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## Capital Requirements for Government Bonds - Implications for Financial Stability

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# Capital Requirements for Government Bonds

## - Implications for Financial Stability

Ulrike Neyer\*      André Sterzel<sup>†</sup>

February 21, 2017

### Abstract

Banks hold relatively large amounts of government bonds. Large sovereign exposures reinforce possible financial contagion effects from sovereigns to banks and are a risk for financial stability. Using a theoretical model, we find that the introduction of capital requirements for government bonds induce banks to decrease their investment in government bonds and to increase their investment in high yield assets. This investment shift and a higher amount of capital imply that banks' balance sheets become more resilient against sovereign debt crises. However, the extent of this effect depends on the type of the introduced capital regulation. Furthermore, we emphasize the role of the central bank in this context.

*JEL classification:* G28, G21, G01.

*Keywords:* Government bonds, banking regulation, capital adequacy rules, sovereign risk, lender of last resort.

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# 1 Introduction

The recent financial crisis has highlighted the strong link between sovereigns and banks in both directions. Serious doubts about the solvency of some EU Member States have put pressure on banks' balance sheets. Bank bailouts in turn have strained public finances. This reinforcing feedback loop has led to risks for financial stability and macroeconomic development. Besides the concept of the European banking union, there is an ongoing debate about the preferential treatment of government bonds to weaken the sovereign-bank nexus. Concerning this, there is a report from the European Systemic Risk Board (2015) which points out the need to reform the regulatory treatment of government bonds in banking regulation. Moreover, the following quote from Jens Weidmann, the president of the Deutsche Bundesbank, also stresses the need for a reform of the current regulatory system in the European Union:

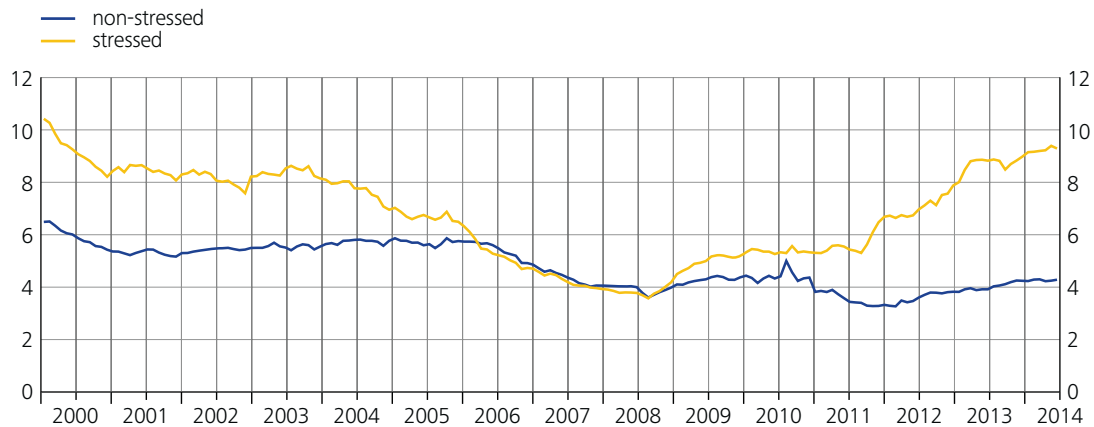
There is one field in regulation, however, where too little has been done so far - the treatment of sovereign exposures in banks' balance sheets. A banking system can only truly be stable if the fate of banks does not hinge on the solvency of their national sovereigns. Thus, I have been advocating, for quite some time now, a phasing-out of the preferential treatment of sovereign borrowers over private debtors.<sup>1</sup>

In regulatory terms sovereign assets receive a privileged treatment compared to other issuers, in the fields: capital regulation, liquidity requirements and large exposure regimes. This paper is concerned especially with the preferential treatment in capital regulation. Although the default probability of some EU Member States cannot be neglected, government bonds exposures of banks assigned a zero risk weight in capital regulation<sup>2</sup>. Moreover it can be seen that the holdings of sovereign debt in stressed EU countries doubled over the last years (Table 1). As a result the interaction of the zero risk weights, the non-neglectable default risk of some EU member states, and the size of government bond exposures in banks' balance sheets, might endanger financial stability.

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<sup>1</sup>The entire speech by Jens Weidmann is available at [https://www.bundesbank.de/Redaktion/EN/Reden/2016/2016\\_05\\_11\\_weidmann.html](https://www.bundesbank.de/Redaktion/EN/Reden/2016/2016_05_11_weidmann.html)

<sup>2</sup>See CRD IV (Capital Requirement Directive) which implemented the Basel III accords in EU law.



Source: SDW.

\* Note: The stressed euro area countries are Greece, Ireland, Italy, Portugal and Spain.

\*\* Note: The non-stressed euro area countries are Austria, Belgium, Finland, France, Germany and the Netherlands.

Figure 1: Total euro area sovereign debt holdings of MFIs as a percentage of total assets

There is some empirical evidence that banks' large holdings of government bonds are driven by the preferential treatment in capital and liquidity regulation (Bonner (2014), Acharya and Steffen (2015), Gennaili et al. (2014a)). Against this background our findings are consistent with the empirical observations and hence, we show that a zero risk weight for sovereign debt encourages banks to increase their investment in government bonds in comparison to a regulation regime where sovereign assets are subject to capital regulation.

By using a theoretical model, the aim of this paper is to show how capital requirements for government bonds influence the investment and capital structure of banks related to a capital ratio where sovereign assets are not considered. It is shown that a phasing-out of the preferential treatment of sovereign borrowers leads to an investment shift. Accordingly, if government bonds are subject to capital regulation, banks increase their investment in high yield assets, decrease their investment in government bonds and the additional risk weight for sovereign assets leads to more equity capital in the whole banking sector. The intuition behind this is, that the investment in government bonds is associated with capital costs and therefore banks try to avoid these costs and decrease their sovereign investments. In our model the decrease in government bonds is limited due to the fact that these bonds are highly liquid and banks need liquid assets to hedge liquidity uncertainty. In order to fulfil the additional capital costs related to sovereign assets, banks are forced to increase

their risk taking behavior to yield higher returns and cover with these returns the increased capital costs.

Based on these results we then investigate the shock absorbing capacity if a government bond shock occurs in both regulation approaches. As a result, spillover effects from sovereigns to banks become weaker if sovereign assets become subject to capital regulation. This effect is driven by the asset shift based on the risk weights for government bonds. Due to the fact that banks decrease their government bonds holdings, banks are confronted with lower government bond losses. Furthermore, we emphasise the importance of the central bank to act as a lender of last resort in this context. The aim of the central bank in our model is to avoid insolvencies due to liquidity issues. Therefore the central bank provides liquidity in exchange for assets which are not affected by the government bond shock - in our case high-yield assets. As a result, a higher liquidity provision by the central bank and lower government bond losses, make the system more resilient to sovereign debt crisis, if government bonds have to be covered with equity capital. However, how strong the asset shift is, and therefore the shock absorbing capability, relies on the risk weight for sovereign assets.

Our paper contributes to three strands of literature. The first strand investigates the interactions between sovereign default and the stability of the banking sector. Bolton and Jeanne (2011) analyse how sovereign solvency issues influence the banking sector in financially integrated economies. Gennaii et al. (2014b) also consider a model which analyses the contagion from sovereigns to banks in a nondiscriminatory way. Both works point out that public default risk weakens the banks' balance sheets. The second area of research examines in how far capital regulation affects banking behavior. Blum (1999) analyses the risk taking behavior of banks under capital requirements in a dynamic framework. It is pointed out, that a binding risk adjusted capital rule may increase banks' risk taking behavior. In the same context Hyun and Rhee (2011) conclude that a binding risk adjusted capital ratio may reduce banks' loan supply. Our work combines both strings of research and, in addition, considers a third strand of literature, the literature of financial stability. Especially, we base our work on analyses dealing with banking crisis and systemic risk. The banking sector is modelled similar to the one in Allen and Carletti (2006) and Heyde

and Neyer (2010). However, in their particular model banks may invest in a safe and a risky long-term asset. In our model, there is no safe long-term asset, but both long-term assets, loans and government bonds, are risky.

The rest of the paper is structured as follows. Section 2 describes the model setup. Section 3 starts with the analysis of bank's optimal behavior in case sovereign bonds are not subject to capital regulation. Then a banks' optimal behavior in the presence of positive risk weights for sovereign assets is analysed. Building on these analyses, section 4 discusses the consequences of capital requirements for government bonds for financial stability. The final section summarizes the paper.

## 2 Model

### 2.1 Technology

We consider three dates,  $t = 0, 1, 2$  and a single all-purpose good that can be invested or consumed. At date 0, the all-purpose good can be invested in three types of assets: a short-term asset and two long-term assets. The short-term asset represents a simple storage technology i.e. one unit at date 0 returns one unit at date 1. The two long-term assets are government bonds and loans. However, unlike in other theoretical works, government bonds are not completely safe but yield a random return  $S$ . With probability  $1 - \beta$  the investment fails and one unit of the all-purpose good invested in government bonds at date 0 produces only  $l < 1$  units of this good at date 2. With probability  $\beta$ , the investment succeeds and produces  $h > 1$  units at date 2. A government bond is a liquid asset and can be traded at price  $p$  on an interbank market at date 1. The loan portfolio yields a random return  $K$ . If the loan investment succeeds, one unit invested at date 0 generates a return of  $H > h > 1$  at date 2 with probability  $\alpha < \beta$ . With probability  $(1 - \alpha)$  the investment fails and produces only  $L < l < 1$  units at date 2. Main characteristics of the loan portfolio are that it is the asset with the highest expected return as  $E(K) > E(S) > 1$ , the highest risk as the variance  $Var(K) > Var(S)$ , and that it is illiquid as loans cannot be traded on an interbank market. Hence, even if a bank goes bankrupt at date 1, the proceed of the loan portfolio is 0 like in Allen and Carletti (2008).

Moreover, banks discover whether the long term assets succeed or fail at date 2. Table 1 summarizes the returns on the different types of assets.

	Return at date 1	Return at date 2	
Short-term asset	1		
Government bonds	$p$	$\left. \begin{array}{ll} h & \beta \\ l & (1 - \beta) \end{array} \right\}$	$E(S) > 1$
Loan portfolio	0	$\left. \begin{array}{ll} H & \alpha \\ L & (1 - \alpha) \end{array} \right\}$	$E(K) > E(S), Var(K) > Var(S)$

Table 1: Return on the Different Types of Assets (Investment at Date 0: 1 Unit)

## 2.2 Agents and Preferences

In our model, there are three types of agents: a continuum of risk averse consumers normalized to one, a large number of banks and a large number of risk neutral investors. Each consumer is endowed with one unit of the all-purpose good at date 0. Like in Diamond and Dybvig (1983) the consumers can be categorized into two groups. One group values consumption only at date 1 (early consumers), the other group only at date 2 (late consumers). We assume both groups to be of the same size. Consequently, the probability of being an early consumer is  $\gamma = 0.5$  and the probability of being a late consumer is  $(1 - \gamma) = 0.5$ . Denoting a consumer's consumption by  $c$ , his utility of consumption is described by

$$U(c) = \ln(c). \tag{1}$$

However, at date 0 each consumer is unsure about its liquidity preference. He does not know whether he is an early or late consumer. Therefore, he concludes a deposit contract with a bank.

According to this contract, he deposits his one unit of the all-purpose good with the bank at date 0 and can withdraw  $c_1^*$  units of the all-purpose good at date 1 or  $c_2^*$  units of this good at date 2. As we have a competitive banking sector, each bank invests in the

short-term asset and the two long-term assets in order to maximize depositors' expected utility. If a bank did not maximize the consumers' utility, another bank would step in and attract away the depositors.

While there is no aggregate liquidity risk, the fraction of early consumers is  $\gamma = 0.5$  for sure, banks are subject to idiosyncratic liquidity risk. Accordingly, they do not know their individual proportion of early consumers. A fraction  $\lambda$  of banks has a proportion  $\gamma_1$  of early consumers and a fraction  $(1 - \lambda)$  of  $\gamma_2$ , so that  $\gamma = 0.5 = \lambda\gamma_1 + (1 - \lambda)\gamma_2$ . As in Allen and Carletti (2006) we assume the extreme case in which  $\gamma_1 = 0$  and  $\gamma_2 = 1$ , so that it turns out for a single bank that either all depositors are early consumers or that all depositors are late consumers.<sup>3</sup> Because of this strong assumption, 50% of all banks are "early" banks, with only early consumers and 50% of all institutions are "late" banks with only late consumers.

As there is no aggregate risk, a bank's liquidity risk can totally be hedged. In our model, the possible hedging takes place via an interbank market for government bonds: All banks may invest into government bonds at date 0. At date 1, when each bank has learnt whether it is a late or an early bank, it sells or buys government bonds on the interbank market at price  $p$  to balance its liquidity position.

In addition, to the funds from consumers, banks have the possibility to raise funds (equity capital) from risk neutral investors. These investors are endowed with an unbounded amount of capital  $W_0$  at date 0. The contract concluded between a bank and an investor defines the units of the all-purpose good (equity capital) which are provided at date 0 ( $e_0^* \geq 0$ ) and the units which are repaid and consumed by the investor at date 1 and date 2 ( $e_1^* \geq 0$  and  $e_2^* \geq 0$ ). The utility function of a risk neutral investor is given by

$$U(e_0, e_1, e_2) = \rho(W_0 - e_0) + e_1 + e_2 \quad (2)$$

with the parameter  $\rho$  representing the investor's opportunity costs of investing into the banking sector.

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<sup>3</sup>The main reason for these strong assumptions is to keep the following optimization problem as simple as possible. The optimization problem in section 4 without this assumption would be:  $EU = \lambda\gamma_1 \ln(c_1) + (1 - \lambda)\gamma_2 \ln(c_1) + \lambda(1 - \gamma_1)[\alpha\beta \ln(c_{2Hh}^*) + \alpha(1 - \beta)\ln(c_{2HI}^*) + (1 - \alpha)\beta \ln(c_{2Lh}^*) + (1 - \alpha)(1 - \beta)\ln(c_{2LI}^*)] + (1 - \lambda)(1 - \gamma_2)[\alpha\beta \ln(c_{2Hh}^*) + \alpha(1 - \beta)\ln(c_{2HI}^*) + (1 - \alpha)\beta \ln(c_{2Lh}^*) + (1 - \alpha)(1 - \beta)\ln(c_{2LI}^*)]$ . Given  $\gamma_1 = 0$  and  $\gamma_2 = 1$  the first and the last term of the equation can be eliminated.



### 2.3 Optimization Problem

As ex-ante, i.e. at date 0, all banks are identical, we can consider a representative bank when analyzing the banks' optimal behaviour at date 0. Deposits are exogenous and equal to one. The bank has to decide on units  $x$  to be invested in the short-term asset, on units  $y$  to be invested in government bonds, on units  $u$  to be invested in loans and on units  $e_0$  to be raised from the risk neutral investors. A bank's optimal behaviour requires to maximize the expected utility of its risk averse depositors. Consequently, a bank's optimization problem reads

$$\begin{aligned} \max EU = & \lambda \ln(c_1) + (1 - \lambda)[\alpha \beta \ln(c_{2Hh}) + \alpha(1 - \beta) \ln(c_{2Hl}) \\ & + (1 - \alpha) \beta \ln(c_{2Lh}) + (1 - \alpha)(1 - \beta) \ln(c_{2Ll})] \end{aligned} \quad (3)$$

$$\text{with } c_1 = x + y\tilde{p}, \quad (4)$$

$$c_{2Hh} = uH + \left(\frac{x}{\tilde{p}} + y\right)h - e_{2Hh}, \quad (5)$$

$$c_{2Hl} = uH + \left(\frac{x}{\tilde{p}} + y\right)l - e_{2Hl}, \quad (6)$$

$$c_{2Lh} = uL + \left(\frac{x}{\tilde{p}} + y\right)h - e_{2Lh}, \quad (7)$$

$$c_{2Ll} = uL + \left(\frac{x}{\tilde{p}} + y\right)l - e_{2Ll}, \quad (8)$$

$$\begin{aligned} \text{s.t. } \rho e_0 = & \lambda(\alpha e_{2H} + (1 - \alpha)e_{2L}) + (1 - \lambda)(\alpha \beta e_{2Hh} \\ & + \alpha(1 - \beta)e_{2Hl} + (1 - \alpha)\beta e_{2Lh} + (1 - \alpha)(1 - \beta)e_{2Ll}), \end{aligned} \quad (9)$$

$$CR^{min} = \frac{e_0}{\phi_x x + \phi_y y + \phi_u u}, \quad (10)$$

$$e_0 + 1 = x + y + u, \quad (11)$$

$$x, y, u, e_0, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll} \geq 0. \quad (12)$$

Equation (3) describes the expected utility of the bank's depositors. With probability  $\lambda = 0.5$  the bank is an early bank, i.e. all of its depositors are early consumers and withdraw their deposits at date 1. In this case, the bank will use the proceeds of the short-term asset ( $x \cdot 1$ ) and of selling all its government bonds on the interbank market

$(y \cdot \tilde{p})$ , with  $\tilde{p}$  denoting the equilibrium price for a government bond, to satisfy its depositors as formally revealed by (4).

Also with probability  $(1 - \lambda = 0.5)$ , the bank is a late bank, i.e. all of its depositors are late consumers and withdraw their deposits at date 2. The consumption level of a late consumer depends on the returns on the bank's investments in government bonds and loans. As the probabilities of success of these investments,  $\alpha$  and  $\beta$ , are independent, we can identify four possible states: both investments succeed, only the investment in the government bonds succeeds, only the investment in the loan portfolio succeeds, or both investments fail. We denote these four states simply as  $Hh$ ,  $Hl$ ,  $Lh$   $Ll$ . Equations (5) to (8) represent the consumption levels of late depositors in these possible states. The first term on the right hand side in each of these equations represents the proceeds from the investment in loans, the second from the investment in government bonds. Note that the quantity of government bonds a late bank holds at date 2 consists of the units  $y$  it invested itself in government bonds at date 0, and of those it has bought on the interbank market at price  $\tilde{p}$  in exchange for its units of the short-term asset  $x$ , at date 1. The last term depicts the amount a bank has to pay to the risk neutral investors at date 2. Note that raising funds from the investors may increase the expected utility of the risk averse depositors, as part of the risk involved with the investments in the long-term assets are then shifted to the risk neutral investors. Due to their risk neutrality, they are indifferent of whether to consume at date 1 or at date 2. Consequently, optimal consumer contracts require  $e_1^* = 0$ .

Equation (9) represents the investors incentive compatibility constraint. Investors are only willing to provide capital to the banking sector, if, at least, their opportunity costs  $\rho$  are covered. With probability  $\lambda = 0.5$  the bank is an early bank. Then, it uses its total amount of  $x$  including those units obtained in exchange for its total amount of government bonds on the interbank market to satisfy all its depositors at date 1, while investors receive the total proceeds from loans  $e_{2H} = uH$ , or  $e_{2L} = uL$  at date 2. With probability  $(1 - \lambda) = 0.5$ , the bank is a late bank. Then, investors receive the residual returns from the bank's investment in long-term assets, i.e. those returns not being used for satisfying the bank's depositors.

Constraint (10) captures the capital requirements the bank may face. They are expressed as a minimum capital ratio  $CR^{min}$  of the bank's equity  $e_0$  to its (risk-)weighted assets  $\phi_x x + \phi_y y + \phi_u u$ . If  $\phi_x = \phi_y = \phi_u = 0$ , there will be no capital requirements. If  $\phi_x = \phi_y = \phi_u = 1$ , the required ratio will correspond to a simple leverage ratio. If  $\phi_x = \phi_y = 0$  and  $\phi_u > 0$ , there will be a privileged treatment of (risky) government bonds in financial regulation. This privileged treatment will be repealed if  $\phi_x = 0$  and  $\phi_u > \phi_y > 0$ . Then, also risky government bonds have to be covered by equity capital. That  $\phi_u > \phi_y <$  reflects that sovereign bonds are less risky than loans. The budget constraint is represented in equation (11), and the last constraint (12) represents the non negativity constraint.

### 3 Interbank Market for Government Bonds

Before solving the optimization problem in the next section, we will have a closer look at the interbank market for government bonds. For the sake of simplicity we assume that if a consumer invested himself in government bonds and if he became a late consumer, his expected utility from this investment would be (weakly) higher than from an investment in the short-term asset, i.e.

$$\beta \ln(h) + (1 - \beta) \ln(l) \geq \ln(1) = 0. \quad (13)$$

At date 1, each bank has learnt whether it is a late or an early bank and thus will buy or sell government bonds on the interbank market. However, late banks will only buy government bonds in exchange for their short-term asset if this does not imply that the utility of their depositors becomes lower compared to the alternative of storing the short-term asset until date 2. This induces that there is a maximum price

$$p^{max} = h^\beta l^{(1-\beta)} \quad (14)$$

late banks are willing to pay for a bond.<sup>4</sup> If  $p \leq p^{max}$ , a late bank wants to sell the total amount of its short-term asset in exchange for government bonds as government bonds

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<sup>4</sup>Equation (14) is derived from:  $\ln(1) = -\ln(p^{max}) + \beta \ln(h) + (1 - \beta) \ln(l)$

yield a (weakly) higher expected utility for their depositors. If  $p > p^{max}$ , a late bank does not want to sell any unit of its short-term asset in exchange for government bonds. Note, that at date 0, all banks are identical and solve the same optimization problem. Accordingly, for all banks the optimal quantities invested in the short-term asset and the long-term assets are identical. We denote these optimal quantities by  $x^*$ ,  $y^*$ , and  $u^*$ . Considering furthermore, that the number of depositors is normalized to one, the optimal quantities of each individual bank correspond to the respective aggregate quantities. As half of the banks are late banks, the aggregate demand function for government bonds at date 1 is

$$y^D = \begin{cases} 0.5 \frac{x^*}{p} & \text{if } p \leq p^{max}, \\ 0 & \text{if } p > p^{max}. \end{cases} \quad (15)$$

Figure 2, illustrates this demand function. The jump discontinuity at  $p^{max}$  results from the fact that for  $p \leq p^{max}$  late banks want to sell their total amount of liquidity  $x^*$  in exchange for government bonds. The negative slope of the demand curve is due to the fact that the amount of liquidity in the banking sector used for buying government bonds is limited to  $0.5x^*$ . Consequently, a higher price  $p$  implies that less government bonds can be bought.

Independently of the price, early banks want to sell all their government bonds at date 1 as early consumers only value consumption at this state. Therefore, we get the totally price inelastic aggregate supply curve

$$y^S = 0.5y^* \quad (16)$$

which is also illustrated in Figure 2.

Considering (15) and (16), the market clearing condition becomes

$$\frac{x^*}{\tilde{p}} = y^*. \quad (17)$$

As there is no aggregate uncertainty and as all banks solve the same optimization problem at date 0, aggregate supply and demand and thus  $\tilde{p}$  are known at date 0. In addition,

the following considerations allow us to set  $\tilde{p} = 1$  when solving the optimization problem. If  $p < 1$ , the return on government bonds at date 1 would be negative and thus smaller than on the short-term asset. Consequently, at date 0 no bank would invest in government bonds but in the short-term asset instead as then independently of whether a bank would become an early or a late bank, the repayments to its depositors would be higher. However, if no bank buys government bonds at date 0, there will be no supply of government bonds and thus no interbank market for government bonds with a positive price at date 1.

If  $p > 1$ , the (liquid) government bond would be worth more than the short-term asset at date 1. Therefore, no bank would invest in the short-term asset at date 0 but in government bonds instead. This behavior would allow higher repayments to the depositors of an early, as well as of a late bank. However, if at date 0 no bank invests in the short-term asset but in government bonds instead, there will be no demand for government bonds at date 1, and thus no interbank market with a positive price. Consequently the only possible equilibrium price at date 1 is  $\tilde{p} = 1$ . Note that due to (13) and (14),  $p^{max} \geq 1$ , which implies that the interbank market is always cleared.

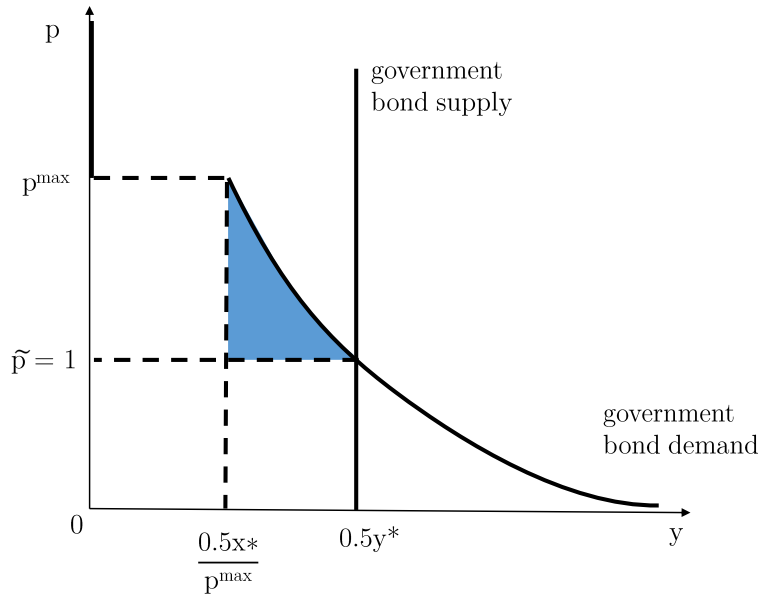


Figure 2: Interbank Market for Government Bonds at Date 1

Furthermore the aggregate demand function allows us to derive the surplus of the banking sector which serves in section 5 as a measure for the shock absorbing ability of the banking sector. Crucial is, the higher the expected utility of government bonds and

hence the maximum price late banks are willing to pay, the higher is the surplus of the late banks and accordingly the shock absorbing ability. In Figure 2 the blue area reflects this measure. Formally, it is given by

$$\mathcal{S} = \int_1^{p^{max}} 0.5x^* \left( \frac{1}{p} - \frac{1}{p^{max}} \right) dp = 0.5x^* \left[ \ln(p^{max}) + \frac{1}{p^{max}} - 1 \right]. \quad (18)$$

## 4 Banks' Optimal Investment and Financing Behavior

The purpose of this section is to analyse the influence of different forms of capital requirements on the banks' optimal financing and investment behaviour. To capture the importance of equity capital in our model, we suppose as a starting point that there are no investors, so that banks do not have the possibility to raise capital. After this, we consider the case in which investors make equity capital available for banks, but the banking sector is not subject to capital regulation. Comparing these two scenarios allow us to analyse the role of capital.

Based on these considerations we then analyse two different regulation regimes. In the first approach there is a binding capital ratio only for loans, whereas (risky) sovereign bonds are not included in capital regulation. In the second approach, in addition to loans, also sovereign bonds have to be backed with equity capital. Modelling a setting with a zero risk weight for government bonds and then taking into account a capital ratio where we introduce a positive risk weight for these bonds, allows us to analyse the balance sheet adjustments, when the preferential treatment for sovereign debt is abolished in financial regulation.

For demonstrating a bank's optimal investment and financing structure under different regulation regimes, we make use of a numerical example similar to the one used by Allen and Carletti (2006). We assume the following values. The government bond returns  $h = 1.3$  with probability  $\beta = 0.98$  and  $l = 0.3$  with probability  $(1 - \beta) = 0.02$ . Consequently, the investment of one unit at date 0 yields the expected return  $E(S) = 1.28$  at date 2. Loans are also state dependent and return at date 2. They return  $H = 1.54$  with probability  $\alpha = 0.93$  and they fail and yield  $L = 0.25$  with probability  $(1 - \alpha) = 0.07$ .

Hence, the expected loan return at date 2 is  $E(K) = 1.449$ . Investors' opportunity costs are  $\rho = 1.5$ .

#### 4.1 No Equity Capital

First, we analyse a scenario with no equity capital. Then, the constraints (9) as well as (10) are omitted, all  $e_{(\cdot)} = 0$ , and the budget restriction (11) becomes:  $x + y + u = 1$ . The optimal solution in this case is:

Balance Sheet			
A			L
$x^* = 0.5$	$D =$		1
$y^* = 0.5$			
$u^* = 0$			
$\Sigma$	1	$\Sigma$	1

Deposit Contracts:

$$c_1^* = 1 \quad c_{2h}^* = 1.3 \quad c_{2l}^* = 0.3$$

$$EU = 0.1165$$

Table 2: No Equity Capital, Banks' Optimal Balance Sheet Structure and Repayments to the Depositors

Without equity capital, banks invest their total amount of deposits in the short-term asset, in government bonds and nothing in loans. Basically, loans have two effects on consumers' consumption: First, the relatively high expected loan return at date 2 increases the expected consumption at date 2,  $E[c_2]$ .<sup>5</sup> Second, due to the budget constraint,  $x + y + u = 1$ , banks have to decide how to split the funds provided by the depositors. Hence, high loan investment implies low investment into liquid assets. As early consumers get repaid with the proceeds of the liquid assets and loans are worthless for them, early consumption is low if banks are strongly invested in loans.

In our numerical example, the utility loss when banks invest in loans (high illiquidity) exceeds the utility gain (higher expected returns at date 2). In consequence, the marginal utility of loans is lower than the marginal utility of sovereign bonds. In other words, loan investment is not beneficial for banks due to the high illiquidity with respect to government

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<sup>5</sup> $E[c_2] = \alpha\beta c_{2Hh}^* + \alpha(1 - \beta)c_{2Hl}^* + (1 - \alpha)\beta c_{2Lh}^* + (1 - \alpha)(1 - \beta)c_{2Ll}^*$ .

bonds. Formally we obtain a corner solution as the non negativity constraint for loans becomes binding.<sup>6</sup>

Moreover, banks split their investment into the liquid assets equally in the short asset and in government bonds,  $x^* = y^*$ . Considering that, there is no aggregate liquidity uncertainty, banks know supply and demand in the government bond market and therefore the equilibrium price at date 1,  $\tilde{p} = 1$ . As 50% of the banks are early banks and 50% of the banks are late banks, banks invest the identical amount in government bonds and the short-term asset, to be able to hedge the idiosyncratic liquidity risk completely by trading government bonds on the interbank market at date 1.<sup>7</sup>

Even though  $c_{2l}^*$  is lower than the consumption at date 1,  $c_1^*$ , there will not be a bank run and consumers will accept this form of deposit contract. This is the case as long as the utility of the expected late consumption is higher than the utility of early consumption  $U(c_1^*) \geq U(E[c_2^*])$ .<sup>8</sup>

## 4.2 With Equity Capital

If banks have the opportunity to raise equity capital from investors, but do not face a binding minimum capital ratio,  $CR^{min} = 0$ , we will get the following solutions:

The results show, that even if the banking sector is not subject to capital requirements, it is optimal for the representative bank to raise equity capital from investors. Moreover, banks start to invest in loans and they invest less in liquid assets related to the case without equity capital. Overall, depositors' expected utility can be increased if equity capital is available.

Raising equity capital from the investors is associated with costs for banks. The investors' per unit opportunity costs are  $\rho$ . In order to make the investors willing to supply  $e_0^*$  to the banking sector, the banks thus have to pay at least  $\rho e_0^*$  to the investors. We denote  $\rho e_0^*$  as gross capital costs for banks. Since  $\rho > E(K)$  equity capital is costly in the sense that the payments to the investors for one unit capital exceed the expected return

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<sup>6</sup>Note, if  $E(K)$  were sufficiently high, banks would invest in loans. However, we are not interested in the absolute investment in each asset class. Hence, we want to show the investment shift under different regulation regimes.

<sup>7</sup>Given, that these assumptions hold also in the sections 4.2-4.4 banks invest always the identical amount in government bonds and the short-term asset.

<sup>8</sup>Note, that the proceeds of the long-term assets become public at date 2 and at date 1 the consumers do not know if they succeed or fail. Therefore late consumers based their decision on expectations.



Balance Sheet

A		L	
$x^* =$	0.4544	$e_0^* =$	0.0853
$y^* =$	0.4544	$D =$	1
$u^* =$	0.1765		
$\sum$	1.0853	$\sum$	1.0853

Contracts with Investors:

$$\begin{array}{cccc} \text{early banks:} & e_{2H}^* = 0.2718 & e_{2L}^* = 0.0441 & \\ \hline \text{late banks:} & e_{2Hh}^* = 0 & e_{2Hl}^* = 0 & e_{2Lh}^* = 0 \quad e_{2Ll}^* = 0 \end{array}$$

Deposit Contracts:

$$c_1^* = 0.9088 \quad c_{2Hh}^* = 1.4536 \quad c_{2Hl}^* = 0.5445 \quad c_{2Lh}^* = 1.2256 \quad c_{2Ll}^* = 0.3168$$

$$EU = 0.1231$$

Table 3: Banks' Optimal Balance Sheet Structure With Equity Capital, Repayments to the Depositors and Investors

even from the most profitable asset - in our case loans. Although capital is costly, banks have an incentive to raise capital from the investors because it increases the consumers' expected utility. The reason is, that the liquidity risk combined with loans can be partially shifted to the risk neutral investors.

In contrast to the case without equity capital, loans become more attractive than government bonds as loans can be financed with equity capital and the liquidity risk associated with loans can be partially transferred to the risk neutral investors. Formally, the utility loss when investing into loans can be mitigated (due to the liquidity risk shift) whereas the utility gain (high expected returns at date 2) remains constant. This implies that the marginal utility of loans increases and banks start investing in loans and reduce their sovereign debt holdings until the marginal utility of both asset classes is the same.

Hereinafter we will have a closer look at the relation between equity capital, loans and thus the contracts with the investors.<sup>9</sup> Concerning that equity capital is costly, banks have two possibilities to back their gross capital costs. First, they can increase their loans disproportionately to the amount of capital and cover the capital costs only with loan

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<sup>9</sup>See equation (9)

returns if it turns out that the bank is an early bank.<sup>10</sup> However, if a bank becomes a late bank, the investors will not receive anything and the total returns from government bonds and loans are paid out to the late depositors.

Second, banks do not have to cover necessarily the total gross capital costs with loan returns if it turns out that they are early banks. They can also decide to increase their loans proportional or less than the amount of equity capital. In that case the expected returns from loans do not cover the opportunity costs of investors as  $E(K) < \rho$ . Accordingly, it is not sufficient to repay the investors only in case it turns out that a bank is an early bank, but also in case the bank becomes a late bank they have to make a payment to the investors. Then, late consumers need to share the returns from the long-term assets with the investors. This reduces consumers' expected consumption at date 2,  $E[c_2]$ . Whereas in the case the late consumers have to share the returns with the investors the expected utility decreases, the utility is not affected if early banks repay the investors with loan returns as loans are not worth for them. Therefore, it is optimal for the banks to increase loans stronger than equity capital and decrease their liquid asset holdings in order to cover the gross capital costs.

Considering the consumption of the early consumers, it can be seen that  $c_1^*$  is lower than the consumers deposit at a bank at date 0. Although the return is negative, they are willing to provide funds to the banking sector as long as banks are able to increase depositors' expected utility. Again, ex ante, all consumers are identical and do not know if they will be early or late consumers. Therefore, they will accept a negative return in the first period, if in general the expected return  $EU$  differs from zero.

### 4.3 Zero Risk Weight for Government Bonds

In order to illustrate how banks respond to different forms of capital regulation, constraint (10) becomes important. As we want to analyse in this section a capital regulation approach with a preferential treatment of (risky) sovereign bonds, only loans are subject to financial regulation. Thus, we introduce a risk weight of 1 for this asset class i.e.

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<sup>10</sup>Deriving from equation (9) is, that the investment into loans needs to be  $\frac{\rho}{\lambda[\alpha(H-L)+L]}$  times higher than the amount of equity capital, in order to meet the capital claims only with loan returns in case banks are early ones. As long as  $E(K) > \rho$  the counter is higher than the denominator and therefore the multiplier is greater than 1.

$\phi_x = \phi_y = 0$  and  $\phi_u = 1$ . To determine the impact of binding capital requirements<sup>11</sup> for loans, the minimum capital ratio is:  $CR^{min}|_{\phi_y=0} = \frac{e_0}{u} = 0.58$  and the results under this additional constraint are shown in table 4.

Balance Sheet					
A			L		
$x^*$	= 0.4581	↑	$e_0^*$	= 0.1157	↑
$y^*$	= 0.4581	↑	$D$	= 1	
$u^*$	= 0.1995	↑			
$\Sigma$	1.1157	↑	$\Sigma$	1.1157	↑

Contracts with Investors:

early banks:	$e_{2H}^* = 0.3072$	↑	$e_{2L}^* = 0.0499$	↑		
late banks:	$e_{2Hh}^* = 0.0635$	↑	$e_{2Hl}^* = 0$		$e_{2Lh}^* = 0$	$e_{2Ll}^* = 0$

Deposit Contracts:

$$c_1^* = 0.9162 \uparrow \quad c_{2Hh}^* = 1.4348 \downarrow \quad c_{2Hl}^* = 0.5821 \uparrow \quad c_{2Lh}^* = 1.2409 \uparrow \quad c_{2Ll}^* = 0.3247 \uparrow$$

$$EU = 0.1224 \downarrow$$

Table 4: Binding Capital Ratio for Loans, Banks' Optimal Balance Sheet Structure, Re-payments to the Depositors and Investors

The arrows in table 4 display the impact of  $CR^{min}|_{\phi_y=0}$  on the balance sheet positions, payments to investors and depositors, compared to the situation without capital regulation. It turns out, that banks increase their investment in government bonds, loans and the short-term asset. On the liability side banks raise equity capital and the expected utility decreases.

If banks have to back their risk weighted assets with equity capital, the gross capital costs are given by:

$$e_0\rho = \phi_x x CR^{min} \rho + \phi_y y CR^{min} \rho + \phi_u u CR^{min} \rho. \quad (19)$$

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<sup>11</sup>If banks do not face binding capital requirements (section 4.2) they chose a optimal capital ratio of:  $CR^{opt} = \frac{e_0^*}{u^*} = \frac{0.0853}{0.1765} = 0.48$ . In order to analyse the impact of a binding capital ratio the following assumption must hold:  $CR^{min} > CR^{opt}$ . We consider a binding capital ratio which is 20% higher than  $CR^{opt}$ .

Each term on the RHS reflects the capital costs for each asset class i.e. for the short-term asset, sovereign bonds and loans. As  $\phi_x = \phi_y = 0$ , the gross capital costs under this regulation approach, are:  $e_0\rho = \phi_u u C R^{min} \rho$ , which reveals that only loans are associated with capital costs. To fulfil a binding capital ratio for loans, banks have two possibilities in order to meet the requirement. Hence, they can increase equity capital relatively to loans or they can decrease loans relatively to capital.

Considering the budget constraint (11), if banks granting less loans c.p., they need to increase their investment into sovereign bonds and the short asset, to rebalance their balance sheets. As early consumers get repaid with these returns, the early consumers' consumption increases. Formally, if banks decrease their loan investment and increase their investment into liquid assets, the marginal utility of loans increases and the marginal utility of government bonds decreases.<sup>12</sup> Related to the late consumption, a decrease in loans relatively to equity capital implies a decrease in consumption *in each state* at date 2.

However, if banks respond to the binding capital ratio for loans with an increase in equity capital relatively to loans, they are also forced to increase their investment into liquid assets.<sup>13</sup> As in the previous case, this leads to an increase in early consumers' consumption. Moreover, an increase in equity capital also rise the gross capital costs. These additional capital claims will be backed by additional payments to the investors *in state Hh*. Accordingly, the consumers benefit from investors' capital in all states where at least one asset fails but they have to share their returns with the investors, if both assets succeed.

Due to the risk aversion of the consumers the banks increase capital instead of decreasing loans as the utility loss due to an increase in equity capital is lower than due to a reduction in loans. However, even though banks have to back their loans with equity capital, it is optimal for them to raise more capital and even increase their loan investment instead of investing the total additional capital totally in government bonds and the short asset. The reason is, that the utility loss due to higher capital which arises with an

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<sup>12</sup>The introduction of a binding capital ratio implies that banks can not achieve the case where the marginal utility of both assets are equal as in section 4.1.

<sup>13</sup>Note, that banks can not invest their additional funds from the investors completely in loans as banks need more equity capital than loans to fulfil the binding capital ratio.

additional loan investment are lower as the utility loss, driven by the losses which arise if banks invest their total additional funds in liquid assets, which have lower returns.

Formally an increase in loans and government bonds leads to a decrease in marginal utility of loans and the marginal utility of sovereign bonds. Considering the expected utility, introducing a binding capital ratio for loans, leads to a decrease in expected consumption and therefore to a reduction in expected utility. The reason is, that the decrease in late consumers' consumption due to higher capital costs, exceeds the increase in early consumer's consumption driven by more investment in liquid assets.

#### 4.4 Positive Risk Weight for Government Bonds

After analysing a regime where (risky) sovereign exposures are not subject to capital regulation, in this section we investigate a regulation approach where banks face a minimum capital ratio which includes a positive risk weight for sovereign debt. In our case, a 5% risk weight for government bonds  $\phi_y = 0.05$  is introduced and the capital ratio, the risk weight for loans as well as the risk weight for the short-term asset are the same as in the previous section.<sup>14</sup> Hence, the capital regulation constraint is:  $CR^{min} = \frac{e_0}{u+0.05y} = 0.58$  and we obtain the following results:

It turns out, that a binding capital ratio also for sovereign bonds leads to a portfolio shift. The arrows in brackets display this portfolio shift with respect to the scenario without capital requirements for government bonds (section 4.3). Banks decrease their investment in liquid assets and increase their loan investment. Moreover, they raise more equity capital and the expected utility decreases.

The gross capital costs under this regulation approach become:  $e_0^*\rho = \phi_y y CR^{min} \rho + \phi_u u CR^{min} \rho$ . Accordingly, banks have three possibilities to meet the newly introduced capital requirements. First, they can raise equity capital relatively to loans and government bonds. Secondly, they can decrease their government bond holdings relatively to the total amount of equity capital. Thirdly, banks can reduce their loans relatively to the amount of investors' capital.

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<sup>14</sup>Note, that the risk weights  $\phi_y$  and  $\phi_u$  do not exactly reflect the risks of each asset class. For our purpose, the exact risk weights for each asset class is not crucial. We want to show how banks change their behaviour if they are faced a positive risk weight for sovereign bonds. Moreover, if the risk weight for sovereign bonds are too high, it is no longer optimal for the banks to invest in long-term assets, as the capital costs are too high and they only invest in the short-term asset.

Balance Sheet

A		L	
$x^* = 0.4508$	(↓)	$e_0^* = 0.1668$	(↑)
$y^* = 0.4508$	(↓)	$D = 1$	
$u^* = 0.2652$	(↑)		
$\sum$	1.1668 (↑)	$\sum$	1.1668 (↑)

Contracts with Investors:

early banks:	$e_{2H}^*=0.4084$ (↑)	$e_{2L}^* = 0.0663$ (↑)		
late banks:	$e_{2Hh}^*=0.1586$ (↑)	$e_{2Hl}^*=0$	$e_{2Lh}^*=0$	$e_{2Ll}^*=0$

Deposit Contracts:

$$c_1^* = 0.9016 \text{ (↓)}$$

$$c_{2Hh}^* = 1.4219 \text{ (↓)} \quad c_{2Hl}^* = 0.6789 \text{ (↑)} \quad c_{2Lh}^* = 1.2384 \text{ (↓)} \quad c_{2Ll}^* = 0.3368 \text{ (↑)}$$

$$EU = 0.1117 \text{ (↓)}$$

Table 5: Capital Ratio for Loans and Government Bonds, Banks' Optimal Balance Sheet Structure, Repayments to the Depositors and Investors

Before analysing how banks respond to the positive risk weight for sovereign bonds, it is important to have a closer look at the role of capital with respect to government bonds. Again, there is a strong link between equity capital and loans, since liquidity risks associated with loans can be shifted to the investors. This increases consumers utility although equity capital is costly (section 4.2). In contrast, government bonds can be traded on the interbank market and there is no liquidity risk. Therefore, financing these bonds with equity capital forces down the expected return and reduces consumers' expected utility as equity capital is costly and liquidity risk shifting is obsolete. Hence, the introduction of a risk weight for sovereign bonds c.p. make these assets less attractive in comparison to loans.

Thus, in order to fulfil the binding capital ratio banks decrease their government bond holdings as well as their investment in the short asset. However, due to the idiosyncratic liquidity uncertainty, banks need liquid assets to hedge this risk. Accordingly, it is not optimal for them to relinquish completely on government bonds. Hence, they need to hold an additional amount equity capital for their sovereign holdings,  $\phi_y y CR^{min}$  or they need

to decrease their loan investment to release equity capital. The latter possibility is not optimal, as this induces an increase in liquid assets to balance the balance sheet. This is not optimal as due to the capital requirements for government bonds, loans become more profitable than sovereign bonds and therefore a reduction in loans and an increase in government bonds lead to a strong utility loss.

Accordingly, banks respond with an increase in equity capital to back the additional capital claims for sovereign bonds. As banks decrease their investment in liquid assets (asset side) and raise more equity capital (liability side), they need to increase their loan investment with respect to the budget constraint (11). An increase in loans implies that banks need to increase their amount of capital even more, given that loans have a higher risk weight than government bonds. Granting more loans reinforce the increase in capital much stronger than the direct effect driven by the positive risk weight for sovereign debt.

Formally, a decrease in sovereign bonds increases the marginal utility of bonds and granting more loans reduces the marginal utility of loans. Considering the expected utility,  $EU$ , introducing a positive risk weight for sovereign assets leads to a decrease in expected utility. This is driven by a lower (expected) consumption for early and late consumers. The lower consumption for early depositors is caused by a lower investment in government bonds and the short-term asset as the positive risk weight reduces the profitability of sovereign bonds. The lower expected consumption for late consumers is driven by higher capital costs driven by an increase in loans.

Our results suggest, that the  $CR^{min}|_{\phi_y > 0}$  encourage banks to increase their investment in loans and decrease their sovereign debt holdings. How strong the investment shift is, depends on the risk weight for government bonds. The more capital banks have to hold for their sovereign debt exposure, the stronger is the investment shift as sovereign bonds become less profitable in comparison to loans. Accordingly, the abolishment of the preferential treatment of government bonds weaken banks' incentives to invest in these bonds. Hence, the vicious cycle between sovereigns and banks will be mitigated at least in one direction, whereas the privileged treatment of sovereign bonds reinforce the link between sovereigns and banks.

## 5 Financial Stability

The aim of this paper is to show the resilience of the banking sector under different regulation approaches in case of a sovereign debt crisis. In order to analyse the impact of sovereign solvency doubts on financial stability<sup>15</sup>, we investigate the influence of a government bond shock after the investment decision on banking behaviour. It turns out that a government bond shock may lead to liquidity issues in the banking sector. To avoid insolvencies due to liquidity issues, the central bank as lender of last resort (LOLR) becomes important. We conclude that the abolishment of the preferential treatment of sovereign assets in financial regulation increases financial stability with respect to sovereign crisis.

### 5.1 Government bond shock

Let us assume, that the banking sector is hit by a government bond shock in form of a higher default probability, after the investment decision at date 0. Table 6 summarizes the asset returns after the shock.

	Return at date 1	Return at date 2	
Short-term asset	1		
Government bond	$h$	$\bar{\beta}$ ↓	} $\overline{E(S)}$ ↓
	$l$	$\overline{(1 - \beta)}$ ↑	
Loans	$H$	$\alpha$	} $E(K)$
	$L$	$(1 - \alpha)$	

Table 6: Returns on the Different Types of Assets After the Shock (Investment at Date 0: 1 Unit)

The government bond returns  $h$  and  $l$  are not affected. While the probability of a sovereign default  $\overline{(1 - \beta)}$  increases, the likelihood that the bond succeed ( $\bar{\beta}$ ), decreases and hence the expected return  $\overline{E(S)}$  also decreases.<sup>16</sup> Given that  $\alpha$  and  $\beta$  are independent, the loan portfolio is not affected by the government bond shock. Moreover, the government bond shock does not influence the return of short-term asset. As in Allen and Gale

<sup>15</sup>The ECB defines financial stability as a condition in which the financial system - intermediaries, markets and market infrastructures - can withstand shocks without major distribution in financial intermediation and the general supply of financial services.

<sup>16</sup>Note that, the bar indicates the respective variables after the shock.



(2000) or in Heyde and Neyer (2010), the asset shock occurs absolutely unforeseen and is completely unanticipated by the banking sector at date 0.<sup>17</sup>

Since a government bond shock may influence the banks' trading behaviour on the interbank market at date 1, in the following we analyse the government bond supply and the government bond demand after the shock. Concerning equation (16), the government bond supply is not affected by the government bond shock. Again, the government bond supply comes from the early banks with only early consumers who value the consumption good only at date 1. As a consequence, early banks are forced to sell their whole stock of government bonds to receive liquidity in order to fulfil the contracts with the early consumers irrespective of the size of the shock.

However, concerning equation (15), the government bond demand - which comes from the late banks - changes, caused by a government bond shock. The reason is, that the maximum price for sovereign debt late banks are willing to pay at date 1 decreases, as the expected return in the next period decreases driven by the shock.<sup>18</sup> In section 3 we have argued that the interbank market will be cleared as long as  $p^{max} \geq 1$ . In this context, the critical default probability of sovereign bonds

$$\overline{(1-\beta)}^{crit} \leq 1 + \frac{\ln(l)}{\ln(h) - \ln(l)}, \quad (20)$$

insures that the market clearing condition holds and the equilibrium price even after the shock is still one. Obviously, the returns in each state ( $h$  and  $l$ ) have a positive effect on the critical default probability of sovereign bonds,  $\frac{\partial \overline{(1-\beta)}^{crit}}{\partial l} > 0$ ,  $\frac{\partial \overline{(1-\beta)}^{crit}}{\partial h} > 0$ . I.e. the higher the government bond returns, the higher the critical default probability.

With respect to the threshold (20), we distinguish between a small sovereign bond shock and a large government bond shock. The small government shock does not affect the market clearing condition and hence the equilibrium price. However, a large sovereign shock forces down the equilibrium price and the market clearing condition no longer holds.

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<sup>17</sup>Allen and Gale (2000) model a liquidity shock, which is expected with 0% probability at date 0 and hence, does not change the investment decision of the representative bank. However, Heyde and Neyer (2010) model an asset shock which occurs after the banks' investment decision. Here, the net return of the state depend loan portfolio is lower when the investment fails than expected at date 0.

<sup>18</sup>See equation (14).

### 5.1.1 Small Government Bond Shock

As long as  $\overline{(1-\beta)} \leq \overline{(1-\beta)}^{crit}$  and thus  $\overline{p^{max}} \geq 1$ , the interbank market is cleared ( $y^S = \overline{y^D}$ ) and thus  $\tilde{p} = 1$ . Figure 3 illustrates the interbank market for government bonds in that case.

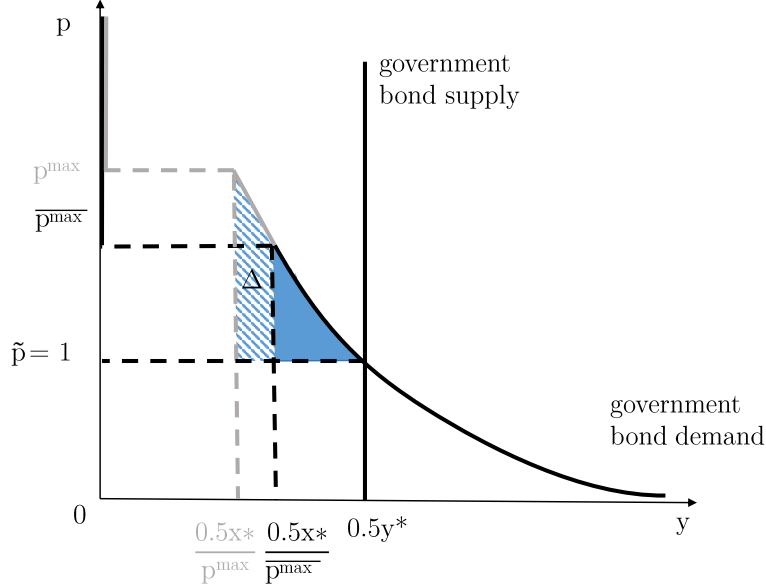


Figure 3: Interbank Market for Government Bonds at Date 1;  $\overline{(1-\beta)} \leq \overline{(1-\beta)}^{crit}$

It can be seen, that the government bond supply does not change, while the government bond demand is influenced by the government bond shock. The increased sovereign default probability forces down the maximum price late banks are willing to pay for sovereign bonds. Hence, due to the negative slope of the demand curve, a lower maximum price for government bonds implies a higher demand. Again, at the price  $\overline{p^{max}}$  late banks are indifferent of holding the liquid asset for another period, or buying government bonds with their whole amount of liquid assets at date 1. As late banks' amount of the short-term asset is limited  $0.5x^*$ , a lower price for government bonds implies a higher sovereign debt demand.

Since the maximum price late banks are willing to pay for sovereign debt decreases, also the surplus of the late banks declines  $\overline{\mathcal{S}}$ . The late banks' surplus, which serves in this model as a measure for the shock absorbing capacity is illustrated by the blue area in figure 3 and formally given by equation (18). Accordingly, a government bond shock in

form of  $\overline{(1 - \beta)} \leq \overline{(1 - \beta)}^{crit}$  leads to a loss of utility for the late banks, which is illustrated by the blue dotted area in figure 3 and defined as

$$\Delta = \mathcal{S} - \overline{\mathcal{S}} = 0.5x \left[ \ln(p^{max}) - \ln(\overline{p^{max}}) + \frac{1}{p^{max}} - \frac{1}{\overline{p^{max}}} \right]. \quad (21)$$

Obviously, the higher the sovereign bond default probability, and thus, the higher the gap between  $p^{max}$  and  $\overline{p^{max}}$ , the higher is the utility loss and consequently the remaining shock buffer. However, if  $\overline{p^{max}} = \tilde{p} = 1$ ,  $\overline{\mathcal{S}} = 0$  and hence  $\Delta = \mathcal{S}$  the late banks do not gain any surplus and the shock absorbing capability is exhausted.

Since the equilibrium price is not affected by the shock, the payments to the early- and late consumers (4)-(8) are as contractually agreed at date 0. Therefore, a small government bond shock does not lead to liquidity issues in the banking sector. However, the late depositors suffer as the shock influences the expected consumption at date 2. While the payments to the late consumers remain constant, the probabilities of each state are affected by the government bond shock,  $\alpha\overline{\beta}$ ,  $\alpha\overline{(1 - \beta)}$ ,  $(1 - \alpha)\overline{\beta}$  and  $(1 - \alpha)\overline{(1 - \beta)}$  and hence the expected consumption at date 2 decreases. Moreover, the higher sovereign bond default probability does not change the early consumers' consumption, as  $c_1^*$  is not state dependent and  $\tilde{p} = 1$ .

Given that the investors get paid in form of  $e_{2Hh}$  in all scenarios with a binding capital ratio, they suffer, as their expected payments at date 2 decrease. Note, that the shock does not change the repayment in state  $Hh$ , however, it decreases the likelihood government bonds and loans succeed,  $\alpha\overline{\beta} < \alpha\beta$ .

### 5.1.2 Large Government Bond Shock

If  $\overline{(1 - \beta)} > \overline{(1 - \beta)}^{crit}$ , late banks are no longer willing to pay one unit of their consumption good for one government bond. Accordingly, because of a higher default probability, the maximum price late banks are willing to pay for sovereign bonds is lower than one  $\overline{p^{max}} < 1$ . This implies that there is an excessive demand in equilibrium which can not be removed by an increase in  $p$ . Therefore, the maximum price late banks are willing to pay is similar to the equilibrium price  $\overline{p^{max}} = \tilde{p} < 1$  and the trading volume is determined by the government bond supply from the early banks. In that case, the shock absorbing

capability is exhausted  $\Delta = \mathcal{S}$ . Figure 4 illustrates the interbank market for government bonds in that case.

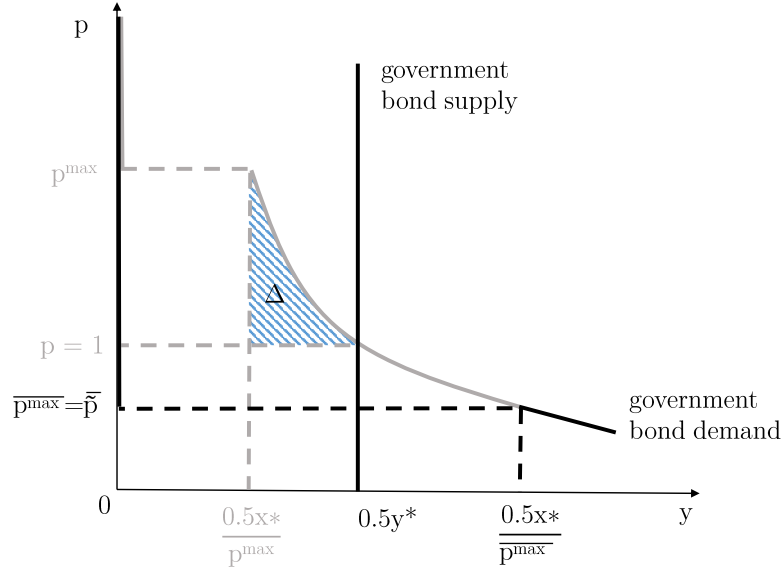


Figure 4: Interbank Market for Government Bonds at Date 1;  $\overline{(1-\beta)} > \overline{(1-\beta)}^{crit}$

The price decline for government bonds induces that a large government bond shock thus affects the repayments to the consumers. Hence, the payment to the early consumers after a large shock is:

$$\bar{c}_1 = x + y \cdot \bar{p} < c_1^* \quad \text{with:} \quad \bar{p} = \overline{p^{max}} < 1. \quad (22)$$

The first term on the RHS represents the return of the short asset ( $x \cdot 1$ ) and has not changed due to the shock.<sup>19</sup> The second term represents the proceeds of selling government bonds on the interbank market. Driven by the government bond shock, the proceeds of selling sovereign claims are lower as anticipated at date 0 ( $\bar{p} < \tilde{p} = 1$ ). Therefore, early banks cannot fulfil the contracts with their depositors i.e. they are illiquid. Since loans are worthless at date 1, early banks are insolvent, as they are no assets left which can be converted into liquidity. The insolvency of the early banks at date 1 implies for the investors that they will not get any returns at date 2.

Based on the price decline for sovereign assets, the consumption in each state for late consumers increases whereas the probabilities of each state are affected by the government

<sup>19</sup>See equation 4.

bond shock,  $\alpha\bar{\beta}$ ,  $\alpha\overline{(1-\beta)}$ ,  $(1-\alpha)\bar{\beta}$  and  $(1-\alpha)\overline{(1-\beta)}$ . The consumption of the late depositors changes as follows:

$$\begin{aligned}
\overline{c_{2Hh}} &= x \cdot (1 - \bar{p}) + (x + y)h + H \cdot u - e_{2Hh}, \\
\overline{c_{2Hl}} &= x \cdot (1 - \bar{p}) + (x + y)l + H \cdot u - e_{2Hl}, \\
\overline{c_{2Lh}} &= x \cdot (1 - \bar{p}) + (x + y)h + L \cdot u - e_{2Lh}, \\
\overline{c_{2Ll}} &= x \cdot (1 - \bar{p}) + (x + y)l + L \cdot u - e_{2Ll},
\end{aligned} \tag{23}$$

$$\text{with: } \bar{p} = \overline{p^{max}} < 1.$$

Obvious is, that for a price  $\bar{p} = \overline{p^{max}} < 1$  late banks do not have to spend their total amount of their short-term asset in exchange for the government bonds from early banks. The first term in the equations given by (23), represents the remaining amount of the short-term asset in late banks' portfolios. The lower the price for government bonds, the higher the amount of liquidity.

The second term contributes the total amount of government bonds late banks hold after the trade on the interbank market. This amount is constant in any case i.e. even though the banks could purchase more sovereign bonds with their amount of the short asset (excessive sovereign bond demand), early banks are the only supplier on the interbank market and the equilibrium is determined by the government bond supply from the early banks (see figure 3).

The third term in the equations given by (23) represents the state dependent loan return and the fourth term illustrates the repayments to the investors. Given that loan returns and sovereign bond returns are independent, the shock has no impact on loans. The payment to the investors  $e_{2Hh}$  is unchanged whereas the probability of occurrence  $\alpha\bar{\beta}$  decreases, what forces down the expected payment to the investors at date 2.

Although the consumption in each state,  $Hh$ ,  $Hl$ ,  $Lh$  and  $Ll$  increases, the expected consumption also for late consumers declines. This is driven by two factors. First, only 50% of the government bonds are traded on the interbank market at date 1 for a lower price,  $p^{max}$ . The remaining 50% are bought at date 0 and the shock can not be hedged

which forces down the consumption. Second, the probabilities of each state are affected by the shock,  $\alpha\bar{\beta} < \alpha\beta$  what also reduces the expected consumption.

The key insight of this section is that sovereign solvency doubts may lead to a price decline for sovereign assets and as a consequence to liquidity issues in the banking sector.

Figure 5 summarizes the main results.

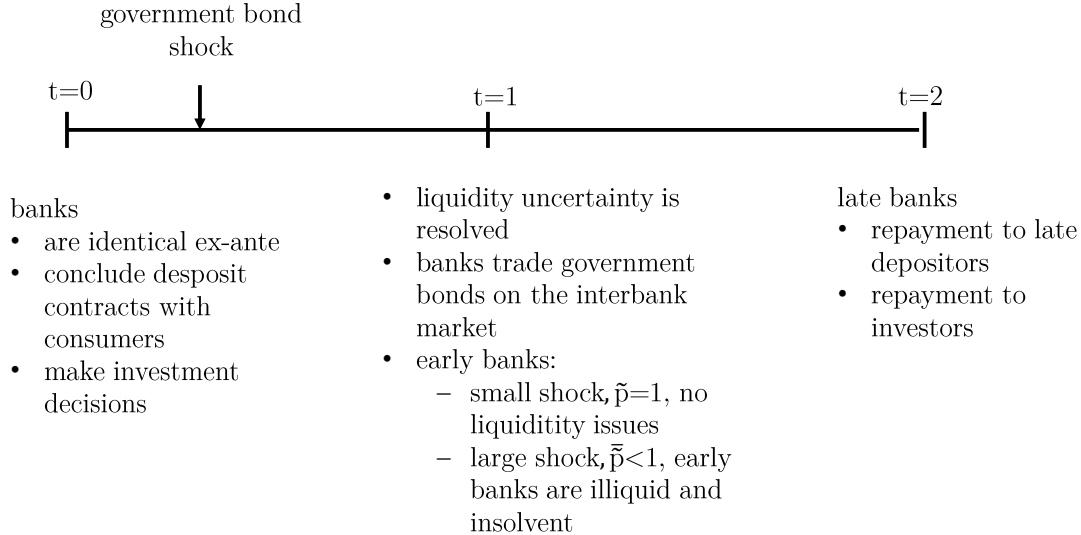


Figure 5: Timeline Government Bond Shock

## 5.2 Central Bank as Lender of Last Resort

As we have seen in the previous analyses, an asset shock in form of an increased sovereign failure probability, may lead to liquidity issues for early banks. Due to the assumption that loans are worthless at date 1, the sovereign bond shock does also lead to bank insolvencies, since the contracts with the early depositors can no longer be fulfilled. To avoid bankruptcies due to liquidity issues we introduce the central bank as lender of last resort (LOLR).<sup>20</sup> In order to act as LOLR, the central bank has the assignment to provide liquidity to troubled banks against sufficient collateral. In our model the central bank

<sup>20</sup>Bagehot (1873): In a liquidity crisis, a central bank should lend freely, at a high rate of interest relative to the pre-crisis period, to any borrower with good collateral. Freixas et al. (2002): discretionary provision of liquidity to a financial institution (or the market as a hole) by the central bank in reaction to an adverse shock which causes abnormal increase in demand for liquidity which cannot be met from an alternative source.

supplies liquidity in exchange for loans. To ensure that the central bank provides as little liquidity as possible but as much as necessary, the following condition must hold

$$\ln\left(\frac{\phi}{u}\right) \leq \alpha \cdot \ln(H) + (1 - \alpha) \cdot \ln(L). \quad (24)$$

The RHS of inequality (24) displays the expected utility of the loan return at date 2. The LHS of inequality (24) reveals the utility from the liquidity supplied by the central bank at date 1 in exchange for one unit loans. As long as the expected loan return at date 2 is equal or higher compared to the utility a bank gains if it sells one unit loan to the central bank at date 1, late banks do not have an incentive to sell their loan portfolios to the central bank. However, early banks sell their loans to the central bank in order to receive liquidity and fulfil the contracts with their depositors. Accordingly inequality (24) ensures that only the troubled banks demand liquidity from the central bank and the sound banks do not have an incentive to sell their loans. By rearranging inequality (24) we derive the maximal amount of liquidity the central bank is willing to supply to the banking sector

$$\phi = H^\alpha L^{(1-\alpha)} u. \quad (25)$$

After analysing the liquidity supply from the central bank, we now analysing the liquidity demand. As we have seen, the government bond shock leads to a lower consumption for early consumers. Thus, the difference between the contractually agreed consumption at date 0 and the lower consumption after the shock determines the aggregate liquidity demand from the early banks to avoid insolvencies

$$\tau = c_1^* - \bar{c}_1 = y^*(\tilde{p} - \bar{p}). \quad (26)$$

More precisely, the liquidity demand is determined by the total amount of government bonds in the banking sector and the price decline for government bonds. Due to the limited amount of liquidity provision by the central bank, the liquidity issues into the banking sector can only be resolved as long as:  $\tau \leq \phi$  i.e., if the price decline of government bonds is that strong that the collateral is not sufficiently high and hence the liquidity provided by

the central bank is not high enough to compensate the government bond losses, troubled banks are insolvent.

Bringing together liquidity demand and supply, we observe the critical price for sovereign bonds:

$$\bar{p}_{crit} \geq \frac{y^* \cdot \tilde{p} - \phi}{y^*} \quad \text{with: } \tilde{p} = 1. \quad (27)$$

This threshold reveals the lowest equilibrium price for government bonds after the shock, which leads to a loss for early banks that can be absorbed by the liquidity provision by the central bank. Moreover, the critical price for government bonds decreases in the amount of liquidity provided by the central bank  $\frac{\partial \bar{p}_{crit}}{\partial \phi}$  i.e., the higher the amount of provided liquidity, the lower can be the equilibrium price for government bonds without early banks insolvencies. Equation (25) reveals that the amount of liquidity is determined by the amount of collateral  $u$ . Consequently, the stronger banks are invested in loans, the higher will be the liquidity supply by the central bank. Moreover, there is a positive relationship between the total amount of government bonds in banks' balance sheets and the critical equilibrium price  $\frac{\partial \bar{p}_{crit}}{\partial y^*}$ . Thus, the more government bonds the banks hold, the higher will be the losses driven by a government bond shock.

Given the critical equilibrium price for government bonds, we can also determine the government bond default probability which leads to early banks' insolvencies (the liquidity demand exceeds the liquidity supply from the central bank)

$$\overline{(1 - \beta)}^{ins} = 1 - \left( \frac{\ln(\bar{p}_{crit}) - \ln(l)}{\ln(l) - \ln(h)} \right). \quad (28)$$

This implies that a low critical equilibrium price for sovereign debt is accompanied with a higher default probability. Consequently the more collateral the banks hold, the stronger can be the price decline for government bonds without any insolvencies. Therefore the default probability can decrease the more, the lower the price for government bonds that can be absorbed by the central bank.

Figure 6 illustrates the critical default probabilities in both regulation regimes.



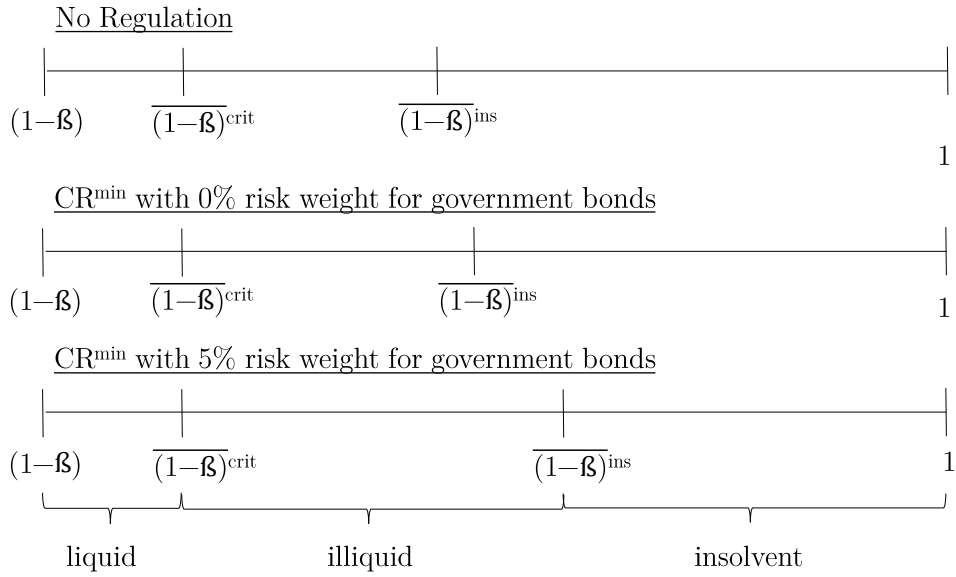


Figure 6: Shock Absorbing Capacity

As mentioned before, if  $(1 - \beta) < \overline{(1 - \beta)} \leq \overline{(1 - \beta)}^{crit}$ , the equilibrium price for government bonds is still 1 and hence a small government bond shock does not lead to liquidity issues in the banking sector. Note, that  $\overline{(1 - \beta)}^{crit}$  is equal in every scenario - with and without regulation -, as the threshold is only driven by government bond returns.<sup>21</sup>

However, if  $\overline{(1 - \beta)} > \overline{(1 - \beta)}^{crit}$ , the maximum price late banks are willing to pay for government bonds decreases and the equilibrium price for government bonds is  $\bar{p} = \overline{p^{max}} < 1$ . In that case early banks are forced to sell their sovereign assets for a lower price than 1 and they can not fulfill the contracts with their depositors. Accordingly early banks are illiquid and the central bank supplies liquidity to these banks to avoid insolvencies due to liquidity issues. The banks' asset structure determine the amount of liquidity supplied by the central bank and the government bonds losses causes by a price decline.

As analysed in section 4, the asset structure differentiate in both regulation approaches. It is shown, that under the capital ratio with positive risk weights for sovereign debt, the highest amount is invested in loans and the lowest amount is invested in sovereign bonds. As a result, the central bank supplies the highest amount of liquidity and the losses caused by the government bonds shock are the lowest. Consequently, under a capital ratio with

<sup>21</sup>see equation (20).

positive risk weights for sovereign assets, the highest sovereign price decline - caused by higher sovereign failure probability  $((\overline{1 - \beta})^{ins})$  - can be adsorbed.

Considering the regulation regime with a zero risk weight for government bonds, it can be seen, that the strong investment in government bonds reinforce the contagion from sovereign to banks. In addition to that effect, the loan investment is lower than in the regulation approach with positive risk weights for sovereign bonds. This implies a lower liquidity provision by the central bank. Consequently a government bond shock which can be absorbed under a capital ratio with positive risk weights for sovereign assets, would lead to insolvencies in a regulation regime with a favourable treatment of sovereign assets.

If the sovereign bond failure probability is higher than  $(\overline{1 - \beta})^{ins}$ , early banks are insolvent. This effect is driven by the fact that the government bond losses exceed the maximum amount of liquidity the central bank provides. Our results suggest that positive risk weights for government bonds incentivize banks to reallocate their portfolio. Due to lower government bond holdings there is a weaker connection from sovereigns to banks and the banking sector is more resilient against sovereign debt crisis.

## 6 Conclusion

In a couple of countries, the banking sector holds a large amount of government bonds. Large sovereign bond exposures imply that the banking sector is strongly affected by an increase in sovereign risk which may have systemic implications. There exists a non-negligible channel of financial contagion from sovereigns to banks. In financial regulation, sovereign borrowers receive a privileged treatment as banks do not have to back sovereign debt with equity capital. Against this background, this paper analyzes in how far the introduction of capital requirements for government contribute to a reduction of systemic risk, to an increase in financial stability.

Using a theoretical model, we analyze the effect of a risk adjusted capital ratio with positive risk weights for sovereign exposure for financial stability. Our model reveals that introducing positive risk weights for government bonds have a positive effect on the banks' ability of absorbing shocks to sovereign bonds. The possible financial contagion effects from sovereigns to banks become weaker, so that in this context capital requirements for

government bonds make a positive contribution to financial stability. There are two aspects being responsible for this positive effect. First, the introduction of capital requirements for sovereign bonds implies that banks decrease their investment in government bonds and increase their investment in high yield assets. This investment shift is necessary as banks need higher returns to satisfy the relatively high return requirements of capital investors. This investment shift implies that the banks' sovereign risk exposure decreases. Second, the higher bank capital and the higher amount of high-yield assets, that are not affected by the shock, increase the banks' shock absorbing capacity if we introduce the central bank as lender of last resort. The central bank buys high yield assets in exchange for liquidity and therefore avoid insolvencies due to liquidity issues. In addition to that, we find that there is the strongest link between sovereign and banks in a regulation regime with a zero risk weight for government bonds.

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