\sigma + \tau)}{\rho + \sigma + (1 - \rho)\tau}.$$

With our point estimates of the parameters, $\chi = -0.79$. Using the simulated data from our model, we calculate the consumption response to permanent shocks, $\xi_C^\eta = 0.57$ for ages 26–60. Thus, we obtain $\chi' = 0.19$ and $\frac{\chi'}{\rho} = 0.08$.

Table 2: Decomposition of Insurance Effects under Progressive Taxation: United States.

	Total	Direct	Indirect	
			Wealth	Labor
Consumption insurance	$(1 - \tau)(1 - \frac{\chi'}{\rho} - \frac{\chi - \chi_\infty}{\rho})$	$-\tau$	$-\frac{\chi'}{\rho}$	$-\frac{\chi - \chi_\infty}{\rho}$
Against Perm. Shock	0.57	-0.11	-0.08	-0.28
Contribution*	43%	11%	7%	25%
Against Tran. Shock	0.10	-0.11	-0.61	-0.28
Contribution*	90%	11%	54%	25%
			$\log C$	$\log H$
Response to 1% Wage Shock			$\frac{(1-\tau)(1-\chi-\chi')}{\rho}$	$\frac{(1-\tau)(\chi+\chi')}{\sigma+\tau}$
Permanent			0.57%	-0.21%
Transitory			0.10%	0.26%

* Contributions are calculated by τ , $(1 - \tau)\frac{\chi'}{\rho}$, $(1 - \tau)\frac{\chi - \chi_\infty}{\rho}$ for progressive taxation, wealth, and labor supply, respectively.

Recall that $\frac{\chi'}{\rho}$ measures the effect of wealth on the current consumption response to post-government wage shocks. This estimate implies that wealth reduces the consumption response by 8%. Similarly, we can calculate the consumption response to

transitory shocks over the working life and decompose the partial insurance channels.

Table 2 summarizes the decomposition of consumption partial insurance channels under progressive taxation. The first result is that the consumption response to permanent wage shocks is much less than one-to-one, on average, suggesting that there is much partial insurance. The second result is that wealth is quantitatively more important than labor in insuring consumption against transitory shocks, but less so in insuring against permanent shocks. Intuitively, the size of wealth typically eclipses the size of transitory shocks. However, it might not be enough to buffer against permanent shocks, because permanent shocks affect wages throughout the life cycle.

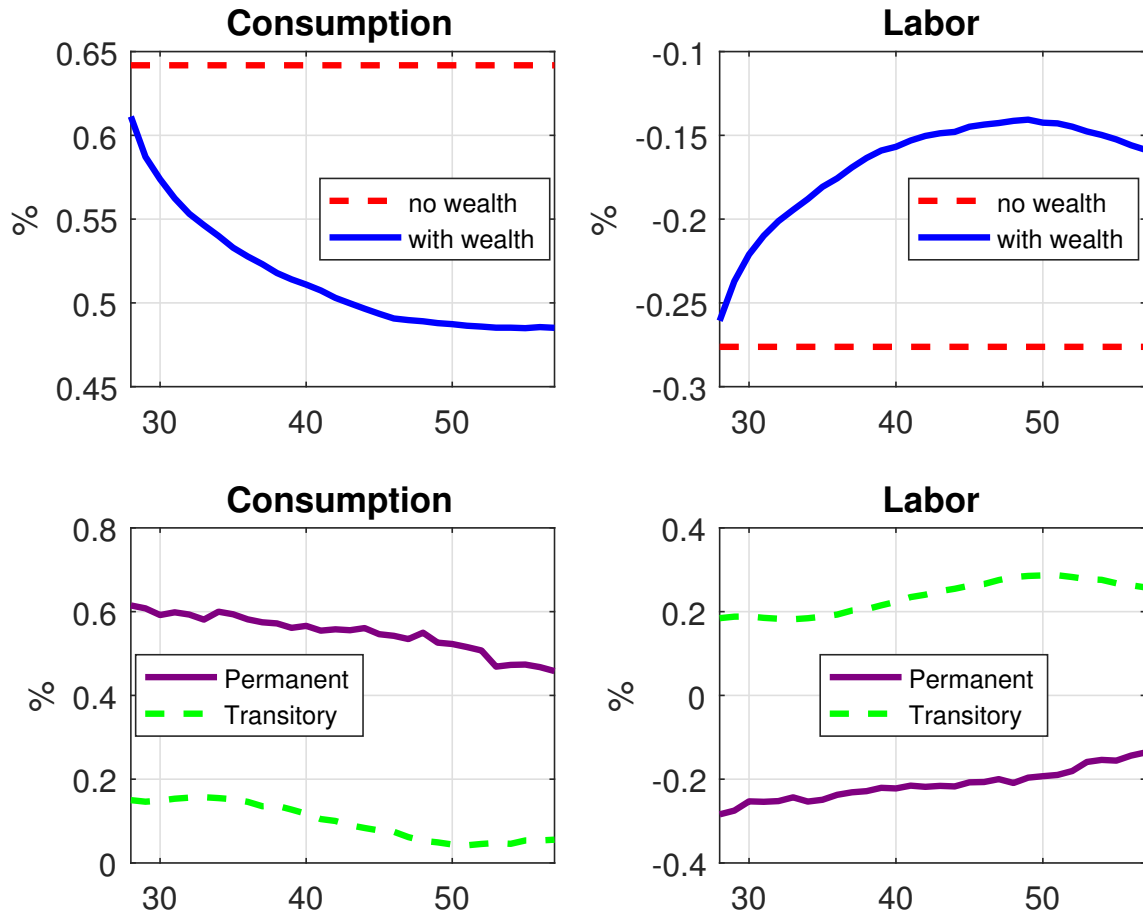
4.2 The Role of Wealth

In the absence of wealth, equations (11) and (12) characterize consumption and labor supply responses to wage shocks. In particular, the responses do not vary over the life cycle. However, in the presence of wealth accumulation, consumption responds less and labor supply responds more, on average. As households hold more wealth over the life cycle, the effect of wealth becomes more pronounced.

The first row of Figure 5 shows the age profiles of consumption and labor supply responses to a 1% wage shock over the life cycle, where shocks include both permanent or transitory shocks. As is evident in the graphs, wealth accumulation over the life cycle diminishes the consumption response, but increases the labor supply response to a wage shock. On average, labor supply responds negatively to a wage shock.

The second row of Figure 5 investigates how wealth affects the consumption and labor supply responses to different types of wage shocks. As wealth is accumulated, consumption responds less to both permanent and transitory shocks; in contrast, labor supply responds less to permanent shocks, but more to transitory shocks.

Figure 5: Response to a 1% Wage Shock



Notes: We use $\frac{cov(\log C_t, \log W_t)}{var(\log W_t)}$ and $\frac{cov(\log H_t, \log W_t)}{var(\log W_t)}$ for consumption and labor responses to wage shocks, respectively. For responses to permanent shocks, we replace $\log W_t$ with η_t ; for transitory shocks, we replace $\log W_t$ with ϵ_t .

Note that labor supply responds negatively to permanent shocks, but positively to transitory shocks. Intuitively, if there is a negative permanent wage shock, households would work more to maintain consumption; at the same time, if there is more wealth to finance future consumption, labor has to adjust by less. If the negative shock is only transitory, households would work less because it is a good time to enjoy some leisure; if there is more wealth, households can afford to work even less.

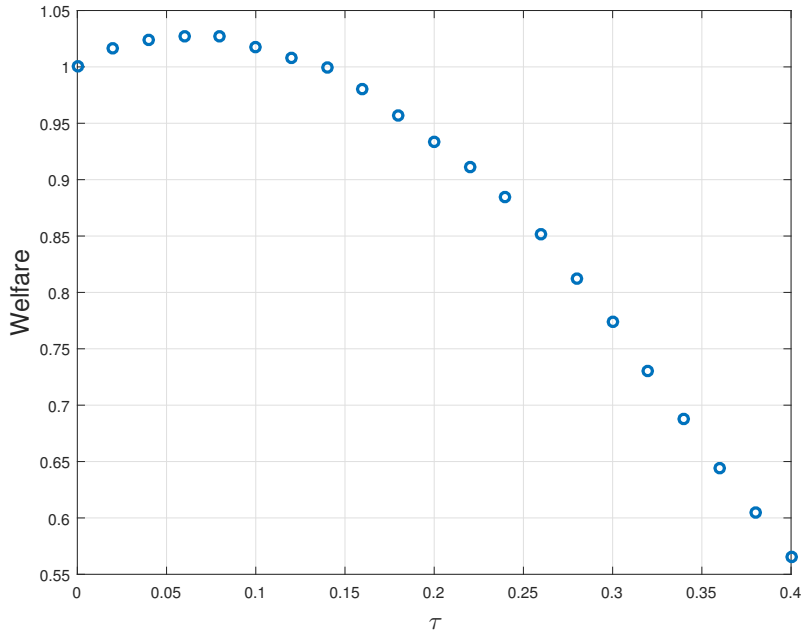
Figure 5 highlights the role of wealth in changing the consumption or labor supply response to wage shocks. As wealth accumulation happens over the life cycle, there is more consumption insurance. Importantly, it shifts the consumption response to transitory shocks to the labor supply response. Recall that equation (8) indicates that transitory shocks to wages have to be borne by either consumption or labor. Wealth affects such an allocation. With more wealth, transitory shocks look more “insurable.”

4.3 Welfare Implications

We calculate the welfare defined in equation (16) under different degrees of progressivity, holding labor income tax revenue constant. Specifically, we first use our structural model to calculate the net tax revenue for households aged 26–60 under the current US tax system. Then, as we vary the degree of progressivity τ , we numerically estimate λ , such that the net tax revenue is fixed. Finally, we calculate welfare under various combinations of τ and λ . For the purpose of comparison, we normalize the welfare under the proportional tax ($\tau = 0$) to 1.

Figure 6 contrasts welfare against the degree of progressivity, holding tax revenue constant. The relationship between progressivity and welfare is not monotonic: as the degree of progressivity increases, welfare *increases* at first, but then declines sharply, suggesting that limited progressivity is desirable, but too much progressivity severely distorts the labor supply and reduces lifetime resources for consumption. When we

Figure 6: Progressivity and Welfare

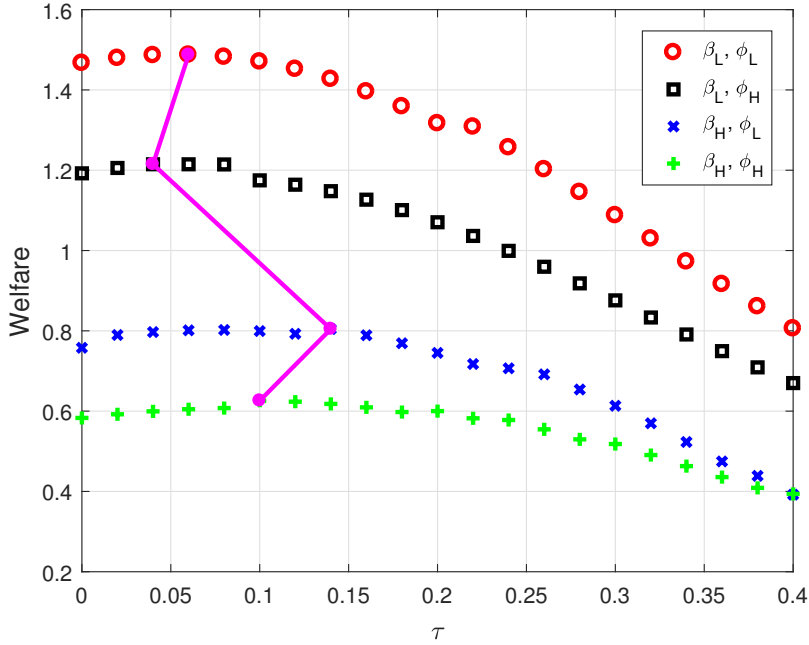


solve the problem of maximizing welfare by choosing τ , we find numerically that the optimal progressivity $\tau_{opt} = 0.06$. As a comparison, our estimate of the degree of progressivity of the current tax system in the United States is $\tau^{US} = 0.11$.

Heathcote et al. (forthcoming) find that the optimal value for the United States is $\tau_{opt}^{US} = 0.062$ in a general equilibrium environment, where consumption, labor supply, human capital, and public goods shape the optimal degree of progressivity. Interestingly, our estimate is similar to theirs, although we emphasize matching the wealth and labor supply distributions over the life cycle in a partial equilibrium framework.

Note that we have assigned equal weights in our welfare definition to households with different preferences and different initial wealth. This definition might have masked the impact of household heterogeneity on welfare. Therefore, we need to investigate the relationship between welfare and progressivity for households with

Figure 7: Progressivity and Welfare by Preference

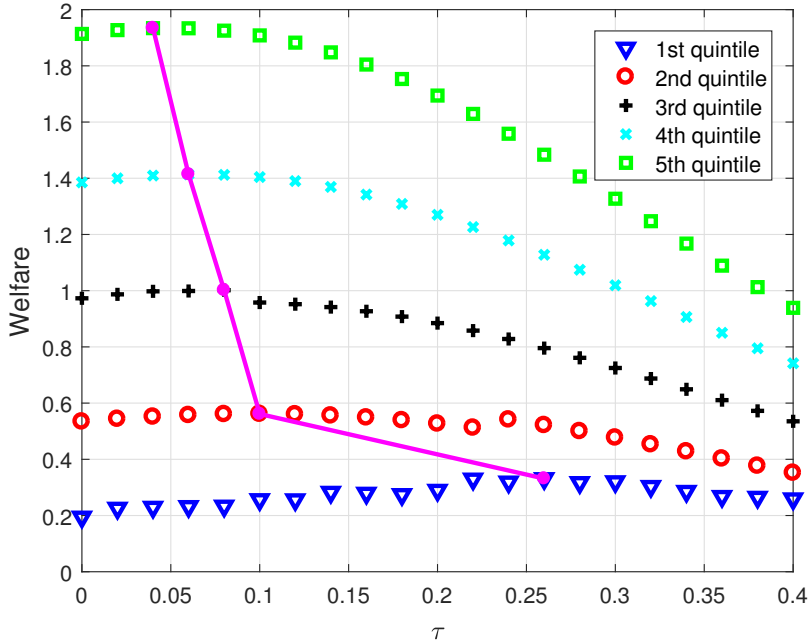


Notes: (β_L, ϕ_L) = (low patience, low disutility of labor), (β_L, ϕ_H) = (low patience, high disutility of labor), (β_H, ϕ_L) = (high patience, low disutility of labor), (β_H, ϕ_H) = (high patience, high disutility of labor).

different preferences and different initial wealth.

In the estimated version of our model, we have assumed that there are two types of discount factors and two types of disutility of labor. As such, there are four types of households. Figure 7 plots the relationship between welfare and progressivity for these four types of households, where β_L and β_H denote low and high patience, respectively, and ϕ_L and ϕ_H denote low and high disutility of labor, respectively. The solid line connects the optimal degree of progressivity for each type. It is quite robust that a little progressivity is desirable, but too much might be too distortionary. Moreover, and perhaps surprisingly, more patient households prefer higher progres-

Figure 8: Progressivity and Welfare by Initial Wealth Quintiles



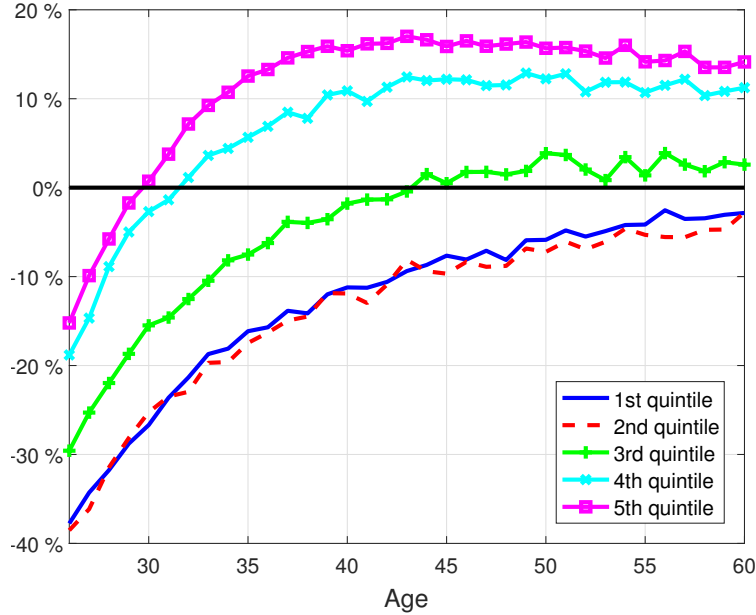
sivity, and households with low disutility of labor prefer higher progressivity.¹⁶

Figure 8 plots the relationship between welfare and progressivity for households at different quintiles of initial wealth distribution. The solid line connects the optimal degree of progressivity for households at different quintiles. The pattern of the relationship is generally preserved, regardless of initial wealth. However, the wealthier households are, the less progressivity they prefer. Based on our model, the current degree of progressivity in the US tax system, where $\tau = 0.11$, is optimal for households in the fourth quintile of wealth, too low for less wealthy households, and somewhat high for wealthier households.

Figure 7 and 8 together emphasize that the optimal degree of progressivity depends on the distribution of heterogeneity in household preferences, as well as on the

¹⁶This is because under progressive taxation, total labor supply is higher and the average tax rate is lower.

Figure 9: From Proportional to Progressive:
Average Tax Revenue Change by Initial Wealth Quintiles



distribution of wealth. Thus, there is no single optimal degree of progressivity for all.

Another way to see how progressive taxation disproportionately affects households at different quintiles of initial wealth distribution is to consider the tax burden shared by each group. We use net tax revenue per age under proportional taxation as a benchmark, because it does not vary by initial wealth, and then determine how much it will change if taxation changes from proportional to progressive, holding total revenue constant. Figure 9 displays the results, showing that households with high initial wealth incur more tax and essentially subsidize households with low initial wealth, because wealth and income are positively correlated. Regardless of initial wealth, households of old age incur more tax and subsidize households of young age, because wage and income increase over the life cycle.

5 Conclusion

This study examines consumption insurance and optimal progressive taxation through the lens of a heterogeneous-agent life-cycle model that matches the empirical distributions of wealth and labor supply. Partial insurance of consumption against wage shocks in the model is achieved through progressive taxation, labor supply adjustment, and precautionary wealth accumulation. Our estimated model suggests that over the life cycle of US households, on average, 43% of permanent wage shocks are insured against, of which 11% is by progressive taxation, 7% by wealth, and 25% by labor supply adjustment. 90% of transitory wage shocks are insured against, of which 11% is by progressive taxation, 54% by wealth, and 25% by labor supply adjustment. Wealth is more important in insurance against transitory shocks than it is for permanent shocks. The optimal degree of progressivity for the United States is less progressive than the current US tax system. However, it differs by household heterogeneity in preferences and initial wealth. Households that are more patient, more willing to work, and less wealthy prefer more progressivity. A comparison with the results from the same model estimated with German data shows that the wealth channel is more important in achieving consumption insurance in Germany. The optimal degrees of progressivity in both countries are similar.

References

- BÉNABOU, R. (2002): “Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?,” *Econometrica*, 70(2), 481–517. Cited on pages 4 and 9.
- BLUNDELL, R., L. PISTAFERRI, AND I. PRESTON (2008): “Consumption Inequality and Partial Insurance,” *American Economic Review*, 98(5), 1887–1921. Cited on pages 2, 5, 6, 20, and 42.
- CAGETTI, M. (2003): “Wealth Accumulation over the Life Cycle and Precautionary Savings,” *Journal of Business & Economic Statistics*, 21(3), 339–353. Cited on page 6.
- CARROLL, C. D. (1992): “The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence,” *Brookings Papers on Economic Activity*, 1992(2), 61–156. Cited on pages 3, 6, 20, and 21.
- (2006): “The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems,” *Economics Letters*, 91(3), 312–320. Cited on page 46.
- CARROLL, C. D., AND A. A. SAMWICK (1998): “How Important Is Precautionary Saving?,” *Review of Economics and Statistics*, 80(3), 410–419. Cited on page 5.
- CHETTY, R., A. GUREN, D. MANOLI, AND A. WEBER (2011): “Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins,” *American Economic Review*, 101(3), 471–475. Cited on page 23.

- CONESA, J. C., S. KITAO, AND D. KRUEGER (2009): “Taxing Capital? Not a Bad Idea after All!” *American Economic Review*, 99(1), 25–48. Cited on page 5.
- CONESA, J. C., AND D. KRUEGER (2006): “On the Optimal Progressivity of the Income Tax Code,” *Journal of Monetary Economics*, 53(7), 1425–1450. Cited on page 5.
- ENGEN, E. M., AND J. GRUBER (2001): “Unemployment Insurance and Precautionary Saving,” *Journal of Monetary Economics*, 47(3), 545–579. Cited on page 5.
- FEHR, H., M. KALLWEIT, AND F. KINDERMANN (2013): “Should Pensions be Progressive?,” *European Economic Review*, 63, 94–116. Cited on page 5.
- FELDSTEIN, M. S. (1969): “The Effects of Taxation on Risk Taking,” *Journal of Political Economy*, 77(5), 755–764. Cited on page 9.
- FLODEN, M., AND J. LINDE (2001): “Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance?,” *Review of Economic Dynamics*, 4(2), 406–437. Cited on page 5.
- GOURINCHAS, P.-O., AND J. A. PARKER (2002): “Consumption Over the Life Cycle,” *Econometrica*, 70(1), 47–89. Cited on page 6.
- GRUBER, J., AND A. YELOWITZ (1999): “Public Health Insurance and Private Savings,” *Journal of Political Economy*, 107(6), 1249–1274. Cited on page 5.
- HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2014): “Consumption and Labor Supply with Partial Insurance: An Analytical Framework,” *American Economic Review*, 104(7), 2075–2126. Cited on pages 2, 5, 9, and 13.
- (forthcoming): “Optimal Tax Progressivity: An Analytical Framework,” *Quarterly Journal of Economics*. Cited on pages 6, 19, and 30.

- HRYSHKO, D. (2012): “Labor Income Profiles are not Heterogeneous: Evidence from Income Growth Rates,” *Quantitative Economics*, 3(2), 177–209. Cited on pages 6 and 21.
- HUBBARD, R. G., J. SKINNER, AND S. P. ZELDES (1995): “Precautionary Saving and Social Insurance,” *Journal of Political Economy*, 103(2), 360–399. Cited on page 5.
- JAPPELLI, T., AND L. PISTAFERRI (2010): “The Consumption Response to Income Changes,” *Annual Review of Economics*, 2(1), 479–506. Cited on page 2.
- KAPLAN, G., AND G. L. VIOLANTE (2010): “How Much Consumption Insurance Beyond Self-Insurance?,” *American Economic Journal: Macroeconomics*, 53–87. Cited on pages 2 and 5.
- KEANE, M., AND R. ROGERSON (2012): “Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom,” *Journal of Economic Literature*, 50(2), 464–476. Cited on page 23.
- KRUEGER, D., AND F. PERRI (2011): “How Do Households Respond to Income Shocks?,” *University of Pennsylvania, Mimeo*. Cited on page 2.
- LILLARD, D., P. GILES, M. GRABKA, AND N. NARGIS (2002): “Codebook for the Cross-National Equivalent File 1980 - 2000, BHPS - GSOEP - PSID - SLID,” Technical report, College of Human Ecology, Cornell University, Ithaca. Cited on page 40.
- LOW, H., AND D. MALDOOM (2004): “Optimal Taxation, Prudence and Risk-sharing,” *Journal of Public Economics*, 88(3), 443–464. Cited on page 5.

- MEGHIR, C., AND L. PISTAFERRI (2004): “Income Variance Dynamics and Heterogeneity,” *Econometrica*, 72(1), 1–32. Cited on pages 3 and 6.
- MIRPLEES, J. A. (1974): “Notes on Welfare Economics, Information, and Uncertainty,” in *Essays on Economic Behavior under Uncertainty*, ed. by M. Balch, D. McFadden, and S. Wu, 243–261. North-Holland, Amsterdam. Cited on page 5.
- THE PANEL STUDY OF INCOME DYNAMICS (2016): “Online Documentation,” Website, University of Michigan. Cited on page 39.
- VARIAN, H. R. (1980): “Redistributive Taxation as Social Insurance,” *Journal of Public Economics*, 14(1), 49–68. Cited on page 5.
- WAGNER, G. G., J. R. FRICK, AND J. SCHUPP (2007): “The German Socio-Economic Panel Study (SOEP): Scope, Evolution and Enhancements,” *Journal of Applied Social Science Studies*, 127(1), 139–169. Cited on page 39.

Appendix

A PSID Data and SOEP Data

This analysis is based on data from the Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (SOEP).

The PSID data set is based on a sample of roughly 5,000 households that were interviewed in 1968. From 1997 on data are only available every second year (1999, 2001, etc.).¹⁷ Of these, about 3,000 were sampled to be representative of the nation as a whole and about 2,000 were low-income families that had been interviewed previously as part of the Census Bureau's Survey of Economic Opportunity. The members of these households have been tracked since then. Wealth data comparable to those from the SOEP are available in 1984, 1989, 1994, 1999, 2001, 2003, 2005, 2007, 2009, 2011, and 2013. We use all waves available to estimate the permanent income and income uncertainty measures.

The SOEP is a representative annual household panel survey with about 20,000 observations per year in Germany that started with roughly 6,000 households in 1984. [Wagner et al. \(2007\)](#) provide a detailed description of the data. As for the PSID, we use all waves available to estimate the permanent income and income uncertainty measures. We use wealth data that are available in the 2002, 2007, and 2012 waves. The SOEP questionnaire for these waves included a special module that collected information about private wealth. Similarly as in the PSID, the surveys asked about the market value of personally owned real estate (owner-occupied housing, other property, mortgage debt), financial assets, tangible assets, private life and pension insurance, consumer credit, and private business equity (net market value; i.e., own share in case of a business partnership). The wealth balance sheets referred to the

¹⁷For a more detailed description, see [The Panel Study of Income Dynamics \(2016\)](#).

personal level, so in the case of jointly owned assets, the survey explicitly asked about each person's individually owned shares.

We use the Cross-National Equivalent File (CNEF) that contains equivalently defined variables for the Panel Study of Income Dynamics and the German Socio-Economic Panel.¹⁸ In each year, we aggregated data on hours worked, labor income, and wealth to the household level, and deflated to 2005 purchasing power parity prices in Euro using the consumer price index provided by the Federal Statistical Offices.

Our measure of labor income, available from 1970 to 2013 in the PSID and from 1984 to 2013 in the SOEP, combines annual household pre-government labor income that a household received in the calendar year prior to the survey year. More specifically, labor income is the sum of income from primary job, secondary job, self-employment, service pay, 13th month pay, 14th month pay, Christmas bonus pay, holiday bonus pay, miscellaneous bonus pay, and profit-sharing income. Wages are calculated as household labor income divided by household hours of work.

We define as household heads the individual who has the highest earnings and highest wage in each year. If two individuals have identical values on these variables, we assign the person defined by the SOEP or PSID as the head as household head.

We drop individuals with non-positive hours of work, negative wages, or wages larger than 150 Euro. Moreover, we drop individuals that have the same age in two different years. Then, our SOEP sample includes 3,696 in 1984 and 5,565 in 2013. The PSID sample includes 2,782 households in 1971, and 5,456 in 2013.

¹⁸For a more detailed description, see [Lillard et al. \(2002\)](#).

B Estimation of Idiosyncratic Risks

Let $\Delta^k w_t = \log(W_t) - \log(W_{t-k})$ denote the wage growth at period t over the past k -year horizon¹⁹, and $(\Delta^k w_t)_{t,k}$ denote the vector of wage growth at different periods over all possible horizons.²⁰ If the true wage process follows that in (5) and (6), the theoretical variance–covariance matrix of $(\Delta^k w_t)_{t,k}$ follows

$$\text{cov}\{\Delta^k w_t, \Delta^p w_s\} = \begin{cases} \sigma_{\eta, s-p+1}^2 + \cdots + \sigma_{\eta, t}^2 & \text{if } s > t > (s-p) > (t-k) \\ \sigma_{\eta, s-p+1}^2 + \cdots + \sigma_{\eta, t}^2 + \sigma_{\epsilon, s-p}^2 & \text{if } s > t > (s-p) = (t-k) \\ \sigma_{\eta, t-k+1}^2 + \cdots + \sigma_{\eta, t}^2 & \text{if } s > t > (t-k) > (s-p) \\ -\sigma_{\epsilon, t}^2 & \text{if } s > t = s-p > (t-k) \\ 0 & \text{if } s > (s-p) > t > (t-k) \\ \sigma_{\eta, t-k+1}^2 + \cdots + \sigma_{\eta, t}^2 + \sigma_{\epsilon, t}^2 & \text{if } s = t > (t-k) > (s-p) \\ \sigma_{\eta, t-k+1}^2 + \cdots + \sigma_{\eta, t}^2 + \sigma_{\epsilon, t}^2 + \sigma_{\epsilon, t-k}^2 & \text{if } s = t > (t-k) = (s-p) \end{cases} \quad (17)$$

where we show the covariances when $t \leq s$ only because the variance–covariance matrix is symmetric.

The variance of transitory shocks at period t can be identified by

$$\sigma_{\epsilon, t}^2 = -\text{cov}\{\Delta^k w_t, \Delta^p w_{t+p}\} \quad k \geq 1, p \geq 1,$$

and the variance of permanent shocks at period t can be identified by

$$\sigma_{\eta, t}^2 = \text{cov}\{\Delta^k w_t, \Delta^p w_{t+i}\} - \text{cov}\{\Delta^k w_{t-1}, \Delta^p w_{t-1+i}\} \quad k \geq 1, p \geq 1, i \geq 1.$$

In other words, to identify the variance of transitory shocks at period t , we need at least wages at periods $t-1, t, t+1$; to identify the variance of permanent shocks at period t , we need at least wages at periods $t-2, t-1, t, t+1$. However, identifica-

¹⁹The life cycle starts at period 1, and $1 \leq k \leq (t-1)$.

²⁰If there are T periods of wages, the length of the vector would be $\frac{T(T-1)}{2}$.

tion of a particular variance is achieved not only through one covariance restriction, but through all possible covariance restrictions implied by (17).²¹ The inclusion of covariance restrictions on long-term ($k > 1$) wage growth in $(\Delta^k w_t)_{t,k}$ is important because it ensures that the effect of permanent shocks stays, and that of transitory shocks diminishes in the covariance term.

To estimate the variances of transitory and permanent shocks, we need to construct the empirical counterpart of the variance-covariance matrix of $(\Delta^k w_t)_{t,k}$.

Suppose the panel data contain T^y years of data. Let the life cycle start from period 1 and end at period T . For simplicity, we assume that $T > T^y$, as is the case in SOEP and PSID. For a typical household, we can observe its wages up to T^y years. Let $\Delta^k w_{i,t} = \log(W_{i,t}) - \log(W_{i,t-k})$ denote the k -year wage growth for household i at period t , where $1 \leq k \leq \min\{t, T^y\} - 1$. If there are no missing data for the household, we would have $X = \frac{T^y(T^y-1)}{2} + (T - T^y)(T^y - 1)$ wage growth rates at different ages over different horizons,²² from which we would construct $\frac{X(X+1)}{2}$ distinct empirical moments for all possible $\text{cov}\{\Delta^k w_t, \Delta^p w_s\}$.

Let the $(X \times 1)$ column vector $\Delta \mathbf{w}_i = (\Delta^k w_t)_{t,k}$ denote wage growth over all available horizons at period t for household i . To deal with missing data, we define \mathbf{d}_i conformably with $\Delta \mathbf{w}_i$, where $d_{i,t}^k = 1$ if $\Delta^k w_{i,t}$ is not missing, and $d_{i,t}^k = 0$ otherwise. Following the notation in [Blundell et al. \(2008\)](#), we construct empirical

²¹For instance, the variance of transitory shocks at period t can also be identified by

$$\sigma_{\epsilon,t}^2 = \text{cov}\{\Delta^k w_t, \Delta^p w_t\} - \text{cov}\{\Delta^k w_t, \Delta^p w_{t+i}\} \quad p > k, i \geq 1,$$

and the variance of permanent shocks at period t can also be identified by

$$\sigma_{\eta,t}^2 = \text{cov}\{\Delta^1 w_t, \Delta^p w_{t+i}\} \quad p \neq s - (t - 1),$$

etc.

²²That is, at period t , where $2 \leq t \leq T^y$, we would have wage growth over a 1-year to $(t-1)$ -year horizon, with a total of $\frac{T^y(T^y-1)}{2}$ wage growth rates; if $t > T^y$, we would have wage growth over a 1-year to $(T^y - 1)$ -year horizon, with a total of $(T - T^y)(T^y - 1)$ wage growth rates.

moments that contain estimates of $\text{cov}\{\Delta^k w_t, \Delta^p w_s\}$

$$\mathbf{m} = \text{vech} \left\{ \left(\sum_{i=1}^N \Delta \mathbf{w}_i \Delta \mathbf{w}_i' \right) \oslash \left(\sum_{i=1}^N \mathbf{d}_i \mathbf{d}_i' \right) \right\},$$

where \oslash denotes elementwise division. Let $\mathbf{m}^*(\sigma_{\eta,t}^2, \sigma_{\epsilon,t}^2)$ denote the corresponding theoretical moments implied by (17). We estimate the variances of permanent and transitory shocks by solving the problem

$$\min_{\{\sigma_{\eta,t}^2\}, \{\sigma_{\epsilon,t}^2\}} (\mathbf{m} - \mathbf{m}^*)' \Lambda (\mathbf{m} - \mathbf{m}^*),$$

where Λ is a weighting matrix. In practice, we choose an identity weighting matrix $\Lambda = \mathbf{I}$ (i.e., equally weighted minimum distance).

C Numerical Solution to the Household's Problem

Working life

Let the value function before retirement be $V_{it}(A_{it}, Z_{it}, \epsilon_{it})$. Then we have the recursive relation

$$V_{it}(A_{it}, Z_{it}, \epsilon_{it}) = u(C_{it}, H_{it}) + \beta E_t V_{it+1}(A_{it+1}, Z_{i,t+1}, \epsilon_{i,t+1}).$$

The first-order conditions with respect to C_t and H_t are

$$\begin{aligned} u^C(C_{it}, H_{it}) &= (1 + \tau_c)(\beta E_t V_{it+1}^A + \mu_t), \\ -u^H(C_{it}, H_{it}) &= (W_t - TX'(W_t H_t) W_t)(\beta E_t V_{it+1}^A + \mu_t), \end{aligned}$$

where μ_t is the Lagrange multiplier on the constraint of $A_{it+1} \geq -\underline{a}Z_{it}$. The envelope

condition is

$$V_{it}^A = \beta(1 + r(1 - \tau_A))E_t V_{it+1}^A. \quad (18)$$

As such, we have

$$\begin{aligned} u^C(C_{it}, H_{it}) &= \beta(1 + r(1 - \tau_A))E_t u^C(C_{it+1}, H_{it+1}) + (1 + \tau_c)\mu_t, \\ u^C(C_{it}, H_{it}) &= -\frac{(1 + \tau_c)u^H(C_{it}, H_{it})}{W_{it} - TX'(W_{it}H_{it})W_{it}}. \end{aligned}$$

Given the specific forms of the utility function and the tax function, we have

$$\begin{aligned} u^C &= C^{-\rho}, \\ u^H &= -\phi H^\sigma, \\ TX'(W_{it}H_{it}) &= 1 - \lambda(1 - \tau)(W_{it}H_{it})^{-\tau}. \end{aligned}$$

The key equations for solving a household's problem are

$$\begin{aligned} C_{it}^{-\rho} &= \beta(1 + r(1 - \tau_A))E_t [C_{it+1}^{-\rho}] + (1 + \tau_c)\mu_t, \\ C_{it}^{-\rho} &= \frac{(1 + \tau_c)\phi}{\lambda(1 - \tau)} H_{it}^{\sigma+\tau} W_{it}^{\tau-1}. \end{aligned}$$

Retirement

After retirement, households have no labor supply choices. Let $V_{it}^R(M_{it})$ be the value function of households after retirement. Then, we have the following recursive formulation:

$$V_{it}^R(M_{it}) = u^R(C_{it}) + \beta (p_t^S V_{i,t+1}^R(M_{i,t+1}) + (1 - p_t^S)u^B(M_{i,t+1})).$$

The first-order condition follows

$$(u^R)'(C_{it}) = \beta(1 + r(1 - \tau_A)) (p_t^S (u^R)'(C_{i,t+1}) + (1 - p_t^S)(u^B)'(M_{i,t+1})),$$

Given the specific function forms of $u^R(\cdot)$ and $u^B(\cdot)$,

$$C_{it}^{-\rho} = \beta(1 + r(1 - \tau_A)) (p_t^S C_{i,t+1}^{-\rho} + (1 - p_t^S) \phi M_{i,t+1}^{-\rho}).$$

In the last possible period of the life cycle, because households die with certainty, the budget constraint is as follows:

$$M_{i,T+1} = (1 + r(1 - \tau_A))(M_{i,T} - C_{i,T}).$$

Combined with the first-order condition

$$C_{iT}^{-\rho} = \beta(1 + r(1 - \tau_A)) \phi M_{i,T+1}^{-\rho},$$

we obtain the last period's consumption function

$$C_{iT} = \frac{1 + r(1 - \tau_A)}{1 + r(1 - \tau_A) + (\beta(1 + r(1 - \tau_A))\phi)^{\frac{1}{\rho}}} M_{iT}.$$

It is apparent that ϕ affects the marginal propensity to consume. The higher ϕ is, the more bequests households will leave. One subtlety here is that in the model we have assumed that it is possible for households to leave a bequest every period after retirement. This is important for the model's capability to generate the empirical wealth accumulation over the life cycle, especially the level of wealth close to retirement. Otherwise, had we assumed households could only leave a bequest in the very last period (T), because of discounting, the bequest motive would have a limited

impact on the level of wealth close to retirement, and the model would not be able to capture the empirical wealth profile over the life cycle.

Endogenous Gridpoints Method

We use the backward policy function iteration to solve the households problem: Given the policy rules $C_{t+1}^*(\cdot)$ and $H_{t+1}^*(\cdot)$ in period $t + 1$, we calculate the policy rules in period t . To speed up the numerical solution, we use a variant of the endogenous gridpoints method first proposed by [Carroll \(2006\)](#). For example, for policy rules before retirement, given A_{t+1}, Z_t, ϵ_t , and the optimal decision $C_{t+1}^*(A_{t+1}, Z_{t+1}, \epsilon_{t+1})$, we will be able to calculate $E_t[C_{t+1}^{-\rho}(A_{t+1}, Z_{t+1}, \epsilon_{t+1})]$ and hence C_t and H_t through the first-order conditions. The endogenous grid is then calculated by $A_t = \frac{1}{(1+r(1-\tau_A))}(A_{t+1} + (1 + \tau_c)C_t - (W_t H_t - TX(W_t H_t)))$. With $C_t(A_t, Z_t, \epsilon_t)$ and $H_t(A_t, Z_t, \epsilon_t)$ on the endogenous grid, we can interpolate them to obtain their values on the pre-specified exogenous grid.

D Choices of Nuisance Parameters

Life span

We assume that the life cycle starts at age 26 (period 0). Households work for $T^w = 40$ years and retire at age 65. They live for $T = 65$ years after beginning their life cycle and die with certainty at age 90.

The choice of when a life cycle begins is primarily based on data availability and the fact that most households finish school before 26. The choice of how long households might live after retirement will influence the estimation of household preference, in particular the time preference rate, because part of wealth accumulation is saving for retirement. We choose T to be big enough – well above average life expectancy – so that its impact on wealth accumulation before retirement is small.

Initial distribution of wealth and wage

A simulation of the model requires the distribution of wealth and income at the beginning of life cycle. To obtain the initial distribution, we sort households between ages 20–24 by their net worth and group them into five equal-sized groups. For each group, we calculate its median net worth and median wage. We randomly assign households at age 26 to different net worth quintiles and use its median net worth and median wage as the starting values of wealth and wage. With these initial distributions, we simulate households' consumption, labor supply, and wealth accumulation dynamics until they reach the age of 65.

Normalization of labor

We normalize H_{it} by the typical number of working hours in a year ($8 \times 5 \times 52 = 2,080$ hours for a single worker). As such, W_{it} should be interpreted as annual wage. We set the maximum working hours to be the total hours available so that $H_{it} \leq \bar{H} = 3$.

Interest rates and other taxes

We use the historical average of interest rates $r = 3\%$, capital tax $\tau_A = 15\%$, and consumption tax $\tau_C = 6\%$.

Mortality rates

For the probability of survival after retirement, we use the life tables in the Human Mortality Database (HMD). The life tables include survival probabilities and life expectancies that vary by age and are available for the years from 1991 to 2012. We first calculate the age effects of the conditional probabilities of survival from 1991 to 2012 and then impute the probability of survival from age 65 on.

E Results for Germany

In this section, we re-estimate our model for Germany, where the tax schedule is notably more progressive, using data from SOEP. We examine whether the strength of consumption partial insurance channels and whether the optimal degree of welfare differ drastically between the two countries. One of the purposes of this exercise is to assess how robust our model and estimation techniques are in applying to different datasets.

Based on our parameter estimates, we find that consumption partial insurance through the wealth channel is more important in Germany, mainly because of less elastic labor supply. Compared to a revenue-neutral proportional tax, progressive taxation in Germany leads to less wealth accumulation, less consumption, and less labor supply, in contrast to our findings for the United States. The optimal degree of progressive taxation is similar in the two countries.

E.1 Progressive Taxation

Table 3 contrasts our estimates of the progressivity of Germany with that of the US. The tax part is where post-government income is lower than pre-government income. In other words, households incur net taxation. The transfer part is where post-government income is higher than pre-government income and households are net recipients of government transfers. Regardless of which part of tax schedule, taxation in Germany is much more progressive than in the US.

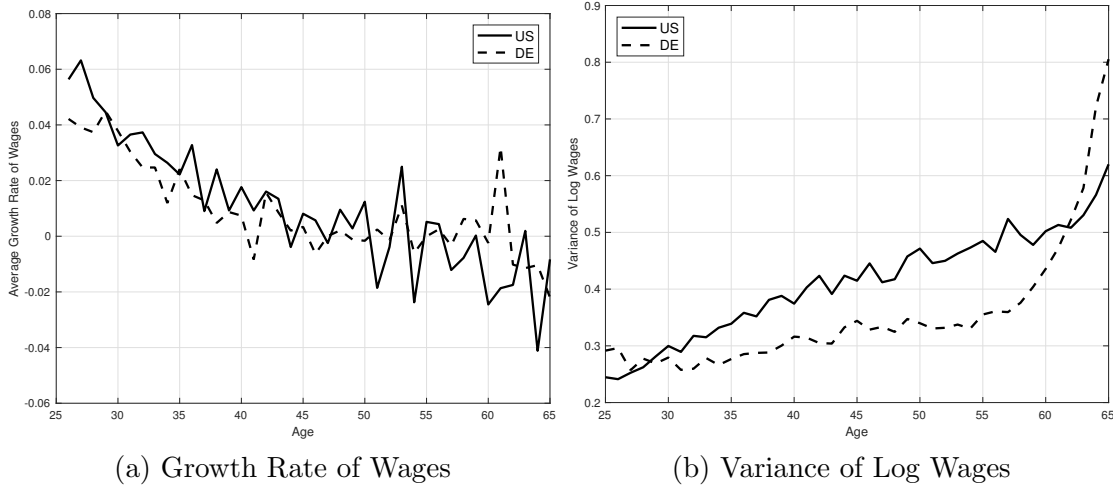
E.2 Wage Process

Figure 10 compares average growth rates of wages γ_t in the left hand graph and the variances of $\log W_{it}$ in the right hand graph. Compared to Germany, US Wage growth is stronger for young households but becomes weaker as they wage. Household

Table 3: A Comparison of Progressivity: US vs. Germany

Parameter	Tax		Transfer	
	τ	λ	τ	λ
US	0.11	2.95	0.67	655
Germany	0.22	7.87	0.82	3296

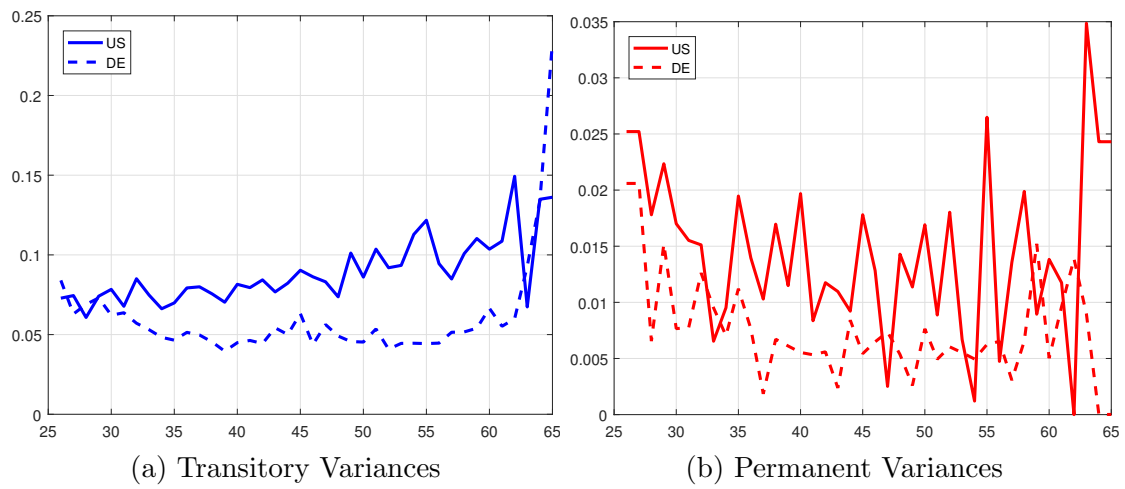
Figure 10: Growth Rate of Wages by Age and Variance of Log Wages



wages differ at a faster rate in the US over most of the life cycle. However, close to retirement, German wages show a prominent increase in dispersion.

Figure 11 compares idiosyncratic risks in US and German wage processes. Variances of both permanent and transitory shocks to wages are lower in Germany, though they are of similar patterns to the US.

Figure 11: Permanent and Transitory Variances by Age



E.3 Preference

Table 4: Parameter Estimates: A Comparison between the US and Germany

	Parameter	US Est.	US Std.	DE Est.	DE Std.
Preference					
Discount factor mean	$\bar{\beta}$	0.991	(0.001)	0.954	(0.003)
Discount factor std	$\text{std}(\beta)$	0.027	(0.003)	0.015	(0.002)
Coefficient of relative risk aversion	ρ	2.47	(0.309)	1.29	(0.147)
Inverse of Frisch elasticity	σ	2.41	(0.829)	3.28	(1.07)
Disutility of labor mean	$\bar{\phi}$	-0.617	(0.389)	-0.386	(0.168)
Disutility of labor std	$\text{std}(\phi)$	0.695	(0.150)	0.742	(0.189)
Borrowing constraint	\underline{a}	0.0136	(0.003)	0.0287	(0.028)
Bequest	φ	1.31	(0.539)	0.748	(0.538)

Standard errors are based on 100 sample bootstraps.

Table 4 displays the parameter estimates of the model. Most of the estimates are in line with estimates from the previous literature.

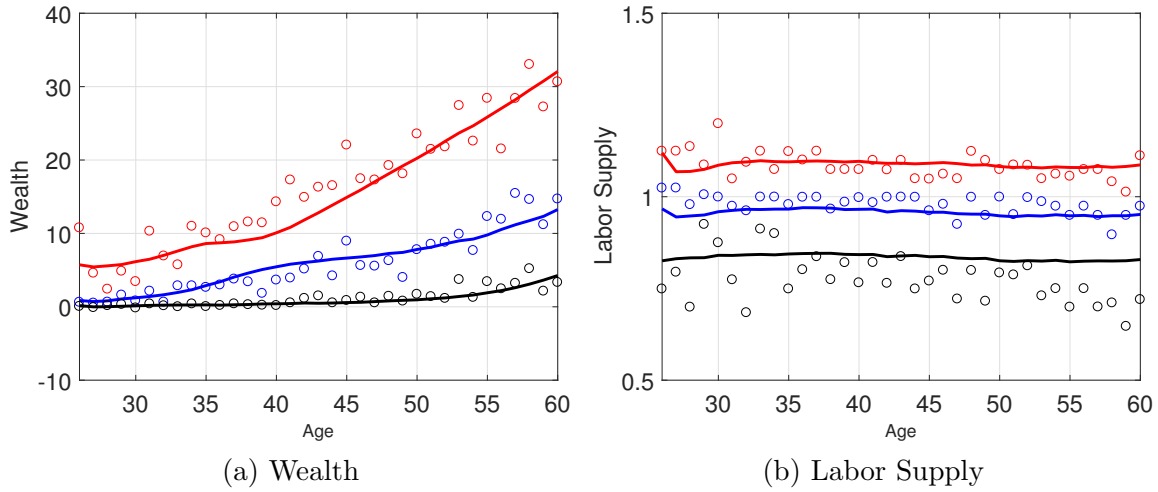
Figure 12 shows that our model is able to capture wealth and labor supply distributions over the life cycle reasonably well for Germany. Interestingly, wealth holdings are much lower in Germany than in the US.

Labor supply is slightly decreasing in both countries over the life cycle. However, at a given age German labor supply has a wider distribution than in the US, an indication of the importance of using heterogeneity in disutility of labor to capture the distribution of labor supply.

E.4 Insurance Effects under Progressive Taxation

Table 5 shows that consumption partial insurance in the US and in Germany is similar whether it is against permanent or transitory wage shocks. However, the strength of insurance channels is quite different. In Germany, because taxation is

Figure 12: Profiles Over the Life Cycle: Model vs. Data



Notes: For each age, red, blue, and black circles are the 75th, 50th, and 25th percentiles of data, respectively, while solid lines of the same color are the corresponding percentiles of the simulated data from the model.

more progressive, the direct insurance effect of progressive taxation is stronger. In addition, wealth plays a more important role than labor in achieving consumption insurance. This is because our estimate of Frisch elasticity for Germany is lower (σ is higher).

E.5 Welfare Implications

Figure 13 compares the relationship between welfare and progressivity in the United States and in Germany. Interestingly, the optimal degree of progressivity is about $\tau = 0.06$ for both countries, in spite of differences in preference and distributions of wealth, labor and wage. For Germany, welfare declines more quickly with progressivity after the optimum. Given that current degree of progressivity for Germany $\tau^{\text{DE}} = 0.22$, Germany is farther away from its optimum than the US.

Table 5: Decomposition of Insurance Effects under Progressive Taxation.

	Total	Direct	Indirect	
			Wealth	Labor
Consumption insurance	$(1 - \tau)(1 - \frac{\chi'}{\rho} - \frac{\chi - \chi_\infty}{\rho})$	$-\tau$	$-\frac{\chi'}{\rho}$	$-\frac{\chi - \chi_\infty}{\rho}$
Against Perm. Shock (Germany)	0.61	-0.22	-0.17	-0.05
Against Perm. Shock (US)	0.57	-0.11	-0.08	-0.28
Against Tran. Shock (Germany)	0.11	-0.22	-0.80	-0.05
Against Tran. Shock (US)	0.10	-0.11	-0.61	-0.28
Response to 1% Wage Shock			$\frac{\log C}{(1-\tau)(1-\chi-\chi')}$	$\frac{\log H}{(1-\tau)(\chi+\chi')}$
Permanent (Germany)			0.61%	-0.001%
Permanent (US)			0.57%	-0.21%
Transitory (Germany)			0.11%	0.18%
Transitory (US)			0.10%	0.26%

Figure 13: Progressivity and Welfare

