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Injunctions against false advertising

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Preliminary version

Abstract

We consider a situation of duopolistic competition in which one firm may (falsely) advertise a higher product quality. Consumers are heterogeneous in that one group forms rational beliefs about the advertised good’s quality, whereas some consumers are naïve in that they fully trust the advertisements. We compare two scenarios in which either the competitor or a government agency has the right to file an injunction suit against the advertising firm. From a welfare perspective, we show that it may be optimal either to have the competitor or the government agency as plaintiff where optimality depends on the share of naïve consumers and the trial costs in a non-trivial way. Consumers prefer the social planner (competitor) as plaintiff when the share of naïve consumers and trial costs are low (high).

Keywords: injunction suits, false advertising, law enforcement, naïve consumers, product differentiation.

JEL classification: K41, K42, L13, L15.

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1 Introduction

In this paper, we analyze injunction suits in the context of false advertising. Advertising is a fundamental strategic variable for firms, which they use in an attempt to gain a competitive edge over their competitors. To this end, firms advertise product attributes such as durability, effectiveness, environmental safety, origin, etc. to convince consumers of their high quality. In most countries, firms are restricted in that they cannot make incorrect claims or use false advertising. For example, in the European Union, Directive 2006/114/EC sets the rules concerning misleading and comparative advertising. According to EU law, misleading advertising is “any advertising which in any way, including its presentation, deceives or is likely to deceive the persons to whom it is addressed or whom it reaches and which, by reason of its deceptive nature, is likely to affect their economic behaviour or which, for those reasons, injures or is likely to injure a competitor” (Article 2 (a)). Moreover, Article 5 requires the member states to “set up adequate mechanisms that enable persons and organizations having a legitimate interest to bring an action to the competent courts or administrative authorities for the cessation and/or the prohibition of misleading or unlawful comparative advertising”.¹ This means that, in principle, government agencies as well as competitors or consumer protection agencies may take legal action against misleading advertising in Europe. However, as the European Commission (2012) points out, the implementation of the directive differs across the member states. In some countries, public authorities can take action against rogue traders (e.g., in France, Italy, and the United Kingdom), whereas in other countries, only affected companies or specific associations can seek court action (e.g., in Austria and Germany).

In the United States, the Federal Trade Commission (FTC) and its Division of Advertising Practices enforce the truth-in-advertising laws across different media outlets.² Moreover, alongside its authority to investigate law violations by individuals and businesses, the FTC also has federal rule-making authority to issue industry-wide regulations.

In general, firms engaging in false advertising may be forced to stop their misleading ad campaign and may be subject to fines and/or compensation payments to consumers. Interestingly, such sanctions and fines do not seem to prevent firms from engaging in false advertising as highlighted by a number of recent cases in which firms were found guilty of misconduct. The range of products for which firms have invented deceptive advertising campaigns, which were later challenged by the FTC, includes such diverse products as dog food³, allegedly mosquito-repellent wristbands⁴, and sports shoes⁵.⁶ Moreover, there is a number of pending cases. The most prominent recent case is Volkswagen’s decept-

²The FTC “authorizes the filing of a complaint when it has ‘reason to believe’ that the law has been or is being violated and it appears to the Commission that a proceeding is in the public interest” (ftc.gov).
tive “Clean Diesel” ad campaign, presumably resulting in billions of US$ in fines and compensations.\textsuperscript{7}

In Germany consumer protection agencies took legal action against producers of jam (Darbo case) and tea\textsuperscript{8} for what they considered misleading labeling. In the UK, the Advertising Standards Authority (ASA) stepped in to ban an advert by supermarket chain Tesco in response to the horsemeat scandal; the advert suggested the problem affected “the whole food industry”.\textsuperscript{9}

As mentioned in the above directive and different form these examples, legal action can also be initiated by competitors. In the United States, a claim can be made against a defendant for false or misleading advertising under Section 43(a) of the Lanham Act. In this case, the following elements are met and the plaintiff must show: (1) defendant made false or misleading statements as to his own products (or another’s); (2) actual deception, or at least a tendency to deceive a substantial portion of the intended audience; (3) deception is material in that it is likely to influence purchasing decisions; (4) the advertised goods travel in interstate commerce; and (5) a likelihood of injury to plaintiff.

For example, in the 2014 case \textit{POM Wonderful LLC v. Coca-Cola Co}, the Supreme Court—even though food and beverage labels are subject to Food and Drug Administration (FDA) regulation—permitted producer POM to proceed with its false advertising claim that “one of Coca-Cola’s juice blends mislead consumers into believing the product consists predominantly of pomegranate and blueberry juice when it in fact consists predominantly of less expensive apple and grape juices, and that the ensuing confusion causes POM to lose sales”.\textsuperscript{10} This case also highlights another important aspect: the potential difference between a regulator’s and a competitor’s incentive to file suit. As a matter of fact, Coca-Cola’s labeling complied with FDA requirements.

In \textit{C-356/04 Lidl Belgium}, the Court of Justice of the European Union (CJEU) ruled in favor of retail store chain Lidl which considered its competitor’s advertising relying on price comparisons to be misleading.

Despite this widespread use of false advertising, firms’ possibility to bring cases against their competitors in court, and the attempts by government bodies and consumer protection agencies to protect consumers from fraud and deception in the marketplace, there is only little theoretical evidence on the economic effects and implications of false advertising. In this paper, we aim to add to the literature. In particular, we are interested in the incentives of different parties (either a competitor interested in expected profit maximization or a government agency interested in maximizing total welfare) to go to court to stop a possibly deceptive advertising campaign. From a social welfare and consumer point of view, these incentives have important implications with regard to who should be given the right to go to court in the first place. Moreover, we analyze how consumer rationality and trial costs impact on these outcomes.

In order to answer these questions, we analyze a modified duopoly market with product


\textsuperscript{8}Bundesverband der Verbraucherzentralen und Verbraucherverbände – Verbraucherzentrale Bundesverband e.V. v Teekanne GmbH & Co. KG.

\textsuperscript{9}See http://www.telegraph.co.uk/finance/newsbysector/retailandconsumer/10286180/Tesco-rapped-over-misleading-horsemeat-ads.html.

\textsuperscript{10}See https://www.supremecourt.gov/opinions/13pdf/12-761_6k47.pdf, p. 1.
differentiation à la Hotelling (1929). In this market, one firm has private information with regard to whether a quality-increasing innovation has been successful or not. The firm may decide to (falsely) advertise a high product quality and the competitor or a government agency may challenge this claim at a cost by filing an injunction suit. It is further assumed that there is a group of rational consumers, who take into account that advertising may convey wrong information, and a group of naïve consumers, who fully trust commercial ads not challenged before court.

We show that in general, there is an equilibrium in mixed strategies in which the firm sometimes advertises a high quality despite an unsuccessful innovation outcome, and the plaintiff sometimes files a suit when trial costs are low. When trial costs are high, the plaintiff refrains from filing a suit and the innovating firm always advertises a high quality (independently of whether the innovation was successful or not). With regard to the impact of the share of naïve consumers, it turns out that for given trial costs, a competitor is more likely to file a suit when the share of naïve consumers increases. When the government agency must authorize the filing of a suit, for some intermediate trial costs, a higher share of naïve consumers first increases the likelihood of a trial before it decreases again. A comparison of the incentives to go to court yields that the government agency is more likely to initiate legal action for low shares of naïve consumers and low trial costs; otherwise, the competitor is more likely to take legal action.

This finding also has important implications for consumers. As their expected welfare is only driven by the likelihood of legal action, they prefer a government agency as plaintiff for low shares of naïve consumers and low trial costs. Different from that, total welfare—due to the potential costs involved—is affected by whether an injunction suit has been initiated or not. We find that expected social welfare is higher for the government agency as plaintiff when the share of naïve consumers and trial costs are low and when trial costs are (very) high. In the first case, the competitor has too little incentive to go to court, whereas in the second case, a government body would save on the trial costs. For low to intermediate trial costs and a sufficiently high share of naïve consumers, we show that expected social welfare is higher for the competitor as plaintiff.

Our paper adds to the literature on false and misleading advertising. A closely related paper is Rhodes & Wilson (2015). The authors analyze a model in which a regulator is able to punish false claims in a situation with rational consumers. In their setup, false advertising can have a beneficial effect (‘price’ effect) by counteracting market power, as the high-quality type will choose a lower price if he cannot signal his type. In consequence firms favor stricter regulations (higher fines) than consumers do. At the same time, the negative consequences of false advertising are due to the ‘persuasion’ effect, making consumers misperceive the true actual quality. The authors provide conditions under which weak, rather than strong, regulation can be optimal for consumers and society due to the positive effects of false advertising in counteracting firms’ market power. Different from their study which mainly deals with a monopoly setup and assumes rational consumers throughout the analysis, we focus on competition and allow for the competitor as plaintiff and for naïve consumers.

Glaeser & Ujhelyi (2010) analyze various responses to regulating misinformation (counter-advertising, taxing, and regulating the product). The authors assume that consumers naïvely believe all advertising claims (see also Hattori & Higashida, 2012). Their results
suggest that a certain amount of misinformation about product quality may improve social welfare under imperfect competition. Despite the fact that misinformation tends to reduce consumer surplus (as consumers buy products they would not have purchased in the first place), misinformation mitigates the problem of underconsumption of products resulting from imperfect competition. As a consequence, government regulations benefit consumers but may harm firms and overall social welfare. Different from their study, we allow for different shares of naïve consumers.

Hattori & Higashida (2015) set up a model of price and misleading advertising competition between two firms producing horizontally and vertically differentiated brands to analyze the allocative implications of misinformation and related regulatory policies (advertising taxes, unit and ad valorem taxes on production). Similar to Glaeser & Ujhelyi (2010), misinformation has two effects which go in opposite directions from a social welfare point of view: misinformation distorts consumers’ decision-making but can correct inefficiencies resulting from the misallocation of goods. It is shown that advertising competition may create a prisoner’s dilemma for firms and reduce welfare but may benefit certain consumer groups: smart consumers who are not affected by misinformation and those with a preference for low-quality brands. Different from their study, advertising is not always misleading in our case but may be correct. Furthermore, fully rational consumers in our setup may not know the product quality with certainty, and we analyze parties’ incentives to take legal action.

The paper proceeds as follows. Section 2 presents the model. We derive the equilibrium in Section 3 and compare the incentives to file an injunction suit. In Section 4, we compare the implications for social welfare and consumer surplus. Section 5 concludes.

2 The model

We consider a model of horizontal product differentiation à la Hotelling (1929) with two firms, 1 and 2. The firms are located at the extremes of a linear city of unit length, with firm 1 being located at \( L_1 = 0 \) and firm 2 at \( L_2 = 1 \). Firm 2 offers a product of normal quality which consumers value at \( v, v > 0 \). Firm 1 invests into product innovation which comes at no cost and—if successful—results in a higher product quality.\(^{11}\) With probability \( 1/2 \), firm 1 is successful and produces a good of high quality which is valued at \( v + 1 \) by consumers. With probability \( 1/2 \), innovation is not successful and product quality remains at the normal level. Whether a product innovation has been achieved is firm 1’s private information and can thus be observed neither by consumers nor by the competitor but may be verified at some costs in court.\(^{12}\) For both firms and both quality levels, marginal costs of production are normalized to zero.

Consumers of mass one are uniformly distributed along the line. Each consumer buys one unit either from firm 1 or from firm 2, i.e., the market is covered.\(^{13}\) Consumers know

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\(^{11}\)Results would not change for positive investment costs as long as the level of costs is exogenous and not too high.

\(^{12}\)Another interpretation is that the competitor—due to its expertise—may be able to verify the actual quality provided (possibly at some cost) but still has to go to court or convince the authorities that product quality has been incorrectly advertised.

\(^{13}\)To this end, we make the standard assumption that the valuation \( v \) is sufficiently large.
that firm 2 offers a product of normal quality, whereas firm 1 may offer a high-quality product. A consumer located at \( x \) who buys from firm \( i \) (with \( i \in \{1, 2\} \)) pays price \( p_i \) and bears linear transport costs \( |L_i - x| \).\(^{14}\) We consider two types of consumers. At each point along the line, a share \( q \) of consumers is naïve in a sense made precise below (subscript \( n \)). The remaining share \( 1 - q \) of consumers is rational (subscript \( r \)). Consumers’ belief with regard to the probability that firm 1 offers a product of high quality is denoted by \( \psi_j \), with \( 0 \leq \psi_j \leq 1 \), \( j = n, r \).

After the realization of the possible product innovation, firm 1 has the choice whether or not to (possibly falsely) advertise a high product quality to customers. Advertising is assumed costless per se.\(^{15}\) Note that attention will be restricted to equilibria in which a firm serving high quality always advertises and if consumers do not observe advertising activity by firm 1, they expect the firm to offer a product of normal quality (superscript \( N \)), i.e., \( \psi_n = \psi_r = \psi^N = 0 \).\(^{16}\)

We compare two different scenarios regarding law enforcement. In both scenarios, in the event that firm 1 advertises a high quality, it may face an injunction suit. In the first scenario, the competitor can initiate the lawsuit. In the second scenario, the decision to file an injunction suit is taken by a government agency which aims at maximizing expected social welfare. The overall costs for both parties, plaintiff and defendant, in a lawsuit are denoted by \( k \), \( k > 0 \), which includes the costs of verifying quality. The cost-sharing rule applied corresponds to the English rule. If the firm offers only normal quality but has chosen to advertise, the injunction is confirmed and the sued firm bears the cost \( k \).\(^{17}\) Because high quality has not been verified in court and advertisements are discontinued, all consumers update their beliefs about the probability that firm 1 serves high quality such that \( \psi_n = \psi_r = \psi^N = 0 \). If the sued firm indeed offers high quality, an injunction will not be granted and the costs \( k \) have to be borne by the plaintiff. Because quality has been verified in court, all consumers update their beliefs about the probability that firm 1 serves high quality such that \( \psi_n = \psi_r = \psi^H = 1 \).

If firm 1 advertises a high quality (superscript \( A \)) and is not challenged by an injunction suit, consumers do not receive any further information in addition to observing an advertisement. In this case, naïve consumers are assumed to fully believe in firm 1’s advertising message and hence \( \psi_n = \psi_r = 1 \). In contrast, rational consumers update their beliefs taking into account that the firm may have falsely advertised a high quality such that \( \psi_r = \psi_r^A \), where \( 1/2 \leq \psi_r^A \leq 1 \).\(^{18}\) Figure 1 summarizes the beliefs of the two different consumer groups.

The advertising and court decisions are followed by competition in prices. Finally, given firms’ advertising and pricing decisions (possibly) together with the findings in the

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\(^{14}\)This specification means that we consider a transport-cost parameter equal to one.

\(^{15}\)We mainly require that advertisement costs do not depend on the product’s true quality. By this assumption, signaling via advertising expenditures is excluded.

\(^{16}\)This assumption has been employed in other recent contributions on misleading advertising as well (e.g., Corts 2013). By doing so, we exclude the existence of some less plausible but possible equilibria.

\(^{17}\)Note that our results do not change qualitatively when we assume that trial costs are equally split between the parties independent of the court decision (American rule).

\(^{18}\)The process of updating beliefs by rational consumers is described in more detail in the following section where we establish the equilibrium. The lower bound on \( \psi_r^A \) follows from the assumption that firms offering high quality always advertise and the success probability is equal to 1/2.
injunction suits, consumers decide which firm to buy from and profits are realized.

For the reader’s ease, we summarize the timing of the game below:

1. Firm 1 learns its quality level and decides whether or not to (truthfully or falsely) advertise high quality of its product.

2. In the event of advertising initiated by firm 1, in scenario 1 (2) firm 2 (the governmental agency) can file an injunction suit focusing on false advertising claims regarding firm 1’s quality. In case of a lawsuit, a correct ruling is rendered by the court.

3. Firms set prices for their products taking into account consumers’ beliefs about product quality.

4. Consumers decide which firm to buy from.

We now turn to the equilibrium analysis.

3 Equilibrium

3.1 Price competition

We solve the model by backward induction and start by analyzing consumers’ purchase decisions in the last stage. Given firm 1’s advertising strategy, the decision about filing an injunction suit, and the resulting belief $\psi_j$ ($\psi_j \in \{\psi^N, \psi^H, \psi^A\}$ with $j = n, r$), a consumer of type $j$ who is indifferent between buying from firm 1 and firm 2 is located at $\tilde{x}_j$, where

$$v + \psi_j - p_1 - \tilde{x}_j = v - p_2 - (1 - \tilde{x}_j) \Rightarrow \tilde{x}_j = \frac{1}{2} + \frac{\psi_j - p_1 + p_2}{2}.$$
As a result, the demands realized by firm 1 and firm 2 for this consumer type amount to $\tilde{x}_j$ and $1 - \tilde{x}_j$.  

Given the indifferent consumer’s location, firms’ profit levels are given by

$$\pi_1 = p_1 \left( q\tilde{x}_n + (1 - q)\tilde{x}_r \right) = p_1 \left( \frac{1}{2} + \frac{\Psi - p_1 + p_2}{2} \right)$$

and

$$\pi_2 = p_2 \left( q(1 - \tilde{x}_n) + (1 - q)(1 - \tilde{x}_r) \right) = p_2 \left( \frac{1}{2} - \frac{\Psi - p_1 + p_2}{2} \right),$$

where $\Psi := q\psi_n + (1 - q)\psi_r$ can be interpreted as the weighted belief about firm 1’s quality improvement.

Firms compete in prices. Starting from (1) and (2), we obtain equilibrium prices as

$$p_1 = 1 + \frac{\Psi}{3}$$

and

$$p_2 = 1 - \frac{\Psi}{3}. \tag{4}$$

The results show that the difference in prices is increasing in the weighted belief about firm 1’s quality improvement. Given equilibrium prices, the indifferent consumers’ locations result as

$$\tilde{x}_j = \frac{1}{2} + \frac{3\psi_j - 2\Psi}{6}. \tag{5}$$

As $0 \leq \psi_r \leq \Psi \leq \psi_n \leq 1$, firm 1 always serves weakly more than half of the naïve consumers, whereas less than half of the rational consumers choose the product of firm 1 if beliefs differ starkly and the share of naïve consumers is relatively large. Nevertheless, for some positive belief about the quality improvement, firm 1 always serves more than half the market because of

$$q\tilde{x}_n + (1 - q)\tilde{x}_r = \frac{1}{2} + \frac{\Psi}{6} \geq \frac{1}{2} \tag{6}$$

To conclude the description of price competition, we report equilibrium profits which are given by

$$\pi_1 = \frac{(3 + \Psi)^2}{18} \tag{7}$$

and

$$\pi_2 = \frac{(3 - \Psi)^2}{18}. \tag{8}$$

Profits are increasing (decreasing) in expected quality of firm 1’s product for firm 1 (firm 2).

---

We implicitly assumed that both firms serve both groups of consumers, i.e., $\tilde{x}_j \in [0, 1]$. As will become clear later, this is indeed the case in any equilibrium considered.

Second-order conditions for a profit maximum are satisfied. See chapter 3.1.4 in Belleflamme & Peitz (2010) for a similar treatment.
3.2 Injunction suits and advertising

In the following, we will analyze firm 2’s and the governmental agency’s incentive to file an injunction suit (Section 3.2.2) and firm 1’s incentive to advertise a high-quality product although the product is only of normal quality (Section 3.2.3). The two decisions determine the equilibrium outcomes as described in Section 3.2.4. In preparation, we summarize profits, consumer surplus and social welfare for the possible informational situations in Section 3.2.1.

3.2.1 Profits, consumer surplus, and social welfare

In order to analyze the decisions taken by the agents involved, three different cases for consumers’ beliefs are relevant: (i) beliefs in the event that no advertising is observed at all or advertising had to be stopped after an injunction had been granted, (ii) advertising can be upheld after an injunction suit, and (iii) advertising takes place and no injunction suit is filed. In the following, we will derive profits, consumer surplus, and social welfare (independent of legal costs $k$) in the three cases. The results constitute the base for the derivation of the equilibrium and the comparison of social welfare for the different possible plaintiffs.

Case (i) is associated with firm 1 being correctly identified as a normal quality firm. Beliefs are given by $\psi_n = \psi_r = \Psi = 0$ which results in profits of

$$\pi_1^N = \pi_2^N = \pi^N = \frac{1}{2}. \quad (9)$$

All consumers buy a normal-quality good from the firm nearest to their location (i.e., $\tilde{x}_n = \tilde{x}_r = 1/2$) and pay an equilibrium price equal to $p_1 = 1$. Accordingly, consumer surplus amounts to

$$CS^N = v - 1 - 2 \int_0^{1/2} x dx = v - \frac{5}{4}. \quad (10)$$

We assume social welfare as the (unweighted) sum of consumer surplus and firm profits. Therefore, social welfare in the first case is given by

$$SW^N = v - \frac{1}{4}. \quad (11)$$

For case (ii) firm 1 is correctly predicted to offer high quality and $\psi_n = \psi_r = \Psi = 1$. In this case, profits are given by

$$\pi_1^H = \frac{8}{9} \quad (12)$$

and

$$\pi_2^H = \frac{2}{9}. \quad (13)$$

For both naïve and rational consumers, the indifferent consumer is located at $\tilde{x}_j = 2/3$ (i.e., two thirds of consumers enjoy the gross valuation $v + 1$ instead of $v$) and prices
amount to \( p_1 = 4/3 \) and \( p_2 = 2/3 \). Accordingly, we obtain consumer surplus as

\[
CS^H = v + \frac{2}{3} - \frac{24}{33} - \frac{12}{33} - \left( \int_0^{\frac{2}{3}} x \, dx + \int_{\frac{2}{3}}^1 x \, dx \right) = v - \frac{13}{18}. \tag{14}
\]

Social welfare amounts to

\[
SW^H = v + \frac{7}{18}. \tag{15}
\]

Finally, with advertising but no injunction suit, case (iii), we obtain \( \psi_n = 1 \) and \( \psi_r = \psi^A_r \) (to be determined below), with corresponding profit levels equal to

\[
\pi_1^A(\psi^A_r) = \frac{(3 + \Psi^A)^2}{18} \tag{16}
\]

and

\[
\pi_2^A(\psi^A_r) = \frac{(3 - \Psi^A)^2}{18}, \tag{17}
\]

where \( \Psi^A := q + (1 - q)\psi^A_r \).

The real quality of firm 1’s product may be either high or normal in this case, where the probability for high quality must coincide with the belief \( \psi^A_r \) of rationale consumers. With \( \tilde{x}_n = 1 - \Psi^A/3 \) and \( \tilde{x}_r = 1/2(1 + \psi^A_r) - \Psi^A/3 \) and prices equal to \( p_1 = 1 + \Psi^A/3 \) and \( p_2 = 1 - \Psi^A/3 \), the expected consumer surplus amounts to

\[
CS^A(\psi^A_r) = v + \psi^A_r \left( \frac{1}{2} + \frac{\Psi^A}{6} \right) - \left( 1 + \frac{\Psi^A}{3} \right) \left( \frac{1}{2} + \frac{\Psi^A}{6} \right) - \left( 1 - \frac{\Psi^A}{3} \right) \left( \frac{1}{2} - \frac{\Psi^A}{6} \right) \\
- \left( q \left( \int_0^{\tilde{x}_n} x \, dx + \int_{\tilde{x}_n}^{1} x \, dx \right) + (1 - q) \left( \int_0^{\tilde{x}_r} x \, dx + \int_{\tilde{x}_r}^{1} x \, dx \right) \right) \\
= v - \frac{5}{4} + \psi^A_r(18 + \psi^A_r) - q(1 - \psi^A_r)(9 - 5\psi^A_r) + 4q^2(1 - \psi^A_r)^2 \\
= v - \frac{5}{4} + \psi^A_r(18 + \psi^A_r) - q(1 - \psi^A_r)(9 - 5\psi^A_r) + 4q^2(1 - \psi^A_r)^2 \tag{18}
\]

Expected social welfare as the sum of consumer and producer surplus amounts to

\[
SW^A(\psi^A_r) = v - \frac{1}{4} + \frac{\psi^A_r}{2} + \frac{5(\psi^A_r)^2 - q(1 - \psi^A_r)(9 - 13\psi^A_r) + 8q^2(1 - \psi^A_r)^2}{36}. \tag{19}
\]

Interestingly, whereas aggregate consumer surplus in this case decreases in the share of na"ive consumers, social welfare increases in \( q \) for either a sufficiently large \( \psi^A_r \) or \( q \). This is due to the fact that, for a given probability of high quality, by charging a higher price in comparison to its competitor, firm 1 induces a distortion in rational consumers buying decision from a social point of view. This distortion of too low consumption of the (potentially) higher quality good may be mitigated by more na"ive consumers who are more inclined to choose firm 1’s product.\(^2\)

\(^2\)Both consumer surplus and social welfare increase in the belief \( \psi^A_r \) reflecting that it indicates a higher probability for the product of firm 1 being of high quality.
3.2.2 Decision on filing an injunction suit

We can now investigate firm 2’s decision to file an injunction suit and compare it to the decision problem of a governmental agency in the event that advertising is used by firm 1. When deciding on an injunction suit, the actors take rational consumers’ belief $\psi_r^A$ as given. Furthermore, with rational decision-makers, $\psi_r^A$ must coincide with their own assessment of the probability that the advertisement campaign has been initiated by a high-quality firm.

Firm 2 as the plaintiff

Observing an advertisement by firm 1, firm 2 has to decide whether to file an injunction suit. If successful, consumers’ beliefs about firm 1’s product quality are corrected downwards yielding an additional profit for firm 2 which obtains $\pi_N$. If the injunction is dismissed, both types of consumers know that firm 1 offers a product of high quality reducing firm 2’s profit to $\pi_H^2$ and in addition firm 2 has to bear the trial costs $k$. Not filing an injunction suit results in profits equal to $\pi_A^2(\psi_r^A)$. The increase in expected profits from filing an injunction suit is given by

$$
\Delta_{F2}(\psi_r^A) = \psi_r^A (\pi_H^2 - k) + (1 - \psi_r^A) \pi_N - \pi_2^A (\psi_r^A) \\
= (1 - \psi_r^A) (6q + \psi_r^A (1 - q)^2 - q^2) - \psi_r^A k.
$$

(20)

Accordingly, (not) filing an injunction suit is optimal for firm 2 for $\Delta_{F2} > 0 (\Delta_{F2} < 0)$. For $\Delta_{F2}$ firm 2 is just indifferent between filing an not filing an injunction suit. For $\psi_r^A = 1$, i.e., rational beliefs suggest only high-quality firms advertise, $\Delta_{F2} = -k < 0$ and no injunction suit is filed. In addition, the more likely it is that the advertisement comes from a firm offering only normal quality (decrease in $\psi_r^A$), the more profitable an injunction suit gets in expectation as$^{22}$

$$
\frac{\partial \Delta_{F2}}{\partial \psi_r^A} = \frac{1 - 2q(4 - q) - 2\psi_r^A (1 - q)^2}{18} - k < \frac{\partial \Delta_{F2}}{\partial \psi_r^A} \bigg|_{\psi_r^A = \frac{1}{2}, k = 0} = -q(6 - q) < 0.
$$

(21)

The maximal level of $\Delta_{F2}$ results for the minimum feasible level of $\psi_r^A$ that is given by the success probability of $1/2$ for a product innovation. Consequently, we obtain an upper bound for trial costs $\hat{k}_{F2}$ such that for higher trial costs an injunction will neither be filed:

$$
\hat{k}_{F2} := \frac{1 + q(10 - q)}{36}.
$$

(22)

Further note that filing an injunction suit becomes more profitable for firm 2 in expected terms the higher the share of naïve consumers:

$$
\frac{\partial \Delta_{F2}}{\partial q} = \frac{(1 - \psi_r^A) (3 - \Psi^A)}{9} > 0.
$$

(23)

$^{22}$Note that $\psi_r^A \geq 1/2$. 

11
The higher the share of naïve consumers, the higher is the additional profit from correcting expectations downwards in the event of a successful lawsuit and the lower is the additional loss if an injunction is not granted by the court.

Government agency as the plaintiff

Turning to the government agency, the expected increase in social welfare from an injunction suit amounts to

\[ \Delta_{SW} = \psi_r^A SW^H + (1 - \psi_r^A) SW^N - k - SW^A \]

\[ = \frac{(1 - \psi_r^A) \left(5\psi_r^A + q \left(9 - 13\psi_r^A\right) - 8q^2 \left(1 - \psi_r^A\right)\right)}{36} - k. \]  

(24)

As for firm 2, in the event of \( \psi_r^A = 1 \), the expected gain in welfare is negative (\( \Delta_{SW} = -k < 0 \)) and no injunction suit will be filed. Furthermore, as for firm 2, an injunction suit becomes more favorable, the lower beliefs \( \psi_r^A \) are that

\[ \frac{\partial \Delta_{SW}}{\partial \psi_r^A} = \frac{(1 - 2\psi_r^A)(5 + 8q^2 - 13q) - q(9 - 8q)}{36} < 0. \]

(25)

Therefore, as for firm 2 we find a critical cost level \( \hat{k}_{SW} \) such that an injunction suit will never be filled for any theoretically possible belief by the government agency. The critical cost level amounts to

\[ \hat{k}_{SW} := \frac{5 + q(5 - 8q)}{144}. \]

(26)

Finally, an increase in the share of naïve consumers changes the increase in expected welfare according to

\[ \frac{\partial \Delta_{SW}}{\partial q} = \frac{(1 - \psi_r^A) \left(9 - 13\psi_r^A - 16q \left(1 - \psi_r^A\right)\right)}{36} \]

which is larger than zero for low values of \( q \) and \( \psi_r^A \) but is negative otherwise. That is, for given a given belief an increase in the share of naïve consumers increases incentives for filing an injunction suit for firm 2 but can decrease incentives for the government agency. This is due to the fact that social welfare in the absence of an injunction suit can increase in the share of naïve consumers.

Comparison of filing decisions

We are now in a position to compare incentives for filing an injunction suit for firm 2 and a governmental agency. To do so, we calculate the difference in the expected gains from

\[^{23}\text{Note that the government agency takes into account trial costs } k \text{ as social costs irrespective of the outcome of a trial.}\]
an injunction suit

\[ \Delta_\Delta = \Delta_{F2} - \Delta_{SW} \]

\[ = (1 - \psi^A_r)q(1 + 2q(1 - \psi^A_r) + 3\psi^A_r) + (1 - \psi^A_r)k \] (28)

which can be smaller or larger than zero. The last term indicates that the government agency takes the trial costs into account independent of the trial outcome. The first term summarizes differences in the changes of firm 2’s profit and social welfare (other than trial costs). The gain from filing an injunction suit may be higher or lower for firm 2 in comparison to the governmental agency depending on the parameter values.\(^{24}\) Especially, for a given belief \(\psi^A_r\) we have

\[ \frac{\partial \Delta_\Delta}{\partial q} = \frac{(1 - \psi^A_r)(1 + 4q(1 - \psi^A_r) + 3\psi^A_r)}{12} > 0, \] (29)

that is, an increase in the share of naïve consumers makes an injunction suit more favorable for firm 2 in comparison to the government agency. Indeed, for only naïve consumers, \(q = 1\),

\[ \Delta_\Delta|_{q=1} = (1 - \psi^A_r)\frac{1}{4} + (1 - \psi^A_r)k > 0 \] (30)

the gain from an injunction suit is always higher for firm 2 than for the government agency. This is true because the expected change in firm 2 profits is higher than the change in social welfare and firm 2 neglects trial costs in the case of winning. Instead, for only rationale consumers., \(q = 0\),

\[ \Delta_\Delta|_{q=0} = -(1 - \psi^A_r)\frac{1}{12} + (1 - \psi^A_r)k \] (31)

which is smaller zero implying higher expected gains from an injunction suit for the government agency for low values of trial costs. In this case, the expected change from an injunction suit in social welfare is larger than the gain in profits for firm 2 and this difference can even outweigh the fact that the government agency always takes into account trial costs.

We may also compare the critical cost levels \(\hat{k}_{F2}\) and \(\hat{k}_{SW}\). The difference amounts to

\[ \hat{k}_{F2} - \hat{k}_{SW} = \frac{-1 + q(35 + 4q)}{144} =: \Delta_k. \] (32)

For the extremes of \(q = 1\), only naïve consumers, and \(q = 0\), only rational consumers, we find

\[ \Delta_k|_{q=1} = \frac{19}{72} > 0 \]

and

\[ \Delta_k|_{q=0} = -\frac{1}{144} < 0. \]

\(^{24}\)Different gains do not necessarily result in different actions as the decision to file an injunction suit is a binary decision. However, differences in expected gains are necessary to induce different filing decisions.
Accordingly, with only naïve consumers, there is a range for trial costs $k$ for which the social planner would never file an injunction suit, whereas firm 2 might do so. In contrast, with only naïve consumers, it might be the case that firm 2 will never file a lawsuit for some values of $k$ for which the social planner might nevertheless go to court. Finally, since

$$\frac{\partial \Delta_k}{\partial q} = \frac{35 + 8q}{144} > 0,$$

there exists exactly one value for the share of naïve consumers $q$ for which the two critical values for trial costs coincide.

So far, our results can be summarized as follows:

**Lemma 1** For given beliefs about firm 1’s product quality in the event of advertising, the expected gain from filing an injunction suit may be higher for firm 2 or the government agency. The difference in expected gains increases in favor of firm 2 for a higher share of naïve consumers. This is mirrored in a higher (smaller) critical level for trial costs for firm 2 in comparison to the social planner for a large (small) share of naïve consumers.

### 3.2.3 Advertising decision

Firm 1 always advertises when a high quality innovation was achieved. In the event of no successful product innovation, no advertising assures a profit level of $\pi^N$. With advertising profits increase to $\pi_1^A(\psi^A)$ as long as no injunction suit is filed, whereas if an injunction suit is filed, firm 1’s profits equal $\pi^N$ and it has to bear the trial costs $k$. In consequence, the expected gain from advertising for a firm 1 offering normal quality is given by

$$\Delta_{F1} := (1 - \beta) \left( \pi_1^A(\psi^A) - \pi^N \right) - \beta k,$$

where $\beta$ is the probability of being taken to court. It chooses to advertise (not to advertise) for sure if the difference $\Delta_{F1}$ is larger (smaller) than zero. For $\Delta_{F1} = 0$ a firm 1 offering normal quality is indifferent between advertising and not advertising. Expected profits from advertising decrease with the probability of an injunction suit and increase in the belief $\psi^A$ and the share of naïve consumers for $\beta < 1$.

### 3.2.4 Equilibrium

For either the social planner or firm 2 as the potential plaintiff, the equilibrium of the game can take on two forms: either an equilibrium in mixed strategies or an equilibrium in which an injunction suit is never filed and therefore firm 1 always advertises. In any equilibrium, the equilibrium value of $\psi^A$ is determined by the advertisement decision.

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25Given our assumptions regarding beliefs this is indeed a dominant strategy for a firm offering high quality.

26In contrast, a pure strategy equilibrium in which firm 1 never advertises after unsuccessful product innovation or in which the plaintiff always files suit cannot exist. In the absence of false advertising the plaintiff would never file suit to which misleading advertising is a best response resulting in a contradiction. Likewise, if the plaintiff always files suit no misleading advertisement will be observed to which filing no suits is a best response.
of firm 1. Denoting by $\alpha$ the probability that a firm 1 with normal quality chooses to advertise we obtain

$$\alpha = \frac{1 - \psi^A_r}{\psi^A_r} \iff \psi^A_r = \frac{1}{1 + \alpha}. \quad (34)$$

Hence, in equilibrium there is an inverse relationship between consumers’ beliefs about quality and firm 1’s advertising decision: More advertising results in a lower belief that firm 1 offers a high-quality product and vice versa.

For a mixed-strategy equilibrium, $\psi^A_r \in (1/2, 1)$ must hold and the equilibrium is derived from $\Delta_{F1} = 0$ and $\Delta_{F2} = 0$ or $\Delta_{SW} = 0$ depending on the plaintiff considered. If $\Delta_{F2} < 0$ (or $\Delta_{SW} < 0$) for every value of $\psi^A_r \in (1/2, 1)$, the corner solution of $\beta = 0$ and $\psi^A_r = 1/2$ results, which is the possible pure-strategy equilibrium. The pure-strategy equilibrium results if $k > \hat{k}_{F2}$ ($k > \hat{k}_{SW}$).

From $\Delta_{F2} = 0$, we obtain for $\psi^A_r$

$$\psi_{F2} = \frac{1 - 2q(4 - q) - 18k + \sqrt{(1+4q)^2 - 36k(1-2q(4-q)) + 324k^2}}{2(1-q)^2} \quad (35)$$

and therefore as $\partial \Delta_{F2}/\partial \psi^A_r < 0$ in equilibrium

$$\psi^A_r = \max \left\{ \psi_{F2}, \frac{1}{2} \right\} \quad (36)$$

if it is firm 2’s decision whether or not to file an injunction suit.\(^{28}\)

In the scenario in which the social planner acts as the plaintiff, we solve for $\Delta_{SW} = 0$ and obtain

$$\psi_{SW} = \frac{5 - 2q(11 - 8q) + \sqrt{(5 - 4q)^2 - 144k(1 - q)(5 - 8q)}}{2(1-q)(5-8q)} \quad (37)$$

In this scenario, since $\partial \Delta_{SW}/\partial \psi^A_r < 0$, the equilibrium is characterized by\(^{29}\)

$$\psi^A_r = \max \left\{ \psi_{SW}, \frac{1}{2} \right\}. \quad (38)$$

To find the equilibrium probability of an injunction suit, we may first state that no injunction suits are filed in a pure-strategy equilibrium in which $\psi^A_r = 1/2$ results, i.e., $\beta = 0$ in this case. Otherwise, in a mixed-strategy equilibrium, we have to plug in the equilibrium belief $\psi^A_r$ into (33) and solve for $\beta$

$$\beta \left( \psi^A_r \right) = 1 - \frac{k}{\pi_1^A(\psi^A_r) - \pi^N + k}$$

$$= 1 - \frac{18k}{18k + (3 + \psi^A_r + q(1 - \psi^A_r)^2) - 9} \quad (39)$$

\(^{27}\)In the following, we will focus on the equilibrium value of $\psi$ instead of $\alpha$.

\(^{28}\)For $q = 1$ ($q = 0$) we obtain $\psi^A_r = \max \{5/(5+k), 1/2\}$ ($\psi^A_r = \max \{1 - 18k, 1/2\}$).

\(^{29}\)For $q = 1$ ($q = 0$), we obtain $\psi^A_r = \max \{1 - 36k, 1/2\}$ ($\psi^A_r = \max \{1/2 + \sqrt{25 - 720k}/10, 1/2\}$).
Note that $\partial \pi_A^1/\partial \psi_r > 0$ and accordingly $\partial \beta/\partial \psi_r > 0$.

The following lemma summarizes the equilibrium:

**Lemma 2** The equilibrium of the advertising-injunction suit game is either in pure strategies with no injunction suits and all firms advertising or in mixed strategies. For firm 2 as potential plaintiff the equilibrium is given by $\psi_r^A = 1/2$ and $\beta = 0$ for $k \geq \hat{k}_F^2$ and $\psi_r^A = \psi_F^2$ and $\beta = \beta(\psi_F^2)$ for $k < \hat{k}_F^2$. In the scenario with the social planner as potential plaintiff, the equilibrium is described by $\psi_r^A = 1/2$ and $\beta = 0$ for $k \geq \hat{k}_{SW}$ and $\psi_r^A = \psi_{SW}$ and $\beta = \beta(\psi_{SW})$ for $k < \hat{k}_{SW}$.

Figure 2 illustrates these findings.

![Figure 2: Rational consumers’ beliefs and the scope of legal action (blue) under both plaintiff scenarios.](image)

To conclude this section, we will briefly discuss the equilibrium outcomes in the two plaintiff regimes. With respect to comparative statics in a mixed-strategy equilibrium we can establish that $30$

$$\frac{\partial \psi_F^2}{\partial q} > 0, \quad \frac{\partial \psi_F^2}{\partial k} < 0.$$  

That is a higher share of naïve consumers makes firm 2 more aggressive with regard to filing an injunction suit such that the equilibrium probability of a misleading advertisement decreases with increases the probability of high quality given an advertisement. Second, an increase in trial costs makes firm 2 more reluctant with respect to filing an injunction suit resulting in a higher probability of a misleading advertisement (and therefore a lower probability of high quality in the case of advertising). Due to $\partial \beta/\partial \psi_r > 0$, an increase in the share of naïve consumers will be accompanied by a higher probability of an injunction suit given advertising and a lower probability of an injunction suit for higher trial costs.

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30The following calculations are available from the authors by request.
With the governmental agency as potential plaintiff, we find for a mixed-strategy equilibrium

\[
\frac{\partial \psi_{SW}}{\partial q} > (\leq 0) \text{ for } q \text{ small and } k \text{ large (otherwise), } \frac{\partial \psi_{SW}}{\partial k} < 0.
\]

Accordingly, we have that an increase in the share of naïve consumers may either lead to a lower (higher) probability of misleading advertisement. As shown before, the expected gain from an injunction suit for the government agency may be decreasing in the share of naïve consumers. In this case, the agency becomes less aggressive and the likelihood of misleading advertising increases (and \(\psi_A^v\) falls). Regarding trial costs, no qualitative differences arise in comparison to firm 2 as potential plaintiff.

Figure 3 summarizes the difference in the equilibrium levels for the two potential plaintiffs. Given a very limited number of naïve consumers in the market, less false advertising is observed when the social planner can file an injunction suit compared to the case in which the competitor can do so (green and red areas). For relatively high legal costs and a very small fraction of naïve consumers, the competitor would refrain from filing suit altogether and only the government agency may file suit (red area). In this situation, the competitor can gain relatively little, as most consumers perfectly update their beliefs (see also Lemma 1). This is anticipated by firm 1 which more often opts for false advertising.

As the share of naïve consumers increases, the opposite is true. Now, firm 2 can benefit more in terms of higher profits, as consumers are less likely to accurately update their beliefs but simply trust the advertising message. This means that many of them go to the competitor expecting a better quality. In this case, the competitor is more aggressive, resulting in a lower equilibrium probability for misleading advertisements (blue and orange areas). Again for relatively high trial costs, it is only the competitor who may file a lawsuit (orange area).

4 Comparisons

After having established the differences in the equilibrium outcomes for the two possible plaintiffs, we are now in a position to provide a welfare comparison. We first investigate social welfare as the sum of producer and consumer surplus before turning explicitly to consumer welfare.

4.1 Welfare

In order to assess which of the two parties, competitor or government agency, should take the position of filing an injunction suit against firm 1 from a social welfare point of view, we need to evaluate the two expected levels of social welfare level. As described in Section 3.2, three possible informational states can be distinguished in the event of advertising taking place. First, firm 1 may have been successful with its innovation activity which becomes public knowledge due to an injunction suit being filed. In this case, recognizing trial costs welfare, amounts to \(SW^H - k\). The probability for the state to occur is given
Figure 3: Comparison of rational consumers’ beliefs and the scope of legal action under both plaintiff scenarios.

Note: blue: $\psi_{F_2}^A > \psi_{SW}^A > 1/2$, $\beta_{F_2} > \beta_{SW} > 0$; orange: $\psi_{F_2}^A > \psi_{SW}^A = 1/2$, $\beta_{F_2} > \beta_{SW} = 0$; green: $\psi_{SW}^A > \psi_{F_2}^A > 1/2$, $\beta_{SW} > \beta_{F_2} > 0$; red: $\psi_{SW}^A > \psi_{F_2}^A = 1/2$, $\beta_{SW} > \beta_{F_2} = 0$; white: $\psi_{SW}^A = \psi_{F_2}^A = 1/2$, $\beta_{F_2} = \beta_{SW} = 0$ (always false advertising, no injunction suits).

by $\beta(\psi_r^A)/2$. Second, the injunction suit may have revealed that firm 1 used misleading advertising. In this case, welfare amounts to $SW^N - k$. The ex-ante probability for this state to incur is given by $\alpha \beta(\psi_r^A)/2 = \beta(1 - \psi_r^A)/(2\psi_r^A)$. Finally, an advertising has been used but no injunction suit has been filed. In this case, firm 1 offers high quality with probability $\psi_{F_2}^A$ and expected social welfare amounts to $SW^A$. The ex-ante probability for this state is given by $1/2(1 + \alpha)(1 - \beta(\psi_r^A)) = (1 - \beta(\psi_r^A))/(2\psi_r^A)$. In addition, no advertising may be observed in the first place. In this case, firm 1 is correctly perceived to serve normal quality and social welfare amounts to $SW_N$. This state occurs with probability $1/2(1 - \alpha) = (2\psi_r^A - 1)/\psi_r^A$. Accordingly, we obtain

$$
\mathbb{E}[SW] = \frac{\beta(\psi_r^A)}{2} \cdot (SW^H - k) + \frac{\beta(\psi_r^A)(1 - \psi_r^A)}{2\psi_r^A} \cdot (SW^N - k) + \frac{1 - \beta(\psi_r^A)}{2\psi_r^A} \cdot SW^A(\psi_r^A) + \frac{2\psi_r^A - 1}{2\psi_r^A} \cdot SW^N.
$$

The last expression may be rearranged to

$$
\mathbb{E}[SW] = \frac{1}{2} \left[ \beta(\psi_r^A)SW^H + (1 - \beta(\psi_r^A))SW^{A,H}(\psi_r^A) \right] + \frac{1 - \psi_r^A}{2\psi_r^A} \left[ \beta(\psi_r^A)SW^N - (1\beta(\psi_r^A))SW^{A,N} \right] + \frac{2\psi_r^A - 1}{2\psi_r^A} SW_N - \frac{\beta(\psi_r^A)}{2\psi_r^A} k.
$$

(41)
where $SW^{A,H}$ ($SW^{A,N}$) describes social welfare in the state of advertising and no injunction suit if the true quality is high (normal). The last term in the above equation depicts expected trial costs. $SW^{A,H}$ and $SW^{A,N}$ depend on $\psi_r^A$ only because of the effect of consumers beliefs on overall transportation costs. Transportation costs are affected directly by consumer beliefs for rational consumers and indirectly by the change in prices for both groups of consumers.

Given the equilibrium outcomes in the two scenarios, we get the expected welfare levels $\mathbb{E}[SW_{F2}]$ and $\mathbb{E}[SW_{SW}]$. The results are as follows:

**Proposition 1** Expected social welfare is higher for the government agency as a plaintiff as long as the government agency is more aggressive with regard to filing an injunction suit. If firm 2 is the more aggressive plaintiff, we have that expected social welfare is higher for firm 2 as plaintiff if both the government agency and firm 2 will file suit with some positive probability. If only firm 2 will sometimes files suit, expected social welfare is higher for firm 2 (the government agency) as plaintiff for intermediate (high) trial costs.

The question of who is the more aggressive plaintiff depends to a large extent on the shares of rational and naïve consumers. From Lemma 2 and Proposition 1, we can state the following:

**Corollary 1** For a low share of naïve consumers, the government agency is more aggressive and expected social welfare is higher with the government agency as the plaintiff. For higher shares of naïve consumers, firm 2 is the more aggressive plaintiff and expected social welfare is higher for firm 2 (the government agency) as plaintiff for low to intermediate (high) costs of trials.

Figure 4 compares both levels. As the proposition and a comparison with Figure 3 highlight, the result is mainly driven by the plaintiff’s incentives to file an injunction suit against firm 1. We observe that when there is less false advertising under the social planner, i.e., the social planner is more aggressive regarding filing an injunction suit (high $\psi_r^A$), the expected welfare is higher under a government agency as plaintiff (see orange and brown area). In a mixed-strategy equilibrium in which there is less false advertising for firm 2 as plaintiff, expected social welfare is higher for firm 2 as plaintiff (blue area). When only firm 2 has an incentive to file injunction suits, social welfare is higher for firm 2 as plaintiff as long as trial costs are not too large (green area); otherwise social welfare is higher for the social planner as (inactive) potential plaintiff (red area).

The results can be explained by resorting to equation (41). Social welfare is described as a function of $\psi^A$ and therefore the extent of false advertising. In slight abuse of the model, considering an exogenous change in rational consumers beliefs, we obtain

$$
\frac{d\mathbb{E}[SW]}{d\psi^A_r} = \frac{\partial \mathbb{E}[SW]}{\partial \beta(\psi^A_r)} \beta'(\psi^A_r) + \frac{1 - \beta(\psi^A_r)}{2(\psi^A)^2} \left[ SW^N - SW^{A,N} \right] + \frac{\beta(\psi^A_r)}{2(\psi^A)^2} k + \frac{1 - \beta(\psi^A_r)}{2(\psi^A)^2} \frac{\partial SW^{A,H}(\psi^A_r)}{\partial \psi^A_r} \left(42\right)
$$

where $\frac{\partial SW^{A,H}(\psi^A_r)}{\partial \psi^A_r} = \frac{\partial SW^{A,N}(\psi^A_r)}{\partial \psi^A_r}$ has been used. The first line describes the indirect effect resulting from an adjustment in the probability of filing suit whereas
Figure 4: Comparison of expected social welfare under both plaintiff scenarios.

Note on the left panel: In the orange area: $E[SW_{SW}] > E[SW_{F2}]$ and $\psi_{r,SW}^A > \psi_{r,F2}^A = 1/2$; in the brown area: $E[SW_{SW}] > E[SW_{F2}]$ and $\psi_{r,SW}^A > \psi_{r,F2}^A > 1/2$; in the blue area: $E[SW_{SW}] < E[SW_{F2}]$ and $1/2 < \psi_{r,SW}^A < \psi_{r,F2}^A$; in the green area: $E[SW_{SW}] < E[SW_{F2}]$ and $1/2 = \psi_{r,SW}^A < \psi_{r,F2}^A$; in the red area: $E[SW_{SW}] > E[SW_{F2}]$ and $1/2 = \psi_{r,SW}^A < \psi_{r,F2}^A$.

the second line reports the overall direct effect. The first two terms in line two are unambiguously positive and indicate that, first, with less false advertising the welfare loss due to the distortion in consumer choice and the accompanying higher transportation costs is reduced, and second, a decrease in false advertising per se reduces the number of suits and therefore trial costs. The last term in line two of equation (42) describes the change in transportation costs in the event of advertising but no injunction suit. Whereas the latter effect can be negative, the overall direct effect of less false advertising (an increase in $\psi_{r}^A$) is always positive.

If the government agency is more aggressive with respect to injunction suits than firm 2, the equilibrium with the government agency displays lower false advertisement and a higher probability for injunction suits in comparison to the one with firm 2 as plaintiff. Since with respect to the filing decision, the government agency which consider social welfare is just indifferent it holds that $\partial E[SW]/\partial \beta(\psi_{r}^A) = 0$. Accordingly, welfare is strictly higher for the government agency as plaintiff due to $\psi_{r,SW}^A > \psi_{r,F2}^A$.

If firm 2 is more aggressive expected social welfare may be higher or lower for firm 2 as plaintiff. As long as a mixed-strategy equilibrium results for both types of plaintiffs (blue area in Figure 3), expected welfare is necessarily higher for firm 2 as plaintiff. Again, it holds that $\partial E[SW]/\partial \beta(\psi_{r}^A) = 0$, whereas the direct effect of a lower number of false advertisement leads to a higher welfare level with firm 2 as the plaintiff (since $\psi_{r,F2}^A > \psi_{r,SW}^A$). Although the government agency considers social welfare when deciding about an injunction suit, it is constrained by a commitment problem. Ideally, it would commit to a higher rate of injunction suits in order to make false advertising less attractive.
Given the government agencies commitment problem, it may therefore further welfare by appointing the more aggressive firm 2 as the plaintiff.

The positive effect of a lower probability for false advertisements is still present when the government agency would abstain completely from filing injunction suits because of the high trial costs whereas firm 2 still files an injunction suit with positive probability. However, the direct positive effect of fewer false advertisement is no counteracted by a negative effect on social welfare since \( \partial \mathbb{E}[SW]/\partial \beta(\psi_r^A) < 0 \) and \( \beta(\psi_{r,F2}^A) > \beta(\psi_{r,SW}^A) = 0 \). Given the extent of false advertising, the gain in social welfare from an injunction suit is more than offset by trial costs. Given the positive and negative effects of the more aggressive behavior of firm 2 in comparison to the government, we find that for intermediate levels of trial costs expected welfare is still higher for firm 2 as plaintiff whereas the opposite results for high levels of trial costs (green and red area in Figure 3).

### 4.2 Consumers

We now turn attention to consumer welfare. As in the previous section, four possible states can emerge. However, since consumers are not directly affected by trial costs, the states of detected false advertising and no advertising yield the same outcome regarding consumer welfare. Accordingly, the ex-ante expected consumer surplus is given by

\[
\mathbb{E}[CS] = \frac{\beta(\psi_r^A)}{2} \cdot CS^H + \frac{1 - \beta(\psi_r^A)}{2\psi_r^A} \cdot CS^A(\psi_r^A) + \left(1 - \frac{1 - \beta(\psi_r^A)(1 - \psi_r^A)}{2\psi_r^A}\right) \cdot CS^N.
\]

Figure 5 illustrates the comparison of expected consumer surplus under both plaintiff regimes. The findings indicate that consumers always prefer the more aggressive plaintiff.\(^{31}\)

### 5 References


\(^{31}\)This results holds also for the subgroups of rationale and naïve consumers. The result is due to the fact that consumers to not have to bear trial costs.
Figure 5: Comparison of expected consumer surplus under both plaintiff scenarios.

Note: In the orange area, it holds that $E[CS_{F2}] < E[CS_{SW}]$, whereas we have $E[CS_{F2}] > E[CS_{SW}]$ in the blue area.