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## Regulating False Disclosure

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# Regulating False Disclosure\*

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## Abstract

Firms can communicate private information about their own product quality through a combination of pricing and direct disclosure where disclosure may be deliberately false. We examine the effect of regulation that penalizes false disclosure by firms in a competitive setting. The cost of false disclosure influences the mix of direct, costly information provision and price signaling in the market, and thereby market outcomes. Stronger regulation reduces the reliance on pricing as the means of credibly conveying quality, reducing prices, market power and the consumption distortion associated with price signaling. Despite this, the gain in social surplus is always outweighed by the cost of direct disclosure unless disclosure itself is relatively inexpensive and the penalty for false disclosure is sufficiently high. Weak regulation of false disclosure is always worse than no regulation. Even high quality firms may suffer from regulation of false disclosure (and may lobby against it).

**JEL Classification:** L13, L15, D82, D43.

**Key-words:** Price Signaling; Disclosure; Product Quality; Oligopoly.

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# 1 Introduction

On the basis of information firms provide, consumers infer which product best to buy. That inference is not always easy as firms may lie about the direct information they provide about their product quality, while at the same time price may also act as a signal of quality. Firms, from their side, strategically provide information and set prices so as to make consumers buy their product. This paper analyzes the effect of regulating the provision of false content (by increasing the cost for firms of providing false information) in competitive markets where price and direct information provision can be used to signal true quality.

Around the world, government agencies have introduced fines for false disclosure. For instance, in the US, the FTC Policy Statement Regarding Advertising Substantiation states that “a firm’s failure to possess and rely upon a reasonable basis for objective claims constitutes an unfair or deceptive act or practice in violation of Section 5 of the FTC Act”.<sup>1</sup> In Canada, section 74.01(1)(a) of the Competition Act prohibits any “representation to the public that is false or misleading in a material respect”. A court may order a violating firm to pay significant damages. In Europe in November 2016, the European Commission has updated the 2006 Directive on Misleading and Comparative Advertising, making it unlawful to engage in false statements concerning product quality.<sup>2</sup> The rationale behind these laws seems to be that consumers make better, more informed choices, if firms are penalized for making false statements yielding more efficient markets. This paper asks the question if, and when, fines are optimal, taking account of the fact that firms have alternative ways to signal their quality to consumers and that consumers may also rely on these alternative signals, such as price.

This paper provides two narratives addressing this question. First, an intermediate cost of false disclosure is never welfare optimal. To understand this first point we show that in the absence of regulation, direct information provision is not credible and that firms will not use direct disclosure, relying on price signaling instead. Price signaling comes at the welfare loss of a consumption distortion as consumers rationally (and knowingly) buy low quality if both low and high quality firms are around, even if high quality generates more social surplus. By increasing the cost of providing false information from initially low levels, a regulator influences the mix of direct, costly information provision and price

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<sup>1</sup>See, e.g., the FTC Policy Statement on Deception (1982) at: <http://www.ftc.gov/bcp/policystmt/ad-decept.htm>

<sup>2</sup>See, e.g., the EU policy statement on [http://ec.europa.eu/consumers/consumer\\_rights/unfair-trade/false-advertising/index\\_en.htm](http://ec.europa.eu/consumers/consumer_rights/unfair-trade/false-advertising/index_en.htm)

signaling that is used in the market, but, importantly, it does not immediately reduce the consumption distortion. In terms of welfare losses, this implies that at intermediate levels of the cost of false disclosure, the direct disclosure costs are simply added to the consumption distortion.

A second narrative is that a high quality firm will only want to disclose directly if these disclosure costs are low enough. If high quality firms disclose at higher disclosure cost they can only acquire enough rent to cover these costs if they take over the market in case their competitor is low quality. However, this would induce low quality firms to compete severely on price, creating such a low margins for the high quality firms that they are better off randomizing their disclosure decision, leaving the market to the low quality firm in case they do not disclose directly. Thus, the consumption distortion is never fully eliminated.

Combining these two narratives, it becomes clear that fining false disclosure can be welfare increasing, but only if the fines are high enough and the disclosure cost firms incur to directly inform consumers of their quality is small enough. No regulation of false disclosure is better than intermediate fines and is optimal if the direct disclosure cost is not too small. Even high quality firms may suffer from regulation of false disclosure (and may lobby against it), and if they do, such lobbying could be pro-social.

The market environment we study is a symmetric incomplete information Bertrand duopoly where products differ in quality. A firm knows its true product quality, but it is not observed by the rival firm or the buyers. High quality is more costly to produce than low quality, but (importantly) generates more surplus. In a world where both low and high quality would be around, it is welfare optimal that consumers buy high quality and in the absence of incomplete information high quality firms would exercise their natural competitive advantage and supply the whole market. Firms can use both pricing and direct communication or disclosure to convey private information about their own product quality. Disclosure is costly and not necessarily credible as the information provided is not verifiable; a low quality firm may lie and misrepresent its product quality. False disclosure comes at an additional cost that we consider a policy parameter that is set by a "regulator". Firms simultaneously make their disclosure and pricing decisions; buyers use these to update their beliefs about each firm's true product quality, and then make their purchase decisions. To prevent us focusing on equilibria that rely on questionable out-of-equilibrium beliefs, we impose the D1 refinement, known to severely restrict the out-of-equilibrium beliefs that can be used to sustain equilibrium.<sup>3</sup>

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<sup>3</sup>It is well-known that under convenient choices of out-of-equilibrium beliefs, multiple outcomes can be

A D1 equilibrium in our environment always exists and is fully revealing. In equilibrium, consumers make choices as if they were *fully informed*: they either correctly interpret price as a signal of quality, and/or they are directly informed by high quality firms that engage in costly disclosure. If they buy low quality, they do so because of the substantially lower price. Nevertheless, the cost of false disclosure has a profound impact on the nature of equilibrium even though it does *not* lead to *more informed* consumer choices. To understand the impact, it is important to see that in the absence of regulation, firms use price to signal quality, leading to a consumption distortion (as too many consumers buy low quality where high quality generates more surplus) and high prices (needed to prevent low quality firms to imitate high quality prices), leading to low levels of consumer surplus.

Regulation affects the structure of equilibria through the kind of beliefs that can be supported to sustain equilibrium outcomes. When the cost of false disclosure increases from initially low levels, it becomes more costly for low quality firms to (falsely) disclose their products are of high quality. This gives high quality firms an incentive to incur the disclosure cost to directly reveal they sell high quality. If the cost of false disclosure remains relatively low, the main impact is to intensify price competition, with high quality firms starting to advertise (with positive probability), without resolving the consumption distortion: consumers continue to buy low quality as low quality sellers continue to price significantly below their high quality competitors. This is good for consumers, but total welfare decreases as the wasteful advertisement expenditure does not have a positive side effect.

If the cost of false disclosure increases further, high quality firms are incentivized to disclose more often, reducing low quality firms' profit to zero, thereby gradually reducing the consumption distortion of the pure price signaling outcome. Regulation influences the ability of the high quality firm to exercise its natural competitive advantage in garnering market share. There remains a welfare loss, however. The nature of the loss changes, however, from the consumption distortion when regulation is low to wasteful advertisement expenditure when regulation is high. Increasing the cost of false disclosure is welfare improving only if disclosure itself is not too costly and the cost of false disclosure is substantial. Weak regulation of false disclosure is worse than no regulation at all. If the cost

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sustained in asymmetric information games, and our game is no exception. By imposing the D1 criterion, we require out-of-equilibrium beliefs to satisfy a severe condition, yielding unique equilibrium predictions for many parameter values. In this way we make sure we do not focus on outcomes that can only be sustained as equilibria for dubious assumptions regarding these beliefs. The equilibria we focus on remain equilibria under weaker conditions on the out-of-equilibrium beliefs.

of direct disclosure is relatively large, increasing the cost of false disclosure reduces welfare as the consumption distortion is never fully eliminated and the disclosure cost outweighs the efficiency gains.

Even though we mainly use the terms "disclosure cost" and "the cost of false disclosure", there are many other interpretations of the model. Disclosure may be alternatively interpreted as informative advertising, or as asking third-party certification agencies to certify the quality of the product. The fact that low quality firms may engage in false advertising or get a high quality certificate may reflect enforcement problems (or other weaknesses) in the regulation of advertisement content or the imperfect ability of third party certifiers or label providers to verify true product quality (or bribery of these institutions). In this vein, increasing the cost of false disclosure can then be interpreted as bringing out the impact of strengthening the regulation concerning advertisement content, increasing liability, or the impact of better regulation of third party certifiers and rating agencies that make it more difficult for them to certify or rate falsely.

Our paper is related to several strands of literature. From a technical point of view, our model is based on the competitive price signaling model of Janssen and Roy (2010) where we add the possibility of direct disclosure and an extra cost of false disclosure. Janssen and Roy (2015) also considered the possibility for firms to directly disclose their private information, but in that paper firms could not lie about the quality of their product nullifying the impact of a policy fining false disclosures.

From a content point of view, papers that are close to ours, include Piccolo et al. (2015, 2016), Rhodes and Wilson (2015) and Drugov and Troya Martinez (2014). These papers' main aim is to explain why false advertisement may occur in equilibrium. Rhodes and Wilson (2015) have a monopoly model where the production cost is independent of quality and consumers out-of-equilibrium beliefs only depend on the disclosure statements, but not on price. They show that there exists an (undefeated) equilibrium where low and high quality firms choose the same actions and consumers may be deceived.<sup>4</sup> Piccolo et al. (2015, 2016) study a duopoly model where it is known that one quality produces high quality, whereas the other produces low quality. What consumers do not know is which firm produces high quality. Moreover, they do not impose severe restrictions on out-of-equilibrium beliefs so that they can sustain pooling equilibria by punishing deviations with a convenient choice of these beliefs. Drugov and Troya Martinez (2014) present a persuasion

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<sup>4</sup>The notion of undefeated equilibrium, pioneered by Mailath and Sobel (1993) cannot be fruitfully applied to our context with two senders.

game where a monopolist seller sells at a fixed price, but can choose the precision and the bias of his signal. As explained above, our aim is not to explain that firms may engage in false advertising.<sup>5</sup> Rather, we show that even if firms do not engage in false disclosure, a policy of changing the cost of false disclosure may have a real impact on market outcomes (for better or for worse).

The paper is also related to the literature on communication with lying cost (see, e.g. Kartik, Ottaviani and Squintani (2007) and Kartik (2009)). That literature shows that "inflated language" and incomplete separation of types are natural outcomes. In contrast, in our setting where firms have an alternative means of signaling (through prices), separation always occurs and low quality firms will never want to disclose false information.

Our paper is also of interest in the light of the fact that it effectively is a signaling game with multiple signals. By allowing false disclosure, but at an additional cost, there effectively is a cost difference between the two types of firms in using direct disclosure as a signal. Thus, our model is a signalling model where firms may use multiple signals (price and direct disclosure) and the question is when do they want to use which signal and when do they want to combine different signals, and how? Direct disclosure can be a signal of quality if the cost difference (that is the cost of false disclosure) is high enough. Pure price signaling occurs when the cost of false disclosure is relatively small (or the disclosure cost is high). In between, both signals are used. The literature on signaling games with multiple signals (see, e.g. Ramey (1996) for an early example) mainly studies the case of a continuum of types, whereas here we deal with finite types and one of the signals is binary.

Another related literature is that where firms advertise content. A key paper here is Anderson and Renault (2006). Anderson and Renault consider a monopoly model where a firm sells a (search) good for which consumers have different valuations (match values). A firm may advertise price and/or content. Without advertising the consumers have to incur a search cost to learn both price and match value. Our context is different in that we have a competitive model with vertical product differentiation where consumers are informed about prices without having to incur a search cost. In addition, we study experience, rather than search, goods where consumers observe the true quality of the good only after they consume it.

There is, of course, a very large literature on signaling quality through price and/or

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<sup>5</sup>As explained in the next Section, we could explain the existence of false advertising in equilibrium if we allow low and high quality to be produced at the same cost, or by imposing less strict conditions on the out-of-equilibrium beliefs.

advertising (see, e.g., Milgrom and Roberts (1986) and Bagwell and Riordan (1991)) or on quality disclosure (see, e.g., Viscusi (1978), Grossman (1981), Milgrom (1981) and Jovanovic (1982)). Unlike much of the disclosure literature, in our paper disclosure is not verifiable and not fully credible; a low quality firm may lie and misrepresent its product quality. In contrast to the advertising signaling literature, it is not just the amount of money spend on advertising, but it is also the content that is important as it is more costly for a low, than for a high, quality firm to inform consumers its quality is high.

The rest of the paper is organized as follows. Section 2 presents the details of the model and the D1 equilibrium notion. It also presents preliminary results, namely that the equilibrium is always fully revealing indicating that regulating false disclosure does not help rational consumers in making more informed decisions, and some of the properties that any fully revealing equilibrium must satisfy. Fully revealing equilibria come in three different types: (i) price signalling equilibrium, (ii) an equilibrium where high quality discloses for sure, and (iii) a mixed equilibrium where high quality randomizes its disclosure decision. Each of the next three sections discusses these different types of equilibria in more detail. Section 3 discusses the nature of a pure price signalling equilibrium and when it exists. Section 4 characterizes all equilibria where high quality directly discloses its quality and the parameter regions where these equilibria exist. Section 5 analyzes the mixed equilibria. Section 6 concludes with a summary and a discussion.

## 2 Model

There are two firms,  $i = 1, 2$ , in the market. Each firm's product may be of either high ( $H$ ) or low ( $L$ ) quality. As in Janssen and Roy (2010), the true product quality is known only to the firm that supplies the product; it is not known to the rival firm or to consumers. It is common knowledge that the *ex ante* probability that a firm's product is of high quality is  $\alpha \in (0, 1)$ . The products of the firms are not differentiated in any dimension other than quality. Firms supply their output at constant unit cost that is equal to  $c_H$  or  $c_L$ , with  $c_H > c_L$ .<sup>6</sup> The unit cost subsumes both current production cost (including the cost of compliance with any form of prevalent regulation) as well as the expected future costs related to selling the product (such as future liability for damage caused).

There is a unit mass of identical consumers in the market; consumers have unit demand

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<sup>6</sup>If  $c_H = c_L$ , then false disclosure can be an outcome in a D1 equilibrium.



and each consumer's valuation of a product of quality  $s$  is given by  $V_s$ ,  $s = H, L$ , where

$$V_H > V_L, V_s > c_s, s = L, H.$$

We focus on the more interesting case where the quality premium  $V_H - V_L$  that buyers are willing to pay for the high quality product exceeds the cost difference:

$$\Delta V = V_H - V_L > \Delta c = c_H - c_L. \quad (1)$$

As the high quality product creates more social surplus than the low quality product, we will say that there is a consumption distortion if consumers buy low quality even if high quality is around. It follows there can only be a consumption distortion in the state of the world where one firm produces low quality and the other high quality, which arises with probability  $2\alpha(1 - \alpha)$ .<sup>7</sup> To ensure full market coverage (i.e., all buyers buy), we assume  $\Delta V \leq \frac{V_H - c_L}{2}$ . We also define the parameter  $\lambda > 1$  by  $\lambda = \frac{\Delta V}{\Delta c}$ , capturing the competitive advantage of a high quality seller over a low quality rival.

Firms have the option of directly communicating their product quality or type to all buyers by sending a message about their type; the (fixed) cost of sending this message is denoted by  $D > 0$ . This communication may take many different forms: advertising, self labeling of the product, third party certification or labeling, rating by an external agency to which the firm supplies data on product attributes and indeed, any other form of voluntary disclosure. As no firm would like to incur the communication cost to say that its product is low quality, a firm either sends no message or sends a message claiming that its product is of high quality. In the rest of the paper we will use direct disclosure and advertising as synonymous. A firm may randomize its disclosure decision and we denote by  $\gamma_H$  the probability that the high quality type discloses (and claims to be of high quality). Such disclosure is *not* necessarily credible or verifiable. In particular, it is possible for a low quality firm to lie or disclose incorrectly and claim that its product is of high quality. However, false disclosure creates an additional fixed cost  $f > 0$ .<sup>8</sup> If  $f$  is large enough, a firm would never lie and so communication would be equivalent to credible and verifiable disclosure.

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<sup>7</sup>Under complete information, the high quality producer would have a competitive advantage over a low quality rival, reducing the latter's market share to zero if the firms engage in price competition.

<sup>8</sup>Strict liability could make  $f$  very close to  $V_H - V_L$ , but never exceeding it. Thus, the paper can be interpreted as investigating the effects of increasing liability.

Formally, the game proceeds in three stages. First, nature independently draws the type (or quality)  $\tau_i$  of each firm  $i$  from a distribution that assigns probabilities  $\alpha$  and  $1 - \alpha$  to  $H$  and  $L$  respectively; the realization of  $\tau_i$  is observed only by firm  $i$ . Next, both firms (having observed their own types), simultaneously decide on their price  $p$  and whether or not to disclose (send message  $m \in \{0, 1\}$  to all buyers, where 0 means no message and 1 means the message "I produce high quality"). After observing all advertised messages and prices, consumers decide whether to buy and if so, from which firm. The payoff of each firm is its expected profit net of any disclosure cost and, where relevant, the cost of false disclosure. The payoff of each consumer is her expected net surplus.

The solution concept used is that of symmetric perfect Bayesian equilibrium (PBE) where the out-of-equilibrium beliefs satisfy a version of the D1 criterion (Cho and Kreps, 1987). In what follows, we simply refer to this as "equilibrium". The D1 criterion has been developed for signaling games with one sender. Retaining tractability, we adapt the D1 criterion to our model as follows.

Consider firm  $i$  unilaterally deviating to a strategy  $(p, m)$  outside the support of its equilibrium strategy. Given the (possibly mixed) equilibrium strategies of other firms, each profile of prior beliefs that buyers may possibly have about the type of firm  $i$  (following this deviation) and each profile of best responses of buyers (based on every such belief profile) defines a certain expected quantity sold by firm  $i$  at price  $p$  and message  $m$ . Let  $B_i(p, m)$  be the set of all possible expected quantities sold (by firm  $i$  at price  $p$  and message  $m$  that can be generated in this manner by considering all possible beliefs and best responses of buyers. Each  $q_i \in B_i(p, m) \subset [0, 1]$  is a quantity that firm  $i$  can "expect" to sell at price  $p$  and message  $m$  for some profile of beliefs of buyers about firm  $i$ 's type and for some configuration of optimal choices of buyers (that depends on realizations of prices charged by other firms) when other firms play according to their equilibrium strategy.

In the spirit of the D1 criterion, we compare the subsets of expected quantities in  $B_i(p, m)$  for which it is gainful for different types of firm  $i$  to deviate to price  $p$  and message  $m$ . More precisely, consider any perfect Bayesian equilibrium where the equilibrium profit of firm  $i$  when it is of type  $\tau$  is given by  $\pi_\tau^{i*}$ ,  $\tau = H, L$ . Consider any  $p \in [0, V_H]$  and  $m \in \{0, 1\}$  outside the support of the equilibrium strategy of firm  $i$  and denote by  $\pi_\tau^i(p, m; q_i)$  the profit firm  $i$  of type  $\tau$  makes when he sells quantity  $q_i$ . For example,  $\pi_L^i(p, 1; q_i) = (p - c_L)q_i - D - f$ . If for  $\tau, \tau' \in \{H, L\}$ ,  $\tau' \neq \tau$ ,

$$\{q_i \in B_i(p) : \pi_\tau^i(p, m; q_i) \geq \pi_\tau^{i*}\} \subset \{q_i \in B_i(p) : \pi_\tau^i(p, m; q_i) > \pi_{\tau'}^{i*}\}$$

where " $\subset$ " stands for strict inclusion, then the D1 refinement suggests that the out-of-equilibrium beliefs of buyers (upon observing a unilateral deviation by firm  $i$  to price  $p$ ) should assign zero probability to the event that firm  $i$  is of type  $\tau$  and thus (as there are only two types), assign probability one to firm  $i$  being of type  $\tau'$ .

Having defined the equilibrium notion, we now show that the equilibrium has to be fully revealing, and we further characterize some properties of any fully revealing equilibrium. That equilibria must be fully revealing is important as it implies in equilibrium consumers are not misled by the content of advertising. Even if the cost of false advertising  $f$  is arbitrarily small the low quality seller will never pretend, through advertising, to be of high quality.

To see the main argument, suppose there exists a partially pooling equilibrium where with at least some positive probability low and high quality firms pool on advertising a price  $p$  with a certain strictly positive probability. In such an equilibrium the firms make a profit equal to

$$\Pr(p)(p - c_L) - D - f,$$

and

$$\Pr(p)(p - c_H) - D,$$

respectively, where  $\Pr(p)$  is the probability a seller sells when setting this price. If a consumer observes an advertised price of  $p + \varepsilon$  one can show that it has to be an out-of-equilibrium price (see the proof of Appendix 1 for details). If a consumer buys at that price with probability  $\Pr(p + \varepsilon)$ , then the low and high quality firms have an incentive to deviate if

$$\Pr(p + \varepsilon)(p + \varepsilon - c_L) - D - f > \Pr(p)(p - c_L) - D - f,$$

for the low quality firm, and

$$\Pr(p + \varepsilon)(p + \varepsilon - c_H) - D > \Pr(p)(p - c_H) - D$$

for the high quality firm. Thus, the critical probability  $\Pr_\tau(p + \varepsilon)$  for the deviation to be profitable for type  $\tau$  is given by

$$\Pr_\tau(p + \varepsilon) = \frac{\Pr(p)(p - c_\tau)}{p + \varepsilon - c_\tau}, \quad i = L, H,$$

and for any  $\Pr(p + \varepsilon) > \Pr_\tau(p + \varepsilon)$  the deviation is profitable for type  $\tau$ . As  $c_H > c_L$

it follows that  $\Pr_H(p + \varepsilon) < \Pr_L(p + \varepsilon)$  so that high quality has an incentive to deviate for a wider range of consumer responses.<sup>9</sup> The D1 refinement then requires consumers to believe that it is the high quality firm that has deviated if they observe a price  $p + \varepsilon$  being advertised.<sup>10</sup> If consumers would buy with some positive probability after observing price  $p$  being advertised and believing the quality is some weighted average of low and high quality, they will certainly buy with a strictly higher probability at price  $p + \varepsilon$  believing this price is set by a high quality firm. Thus, firms want to deviate and shift the probability mass from advertising a price  $p$  to advertising a price  $p + \varepsilon$ , contradicting that both firms advertising a price  $p$  with strictly positive probability can be part of an equilibrium.

The next proposition summarizes:

**Proposition 1** *There does not exist a pooling or semi-pooling D1 equilibrium.*

We next consider the properties that any fully revealing equilibrium has to satisfy. There are three types of fully revealing equilibria: (i) a pure price signalling equilibrium, (ii) a disclosure equilibrium, or (iii) a mixed signalling equilibrium where high quality randomizes between direct disclosure and price signaling. Janssen and Roy (2010) show that in a pure price signaling equilibrium, low quality firms randomize their prices over an interval  $[\underline{p}_L, \bar{p}_L]$ , while the high quality firm sets a deterministic price  $p_H^{ND}$  with  $\bar{p}_L = p_H^{ND} - \Delta V$  and  $\underline{p}_L = \alpha \bar{p}_L + (1 - \alpha)c_L$ . The next Proposition shows that some of these properties can be generalized to hold for the other two types of equilibria in our model. We use superscripts  $D$  and  $ND$  to indicate whether prices are advertised or not.

**Proposition 2** *The following properties hold for any fully revealing (symmetric perfect Bayesian D1) equilibrium:*

- (a) *Low quality types do not advertise and in particular, no cost of false advertising is incurred in equilibrium;*
- (b) *If  $\gamma_H \in [0, 1)$ , then a high quality firm does not randomize over prices when it does not advertise and sells only if the rival firm is a high quality type; Moreover, it advertises lower prices than the non-advertised prices, i.e.,  $\bar{p}_H^D \leq p_H^{ND}$ , while low quality firms randomize over a set of prices whose upper bound  $\bar{p}_L$  satisfies  $c_L < \bar{p}_L \leq p_H^{ND} - \Delta V$  without a mass point at  $\bar{p}_L$ ;*

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<sup>9</sup>Note that a pooling equilibrium where consumers are deceived by false advertising exists if  $\Delta c = 0$ . This clarifies the importance of the common cost assumption in Rhodes and Wilson (2015) and Piccolo et al. (2015a,b) to argue that false advertising may occur in equilibrium.

<sup>10</sup>Note that this argument nicely illustrates the idea that (even out-of-equilibrium) higher advertised prices signal higher quality.

(c) *If low quality types make positive profits, then they must randomize over a continuous support without mass points.*

The Proposition makes clear that as long as the low quality type makes positive profits, important parts of the pure price signaling equilibrium remain valid. If a high quality firm does not advertise, it only makes positive sales if the competitor is also a non-advertising high quality firm: the Proposition argues that a low quality competitor undercuts by a large enough margin to have consumers buy from the competitor, while a high quality competitor sets lower prices. The Proposition also makes clear that as long as there is a positive probability that a high quality firm does not advertise, a low quality firm makes positive profits and must randomize. This also implies there is a consumption distortion as low quality firms must make positive sales when they set a price equal to the upper bound of the price distribution and in that case they only sell if the rival is a high quality firm. High quality firms randomizing between advertising and not advertising implies that they must make higher sales when advertising as otherwise they cannot be indifferent between the two actions. Finally, the Proposition also points towards the fact that low quality firms will only be pushed towards pricing at marginal cost if high quality firms always disclose.

### 3 Pure Price Signaling

To understand the role of disclosure and what policy can achieve by making false disclosure more costly, it is important to understand how price signaling works and under what conditions firms abstain from direct disclosure. It is clear that a price signaling equilibrium has to be identical to the equilibrium analyzed in Janssen and Roy (2010), hereafter JR(2010). JR(2010) show that if  $\Delta V > \Delta c$  and  $\Delta V \leq \frac{V_H - c_L}{2}$ , the unique symmetric D1 equilibrium outcome is one where high quality firms charge a deterministic high price  $p_H^{ND}$  and low quality firms randomize their prices over an interval  $[\underline{p}_L, \bar{p}_L]$  according to a distribution function  $F_L$  where  $p_H^{ND} = c_L + 2\Delta V$ ,  $\bar{p}_L = p_H^{ND} - \Delta V = c_L + \Delta V$  and for all  $p \in [\underline{p}_L, \bar{p}_L]$

$$F_L(p) = 1 - \frac{\alpha}{1 - \alpha} \left[ \frac{\Delta V}{p - c_L} - 1 \right] = \frac{1}{1 - \alpha} - \frac{\alpha \Delta V}{(1 - \alpha)(p - c_L)}.$$

Further, all consumers buy. If one firm charges  $p_H^{ND}$  and the other firm charges a price in  $[\underline{p}_L, \bar{p}_L]$ , buyers always buy from the latter; when both firms charge prices in  $[\underline{p}_L, \bar{p}_L]$ , buyers buy from the lower priced firm. Thus, in the price signaling equilibrium there is a

consumption distortion in that when both low and high quality firms exist in the market, consumers buy low quality, while the surplus generated by high quality is larger. Also, the market equilibrium exhibits a fair amount of market power with  $\bar{p}_L - c_L = \Delta V$ , which is necessary to keep low quality firm from imitating the high quality price.

JR(2010) show that the high quality firm does not have an incentive to undercut as the D1 logic implies that any firm charging price  $p \in (\bar{p}_L, p_H)$ , is believed to have low quality with probability one so that no consumer would buy at these prices as they would buy from the competitor instead. For this equilibrium, but also for other equilibria discussed in the next Sections, it is important to understand the role of the D1 refinement. JR(2010) show that there are multiple price signaling equilibria in their model that are identical up to the choice of  $p_H^{ND}$  and that any  $p_H^{ND} \geq c_L + 2\Delta V$  results in an equilibrium. the D1 equilibrium selects the most competitive of these equilibria and in it the low quality firm is just indifferent between choosing a  $p \in [\underline{p}_L, \bar{p}_L]$  and imitating high quality and setting  $p_H^{ND}$ . As prices are relatively low, low quality has relatively more incentive to deviate compared to high quality given their lower cost of production.

As there is a consumption distortion, but no disclosure costs are incurred, the welfare loss, compared to the first-best, equals

$$WL = 2\alpha(1 - \alpha)(\Delta V - \Delta c). \quad (2)$$

Consumer surplus can be calculated to be equal to<sup>11</sup>

$$(V_L - c_L) - \alpha(2 - \alpha)\Delta V,$$

while firm profits equal  $\pi_H^* = \alpha(\Delta V - \Delta c/2)$  and  $\pi_L^* = \alpha\Delta V$ . It is clear that both total surplus, consumer surplus and firms' profits are independent of  $f$  and  $D$  whenever the pure price signaling equilibrium exists.

It remains to be seen for which parameter values  $D$  and  $f$  the pure price signaling equilibrium exists. The main reason why such an equilibrium may not exist for some

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<sup>11</sup> Consumer surplus is equal to

$$\begin{aligned} & (1 - \alpha)^2(V_L - E \min p_L) + 2\alpha(1 - \alpha)(V_L - E p_L) + \alpha^2(V_H - p_H) \\ &= (V_L - c_L) - 2\alpha(1 - \alpha)\Delta V \left(1 - \frac{\alpha\Delta V}{1 - \alpha} \ln \frac{1}{\alpha}\right) - 2\alpha^2\Delta V \ln \frac{1}{\alpha} - \alpha^2\Delta V, \end{aligned}$$

which can be rewritten as (2).

parameter values is that the high quality firm may have an incentive to deviate by directly disclosing its quality if he is believed to be a high quality firm by consumers. Both the disclosure cost  $D$  and the cost of false disclosure  $f$  play an important role in this respect:  $D$  should not be too high for the deviation to be profitable, whereas  $f$  should not be too small for consumers to infer that a low quality firm would have less incentives to be responsible for the deviation.

To determine more precisely when a pure price signaling equilibrium exists, define two critical levels of the disclosure cost  $D_1$  and  $D_2$  as follows:

$$D_1 = \alpha(\Delta V - \frac{\Delta c}{2}) \text{ and } D_2 = \Delta V - (1 - \frac{\alpha}{2})\Delta c$$

It is easy to check that  $0 < D_1 < D_2$ . For each  $D \in (0, D_2)$  define  $f^*(D)$ , a critical level of the cost of false disclosure, by

$$\begin{aligned} f^*(D) &= \frac{D - \frac{\alpha}{2}\Delta V}{\lambda - 1}, \text{ for } D_1 \leq D < D_2 \\ &= \frac{D}{2\lambda - 1}, \text{ for } 0 \leq D \leq D_1 \end{aligned}$$

Observe that  $f^*(D)$  is continuous and strictly increasing in  $D$  on  $(0, D_2)$ ,  $f^*(D) \rightarrow 0$  as  $D \rightarrow 0$ , and  $f^*(D_2) = (1 - \frac{\alpha}{2})\Delta c$ . For later reference, we will denote the inverse of  $f^*(D)$  by  $D^*(f)$ .

We can then state the main proposition regarding pure price signaling equilibria:

**Proposition 3** *A symmetric equilibrium with pure price signaling i.e., one where firms do not disclose directly, exists if, and only if, either (i)  $D \geq D_2$ , or (ii)  $D < D_2$  and  $f \leq f^*(D)$ .*

Figure 1 depicts the region where a pure price signaling equilibrium exist. For  $D < D_2$ , the bold line represents the function  $f^*(D)$ . A symmetric pure price signaling equilibrium exists only to the left of and above the bold line.

INSERT FIGURE 1 HERE

The parameter region for a pure price signaling equilibrium to exist is readily understood. If the advertising cost  $D$  is too large (larger than  $D_2$ ), then it is clear that it never pays to disclose and we can only have pure price signaling. The specific value of  $D_2$  derives from the fact that if to incur the fixed disclosure cost it is best to do so by

taking over the whole market. Even if consumers believe it is the high quality firm that has deviated (and he can only do so by incurring the disclosure cost), the high quality firm has to set a price as low as  $\underline{p}_L + \Delta V$  to take over the market, resulting in a profit of  $\underline{p}_L + \Delta V - c_H - D = (1 + \alpha)\Delta V - \Delta c - D$ . This is not higher than the high quality equilibrium profit of  $\alpha\Delta V - \frac{\alpha}{2}\Delta c$  if  $D \geq D_2$ .

If the disclosure cost is smaller, then it may be optimal for a high quality firm to disclose its private information directly (and undercutting the high quality price), if this double signal is correctly interpreted (by consumers as coming from a high quality firm. In terms of the D1 logic, this implies that a low quality firm does *not* have a stronger incentive to send such a message than a high quality firm. Intuitively, this depends on the cost of false disclosure  $f$ : if this cost is large, then low quality firms will not have an incentive to send such a signal and high quality firms will then find it optimal to deviate; if this cost is small, then consumers will believe that low quality firms are trying to mislead them, will therefore not buy from a firm sending this double signal, making it unprofitable for high quality firms to deviate.

The critical value  $f^*(D)$  is determined by two equations, depending on the parameter values. The condition  $D \leq (2\lambda - 1)f$  yields that a deviation to a price in the interval  $(\bar{p}_L, p_H^{ND})$  will always be interpreted as coming from a low quality firm, making such a deviation unprofitable. If  $D > (2\lambda - 1)f$  some deviations will be interpreted as coming from high quality firms, but not all of these deviations are profitable. This requires a second condition when  $D_1 \leq D < D_2$ .

## 4 Pure Disclosure Signaling

We start the analysis of this Section by analyzing our first narrative in more detail, namely that intermediate levels of regulation increase the welfare loss relative to the pure price signaling equilibrium as the consumption distortion is not eliminated, while high quality firms engage in wasteful disclosure expenditures. We focus here on the direct disclosure cost  $D$  being small so that for intermediate levels  $f$  of regulation high quality firms disclose in equilibrium, while low quality firms make positive profits, capturing the market even if high quality firms are present. Later we also show that no other equilibria exist for these parameter values.

Given our characterization result in Proposition 2 low quality firms randomize over an interval  $[\underline{p}_L, \bar{p}_L]$  if they make positive profits. We will focus on an equilibrium where



a high quality firm chooses a deterministic price  $p_H^D$  and that  $\bar{p}_L^D = p_H^D - \Delta V$ .<sup>12</sup> Thus, the equilibrium structure is very similar to that of the pure price signaling equilibrium, with low quality firms selling to all consumers if the competitor produces high quality. In particular, the equilibrium profits are given by

$$\pi_H^* = \frac{\alpha}{2}(p_H^D - c_H) - D = \pi_H^D - D$$

and

$$\pi_L^* = \alpha(p_H^D - \Delta V - c_L).$$

The main remaining issue is how to determine  $p_H^D$ . An important consideration is that low quality firms should not have an incentive to imitate the high quality behavior. This implies a lower limit on  $p_H^D$ :

$$\frac{\alpha}{2}(p_H^D - c_L - 2\Delta V) + D + f \geq 0.$$

To determine  $p_H^D$  we have to consider the main other deviations, namely for any of the firms to set an out-of-equilibrium price  $\hat{p} \in (\bar{p}_L^D, p_H^D)$ . This can be accompanied both by direct disclosure and by not disclosing. It is clear that if consumers believe that a high quality firm has deviated, then they will buy and this makes such a deviation profitable. Using the D1 logic explained in the previous Section, the requirement that consumers think that a deviating firm that discloses and chooses  $\hat{p} \in (\bar{p}_L^D, p_H^D)$  is of low quality implies that a low quality firm should be indifferent between its equilibrium strategy and mimicking the high quality behavior. This results in the requirement that

$$\tilde{p}_H^D = c_L + 2\Delta V - \frac{2(D + f)}{\alpha}. \quad (3)$$

As low quality firms make nonnegative profits if, and only if,  $\bar{p}_L > c_L$  it is clear that given this price  $\tilde{p}_H^D$  this is guaranteed if, and only if,

$$D + f \leq \frac{\alpha}{2}\Delta V. \quad (4)$$

In the proof of the next Proposition we show that the requirement that consumers think that a deviating firm that does *not* disclose and chooses  $\hat{p} \in (\bar{p}_L^D, p_H^D)$  is of low quality is

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<sup>12</sup>In the proof of Proposition 4 it becomes clear that there cannot be other equilibria where the high quality always discloses and the low quality firm makes positive profit.

equivalent to requiring

$$D \leq \left( \frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1 \right) f, \quad (5)$$

while both restrictions together imply that the high quality firms profits are nonnegative as well.

**Proposition 4** *There exists a symmetric equilibrium where high quality firms disclose for sure and charge a deterministic price given by (3), while low quality firms do not disclose and randomize prices over the interval  $[\underline{p}_L, \bar{p}_L]$ , with  $\bar{p}_L = p_H - \Delta V > c_L$ , if, and only if, (4) and (5).*

The main differences between this pure disclosure equilibrium and the pure price signaling equilibrium is that the high quality firm sets a price that is  $\frac{2(D+f)}{\alpha}$  lower than in the pure price signaling equilibrium, while it has to disclose as the disclosure cost  $D$  is relatively low compared to the cost of false disclosure  $f$ . It immediately follows that *all* prices are lower than in the pure price signaling equilibrium (see footnote XX for details). This causes consumer surplus to be larger and to be increasing in both  $D$  and  $f$  in the whole range where this equilibrium exists. The situation with respect to total surplus is quite the opposite: as the consumption distortion is unaffected, while high quality firms engage in wasteful advertising, the total welfare loss equals  $2\alpha(1-\alpha)(\Delta V - \Delta c) + 2\alpha D$ , which is larger than in the pure price signaling equilibrium. Finally, profits of both types of firms are decreasing in  $f$  and  $D$ . Thus, both types of firms are better off in the pure price signaling equilibrium than in this pure disclosure equilibrium and total surplus is also lower. Thus, when  $D$  is small, regulation does not necessarily eliminate the consumption distortion of the pure price signaling equilibrium and, in particular, an intermediate level of regulation is worse than no regulation at all. In the next Section we show that a similar statement holds true when  $D$  is larger.

Let us next consider whether an equilibrium without consumption distortion exists and if so, how severe the regulation should be. It is easy to see that if a low quality firm only sells in case the competitor also sells low quality products, then a Bertrand competition argument can be used to argue that  $p_L = c_L$ . Moreover, if the competitor has low quality a high quality firm will only sell if his maximum price equals  $c_L + \Delta V$ . High quality firms setting a price  $c_L + \Delta V$  for sure can only be part of a non-distortionary equilibrium if consumers buy high quality in the state where both low and high quality firms are active. In that case low quality firms do not have an incentive to imitate the high quality firms if  $f$

is such that  $D + f \geq (1 - \frac{\alpha}{2}) \Delta V$ . To prevent high quality firms to undercut, it should be the case that, however, that consumers believe that prices lower than  $c_L + \Delta V$  accompanied by a disclosure statement are set by a low quality firm. It is clear that for large values of  $f$  low quality firms never have an incentive to disclose and, therefore, such a pure pricing strategy cannot be sustained in a reasonable equilibrium. In fact, out-of-equilibrium beliefs that are consistent with the D1 logic imply that if  $D + f > (1 - \frac{\alpha}{2}) \Delta V$  consumers should believe that such a deviation comes from a high quality firm, making the deviation profitable.

This implies that as  $f$  is large enough high quality firms should randomize their pricing decision in any equilibrium: If the rival produces low quality, a disclosing high quality firm will always sell as long as his price is smaller than  $c_L + \Delta V$ , while if the rival has high quality, there is Bertrand competition. There are two types of these equilibria: one where the high quality firm randomizes with a continuous distribution over the interval of prices  $[\underline{p}_H^D, \bar{p}_H^D]$  and another one where the high quality price distribution has, in addition, a mass point at a price  $\hat{p}_H^D$  that is smaller than  $\underline{p}_H^D$ . In fact, considering this second type of pricing strategy allows us to have a non-distortionary equilibrium for slightly lower values of  $f$  than considered above.

It is clear that by charging the upper bound  $\bar{p}_H^D = c_L + \Delta V$  the high quality firm will only sell in case the competitor is of low quality, implying that it makes a profit of  $(1 - \alpha)(\Delta V - \Delta c) - D$ . A first condition for this equilibrium type to exist is that this is nonnegative, i.e.,

$$D \leq (1 - \alpha)(\Delta V - \Delta c). \quad (6)$$

A second condition is that low quality firms should not have an incentive to imitate prices that are set in equilibrium by high quality firms. As the high quality firms are indifferent over a set of prices and as  $c_H > c_L$  it follows that for the low quality firm the most profitable deviation is to deviate to the lowest price in the equilibrium support. In the proof of the next Proposition we show that the lowest values of  $f$  where we can support such an equilibrium is where  $\hat{p}_H^D$  is charged with probability 1, resulting in a profit equal to  $(1 - \alpha + \frac{\alpha}{2})(\hat{p}_H^D - c_H) - D$ . As a high quality firm can anyway get a profit of  $(1 - \alpha)(\Delta V - \Delta c) - D$  by charging  $c_L + \Delta V$  it follows that in this case  $\hat{p}_H^D = c_H + \frac{(1 - \alpha)(\Delta V - \Delta c)}{1 - \frac{\alpha}{2}}$ . For the low quality firm it is not gainful to deviate to this price if

$$D + f \geq (1 - \alpha)\Delta V + \frac{\alpha}{2}\Delta c. \quad (7)$$

These arguments establish the main parts of the following proposition. The remaining elements of the proof are in the Appendix.

**Proposition 5** *There exists an equilibrium where high quality firms disclose and randomize prices with  $\bar{p}_H = c_L + \Delta V$ , while low quality firms set  $p_L = c_L$ , and only sell in the state where both firms produce low quality if, and only if (6) and (7) hold.*

Thus, regulation can eliminate the consumption distortion if it is sufficiently harsh and as long as  $D$  is sufficiently small. In the parameter region considered in Proposition 5 if  $f$  becomes larger the mass point at  $\hat{p}_H^D$  becomes smaller and, eventually, if  $D + f > (1 - \alpha)\Delta V + \alpha\Delta c$  the high quality firm randomizes with a continuous distribution over the interval of prices  $[\underline{p}_H^D, \bar{p}_H^D]$ . Further strengthening the regulation of false disclosure has no effect on the market outcome. It is clear that the market outcome is fairly competitive. Under full information, high quality firms set  $p_H = c_L + \Delta V$  in case there is one high and one low quality firm in the market. Because of asymmetric information, the high quality price is smaller as the firm may be in competition with another high quality firm. Also, the high quality prices are strictly smaller and consumer surplus is strictly larger than in any of the other equilibria we have characterized so far. As there is no consumption distortion, the total welfare loss is constant at  $2\alpha D$ .

Given the discussion so far, the intermediate region is easily understood. If (4) does not hold, then low quality firms make zero profits. At the threshold  $D + f = \frac{\alpha}{2}\Delta V$ , low quality sells at  $p_L = c_L$ , while high quality discloses and sets  $p_H = c_L + \Delta V$ , but consumers still buy low quality if both are available (as was the case if (4) holds). On the other hand, if  $D + f > (1 - \frac{\alpha}{2})\Delta V$  firms set the same prices, but consumers buy high quality if both are available. In the region in between both types of firms keep setting the same prices and consumers buy at the firm setting a price  $c_L + \Delta V$  with probability  $\beta$  if one firm sets this price, while the other firm sets  $c_L$  (and consumers believe both qualities are available). To guarantee that high quality firms do not have an incentive to undercut, it should be the case that consumers believe that if they observe a price  $p$  in the interval  $(c_H, c_L + \Delta V)$  it is set by a low quality firm. Using the D1 logic explained in the previous Section, this implies that the low quality should be indifferent between setting  $p_L = c_L$  and advertising and setting  $p_H = c_L + \Delta V$ . Thus, as low quality then sells with probability  $(1 - \alpha)\beta + \frac{\alpha}{2}$  if it imitates the high quality price, this determines  $\beta$  to be equal to

$$\beta = \frac{D + f}{(1 - \alpha)\Delta V} - \frac{\alpha}{2(1 - \alpha)} \quad (8)$$

For this equilibrium to exist we should have that  $0 \leq \beta \leq 1$ , which translates into

$$\frac{\alpha}{2}\Delta V \leq D + f \leq \left(1 - \frac{\alpha}{2}\right)\Delta V, \quad (9)$$

and that  $\pi_H^* \geq 0$ , which implies that

$$D \leq (\lambda - 1)f. \quad (10)$$

These arguments establish the main parts of the following proposition. The remaining elements of the proof are in the Appendix.

**Proposition 6** *There exists a D1 equilibrium with  $p_L = c_L$ , and  $p_H = c_L + \Delta V$  and consumers buying high quality with probability  $\beta$  given by (8), if and only if, (9) and (10) hold.<sup>13</sup>*

As according to (8),  $\beta$  is increasing in  $D$  and  $f$  (keeping  $D$  constant) the consumption distortion becomes smaller when one of these parameters becomes larger, while wasteful disclosure expenditures remain unaffected at  $2\alpha D$ . Thus, the welfare loss decreases when increasing  $f$ . As at these prices consumers are indifferent between buying high and low quality, consumer surplus remains constant. As high quality firms make a profit per unit of  $\Delta V - \Delta c$  and they sell with probability  $(1 - \alpha)\beta + \alpha/2$ , their profits are increasing if  $f$  increases, while  $\pi_L = 0$ . Thus, in this parameter region, firms favor more regulation and this improves social welfare.

Having characterized three disclosure equilibria, where the high quality firm chooses to always disclose its information directly, we can now characterize the full parameter region where pure disclosure equilibria exist. It turns out no other pure disclosure equilibria exists. So, define  $\tilde{D}(f)$  as

$$\tilde{D}(f) = \begin{cases} \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right)f & \text{if } 0 \leq f \leq \frac{\alpha}{2}\Delta c \\ (\lambda - 1)f & \text{if } \frac{\alpha}{2}\Delta c < f \leq (1 - \frac{\alpha}{2})\Delta c \\ (1 - \frac{\alpha}{2})\Delta V - f & \text{if } (1 - \frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c \\ (1 - \alpha)(\Delta V - \Delta c) & \text{if } f > \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c. \end{cases}$$

It is not difficult to see that  $\tilde{D}(f)$  is continuous in  $f$ . If  $0 \leq f \leq \frac{\alpha}{2}\Delta c$  the value

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<sup>13</sup>Considering Proposition 5 and Proposition 7 it is clear that there is a region where  $(1 - \alpha)\Delta V + \frac{\alpha}{2}\Delta c < D + f < (1 - \frac{\alpha}{2})\Delta V$  where the two equilibria with  $p_L = c_L$  overlap.

of  $\tilde{D}(f)$  follows from the characterization of the distortionary disclosure equilibrium in Proposition 4. For these values of  $f$  (5) implies that (4) holds and  $\tilde{D}(f)$  is the RHS of (5). If  $f > (1 - \frac{\alpha}{2})\Delta c$  the value of  $\tilde{D}(f)$  is determined by the parameter region of the distortionary disclosure equilibrium in Proposition 5 as  $\max((1 - \frac{\alpha}{2})\Delta V - f, (1 - \alpha)(\Delta V - \Delta c))$ . This yields the last two components of  $\tilde{D}(f)$ : if  $f < \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$  the first term is larger, whereas the second term is larger when  $f$  is larger. If  $\frac{\alpha}{2}\Delta c < f \leq \frac{\alpha}{2}\Delta V$  the distortionary disclosure equilibrium of Proposition 4 still exists, but the upper bound of  $D$  where a pure disclosure equilibrium exists is not anymore determined by the partial disclosure equilibrium of Proposition 6.

We depict the function  $\tilde{D}(f)$  in Figure 2 and state the following result.

INSERT FIGURE 2 HERE

**Proposition 7** *An equilibrium where high quality always discloses exists if, and only if,  $D \leq \tilde{D}(f)$ .*

It is easy to see that  $\tilde{D}(f) < D^*(f)$ . Thus, the pure price signaling equilibria and the pure disclosure equilibria are mutually exclusive. Moreover, there also should exist mixed disclosure equilibria to have an existence result for all parameter values. These equilibria will be discussed in the next two Sections together with a comparative statics and welfare analysis of the impact of changes in  $f$ .

## 5 Regulating false disclosure when disclosure costs are small

In the previous Sections, we have discussed two extreme cases of equilibria: one pure price signaling equilibrium where high quality does not direct disclose, and a class of equilibria where the high quality discloses for sure. we have seen in Figure 2 that these equilibria are mutually exclusive in the sense that the pure price signaling equilibrium exists if, and only if,  $D \geq D^*(f)$ , whereas the pure disclosure equilibria exist if, and only if,  $D \leq \tilde{D}(f)$ , with  $\tilde{D}(f) < D^*(f)$  for any  $f$ . For  $f$  approaching 0, both  $\tilde{D}(f)$  and  $D^*(f)$  are approaching 0. In this section, we perform a full comparative statics and welfare analysis of the impact of changes in  $f$  when the direct disclosure cost  $D$  is relatively small. To do so, we first have to provide an equilibrium analysis when  $D$  is small and  $f$  is also small such that  $\tilde{D}(f) < D < D^*(f)$ . In particular, we focus on  $f$  values in the range  $0 < f < \frac{\alpha\Delta c}{2}$ .

For this region of  $f$  values we showed in the previous two sections that the pure price signaling equilibrium the pure disclosure equilibrium looked very similar to each other:

the high quality firm sets a deterministic price and the low quality firm randomizes so as to sell to consumers if the rival is of high quality. We now show that in between there exists an equilibrium that naturally transits between these two equilibria, namely a mixed disclosure equilibrium where the high quality firm sets a deterministic price that is independent of whether or not he discloses, *i.e.*,  $p_H^D = p_H^{ND} = p_H$ , while if both firms set  $p_H$  out of which one discloses and the other does not, consumers buy from the disclosing firm with probability  $\beta$ . Thus, high quality firms randomize their disclosure decision, relying on both signaling mechanisms we have encountered before. As the equilibrium is separating and consumers anyway infer that the firm is of high quality even if it does not disclose, consumers are indifferent and hence may randomize their purchasing decision. As disclosure comes at an additional cost, it is clear that  $\beta > .5$  to make the high quality type indifferent. The other features are the same as in the pure price signaling equilibrium and the pure disclosure equilibrium of Proposition 4: low quality randomizes over the interval  $[\underline{p}_L, \bar{p}_L]$ , with  $\bar{p}_L = p_H - \Delta V$ .

In the proof of the next Proposition we show that all conditions of a D1 equilibrium can be fulfilled and that they imply that

$$\beta = \frac{1}{2} + \frac{f}{\alpha \Delta c}, \quad \gamma_H = \alpha \frac{\Delta V}{D + f} - \frac{\alpha \Delta c}{2f}$$

and

$$p_H = c_H + \Delta c \frac{D}{f}.$$

It follows that indeed  $0.5 < \beta < 1$  for  $0 < f < \frac{\alpha}{2} \Delta c$  and that high quality firms make positive profit. Also, the gradual transition to the pure disclosure and the pure price signaling equilibria becomes transparent from these equations. Substituting  $D = D^*(f) = (2\lambda - 1)f$ , one gets  $\gamma_H = 0$  and  $p_H = c_L + 2\Delta V$ , which characterizes the pure price signaling equilibrium. Substituting  $D = \tilde{D}(f) = \left( \frac{2\alpha \Delta V}{\alpha \Delta c + 2f} - 1 \right) f$ , one gets  $\gamma_H = 1$  and  $p_H = c_L + 2\Delta V - \frac{2(D+f)}{\alpha}$ , which characterizes the pure disclosure equilibrium.

**Proposition 8** *There exists an equilibrium where high quality firms choose to directly disclose with probability  $0 < \gamma_H < 1$  and chooses a price  $p_H = c_H + D\Delta c/f$  independent of whether or not it discloses, while low quality firms randomize over the interval  $[\underline{p}_L, \bar{p}_L]$ , with  $\bar{p}_L = p_H - \Delta V$ , if and only if,  $\tilde{D}(f) < D < D^*(f)$  and  $0 < f < \frac{\alpha \Delta c}{2}$ .*

In terms of welfare properties, this equilibrium shares many features with the pure

disclosure equilibrium that exists for smaller values of  $D$ . The price of the high quality firm is decreasing in  $f$  implying that *all* prices are decreasing in  $f$  (and lower than in the pure price signaling equilibrium). Consumer surplus is therefore increasing in  $f$ , while total surplus is decreasing in  $f$ : as the consumption distortion is unaffected, while high quality firms engages more and more in wasteful advertising, the total welfare loss equals  $2\alpha(1-\alpha)(\Delta V - \Delta c) + 2\alpha\gamma_H D$ . Finally, profits of both types of firms are decreasing in  $f$ . The main difference with the welfare properties of the pure disclosure equilibrium is the impact of  $D$ : the impact of  $D$  on prices is the opposite to the impact of  $f$ , whereas the impact goes in the same direction in the pure disclosure equilibrium. In terms of profits and total surplus, it is clear that the mixed equilibrium is in between the pure price signaling equilibrium and the distortionary pure disclosure equilibrium.

Having characterized this equilibrium, we are now in the position to perform a full comparative statics and welfare analysis for small values of  $D$ . In particular, we ask how welfare changes with the policy parameter  $f$  and whether or not the incentives of firms are aligned with total surplus. In particular, we focus in this Section on the parameter range where  $0 < D < (\Delta V - \Delta c) \min\{\frac{\alpha}{2}, 1 - \alpha\}$ . The range is chosen in such a way that the transition between different types of equilibria is such that when increasing  $f$  starting at  $f = 0$  one passes through regions where the pure price signaling equilibrium, the mixed equilibrium of Proposition 8 and different types of pure disclosure equilibria exist.

If  $f$  is small (such that  $D > D^*(f)$ ) we have a pure price signaling equilibrium where the welfare loss equals the consumption distortion  $2\alpha(1-\alpha)(\Delta V - \Delta c)$  and profits are equal to  $\frac{\alpha}{2}(p_H^{ND} - c_H) = \alpha(\Delta V - \Delta c/2)$  and  $\alpha(\bar{p}_L - c_L) = \alpha\Delta V$  for the high and low quality firms, respectively. In this Section we have seen that when  $f$  increases such that  $\tilde{D}(f) < D < D^*(f)$  we transit to the equilibrium of Proposition 8 where the prices are decreasing in  $f$ , the consumption distortion is not resolved and high quality firms increase their direct disclosure efforts as  $f$  increases. That is, high and low quality profits and total surplus are all decreasing in  $f$ . This decrease in profits continues, while total surplus remains constant when  $f$  increases further such that  $D < \tilde{D}(f)$  entering the region of the distortionary pure disclosure equilibrium of Proposition 4. In the whole region until  $f$  reaches  $f = \frac{\alpha}{2}\Delta V - D$  the consumption distortion is not resolved and high quality firms disclose with probability 1 so that the welfare loss compared to the first best is at its highest and equals  $2\alpha[(1-\alpha)(\Delta V - \Delta c) + D]$ , while firms' profits reach a minimum: the low quality firms' profits are 0, whereas the high quality firms' profits equal  $\frac{\alpha}{2}(\Delta V - \Delta c) - D$ .

When  $f$  increases further such that  $f > \frac{\alpha}{2}\Delta V - D$  welfare starts increasing. We are



first in the region of Proposition 7 where prices are equal to  $p_L = c_L$  and  $p_H = c_L + \Delta V$  and independent of the disclosure cost  $f$ , while the consumption distortion gradually disappears when  $f$  increases as consumers shift to buying high quality when both low and high quality firms exist in the market, while the high quality firm's profit gradually increases until it reaches  $(1 - \frac{\alpha}{2})(\Delta V - \Delta c) - D$  and total surplus reaches  $2\alpha D$  when  $f$  reaches  $(1 - \frac{\alpha}{2})\Delta V - D$ . As  $D < (1 - \alpha)(\Delta V - \Delta c)$  total surplus is larger than in the pure signaling equilibrium, while low quality profits are lower and high quality profits are larger if, and only if,  $(1 - \frac{\alpha}{2})(\Delta V - \Delta c) - D > \alpha(\Delta V - \Delta c/2)$ , or  $D < (\Delta V - \Delta c) - \frac{3\alpha}{2}\Delta V$ , i.e., if and only if,  $\alpha$  is relatively small. When  $f$  increases further beyond  $(1 - \frac{\alpha}{2})\Delta V - D$  total surplus does not change anymore, while consumer surplus increases at the expense of high quality profits. The latter decreases again to  $(1 - \alpha)(\Delta V - \Delta c) - D$ .

Thus, for small  $D$  it is clear that total surplus is maximized for large  $f$ , while an intermediate value of  $f$  is worse than no cost of false disclosure. Low quality profits are decreasing in  $f$ , whereas high quality profits is non-monotonic in roughly the same way as total surplus; it first decreases and then increases in  $f$ . The main difference is that high quality profits are decreasing for larger values of  $f$ , while they may have their global maximum at low values of  $f$ .

## 6 Regulating false disclosure when disclosure costs are larger

In this Section we perform a similar analysis when the direct disclosure cost is larger. In particular, we focus on intermediate  $D$  values,  $(\lambda - 1)(1 - \frac{\alpha}{2})\Delta c < D < D_2$ . The lower bound of this region is the maximum value of  $\tilde{D}(f)$ , while the upper bound is the maximum value of  $D^*(f)$ .<sup>14</sup> Thus, this region is such that for small values of  $f$  one first passes through the pure price signaling equilibrium and the mixed equilibrium of Proposition 8 in a very similar way as in the previous section. The main difference is that the region of pure disclosure equilibria is never crossed.

We proceed as follows. We first prove an existence result: for all values of  $f$  and  $D$  a D1 equilibrium exists. As we focus here on intermediate  $D$  values such that for larger  $f$  values  $\tilde{D}(f) < D < D^*(f)$  the existence result implies that in this region the equilibrium

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<sup>14</sup>The comparative statics of  $D$  is either uninteresting (for  $D > \max D^*(f)$  there is only a pure price signaling equilibrium that does not change with changes in  $f$ ), or a somewhat complicated combination of the comparative statics under the two regions that we analyze (as there may be several transitions between the pure disclosure and mixed equilibria).

must have  $0 < \gamma_H < 1$  for all  $f$  values such that  $D < D^*(f)$ . Thus, this Section illustrates our second narrative that a high quality firm does not disclose for sure if the disclosure costs are not small. We then show that all these mixed disclosure equilibria generate a total surplus that is smaller than that generated in the pure price signaling equilibrium. As this latter equilibrium exist for all values of  $D$  whenever  $f = 0$ , we conclude that high fines are not optimal at intermediate levels of the direct disclosure cost.

In the previous Sections, we have demonstrated that a pure price signaling equilibrium exists for  $D \geq D^*(f)$ , while an equilibrium with high quality firms always disclosing exists if  $D \leq \tilde{D}(f)$ . The next Proposition establishes that equilibrium existence is also guaranteed if  $\tilde{D}(f) < D < D^*(f)$ .

**Proposition 9** *A D1 equilibrium always exists.*

In our model, the signaling space consists of a combination of a continuous price signal and a binary signal of whether or not to send a direct disclosure message. The overall proof is constructive by showing which equilibria exist for which parameter values.<sup>15</sup> Equilibrium is not always uniquely determined as there are parameter values for which different D1 equilibria co-exist. However, this only arises for small  $D$  values and intermediate  $f$  values. For other parameter values, equilibrium is unique.

Proposition 2b establishes that if high quality does not disclose with positive probability, then a low quality firm captures the market if the competitor sells high quality and does not disclose. By leaving the market to low quality firms by not disclosing, a high quality firm softens price competition, creating enough rent to cover the intermediate level of disclosure costs. Thus, in every equilibrium for intermediate  $D$  values and all  $f$  such that  $D < D^*(f)$ , there is a positive probability that high quality does not advertise and cedes the market to the low quality competitor. In welfare terms, this also implies that the consumption distortion is never fully eliminated. Denoting by  $q_L^e$ , respectively  $q_{L/D}^e$ , the

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<sup>15</sup>To provide one example, we show that for  $f > (1 - \frac{\alpha}{2})\Delta c$  the equilibrium combines features of the pure price signaling equilibrium and the non-distortionary disclosure equilibrium. Proposition 5 has shown that a nondistortionary disclosure equilibrium exists with  $\bar{p}_H^D = \underline{p}_L + \Delta V$ , where the high quality firm sells if the competitor is low quality (and in that equilibrium  $\underline{p}_L = \bar{p}_L = c_L$ ), while Proposition 3 established that for  $D > D^*(f)$  a pure price signaling equilibrium exists with  $p_H^{ND} = c_L + 2\Delta V$  and the low quality firm randomizing over the interval  $[\underline{p}_L, \bar{p}_L]$ , with  $\bar{p}_L = p_H^{ND} - \Delta V$ . In the intermediate range, the equilibrium has  $\gamma_H \in (0, 1)$ : low quality sells only when the competitor has low quality or when it has high quality and does not disclose. As the equilibrium profit for the low quality firms is positive, he randomizes with  $c_L < \underline{p}_L < \bar{p}_L$ .

expected quantity sold by a low quality firm if the competitor is a high quality firm, respectively a high quality disclosing firm, the expression for welfare loss in a mixed equilibrium can be written as

$$\begin{aligned}
WL &= 2\alpha[\gamma_H D + (1 - \alpha)q_L^e(\Delta V - \Delta c)] \\
&= 2\alpha[D\gamma_H + (1 - \alpha)(\Delta V - \Delta c)\{(1 - \gamma_H) + \gamma_H q_{L/D}^e\}] \\
&= -2\alpha\gamma_H[(1 - \alpha)(\Delta V - \Delta c)(1 - q_{L/D}^e) - D] + 2\alpha(1 - \alpha)(\Delta V - \Delta c)
\end{aligned}$$

so that the welfare loss is strictly larger than that in the pure price signaling equilibrium if, and only if,

$$2\alpha\gamma_H[(1 - \alpha)(\Delta V - \Delta c)(1 - q_{L/D}^e) - D] < 0.$$

As for any  $D < D^*(f)$  we have  $\gamma_H > 0$  and  $D > (1 - \alpha)(\Delta V - \Delta c)$ , it follows that this inequality holds. In other words, no matter how large the cost of false disclosure, the pure signaling equilibrium is welfare dominant when the disclosure cost  $D$  is in the intermediate region.

Interestingly, in the mixed equilibrium profits of firms are not larger than  $\frac{\alpha}{2}(1 - \gamma_H)(p_H^{ND} - c_H) = \alpha(1 - \gamma_H)(\Delta v - \Delta c/2)$  and  $\alpha(1 - \gamma_H)(\bar{p}_L - c_L) = \alpha(1 - \gamma_H)\Delta V$  for the high and low quality firms, respectively, implying that both types of firms make less profits if  $f$  is large than in the pure price signaling equilibrium (if  $f$  is small). Thus, for this parameter region it is in the interest of both types of firms to oppose regulation, in line with social welfare.

## 7 Discussion and Conclusion

This paper explores the economic implications of policies aiming at increasing the cost of false disclosure. Throughout the world, governments penalize false advertising and the question we ask is whether it is optimal to impose a fine on false disclosure, and if so, how high this fine should be? In answering this question, we think it is important to consider that firms have alternative means to signal their quality. In particular, firms may signal their quality through the prices they choose. Direct disclosure, by means of advertising, or otherwise, is then simply an alternative way to signal quality, where the only disclosure cost for the high quality firm is the direct cost of sending the signal, whereas the cost of

false disclosure is the additional cost of sending this signal for the low quality firm.

Equilibria are always fully revealing, indicating that even in the absence of direct disclosure consumers correctly infer, before purchase, the quality they buy. False disclosure never arises in the equilibria we focus on. Nevertheless, imposing a fine on false disclosure has profound effects on market outcomes through the credibility of the information provided through direct disclosure. In the absence of a fine on false disclosure, firms reveal quality through price signaling resulting in a consumption distortion where consumers buy low quality even if high quality is available and yields higher social surplus.

We show that the welfare effect of increasing the cost of false disclosure critically depends on the disclosure cost. If the direct disclosure cost is small, imposing a sufficiently high fine for false disclosure is welfare optimal. The welfare effects are, however, non-monotonic in this case, as an intermediate fine will be detrimental inducing costly disclosure without eliminating the consumption distortion. If the disclosure cost is larger, total surplus decreases in the fine for false disclosure as the reduction in consumption distortion does not compensate the direct disclosure cost incurred by high quality firms. Thus, the welfare implications of introducing a fine for false disclosure are very sensitive to the specifics of the market environment and a general fine covering all markets does not seem to be optimal.

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We begin by characterizing the class of *symmetric* Perfect Bayesian Equilibria where out of equilibrium beliefs satisfy the *D1 criterion* (hereafter, "equilibrium" refers to only such an equilibrium).

**Proposition 1** *Every equilibrium is fully revealing; there is no pooling or partially pooling equilibrium.*

**Proof.** Suppose to the contrary there is a pooling or partially pooling equilibrium. First, suppose that both types pool (at least partially) by advertising and charging common prices with strictly positive probability in this equilibrium. Let  $S_\tau^A$  denote the essential support of the distribution of equilibrium prices of type  $\tau$  when it advertises. Then, for  $S = S_H^A \cap S_L^A$ ,

$$\Pr\{p_H \in S, p_L \in S, \text{ both types advertise}\} > 0.$$

As high and low quality types differ in their marginal cost, they cannot both be indifferent between two pooling prices. Hence,  $S$  is a singleton  $\{\bar{p}\}$  and both types assign a probability mass point at  $\bar{p}$ . It is easy to see that prices slightly above  $\bar{p}$  are not in  $S_L^A$ ; at such price a low quality firm (being revealed as a low quality type for sure) would earn strictly smaller profit than at  $\bar{p}$ . Further, prices slightly above  $\bar{p}$  cannot be in  $S_H^A$ ; at such price a firm (being revealed as a high quality type for sure) would earn strictly higher profit than at pooling price  $\bar{p}$  (by attracting all buyers in the state where rival charges  $\bar{p}$ ) so that a firm can strictly increase its payoff by shifting probability mass from pooling price  $\bar{p}$  to a price just above  $\bar{p}$  (while continuing to advertise). So, prices slightly above  $\bar{p}$  (when combined with advertising) are out of equilibrium. We now argue that the D1 criterion implies that out of equilibrium beliefs must regard a firm that advertises and charges price just above  $\bar{p}$  as being of high quality with probability one. This in turn implies that a firm (of either type) can strictly increase its payoff by deviating to a price just above  $\bar{p}$  (while continuing to advertise). To see this implication of the D1 criterion, choose  $\epsilon > 0$  arbitrarily small and consider an out of equilibrium price  $\bar{p} + \epsilon$ . Let  $q_\tau(\bar{p} + \epsilon)$  be the minimum expected quantity that makes a type  $\tau$  seller indifferent between deviating to price  $\bar{p} + \epsilon$  while advertising and its equilibrium strategy i.e.,

$$\begin{aligned} (\bar{p} + \epsilon - c_H)q_H(\bar{p} + \epsilon) - A &= \pi_H^* \\ (\bar{p} + \epsilon - c_L)q_L(\bar{p} + \epsilon) - A - f &= \pi_L^* \end{aligned}$$

where  $\pi_\tau^*$  is the equilibrium payoff of type  $\tau$ . However, given that in equilibrium both types charge  $\bar{p}$  with strictly positive probability, denoting by  $q(\bar{p})$  the quantity sold by a firm at  $\bar{p}$  (with advertising), we have:

$$\pi_H^* = (\bar{p} - c_H)q(\bar{p}) - A, \quad \pi_L^* = (\bar{p} - c_L)q(\bar{p}) - A - f$$

Thus,

$$q_H(\bar{p} + \epsilon) = \frac{(\bar{p} - c_H)q(\bar{p})}{(\bar{p} + \epsilon - c_H)}$$

and

$$q_L(\bar{p} + \epsilon) = \frac{(\bar{p} - c_L)q(\bar{p})}{(\bar{p} + \epsilon - c_L)}$$

Observe that  $\frac{(\bar{p} - c)q(\bar{p})}{(\bar{p} + \epsilon - c)}$  is strictly decreasing in  $c$  and therefore,

$$q_H(\bar{p} + \epsilon) < q_L(\bar{p} + \epsilon)$$

so that  $H$  type can gain from deviation to price  $\bar{p} + \epsilon$  for a larger set of expected quantities sold and the D1 criterion requires that any firm deviating to such a price must be regarded as high quality with probability one. Thus, a full or partially pooling equilibrium with both types pooling by advertising and charging common prices cannot exist. A very similar argument can be constructed to show that a full or partially pooling equilibrium with both types pooling by not advertising and charging common prices cannot exist (the calculations are very similar, with  $A$  and  $f$  being zero). ■

**Lemma 2** *The following properties hold for any equilibrium:*

(a) *Low quality types do not disclose and in particular, no cost of false disclosure is incurred in equilibrium.*

(b) *Suppose high quality types disclose with probability  $\gamma_H \in [0, 1)$ . When it does not disclose, a high quality firm does not randomize over prices i.e., charges a deterministic price  $p_H^{NA}$  and sells only if the rival firm is a high quality type (i.e., it cedes the entire market to its rival in the event that the latter is of low quality creating a consumption distortion). Further, if  $\gamma_H \in (0, 1)$  then,  $\bar{p}_H^A$  the upper bound of the support of prices charged by a high quality type when it discloses satisfies*

$$\bar{p}_H^A \leq p_H^{NA}.$$

*Low quality firms randomize over a set of prices whose upper bound  $\bar{p}_L$  satisfies:*

$$\bar{p}_L \leq p_H^{NA} - \Delta V$$

*and there is no mass point at  $\bar{p}_L$ . If high quality types disclose and charge a price  $\tilde{p}$  with strictly positive probability, then  $\tilde{p}$  must be the lowest price charged in equilibrium by a high quality type. Both types earn strictly positive profit.*

**Proof.** (a) In a fully revealing equilibrium where a low quality type discloses with positive probability and charges some price  $p$  and sells expected quantity  $q$

$$(p - c_L)q \geq A.$$

In particular,  $p > c_L$ . If the firm does not disclose and charges price  $p - \epsilon$  for  $\epsilon > 0$  arbitrarily small, the expected quality of buyers cannot be any worse and the firm sells an expected quantity of at least  $q$ . As  $A > 0$ , this deviation is strictly gainful for  $\epsilon$  small enough.

(b) Consider an equilibrium where high quality firms disclose with probability  $\gamma_H \in [0, 1)$ . Let  $\bar{p}_H^A, \underline{p}_H^A$  ( $\bar{p}_H^{NA}, \underline{p}_H^{NA}$ ) be the supremum and the infimum of the essential support of prices charged by a high quality firm when it discloses (does not disclose). In the arguments that follow when we talk about expected quantities sold at these upper and lower bound prices when there is no probability mass point at the bounds we really mean the limiting (expected) quantity sold as prices (from within the support) approach these bounds.

We first show that if  $\gamma_H > 0$ ,  $\bar{p}_H^A \leq \bar{p}_H^{NA}$ . Suppose to the contrary that  $\bar{p}_H^A > \bar{p}_H^{NA}$ . Then,  $\bar{p}_H^A$  is the upper bound of high quality prices. As  $A > 0$ , a high quality firm must sell strictly positive expected quantity at price  $\bar{p}_H^A$  to cover disclosure cost and therefore, must sell strictly positive expected quantity at price  $\bar{p}_H^{NA} < \bar{p}_H^A$ . As the low quality firm can always imitate price  $\bar{p}_H^{NA} \geq c_H > c_L$  without disclosing, it follows that  $\pi_L^* > 0$ . This implies that  $\bar{p}_L > c_L$  and as there is no probability mass point at  $\bar{p}_L$  at  $\bar{p}_L$  low quality firms sell with strictly positive probability in



the event where rival is of high quality type (if it only sells when rival is of low quality type, it will sell zero at  $\bar{p}_L$  earning zero profit). As  $\bar{p}_H^A$  is the upper bound of high quality prices, we must have  $\bar{p}_L \leq \bar{p}_H^A - \Delta V$ . Thus, at price  $\bar{p}_H^A$  a high quality firm that discloses sells zero when the rival is of low quality type. Therefore, the only way they can sell a strictly positive quantity at  $\bar{p}_H^A$  is if high quality types put a strictly positive probability mass  $\sigma > 0$  at price  $\bar{p}_H^A$  (when it discloses) and the equilibrium profit of the high quality firm must be  $\pi_H^* = (\bar{p}_H^A - c_H) \frac{\alpha\sigma}{2} - A$ . But this implies prices slightly below  $\bar{p}_H^A$  are not in the support of a high quality type's price distribution (when it discloses); as such a price would yield strictly higher payoff than  $\bar{p}_H^A$ . To deter the high quality firm from disclosing and deviating to a price slightly below  $\bar{p}_H^A$ , out of equilibrium beliefs must assign sufficiently high probability that the deviating firm is of low quality type. It can be checked that such a belief cannot be consistent with D1 refinement if low quality type strictly prefers to not imitate the action of disclosing (falsely) and charging  $\bar{p}_H^A$ . This implies that a low quality type must be indifferent between its equilibrium strategy and deviating to advertising and charging  $\bar{p}_H^A$  i.e.,

$$\begin{aligned}\pi_L^* &= (\bar{p}_H^A - c_L) \frac{\alpha\sigma}{2} - A - f = \pi_H^* + \Delta c \frac{\alpha\sigma}{2} - f \\ &= (\underline{p}_H^{NA} - c_H) q(\underline{p}_H^{NA}) + \Delta c \frac{\alpha\sigma}{2} - f\end{aligned}\quad (1)$$

where  $q(\underline{p}_H^{NA})$  is the expected quantity sold by the high quality type at price  $\underline{p}_H^{NA}$  when it does not advertise. Observe that as  $\underline{p}_H^{NA} < \bar{p}_H^A$ ,  $q(\underline{p}_H^{NA}) \geq \alpha\sigma$ . Further, the incentive constraint of the low quality type implies:

$$\pi_L^* \geq (\underline{p}_H^{NA} - c_L) q(\underline{p}_H^{NA}) = (\underline{p}_H^{NA} - c_H) q(\underline{p}_H^{NA}) + \Delta c q(\underline{p}_H^{NA}) \geq (\underline{p}_H^{NA} - c_H) q(\underline{p}_H^{NA}) + \Delta c \alpha\sigma$$

Thus, from (1)

$$\pi_L^* = (\underline{p}_H^{NA} - c_H) q(\underline{p}_H^{NA}) + \Delta c \frac{\alpha\sigma}{2} - f \leq \pi_L^* - \Delta c \frac{\alpha\sigma}{2} - f,$$

a contradiction. Thus, if  $\gamma_H > 0$ ,  $\bar{p}_H^A \leq \bar{p}_H^{NA}$ . This implies that for  $\gamma_H \in [0, 1]$ ,  $\bar{p}_H^{NA}$  is the highest price charged by the high quality type

Next, we show that for  $\gamma_H \in [0, 1]$  the high quality type does not randomize over prices when it does not disclose. Suppose not. Then,  $\bar{p}_H^{NA} > \underline{p}_H^{NA}$ ; this implies that high quality type sells strictly positive expected quantity at price  $\underline{p}_H^{NA} \geq c_H > c_L$  (for instance, in the state where rival is of high type and does not disclose) and high quality types earn expected profit  $\pi_H^* > 0$  (note that  $\bar{p}_H^{NA} > c_H$  as  $\underline{p}_H^{NA} \geq c_H$ ). As a consequence, the low quality type can earn strictly positive profit by imitating the high type's action and hence,  $\pi_L^* > 0$ . Then, using identical arguments as above, one can show that  $\bar{p}_L \leq \bar{p}_H^{NA} - \Delta V$  and there is no probability mass point at  $\bar{p}_L$ . Therefore, at price  $\bar{p}_H^{NA}$ , the high quality type is undercut with probability one in the state where rival is low quality type. However, as  $\pi_H^* > 0$ ,  $q(\bar{p}_H^{NA}, NA)$ , the expected quantity sold by the high quality type at  $\bar{p}_H^{NA}$  when it does not disclose, is strictly positive. Let  $\xi_{NA} \geq 0$  and  $\xi_A \geq 0$  be the respective probability masses, if any, placed by the high quality type at the price  $\bar{p}_H^{NA}$  in the states where it does not disclose and where it discloses (this allows for the possibility that  $\bar{p}_H^A = \bar{p}_H^{NA}$ ; if  $\bar{p}_H^A < \bar{p}_H^{NA}$  then necessarily  $\xi_A = 0$ ). Using symmetry of the equilibrium, there exists  $\beta \in [0, 1]$  such that

$$\begin{aligned}q(\bar{p}_H^{NA}, NA) &= \alpha \left[ \frac{1 - \gamma_H}{2} \xi_{NA} + \hat{\beta} \gamma_H \xi_A \right] \\ q(\bar{p}_H^{NA}, A) &= \alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \hat{\beta})(1 - \gamma_H) \xi_{NA} \right]\end{aligned}$$

where  $q(\bar{p}_H^{NA}, A)$  is the expected quantity sold by the high quality firm at price  $\bar{p}_H^{NA}$  when it discloses. Then

$$\pi_H^* = (\bar{p}_H^{NA} - c_H)\alpha \left[ \frac{1 - \gamma_H}{2} \xi_{NA} + \beta \gamma_H \xi_A \right] = (\underline{p}_H^{NA} - c_H)q(\underline{p}_H^{NA}, NA)$$

where  $q(\underline{p}_H^{NA}, NA)$  is the expected quantity sold by the high quality firm at price  $\underline{p}_H^{NA}$  when it does not disclose. Note that as  $\underline{p}_H^{NA} < \bar{p}_H^{NA}$ ,

$$q(\underline{p}_H^{NA}, NA) \geq \alpha((1 - \gamma_H)\xi_{NA} + \gamma_H \xi_A) \quad (2)$$

Consider the case where  $\gamma_H \xi_A > 0$ . This implies that  $\bar{p}_H^A = \bar{p}_H^{NA}$  and  $\xi_A > 0$ . Using similar argument as above, prices slightly below  $\bar{p}_H^{NA}$  are not in the support of the equilibrium pricing strategy of high quality types and out of equilibrium beliefs that deter high quality types from disclosing and undercutting from  $\bar{p}_H^{NA}$  can be consistent with the D1 criterion only a low quality type is indifferent between its equilibrium strategy and deviating to disclosing (falsely) and charging  $\bar{p}_H^{NA}$ :

$$\begin{aligned} \pi_L^* &= (\bar{p}_H^{NA} - c_L)\alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] - A - f \\ &= (\bar{p}_H^{NA} - c_H)\alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] + \Delta c \alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] - A - f \\ &= \pi_H^* + \Delta c \alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] - f \\ &= (\underline{p}_H^{NA} - c_H)q(\underline{p}_H^{NA}, NA) + \Delta c \alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] - f \\ &= (\underline{p}_H^{NA} - c_L)q(\underline{p}_H^{NA}, NA) - \Delta c q(\underline{p}_H^{NA}, NA) + \Delta c \alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] - f \\ &\leq (\underline{p}_H^{NA} - c_L)q(\underline{p}_H^{NA}, NA) - \Delta c \alpha((1 - \gamma_H)\xi_{NA} + \gamma_H \xi_A) + \Delta c \alpha \left[ \frac{1}{2} \gamma_H \xi_A + (1 - \beta)(1 - \gamma_H)\xi_{NA} \right] - f, \text{ using (2)} \\ &\leq \pi_L^* - \Delta c \alpha \left( \frac{1}{2} \gamma_H \xi_A + (1 - \gamma_H)\beta \xi_{NA} \right) - f, \end{aligned}$$

a contradiction (the last inequality uses the incentive constraint of the low quality type to not imitate  $\underline{p}_H^{NA}$  without disclosing). Now, consider the case where  $\gamma_H \xi_A = 0$ . As  $q(\bar{p}_H^{NA}, NA) > 0$ , we have  $\xi_{NA} > 0$  and

$$q(\bar{p}_H^{NA}, NA) = \frac{\alpha}{2}(1 - \gamma_H)\xi_{NA}$$

Again, low quality firm must be indifferent between its equilibrium strategy and deviating to  $\bar{p}_H^{NA}$  without disclosing:

$$\begin{aligned} \pi_L^* &= (\bar{p}_H^{NA} - c_L) \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2} = (\bar{p}_H^{NA} - c_H) \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2} + \Delta c \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2} \\ &= (\underline{p}_H^{NA} - c_H)q(\underline{p}_H^{NA}, NA) + \Delta c \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2} = (\underline{p}_H^{NA} - c_L)q(\underline{p}_H^{NA}, NA) - \Delta c q(\underline{p}_H^{NA}) + \Delta c \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2} \\ &\leq \pi_L^* - \Delta c \left( q(\underline{p}_H^{NA}, NA) - \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2} \right) \leq \pi_L^* - \Delta c \frac{\alpha(1 - \gamma_H)\xi_{NA}}{2}, \end{aligned}$$

a contradiction (the second to last inequality follows from the incentive constraint of the low quality type to not imitate  $p_H^{NA}$  and the last inequality follows from  $q(p_H^{NA}, NA) \geq \alpha(1 - \gamma_H)\xi_{NA}$ ). Thus, we have shown that the high quality type cannot randomize and must charge a deterministic price  $p_H^{NA}$  when it does not disclose. Further,  $p_H^{NA} = \bar{p}_H^{NA} = \underline{p}_H^{NA} \geq \bar{p}_H^A$  i.e., high quality never charges a higher price than  $p_H^{NA}$  when it discloses. As this is a symmetric equilibrium, at price  $p_H^{NA}$  (and without disclosing), the high quality firm sells with strictly positive probability (definitely in the state where rival is of high quality and does not advertise). As  $p_H^{NA} \geq c_H > c_L$ , the low quality type can earn strictly positive profit by imitating this price  $p_H^{NA}$  (without disclosing) and thus,  $\pi_L^* > 0$ . Using very similar arguments as above, low quality types must randomize over prices and there cannot be a mass point at the upper bound of low quality prices; further, at  $\bar{p}_L$  low quality types must sell with strictly positive probability in the state where rival is high type and as  $p_H^{NA}$  is the upper bound of the support of high quality prices,

$$\bar{p}_L \leq p_H^{NA} - \Delta V. \quad (3)$$

Therefore, at price  $p_H^{NA}$ , the high quality type loses all market in the state where rival is low quality type. Note that  $\pi_L^* > 0$  implies that  $\bar{p}_L > c_L$  so that  $p_H^{NA} \geq c_L + \Delta V > c_H$  so that  $\pi_H^* > 0$ . We now show that if a high quality type charges a price  $\tilde{p}$  with strictly positive probability in the event that it discloses, then  $\tilde{p}$  must be the lowest price charged in equilibrium by a high quality type. From above, discussion we know that  $\underline{p}_H^{NA} \geq \tilde{p}$ . It remains to be shown that also  $\underline{p}_H^A \geq \tilde{p}$ . Suppose to the contrary that  $\underline{p}_H^A < \tilde{p}$ . Then there exists  $\hat{p} \in [\underline{p}_H^A, \tilde{p})$  such that

$$\pi_H^* = (\hat{p} - c_H)q(\hat{p}) - A = (\tilde{p} - c_H)q(\tilde{p}) - A$$

where  $q(p)$  is the expected quantity sold in equilibrium at price  $p$  by a disclosing firm. As  $\hat{p} < \tilde{p}$  and  $(\hat{p} - c_H)q(\hat{p}) = (\tilde{p} - c_H)q(\tilde{p})$

$$q(\hat{p}) > q(\tilde{p})$$

As there is a strictly positive probability mass at  $\tilde{p}$ , it must be that undercutting is prevented by out-of-equilibrium beliefs and as before, this is consistent with the D1 criterion only if

$$\pi_L^* = (\tilde{p} - c_L)q(\tilde{p}) - A - f$$

However, from the incentive constraint of the low quality type:

$$\pi_L^* \geq (\hat{p} - c_L)q(\hat{p}) - A - f$$

so that

$$(\hat{p} - c_L)q(\hat{p}) \leq (\tilde{p} - c_L)q(\tilde{p})$$

However,

$$\begin{aligned} & (\hat{p} - c_L)q(\hat{p}) - (\tilde{p} - c_L)q(\tilde{p}) = (\hat{p} - c_H)q(\hat{p}) - (\tilde{p} - c_H)q(\tilde{p}) + \Delta c(q(\hat{p}) - q(\tilde{p})) \\ & = \Delta c(q(\hat{p}) - q(\tilde{p})) > 0, \end{aligned}$$

a contradiction. ■

**Lemma 3** *In a pure price signaling equilibrium (where no disclosure occurs), high quality types charge a deterministic high price*

$$p_H = c_L + 2\Delta V$$

and sell only in the state where rival is of high quality type while low quality types randomize their prices over an interval  $[\underline{p}_L, \bar{p}_L]$  according to a (continuous) distribution function  $F_L$  where

$$\bar{p}_L = p_H - \Delta V$$

and for all  $p \in [\underline{p}_L, \bar{p}_L]$

$$F_L(p) = 1 - \frac{\alpha}{1 - \alpha} \left[ \frac{\Delta V}{p - c_L} - 1 \right] = \frac{1}{1 - \alpha} - \frac{\alpha \Delta V}{(1 - \alpha)(p - c_L)}.$$

The equilibrium (expected) profits for the two types are given by

$$\pi_H^* = \frac{\alpha}{2}(2\Delta V - \Delta c), \pi_L^* = \alpha \Delta V$$

Further, all consumers buy but there is a consumption distortion in that when both low and high quality firms exist in the market, consumers buy low quality, while the surplus generated by high quality is larger. Low quality types are indifferent between imitating and not imitating the price set by the high quality firm.

**Proof.** The equilibrium must be identical to the unique symmetric D1 equilibrium in Janssen and Roy (2010). ■

**Lemma 4** (i) A symmetric pure price signaling equilibrium exists if

$$\frac{A}{f} \geq 2\lambda - 1 \quad (4)$$

(ii) Suppose that (4) does not hold. A symmetric pure price signaling equilibrium exists if, and only if, one of the following holds:

(ii.a)

$$A_2 \leq A < f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V$$

(ii.b)

$$A \geq \max \left\{ f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V, \frac{\alpha}{2}\Delta V + f[\lambda - 1] \right\}$$

**Proof.** In view of Lemma 3, we focus on a candidate equilibrium as described in the statement of the lemma. To establish the existence of such an equilibrium we fix out of equilibrium beliefs of buyers to be such that any firm that does *not* disclose and deviates to a price in  $(\bar{p}_L, p_H)$  is perceived as being a low quality type for sure. As this has been shown to be a D1 perfect Bayesian equilibrium in the special case where firms are not allowed to disclose, to show that this is an equilibrium in our model all we need to check is that neither type has any incentive to deviate by *disclosing*. It is easy to check that given the rival firm's strategy, if a firm of either type deviates and discloses, it will never gain by charging a price  $\tilde{p} > p_H$  (sells zero). Further, given that neither type has a strict incentive to imitate the other's equilibrium price in the game where they do not have the option of disclosing, neither type can gain by engaging in costly disclosure and charging a price  $\tilde{p} = p_H$  or a price  $\tilde{p} \leq \bar{p}_L$ . Consider an out of equilibrium action by a firm that discloses and charges price  $\tilde{p} \in (\bar{p}_L, p_H)$ . Such a deviation cannot be gainful (for either type) if out of equilibrium beliefs

associate this deviation with being a low quality firm for sure. We first examine the restriction imposed by D1 refinement on buyers' beliefs following such a deviation. Let  $q_H(\hat{p}), q_L(\hat{p})$  be the expected quantity that a  $H$  and a  $L$  type firm must sell respectively in order to be indifferent between this deviation and not deviating from their equilibrium strategies. If  $q_H(\hat{p}) < q_L(\hat{p})$ , then D1 refinement suggests that buyers must assign probability one to the deviating firm's type being of  $H$  type; if  $q_H(\hat{p}) > q_L(\hat{p})$ , the belief must assign probability one to  $L$  type. It is easy to check that:

$$q_H(\hat{p}) = \frac{\pi_H^* + A}{(\hat{p} - c_H)}, q_L(\hat{p}) = \frac{\pi_L^* + A + m}{(\hat{p} - c_L)}$$

so that

$$q_H(\hat{p}) < q_L(\hat{p})$$

if, and only if,

$$\frac{\hat{p} - c_L}{\hat{p} - c_H} < \frac{\pi_L^* + A + f}{\pi_H^* + A} \quad (5)$$

Let  $S$  be defined by:

$$S = \{ \hat{p} \in (\bar{p}_L, p_H) : (5) \text{ holds} \}$$

Observe that the left hand side of (5) is continuous and strictly decreasing in  $\hat{p}$ .  $S$  is an interval.  $S$  is nonempty if, and only if,

$$\frac{p_H - c_L}{p_H - c_H} < \frac{\pi_L^* + A + f}{\pi_H^* + A}$$

which (using the fact that  $\pi_H^* = \frac{\alpha}{2}(p_H - c_H)$  and  $\pi_L^* = \alpha\Delta V = \frac{\alpha}{2}(p_H - c_L)$ ) reduces to:

$$\frac{\pi_L^*}{\pi_H^*} < \frac{\pi_L^* + A + f}{\pi_H^* + A}$$

that further reduces to,

$$\frac{A}{f} < 2\lambda - 1. \quad (6)$$

Observe that the right hand side of (6) is strictly positive under assumption (??). (6) is a necessary condition for any gainful deviation by a  $H$  type firm; if it does not hold than charging  $\hat{p} < p_H$  will necessarily be associated with being a low quality firm with probability one so that no gainful deviation is possible. This yields part (i) of the lemma.

We now establish part (ii) of the lemma. In doing so, we will work with the most pessimistic beliefs consistent with D1 criterion so that our conditions for existence of a symmetric pure price signaling equilibrium are both necessary and sufficient. Assume that (6) holds. In that case, the set  $S$  is a non-empty interval. As the expression on the left hand side of (5) is continuous and strictly decreasing in  $\hat{p}$  and diverges to  $+\infty$  as  $\hat{p} \downarrow c_H$ , there exists a unique  $p_0 \in (c_H, p_H)$  such that

$$\frac{p_0 - c_L}{p_0 - c_H} = \frac{\pi_L^* + A + f}{\pi_H^* + A} \quad (7)$$

Simplifying (7) we have

$$p_0 = \frac{c_H(\pi_L^* + A + f) - c_L(\pi_H^* + A)}{\pi_L^* - \pi_H^* + f} \quad (8)$$

which reduces to

$$p_0 = \frac{(\alpha\Delta V + A + f)\Delta c}{f + \frac{\alpha}{2}\Delta c} + c_L \quad (9)$$

It is easy to see that the deviating firm will not set price cannot be below  $\underline{p}_L + \Delta V$  because if the latter price is in  $S$ , the firm sells to all buyers with probability one. Note that

$$\underline{p}_L + \Delta V = c_L + (1 + \alpha)\Delta V > c_L + \Delta V = \bar{p}_L$$

Therefore, without loss of generality, we confine attention to deviation price  $\hat{p}$  satisfying

$$\begin{aligned} \hat{p} &\geq \underline{p}_L + \Delta V, \text{ if } p_0 < \underline{p}_L + \Delta V \\ &> p_0, \text{ if } p_0 \geq \underline{p}_L + \Delta V. \end{aligned} \quad (10)$$

Using (9), one can show that:  $p_0 \geq \underline{p}_L + \Delta V$  if, and only if,

$$A \geq f[(1 + \alpha)\lambda - 1] - \left(\frac{\alpha(1 - \alpha)}{2}\right)\Delta V \quad (11)$$

Now, deviation to any price  $\hat{p}$  satisfying (10) is strictly gainful for a type  $H$  firm if, and only if,

$$\begin{aligned} \pi_H^* &< (\hat{p} - c_H)[\alpha + (1 - \alpha)F(\hat{p} - \Delta V)] - A \\ &= \alpha(\hat{p} - c_H) \left[ \frac{\Delta V}{\hat{p} - \Delta V - c_L} \right] - A \\ &= \frac{\hat{p} - c_H}{\hat{p} - \Delta V - c_L} \pi_L^* - A \end{aligned}$$

i.e.,

$$\frac{\Delta V - \Delta c}{\hat{p} - c_H} > 1 - \frac{\pi_L^*}{\pi_H^* + A} \quad (12)$$

Observe that the left hand side of (12) is strictly decreasing in  $\hat{p}$ . Deviation to any price  $\hat{p}$  satisfying (10) is strictly gainful for a  $L$  type firm if, and only if,

$$\begin{aligned} \pi_L^* &< (\hat{p} - c_L)[\alpha + (1 - \alpha)F(\hat{p} - \Delta V)] - A - f \\ &= \alpha(\hat{p} - c_L) \left[ \frac{\Delta V}{\hat{p} - \Delta V - c_L} \right] - A - f \\ &= \frac{\hat{p} - c_L}{\hat{p} - \Delta V - c_L} \pi_L^* - A - f \end{aligned}$$

i.e.,

$$\hat{p} - c_L < [1 + \frac{\pi_L^*}{A + f}]\Delta V \quad (13)$$

First, consider the case where

$$A < f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V \quad (14)$$

i.e. (11) does not hold so that

$$p_0 < \underline{p}_L + \Delta V.$$

In this case, there is a strictly gainful deviation for at least one type i.e., either (12) or (13) holds at some  $\hat{p} \in S$  satisfying (10) if, and only if, either (12) or (13) holds at  $\hat{p} = \underline{p}_L + \Delta V$  i.e., either

$$\frac{\Delta V - \Delta c}{\underline{p}_L + \Delta V - c_H} < 1 - \frac{\pi_L^*}{\pi_H^* + A} \quad (15)$$

or,

$$\underline{p}_L + \Delta V < [1 + \frac{\pi_L^*}{A + f}] \Delta V + c_L \quad (16)$$

Using  $\underline{p}_L - c_L = \pi_L^*$ , (15) reduces to

$$A < \left( \lambda - (1 - \frac{\alpha}{2}) \right) \Delta c = A_2 \quad (17)$$

and (16) reduces to:

$$A + f < \lambda \Delta c. \quad (18)$$

It is easy to check that under (14), (18) implies (17). This is obvious if  $f > (1 - \frac{\alpha}{2}) \Delta c$ . On the other hand, if  $f \leq (1 - \frac{\alpha}{2}) \Delta c$ , we have from (14):

$$\begin{aligned} A &< f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2} \Delta V \\ &\leq (1 - \frac{\alpha}{2}) \Delta c [(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2} \Delta V \\ &= \left( \lambda - (1 - \frac{\alpha}{2}) \right) \Delta c = A_2 \end{aligned}$$

so that (17) holds. Thus, we have shown that if (6) and (14) hold, then a symmetric pure price signaling equilibrium exists if, and only if,

$$A \geq A_2.$$

Continue to assume that (6) holds and now suppose that (14) does not hold i.e.,

$$A \geq f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2} \Delta V \quad (19)$$

so that  $p_0 \geq \underline{p}_L + \Delta V$ . In this case, there is strictly gainful deviation for at least one type i.e., either (12) or (13) holds at  $\hat{p} = p_0$  i.e., either

$$\frac{\Delta V - \Delta c}{p_0 - c_H} > 1 - \frac{\pi_L^*}{\pi_H^* + A} \quad (20)$$

or,

$$p_0 < [1 + \frac{\pi_L^*}{A + m}] \Delta V + c_L \quad (21)$$

Using (7), (20) reduces to:

$$p_0 < \frac{\Delta V}{A + f} (\pi_H^* + A) + c_H \quad (22)$$

and using (9) this reduces to:

$$A < \frac{\alpha}{2}\Delta V + f[\lambda - 1] \quad (23)$$

Using (9), one can check that (21) also reduces to (23). Thus, we have shown that if (6) holds and (14) does not hold, then a symmetric pure price signaling equilibrium exists if, and only if,

$$A \geq \frac{\alpha}{2}\Delta V + f[\lambda - 1] \quad (24)$$

This completes the proof of part (ii) of the lemma. ■

Define two critical levels of the disclosure cost  $A_1$  and  $A_2$  as follows:

$$A_1 = \alpha(\Delta V - \frac{\Delta c}{2}) \text{ and } A_2 = \Delta V - (1 - \frac{\alpha}{2})\Delta c$$

It is easy to check that  $0 < A_1 < A_2$ . For each  $A \in (0, A_2)$  define  $f^*(A)$ , a critical level of the cost of false disclosure, by

$$\begin{aligned} f^*(A) &= \frac{A - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c} \Delta c, \text{ for } A_1 \leq A < A_2 \\ &= \frac{A\Delta c}{2\Delta V - \Delta c}, \text{ for } 0 \leq A \leq A_1 \end{aligned}$$

Observe that  $f^*(A)$  is continuous and strictly increasing in  $A$  on  $(0, A_2)$ ,  $f^*(A) \rightarrow 0$  as  $A \rightarrow 0$ , and  $f^*(A_2) = (1 - \frac{\alpha}{2})\Delta c$ .

**Proposition 5** *A symmetric equilibrium with pure price signaling i.e., one where firms do not disclose directly, exists if, and only if, either (i)  $A \geq A_2$ , or (ii)  $A < A_2$  and  $f \leq f^*(A)$ .*

**Proof.** We first show that a symmetric pure price signaling equilibrium exists if  $A \geq A_2$ . Using part (ii.a) of Lemma 4, we know this true if

$$A < f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V$$

Now, suppose\*

$$A \geq f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V \quad (25)$$

It is easy to check that if

$$f \leq (1 - \frac{\alpha}{2})\Delta c,$$

then

$$\frac{\alpha}{2}\Delta V + f[\lambda - 1] \leq \left[ \lambda - (1 - \frac{\alpha}{2}) \right] \Delta c = A_2$$

so that  $A \geq A_2$  implies that the condition in part (ii.b) of Lemma 4 is satisfied and a symmetric pure price signaling equilibrium exists. On the other hand, if

$$f > (1 - \frac{\alpha}{2})\Delta c$$



then

$$\alpha\lambda f - \alpha(1 - \frac{\alpha}{2})\Delta V > 0$$

so that from (25),

$$\begin{aligned} A &\geq f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V \\ &= \frac{\alpha}{2}\Delta V + f[\lambda - 1] + \{\alpha\lambda f - \alpha(1 - \frac{\alpha}{2})\Delta V\} \\ &> \frac{\alpha}{2}\Delta V + f[\lambda - 1] \end{aligned}$$

which implies that once again the condition in part (ii.b) of Lemma 4 is satisfied and a symmetric pure price signaling equilibrium exists. Next, we show that if  $A < A_2$ , then a symmetric pure price signaling equilibrium exists if, and only if,  $f \leq f^*(A)$ . To do this first consider  $A \in [A_1, A_2)$ . Then,

$$f^*(A) \leq \lim_{A \uparrow A_2} f^*(A) = (1 - \frac{\alpha}{2})\Delta c$$

so that  $f \leq f^*(A)$  implies

$$f \leq (1 - \frac{\alpha}{2})\Delta c$$

and using this,

$$f[(1 + \alpha)\lambda - 1] - \frac{\alpha(1 - \alpha)}{2}\Delta V \leq \frac{\alpha}{2}\Delta V + f[\lambda - 1]. \quad (26)$$

Further, for  $A \in [A_1, A_2)$ ,  $f^*(A) = \frac{A - \frac{\alpha}{2}\lambda\Delta c}{\lambda - 1}$  so that

$$f \leq f^*(A) \Leftrightarrow A \geq f(\lambda - 1) + \frac{\alpha}{2}\Delta V \quad (27)$$

Combining (26) and (27), we see that the condition in part (ii.b) of Lemma 4 is satisfied and a symmetric pure price signaling equilibrium exists. On the other hand, if  $f > f^*(A)$ , then (using (27)):

$$A < f(\lambda - 1) + \frac{\alpha}{2}\Delta V \quad (28)$$

and thus the condition in part(ii.b) of Lemma 4 does not hold. As  $A < A_2$ , using part (ii.a) of Lemma 4 does not hold. Further, for  $A \in [A_1, A_2)$ ,

$$f^*(A) \geq f^*(A_1) = \frac{\alpha(\lambda - \frac{1}{2})\Delta c - \frac{\alpha}{2}\lambda\Delta c}{\lambda - 1} = \frac{\alpha}{2}\Delta c$$

so that  $f > f^*(A)$  implies

$$f > \frac{\alpha}{2}\Delta c \quad (29)$$

so that using (28) and (29) we have

$$\frac{A}{f} < (\lambda - 1) + \frac{\alpha}{2f}\Delta V < 2\lambda - 1$$

which implies that the condition in part (i) of Lemma 4 does not hold. It follows that no symmetric pure price signaling equilibrium exists. Finally, consider  $A \in (0, A_1)$ . Suppose  $f \leq m^*(A)$ . As  $f^*(A) = \frac{A}{2\lambda-1}$ ,

$$f \leq \frac{A}{2\lambda-1}$$

i.e., the condition in part (i) of Lemma 4 holds and therefore, a symmetric pure price signaling equilibrium exists. Now, suppose  $f > f^*(A)$ . Then,

$$A < (2\lambda - 1)f. \quad (30)$$

As  $A < A_2$ , the condition in part (ii.a) of Lemma 4 does not hold. Further, for  $A < A_1 = \alpha(\lambda - \frac{1}{2})\Delta c$ ,

$$f^*(A) = \frac{A}{2\lambda-1} > \frac{A - \frac{\alpha}{2}\Delta V}{\lambda-1}$$

so that  $f > f^*(A)$  implies

$$f > \frac{A - \frac{\alpha}{2}\Delta V}{\lambda-1}$$

so that the condition in part(ii.b) of Lemma 4 does not hold. Thus, a symmetric pure price signaling equilibrium does not exist. This concludes the proof. ■

**Lemma 6** (i) *The conditions:*

$$A + f \leq \frac{\alpha}{2}\Delta V \quad (31)$$

$$\frac{A}{f} \leq \left( 2\lambda - 1 - \frac{2(A+f)}{\alpha\Delta c} \right) \quad (32)$$

are necessary and sufficient for existence of an equilibrium (hereafter, equilibrium P1) where high quality firms disclose for sure and charge a deterministic price given by

$$p_H^A = c_L + 2\Delta V - \frac{2(A+f)}{\alpha}$$

while low quality firms do not disclose and randomize prices over the interval  $[\underline{p}_L, \bar{p}_L]$ , with  $\underline{p}_L = \alpha\bar{p}_L + (1-\alpha)c_L > c_L$  and  $\bar{p}_L = p_H - \Delta V$ . In this equilibrium buyers buy high quality product only in the state where both firms are of high quality leading to a consumption distortion (in addition to the distortion arising through firms incurring a disclosure cost  $A$ ).

(ii) *The conditions*

$$\frac{\alpha}{2}\Delta V \leq A + f \leq \left( 1 - \frac{\alpha}{2} \right) \Delta V \quad (33)$$

$$A \leq f(\lambda - 1) \quad (34)$$

are necessary and sufficient for the existence of an equilibrium (hereafter, equilibrium P2) where high quality firms disclose for sure and charge a deterministic price  $p_H = c_L + \Delta V$  and low quality firms charge a deterministic price  $p_L = c_L$ ; in the state where one firm is of high quality type and the other is of low quality type, buyers buy high quality with probability  $\beta \in [0, 1]$  given by

$$\beta = \frac{A + m}{(1-\alpha)\Delta V} - \frac{\alpha}{2(1-\alpha)}$$

(iii) The conditions

$$A \leq (1 - \alpha)(\Delta V - \Delta c) \quad (35)$$

$$A + f \geq [(1 - \alpha)\lambda + \frac{\alpha}{2}]\Delta c \quad (36)$$

are necessary and sufficient for an equilibrium (hereafter, equilibrium P3) where high quality firms disclose for sure and randomize prices over a support whose upper bound  $\bar{p}_H = c_L + \Delta V$  while low quality firms set a deterministic price  $p_L = c_L$ , and only sell in the state where both firms produce low quality (no consumption distortion). High quality prices are distributed continuously over an interval  $[p_H, \bar{p}_H]$  if

$$A + f \geq (1 - \alpha)\Delta V + \alpha\Delta c \quad (37)$$

and there is strictly positive probability mass on the lower bound of high quality prices if

$$(1 - \alpha)\Delta V + \alpha\Delta c > A + f \geq [(1 - \alpha)\lambda + \frac{\alpha}{2}]\Delta c \quad (38)$$

**Proof.** (i) The profits in such an equilibrium are given by

$$\pi_H^* = \frac{\alpha}{2}(p_H^A - c_H) - A = \pi_H^A - A$$

and

$$\pi_L^* = \alpha(p_H^A - \Delta V - c_L).$$

Further to ensure that that low quality firms should not have an incentive to imitate the high quality behavior, we need a lower bound on  $p_H^A$ :

$$\frac{\alpha}{2}(p_H^A - c_L - 2\Delta V) + A + f \geq 0. \quad (39)$$

To determine  $p_H^A$  we have to consider the main other deviations, namely for any of the firms to set an out-of-equilibrium price  $\hat{p} \in (\bar{p}_L^A, p_H^A)$ . It is clear that if consumers believe that a high quality firm has deviated, then they will buy and this makes such a deviation profitable. Therefore, this equilibrium can be sustained if and only if out of equilibrium beliefs regard deviating firm that discloses and chooses  $\hat{p} \in (\bar{p}_L^A, p_H^A)$  as being of low quality with sufficiently high probability. This is consistent with the D1 criterion if, and only if, a low quality firm is indifferent between its equilibrium actions and mimicking the high quality behavior i.e.,

$$\pi_L^* = \alpha(p_H^A - \Delta V - c_L) = \frac{\alpha}{2}(p_H^A - c_L) - A - f$$

which yields

$$p_H^A = c_L + 2\Delta V - \frac{2(A + f)}{\alpha} \quad (40)$$

To see this let  $q_H(\hat{p}), q_L(\hat{p})$  be the expected quantity that a  $H$  and a  $L$  type firm must sell respectively in order to be indifferent between this deviation and not deviating from their equilibrium strategies. If  $q_H(\hat{p}) < q_L(\hat{p})$ , then D1 refinement suggests that buyers must assign probability one to the deviating firm's type being of  $H$  type (if  $q_H(\hat{p}) > q_L(\hat{p})$ , the belief must assign probability one to  $L$  type). It is easy to check that:

$$q_H(\hat{p}) = \frac{\pi_H^A}{(\hat{p} - c_H)}, q_L(\hat{p}) = \frac{\pi_L^* + A + f}{(\hat{p} - c_L)}$$

so that

$$q_H(\hat{p}) \geq q_L(\hat{p})$$

if, and only if,

$$\frac{\hat{p} - c_L}{\hat{p} - c_H} \geq \frac{\pi_L^* + A + f}{\pi_H^A} \quad (41)$$

Observe that the left hand side of (41) is continuous and strictly decreasing in  $\hat{p}$ . Thus, for the above to be an equilibrium, and that (41) holds for all deviations, we should have (41) holds for  $\hat{p} = p_H^A$ . Thus, we have that the following must hold:

$$\frac{p_H^A - c_L}{p_H^A - c_H} \geq \frac{\alpha(p_H^A - \Delta V - c_L) + A + f}{\frac{\alpha}{2}(p_H^A - c_H)}.$$

which holds, if and only if,

$$\frac{\alpha}{2}(p_H^A - c_L - 2\Delta V) + A + f \leq 0.$$

Combining with (39) we have (40).

We also need to ensure that firms do not deviate to out-of-equilibrium price  $\hat{p} \in (\bar{p}_L^A, p_H^A)$  without disclosing which is possible if, and only if, out of equilibrium beliefs regard deviating firm that does not disclose and chooses  $\hat{p} \in (\bar{p}_L^A, p_H^A)$  as being of low quality with sufficiently high probability. We show that such a requirement is consistent with the D1 criterion if, and only if, (32) holds. The condition to be satisfied is that for all  $\hat{p} \in (\bar{p}_L^A, p_H^A)$

$$\frac{\hat{p} - c_L}{\hat{p} - c_H} \geq \frac{\pi_L^*}{\pi_H^A - A}$$

or

$$\frac{p_H^A - c_L}{p_H^A - c_H} \geq \frac{\alpha(p_H^A - \Delta V - c_L)}{\frac{\alpha}{2}(p_H^A - c_H) - A}.$$

Given (40) this reduces to

$$f(p_H^A - c_H) \geq A\Delta c.$$

and using (40) again, we have:

$$f \left( 2\Delta V - \Delta c - \frac{2(A + f)}{\alpha} \right) \geq A\Delta c$$

i.e., condition (32). Finally, we need to ensure that both types make negative profits. Given (40),  $\pi_L^* \geq 0$  if, and only if,

$$A + f \leq \frac{\alpha}{2}\Delta V.$$

i.e., (31) holds. The condition  $\pi_H^* \geq 0$  is equivalent to  $2A + f \leq \alpha(\lambda - \frac{1}{2})\Delta c$ , which is implied by  $\pi_L^* \geq 0$  as long as

$$A \leq \frac{\alpha}{2}(\Delta V - \Delta c)$$

which, in turn, is implied by (31) and (32); observe that if  $f \geq \frac{\alpha}{2}\Delta c$ , then (31) implies (32), while the reverse is true if  $f \leq \frac{\alpha}{2}\Delta c$ . Thus, it follows that the condition  $\pi_H^* \geq 0$  is satisfied.

(ii) To deter deviation by high quality firm to a price below  $p_H = c_L + \Delta V$  with disclosure, we need out of equilibrium beliefs to regard such a deviating firm to be of low quality (with sufficiently high probability); following similar arguments as in part (i), such beliefs are consistent with the D1 criterion if and only if low quality type is indifferent between setting  $p_L = c_L$  and deviating to  $p_H = c_L + \Delta V$  with false disclosure:

$$\left[ (1 - \alpha) \beta + \frac{\alpha}{2} \right] \Delta V - A - f = 0.$$

This determines  $\beta$  to be equal to

$$\beta = \frac{A + m}{(1 - \alpha) \Delta V} - \frac{\alpha}{2(1 - \alpha)}$$

As for deviation to price below  $p_H = c_L + \Delta V$  without disclosure, as low quality type earns zero profit, out of equilibrium beliefs that assign probability one to such a firm being of low quality type is always consistent with the D1 criterion. As  $0 \leq \beta \leq 1$ , we must have (33). Further  $\pi_H^* \geq 0$  if, and only if,

$$\left[ (1 - \alpha) \gamma + \frac{\alpha}{2} \right] (\Delta V - \Delta c) - A \geq 0,$$

which reduces to (34).

(iii) As high quality type is undercut with probability one at price  $\bar{p}_H = c_L + \Delta V$  in the state where rival is of high quality type, the equilibrium profit of high quality type is given by:

$$\pi_H = (\Delta V - \Delta c)(1 - \alpha) - A$$

and this is non-negative if, and only if, (35) holds. To deter high quality firm from deviating to not disclosing, out-of-equilibrium beliefs must regard any such deviating firm as low quality with sufficiently high probability and this is consistent with the D1 criterion as low quality firms earn zero profit in this equilibrium. First, consider the equilibrium where high quality price distribution has no mass point. The equilibrium distribution of high quality prices is given by  $F$  where:

$$(p_H - c_H)[(1 - \alpha) + \alpha(1 - F(p_H))] = (\Delta V - \Delta c)(1 - \alpha), p_H \in [\underline{p}_H, \bar{p}_H] \quad (42)$$

and

$$\underline{p}_H = (\Delta V - \Delta c)(1 - \alpha) + c_H$$

If a low quality firm advertises and charges  $p_H \in [\underline{p}_H, \bar{p}_H]$ , then his deviation profit is

$$\begin{aligned} & (p_H - c_L)[(1 - \alpha) + \alpha(1 - F(p_H))] - A - f \\ = & (\Delta V - \Delta c)(1 - \alpha) \left[ \frac{p_H - c_L}{p_H - c_H} \right] - (A + f) \\ \leq & (\Delta V - \Delta c)(1 - \alpha) \left[ \frac{\underline{p}_H - c_L}{\underline{p}_H - c_H} \right] - (A + f) \\ = & \underline{p}_H - c_L - (A + f) \\ = & \Delta V(1 - \alpha) + \alpha \Delta c - (A + f) \end{aligned}$$

so that the deviation is not gainful if  $f \geq \Delta V(1 - \alpha) + \alpha \Delta c - A$  i.e., (37). It is easy to check that under (37), a low quality firm will not advertise and deviate to a price less than  $\underline{p}_H$  even if

he is thought of as a high quality firm with probability one. Under the out-of-equilibrium beliefs, a low quality firm will not gain strictly (in fact, sells zero) if it does not advertise and charges a price higher than  $c_L$ . Finally, it is easy to check that a high quality firm has no incentive to deviate unilaterally. Next, consider the equilibrium where high quality price distribution has a probability mass point; from Lemma 2 the mass point can only be at the at the lower bound of its price distribution and this lower bound price must be an isolated point in the support. In particular, high quality types randomize over prices in the interval  $[\underline{p}_H, c_L + \Delta V)$  with probability  $\kappa$ , and with probability  $1 - \kappa$ , they charge a deterministic price equal to  $\hat{p}_H < \underline{p}_H$ . High quality type is indifferent between prices in its equilibrium strategy if, and only if,

$$\underline{p}_H = c_H + \frac{1 - \alpha}{1 - \alpha + \alpha\kappa}(\Delta V - \Delta c)$$

and

$$\hat{p}_H = c_H + \frac{1 - \alpha}{1 - \alpha + \alpha\frac{1+\kappa}{2}}(\Delta V - \Delta c).$$

Out of equilibrium beliefs that regards any firm that does not disclose as a low quality firm for sure is consistent with D1 criterion as low quality firms earn zero profit in equilibrium; this deters the high quality type from deviating to not disclosing. To deter high quality from deviating from price  $\hat{p}_H$  while disclosing, out of equilibrium beliefs must regard such a deviant as low quality with sufficiently high probability and this is consistent with D1 if and only low quality types are indifferent their equilibrium strategy and and imitating the high quality's action of disclosing and charging the deterministic price  $\hat{p}_H$ . This yields

$$\left(1 - \alpha + \alpha\frac{1 + \kappa}{2}\right) \Delta c + (1 - \alpha)(\Delta V - \Delta c) - A - m = 0,$$

so that

$$\kappa = \frac{2(A + m - (1 - \alpha)\Delta V - \frac{\alpha}{2}\Delta c)}{\alpha\Delta c}.$$

$\kappa \in [0, 1]$  if, and only if, (38) holds. As high quality type is indifferent over prices in the range  $[\underline{p}_H, c_L + \Delta V)$  and the price  $\hat{p}_H$ , it is easy to show that a low quality type (lower marginal cost) strictly prefers to sell at the lowest price  $\hat{p}_H$  than any price in the range  $[\underline{p}_H, c_L + \Delta V)$  and at that price it earns the same payoff as its equilibrium strategy. Therefore, a low quality type can only be worse off by disclosing and deviating to any price in the interval  $[\underline{p}_H, c_L + \Delta V)$ . Further, high quality equilibrium profit is non-negative if, and only if, (35) holds. It is easy to check now that no other deviation can be gainful for either type. This completes the proof. ■

**Proposition 7** (a) A pure disclosure equilibrium (where high quality firms disclose for sure) exists if, and only if,  $A \leq \tilde{A}(f)$  where

$$\tilde{A}(f) = \left\{ \begin{array}{ll} \left( \frac{2\alpha\lambda\Delta c}{\alpha\Delta c + 2f} - 1 \right) f & \text{if } 0 \leq f < \frac{\alpha}{2}\Delta c \\ (\lambda - 1)f & \text{if } \frac{\alpha}{2}\Delta c \leq f < (1 - \frac{\alpha}{2})\Delta c \\ (1 - \frac{\alpha}{2})\Delta V - f & \text{if } (1 - \frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c \\ (1 - \alpha)(\Delta V - \Delta c) & \text{if } f > \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c \end{array} \right\}$$

(b) A non-distortionary pure disclosure equilibrium (i.e., one where buyers always buy high quality as long as there is a high quality seller in the market) exists if, and only if, in addition to  $A \leq \tilde{A}(f)$ .

$$f \geq \left(1 - \frac{\alpha}{2}\right) \Delta c \quad (43)$$

and

$$(1 - \alpha)(\Delta V - \Delta c) \geq A \geq \max\{(1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} - f, 0\} \quad (44)$$

(c) A fully distortionary pure disclosure equilibrium (i.e., one where buyers always buy low quality as long as there is a low quality seller in the market) exists if, and only if, in addition to  $A \leq \tilde{A}(f)$

$$f \leq \frac{\alpha}{2} \Delta V \quad (45)$$

and

$$A \leq \frac{\alpha}{2} \Delta V - f, \text{ for } \frac{\alpha}{2} \Delta c \leq f \leq \frac{\alpha}{2} \Delta V \quad (46)$$

(d) A partially distortionary pure disclosure equilibrium (i.e., where a low quality seller shares the market with a high quality rival) exists, if, and only if, in addition to  $A \leq \tilde{A}(f)$

$$\frac{\alpha}{2} \Delta c \leq f < \left(1 - \frac{\alpha}{2}\right) \Delta V, A > \frac{\alpha}{2} \Delta V - f \quad (47)$$

and

$$A < \left(1 - \frac{\alpha}{2}\right) \Delta V - f, \text{ if } \left(1 - \frac{\alpha}{2}\right) \Delta c \leq f < \left(1 - \frac{\alpha}{2}\right) \Delta V \quad (48)$$

**Proof.** (a) Consider a pure disclosure equilibrium. There two kinds of such equilibria: (i) Low quality type makes strictly positive profit; (ii) Low quality type makes zero profit.

(i) Consider an equilibrium where low quality types make strictly positive profit. We will show that high quality types must charge a deterministic price. Suppose to the contrary that high quality type randomize over prices and low quality type makes strictly positive profit. Then, a low quality type must sell with strictly positive probability in the state where rival is of high type (if low type sells only when rival is low type, price competition drives low quality price to  $c_L$ ) and randomize its price between a lower bound  $\underline{p}_L$  and an upper bound  $\bar{p}_L$ . Further, in a symmetric equilibrium there cannot be a mass point at  $\bar{p}_L$  so that at price  $\bar{p}_L$ , a low quality type sells only in the state where rival is high type. As low type must sell with strictly positive probability at price  $\bar{p}_L$

$$\bar{p}_L \leq \bar{p}_H - \Delta V \quad (49)$$

We now claim that in such an equilibrium high quality type must charge a deterministic price. Using Lemma 2, the high type can have a mass point at only the lowest bound of the support of its prices  $\underline{p}_H$ . Thus, if high quality type randomizes over prices then at the upper bound  $\bar{p}_H$  of the support of high quality prices, it sells only in the state where rival is low quality. As the gross profit for a high quality type must cover disclosure cost, at price  $\bar{p}_H$  a high quality seller must sell strictly positive quantity in the state where rival is low quality type so that

$$\bar{p}_H \leq \bar{p}_L + \Delta V \quad (50)$$

so that using (49), we have

$$\bar{p}_H = \bar{p}_L + \Delta V \quad (51)$$

As there is no mass point at  $\bar{p}_H$  or  $\bar{p}_L$ , low quality type is undercut with probability one at price  $\bar{p}_L$  and must earn zero expected profit in equilibrium, a contradiction. Thus, high quality type must charge a deterministic price  $\tilde{p}_H$ . Further, (51) must still hold i.e.,

$$\bar{p}_L = \tilde{p}_H - \Delta V.$$

The unique equilibrium of this kind is equilibrium P1 characterized in Lemma 6 and it exists if, and only if,

$$A + f \leq \frac{\alpha}{2} \Delta V \quad (52)$$

and

$$\frac{A}{f} \leq \left[ 2\lambda - 1 - \frac{2(A+f)}{\alpha \Delta c} \right] \quad (53)$$

It is easy to check that (52) and (53) hold if, and only if,

$$0 < f \leq \frac{\alpha}{2} \Delta V \quad (54)$$

and

$$A \leq \left[ \frac{2\alpha \Delta V}{\alpha \Delta c + 2f} - 1 \right] f \text{ if } 0 \leq f < \frac{\alpha}{2} \Delta c \quad (55)$$

$$\leq \frac{\alpha}{2} \Delta V - f \text{ if } \frac{\alpha}{2} \Delta c \leq f \leq \frac{\alpha}{2} \Delta V \quad (56)$$

and these are the necessary and sufficient for an equilibrium where (high quality types disclose for sure and) low quality types make strictly positive profit.

(ii) Now, consider an equilibrium where (high quality types disclose for sure and) low quality types make zero profit. As the equilibrium is symmetric, a low quality firm must sell in the state where rival is of low quality type, and as they earn zero profit, low quality types must charge a deterministic price  $c_L$ . The upper bound  $\bar{p}_H$  of the support of the distribution of high quality prices must satisfy

$$\bar{p}_H \leq c_L + \Delta V \quad (57)$$

for otherwise, a low quality firm can deviate to a price above  $c_L$  and sell strictly positive expected quantity. There are two sub-cases: (ii.a) Low quality type sells strictly positive quantity in the state where rival is high quality; (ii.b) Low quality type sells only in the state where rival is low quality.

(ii.a) If high quality types randomize over prices, then using Lemma 2, they can have no probability mass on  $\bar{p}_H$  so that (57) implies that low quality type is undercut with probability one in the state where rival is high quality type. Therefore, high quality types must charge a deterministic price  $\tilde{p}_H$  where

$$\tilde{p}_H = c_L + \Delta V.$$

Buyers are then indifferent between buying high quality at price  $\tilde{p}_H$  and low quality at price  $c_L$ . In the state where one firm is of high quality and the other is of low quality, a fraction  $\beta \in [0, 1)$  of buyers buy high quality and the rest buy low quality. The unique equilibrium of this kind is equilibrium P2 characterized in Lemma 6(ii) and it exists if, and only if,

$$\frac{\alpha}{2} \Delta V \leq A + f < \left( 1 - \frac{\alpha}{2} \right) \Delta V \quad (58)$$



and

$$A \leq f(\lambda - 1) \quad (59)$$

In particular,

$$\beta = \frac{1}{1 - \alpha} \left[ \frac{A + f}{\Delta V} - \frac{\alpha}{2} \right] \quad (60)$$

Note that  $\beta = 0$  i.e., low quality type sells to the entire market when rival is high quality in this kind of an equilibrium if, and only if, in addition to (59), the first inequality in (58) holds with an equality i.e.,

$$\frac{\alpha}{2} \Delta V = A + f$$

It is easy to check that the necessary and sufficient conditions (58) and (59) hold if, and only if:

$$\frac{\alpha}{2} \Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right) \Delta V \quad (61)$$

and

$$\frac{\alpha}{2} \Delta V - f \leq A \leq f(\lambda - 1), \text{ if } \frac{\alpha}{2} \Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right) \Delta c \quad (62)$$

$$\frac{\alpha}{2} \Delta V - f \leq A < \left(1 - \frac{\alpha}{2}\right) \Delta V - f, \text{ if } \left(1 - \frac{\alpha}{2}\right) \Delta c \leq f < \left(1 - \frac{\alpha}{2}\right) \Delta V. \quad (63)$$

(ii.b) There are two further sub-cases: (ii.b.1) High quality charges a deterministic price; (ii.b.2) High quality randomizes over prices.

(ii.b.1) It is easy to check that an equilibrium of type (ii.b.1) must be essentially identical to a type (ii.a) equilibrium (i.e., a P2 equilibrium) with  $\beta = 1$ . Once again, as shown in Lemma 6(ii), a necessary and sufficient conditions for this is knife edge: (59) holds and the second inequality in (58) holds with equality i.e.,

$$A + f = \left(1 - \frac{\alpha}{2}\right) \Delta V \quad (64)$$

These, in turn, hold if, and only if. (64) holds and

$$\left(1 - \frac{\alpha}{2}\right) \Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right) \Delta V \quad (65)$$

(ii.b.2) We claim that here too, (57) must hold with equality i.e., the upper bound  $\bar{p}_H$  of the support of the distribution of high quality prices must satisfy

$$\bar{p}_H = c_L + \Delta V. \quad (66)$$

Suppose to the contrary that  $\bar{p}_H < c_L + \Delta V$ . As there can be a mass point only at the lower bound of high quality price distribution (Lemma 2), at price  $\bar{p}_H$  high quality seller sells only in the state where rival is low quality. We now claim that D1 refinement implies that a firm disclosing and charging a price  $\hat{p} \in (\bar{p}_H, c_L + \Delta V)$  must be regarded as a high quality type. To see this consider such a  $\hat{p}$ . Let  $q^\tau(\hat{p})$  be the (expected) quantity that must be sold by a type  $\tau$  at price  $\hat{p}$  to be indifferent between its equilibrium strategy and deviation to disclosing and charging  $\hat{p}$ . Then

$$\begin{aligned} (\bar{p}_H - c_H)(1 - \alpha) - A &= (\hat{p} - c_H)q^H(\hat{p}) - A \\ 0 &= (\hat{p} - c_L)q^L(\hat{p}) - A - f \end{aligned}$$

so that

$$\frac{q^H(\hat{p})}{q^L(\hat{p})} < 1 \quad (67)$$

if

$$(1 - \alpha)((\bar{p}_H - c_H) < (A + f) \left( \frac{\hat{p} - c_H}{\hat{p} - c_L} \right) \quad (68)$$

Observe that  $\left( \frac{\hat{p} - c_H}{\hat{p} - c_L} \right)$  is strictly increasing in  $\hat{p}$  so that (68) must hold for some  $\hat{p} \in (\bar{p}_H, c_L + \Delta V)$  if

$$(1 - \alpha)((\bar{p}_H - c_H) \leq (A + f) \left( \frac{\bar{p}_H - c_H}{\bar{p}_H - c_L} \right)$$

which holds as long as

$$(1 - \alpha)((\bar{p}_H - c_L) - (A + f) \leq 0$$

and this inequality must hold in such an equilibrium to ensure that low quality type does not deviate to disclosing and imitating high quality price  $\bar{p}_H$ . Thus, (67) holds for  $\hat{p} \in (\bar{p}_H, c_L + \Delta V)$  and therefore, D1 criterion implies that out of equilibrium belief should regard any firm that discloses and charges  $\hat{p} \in (\bar{p}_H, c_L + \Delta V)$  as high quality type. But this implies that the high quality type can deviate to such a  $\hat{p}$  while disclosing, sell the same expected quantity as at  $\bar{p}_H$  and earn strictly higher profit, a contradiction. Thus, any equilibrium where high quality randomizes over prices and low quality type sells only in the state where rival is of low quality type, is one where (66) holds. As there can only be a mass point at the lower bound of the high quality distribution, it is easy to check that the high quality type must randomize with a continuous distribution over an interval whose upper bound is  $c_L + \Delta V$  with or without a positive mass at an isolated price strictly below this interval. All equilibria of this kind are of type P3 characterized in Lemma 6(iii) and it is shown that necessary and sufficient conditions for existence of such equilibria are

$$A \leq (1 - \alpha)(\Delta V - \Delta c) \quad (69)$$

$$A + f \geq (1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} \quad (70)$$

It is easy to check that (69) and (70) hold if, and only if,

$$f \geq \left(1 - \frac{\alpha}{2}\right) \Delta c \quad (71)$$

and

$$\max \left\{ \left( (1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \right), 0 \right\} \leq A \leq (1 - \alpha)(\Delta V - \Delta c) \quad (72)$$

Using the necessary and sufficient conditions for all possible pure disclosure equilibria outlined above (in particular, conditions (54)-(56), (61)-(63), (64)-(65), (71)-(72)) one can see that a pure disclosure equilibrium i.e., type (i) or type (ii) equilibrium exists if, and only if,  $A \leq \tilde{A}(f)$ . The details for various ranges of values of  $f$  are as follows:

- (a)  $0 \leq f < \frac{\alpha}{2} \Delta c$  : only an equilibrium of type (i) can hold and it does (for that range of  $f$ ) if, and only if, (55) holds i.e.,  $A \leq \left( \frac{2\alpha\lambda\Delta c}{\alpha\Delta c + 2f} - 1 \right) f$
- (b)  $\frac{\alpha}{2} \Delta c \leq f < (1 - \frac{\alpha}{2}) \Delta c$  : a type (i) equilibrium exists if, and only if (56) holds i.e.,  $A \leq \frac{\alpha}{2} \Delta V - f$  while a type (ii.a) equilibrium holds if and only if (62) holds i.e.,  $\frac{\alpha}{2} \Delta V - f \leq A \leq f(\lambda - 1)$ ;

a type (ii.b) equilibrium does not exist. As  $f \geq \frac{\alpha}{2}\Delta c$  implies  $\frac{\alpha}{2}\Delta V - f \leq f(\lambda - 1)$ , a pure disclosure equilibrium exists if, and only if,  $A \leq f(\lambda - 1)$ .

(c)  $(1 - \frac{\alpha}{2})\Delta c \leq f \leq \frac{\alpha}{2}\Delta V$  : a type (i) equilibrium exists if, and only if (56) holds i.e.,  $A \leq \frac{\alpha}{2}\Delta V - f$  and types (ii.a) or (ii.b.1) equilibria exist if and only if (63) holds i.e.,  $\frac{\alpha}{2}\Delta V - f \leq A \leq (1 - \frac{\alpha}{2})\Delta V - f$ ; thus, a pure disclosure equilibrium exists for all  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ . A type (ii.b.2) equilibrium exists, if and only if (72) holds i.e.,  $(1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq A \leq (1 - \alpha)(\Delta V - \Delta c)$ ; as  $f \leq \frac{\alpha}{2}\Delta V$  implies  $(1 - \alpha)(\Delta V - \Delta c) \leq (1 - \frac{\alpha}{2})\Delta V - f$ , this kind of equilibrium occurs only for  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ . Thus, a pure disclosure equilibrium exists if, and only if,  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ .

(d)  $\frac{\alpha}{2}\Delta V < f \leq \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$  : a type (i) equilibrium does not exist, types (ii.a) or (ii.b.1) equilibria exist if and only if (63) holds which reduces to  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ ; type (ii.b.2) equilibrium exists, if and only if (72) holds i.e.,  $(1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq A \leq (1 - \alpha)(\Delta V - \Delta c)$  as  $f \leq \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$  implies  $(1 - \alpha)(\Delta V - \Delta c) \leq (1 - \frac{\alpha}{2})\Delta V - f$ , this kind of equilibrium occurs only for  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ . Thus, a pure disclosure equilibrium exists if, and only if,  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ .

(e) for  $f > \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$ : a type (i) equilibrium does not exist, types (ii.a) or (ii.b.1) equilibria exist if and only if (63) holds which reduces to  $A \leq (1 - \frac{\alpha}{2})\Delta V - f$ ; while; type (ii.b.2) equilibrium exists, if and only if (72) holds i.e.,  $(1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq A \leq (1 - \alpha)(\Delta V - \Delta c)$ . As for this range of  $f$ ,  $(1 - \frac{\alpha}{2})\Delta V - f < (1 - \alpha)(\Delta V - \Delta c)$  we have that a pure disclosure equilibrium exists if, and only if,  $A \leq (1 - \alpha)(\Delta V - \Delta c)$ .

This concludes the proof of part (a) of the proposition.

(b) Equilibrium is non-distortionary if high quality firm sells to the entire market whenever it faces a low quality rival. Of all the pure disclosure equilibria, only the equilibrium of type (ii.b) above is non-distortionary. Note that such an equilibrium arises only in cases (c) through (e) i.e., only for  $f \geq (1 - \frac{\alpha}{2})\Delta c$ . Using the necessary and sufficient conditions (64) and (65) for an equilibrium of type (ii.b.1), we have that such an equilibrium exists if and only if

$$\left(1 - \frac{\alpha}{2}\right)\Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right)\Delta V, A = (1 - \frac{\alpha}{2})\Delta V - f = \tilde{A}(f)$$

On the other hand, a type (ii.b.2) equilibrium exists, if and only if (72) holds i.e.,  $(1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq A \leq (1 - \alpha)(\Delta V - \Delta c)$  which also requires  $f \geq (1 - \frac{\alpha}{2})\Delta c$  and holds if, and only if,

$$\begin{aligned} f &\geq \left(1 - \frac{\alpha}{2}\right)\Delta c, A \leq (1 - \alpha)(\Delta V - \Delta c) \\ A + f &\geq (1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2}, \text{ for } \left(1 - \frac{\alpha}{2}\right)\Delta c \leq f \leq (1 - \alpha)\Delta V + \frac{\alpha\Delta c}{2}. \end{aligned}$$

This establishes part (b) of the proposition.

(c) Equilibrium is fully distortionary if high quality type does not sell at all in the state where rival is of low type. Of the pure disclosure equilibria, only those of type (i) and of type (ii.a) with  $\beta = 0$  are fully distortionary. As described in items (a) - (c) above, a type (i) equilibrium exists if and only if

$$f \leq \frac{\alpha}{2}\Delta V$$

$$\begin{aligned} A &\leq \tilde{A}(f), \text{ for } f < \frac{\alpha}{2}\Delta c \\ &\leq \frac{\alpha}{2}\Delta V - f, \text{ for } \frac{\alpha}{2}\Delta c \leq f \leq \frac{\alpha}{2}\Delta V \end{aligned}$$

Also, an equilibrium of type (ii.a) with  $\beta = 0$  exists if, and only if,

$$A = \frac{\alpha}{2}\Delta V - f, \frac{\alpha}{2}\Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right)\Delta V$$

which is a subset of the set of parameters for which a type (i) equilibrium exists. This establishes part (c) of the proposition.

(d) Finally, a partially distortionary equilibrium is where buyers both high and low quality products in the state of the world where there the two firms differ in their types. Only a pure disclosure equilibria of type (ii.a) with  $\beta \in (0, 1)$  are partially distortionary and the conditions for such an equilibrium have been outlined above. This concludes the proof. ■

Observe that the conditions for a non-distortionary pure disclosure equilibrium overlap with those for a partially distortionary equilibrium and in particular, these two types of equilibria coexist for a range of values of  $A$  when  $\left(1 - \frac{\alpha}{2}\right)\Delta c \leq f < \left(1 - \frac{\alpha}{2}\right)\Delta V$ . It is easy to see that a non-distortionary equilibrium must necessarily be a pure disclosure equilibrium; this is because as noted in Lemma 2, in any equilibrium where the high quality type does not disclose with strictly positive probability, it does not sell in the state of the world where it does not disclose and faces a low quality rival. It follows therefore:

**Corollary 8** *A non-distortionary equilibrium exists if, and only if,  $A \leq \tilde{A}(f)$  and (43) and (44) hold.*

We now consider mixed disclosure equilibria where high quality types randomize between disclosure and non-disclosure and in particular, high quality types disclose with probability  $\gamma_H \in (0, 1)$ . From Lemma 2, in the event that it does not disclose, a high quality firm charges a deterministic price  $p_H^{NA}$  and sells only if the rival firm is a high quality type (i.e., it cedes the entire market to its rival in the event that the latter is of low quality creating a consumption distortion). Further, if  $\gamma_H \in (0, 1)$  then,  $\bar{p}_H^A$  the upper bound of the support of prices charged by a high quality type when it discloses satisfies

$$\bar{p}_H^A \leq p_H^{NA}.$$

Low quality firms randomize over a set of prices whose upper bound  $\bar{p}_L$  satisfies:

$$\bar{p}_L \leq p_H^{NA} - \Delta V$$

and there is no mass point at  $\bar{p}_L$ . If high quality types disclose and charge a price  $\tilde{p}$  with strictly positive probability, then  $\tilde{p}$  must be the lowest price charged in equilibrium by a high quality type. Both types earn strictly positive profit.

Recall the function  $\tilde{A}(f)$  that marks the (upper) boundary of the region of pure disclosure equilibria and the function  $f^*(A)$  that marks the (right) boundary of the region of pure price signaling equilibria.

**Proposition 9** *The conditions*

$$\begin{aligned} 0 &< f < \frac{\alpha}{2}\Delta c \\ \tilde{A}(f) &< A \leq f^{*-1}(f) \end{aligned}$$

*are necessary and sufficient for the existence of a mixed disclosure equilibrium where high quality types randomize between disclosure and non-disclosure and further, in the state of the world*

where both firms are of high quality type and only one firm discloses, both firms sell. In any such equilibrium (hereafter, M1 equilibrium), a high quality type charges the deterministic price

$$p_H = c_H + \frac{A}{f} \Delta c$$

whether or not it discloses, while each low quality type randomizes prices over an interval  $[\underline{p}_L, \bar{p}_L]$  where  $\bar{p}_L = p_H - \Delta V$  and sells to all buyers when it faces a high quality rival thus generating full consumption distortion.

**Proof.** Consider a mixed disclosure equilibrium where high quality types disclose with probability  $\gamma_H \in (0, 1)$  and further, with positive probability consumers randomize their purchase decision if both firms have high quality and one discloses and the other does not. We first claim that in such an equilibrium whether or not it discloses a high quality firm must set the same deterministic price. From Lemma 2, the equilibrium must be fully revealing and in the event where the high quality firm does not disclose it will set a deterministic price  $p_H^{NA}$ . Thus, if with positive probability consumers randomize their purchase decision if both firms have high quality and one discloses and the other does not, it must be that a high quality firm charges  $p_H^{NA}$  with strictly positive probability when it discloses. This implies that prices slightly lower than  $p_H^{NA}$  are not in the support of a disclosing high quality firm's pricing strategy (they would yield strictly higher profit than  $p_H^{NA}$ ) and that to deter deviation to such prices, out of equilibrium beliefs must consider a firm that deviates to such a price with or without disclosure as being of low quality type with sufficiently high probability. However, for the latter belief to be consistent with the D1 criterion, a low quality type must be indifferent between its equilibrium strategy, deviating to  $p_H^{NA}$  with (false) disclosure and deviating to  $p_H^{NA}$  without disclosure. If a disclosing high quality firm randomizes over prices strictly  $p_H^{NA}$  i.e., then it must be indifferent between all of these prices which (as  $c_L < c_H$ ) implies that a low quality firm will earn strictly higher profit by deviating to any such price with disclosure rather than deviating to  $p_H^{NA}$  with disclosure and as the latter yields same payoff as its equilibrium profit, the former is a strictly gainful deviation for the low quality type. Thus, disclosing high quality types must charge  $p_H^{NA}$  with probability one whether or not they disclose. Using Lemma 2, at price  $p_H^{NA}$ , high quality type sells only if the rival firm is a high quality type and  $p_H^{NA} \geq \bar{p}_L + \Delta V$ . As high quality types charge a deterministic price  $p_H^{NA}$ , if  $p_H^{NA} > \bar{p}_L + \Delta V$ , a low quality type will always earn strictly higher profit by deviating to a price slightly higher than  $\bar{p}_L$ , and therefore, it must be the case that  $p_H^{NA} = \bar{p}_L + \Delta V$ . In the rest of the proof, let  $p_H = p_H^{NA}$ . When both firms are of high quality, they both charge  $p_H$  (whether or not they disclose) leaving the consumer is indifferent and as this is a symmetric equilibrium, buyers buy from each firm with equal probability when they both disclose or when neither discloses. Recall that we are considering an equilibrium where consumers randomize their purchase decision if both firms charge  $p_H$  and only one discloses; let  $\beta \in (0, 1)$  denote the probability that each buyer buys from the disclosing firm in that state. As mentioned above, low quality should randomize over the interval  $[\underline{p}_L, \bar{p}_L]$ , with  $p_H = \bar{p}_L + \Delta V$ ; it is easy to check that  $\underline{p}_L^A = \alpha \bar{p}_L^A + (1 - \alpha)c_L$  and the equilibrium profits are given by

$$\pi_H^* = \alpha \left( \frac{\gamma_H}{2} + \beta(1 - \gamma_H) \right) (p_H - c_H) - A = \alpha \left( \frac{1 - \gamma_H}{2} + \gamma_H(1 - \beta) \right) (p_H - c_H) \quad (73)$$

and

$$\pi_L^* = \alpha(p_H - \Delta V - c_L), \quad (74)$$

respectively. Again, as argued above, low quality type must be indifferent between its equilibrium strategy and setting  $p_H$  with and without disclosure, we have

$$\pi_L^* = \alpha\left(\frac{\gamma_H}{2} + \beta(1 - \gamma_H)\right)(p_H - c_L) - A - f = \alpha\left(\frac{1 - \gamma_H}{2} + \gamma_H(1 - \beta)\right)(p_H - c_L). \quad (75)$$

Using this in (73), we get

$$\alpha\left(\frac{\gamma_H}{2} + \beta(1 - \gamma_H)\right)\Delta c - f = \alpha\left(\frac{1 - \gamma_H}{2} + \gamma_H(1 - \beta)\right)\Delta c,$$

which yields

$$\beta = \frac{1}{2} + \frac{f}{\alpha\Delta c}. \quad (76)$$

Note that  $\beta > 0.5$  as in order to have an incentive to disclose, a disclosing high quality firm should have a larger market share than a disclosing high quality firm if they set the same price.  $\beta \leq 1$  if, and only if,

$$f < \frac{\alpha}{2}\Delta c \quad (77)$$

From (75) and (76) we have

$$p_H = c_L + \Delta c\left(\frac{A + f}{f}\right) = c_H + \frac{A\Delta c}{f}.$$

It is clear that this is larger than  $c_H$  so that high quality makes positive profit. Substituting the expressions for  $\beta$  and  $p_H$  into

$$\alpha(p_H - \Delta V - c_L) = \alpha\left(\frac{\gamma_H}{2} + \beta(1 - \gamma_H)\right)(p_H - c_L) - A - f$$

we get

$$\gamma_H = \alpha\frac{\Delta V}{A + f} - \frac{\alpha\Delta c}{2f}.$$

so that  $0 < \gamma_H < 1$  if, and only if,

$$f > \frac{A}{2\lambda - 1} \quad (78)$$

and

$$A > f\left(\frac{2\Delta V}{\Delta c + \frac{2f}{\alpha}} - 1\right), \quad (79)$$

Note that (79) is equivalent to

$$A > \tilde{A}(f)$$

under constraint (77). Further,

$$\frac{A}{2\lambda - 1} \geq \frac{\alpha}{2}\Delta c$$

for

$$A \geq \frac{\alpha}{2}\Delta c(2\lambda - 1) = \alpha(\Delta V - \Delta c)$$

so that (78) and (77) can be jointly satisfied only if

$$A < \alpha(\Delta V - \frac{\Delta c}{2}) = A_1$$

for this range of  $A$ ,

$$f^*(A) = \frac{A}{2\lambda - 1}$$

so that (78) is equivalent to

$$A < f^{*-1}(f).$$

Thus,  $0 < f < \frac{\alpha}{2}\Delta c$ , and  $\tilde{A}(f) < A \leq f^{*-1}(f)$  are necessary and sufficient for the existence of this kind of equilibrium. This completes the proof. ■

Next, we consider mixed disclosure equilibria where in the state where both firms are of high quality and only one discloses, buyers buy only from the one that discloses.

**Proposition 10** *Under the conditions*

$$(\lambda - 1)(1 - \alpha)\Delta c < A < A_2 \tag{80}$$

$$f > \left[1 - \frac{\alpha}{2}\right] \Delta c. \tag{81}$$

*there is a mixed disclosure equilibrium (hereafter, M2 equilibrium) where in the state where it discloses, a high quality type randomizes over an interval of prices below  $p_H^N$  (with or without a mass point at the lower bound of the support) and sells to the entire market when the rival is either a low quality firm or a high quality firm that does not disclose.*

**Proof.** We first consider a version of M2 equilibrium where the disclosing high quality firm randomizes with no probability mass point. Let  $\gamma_H \in (0, 1)$  denote the probability of disclosure by a high quality type. As in any mixed disclosure equilibrium, when it does *not* disclose, the high quality firm charges  $p_H^N$  and at this price it only sells in the state where the rival is  $H$  type and does not disclose i.e., it sells with probability  $\frac{\alpha(1-\gamma_H)}{2}$ . Further, a low quality type does not disclose and sells in the state where rival is low quality as well as the state in which rival is high quality and charges  $p_H^N$ . In the M2 equilibrium we construct, the low quality firm randomizes over an interval  $[\underline{p}_L, \bar{p}_L]$  where

$$\bar{p}_L = p_H^N - \Delta V$$

When it discloses, the high quality firm randomizes prices over an interval  $[p_H^A, \bar{p}_H^A]$  where

$$\bar{p}_H^A = \underline{p}_L + \Delta V < p_H^N$$

i.e., buyers are indifferent between buying low quality at the lower bound of low quality prices  $\underline{p}_L$  and the upper bound of high quality prices when the firm discloses. It is easy to see that  $\bar{p}_H^A > \bar{p}_L$ . At price  $\bar{p}_L$  a low quality firm sells with probability  $\alpha(1-\gamma_H)$  i.e., only when rival is high quality but does not disclose. At price  $\underline{p}_L$  a low quality firm sells with probability  $\alpha(1-\gamma_H) + (1-\alpha) = 1-\alpha\gamma_H$ . When it discloses and charges price  $\bar{p}_H^A$ , a high quality firm also sells with probability  $1-\alpha\gamma_H$ , and it sells with probability 1 when it charges  $\underline{p}_H^A$ . The only restriction on out of equilibrium beliefs is

that a firm that does not disclose and charges any price below  $p_H^N$  is deemed to be low quality with probability one. It is easy to check that this restriction is consistent with D1 refinement if and only if a low quality firm is indifferent between charging  $p_H^N$  (without advertising) and sticking to its equilibrium strategy i.e.,

$$(p_H^N - c_L) \frac{\alpha(1 - \gamma_H)}{2} = (\bar{p}_L - c_L) \alpha(1 - \gamma_H)$$

and this yields:

$$p_H^N = 2\Delta V + c_L \quad (82)$$

The equilibrium profit of the high quality firm is therefore:

$$\pi_H^* = (2\Delta V - \Delta c) \frac{\alpha(1 - \gamma_H)}{2} \quad (83)$$

Further:

$$\bar{p}_L = p_H^N - \Delta V = \Delta V + c_L \quad (84)$$

and therefore, the equilibrium profit of the low quality firm is

$$\pi_L^* = \Delta V \alpha(1 - \gamma_H) \quad (85)$$

Further, as

$$(\underline{p}_L - c_L)(1 - \alpha\gamma) = \pi_L \quad (86)$$

we have

$$\underline{p}_L = \left[ \frac{\alpha(1 - \gamma_H)}{1 - \alpha\gamma_H} \right] \Delta V + c_L \quad (87)$$

The upper bound of prices for a high quality firm that discloses is now:

$$\bar{p}_H^A = \underline{p}_L + \Delta V = \left[ \frac{\alpha(1 - \gamma_H)}{1 - \alpha\gamma_H} + 1 \right] \Delta V + c_L \quad (88)$$

which is decreasing in  $\gamma_H$  and converges to  $\bar{p}_L$  as  $\gamma \rightarrow 1$ . The profit of the high quality firm when it discloses and charges price  $\bar{p}_H^A$  is given by

$$(\bar{p}_H^A - c_H)(1 - \alpha\gamma_H) - A \quad (89)$$

and this is equal to  $\pi_H^*$  if, and only if,

$$\gamma_H = \frac{1}{\alpha} \left[ \frac{\Delta V - \Delta c(1 - \frac{\alpha}{2}) - A}{\Delta V - \frac{\Delta c}{2}} \right] \quad (90)$$

It can be checked that our assumption (80) is exactly what is needed to ensure that  $\gamma_H \in (0, 1)$ . Indeed,  $\gamma_H \rightarrow 0$  as  $A \rightarrow [\lambda - (1 - \frac{\alpha}{2})] \Delta c$  and  $\gamma_H \rightarrow 1$  as  $A \rightarrow (\lambda - 1)(1 - \alpha)\Delta c$ . The lower bound  $\underline{p}_H^A$  for high quality price when the firm discloses satisfies:

$$(\underline{p}_H^A - c_H) = (\bar{p}_H^A - c_H)(1 - \alpha\gamma_H) = \pi_H^* + A \quad (91)$$



and this yields:

$$\begin{aligned} p_H^A &= \left[ \left( \frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H} + 1 \right) \Delta V - \Delta c \right] (1-\alpha\gamma_H) + c_H \\ &= 2A + c_H - (1-\alpha)(\lambda-1)\Delta c \end{aligned} \quad (92)$$

The distribution function  $F(\cdot)$  for low quality price satisfies:

$$(p_L - c_L)[\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))] = \pi_L^* = \alpha(1-\gamma_H)\Delta V, p_L \in [\underline{p}_L, \bar{p}_L] \quad (93)$$

The distribution function  $G(\cdot)$  for high quality price when the firm discloses satisfies:

$$\begin{aligned} &(p_H^A - c_H)[(1-\alpha\gamma_H) + \alpha\gamma_H(1-G(p_H^A))] \\ &= \pi_H^* + A \end{aligned} \quad (94)$$

$$= \Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma), p_H^A \in [\underline{p}_H^A, \bar{p}_H^A] \quad (95)$$

This completes the description of the equilibrium. Next, we show that there is no incentive to deviate from this equilibrium. It is easy to check that given the out of equilibrium belief, no high quality firm can strictly gain by deviating from its equilibrium strategy without disclosing. As the high quality firm gets the entire market at price  $\underline{p}_H^A$  when it discloses, it has no incentive to disclose and charge price below  $\underline{p}_H^A$ . Nor can it gain by charging price above  $p_H^N$  (sells zero). It remains to check that a high quality firm cannot gain by disclosing and charging an out of equilibrium price  $p_H^A \in (\bar{p}_H^A, p_H^N)$ . For any such deviation price  $p_H^A$ , there exists  $p_L = p_H^A - \Delta V \in (\underline{p}_L, \bar{p}_L)$ . The deviation profit is given by:

$$\begin{aligned} &[\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_H^A - \Delta V))](p_H^A - c_H) - A \\ &= [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))](p_L + \Delta V - c_H) - A \\ &= \left[ \frac{p_L + \Delta V - c_H}{p_L - c_L} \right] \pi_L^* - A, \text{ using (93),} \end{aligned}$$

and since  $\frac{p_L + \Delta V - c_H}{p_L - c_L}$  is strictly decreasing in  $p_L$  (use (??)) this is

$$\begin{aligned} &\leq \left[ \frac{\underline{p}_L + \Delta V - c_H}{\underline{p}_L - c_L} \right] \pi_L^* - A = [\underline{p}_L + \Delta V - c_H] (1-\alpha\gamma_H) - A \\ &= [\bar{p}_H^A - c_H] (1-\alpha\gamma_H) - A \\ &= \pi_H^* \text{ (use (89) and (90))} \end{aligned}$$

so that the deviation cannot be strictly gainful. We now look at the incentive of a low quality firm to deviate. Whether or not it discloses, the firm will sell zero if it charges price above  $p_H^N$  (even if it is thought of as a high quality firm). Given the out of equilibrium beliefs, if a low quality firm deviates without disclosing and charges price  $\in (\bar{p}_L, p_H^N)$  it will be thought of as a low quality firm and will sell zero. If it charges price  $p_L < \underline{p}_L$  (without disclosing) it will be perceived as a low quality firm but may be able to attract more buyers in the state where rival is high quality and discloses; without loss of generality, consider deviation to  $p_L \in [\underline{p}_H^A - \Delta V, \underline{p}_L)$ . The deviation profit

is then given by

$$\begin{aligned}
& [(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - G(p_L + \Delta V))](p_L - c_L) \\
& = [(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^A))](p_H^A - \Delta V - c_L) \text{ where } p_H^A = p_L + \Delta V \\
& = \left[ \frac{p_H^A - \Delta V - c_L}{p_H^A - c_H} \right] (\pi_H^* + A), \text{ using (94)}
\end{aligned}$$

and since  $\left[ \frac{p_H^A - \Delta V - c_L}{p_H^A - c_H} \right]$  is strictly increasing in  $p_H^A$  (use (??)) this is

$$\begin{aligned}
& \leq \left[ \frac{\bar{p}_H^A - \Delta V - c_L}{\bar{p}_H^A - c_H} \right] (\pi_H^* + A) \\
& = [\bar{p}_H^A - \Delta V - c_L] (1 - \alpha\gamma_H) \text{ (use (89) and (90))} \\
& = \Delta V \alpha (1 - \gamma_H), \text{ using (88)} \\
& = \pi_L^* \text{ (see (85))}
\end{aligned}$$

and thus the deviation is not strictly gainful. We now consider deviation by a low quality firm where it discloses (falsely). If it does so, it cannot gain by charging price below  $\underline{p}_H^A$  as it sells to the entire market at that price. So, consider deviation price  $p_H^A \in [\underline{p}_H^A, \bar{p}_H^A]$  with advertising. The deviation profit is given by

$$\begin{aligned}
& [(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^A))](p_H^A - c_L) - A - f \\
& = \left[ \frac{p_H^A - c_L}{p_H^A - c_H} \right] (\pi_H^* + A) - A - f, \text{ using (94)}
\end{aligned}$$

and as  $\left[ \frac{p_H^A - c_L}{p_H^A - c_H} \right]$  is strictly decreasing in  $p_H^A$ , this is

$$\begin{aligned}
& \leq \left[ \frac{\underline{p}_H^A - c_L}{\underline{p}_H^A - c_H} \right] (\pi_H^* + A) - A - f \\
& = \underline{p}_H^A - c_L - A - f, \text{ using (91)} \\
& = (\underline{p}_H^A - c_H) + \Delta c - A - f \\
& = \pi_H^* + A + \Delta c - A - f \\
& = (2\Delta V - \Delta c) \frac{\alpha(1 - \gamma_H)}{2} + \Delta c - f \\
& = \Delta V \alpha (1 - \gamma_H) + \Delta c (1 - \frac{\alpha(1 - \gamma_H)}{2}) - f \\
& = \pi_L^* + \Delta c (1 - \frac{\alpha(1 - \gamma_H)}{2}) - f
\end{aligned} \tag{96}$$

which is  $\leq \pi_L^*$  if

$$f \geq \Delta c (1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2})$$

which (using (90)) holds if, and only if, we assume that :

$$f \geq \left[ 1 - \frac{\alpha}{2} + \frac{\lambda - (1 - \frac{\alpha}{2})}{2\lambda - 1} \right] \Delta c - \frac{A}{2\lambda - 1} \tag{97}$$

Thus, under (97), deviation by a low quality type to advertising and charging price in  $[p_H^A, \bar{p}_H^A]$  is not gainful. Finally, consider deviation by the same firm to disclosing and setting price  $p_H^A \in (\bar{p}_H^A, p_H^N)$ . The maximum possible deviation profit (i.e., even if the firm is perceived as high quality with prob 1) is given by:

$$\begin{aligned} & [\alpha(1 - \gamma_H) + (1 - \alpha)(1 - F(p_H^A - \Delta V))](p_H^A - c_L) - A - f \\ = & [\alpha(1 - \gamma_H) + (1 - \alpha)(1 - F(p_L))](p_L + \Delta V - c_L) - A - f \\ = & \left[ \frac{p_L + \Delta V - c_L}{p_L - c_L} \right] \pi_L^* - A - f, \text{ using (93),} \end{aligned}$$

and as  $\left[ \frac{p_L + \Delta V - c_L}{p_L - c_L} \right]$  is strictly decreasing in  $p_L$ , this is

$$\begin{aligned} & \leq \left[ \frac{p_L + \Delta V - c_L}{p_L - c_L} \right] \pi_L^* - A - f \\ = & [p_L + \Delta V - c_L] (1 - \alpha\gamma) - A - f, \text{ using (86)} \\ = & [\bar{p}_H^A - c_L] (1 - \alpha\gamma) - A - f \\ = & [\bar{p}_H^A - c_H] (1 - \alpha\gamma) + \Delta c - A - f \\ = & (p_H^A - c_H) + \Delta c - A - f \\ = & (p_H^A - c_L) - A - f \end{aligned}$$

$\leq \pi_L^*$  under condition (97) as was shown above (see arguments following (96)). Thus, we have shown that under assumption (97), there is no incentive to deviate.

Next, we consider a variation of the above M2 equilibrium with the only difference that when a high quality firm discloses, it randomizes prices over an interval  $[p_H^A, \bar{p}_H^A]$  with probability  $1 - \kappa \in (0, 1)$  and chooses a price  $\tilde{p}_H^A \in (c_H, p_H^A)$  with probability  $\kappa$ . As before,

$$\bar{p}_H^A = p_L + \Delta V < p_H^N$$

Also, as before, at price  $\bar{p}_L$  a low quality firm sells with probability  $\alpha(1 - \gamma_H)$  i.e., only when rival is high quality but does not disclose. At price  $p_L$  a low quality firm sells with probability  $1 - \alpha\gamma_H$ . When it discloses and charges price  $\bar{p}_H^A$ , a high quality firm also sells with probability  $1 - \alpha\gamma_H$ , and it sells with probability  $1 - \alpha\kappa\gamma_H$  when it charges  $p_H^A$ . When it discloses and charges  $\tilde{p}_H^A$ , the high quality firm sells with probability  $(1 - \frac{\alpha\kappa\gamma_H}{2})$ . Only restrictions on out of equilibrium beliefs are that : (a) a firm that does not advertise and charges any price below  $p_H^N$  is deemed to be low quality with probability one and (b) any firm advertising and charging price below  $\tilde{p}_H$  is deemed to be low quality with probability one. As before, for restriction (a) to be consistent with D1 refinement we need is that a low quality firm should be indifferent between charging  $p_H^N$  without advertising and sticking to its equilibrium strategy which yields the same expressions for

$$p_H^N = 2\Delta V + c_L. \quad (98)$$

and the equilibrium profit of the high quality firm :

$$\pi_H^* = (2\Delta V - \Delta c) \frac{\alpha(1 - \gamma_H)}{2} \quad (99)$$

Further, as before,

$$\bar{p}_L = p_H^N - \Delta V = \Delta V + c_L \quad (100)$$

$$\pi_L^* = \Delta V \alpha (1 - \gamma_H) \quad (101)$$

$$\underline{p}_L = \left[ \frac{\alpha(1 - \gamma_H)}{1 - \alpha\gamma_H} \right] \Delta V + c_L \quad (102)$$

$$\bar{p}_H^A = \underline{p}_L + \Delta V = \left[ \frac{\alpha(1 - \gamma_H)}{1 - \alpha\gamma_H} + 1 \right] \Delta V + c_L \quad (103)$$

$$\gamma_H = \frac{1}{\alpha} \left[ \frac{\Delta V - \Delta c(1 - \frac{\alpha}{2}) - A}{\Delta V - \frac{\Delta c}{2}} \right] \quad (104)$$

and (80) is exactly what is needed to ensure that  $\gamma_H \in (0, 1)$ . The lower bound  $\underline{p}_H^A$  for high quality price when the firm discloses now satisfies:

$$(\underline{p}_H^A - c_H)(1 - \alpha\kappa\gamma_H) = (\bar{p}_H^A - c_H)(1 - \alpha\gamma_H) = \pi_H^* + A \quad (105)$$

and this yields:

$$\begin{aligned} \underline{p}_H^A &= [\Delta V(1 + \alpha - 2\alpha\gamma_H) - \Delta c(1 - \alpha\gamma_H)] \frac{1}{(1 - \alpha\kappa\gamma_H)} + c_H \\ &= \frac{\pi_H^* + A}{1 - \alpha\kappa\gamma_H} + c_H \end{aligned} \quad (106)$$

As before, the distribution function  $F(\cdot)$  for low quality price satisfies:

$$(p_L - c_L)[\alpha(1 - \gamma_H) + (1 - \alpha)(1 - F(p_L))] = \pi_L^* = \alpha(1 - \gamma_H)\Delta V, p_L \in [\underline{p}_L, \bar{p}_L] \quad (107)$$

The distribution function  $G(\cdot)$  for high quality price on the interval  $[\underline{p}_H^A, \bar{p}_H^A]$  when the firm discloses satisfies:

$$\begin{aligned} & (p_H^A - c_H)[(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - \kappa)(1 - G(p_H^A))] \\ &= \pi_H^* + A \end{aligned} \quad (108)$$

$$= \Delta V(1 + \alpha - 2\alpha\gamma_H) - \Delta c(1 - \alpha\gamma_H), p_H^A \in [\underline{p}_H^A, \bar{p}_H^A] \quad (109)$$

To ensure that restriction (b) on out of equilibrium belief is consistent with D1 we need that a low quality firm should be indifferent between deviating to advertising and charging  $\tilde{p}_H^A$  and sticking to its equilibrium strategy i.e.,

$$(\tilde{p}_H^A - c_L) \left( 1 - \frac{\alpha\kappa\gamma_H}{2} \right) - A - f = \pi_L^* = \Delta V \alpha (1 - \gamma_H) \quad (110)$$

Further, high quality type must be indifferent between choosing  $\tilde{p}_H^A$  while disclosing and other actions in the support of its equilibrium strategy which requires:

$$(\tilde{p}_H^A - c_H) \left( 1 - \frac{\alpha\kappa\gamma_H}{2} \right) = \pi_H^* + A \quad (111)$$

$$= (2\Delta V - \Delta c) \frac{\alpha(1 - \gamma_H)}{2} + A \quad (112)$$

$$= \Delta V(1 + \alpha - 2\alpha\gamma_H) - \Delta c(1 - \alpha\gamma_H) \quad (113)$$

Comparing (111) and (105), we can see that  $\tilde{p}_H^A < \underline{p}_H^A$ . From (110)

$$\begin{aligned}
& \pi_L^* + A + f \\
&= (\tilde{p}_H^A - c_L) \left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) \\
&= (\tilde{p}_H^A - c_H) \left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) + \Delta c \left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) \\
&= \pi_H^* + A + \Delta c \left(1 - \frac{\alpha\kappa\gamma}{2}\right), \text{ using (111)}
\end{aligned}$$

so that

$$\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) = \frac{1}{\Delta c} (\pi_L^* - \pi_H^* + f) \quad (114)$$

which yields:

$$\kappa = \frac{2}{\alpha\gamma_H} \left[1 - \frac{1}{\Delta c} (\pi_L^* - \pi_H^* + f)\right] \quad (115)$$

Further, using (114) in (111) we have:

$$\tilde{p}_H^A = \left[ \frac{\pi_H^* + A}{\pi_L^* - \pi_H^* + f} \right] \Delta c + c_H \quad (116)$$

We need to ensure that  $\kappa \in (0, 1)$  which is satisfied as long as

$$\left(1 - \frac{\alpha\gamma_H}{2}\right) \Delta c - (\pi_L^* - \pi_H^*) < f < \Delta c - (\pi_L^* - \pi_H^*) \quad (117)$$

Note that

$$(\pi_L^* - \pi_H^*) = \frac{\alpha(1 - \gamma_H)}{2} \Delta c < \Delta c \quad (118)$$

and (117) is satisfied as long as

$$\left[1 - \frac{\alpha}{2}\right] \Delta c < f < \left[1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2}\right] \Delta c \quad (119)$$

which (using (104)) reduces to

$$\left[1 - \frac{\alpha}{2}\right] \Delta c < f < \left[1 - \frac{\alpha}{2} + \frac{\lambda - (1 - \frac{\alpha}{2})}{2\lambda - 1}\right] \Delta c - \frac{A}{2\lambda - 1} \quad (120)$$

Observe that the inequalities in (120) can be written as

$$A + f(2\lambda - 1) < [(2 - \alpha)(\lambda - 1) + \lambda] \Delta c \quad (121)$$

$$f > \left[1 - \frac{\alpha}{2}\right] \Delta c \quad (122)$$

This completes the description of the equilibrium. Next, we show that there is no incentive to deviate from this equilibrium. Observe that a high quality firm can never strictly gain by advertising and choosing a price  $p \in (\tilde{p}_H^A, \underline{p}_H^A)$  as it sells the same expected quantity in that case as it would at  $\underline{p}_H^A$ . As the high quality firm gets the entire market at price  $\tilde{p}_H^A$  when it discloses, it has no incentive to disclose and charge price below  $\tilde{p}_H^A$ . Using identical arguments to that in the first part of the

proof, one can check that there is no other gainful deviation for a high quality type. We now look at the incentive of a low quality firm to deviate. Whether or not it discloses, the firm will sell zero if it charges price above  $p_H^N$  (even if it is thought of as a high quality firm). Given the out of equilibrium beliefs, if a low quality firm deviates without disclosing and charges price  $\in (\bar{p}_L, p_H^N)$  it will be thought of as a low quality firm and will sell zero. If it charges price  $p_L < \underline{p}_L$  (without advertising) it will be perceived as a low quality firm but may be able to attract more buyers in the state where rival is high quality and discloses. First, consider deviation to  $p_L \in [\underline{p}_H^A - \Delta V, \underline{p}_L]$ . The deviation profit is then given by

$$\begin{aligned} & [(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - \kappa)(1 - G(p_L + \Delta V))](p_L - c_L) \\ = & [(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - \kappa)(1 - G(p_H^A))](p_H^A - \Delta V - c_L) \text{ where } p_H^A = p_L + \Delta V \\ = & \left[ \frac{p_H^A - \Delta V - c_L}{p_H^A - c_H} \right] (\pi_H^* + A), \text{ using (108)} \end{aligned}$$

and since  $\left[ \frac{p_H^A - \Delta V - c_L}{p_H^A - c_H} \right]$  is strictly increasing in  $p_H^A$  (use (??)) this is

$$\begin{aligned} & \leq \left[ \frac{\bar{p}_H^A - \Delta V - c_L}{\bar{p}_H^A - c_H} \right] (\pi_H^* + A) \\ = & [\bar{p}_H^A - \Delta V - c_L] (1 - \alpha\gamma_H) \text{ (use (103) and (104))} \\ = & \Delta V \alpha (1 - \gamma_H), \text{ using (103)} \\ = & \pi_L^* \text{ (see (101))} \end{aligned}$$

and thus the deviation is not strictly gainful. We now consider deviation by a low quality firm where it discloses (falsely). Consider deviation to price  $p_H^A \in [\underline{p}_H^A, \bar{p}_H^A]$  with disclosure. The deviation profit is given by

$$\begin{aligned} & [(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - \kappa)(1 - G(p_H^A))](p_H^A - c_L) - A - f \\ = & \left[ \frac{p_H^A - c_L}{p_H^A - c_H} \right] (\pi_H^* + A) - A - f, \text{ using (108)} \end{aligned}$$

and as  $\left[ \frac{p_H^A - c_L}{p_H^A - c_H} \right]$  is strictly decreasing in  $p_H^A$ , this is

$$\begin{aligned} & \leq \left[ \frac{\underline{p}_H^A - c_L}{\underline{p}_H^A - c_H} \right] (\pi_H^* + A) - A - f \\ = & (\underline{p}_H^A - c_L)(1 - \alpha\kappa\gamma_H) - A - f, \text{ using (104)} \\ = & (\underline{p}_H^A - c_H)(1 - \alpha\kappa\gamma_H) + \Delta c(1 - \alpha\kappa\gamma_H) - A - f \\ = & \pi_H^* + A + \Delta c(1 - \alpha\kappa\gamma_H) - A - f \\ = & (2\Delta V - \Delta c) \frac{\alpha(1 - \gamma_H)}{2} + \Delta c(1 - \alpha\kappa\gamma_H) - f \\ = & \Delta V \alpha (1 - \gamma_H) + \Delta c(1 - \alpha\kappa\gamma_H - \frac{\alpha(1 - \gamma_H)}{2}) - f \\ = & \pi_L^* + \Delta c(1 - \alpha\kappa\gamma_H - \frac{\alpha(1 - \gamma_H)}{2}) - f \end{aligned} \tag{123}$$

which is  $\leq \pi_L^*$  if

$$\begin{aligned}
f &\geq \Delta c(1 - \alpha\kappa\gamma_H - \frac{\alpha(1 - \gamma_H)}{2}) \\
&= \Delta c \left[ \frac{2}{\Delta c} \{ \pi_L^* - \pi_H^* + f \} - 1 - \frac{\alpha(1 - \gamma_H)}{2} \right], \text{ using (115)} \\
&= \Delta c \left[ \frac{2}{\Delta c} \{ \frac{\alpha(1 - \gamma_H)}{2} \Delta c + f \} - 1 - \frac{\alpha(1 - \gamma_H)}{2} \right], \text{ using (118)} \\
&= \Delta c \left[ \frac{\alpha(1 - \gamma_H)}{2} - 1 \right] + 2f
\end{aligned}$$

which reduces to :

$$f \leq \left[ 1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2} \right] \Delta c$$

that follows from condition (120). It is obvious that deviation to disclosing and setting any price in the segment  $(\tilde{p}_H^A, \underline{p}_H^A)$  cannot be strictly gainful as the maximum amount it can sell (even if it is perceived as high quality) is identical to that at  $\underline{p}_H^A$ . Given restriction (b) on out of equilibrium beliefs, deviating to disclosing and charging a price below  $\tilde{p}_H^A$  will make buyers believe that it is a low quality firm and therefore the firm will and so the deviating firm will earn strictly less profit than it would if it did not disclose and charged the same price; we have already seen that the latter kind of deviation cannot be gainful. Finally, consider deviation by the low quality firm to disclosing and charging price  $p_H^A \in (\bar{p}_H^A, p_H^N)$ . The maximum possible deviation profit (i.e., even if the firm is perceived as high quality with prob 1) is given by:

$$\begin{aligned}
&[\alpha(1 - \gamma_H) + (1 - \alpha)(1 - F(p_H^A - \Delta V))](p_H^A - c_L) - A - f \\
&= [\alpha(1 - \gamma_H) + (1 - \alpha)(1 - F(p_L))](p_L + \Delta V - c_L) - A - f \\
&= \left[ \frac{p_L + \Delta V - c_L}{p_L - c_L} \right] \pi_L^* - A - f, \text{ using (107),}
\end{aligned}$$

and as  $\left[ \frac{p_L + \Delta V - c_L}{p_L - c_L} \right]$  is strictly decreasing in  $p_L$ , this is

$$\begin{aligned}
&\leq \left[ \frac{\underline{p}_L + \Delta V - c_L}{\underline{p}_L - c_L} \right] \pi_L^* - A - f \\
&= \left[ \underline{p}_L + \Delta V - c_L \right] (1 - \alpha\gamma_H) - A - f, \text{ using (??)} \\
&= [\bar{p}_H^A - c_L] (1 - \alpha\gamma_H) - A - f \\
&= [\bar{p}_H^A - c_H] (1 - \alpha\gamma_H) + \Delta c(1 - \alpha\gamma_H) - A - f \\
&= \pi_H^* + A + \Delta c(1 - \alpha\gamma_H) - A - f \\
&= [\underline{p}_H^A - c_H] (1 - \alpha\kappa\gamma_H) + \Delta c(1 - \alpha\gamma_H) - A - f \\
&\leq [\underline{p}_H^A - c_H] (1 - \alpha\kappa\gamma_H) + \Delta c(1 - \alpha\kappa\gamma_H) - A - f, \text{ as } \kappa \in (0, 1) \\
&= [\underline{p}_H^A - c_L] (1 - \alpha\kappa\gamma_H) - A - f
\end{aligned}$$

which is  $\leq \pi_L^*$  under condition (120) as shown above (see arguments following (123)). Thus, we have shown that under condition (120), there is no incentive to deviate. Finally, note that (81) implies that either (97) or (120) holds. This completes the proof. ■

**Corollary 11** *Suppose that  $f > (1 - \frac{\alpha}{2})\Delta c$  and  $A \in (\tilde{A}(f), A_2)$ . Then, a mixed disclosure equilibrium of type M2 exists.*

**Proof.** Follows from Proposition 10 and the fact that by definition,  $\tilde{A}(f) = (\lambda - 1)(1 - \alpha)\Delta c$  for  $f > \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$  and that if  $(1 - \frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$ , then  $\tilde{A}(f) = (1 - \frac{\alpha}{2})\Delta V - f \geq (\lambda - 1)(1 - \alpha)\Delta c$ . ■

**Lemma 12** *Suppose that*

$$\frac{\alpha}{2}\Delta c \leq f \leq (1 - \frac{\alpha}{2})\Delta c \quad (124)$$

*and*

$$\tilde{A}(f) < A < f^{*-1}(f) \quad (125)$$

*Then, a mixed disclosure equilibrium exists.*

**Proof.**

**Lemma 13** *First note that for the range of values of  $f$  satisfying (124),  $\tilde{A}(f) = (\lambda - 1)f$  and  $f^{*-1}(f) = \frac{\alpha}{2}\Delta V + (\lambda - 1)f$  so that (125) implies*

$$(\lambda - 1)f < A < \frac{\alpha}{2}\Delta V + (\lambda - 1)f \quad (126)$$

*Consider the following mixed disclosure equilibrium. Each high quality type discloses with probability  $\gamma_H \in (0, 1)$ ; further, it sets a deterministic price  $p_H^A$  when it discloses and a higher price  $p_H^N > p_H^A$  when it does not disclose. Each low quality type randomizes over two disconnected intervals of prices  $[\underline{p}_L^1, \bar{p}_L^1]$  and  $[\underline{p}_L^2, \bar{p}_L^2]$  with probabilities  $\gamma_L$  and  $1 - \gamma_L$  respectively and a continuous distribution over each interval, where  $\bar{p}_L^1 < \underline{p}_L^2$ . Further,*

$$\bar{p}_L^1 = p_H^A - \Delta V, \bar{p}_L^2 = p_H^N - \Delta V. \quad (127)$$

*The equilibrium expected profits  $\pi_H^*$  and  $\pi_L^*$  then satisfy:*

$$\pi_L^* = (\alpha + (1 - \alpha)(1 - \gamma_L))(\bar{p}_L^1 - c_L) = \alpha(1 - \gamma_H)(\bar{p}_L^2 - c_L) \quad (128)$$

$$\pi_H^* = \left( \frac{\alpha\gamma_H}{2} + \alpha(1 - \gamma_H) + (1 - \alpha)(1 - \gamma_L) \right) (p_H^A - c_H) - A = \frac{\alpha(1 - \gamma_H)}{2} (p_H^N - c_H) \quad (129)$$

*Out of equilibrium beliefs regard any firm charging price  $\in (\bar{p}_L^2, p_H^N)$  without disclosure or price  $\in (\bar{p}_L^1, p_H^A)$  with disclosure as being of low type. Using very similar arguments as in earlier propositions, this kind of belief restriction is consistent with the D1 criterion along as low quality type is indifferent between following its equilibrium strategy and deviating to disclosing and charging  $p_H^A$  or not disclosing and charging  $p_H^N$ :*

$$\pi_L^* = \left( \frac{\alpha\gamma_H}{2} + \alpha(1 - \gamma_H) + (1 - \alpha)(1 - \gamma_L) \right) (p_H^A - c_L) - A - f = \frac{\alpha(1 - \gamma_H)}{2} (p_H^N - c_L) \quad (130)$$



From (127), (128) and (130) we obtain

$$p_H^N = c_L + 2\Delta V, \bar{p}_L^2 = c_L + \Delta V \quad (131)$$

Using (129), (130) reduces to

$$\gamma_L = \frac{1 - \frac{\alpha}{2} - \frac{f}{\Delta c}}{1 - \alpha} \quad (132)$$

and this lies in  $[0, 1]$  under condition (124). From (128) and (129), we obtain:

$$p_H^A - c_H = \Delta V - \Delta c + \frac{\alpha(1 - \gamma_H)\Delta V}{\frac{\alpha}{2} + \frac{f}{\Delta c}} \quad (133)$$

and

$$p_H^A - c_H = \frac{2A + \alpha(1 - \gamma_H)(2\Delta V - \Delta c)}{2\left(\frac{\alpha(1 - \gamma_H)}{2} + \frac{f}{\Delta c}\right)}, \quad (134)$$

that simultaneously determine  $p_H^A$  and  $\gamma_H$ . These yield:

$$\Delta V (\alpha(1 - \gamma_H))^2 + \left(\frac{f}{\Delta c} - \frac{\alpha}{2}\right) \Delta V \alpha(1 - \gamma_H) + 2\left(\frac{\alpha}{2} + \frac{f}{\Delta c}\right) ((\lambda - 1)f - A) = 0$$

It is easy to check that for  $A = (\lambda - 1)f$ ,  $\gamma_H = 1$ , that  $\gamma_H$  is decreasing in  $A$  and that at  $A = (\lambda - 1)f + \frac{\alpha\Delta V}{2}$ ,  $\gamma_H = 0$ . Thus, (126), which follows from condition (125) implies that there exists  $\gamma_H \in (0, 1)$  and therefore, using (133) or (134), there exists  $p_H^A > c_H$  that meets the equilibrium conditions. Note that (133) implies that  $p_H^A$  is strictly decreasing in  $\gamma_H$  so we have

$$p_H^A \leq c_L + \Delta V + \frac{\alpha\Delta V}{\frac{\alpha}{2} + \frac{f}{\Delta c}} < c_L + 2\Delta V = p_H^N$$

using the first inequality in condition (124). Using (127), we obtain  $\bar{p}_L^1$ . The values of  $\underline{p}_L^1$  and  $\underline{p}_L^2$  are determined by:

$$\begin{aligned} (\alpha + (1 - \alpha)(1 - \gamma_L))(\bar{p}_L^1 - c_L) &= (\underline{p}_L^1 - c_L) \\ \alpha(1 - \gamma_H)(\bar{p}_L^2 - c_L) &= (((1 - \alpha)(1 - \gamma_L) + \alpha(1 - \gamma_H))(\underline{p}_L^2 - c_L)) \end{aligned}$$

and using the previous equations one can check that  $\bar{p}_L^1 < \underline{p}_L^2$ . The distribution of low quality prices over the two segments  $[\underline{p}_L^1, \bar{p}_L^1]$  and  $[\underline{p}_L^2, \bar{p}_L^2]$  can now be determined in the usual manner by equalizing the expected profit earned at various prices and it can be shown that the distribution is continuous over each interval. Finally, the out of equilibrium beliefs can be used to show that no type of any firm has a unilateral incentive to deviate.

■

We are now ready to state our main existence result:

**Proposition 14** *An (symmetric fully separating D1) equilibrium exists.*

**Proof.** From Proposition 5, we know that there is a pure price signaling equilibrium if either (i)  $A \geq A_2$ , or (ii)  $A < A_2$  and  $f \leq f^*(A)$ . From Proposition 7, a pure disclosure equilibrium exists if  $A \leq \tilde{A}(f)$ . So we are left with the subset of the parameter space where  $A \in (\tilde{A}(f), A_2)$  and  $f > f^*(A)$ . This is identical to the set  $\{(A, f) : A \in (\tilde{A}(f), f^{*-1}(f)), f \in (0, (1 - \frac{\alpha}{2})\Delta c)\} \cup \{(A, f) : A \in (\tilde{A}(f), A_2), f \geq (1 - \frac{\alpha}{2})\Delta c\}$ . From Corollary 11, a mixed disclosure equilibrium M2 exists on  $\{(A, f) : A \in (\tilde{A}(f), A_2), f \geq (1 - \frac{\alpha}{2})\Delta c\}$ . Further, combining Proposition 9 and Lemma 12, we have existence of mixed disclosure equilibrium on  $\{(A, f) : A \in (\tilde{A}(f), f^{*-1}(f)), f \in (0, (1 - \frac{\alpha}{2})\Delta c)\}$ . ■

Proposition 9 provides necessary and sufficient conditions for a mixed disclosure equilibrium of type M1 where high quality types randomize between disclosure and non-disclosure and further, in the state of the world where both firms are of high quality type and only one firm discloses, both firms sell. The next result provides a necessary condition for a mixed disclosure equilibrium that is not of type M1.

**Lemma 15** *A mixed disclosure equilibrium that is not of type M1 (as described in Proposition 9) exists only if*

$$A > (\Delta V - \Delta c) \min \left\{ (1 - \alpha), \frac{f}{\Delta c} \right\} \quad (135)$$

**Proof.** Consider a mixed disclosure equilibrium that is not of type M1. Then, in the state of the world where there is one disclosing and one non-disclosing high quality firm present in the market, only one of the two firms sells. As  $\bar{p}_H^A \leq p_H^N$ , the only circumstance under which buyers would buy from a non-disclosing firm if  $\bar{p}_H^A = p_H^N$  and the disclosing firm charges  $\bar{p}_H^A$  with probability one. But then the high quality firm would earn higher expected profit by not disclosing with probability one, a contradiction. Thus, in any such equilibrium all buyers buy from the disclosing high quality firm when both firms are of high quality and only one discloses.

Next, we argue that either the disclosing high quality firm sets a deterministic price, or he serves the entire market if the rival firm produces low quality. To see this suppose that with strictly positive probability a disclosing high quality firm randomizes over prices but does not sell to all buyers. Then,  $\bar{p}_H^A > \underline{p}_L + \Delta V$  (note that the high quality type can put a positive probability mass only at the lower bound of the support of prices). First, suppose that  $\underline{p}_L + \Delta V < \bar{p}_H^A \leq \bar{p}_L + \Delta V$ . Then,  $\bar{p}_H^A - \Delta V \in (\underline{p}_L, \bar{p}_L]$ . Further, there exists  $\epsilon > 0$  such that  $\bar{p}_H^A - \epsilon$  is in the interior of the support of the distribution of high quality prices with disclosure and

$$\underline{p}_L < \bar{p}_H^A - \epsilon - \Delta V < \bar{p}_H^A - \Delta V \leq \bar{p}_L \quad (136)$$

The high quality equilibrium profit must be equalized at prices  $\bar{p}_H^A - \epsilon$  and  $\bar{p}_H^A$

$$\begin{aligned} & [(1 - \alpha)(1 - F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha(1 - \gamma_H) + \alpha\gamma_H(1 - F_H(\bar{p}_H^A - \epsilon))] (\bar{p}_H^A - \epsilon - c_H) \\ &= [(1 - \alpha)(1 - F_L(\bar{p}_H^A - \Delta V)) + \alpha(1 - \gamma_H)] (\bar{p}_H^A - c_H) \end{aligned}$$

However, this implies that

$$\begin{aligned}
& [(1-\alpha)(1-F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\bar{p}_H^A - \epsilon))] (\bar{p}_H^A - \epsilon - \Delta V - c_L) \\
= & [(1-\alpha)(1-F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\bar{p}_H^A - \epsilon))] (\bar{p}_H^A - \epsilon - c_H) \\
& - [(1-\alpha)(1-F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\bar{p}_H^A - \epsilon))] (\Delta V - \Delta c) \\
= & [(1-\alpha)(1-F_L(\bar{p}_H^A - \Delta V)) + \alpha(1-\gamma_H)] (\bar{p}_H^A - c_H) \\
& - [(1-\alpha)(1-F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\bar{p}_H^A - \epsilon))] (\Delta V - \Delta c) \\
= & [(1-\alpha)(1-F_L(\bar{p}_H^A - \Delta V)) + \alpha(1-\gamma_H)] (\bar{p}_H^A - \Delta V - c_L) \\
& + [(1-\alpha)(1-F_L(\bar{p}_H^A - \Delta V)) + \alpha(1-\gamma_H)] (\Delta V - \Delta c) \\
& - [(1-\alpha)(1-F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\bar{p}_H^A - \epsilon))] (\Delta V - \Delta c) \\
= & [(1-\alpha)(1-F_L(\bar{p}_H^A - \Delta V)) + \alpha(1-\gamma_H)] (\bar{p}_H^A - \Delta V - c_L) \\
& - [(1-\alpha)(F_L(\bar{p}_H^A - \Delta V) - F_L(\bar{p}_H^A - \epsilon - \Delta V)) + \alpha\gamma_H(1-F_H(\bar{p}_H^A - \epsilon))] (\Delta V - \Delta c) \\
< & [(1-\alpha)(1-F_L(\bar{p}_H^A - \Delta V)) + \alpha(1-\gamma_H)] (\bar{p}_H^A - \Delta V - c_L)
\end{aligned}$$

so that low quality type strictly prefers to charge  $\bar{p}_H^A - \Delta V$  than  $\bar{p}_H^A - \epsilon - \Delta V$  which contradicts (136). Next, suppose that  $\bar{p}_H^A > \bar{p}_L + \Delta V$ . The disclosing high quality firm's profit at any  $p \in (\bar{p}_L + \Delta V, \bar{p}_H^A)$  is

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(p))] (p - c_H) - A = \alpha(1-\gamma_H)(\bar{p}_H^A - c_H) - A \quad (137)$$

while setting a price  $p - \Delta V > \bar{p}_L$  the low quality firm would make a profit of

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(p))] (p - \Delta V - c_L),$$

which using (137) can be rewritten as

$$\alpha(1-\gamma_H)(\bar{p}_H^A - c_H) \frac{p - \Delta V - c_L}{\bar{p}_H^A - c_H},$$

and this is increasing in  $p$ , implying low quality firm would gain by deviating to prices larger than  $\bar{p}_L$ , a contradiction. Thus, if the disclosing high quality firm randomizes, it serves the entire market when the rival firm produces low quality.

First, consider a candidate mixed disclosure equilibrium where high quality discloses and sets a deterministic price  $p_H^A \leq p_H^N$ . The only way this configuration can be sustained as a D1 equilibrium is when consumers believe that a disclosing firm that undercuts  $p_H^A$  or a non-disclosing firm that undercuts  $p_H^N$  is a low quality firm. If  $p_H^A < p_H^N$ , equilibrium requires that low quality randomizes over two disjoint intervals  $[\underline{p}_L^1, \bar{p}_L^1]$  and  $[\underline{p}_L^2, \bar{p}_L^2]$  with  $\bar{p}_L^1 = p_H^A - \Delta V$  and  $\bar{p}_L^2 = p_H^N - \Delta V$ . This is because with at most two different deterministic prices set by high quality firms, the indifference condition for the low quality firm cannot be met if there are more than two disjoint intervals, while  $\bar{p}_L^1 < p_H^A - \Delta V$  or  $\bar{p}_L^2 < p_H^N - \Delta V$  cannot be part of an equilibrium as in that case the low quality firm wants to deviate to prices above the upper bound of the respective interval. Thus, if a high quality firm discloses, then he serves the market if the low quality rival sets prices in the upper interval, but not when low quality sets prices in the lower interval as for all  $p, \tilde{p}$  with  $p < \bar{p}_L^1 < \tilde{p}$ ,  $V_L - p > V_H - p_H^A > V_L - \tilde{p}$ . Let us then consider a candidate equilibrium where high quality chooses  $p_H^N$  with probability  $1 - \gamma_H$  and price  $p_H^A$  with probability  $\gamma_H$ , where  $p_H^A \leq p_H^N$  and low quality randomizes over two disconnected sets  $[\underline{p}_L^1, \bar{p}_L^1]$  and  $[\underline{p}_L^2, \bar{p}_L^2]$  with probabilities  $\gamma_L$  and  $1 - \gamma_L$ ,

respectively. This is exactly the mixed disclosure equilibrium constructed in the proof of Lemma 12. Following the steps in that proof we see that for  $0 \leq \gamma_L \leq 1$  we need that  $\frac{\alpha}{2}\Delta c \leq f \leq (1 - \frac{\alpha}{2})\Delta c$ . Further,  $\gamma_H$  is determined by the solution to

$$\Delta V (\alpha(1 - \gamma_H))^2 + \left( \frac{f}{\Delta c} - \frac{\alpha}{2} \right) \Delta V \alpha(1 - \gamma_H) + 2 \left( \frac{\alpha}{2} + \frac{f}{\Delta c} \right) ((\lambda - 1)f - A) = 0.$$

As  $\frac{f}{\Delta c} \geq \frac{\alpha}{2}$ , we have that a solution with  $1 - \gamma_H \geq 0$  does not exist if  $(\lambda - 1)f < A$  as in that case all three coefficients of this quadratic expression are non-negative and the last term is strictly positive. Thus, a necessary condition for this candidate equilibrium to exist is that  $A > (\lambda - 1)f$ , in addition to  $\frac{\alpha}{2}\Delta c \leq f \leq (1 - \frac{\alpha}{2})\Delta c$ . Therefore, we must have  $A > (\Delta V - \Delta c) \frac{f}{\Delta c}$ . ■

Next, consider a mixed disclosure equilibrium where when the high quality type discloses it randomizes its pricing decision. Using our previous argument, such a firm sells to the entire market if the competitor is of low quality. This requires  $\bar{p}_H^A \leq \underline{p}_L + \Delta V$ . If this behavior constitutes part of an equilibrium, it still must be the case that  $\bar{p}_L = p_H^N - \Delta V$  so that  $\pi_L^* = \alpha(1 - \gamma_H)(\bar{p}_L - c_L)$  and  $\pi_H^* = \frac{\alpha(1 - \gamma_H)}{2}(p_H^N - c_H)$ . Moreover, the D1 refinement requires that low quality is just indifferent between charging  $\bar{p}_L$  and not disclose and set price  $p_H^N$  so that we should have  $\pi_L^* = \frac{\alpha(1 - \gamma_H)}{2}(p_H^N - c_L)$ . Combining these requirements yields:

$$p_H^N = c_L + 2\Delta V \quad (138)$$

so that

$$\pi_H^* = (2\Delta V - \Delta c) \frac{\alpha(1 - \gamma_H)}{2} \quad (139)$$

and

$$\bar{p}_L = p_H^N - \Delta V = c_L + \Delta V.$$

Further, as

$$(1 - \alpha\gamma_H)(\underline{p}_L - c_L) = \alpha(1 - \gamma_H)(\bar{p}_L - c_L)\alpha(1 - \gamma_H)\Delta V$$

we have

$$\underline{p}_L = \left[ \frac{\alpha(1 - \gamma_H)}{1 - \alpha\gamma_H} \right] \Delta V + c \quad (140)$$

The upper bound of prices for a high quality firm that discloses is now:

$$\bar{p}_H^A = \underline{p}_L + \Delta V = \left[ \frac{\alpha(1 - \gamma_H)}{1 - \alpha\gamma_H} + 1 \right] \Delta V + c_L$$

which is decreasing in  $\gamma_H$  and converges to  $\bar{p}_L$  as  $\gamma_H \rightarrow 1$ . The profit of the high quality firm when it discloses and charges price  $\bar{p}_H^A$  is given by

$$(\bar{p}_H^A - c_H)(1 - \alpha\gamma) - A$$

and this is equal to  $\pi_H$  (the profit (139) it gets by not advertising and charging  $p_H^N$  if, and only if,

$$\gamma_H = \frac{1}{\alpha} \left[ \frac{\Delta V - \Delta c(1 - \frac{\alpha}{2}) - A}{\Delta V - \frac{\Delta c}{2}} \right]$$

This candidate equilibrium only exist if  $\gamma_H \in (0, 1)$ , which in particular requires that  $A > (1 - \alpha)(\Delta V - \Delta c)$ .

**Proposition 16** *In the region of the parameter space*

$$\begin{aligned} 0 &< f < \frac{\alpha}{2} \Delta c \\ \tilde{A}(f) &< A \leq f^{*-1}(f) \end{aligned}$$

*the M1 mixed disclosure equilibrium (characterized in Proposition 9) is the the unique equilibrium. In the region of the parameter space where*

$$A \leq \tilde{A}(f)$$

*the pure disclosure equilibria (characterized in Proposition 7) are the only equilibria. Finally, if  $A \leq \tilde{A}(f)$  and  $f > (1 - \frac{\alpha}{2}) \Delta V$ , the fully nondistortionary equilibrium is the unique equilibrium.*

**Proof.** Follows from Proposition 9, Proposition 7 and Lemma 15. ■