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The Improbability of Reswitching, the Certainty of Wicksell-Effects and the Poverty of Production Functions: The Cambridge Critique of Capital Transformed

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***The Improbability of Reswitching, the Certainty of Wicksell-Effects and
the Poverty of Production Functions:***

The Cambridge Critique of Capital Transformed

Contribution to VfS 2017

Abstract

Capital theory has taken a new turn with the theoretical discovery that wage curves tend to get linear in random systems, the larger they are, and with the confirmation that empirical wage curves do not deviate a great deal from linearity. The present paper adds to these results by arguing that reswitching becomes less likely for larger systems, while Wicksell effects are almost surely present. But it can also be shown that the elasticity of substitution is likely to be small in random systems so that a policy to lower real wages will not easily generate much additional employment in a closed economy. A new perspective on employment policies therefore is called for.

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The aim of this paper is to present new results in capital theory which must lead to a reorientation of research within the school of thought that emerged from Sraffa's classic *Production of Commodities by Means of Commodities* (Sraffa 1960). It has been shown that the assumption that the coefficients defining a Sraffa system are random leads to a modification of the critique of capital theory. So-called approximate surrogate production functions may be constructed, if the system exhibits random properties (Schefold 2013). Similarly, the Marxian proposition that profits equal surplus value can be reproduced as a mathematically rigorous theorem with these assumptions (Schefold 2016). Finally, it turns out that neoclassical equilibria with an endogenous composition of the capital stock (the so-called "old" neoclassical equilibrium) then is macro-economically stable and quite similar to the surrogate production function (Schefold 2016a).

One thus seems to get back to the state of economics in the late 19th century, when John Bates Clark postulated the production function, when Engels was convinced that the Marxian transformation of values into prices worked in such a way that the rate of profit in value terms was equal to the rate of profit in price terms and when the old neoclassical equilibrium described a general equilibrium in terms of supply and demand with a uniform rate of profit (Petri 2004). But the situation is more complicated than that. On one hand, what is true for random systems is not true in general so that the question arises whether actual systems have random properties. If they did, wage curves would have to be linear and the eigenvalues of the matrices of single product systems would have to have small moduli of the non-dominant eigenvalues. These properties seem to hold not strictly, but approximately, which is intriguing (see Mariolis Tsoulfidis 2014 and Shaikh 2016). We must confront the difficult question how far this criticism carries, which is relevant also for Keynesian economics. For example, Kaldor, without knowing, assumed linear wage curves turning upwards around a constant maximum rate of profit to represent technical progress (Schefold 1979).

I here want to show, basing myself partly on Schefold 2016a and on Schefold 2013a, that reswitching and reverse capital deepening are so improbable as to be insignificant. Wicksell effects will almost surely occur, if one does not specifically select the standard commodity as the numéraire. The critique of neoclassical theory was based on reverse capital deepening (Harcourt 1972). The main point of neoclassical theory has always been that lowering the real wage would lead to the adoption of more labour-intensive methods. The existing equipment of capital-intensive machinery would gradually be sold and replaced by more labour-intensive equipment of roughly the same value, and thus more people would find employment. It was a matter of changing methods of production and not, as in Wicksell effects, of a revaluation of existing machines which would remain in place. What really impressed the neoclassical economists at the time of the debate of capital theory was the possibility that lowering the real wage and raising the rate of interest might *often*, against their expectations, result in the adoption of more capital-intensive methods.

Now it has turned out, contrary to what at least I used to believe, that such reversals in the choice of technique are rare. But the essential core of the critique remains: the proposition that it suffices to lower real wages in order to increase the level of employment is dubious in closed economies for a new and different reason: the elasticity of substitution is likely to be lower than expected, because far fewer techniques appear on the envelope than there are potential techniques (Schefold 2013b). It is true that something at first sight similar to a wage reduction, namely reducing the exchange rate, may increase employment in open economies. By analogy, internal devaluation (if feasible) by an individual country in a monetary union may be successful, because that represents a reduction of absolute costs without necessarily changing the intensity of capital. This is a different effect to be recognised primarily as part of classical theory, while we are here concerned with the validity of the neoclassical argument which is based on the relative cost of capital and labour. If a country devalues, it reduces not only the cost of labour in terms of international currency, but also the costs of domestically produced capital goods; a new level of international competitiveness is reached,

possibly keeping the same techniques. By contrast, the neoclassical mechanism is supposed to be based on the reduction of the cost of labour by means of substitution, the change of technique involving an *increase* of the ratio of the interest rate to the wage rate.

Wicksell Effects

We begin with the usual system

$$(1+r)\mathbf{A}\mathbf{p} + w\mathbf{l} = \mathbf{p},$$

where \mathbf{A} is a semi-positive indecomposable and productive input-output matrix and \mathbf{l} a positive labour vector. The wage curve results, once a numéraire $\mathbf{d} \geq 0$ is given. Prices in terms of the wage rate $\hat{\mathbf{p}} = \mathbf{p}/w$ are given by

$$\hat{\mathbf{p}} = (\mathbf{I} - (1+r)\mathbf{A})^{-1} \mathbf{l}; \quad 0 \leq r < R;$$

where $R > 0$ is the maximum rate of profit. The wage rate with numéraire \mathbf{d} , $\mathbf{d}\mathbf{p} = 1 = \mathbf{d}\hat{\mathbf{p}}w$, then is $w = 1/\mathbf{d}\hat{\mathbf{p}}$; it is monotonically falling, since the components of $\hat{\mathbf{p}}$ are rising.

If one assumes, as usual, that the numéraire is the net output of the economy and if the analysis is confined to stationary states, activity levels $\mathbf{q} > 0$ are given by $\mathbf{q} = \mathbf{d}(\mathbf{I} - \mathbf{A})^{-1}$, total capital is $K = \mathbf{q}\mathbf{A}\mathbf{p}$, labour employed $L = \mathbf{q}\mathbf{l}$, and the intensity of capital $k = K/L$ results from $y = w + rk$, $k = (y - w)/r$, where $y = \mathbf{d}\mathbf{p}/\mathbf{q}\mathbf{l} = 1/\mathbf{q}\mathbf{l}$ is output per head. This may be represented in a diagram

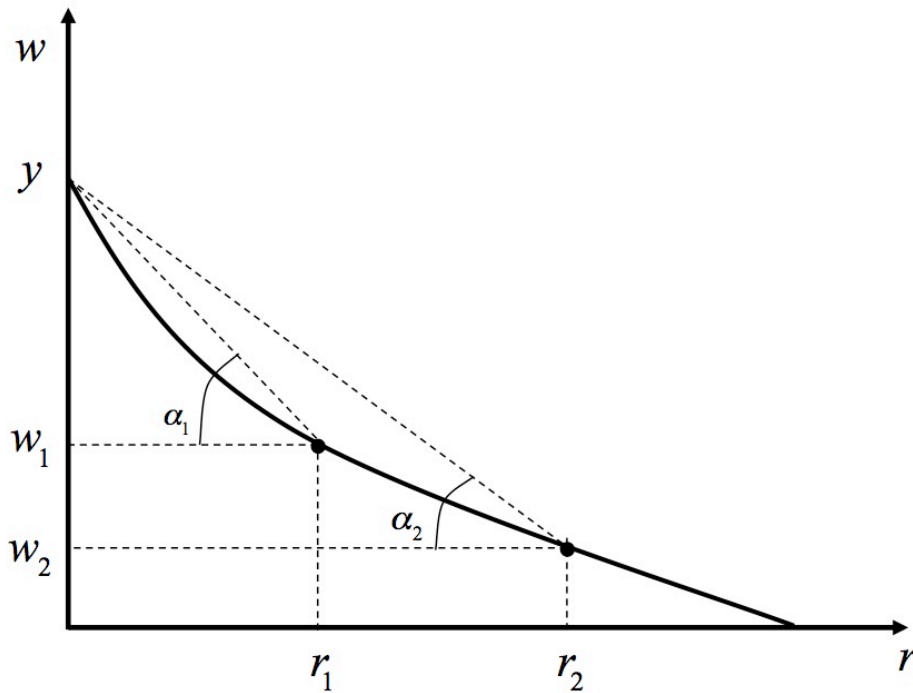


Diagram 1: Wage curve for one technique with neoclassical Wicksell effect. Full employment of labour force. L_1 at r_1 , of labour force $L_2 > L_1$ at r_2 . Capital intensities k_1, k_2 given by $\tan \alpha_i = k_i$.

The comparison of the two situations depicted in diagram 1 shows that the same amount of capital can employ more or less labour simply on account of a revaluation of the technique in actual use. If the wage rate falls from w_1 to w_2 , not the technique changes but the prices, and in such a way that the same amount of capital in terms of numéraire can, with constant returns to scale, lead to an expansion of production, because the employment of capital goods has become cheaper, so that more output of the same composition is produced and more labourers can be employed. But output per head remains constant and the rate of profit rises: the wage is shared among more labourers.

The assumed identity of the value of capital goods in terms of long-run prices in both positions and a given quantity of capital in value terms are problematic, but have been defended by Clark (1899) as caused by the accumulation of replacement funds in the context of technical change: they can be used to gradually introduce a new technique. The same may be argued in the case of Wicksell effects. If the existing capital goods become cheaper, accumulated amortisation funds can be used for expansion. The converse holds, if the capital goods become more expensive, as in the next case. The level of production is in this neoclassical perspective given by the amount of capital, not by effective demand.

An anti-neoclassical Wicksell effect is shown in diagram 2.

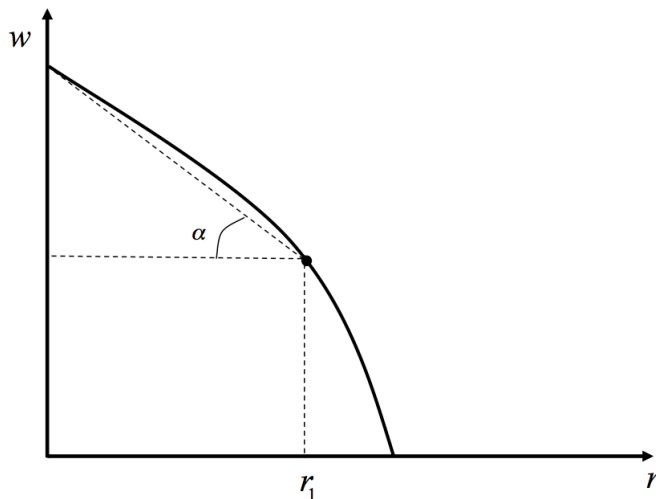


Diagram 2: A wage curve exhibiting an anti-neoclassical Wicksell effect.

Here, the fall of the real wage will render the same kind of equipment more expensive so that fewer workers will find employment, if the amount of capital in terms of numéraire is kept constant. The labour market is unstable, because, if the wage falls in consequence of unemployment, employment must be reduced further.

The limit case between neoclassical Wicksell effects, caused by wage curves which are convex towards the origin as in diagram 1 and anti-neoclassical Wicksell effects, caused by wage curves of opposite curvature, as in diagram 2, is given by the standard commodity s of a given technique (A, l) with $s = q(I - A)$, with $(1 + R)qA = q > 0$, $ql = 1$ and, in consequence, the linear wage curve $w = 1 - (r / R)$. The standard commodity is an eigenvector of the input matrix and the capital-output ratio is the inverse of the maximum rate of profit. There is no Wicksell effect, as one varies the rate of profit, because total outputs and inputs are proportional to the standard commodity.

It follows that the same technique will give rise to both Wicksell effects, if net output and hence the numéraire changes in either direction. To visualise the effect of a variation of the numéraire, we consider the set

$$D = \{ \mathbf{d} \geq 0 \mid \mathbf{d}(\mathbf{I} - \mathbf{A})^{-1} \mathbf{1} = 1 \} = \{ \mathbf{d} \geq 0 \mid w(0) = 1 \};$$

this is the set of all numéraires for which the wage rate at rate of profit zero or output per head is equal to one. It is clear that D is a simplex and that \mathbf{s} , the standard commodity, is an inner point of D , since $\mathbf{s} > 0$.

A wage curve may have inflection points. Whether a wage curve exhibits neoclassical or anti-neoclassical Wicksell effects therefore depends on the level of the rate of profit. This is illustrated by diagram 3

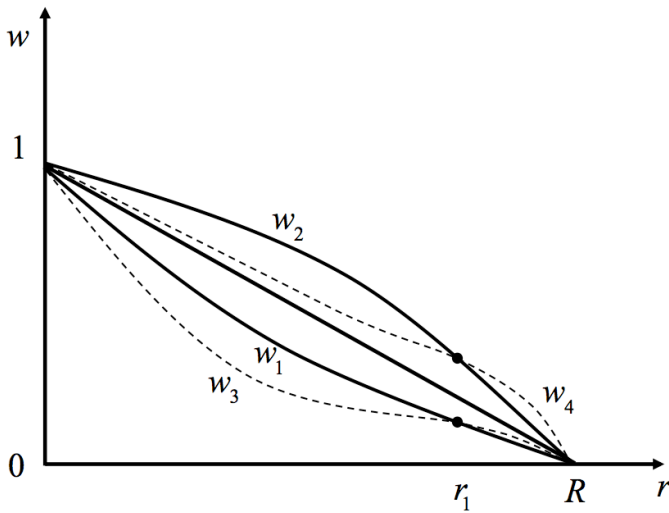


Diagram 3: The same technique gives rise to different Wicksell effects, represented relative to the standard wage curve: w_1 neoclassical throughout, w_3 with inflection point at r_1 in part neoclassical (r low); w_2 anti-neoclassical throughout, w_4 in part anti-neoclassical (r high).

We now choose some r_1 ; $0 < r_1 < R$; we can divide the set D of all numéraires/net outputs for technique (\mathbf{A}, \mathbf{I}) , with $w(0) = 1$, into three sets (assuming that $\hat{\mathbf{p}}(0)$ and $\hat{\mathbf{p}}(r_1)$ are not proportional), according to the level of the wage $w(r)$, associated with \mathbf{d} , at r_1 :

$$\begin{aligned} \{D_s &= \mathbf{d} \in D \mid w = 1 - r_1 / R\}, \\ \{D_n &= \mathbf{d} \in D \mid w < 1 - r_1 / R\}, \\ \{D_a &= \mathbf{d} \in D \mid w > 1 - r_1 / R\}. \end{aligned}$$

D_s is here given by the intersection of the simplex D with the simplex

$$\bar{D} = \{ \mathbf{d} \geq 0 \mid \mathbf{d}(\mathbf{I} - (1 + r_1)\mathbf{A})^{-1} \mathbf{1} = R / (R - r_1) \},$$

and this is a line segment in D which separates D_n from D_a . It is crucial that the standard commodity \mathbf{s} is an inner point of D_s . We here suppose that prices at $r=0$ and $r=r_1$ are not

proportional i.e. we exclude the case of the labour theory of value. We also exclude for simplicity any inflection points of the wage curves involved (as in the case of dotted lines in the diagram 3). Then we may say that the area of the potential numéraires is divided into two areas D_n and D_a yielding neoclassical or anti-neoclassical Wicksell effects. This is shown in diagram 4.

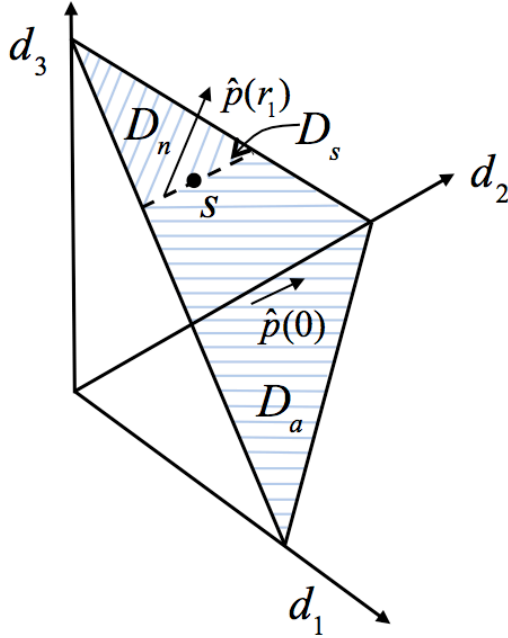


Diagram 4: The numéraire vectors that give rise to neoclassical (D_n) and to anti-neoclassical Wicksell effects (D_a).

We arrive at the main result of this section. Since s is an inner point of D , the probability that we shall have a neoclassical or an anti-neoclassical Wicksell effect is always positive, if we take the measure of the corresponding areas as percentages of the total area of D as measures of this probability. We may denote the probability for anti-neoclassical Wicksell effects as

$$\varphi = \frac{\mu(D_a)}{\mu(D)},$$

with $0 < \varphi < 1$. A priori, all \mathbf{d} are equally probable: we may just as well have a neoclassical as well as an anti-neoclassical Wicksell effect, while the probability of having no Wicksell effect at all is zero, because D_s is of measure zero. This corresponds to empirical findings. Wage curves are never absolutely straight, unless by construction, and the curvature may go either way.

In practice, the numéraire, being by definition the vector of net output, is composed of all consumption goods and, if we also admit growth, investment goods. The curvature for such averages is not as strong as it is when individual commodities are chosen as numéraires. As one moves down the envelope of wage curves engendered by a spectrum of techniques, where the optimum technique is chosen at each rate of profit, the curvatures of the individual wage curves making up the envelope will alternate, but not regularly. A series of anti-neoclassical Wicksell effects will be followed by a series of neoclassical Wicksell effects and *vice versa*, for only one method changes at each switchpoint, but the curvature is determined by the technique as a whole, and so usually several switchpoints must be passed and several methods changed before the sign of

the Wicksell effect changes. If the system is approximately random, one can nevertheless get an approximate surrogate production function, because each switch in itself lowers the intensity of capital, as the wage falls, since reverse capital deepening is unlikely according to the results of the next section. A capital reversal results only, if the anti-neoclassical Wicksell effect is strong enough to raise the intensity of capital sufficiently strongly to more than compensating the capital deepening due to the technical change before the next switch is reached.

In order to see how the character of the Wicksell effects may depend on the numéraire, given net output, it is necessary to transcend the above analysis and to abandon the assumption that the numéraire is equal to net output. More cases then would have to be distinguished, but we shall here be content with looking at one particular example. Let the economy be in standard proportions \mathbf{q} , with $\mathbf{q}\mathbf{l} = 1$, $\mathbf{e}\mathbf{l} = 1$, $\mathbf{q}(\mathbf{I} - \mathbf{A})\bar{\mathbf{p}} = 1$ and $\bar{\mathbf{p}}$ standard prices, hence $w = 1 - r/R$. The capital-labour ratio in standard prices then is

$$\bar{k} = \bar{K}/L = \mathbf{q}\mathbf{A}\bar{\mathbf{p}}/\mathbf{q}\mathbf{l} = \mathbf{q}\mathbf{A}\bar{\mathbf{p}} = (1/R)\mathbf{q}(\mathbf{I} - \mathbf{A})\bar{\mathbf{p}} = 1/R$$

and equal to the capital-output ratio, for output is equal to one in this numéraire. Hence there is no Wicksell effect.

Now we change the numéraire only, without changing the composition of outputs, and define prices in terms of commodity i by $\mathbf{p}^* = \bar{\mathbf{p}}/\bar{p}_i$. Then we have the capital-labour ratio k^* in terms of this numéraire given by $K^* = \bar{K}/\bar{p}_i = 1/R\bar{p}_i$. This will fall with the rate of profit, if \bar{p}_i rises (neoclassical Wicksell effect). That prices may go up or down with r , if they are not equal to labour values, is sufficiently known, but also clear from differentiating $1 = \mathbf{q}(\mathbf{I} - \mathbf{A})\bar{\mathbf{p}}$ with respect to r . If Wicksell effects are thus simply due to numéraire changes, they will be regarded as not so relevant.

Should we not also consider the opposite case: changes in the composition of output, keeping the numéraire constant? Formally, we would find a certain symmetry, with Wicksell effects being due to output-changes in isolation. However, we are here concerned with the neoclassical question of how substitution of capital and labour may keep output constant. Hence a different question becomes important: we must ask to what extent neoclassical assumptions about the substitution of capital and labour can be verified, when techniques change.

Reswitching and Reverse Capital Deepening: Predictions about Technical Change

I repeat my terminology in order to avoid misunderstandings. One speaks of switchpoints, if two wage curves intersect on the envelope. These two wage curves, derived from the same spectrum of techniques, will differ with respect only to one method of production in one industry, since technical choice, as induced by changes in distribution, is generically piecemeal, while other sorts of intersections occur below the envelope. Prices are equal at switchpoints, so that prices change continuously at the switchpoint, while quantities (activity levels) change spasmodically in the transition, for which the composition of net output is given. Reswitching takes place, if the same two wage curves intersect twice on the envelope. This is obviously more rare than reverse capital deepening, which takes place, if two wage curves intersect twice, the switchpoint with the higher rate of profit being on the envelope and with the first intersection being dominated by the wage curve of a third technique, as shown in diagram 5. As is clear from diagram 5, reverse capital deepening is remarkable because the intensity of capital rises instead of falling, as the rate of profit rises beyond the switchpoint at the higher rate of profit.

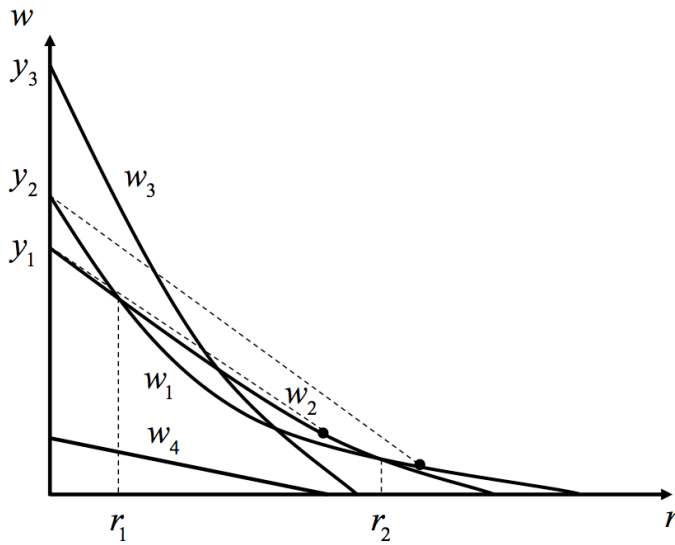


Diagram 5: Reverse capital deepening. The techniques represented by w_1 and w_2 differ only in the method of production in one industry. Hence it would be a matter of reswitching with two switches on the envelope at r_1 and r_2 , but the switch at r_1 is dominated by w_3 . There is a capital reversal at r_2 in that $k_1(r_2 - \varepsilon) < k_2(r_1 + \varepsilon)$ for $\varepsilon \rightarrow 0, \varepsilon > 0, k_i(r) = (1/r)(y_i - w(r))$. Finally, w_4 is an inefficient, but very labour-intensive technique.

Reverse capital deepening is important for two reasons. The essential reason is that one expects, according to the teaching of the mainstream, that lowering the real wage will lead to the adoption of more labour intensive methods, hence to an increase of employment due not to some dubious change of valuation as in a neoclassical Wicksell effect, but because a technique (one or several methods of production in one or several industries) changes. According to the conventional neoclassical theory, lowering the rate of profit by some percentage points will lead to an avalanche of such changes. But reverse capital deepening means that the opposite happens. A change of a method of production takes place, but the intensity of capital rises. Reverse capital deepening also is important for a derived reason: the wage curve diagram looks innocent in that one expects that techniques get more labour intensive as one moves down the envelope, but here, suddenly and surprisingly, the opposite happens.

I use the term “capital reversal” as a general term for what happens at the right-hand side switchpoint in the case of reswitching, for anti-neoclassical Wicksell effects and for reverse capital deepening. Moreover, there can also be capital reversals at the sectoral level, without having a capital reversal at the macro-economic level. In all these cases – though for different reasons – capital per head rises at the macro and/or micro level in response to a rise of the rate of profit. This is discussed in Han and Schefold (2006), but these and yet other cases with multiple switches are not discussed here. In the present paper, we only want to show that reswitching becomes more and more unlikely, the higher the dimension of the system. The attempted demonstration also applies to reverse capital deepening, in that we shall show that the double intersection of two wage curves, with one switch on the envelope, has a probability which tends to zero as the dimension of the system increases. One therefore finds empirically that examples of reswitching are very rare (only one has been found so far, in Han and Schefold 2006), while reverse capital deepening is encountered a little more often, for if there is the double intersection on the envelope, chances are that the first switchpoint at the lower rate of profit is dominated by the wage curve of some other technique. Zambelli (2004, p. 12) has a most interesting diagram of the envelope of empirical wage

curves for thirty countries with 31 sectors. There are as many Wicksell effects, anti-neoclassical and neoclassical, on the envelope of his wage curves as there are switchpoints, but there is no reverse capital deepening, let alone reswitching.

The likelihood of reswitching has first been analysed in Schefold (1976a); it was found that the probability of reswitching was not zero. The likelihood of reswitching and reverse capital deepening has also been discussed by D'Ippolito (1987) and Petri (2011), but they were concerned with two-sector models. It is the point of the present analysis to show that the likelihood of reswitching becomes much smaller for multi-sector models. The likelihood found for two-sector models is not very relevant for the discussion of the policy implications, especially if the capital reversal merely results from aggregation.

The measure of the likelihood for reswitching can be given by considering the set M_1 of conceivable methods of production $\{(\mathbf{a}_0, l_0) \geq 0\}$, which are an alternative to the method employed in the first industry (\mathbf{a}_1, l_1) of a given system (\mathbf{A}, \mathbf{l}) by having a switchpoint – that is by having the same price for the first commodity – at a given rate of profit r_1 . Using

$$M(r) = \{(\mathbf{a}_0, l_0) \geq 0 \mid (1+r)\mathbf{a}_0\hat{\mathbf{p}}(r) + l_0 = (1+r)\mathbf{a}_1\hat{\mathbf{p}}(r) + l_1, 0 \leq r < R\}, \hat{\mathbf{p}} = \mathbf{p} / \mathbf{w};$$

we find that $M_1 = M(r_1)$ is a simplex of dimension n in \mathbb{R}^{n+1} . Consider $M(r_1) \cap M(r_2)$, $r_1 \neq r_2$. This is an intersection of two simplices of dimension n in \mathbb{R}^{n+1} . The simplices are different, if $\hat{\mathbf{p}}(r_1)$ is not proportional to $\hat{\mathbf{p}}(r_2)$, hence essentially, if prices are not proportional to labour values. For more details and a diagrammatic representation see (fig. 1) in Schefold (1976a).

$M(r_1) \cap M(r_2)$ is of dimension $n-1$. It is trivial that $M(r_1) \cap M(r_2)$ contains (\mathbf{a}_1, l_1) for all r_2 . Let $\mu(M)$ be the n -dimensional Euclidean measure of set M . Obviously $\mu(M_1) > 0$, but $\mu(M(r_1) \cap M(r_2)) = 0$. This means that a given alternative method (\mathbf{a}_0, l_0) at r_1 will only incidentally be an alternative also at r_2 ; to get reswitching at two pre-assigned rates of profit is possible only by a fluke. But $M(r_1) \cap M(r_2)$ turns around (\mathbf{a}_1, l_1) as r_2 varies and thus covers open n -dimensional neighbourhoods. (\mathbf{a}_1, l_1) is semi-positive, but not necessarily positive. The case where (\mathbf{a}_1, l_1) is not strictly positive has been drawn in fig. 1 in Schefold (1976a). $M(r_1) \cap M(r_2)$ then covers a triangle, if $n=2$. Generally, the likelihood of getting a reswitch somewhere in $\{0 \leq r_2 < R\}$, $r_1 \neq r_2$, if we let r_2 vary, given r_1 , is not a fluke, since

$$M^* = \bigcup_{\substack{0 \leq r_2 < R \\ r_2 \neq r_1}} [M(r_1) \cap M(r_2)]$$

is n -dimensional and the larger, the larger is the movement of relative prices. The ratio

$$\pi = \frac{\mu(M^*)}{\mu(M(r_1))}$$

can be interpreted as the likelihood of reswitching. Clearly, $\pi > 0$, if relative prices are not constant, but the change of relative prices in $(0, R)$ is limited; so $M(r_1) \cap M(r_2)$ covers only a small part of $M(r_1)$ as r_2 varies; the probability π is small, but not infinitely small. If the change of relative prices is bounded, it turns out that, the larger the dimension of the system, the smaller is the

volume of the set of potential methods which give rise to reswitching relative to the volume of the set of all the potential methods which give rise to one switch. Hence, π must be much smaller than one for any given system with many sectors. One could also show that π diminishes, as the system approximates random properties, but we already know that the wage curves will then tend to be linear, so that π then obviously tends to zero.

A proof that the probability tends to zero as the dimension tends to infinity, if a boundedness condition is fulfilled, has been given in the Appendix in Schefold (2016a). As the motivation for the assumption is formally complicated, I here want to present the idea in more intuitive form. We use the same notation as in Schefold (2016a).

We consider the vertices spanning the simplex $M(r)$ as functions of r . They are denoted by $z_i \mathbf{e}_i$; $i = 1, \dots, n+1$; \mathbf{e}_i being unit vectors in \mathbb{R}^{n+1} .

Let $\bar{\mathbf{p}}, \bar{w}$ be standard prices and wage rate $\bar{w} = 1 - r/R$,

$$\tilde{\mathbf{p}} = \begin{pmatrix} (1+r)\bar{\mathbf{p}} \\ \bar{w} \end{pmatrix}.$$

The vertices of $M(r)$ then fulfil

$$[z_i \mathbf{e}_i - (\mathbf{a}_i, l_i)] \tilde{\mathbf{p}}(r) = 0; \quad i = 1, \dots, n+1;$$

so that they can be calculated, with $i = 2, \dots, n$:

$$\begin{aligned} z_1 &= \frac{1}{1+r}, \\ z_i &= \frac{\bar{p}_1}{(1+r)\bar{p}_i}, \\ z_{n+1} &= \frac{R\bar{p}_1}{R-r}. \end{aligned}$$

The z_i are continuous in $0 \leq r \leq R$, the prices also being continuous functions, except that $z_{n+1} = \hat{p}_1(r)$ diverges to infinity at R . Two of the vertices z_i , z_1 and z_{n+1} , are not in M^* , since $z_1(r)$ and $z_{n+1}(r)$ are strictly monotonous. The methods $z_1 \mathbf{e}_1$ and $z_{n+1} \mathbf{e}_{n+1}$ can only have a switch, but not a reswitch with (\mathbf{a}_1, l_1) .¹ In order to visualise how $M(r)$ shifts with changes of r , we locate the vertices of $M(r)$ in \mathbb{R}^{n+1} in the two-dimensional coordinate hyperplanes H_{ij} of \mathbb{R}_+^{n+1} . Coordinate $n+1$ is the coordinate of the labour input. There are $n(n-1)/2$ such hyperplanes with $i < j$, if one avoids double counting. The line segments $h_{ij}(r)$ connecting $z_i(r) \mathbf{e}_i$ and $z_j(r) \mathbf{e}_j$ in H_{ij} represent the edges of $M(r)$. The movements of $h_{ij}(r)$ show us how the set of potential techniques moves with r ; intersections of $h_{ij}(r_1)$ and of $h_{ij}(r_2)$ span the convex $(n-1)$ -dimensional set $M(r_1) \cap M(r_2)$

¹ This proposition is what survives of Levhari's erroneous proof of the impossibility of reswitching (see the argument on p. 105 of Levhari 1965).

of potential techniques leaving a switch at r_1 and a reswitch at $r_2 \neq r_1$. $M(r_1) \cap M(r_2)$ is, as we saw, not empty, because it contains (\mathbf{a}_1, l_1) .

We assume at first that the labour theory of value holds so that $\bar{\mathbf{p}}(r)$ is constant and $\bar{w} = 1 - r/R$. As the reader will easily verify, $h_{ij}(r_1)$ and $h_{ij}(r_2)$; $i < j$; $0 \leq r_1 < r_2 < R$; will then not intersect, if $j < n+1$, because the line segment $h_{ij}(r)$ is parallel to $h_{ij}(r_1)$ and shifts downwards to the left as r rises with both $z_i(r)$ and $z_j(r)$ falling. But there will be intersections in the coordinate hyperplanes involving the labour dimension $n+1$. These intersections span the $(n-1)$ -dimensional simplex $M(r_1) \cap M(r_2)$, and one shows by means of a short calculation that they remain stationary, if r_2 changes, given r_1 , if and only if relative prices of commodities do not change, that is, if prices equal labour values.

We may illustrate this in a plane diagram, although we are dealing with more than three dimensions, simply by looking at a surface in three dimensions of the $(n+1)$ -dimensional simplex of $M(r)$. This is illustrated in diagram 6. The vectors (\mathbf{a}_1, l_1) and (\mathbf{a}_0, l_0) are here assumed to have semi-positive components in dimensions i, j and $n+1$, and zero components otherwise.

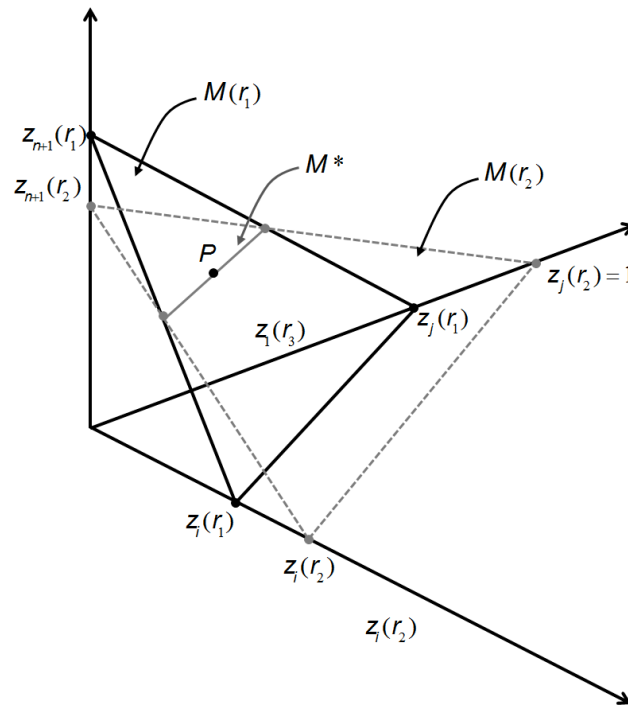


Diagram 6: Diagram 6 shows the surface of $M(r_1)$ as the triangle with black lines in the subspace $i, j, n+1$. The surface of $M(r_2)$ shifts from $r_2 = 0$ towards r_1 ; the two triangles coincide for $r_2 = r_1$. Because the labour theory of value holds, the bases of the triangles in dimensions i, j are parallel lines, and the intersection of the triangles, in M^* , is a constant line segment containing P . P is the original technique (\mathbf{a}_1, l_1) – we assume here that the components of (\mathbf{a}_1, l_1) are positive exactly in the dimensions $i, j, n+1$. M^* in this case consists of the alternative techniques (\mathbf{a}_0, l_0) – with semi-positive components in the same

dimensions – which can be switched with (\mathbf{a}_1, l_1) at all levels of the rate of profit; corresponding wage curves would coincide, which is possible because the labour theory of value holds. If r_2 moves beyond r_1 and we have $r_2 > r_1$, the dotted triangle has its vertex at $z_{n+1}(r_2) > z_{n+1}(r_1)$, and *vice versa* for the coordinates of the bases of the triangles (this is not shown to keep the diagram simple). Clearly, M^* , being here a straight line segment in the three-dimensional subspace, is of measure zero in the two-dimensional $M(r_1)$, and so the probability of reswitching is zero, if the labour theory of value holds.

Next we consider what happens if relative prices change. We now assume that the system is regular in the sense of Schefold (1976), so that the price vectors at any two different levels of the rate of profit are linearly independent. This means that the line of intersection of $M(r_1)$ and $M(r_2)$ turns around P , and we have to see what this implies in different subspaces. We begin with the same subspace in the dimensions $1 \leq i < j < n+1$. We obtain a diagram familiar from Schefold (1976) – diagram 7:

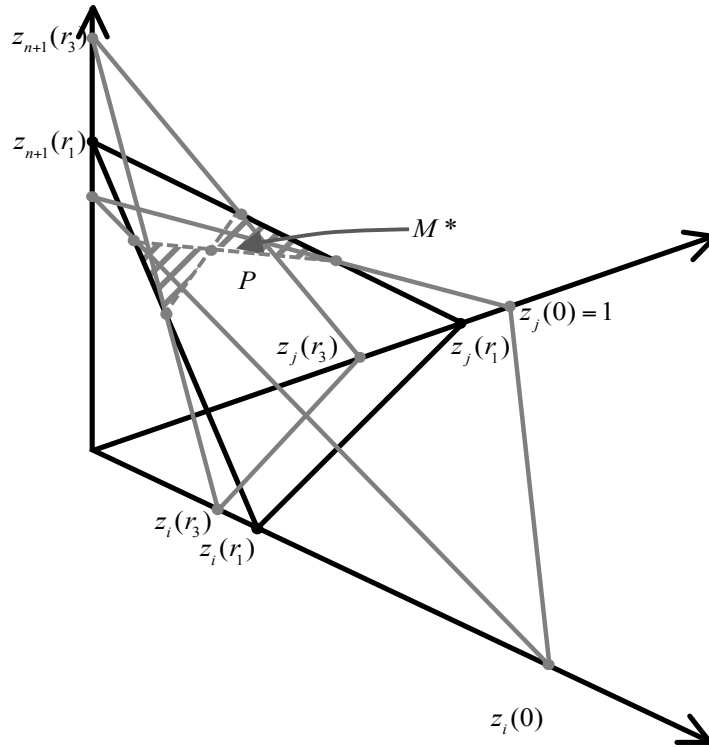


Diagram 7: M^* , the set of all potential techniques with a switch at r_1 and another at some $r_2 \neq r_1$; $0 \leq r_2 \leq R$. $M(r_2)$ is drawn for $r_2 = 0$ and for a large r_2 (denoted r_3).

Diagram 7 shows how the intersection of the surfaces of $M(r_1)$ and $M(r_2)$ turns around P , the point which represents the original technique (\mathbf{a}_1, l_1) , and the potential alternative techniques (\mathbf{a}_0, l_0) cover an entire area. Since the system is assumed to be regular, the same phenomenon will be observed in all the subspaces so that M^* will be of the same dimension as $M(r_1)$ and hence M^* will not be of measure zero.

This may even be illustrated for the case $n=3$. $M(r_1)$ then is a tetrahedron, which may be represented in \mathbb{R}^3 , which intersects in \mathbb{R}^4 with tetrahedron $M(r_2)$, $r_2 \neq r_1$, in a two-dimensional segment of a plane, which can be visualised as a planar cross section of $M(r_1)$ in \mathbb{R}^3 ; it moves around (\mathbf{a}_1, l_1) . It fills a three-dimensional volume M^* for $0 \leq r_2 \leq R$ of star-like shape around (\mathbf{a}_1, l_1) , if (\mathbf{a}_1, l_1) has positive components in four dimensions.

Is it possible that M^* will coincide with $M(r_1)$ in several or even all the subspaces? The coincidence cannot be ruled out for some of the subspaces. To see it, consider what happens if (\mathbf{a}_1, l_1) happens to be on the boundary of $M(r_1)$. The star-like figure of M^* in diagram 7 will then look like a triangle, as drawn for the case $n=2$ in Schefold (1976). If $M(r_1)$ and M^* could coincide in all the subspaces, M^* would cover the same space as $M(r_1)$, their measures would be the same and the likelihood of reswitching would be equal to one. But this is impossible. At least two vertices of $M(r_1)$, with neighbourhoods, are not in M^* ; it follows that M^* is strictly a subset of $M(r_1)$. We find that $M(r_1) - M^*$ contains full n -dimensional neighbourhoods so that the measure of M^* is smaller than that of $M(r_1)$. We have $\mu(M^*) < \mu(M(r_1))$. Now $M(r_1)$ is an n -dimensional simplex in \mathbb{R}^{n+1} spanned by $n+1$ vectors $z_1(r_1)\mathbf{e}_1, \dots, z_{n+1}(r_1)\mathbf{e}_{n+1}$. The volume of a simplex increases with the distance between the vertices spanning it. There must be a smaller simplex M^{***} , geometrically similar to $M(r_1)$, given by $\gamma z_1(r_1)\mathbf{e}_1, \dots, \gamma z_{n+1}(r_1)\mathbf{e}_{n+1}$; $0 < \gamma < 1$; such that $\mu(M^*) = \mu(M^{***})$. We shall say that (\mathbf{A}, l) is bounded by γ , or, simply that it is bounded. The boundedness presupposes that $M(r_1) - M^*$ is n -dimensional, which is geometrically obvious if $(\mathbf{a}_1, l_1) > 0$. If $(\mathbf{a}_1, l_1) \geq 0$, M^* could almost coincide with “much” of $M(r_1)$ only by virtue of extreme movements of $\tilde{\mathbf{p}}(r)$ in the entire nonnegative orthant, of which I do not know how improbable they are.

The existence of a general bound γ ; $0 < \gamma < 1$; will nevertheless be assumed, as this seems justified by our geometrical evidence. If one prefers, it is a conjecture that the existence of γ (the assumption of boundedness) can be replaced by weaker assumptions. Given boundedness, we come directly to the conclusion. Let an Euclidian n -dimensional coordinate system be given in the n -dimensional hyperplane containing $M(r_1)$ and let the vertices $z_i(r_1)\mathbf{e}_i$ of $M(r_1)$ in these coordinates be expressed by $n+1$ n -vectors \mathbf{v}_i . The n -dimensional volume V of $M(r_1)$ then is given by

$$V = \frac{1}{n!} |\det(\mathbf{v}_1 - \mathbf{v}_{n+1}, \dots, \mathbf{v}_n - \mathbf{v}_{n+1})|,$$

the volume V^{***} of M^{***} is given by

$$V^{***} = \frac{1}{n!} |\det(\gamma \mathbf{v}_1 - \gamma \mathbf{v}_{n+1}, \dots, \gamma \mathbf{v}_n - \gamma \mathbf{v}_{n+1})|.$$

The likelihood of reswitching

$$\pi = \mu(M^*) / (\mu M(r_1))$$

therefore can be expressed for systems bounded by γ as

$$V^{***} / V = \gamma^n .$$

Obviously π tends to zero as $n \rightarrow \infty$.

The boundedness condition is also discussed in Schefold (2016a).

The analysis for the likelihood of reswitching and reverse capital deepening, applicable to the entire complex of wage curves derived from a full spectrum of techniques, remains to be worked out. It will – but this is again a conjecture – still turn out that the paradoxes are rare, especially for large systems, although more technical alternatives come up, because the number of wage curves of techniques appearing on the envelope increases only slowly with n and with the number of techniques, as the next session will recall.

Tentative Conclusions for the Theory of Employment

What remains of the critique of capital theory? It certainly has to be modified. Reverse capital deepening has been shown to be necessarily rare, in accordance with the results of Han and Schefold (2006). If reverse capital deepening occurred once in a stream of technical changes, the public would hardly notice, for it would be an isolated event in a world in which technical changes are going on all the time, and every single change would be small. Of greater consequence are the anti-neoclassical Wicksell effects, especially if they should show up in a sequence with a change in distribution. If there is any truth in the idea that the amount of capital limits employment, changing its form slowly and varying in value with distribution, such Wicksell effects would destabilise the labour market. Whether the cause would be identified again is doubtful. The public might attribute a loss of employment, accelerated by the introduction of more labour-intensive techniques turning into more capital-intensive techniques with falling wages, to a lack of effective demand. The anti-neoclassical Wicksell effects imply a negative elasticity of substitution of a production function derived from a wage curve concave to the origin. This would be regarded as a strange occurrence in the present world, where neoclassical economists prefer to attribute a tendency to rising inequality to production functions with elasticities of substitution greater than one, rather than admitting economic power and imperfect competition.

Nonetheless, our findings imply, together with what follows from the hypothesis that techniques are approximately random, that a description of the technological possibilities by means of approximate surrogate production functions is not absurd, and conditions may be such that the properties of an ideal neoclassical world are realised sufficiently well to explain why so many economists adhere to neoclassical doctrine: not only for ideological reasons, not only because they feel that the world must be such, if the market system is to be just and efficient, but because they perceive that reality appears to conform to the picture: profits are maximised, wages seem to be paid to compensate for the contribution made by the worker, people find employment and less developed countries use more labour-intensive methods of production than those more developed – e.g. old cars circulate longer in the underdeveloped world because repair work is cheaper etc.

Another critical finding may perhaps help to reconcile theory and experience. We have found that lowering the real wage rate relative to the rate of interest will in most cases not lead to the paradoxical transition to a more capital-intensive technique. But is the intensity of labour likely to rise fast and significantly? I showed in Schefold (2013b) that if a spectrum of techniques is given which, for combinatorial reasons, gives rise to a very large number of potential techniques, the

number of wage curves actually appearing on the envelope will be quite limited. The main example shall be repeated here.

The spectrum of techniques is a very abstract and loose concept because the techniques not actually used, i.e. the techniques existing only as books of blueprints in the hands of engineers, may not be realistic and may have to be changed before they can be put into practice. But it is a fact of life that the techniques employed in different countries differ, and these can be represented by input-output tables at an intermediate level of aggregation. My preferred example is that of ten countries and of a subdivision of the economy into one hundred sectors. This hypothesis is made because one can think that the entrepreneurs and their engineers in each country within each sector have some knowledge of what is being done in the nine other countries in the same sector, but they know very little about what happens in other sectors: knowledge is dispersed and decentralised. So up to 10^{100} combinations are possible here. How many of them will be efficient and turn up on the envelope? Let the total number of feasible techniques be s , where s could be equal to 10^{100} in principle. We order the wage curves of these s techniques according to the level of the wage at $r = 0$; hence we get $w_1 > w_2 > \dots > w_s$ without loss of generality. If now the assumption is made that the corresponding maximum rates of profit R_1, \dots, R_s are completely uncorrelated with the sequence of the wage rates at $r = 0$, if, in this sense, randomness is introduced by assumption, one can deduce (see Schefold, 2013b) that the expected number of wage curves appearing on the envelope of all the wage curves in $0 \leq r \leq R$ in very good approximation is smaller than, and most equal to, $\ln s$, where \ln denotes the natural logarithm. This means for our example that at most $\ln s$ wage curves will be expected to appear on the envelope, where $\ln s = \ln(10^{100}) = 100 \cdot \ln 10 \approx 243$. The number of efficient curves appearing on the envelope is a very small number in relation to the very large number of wage curves below the envelope, and this small number of efficient techniques is spread over the entire interval between 0 and R , where R is the largest of the maximum rates of profit of the s systems. If the wage rate is lowered in accordance with neoclassical postulates in a situation with unemployment, the rate of profit would have to rise by a few percentage points and, in the end, methods of production would be changed only in a few sectors (remember that technical change along the envelope is piecemeal!), and each of these changes would involve only a small increase in employment, if the value of “capital” was kept constant. One might speak of a ‘poor’ production function: ‘poor’ in – but not devoid of – substitution possibilities. Without going to details about what it would mean to keep “capital” constant, we would find that the gain in employment was small. Zambelli (2014), in the paper referred to, with wage curves derived from 30 countries with 31 sectors, finds that only 64 wage curves out of 30^{31} wage curves are on the envelope.

Does this mean that it is not possible to increase employment by choosing among alternative existing techniques? Of course, not! One can in principle return to old labour-intensive techniques, but they are likely to be not efficient. One would not only have to lower wages, but profits as well, in order to adopt truly labour-intensive techniques. This policy, then, is not so different from that of the Luddites who, in the early phase of the industrial revolution, wanted to smash the machines. Or one can think of Mao Tse Tung’s strategy to have small-scale steel production in the villages. Already in antiquity there was speculation about what it would mean to abandon the plough and to return to the spade.

Keynes proposed not to lower the real wage but the rate of interest in order to induce more investment. However, the best and most urgent investment projects probably already have been realised at any moment of time. Keynes’s own scepticism showed when he ironically suggested (Keynes 1926, p. 129) that one might fill bottles with dollar notes and bury them, to have them dug up again by the unemployed – a process which would mimic in his eyes what happened under the gold standard. He expressed hope that better investment projects would be found; like building roads and houses. That is what everybody hopes, but there are the other difficulties: the additional

investment, if provided by the state, increases the debt, and the response of private investment to a lowering of interest rates may be sluggish.

Keynes thought that the ancient Egyptians were fortunate in that they possessed two activities which they would pursue spontaneously, pyramid building and the seeking of gold, which could be extended almost without limit and created employment (Keynes, 1936, p. 131). We seem not to know whether the ancient Egyptians thought of employment effects when they cared for their dead in this manner, but we do know this in the case of the ancient Chinese, where *Guanzi*, the advisor of a prince, explicitly stated that the requirements for the burial of officials could be increased to have more sculptors for the tombs and more seamstresses to add burial clothing to the coffins and where it was also said that the wise prince should never bar access to the cinnabar mines – the accumulation of cinnabar was somewhat analogous to the accumulation of gold (Schefold 2016c). Employment can in fact be increased not only by inducing the public to consume more but also by compelling it through institutional change to employ people in services, paid for out of private pockets. However, the possibilities to impose such policies are limited for a democratic government. Finally, employment can be increased by fostering growth and technical progress but this, in the end, may lower employment through labour saving techniques. Technical progress essentially is autonomous; it is our fate in that we can do little to influence its characteristics. There are generations which are happy when progress creates a lot of employment, as at the time of mass production of consumer durables like cars and washing machines which created many forms of secondary employment in the form of building roads or having laundry shops in the 1950s, and there are less fortunate generations for whom technical progress renders many workers redundant.

The problem of persistent unemployment worried Wicksell as a prominent neoclassical economist prior to Keynes. According to a summary given by Boianovsky and Trautwein (2003, pp. 423-425), he noted between 1923 and 1926 that „... technical progress could lead to a situation where a small fraction of the labour force is sufficient to operate and maintain the machines. The rest of the workers could therefore be redundant with zero wages ... Moreover, if the workers who operate the machines have better skills than the others, their wages would not tend to zero ... This would fit better the Swedish data showing that real wages of employed workers moved upwards in the 1920s.“ The upward movement of wages caused other observers to interpret the unemployment as voluntary, but even if all wages fall to zero, full employment might not be reached under profit maximisation, if the production function is poor and no efficient techniques can be created from malleable capital. One then is back at the options of expanding effective demand for goods of dubious usefulness like armament, of encouraging the adoption of inefficient techniques or of attempting employment-creating social change like shifting the balance between work and leisure and introducing new services that appear desirable in the perspective of a smaller or a bigger cultural transformation.

The main message of the debate about capital theory was, in my eyes, that the mechanism which is supposed to lead to full employment in the presence of flexible factor prices is tied to assumptions of technology which are not necessarily realistic. This is what we should have learnt from the great debate about capital theory, not any dogmatic statement about a unique truth belonging to any particular school of economics. The sceptical message still stands, and it calls for pragmatic solutions at a different level of abstraction.

(15 February 2017]

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