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Crowdfunding, Efficiency, and Inequality

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Crowdfunding, Efficiency, and Inequality^a

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Abstract

We show how decentralized individual investments can efficiently allocate capital to innovating firms via equity crowdfunding. We develop a model where consumers have privately known consumption preferences and may act as investors. Consumers identify worthwhile investments based on their own preferences and invest in firms whose product they like. An efficient capital allocation is achieved if all groups of consumers have enough wealth to invest. If some groups of consumers cannot invest, capital flows reflect preferences of the wealthy but not future demand. Information gathering by financial intermediaries can improve the allocation of capital when wealth constraints prevent an efficient allocation by consumers.

Keywords: Capital Markets, Crowdfunding, Crowdinvesting, Financial Markets, Financial Intermediation, Information Aggregation, Wealth Inequality, Welfare

JEL Classification: D24, D31, D53, D63, D82, G20

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1 Introduction

Crowdfunding is a new financing form for start-ups, entrepreneurs, artists, and even social organisations that draws on the masses (e.g., consumers, the general public) rather than a few professional financial intermediaries. Crowdfunding encompasses different funding models, in particular equity (as we consider in this paper) and debt contracts for investors, pre-order campaigns where consumers pay for the product upfront, and campaigns that rely on the good will of investors (i.e., donation or reward-based campaigns), and is conducted on Internet platforms (e.g., [Belleflamme et al., 2015](#)).

The aggregate funding volume of crowdfunding has grown considerably in recent years and may soon rival traditional funding forms.¹ While crowdfunding is attracting considerable interest in the financial industry and start-up scene, little is known about its allocative effects compared to other financing forms. This paper theoretically studies if and when equity crowdfunding can aggregate decentralized consumer information and efficiently allocate capital to new firms and technologies.

We consider a situation where a new firm requires funding to produce a new consumer product. One of the main problems in these early funding decisions is whether the new product or technology will be popular among consumers (and, hence, whether the firm will be profitable), i.e., there is aggregate demand uncertainty. From a social welfare perspective, if the product is going to be very popular, then it should obtain a lot of funding to meet the large future demand; if the product is only interesting to a small niche, then it should receive less or no funding. The novelty in crowdfunding is that a representative sample of potential consumers of the new product may participate in the funding decision of the firm—unlike in classical venture capital firms, where a select few decide on funding products that they might never consume themselves.

Our main finding is that equity crowdfunding can resolve the demand uncertainty at the funding stage and attain a Pareto efficient capital allocation. Crowdfunding works well because crowdinvestors can rely on their own consumption preferences when making their investment decision. In equilibrium, more interested consumers also invest more, thus the aggregate investment and hence the production of the new product can closely match future aggregate demand. The efficiency result is not merely driven by a simple pre-order mechanism, as consumers in equity crowdfunding are interested in a financial return and do not pre-order at the funding stage. Our finding is a very positive welfare result for crowdfunding, because it means that equity crowdfunding can achieve socially optimal funding decisions even if the properties of new products are not contractible at the funding stage, which disqualifies pre-order crowdfunding and other more traditional pre-sales. Our

¹“In all, crowdfunding platforms have raised some \$2.7 billion and successfully funded more than a million campaigns in 2012, [...] with an 81% increase to \$5.1 billion expected for 2013. By 2025, the global crowdfunding market could reach between \$90 billion and \$96 billion—roughly 1.8 times the size of the global venture capital industry today” ([Fortune, 2014](#)).

model also provides a theory that explains why investment behavior may be affected by personal consumption preferences for the products that these firms sell: Consumers that like the product tend to believe that others will also like the product, so the firm will have higher future revenues and will be more profitable.

We further find that a Pareto efficient capital allocation cannot generally be achieved if some consumer groups are wealth constrained, so that crowdinvestments reflect the preferences of the wealthy, but not necessarily those of all consumers. Thus, depending on the wealth distribution among consumers, capital may be misallocated, because consumer groups that are unable to invest still consume. Our analysis of the role of the wealth distribution is motivated by the empirical fact that financial wealth is distributed far more unequally than income in most industrialized countries. For example, 60 percent of American households possessed almost no financial wealth (one percent of total financial wealth), while they held about 22 percent of total income late last century (Wolff, 2002).² Thus, a sizable share of households consume and thereby determine the success of new products and technologies, but do not have significant financial wealth and thus cannot participate in the funding decisions. In the presence of wealth constraints, the funding decisions may therefore be biased in a similar way as a poll is biased if certain voter segments do not participate.

Our analysis is based on a Bayesian investment game embedded in a dynamic general equilibrium structure with two periods (funding and consumption stage). The Bayesian game describes an investment process where shares of the firm are directly sold to many small consumers (“the crowd”),³ and the proceeds are used to increase production with a linear production technology. In the first period, consumers can invest in the new firm to increase their income for consumption, or in an outside option. In the second period, the firms sells its supply of the new good on a competitive goods market. Our main result of an efficient capital allocation holds for a broad class of preference distributions and utility functions, and also for debt rather than equity contracts. Moreover, we show that the efficient allocation is implemented in ex post equilibrium, where the informational requirements on investors about the types of others are mild compared to Bayesian equilibrium.

We consider several extensions that largely confirm robustness of our main result. First, we introduce sequential investment campaigns that allow for learning about the state from previous investments. We find that the sequentiality can mitigate the problems of wealth constraints under some conditions, and there exist efficient equilibria in the sequential in-

²More recently, Saez and Zucman (2014) found that the bottom 90 percent of American households owned about 23 percent of wealth, but received 60 percent of income in 2012. A similar disparity of income and wealth distributions can be observed in many other countries (Davies et al., 2011). Moreover, according to Piketty (2014), the inequality of the wealth distribution increased over time in several industrialized nations.

³This kind of crowdinvesting differs from traditional forms of financing such as IPOs, where typically a predetermined share of the firm is sold to larger institutional investors, with assistance of an underwriter. Unlike IPOs, our crowdinvestment process does not determine a share price; rather, crowdinvestors invest an amount of capital and are entitled to a share of firm earnings in proportion to their investment. Otherwise both forms are similar in that equity shares are sold.

vestment model that do not exist in the simultaneous investment model. However, these new equilibria are in weakly dominated strategies, so there may be incentives for strategic waiting in crowdfunding that prevent effective learning from investments. Second, we theoretically and numerically analyze cases of nonlinear production technologies. In line with our main result, we find that welfare is larger for more balanced wealth distributions among consumers. But a first best allocation cannot generally be achieved any more with nonlinear technologies. Third, we let large professional investors compete with the crowd and allow for costly market research about consumer preferences, which closely matches the practice in venture capital firms. We find that the crowd drives out professional investors if there are no consumer wealth constraints, achieving an efficient capital allocation. Professional investors may be active in crowdfunding only if the crowd cannot aggregate all consumer preferences because not all consumers are able to invest. But professional investors and market research cannot completely rectify the capital misallocation that arises when some groups of consumers are unable to invest.

Our model fits crowdfunding campaigns where firms primarily seek funding to expand production and meet demand. Consider three examples out of many on the platforms specializing in equity crowdfunding. The fashion designer firm “ilovegorgeous”, for example, described their campaign on Crowdcube in the UK as “raising to fulfil customer demand by launching a boyswear collection.” The firm sold 15% of its equity and obtained about 233% of the funds it initially asked for. This example highlights that pre-order crowdfunding may not always be suitable, since the new collection had not even been designed. “Kumpan” manufactures electric scooters and sought funding to “expand [...] and meet demand for emission-free mobility” on Companisto in Germany. It offered about 3% equity, at the time of this writing the campaign was still ongoing but already surpassed its funding target. Finally, the US craft beer brewery “Mine Shaft Brewing” (MSB) sought funding to expand production capacity on the US platform Crowdfunder. In their pitch MSB cites a distributor saying “If MSB was producing now [we] would want you in UT, WY, NM”, suggesting that limited production capacity was the only obstacle to profits. MSB did not meet their funding goal, perhaps because the general public was not yet allowed to invest by the regulator,⁴ which may have had similar consequences as the wealth constraints in our model.

To the best of our knowledge, there is no systematic empirical evidence that the funding decisions are different when the wealthy invest on behalf of wealth-constrained consumers, but there is some anecdotal evidence. On the one hand, there are cases where firms successfully turned to the crowd after failed attempts to get financing from traditional financial intermediaries. One such example is ‘Good & Proper Tea’, which sells quality tea from a customized van turned into a tea bar. The founder was unable to secure more than two

⁴In the US, members of the general public had been excluded from investing in start-ups and small enterprises until the Jumpstart Our Business Startups (JOBS) Act, Title III—specifically aimed at enabling equity crowdfunding—went into effect in May 2016. Before May 2016, only accredited individuals exceeding a certain income or asset threshold were allowed to invest.

thirds of the necessary funding from conventional sources, but managed to obtain more than the remaining gap via crowdfunding ([Guardian, 2014](#)). On the other hand, there are also examples where wealthy investors' personal views and preferences seemed not to be aligned with the general customer base, possibly leading to inefficient funding decisions. A case in point is the German Luxury fashion label Escada, which had seen its best days in the 1980s when it provided red carpet dresses for Hollywood actresses. In 2009, however, Escada's dresses were widely perceived as out of fashion and the company had to file for insolvency ([Handelsblatt, 2009](#)). In the same year, the firm was saved by Megha Mittal, daughter in law of billionaire Lakshmi Mittal, whose first act as a CEO was to shop in her own store, taking advantage of the employee rebate. Six years later, the company was again in distress due to unsatisfactory sales ([Handelsblatt, 2015](#)).⁵ This example illustrates the possible impact of the wealth distribution and an exclusion of consumers on welfare and funding success.

1.1 Related literature

It is a fundamental insight in financial economics that asset markets may (under certain conditions) efficiently aggregate decentralized information about relevant fundamentals. This paper contributes to this literature, arguing that a simple investment mechanism can efficiently aggregate information about future demand even when forward markets for innovative products are missing. In equilibrium individuals invest in the products they like to consume. A simple linear example and some first empirical evidence for preference related investment choices can be found in an earlier version of this paper ([Grüner, 2008](#)). The present paper generalizes this theoretical analysis, studying in detail the distributional, technological and institutional conditions for efficient information aggregation, and it specifically addresses equity crowdfunding as a funding mechanism.

Several recent empirical studies investigate the determinants of fundraising success on crowdfunding platforms (e.g., [Agrawal et al., 2011](#); [Ahlers et al., 2012](#); [Mollick, 2014](#); [Li and Martin, 2014](#)). [Agrawal et al. \(2014\)](#) and [Belleflamme et al. \(2015\)](#) review the first empirical findings. According to [Agrawal et al. \(2014\)](#), early results suggest that crowdfunding can replace traditional sources of financing, just as we find in our model. They also remark that, when investments are linked to and motivated by an earlier access to the product, financing by the crowd may be able to provide information about demand to the entrepreneur that would not be available from venture capitalists.

An emerging formal literature studies information aggregation by pre-order crowdfunding

⁵The fact that professional (and typically wealthy) investors may find it difficult to predict the relevant behavior and preferences of other population groups has recently also been noted within the financial community itself. In an attempt to explain the markets' failure to predict the outcome of the British EU-exit referendum, Nicholas Colas, chief market strategist at New York brokerage Convergex, argues that "[t]here are two halves of America that do not talk to each other. Asset owners do not understand what those people who do not have wealth think about the world" ([Financial Times, 2016](#)).

mechanisms (Chang, 2015; Chemla and Tinn, 2016; Ellman and Hurkens, 2015; Schwiendbacher, 2015; Strausz, 2016). A pre-order crowdfunding scheme operates like a set of forward transactions that are all made conditional on the aggregate volume of purchases. This means that the pre-order purchases realize only if investors pledge enough money in the aggregate. The entrepreneur can use the product price and the funding threshold to extract information from customers that is useful for the investment decision.⁶ With binary consumer types efficiency may be reached.⁷

The present paper instead considers an equity crowdfunding mechanism that can be used even when pre-order transactions are not feasible. Thus, our model—and equity crowdfunding more generally—allows agents to fund projects even if they have no interest in acquiring the product, because the investment goal is a financial return and not consumption. Indeed, we show that consumers who are not interested in the product may invest in the project in the inefficient equilibria (i.e., given wealth constraints in some consumer segments or nonlinear production technologies, see section 3.2), but interested consumers endogenously emerge as the only investors in the absence of the frictions. Equity crowdfunding is viewed as more suitable for bigger projects: “Equity crowdfunding is perfect for companies that are looking to raise more capital than those that choose a rewards-based approach. These companies are typically seeking sums higher than \$50k and have achieved social proof and gained enough traction to incentive their backers with the chance to own a small piece of their company as it grows” (Fundable, 2016). Moreover, we consider projects which mainly require funding for production or expansion, whereas the above studies consider projects that require funding to start (i.e., focus on fixed costs). Finally, we further highlight the role of the wealth distribution on funding decisions and include large professional investors in an extension, neither of which is considered in this literature.

Our extension with professional investors relates to the literature comparing market-based and bank-based financial systems (for a review, see Allen and Gale, 2001). Like us, Allen and Gale (1999) consider the problem of financing new technologies, but provide an alternative explanation why market finance (the analogue to our crowdfundinvestment) might emerge instead of financial intermediation. They show that sufficiently strong diversity of

⁶Agrawal et al. (2014) and Belleflamme et al. (2014) made this point informally. Ellman and Hurkens (2015) study how crowdfunding can be used for market testing. Chang (2015), Chemla and Tinn (2016), and Strausz (2016) focus on the trade off between information gathering aggregation and incentives when the entrepreneur can embezzle the funds. Belleflamme et al. (2014) investigate whether an entrepreneur should rather use pre-order or equity-based crowdfunding. In their model, crowdfunders are motivated by “community benefits” (utility from contributing) rather than investment return considerations as in our case.

⁷When the size of the investment is fixed and when there are no variable costs, the problem of designing a pre-order crowdfunding scheme is identical to the problem of providing an indivisible public good with use exclusion. It is well known that efficiency and voluntary participation can generally not both be reached in this case because either some users may be (inefficiently) excluded in equilibrium, or individual contributions are too small (see, e.g., Gailmard and Palfrey, 2005 and the references therein). Efficiency results such as the introductory example in Strausz (2016) thus rely on a discrete distribution of consumer types, whereas our main result holds for continuous type distributions as well.

opinion among traders will favor market finance over intermediation, because an intermediary is more likely to make a suboptimal decision from the perspective of investors. In contrast to their model, our analysis focuses on the effect of the investor information and wealth distribution rather than diversity of opinion on the efficiency of capital allocation and the extent of financial intermediation.

It has also been noted that initial public offerings can aggregate useful information about the future success of projects (e.g., [Benveniste and Spindt, 1989](#)). The present paper studies the case where this information concerns the attractiveness of a firm's products for a population of consumers who also act as investors.

Our paper is also related to [Subrahmanyam and Titman \(1999\)](#), who investigate the firm choice between public (market) financing and private financing (intermediation). The entrepreneur in their model decides whether to go public or use private finance, and he anticipates whether he receives more information about business growth opportunities from the market via stock prices or from the private financier. While the analysis of [Subrahmanyam and Titman \(1999\)](#) explains firm choices between public and private finance based on informational benefits to the firm, we instead focus on the impact of the wealth distribution and information among investors on financial market structure. The second major difference is that the entrepreneur in [Subrahmanyam and Titman \(1999\)](#) chooses the financing form himself, whereas we assume that the entrepreneur offers equity in a crowdinvestment campaign, and competition between many small investors and professional investors determines who holds equity.

Finally, our contribution is related to a large literature that studies the effects of wealth inequality on allocative efficiency and in particular on the functioning of capital markets. A non-exclusive list is [Aghion and Bolton \(1997\)](#), [Banerjee and Newman \(1993\)](#), [de Mesa and Webb \(1992\)](#), [Galor and Zeira \(1993\)](#), [Grüner \(2003\)](#), [Grüner and Schils \(2007\)](#), and [Piketty \(1997\)](#). All these papers emphasize the link between credit market imperfections and agents' investments into private production possibilities. Investors with little wealth either do not get credit for their individual investment projects, or they only get credit at a higher interest rate. This is why the distribution of wealth has macroeconomic implications. In the present paper, we instead consider the link between inequality of the wealth distribution and the investment in firms/technologies seeking funding. Moreover, a major difference of our model compared to existing incomplete markets models is that agents hold private information about consumption needs, which is also a signal about the realization of relevant aggregate uncertainty.

2 Equity crowdfunding: The baseline model

2.1 Consumers and endowments

Consider an economy which is populated by a continuum of consumer-investors indexed by $i \in [0, 1]$, who we will also call ‘crowdinvestors.’ Each consumer has an initial endowment of wealth w_i in period 1 and receives an exogenous income $y_i > 0$ in period 2. Income and wealth are measured in monetary units. Individuals consume in period 2 and use the capital market to increase their income in period 2. They can invest any positive amount of their wealth in the firm producing the new good or in an outside option at the riskless rate R , i.e., one unit invested in period 1 turns into R units in period 2. The riskless rate R is exogenously given. In period 2, two consumption goods are available: consumption c at a normalized price of 1 and the novel consumption good x .

The utility function of consumer i in general terms is

$$u_i(c, x; \theta_i) \tag{1}$$

in consumption of c and the novel good x with parameter θ_i . We assume continuity and monotonicity for u_i :

$$u_i(c, \cdot) > u_i(c', \cdot) \text{ for all } c > c' \geq 0 \text{ and } u_i(\cdot, x) \geq u_i(\cdot, x') \text{ for all } x > x' \geq 0, \tag{2}$$

$$u_i \text{ is continuous.} \tag{3}$$

The strict inequality for c in (2) implies that preferences are non-satiated. Continuity (3) guarantees that an optimum to the consumer demand problem exists. The parameter θ_i is private information of i . For each i , the type $\theta_i \in \Theta_i$ is distributed according to $\phi_i(\theta_i|s)$ depending on a state of the world s with distribution $\sigma(s)$.

There is a spot market for goods c and x in period 2. But there is neither a credit market on which consumers may borrow against future income y_i nor a pre-sale of the innovative good x . A credit market friction is key to our results because, on a perfect credit market, all consumers could borrow against their future income in order to finance investments in the new product. Still, the assumption of no credit markets is stricter than necessary and only made for simplicity here.⁸ Impossibility of pre-sales is an appropriate assumption if the innovative good x has important features that are not contractible at the funding stage, which is the case for many of the investment projects financed by crowdinvestors. Thus, companies cannot finance their investments drawing on the current sales revenues and must rely on external funding.

⁸It is sufficient to assume a wedge between borrowing and saving rates due to credit market frictions (e.g., [Galor and Zeira, 1993](#)), because borrowing requires an excess return from investing in equilibrium, which is incompatible with an efficient capital allocation (Lemma 1). Thus, allowing borrowing in imperfect credit markets does not change our efficiency results.

2.2 Production and the Bayesian investment game

There are $m > 1$ firms which have access to a technology for the production of good x . Each firm produces according to the linear technology:

$$F(X) = X,$$

where $F(X)$ denotes the produced amount (supply) of the novel good and X the aggregate investment made in period 1. Each consumer i may invest any amount $0 \leq \hat{x}_i \leq w_i$ in the firms producing the new good, and the total production of all firms is determined by the aggregate investment

$$X = \int_0^1 \hat{x}_i di.$$

All firms act as price takers in period 2 and distribute profits to all shareholders according to their relative investment shares.

In period 2, consumers receive their exogenous income y_i and the return on their riskless or crowd-investments. Let \tilde{y}_i be the total budget available to consumer i in period 2. At a given spot market price of the novel good p , each consumer solves the following maximization problem:

$$\begin{aligned} \max_{c, x \geq 0} u_i(c, x; \theta_i) \\ \text{s.t. } \tilde{y}_i \geq c + px. \end{aligned} \tag{4}$$

For each i , denote the demand function given relative price p and budget \tilde{y}_i that solves (4) by $x_i(p, \tilde{y}_i; \theta_i)$. The aggregate demand correspondence in state s is therefore

$$x(p) = \int_0^1 \int_{\Theta_i} x_i(p, \tilde{y}_i; \theta_i) \phi(\theta_i | s) d\theta_i di. \tag{5}$$

In this paper, we will focus on the cases where aggregate demand (5) depends on the state of the world s . We will use explicit expressions for demand and distributions from section 2.4 onwards, but for now we think of the distribution of s as aggregate demand uncertainty regarding the novel good x . Thus, ex ante, the novel product may be very popular among consumers or very unpopular, which is exactly the kind of demand uncertainty that venture capital firms face when deciding on whether to fund start-ups.

Since producing firms act as price takers on the product market in period 2, aggregate investment X determines the good's price in equilibrium by equating aggregate demand and supply, $X = x(p)$. The market price $p = p(X, s)$ for the novel good depends on the realization of s (affecting demand) as well as aggregate investment X (affecting supply). The equilibrium return on investment in the production of good x is the firm revenue share

proportional to total investment, which simply equals the good's price,

$$r = \frac{p(X, s) \cdot X}{X} = p(X, s). \quad (6)$$

An equilibrium combines a Walrasian goods market equilibrium in period 2 with a Bayesian Nash equilibrium of the investment game in period 1.

Definition 1. *An equilibrium of the model consists of*

- i. a consumption plan $x_i(p)$ for each consumer,*
- ii. an investment plan $\hat{x}_i(\theta_i)$ for each consumer, and*
- iii. a relative price function $p(X, s)$ for good x ,*

such that

- i. the consumption plan maximizes utility (1) subject to the consumer's period 2 budget constraint,*
- ii. the investment plans constitute a Bayesian Nash equilibrium of the investment game subject to the wealth constraints, taking into account the consumption plans and the relative price $p(X, s)$, and*
- iii. at price $p(X, s)$ the aggregate demand for good x equals supply $F(X)$.*

We save on notation by not including wealth w_i in the investment plan $\hat{x}_i(\theta_i)$, since consumer i 's wealth is already associated with the index i . Since each state leads to a unique aggregate investment, we can define a state price $p(s)$.

A crucial point in this model is that the aggregate demand uncertainty is not easily resolved by asking consumers for their preferences, as is in principle possible on crowdfunding platforms or elsewhere on the internet. In every finite sample of consumers, every consumer who is interested in the new product has an incentive to overstate his interest in order to increase production, thus lowering prices. Hence, crowdfunding can be viewed as a mechanism where expressing interest is costly. We investigate the possibility of costly market research explicitly in section 3.3.

2.3 Main result

The main question is whether crowdfunding can achieve an efficient capital allocation, so that the new product is funded if and to the degree that Pareto-efficiency is attained. The problem is aggregate demand uncertainty: If aggregate demand depends on the realization of s , then Pareto-efficiency requires supply (production) to also react to the state s , which is difficult since preferences $\{\theta_i\}_i$ are private information. Given the linear production

technology, for which one unit of the novel product x has an opportunity cost of R units of c , the Pareto-optimal aggregate investment is such that the good's market clearing price equals R in every state, which would exactly reflect the technological constraints (i.e., the marginal rate of transformation).

Our main result in Proposition 1 states that equity crowdfunding with a linear production technology can achieve a Pareto-optimal allocation if no consumer is wealth constrained. This result does not depend on a specific utility function nor a specific distribution of $\{\theta_i\}_i$. The general idea is that each consumer invests an amount that depends on his own preferences (i.e., θ_i). More specifically, every consumer invests the amount that is necessary to produce the quantity that he will later consume at a relative price of R . Thus, supply always matches the aggregate demand at a relative price of R for every realization of the type profile $\{\theta_i\}_i$, which is Pareto-optimal.

Proposition 1.

Suppose that all consumers have sufficient wealth $w_i \geq \sup_{\theta_i} x_i(p = R, \tilde{y}_i = w_i R + y_i; \theta_i)$. If u_i for all i is continuous, $u_i(c, \cdot) > u_i(c', \cdot)$ for all $c > c' \geq 0$ and $u_i(\cdot, x) \geq u_i(\cdot, x')$ for all $x > x' \geq 0$, then there exists an ex post equilibrium where all consumers invest $\hat{x}_i = x_i(R, w_i R + y_i; \theta_i)$ and consume $x_i(R, w_i R + y_i; \theta_i)$. This equilibrium is Pareto-efficient.

Proof. See appendix. □

According to Proposition 1, crowdinvestments can efficiently replace a missing forward market for good x if all consumers have enough wealth. The more consumers are interested in the novel good, the more they invest in equilibrium, thus increasing production of the good. Consequently, firms do not have to convince third parties that their business idea is worth investing in; instead, the source of funding is consumers who already find the product attractive. Indeed, the feature that consumers are also investors in crowdfunding is key, because this allows investments to react to changes in consumer demand. The equilibrium also explains why crowdfunding success is a valuable signal to obtain alternative funding: “[E]ntrepreneurs often use success on a [crowdfunding platform] to signal their creditworthiness and, thereby, facilitate their access to bank loans or attract venture capitalists” (Belleflamme et al., 2015, p. 18).

The Bayesian Nash equilibrium of the investment game is also an ex post equilibrium. The concept of ex post equilibrium is considerably stronger than Bayesian equilibrium (e.g., Dasgupta and Maskin, 2000; Bergemann and Välimäki, 2002), because every ex post equilibrium is a Bayesian equilibrium but not vice versa.⁹ The ex post equilibrium is conceptually appealing as it does not require players to know the distribution of types or states since the

⁹But note that ex post equilibrium is not identical to the notion of dominant strategy equilibrium in our setting (which is common values at the investment stage). Players still need to be aware of the equilibrium strategy profile.

equilibrium strategy is optimal for every type profile $\{\theta_i\}_i$. Hence, the efficient capital allocation would be achieved even if players had incorrect beliefs about the type distribution. This is a nice feature in the present application, since occasional crowdinvestors may not be as experienced as professional investors typically considered in finance.

Note that crowdfunding with debt rather than equity contracts would yield the same equilibrium outcome if all consumers hold enough wealth. A debt contract would promise a return of R per unit and the firm can repay the investors as long as $p(s) \geq R$, which is always the case in the efficient ex post equilibrium. Thus, payoffs and allocations would be exactly the same as in the model with equity contracts.

It is also interesting that the efficient equilibrium in Proposition 1 *reproduces* the outcome of pre-order crowdfunding, even though there is no formal pre-ordering: Effectively, all consumers pay an amount $x_i(R, w_i R + y_i; \theta_i)$ to the company in period 1, which exactly covers the cost of producing their consumption bundle, and all consumers receive $x_i(R, w_i R + y_i; \theta_i)$ units of the good in period 2. The additional ‘transfers’ in our model—returns on equity to the crowdinvestors and payment to the company when purchasing the good—cancel out exactly. Clearly, as this analogy shows, the efficiency result rests on the linearity of the production function. In the presence of nonlinearities, the resulting equilibrium would generally not be efficient anymore (see section 3.2).

2.4 Equity crowdfunding and wealth constraints

Motivated by the wealth and income distribution statistics from the introduction, we will later analyze situations where some consumers are wealth constrained, unlike in Proposition 1. In this case, and in the other extensions of our baseline model that will follow, the analysis of equilibria becomes considerably more complex. In order to deal with these difficulties, we use a more specific setup with an explicit utility function and distribution of types that permits us to explicitly calculate demand functions and posterior beliefs. We also use this more specific setup because it illustrates the problem of demand uncertainty better and allows us to generalize our analysis in other dimensions later.

Preferences are represented by the following utility function:

$$u(c_i, x_i; \theta_i) = c_i + \theta_i x_i^\alpha, \quad (7)$$

with $0 < \alpha < 1$. The parameter θ_i is private information of consumer i , with $\theta_i \in \{0, 1\}$, i.e., consumers either derive utility from consuming good x , or they do not. Solving the maximization problem yields the individual demand for good x ,

$$x_i(p) = \left(\frac{\alpha \theta_i}{p} \right)^{\frac{1}{1-\alpha}} = \theta_i \left(\frac{\alpha}{p} \right)^{\frac{1}{1-\alpha}}, \quad \theta_i \in \{0, 1\}. \quad (8)$$

Since producing firms act as price takers on the product market in period 2, aggregate investment X determines the good's price in equilibrium according to

$$p = \alpha \left(\frac{\bar{s}}{X} \right)^{1-\alpha}. \quad (9)$$

Based on the wealth and income distribution statistics above, we assume that all consumers have enough income y_i in Period 2 to buy some of the new product even if they do not have any wealth, $y_i \geq \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$ for all i .

Let all consumers $i \in [0, 0.5]$ belong to a consumer group 1 and all consumers in $i \in (0.5, 1]$ belong to a consumer group 2. There is aggregate uncertainty regarding the share of consumers in the population that would like to consume the novel good x . This aggregate demand uncertainty is captured by a state of the world $s = (s_1, s_2)$, where s_1, s_2 are independently drawn according to the distributions

$$s_1 = 2 \cdot \int_0^{0.5} \theta_i di = \begin{cases} \beta > 1/2 & \text{with probability } \frac{1}{2} \\ 1 - \beta & \text{with probability } \frac{1}{2} \end{cases}$$

$$s_2 = 2 \cdot \int_{0.5}^1 \theta_i di = \begin{cases} \beta > 1/2 & \text{with probability } \frac{1}{2} \\ 1 - \beta & \text{with probability } \frac{1}{2}. \end{cases}$$

Thus, s_j is the share of consumers in group j with $\theta_i = 1$ and $\bar{s} = (s_1 + s_2)/2$ is the share of interested consumers in the population. Consequently, there is demand uncertainty because a majority ($s_1 = s_2 = \beta$), a minority ($s_1 = s_2 = 1 - \beta$), or half of the population ($s_1 \neq s_2$) may be interested in the new product. Note the subtle difference between s (a tuple) and \bar{s} (a scalar). In this setup, \bar{s} summarizes everything about state s that affects aggregate demand and thus prices (see (9)).

As before, Pareto-efficiency requires a good's price of R .

Lemma 1. *Suppose aggregate wealth fulfills $\int_0^1 w_i di \geq \beta(\alpha/R)^{\frac{1}{1-\alpha}}$ and the production technology $F(X) = X$ is linear. Then the capital allocation is Pareto-efficient if and only if aggregate investment is $X = \bar{s} \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$. This outcome is realized in a market equilibrium if and only if the good's market clearing price is $p(s) = R$ for all s .*

Proof. See Appendix. □

Consider again the case of sufficient wealth as in Proposition 1. As before, an efficient ex post equilibrium exists (Proposition 2, part (i.)). While there are multiple efficient equilibria,¹⁰ parts (ii.) and (iii.) of Proposition 2 show that all of the efficient equilibria are unique in the aggregate (i.e., in terms of capital allocation, prices, and investment returns). Moreover, one could imagine additional equilibria to the efficient ones if all consumers have

¹⁰For example, since individual investors have zero measure, one could change the investment strategies of finitely many players without affecting the capital allocation.

sufficient wealth, but part (iv.) shows that there are no equilibria where the capital allocation is inefficient. Unlike the ex post equilibrium in Proposition 1, part (iv.) uses the details of the type distribution to pin down the beliefs of the investors in an inefficient equilibrium candidate.

Proposition 2. *Suppose all consumers hold wealth $w_i \geq \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}}$.*

- i. There exists an ex post equilibrium in which all consumers invest an amount $\hat{x}_i = \left(\frac{\theta_i \alpha}{R}\right)^{\frac{1}{1-\alpha}}$ in the capacity for the production of good x . This equilibrium is Pareto-optimal.*
- ii. The ex post equilibrium aggregate investment function $X(s)$ is unique.*
- iii. An equilibrium is efficient if and only if it is an ex post equilibrium.*
- iv. An inefficient equilibrium does not exist.*

Proof. See appendix. □

The intuition that there is no inefficient equilibrium (iv.) is as follows. If there were an inefficient equilibrium, then there would be at least one state where the return on investment exceeds R . Because the preferences of crowdinvestors contain information about the likelihood of such a state—it is more likely that others in the same group are interested in the good if i is interested—they would best respond by increasing investment. Together, the two groups of interested consumers can remove any excess return, because they have enough wealth to arbitrage away mispricing.

Proposition 2 yields a strong model prediction: If all crowdinvestors have enough wealth, then the capital allocation is efficient. Now we investigate the role of wealth constraints in the functioning of equity crowdfunding.

The next result shows that an efficient capital allocation can be achieved if and only if all groups of potential consumers in the population have sufficient wealth. In particular, it is not enough that aggregate wealth in the population is large—the distribution matters. For example, group 1 may be wealthy enough to cover the necessary investments of both groups while group 2 has no wealth. But then the capital allocation would be determined solely by the preferences (i.e., s_1) and information of the wealthy group and not react to preference realizations in the poor group (s_2). Thus, Pareto-efficiency can be attained if and only if all consumer groups have enough wealth to make production react to all preference changes via investments. The result focuses on the distribution of wealth if enough aggregate wealth is available in the population ($\int w_i di \geq \beta(\alpha/R)^{1/(1-\alpha)}$).

Proposition 3. *Suppose w_i is constant within each group of crowdinvestors and $\int w_i di \geq \beta(\alpha/R)^{1/(1-\alpha)}$. Then there exists an efficient equilibrium if and only if consumers in each group hold enough wealth, $w_i \geq (\alpha/R)^{1/(1-\alpha)}$.*

Proof. See appendix. □

Proposition 3 demonstrates the novelty of crowdfunding: Unlike bank finance or start up finance via venture capital, funding decisions are not made by a few big agents or a privileged group, but potentially by a representative sample from the relevant consumer population. The key is that information must be collected from all consumer groups in order to achieve the efficient allocation. This is possible only if the aggregate investment from the given group reflects accurately the consumption need for the new good in that group. It is not crucial that all $\theta_i = 1$ consumers invest the same amount; half of them could just invest twice as much while the other half does not invest. Thus, efficiency is consistent with some wealth inequality within a group, where types are drawn from the same distribution, but not with inequality across groups so that one group has insufficient aggregate wealth.

3 Extensions

3.1 Sequential investments

3.1.1 Setup

On most crowdinvestment platforms irreversible investments can be made in a given timespan. During this investment time frame, the current aggregate investment into a project is observable for potential investors. Thus, sequential and observable investments may allow wealthy crowdinvestors to learn something about the preferences of the poor, and consequently adjust their investment to what they have observed. This could potentially alleviate the inefficiency problem due to wealth constraints that we discussed in the previous section. In order to study this problem, we extend the simultaneous investment game from section 2 to a simple sequential two stage investment game. Generalizing to more than two periods or even continuous time does not change the efficiency results of this section.

We continue with the simple setup from section 2.4, but make the following modification to two investment periods. In $t = 0$, all crowdinvestors may condition their investment plans $\hat{x}_i^{t=0}(\theta_i)$ only on their own private information as before, leading to aggregate investment $X_0 = \int_0^1 \hat{x}_i^0(\theta_i) di$. In $t = 1$, all crowdinvestors may condition their investment plans $\hat{x}_i^1(\theta_i, X_0)$ on their private information and aggregate investment from the previous investment stage. The equilibrium concept from definition 1 can be readily extended to the present setup by replacing the one stage by the two stage investment plans. In equilibrium, investors can adjust their investment to the realization of X_0 , and use the information contained in X_0 about the distribution of θ_i when investing at $t = 1$. Overall investment by crowdinvestor i in the company is $\hat{x}_i^0(\theta_i) + \hat{x}_i^1(\theta_i, X_0)$, i.e., the sum of the individual investments in $t = 0$ and $t = 1$, with $\hat{x}_i^t \geq 0$ as before.

3.1.2 Results

It is straightforward to show that all equilibria from the baseline model can be extended to equilibria in this dynamic model with exactly the same outcomes. Hence, the set of equilibria is weakly larger in the dynamic model.

Proposition 4. *Any equilibrium with investment strategy profile $\{\hat{x}_i(\theta_i)\}_i$ from the baseline model in section 2 can be extended to an outcome-identical equilibrium in the dynamic model.*

Proof. Take any equilibrium investment strategy profile $\{\hat{x}_i(\theta_i)\}_i$ from the baseline model. Consider the following equilibrium candidate for the dynamic model:

$$\begin{aligned}\hat{x}_i^{t=0}(\theta_i) &= 0 \quad \forall i, \\ \hat{x}_i^{t=1}(\theta_i, X_0) &= \hat{x}_i(\theta_i) \quad \forall i.\end{aligned}$$

Since nobody invests in $t = 0$, $X_0 = 0$ in all states, so aggregate investment is uninformative. Consequently, at $t = 1$, investors have the same information they have in the baseline model, so if $\hat{x}_i(\theta_i)$ is an equilibrium strategy in the baseline model, it also must be an equilibrium strategy in the last investment period of the dynamic model.

It remains to be shown that there is no profitable deviation at $t = 0$. A unilateral deviation to $\hat{x}_i^{t=0}(\theta_i) > 0$ at $t = 0$ does not change the information of i nor does it change the investments by other investors (and hence the payoff of i), since i has no mass and so does not affect X_0 . Consequently, i is indifferent between investing earlier or investing according to the equilibrium candidate strategy. \square

The question now is whether efficient equilibria exist in the dynamic model that do not exist in the baseline model (Proposition 3) due to the possibility of learning from and reacting to aggregate investment. Naturally, an efficient equilibrium cannot exist if consumers of one of the groups do not have any wealth. The intuition is quite simple: If investors of a group cannot invest at all, then nothing can be learned about their preferences from observing aggregate investment. However, Proposition 5 shows that efficient equilibria exist if the ‘poor consumer group’ has some wealth so that it can signal their preferences by investing, and the ‘wealthy consumer group’ has enough wealth to cover the rest. The efficient equilibria are coordination equilibria in the sense that the poor consumers first invest and reveal their preference distribution to the wealthy consumers, who later invest on behalf of the poor what these would have invested absent wealth constraints. We demonstrate this sequential revelation of information by poor consumers to wealthy consumers with an example and then give the necessary and sufficient condition for the existence of efficient equilibria in the sequential model.

Example. *Suppose w_i is constant within each group, and consider the case where the poor group 2 has positive but less wealth than necessary to achieve the efficient investment ac-*

cording to Proposition 2,

$$(\alpha/R)^{1/(1-\alpha)} > w_i = \underline{w} > 0 \quad \forall i \in (0.5, 1],$$

and wealth \bar{w} in the wealthy group 1 is sufficient to cover their own investments and the rest of group 2's investment,

$$w_i = \bar{w} \geq ((\alpha/R)^{1/(1-\alpha)} - \underline{w}) \beta + (\alpha/R)^{1/(1-\alpha)} \quad \forall i \in [0, 0.5].$$

Then there exists an efficient equilibrium in the sequential model where only the poor invest at $t = 0$. The investment strategies at $t = 0$ are

$$\begin{aligned} \hat{x}_i^0(\theta_i = 1) &= \underline{w} \quad \forall i \in (0.5, 1], \\ \hat{x}_i^0(\theta_i = 0) &= 0 \quad \forall i \in (0.5, 1], \\ \hat{x}_i^0(\theta_i) &= 0 \quad \forall i \in [0, 0.5], \end{aligned}$$

Here it is crucial that $\theta_i = 0$ and $\theta_i = 1$ types in the poor group invest different amounts so that aggregate investment changes by state (i.e., by the share of $\theta_i = 1$ consumers).

The resulting aggregate investment is $X_0(s_2) = \underline{w}s_2/2$, which is strictly increasing in the realization of $s_2 \in \{1 - \beta, \beta\}$. Thus, s_2 will be revealed at $t = 1$ where X_0 is observable, and wealthy consumers can react to the state. Consider now the following $t = 1$ investment strategy profile:

$$\begin{aligned} \hat{x}_i^1(\theta_i, X_0) &= 0 \quad \forall i \in (0.5, 1], \\ \hat{x}_i^1(\theta_i = 1, X_0 = \underline{w}\beta/2) &= ((\alpha/R)^{1/(1-\alpha)} - \underline{w}) \beta + (\alpha/R)^{1/(1-\alpha)} \quad \forall i \in [0, 0.5], \\ \hat{x}_i^1(\theta_i = 1, X_0 = \underline{w}(1 - \beta)/2) &= ((\alpha/R)^{1/(1-\alpha)} - \underline{w}) (1 - \beta) + (\alpha/R)^{1/(1-\alpha)} \quad \forall i \in [0, 0.5], \\ \hat{x}_i^1(\theta_i = 0, X_0 = \underline{w}\beta/2) &= ((\alpha/R)^{1/(1-\alpha)} - \underline{w}) \beta \quad \forall i \in [0, 0.5], \\ \hat{x}_i^1(\theta_i = 0, X_0 = \underline{w}(1 - \beta)/2) &= ((\alpha/R)^{1/(1-\alpha)} - \underline{w}) (1 - \beta) \quad \forall i \in [0, 0.5], \end{aligned}$$

and invest optimally otherwise (i.e., off the equilibrium path).

Crucially, investments by the the wealthy crowdinvestors $i \in [0, 0.5]$ depend on aggregate investment at $t = 0$, X_0 , and thus on s_2 . In short, all wealthy consumers split the funding gap that is left by the poor consumers, and interested wealthy consumers additionally invest as in the simultaneous investment model.

Aggregate investment in equilibrium is therefore $X = (\alpha/R)^{1/(1-\alpha)}(s_1 + s_2)/2$ in every state, which is efficient. Given the efficient outcome and the implied return of R on investment, all investors are indifferent between individually investing earlier or later, or investing in the safe asset, since no unilateral deviation will change X_0 due to the zero mass of each i . Hence, the described investment strategy profiles constitute a Bayesian Nash equilibrium of the dynamic investment game. Indeed, it is also an *ex post* equilibrium, since a differ-

ent realization of s will lead to a different aggregate investment X_0 to which the wealthy consumers appropriately react. \square

Consequently, the possibility of sequential investments with observable aggregate investment might help achieve the efficient capital allocation even if this would not be possible with simultaneous investments as in the baseline model of section 2. Still, the poor consumer group needs some wealth to invest and thereby signal the preference distribution in their group, so wealth constraints are not completely negated by the possibility of sequential investments.

The next proposition gives the necessary and sufficient condition on the distribution of wealth in the two groups for existence of an equilibrium with wealth-unconstrained efficient allocation. Compared to Proposition 2 in the simultaneous investment model, the aggregate wealth that is required to achieve an efficient capital allocation is lower, because sequential revelation of information allows uninterested consumers ($\theta_i = 0$) to invest as well. Indeed, the necessary aggregate wealth is the bare minimum required to attain a good's price of R in every state. Moreover, compared to the simultaneous investment model, the wealth distribution does not have to be balanced any more due to the possibility of learning from aggregate investment and "investing on behalf of the other group" (see the previous example).

Proposition 5. *Suppose w_i is constant within each group, and without loss of generality call group 2 ($i \in (0.5, 1]$) the "poor consumers". Then there exists an equilibrium with wealth-unconstrained efficient capital allocation in the dynamic model if and only if*

1. *wealth \underline{w} in the poor group 2 is positive,*

$$w_i = \underline{w} > 0 \quad \forall i \in (0.5, 1],$$

2. *and wealth \bar{w} in the wealthy group 1 is sufficient to cover the remaining investment for an efficient capital allocation,*

$$w_i = \bar{w} \geq ((\alpha/R)^{1/(1-\alpha)} - \min\{(\alpha/R)^{1/(1-\alpha)}, \underline{w}\})\beta + \beta(\alpha/R)^{1/(1-\alpha)} \quad \forall i \in [0, 0.5].$$

Proof. See appendix. \square

Propositions 4 and 5 together imply that inefficient equilibria may exist along with efficient learning equilibria if the condition from Proposition 2 is not fulfilled, i.e., if there are binding wealth constraints in some consumer groups but not others. Thus, the welfare prediction of the model is not unique in the case of sequential investments.

A plausible equilibrium refinement in game theory is requiring that no weakly dominated strategies are played in equilibrium. In our sequential investment model, investing at $t = 0$ is a weakly dominated strategy, because an early investor cannot react to a situation where too much was invested at $t = 0$ so that the investment return is below R in any state. A

player who postponed his own investment to $t = 1$ could still react by observing X_0 and not investing.¹¹ Thus, the possibility of other players diluting the equity favors investing at the last possible moment.

Proposition 6. *The only equilibria that are not played in weakly dominated strategies are those with investments only in the last period $t = 1$.*

Proof. Any investment at $t = 0$ can be costlessly postponed to $t = 1$ without changing the informativeness of X_0 since investors are atomistic. If—perhaps off the equilibrium path—a large amount is invested at $t = 0$, so that the investment return is below R in any state, then a player who postponed his own investment to $t = 1$ could still react by observing X_0 and not investing. A player that invested at $t = 0$ cannot. Thus, investing late is at least as good, and for some actions of other players better. \square

To summarize, if we focus on the ex post equilibrium concept for equilibrium selection, then allowing for sequential investments increases the set of situations with an efficient capital allocation. However, efficient equilibria in undominated strategies exist if and only if they exist in the static model of section 2.

3.2 Nonlinear production technology

3.2.1 Setup

In this extension, we investigate how a nonlinear production technology prevents an efficient capital allocation, and compare the welfare properties depending on the wealth distribution.

The production technology is generalized to the case where aggregate investment X translates into supply of the novel good according to the production function

$$F(X) = X^\lambda = \left[\int \hat{x}_i di \right]^\lambda, \quad \lambda > 0.$$

This covers the case of a single firm but also the case of multiple identical firms: The production function is up to a constant factor identical to a situation where $1 \leq M < \infty$ firms receive an $1/M$ -share of the investment and produce, so that aggregate supply is given by

$$F(X) = M \left[\int \hat{x}_i / M di \right]^\lambda = M^{1-\lambda} \left[\int \hat{x}_i di \right]^\lambda.$$

For $0 < \lambda < 1$ the production function is concave (decreasing returns to scale), and for $\lambda > 1$ it is convex (increasing returns to scale). $\lambda = 1$ is the linear case considered throughout the

¹¹Indeed, in parimutuel betting—where as in our case the profits and losses are shared among all who invest—it is typically observed that bettors wait to place their bets until the very last moment in order to be able to react to new information (and not reveal their information to others), see, for example, (Ottaviani and Sørensen, 2009) and the references therein. Ebay is another context where strategic waiting for the last moment to act is common (Roth and Ockenfels, 2002).

main part of the paper. For $\lambda > 1$, we require $1/\lambda > \alpha$, otherwise the planner's problem may have a corner solution.

Consumer demand for given prices remains unchanged. The generalized market clearing condition and spot market price is

$$X^\lambda = \bar{s} \left(\frac{\alpha}{p} \right)^{1/(1-\alpha)} \iff p = \alpha \left(\frac{\bar{s}}{X^\lambda} \right)^{1-\alpha}. \quad (10)$$

3.2.2 The social optimum

We first determine the planner's solution for the optimal aggregate state dependent investment X^* in the novel good knowing s . With binary types, market clearing requires that $x_i = F(X)/\bar{s}$, where \bar{s} is the share of interested consumers, x_i is the symmetric consumption level for $\theta_i = 1$ types in the population, and $F(X)$ is the aggregate supply of the novel good. The cost function for producing χ units of the novel good is

$$C(\chi) = RX = R\chi^{1/\lambda},$$

since every unit of investment X has an opportunity cost of R units of c . The marginal cost is $MC_2 = \chi^{1/\lambda-1}R/\lambda$. In the social optimum, the marginal rate of substitution for a $\theta_i = 1$ consumer has to equal the ratio of marginal costs of production (investment) of the two goods,

$$\begin{aligned} MRS &= \frac{MU_1}{MU_2} = -\frac{1}{\alpha x_i^{\alpha-1}} = -\frac{1}{\alpha(\chi/\bar{s})^{\alpha-1}} = -\frac{MC_1}{MC_2} = -\frac{1}{\chi^{1/\lambda-1}R/\lambda} \\ &\iff \chi^* = \left[\frac{\lambda\alpha}{R} \bar{s}^{1-\alpha} \right]^{\frac{\lambda}{1-\lambda\alpha}} \iff X^* = \chi^{*1/\lambda} = \left[\frac{\lambda\alpha}{R} \bar{s}^{1-\alpha} \right]^{\frac{1}{1-\lambda\alpha}}. \end{aligned} \quad (11)$$

Consequently, the optimal aggregate investment X^* depends nonlinearly on the share of interested consumers \bar{s} whenever $\lambda \neq 1$. If the planner allocates the new good in a competitive goods market, then the Pareto-optimal investment yields a market clearing price p^* , found by equating aggregate demand and supply,

$$\chi^* = \left[\frac{\lambda\alpha}{R} \bar{s}^{1-\alpha} \right]^{\frac{\lambda}{1-\lambda\alpha}} = x_i \bar{s} = \bar{s} \left(\frac{\alpha}{p} \right)^{1/(1-\alpha)} \iff p^* = \alpha \bar{s}^{1-\alpha} \left(\frac{R}{\lambda\alpha \bar{s}^{1-\alpha}} \right)^{\frac{\lambda-\lambda\alpha}{1-\lambda\alpha}},$$

which depends on \bar{s} whenever $\lambda \neq 1$. Thus, unlike in the linear case, we cannot determine efficiency simply by checking state independence of the spot market price.

3.2.3 Crowdfunding and inefficient capital allocation

As a benchmark, consider the investment that consumers would make if the state s were common knowledge. As before, the company distributes all revenues/profits pro rata among

investors. The return on investment into the new company, taking into account the nonlinear production technology, is

$$r(s) = \frac{p(s)F(X)}{X} = p(s)X^{\lambda-1} = \alpha \bar{s}^{1-\alpha} X^{\alpha\lambda-1}.$$

Given the opportunity cost of investment R , any equilibrium investment profile (absent wealth constraints) must equate the return on investment with R , which implies an aggregate market investment X^m ,

$$\alpha \bar{s}^{1-\alpha} X^{\alpha\lambda-1} \stackrel{!}{=} R \iff X^m = \left(\frac{\alpha}{R} \bar{s}^{1-\alpha} \right)^{\frac{1}{1-\alpha\lambda}}.$$

The aggregate market investment X^m differs from the social planner investment X^* in (11) by a factor of $\lambda^{\frac{1}{1-\alpha\lambda}}$, so that $X^m = X^*$ if and only if $\lambda = 1$, i.e., if and only if the production technology is linear. Consequently, even if there was no asymmetric information problem, the market would not achieve an efficient allocation with a nonlinear production technology. In particular, the capital allocation would not be Pareto-efficient even if we allowed for sequential investments as in section 3.1 that reveal the state s .

3.2.4 Equilibrium

In this section, we determine which types of consumers invest in equilibrium depending on the wealth distribution and the production technology. In particular, we consider two cases for the wealth distribution: First the case where both consumer groups have sufficient wealth to bring the crowdinvestment return down to R (“balanced wealth”), and second the case where only one consumer group has sufficient wealth while the other has none (“imbalanced wealth”). Proposition 7 gives the formal result and Figure 1 represents the results graphically. We focus on equilibria where all consumers of the same type make symmetric investments (in case of imbalanced wealth only consumers of the wealthy group invest), and uniqueness refers to this class of equilibria.

If both consumer groups have enough wealth, then only $\theta_i = 1$ types invest in equilibrium if $\lambda \geq 1$ (weakly convex production technology), and both types invest if $0 < \lambda < 1$ (concave production technology). Thus, the production technology determines whether only interested consumers invest in equilibrium or whether also consumers invest who do not intend to buy the product, and the linear production technology is the cutoff.

If only one consumer group has wealth, then only $\theta_i = 1$ types in the wealthy group invest in equilibrium if $\lambda \geq \underline{t}$ with $\underline{t} > 1$ defined below (sufficiently convex production technology), and both types of the wealthy group invest if $\lambda < \underline{t}$ (concave, linear, or slightly convex production technology). Consequently, there is an intermediate range of the production technology $\lambda \in [1, \underline{t})$ where the type of equilibrium (i.e., the type of investing consumers) changes depending on the wealth distribution.

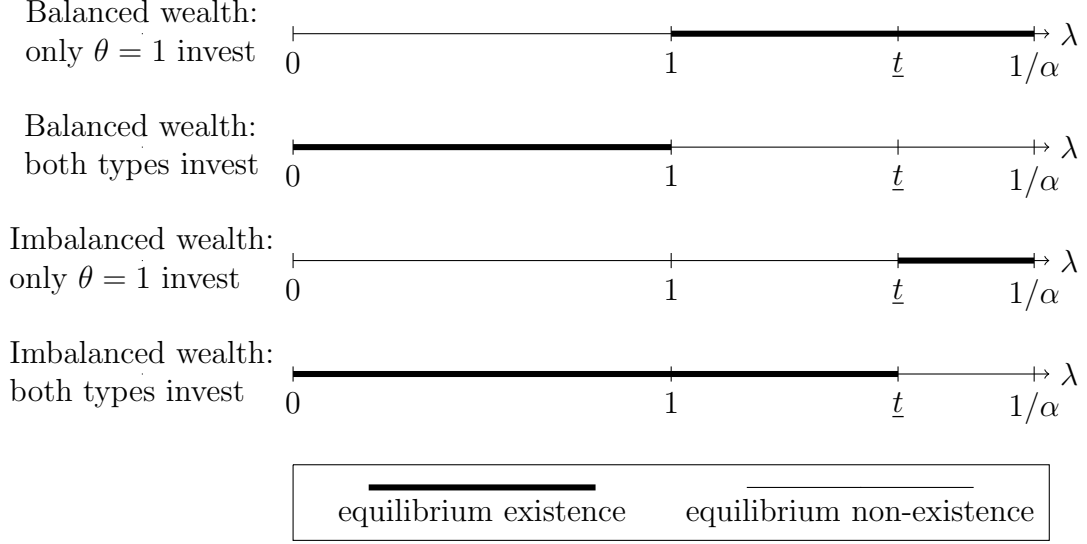


Figure 1: Existence of equilibria in which both types invest or only the interested consumers invest depending on technology parameter λ

The threshold \underline{t} for production parameter λ is:

$$1/\alpha > \underline{t} := \frac{\log \left(\frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}} \right)}{\log \left(\frac{\beta}{1-\beta} \right) \alpha} + 1/\alpha > 1. \quad (12)$$

Proposition 7. *Consider the case of two consumer groups and a nonlinear production technology $F(X) = X^\lambda$.*

- i. Balanced wealth: There exists a unique symmetric equilibrium where all consumers of type $\theta_i = 1$ but not those of type $\theta_i = 0$ invest if and only if $\lambda \geq 1$.*
- ii. Balanced wealth: There exists a unique symmetric equilibrium where all consumers of both types $\theta_i = 0$ and $\theta_i = 1$ invest if and only if $\lambda < 1$.*
- iii. Imbalanced wealth: There exists a unique symmetric equilibrium where all consumers of type $\theta_i = 1$ but not those of type $\theta_i = 0$ of group 1 invest in equilibrium if and only if $\lambda \geq \underline{t}$.*
- iv. Imbalanced wealth: There exists a unique symmetric equilibrium where consumers of both types $\theta_i = 0$ and $\theta_i = 1$ of group 1 invest in equilibrium if and only if $\lambda < \underline{t}$.*

Proof. See appendix. □

3.2.5 Welfare

We explained in section 3.2.3 that crowdfunding does not achieve a Pareto-efficient capital allocation if the production technology is nonlinear. A natural question that follows is: Un-

der which conditions is welfare higher in equilibrium if wealth is balanced among consumer groups? As a welfare metric, we use utilitarian welfare (average population utility).

The quasi-linear utility function with binary types gives us a good setup to think about the aggregate preference realization, which is summarized by the scalar $\bar{s} = (s_1 + s_2)/2$ (share of $\theta_i = 1$ types in the population). From the linear case we learned that welfare is maximized if all consumers have sufficient wealth, so that investments (and thus supply) can react to the preference distribution of all consumer groups, i.e., to s_1 and s_2 . If group 2 is wealth constrained, then aggregate investment depends only on s_1 and does not change in realization $s_2 \in \{1 - \beta, \beta\}$.

The monotonicity of aggregate investment in \bar{s} is the reason why welfare is higher in the absence of wealth constraints for approximately linear production technologies (Proposition 8), since it allows supply to track changes in aggregate demand. In case of wealth imbalance without the monotonicity, there are states with relative scarcity (and little consumption) and states with relative excess supply that reduce social welfare. Indeed, the first best investment in (11) is monotone in \bar{s} for all λ , so monotonicity is necessary but not sufficient for a welfare maximum.

Proposition 8. *Consider the unique symmetric equilibria (Proposition 7). Utilitarian welfare is larger in the case of balanced wealth compared to the case of imbalanced wealth for approximately linear production technologies, i.e., in a neighborhood of $\lambda = 1$.*

Proof. See appendix. □

This result holds because welfare is larger in the balanced wealth case for $\lambda = 1$, and since investment strategies, prices, and indirect utilities are continuous in λ , welfare must also be larger for λ close to 1.

Unlike in the linear production case, where a state independent price of R implied Pareto-efficiency, there is no good indirect way of comparing welfare for $\lambda \neq 1$. Thus, we have to directly compare the long ex ante welfare terms, which we do numerically.

Ex ante welfare is the utilitarian welfare averaged over all four states $(s_1, s_2) \in \{1 - \beta, \beta\}^2$. For every parameter profile $(\alpha, \beta, \lambda, R)$, we determine income $y > 0$ for every consumer so that it is never binding in equilibrium. Moreover, we determine wealth $w(\alpha, \beta, \lambda, R)$ in the balanced wealth case so that it is never binding for the crowdinvestments. In the imbalanced wealth case, we set wealth to $2w(\alpha, \beta, \lambda, R)$ for the wealthy group and to zero for the poor consumer group, so that aggregate wealth is the same (both groups have size $1/2$ as before). Wealth is constant within the group.

Since there is no demand uncertainty for $\beta = 1/2$, we focus on $\beta > 1/2$. The numerical result 1 is based on the following parameter grid G , where (following the Matlab syntax) $\{a : z : b\} := [a, b] \cap \{a + kz\}_{k=0,1,2,\dots}$ are the values for a single variable in the interval

between a and b in steps of z .

$$G := \{\alpha, \beta, \lambda, R :$$

$$\alpha \in \{0.1 : 0.1 : 0.9\}, \beta \in \{0.6 : 0.1 : 0.9\}, \lambda \in \{0.1 : 0.1 : 3\}, R \in \{1 : 0.1 : 3\}, \alpha > 1/\lambda\}.$$

In particular, the grid includes values $\lambda > 1$ and $\lambda < 1$, subject to $\lambda < 1/\alpha$. As the following result shows, balanced wealth is better for social welfare.

Result 1. *For all parameters in the grid G , in particular for both $\lambda < 1$ and $\lambda > 1$, ex ante utilitarian welfare is larger for the economy with balanced wealth compared to the economy with imbalanced wealth.*¹²

The main reason is as before: Investment is monotone in \bar{s} for balanced wealth but not for imbalanced wealth, thus there is less variance in prices and supply, and hence more “consumption smoothing” in the case of balanced wealth with positive welfare consequences given the concavity of the utility function in x .

3.3 Financial intermediaries and market research

3.3.1 The extended model

In this section, we add a financial sector consisting of $N \in \mathbb{N}$ investment funds¹³, indexed by j , with exogenous large endowment $W_j > 0$, who may acquire information about consumer preferences and maximize expected investment returns. They can either make safe investments with return R , or they can invest in the novel consumption good with variable return. These funds may be viewed as arbitrageurs, who arbitrage away excess returns in the investment of the firm producing the novel good.

We assume that investment funds have no information¹⁴ on the realization of consumer preferences (unlike consumers, whose preference θ_i is informative). Funds may acquire information about the realization of preferences in the consumer population to identify worthwhile investment opportunities. This can be thought of as buying market studies which evaluate the revenue potential of the new product or commissioning consumer surveys.

Formally, we represent the “market research” information by two binary and independent signals about the preference realization in the wealthy (1) and poor (2) consumer group,

¹²Matlab scripts of the numerical calculations are available upon request. We briefly explored even wider grids but found no cases that contradicted our result.

¹³We call the financial market intermediaries “investment funds,” but these may be replaced by any other large investing institutional entity, such as banks, venture capital firms, hedge funds, pension funds, or investment banking divisions.

¹⁴This assumption is made to simplify the exposition, and any *imperfect* information about the realization of s for investment funds yields the same results concerning an efficient capital allocation for any $N \in \mathbb{N}$.

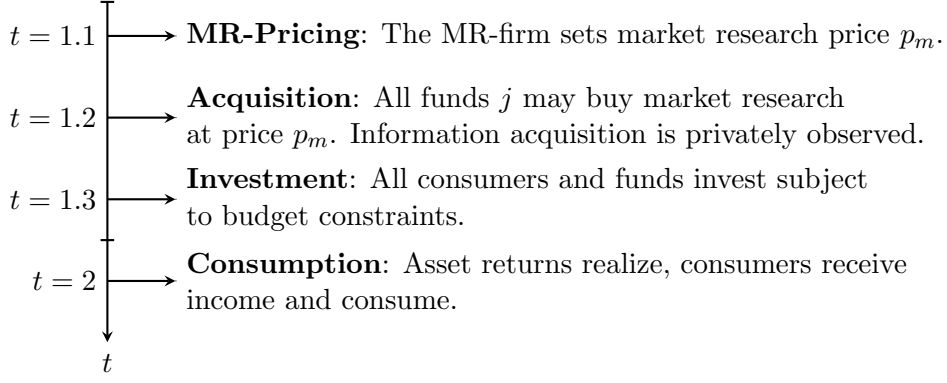


Figure 2: The timing of decisions.

$m \in \{0, 1\} \times \{0, 1\}$. The signal quality is exogenously given by

$$\begin{aligned} \gamma &:= \Pr(m_1 = 1 | s_1 = \beta) = \Pr(m_1 = 0 | s_1 = 1 - \beta) \\ &= \Pr(m_2 = 1 | s_2 = \beta) = \Pr(m_2 = 0 | s_2 = 1 - \beta) > 1/2. \end{aligned}$$

Market research is offered by a monopolist market research (MR) firm, which sells the same signal m to all interested buyers, i.e., signals are perfectly correlated. Neither the assumption that the MR sector is monopolistic nor that signals are perfectly correlated drives our results, as will become clear shortly. For non-triviality, we assume the MR firm can produce market research (i.e., conduct surveys, gather and analyze data) at sufficiently low cost $c > 0$, so that it can always offer market research at positive market research price p_m . If the MR firm sells market research to $0 \leq n \leq N$ funds, then its profit is given by $\pi_{MR} = np_m - \mathbf{1}\{n > 0\}c$.

In contrast to the model of section 2, aggregate investment is now the sum of the infinitesimally small crowdinvestments \hat{x}_i and the investments of the “large” financial sector entities f_j . Thus aggregate investment in good x is

$$X = \int_0^1 \hat{x}_i di + \sum_{j=1}^N f_j.$$

The timing of decisions is displayed in Table 2. And now that we added new players to the game, we extend the equilibrium definition as follows.

Definition 2. *An equilibrium of the extended model consists of*

- i. a market research price p_m set by the MR-firm at $t = 1.1$,*
- ii. an acquisition plan $a_j(p_m) \in \{0, 1\}$ to purchase market research $m \in \{0, 1\} \times \{0, 1\}$ for each investment fund at $t = 1.2$,*
- iii. an investment plan $\hat{x}_i(\theta_i)$ for each consumer at $t = 1.3$,*

iv. an investment plan $f_j(p_m, m_j)$ for each investment fund at $t = 1.3$, where $m_j = m$ iff $a_j = 1$ and $m_j = \emptyset$ iff $a_j = 0$,

v. a consumption plan $x_i(p)$ for each consumer,

vi. a relative price function $p(X, s)$ for good x ,

so that

- i. the market price p_m maximizes expected profits of the market research firm at $t = 1.1$, taking into account $a_j(p_m)$ of all j ,
- ii. the information acquisition plans a_j and investment plans \hat{x}_i and f_j constitute a Bayesian Nash equilibrium of the investment game subject to the wealth constraints, taking into account the consumption plans and the relative price $p(X, s)$,
- iii. the consumption plan x_i maximizes utility subject to the consumer's future budget constraint, and
- iv. at the price $p(X, s)$, the demand for good x equals supply X .

In our extended model, there are two possible sources of inefficiency, (i) that the creation of market research wastes cost $c > 0$ (new), and (ii) that state-contingent investment in the novel product is inefficient in the sense of Lemma 1 (as before). Since we assume that there is sufficient aggregate wealth in the economy to fund production of the efficient consumption in every state and also that utility is transferable, Pareto-efficiency from an ex-ante perspective requires that neither of the two kinds of inefficiencies occur, i.e., requires that no market research is carried out and that the capital allocation is efficient.

Definition 3. *Pareto-efficiency from an ex-ante perspective involves all agents in the economy (consumers, funds, market research firm), and requires that*

- i. the market research cost $c > 0$ is not wasted, and
- ii. the state-contingent capital allocation is efficient (Lemma 1).

The following analysis focuses on the possibility of efficient state-contingent investment, i.e., efficiency of the capital allocation (ii), which is necessary but not sufficient for Pareto-optimality. Our results show that Pareto-efficiency with an unequal wealth distribution fails not only because the market research cost is wasted, but because the capital allocation cannot be efficient even if market research is acquired in equilibrium.

3.3.2 Equilibrium existence

We first establish the existence of an equilibrium in the extended version of our model.

Proposition 9. *An equilibrium in which all crowdinvestors play pure strategies exists.*

Proof. See Appendix. □

The following sections analyze the equilibrium properties, especially with respect to the wealth distribution of crowdinvestors, in more detail.

3.3.3 The impossibility of efficient investment with active funds

To characterize the set of possible equilibria in more detail, we next show that efficient state dependent investment and active funds—i.e., funds which are investing into the new product—are inconsistent. The main obstacle to achieving efficient investment with active investment funds is an informational friction: Funds first have to buy the information that allows them to adjust their investment, but there are no excess returns in an efficient equilibrium that would incentivize them to buy market research. We discuss these obstacles in more detail in section 3.3.5.

Proposition 10. *There exists no equilibrium with an efficient state-dependent capital allocation in which investment funds invest into the new product.*

Proof. See Appendix. □

This result is independent of the wealth distribution of consumers. The proof proceeds in two main steps. First, suppose there is an efficient equilibrium where funds invest. Efficiency implies the investment return is R in every state (Lemma 1). But then it does not pay to buy market research for price $p_m > 0$, since return R can be realized with the alternative investment without this additional cost. Second, given that funds must be uninformed in an efficient equilibrium, their investment is constant over states s . Aggregate investment may still react to changes in s , since consumers may invest depending on their preferences. However, they do not invest as much as they would if investment funds were inactive, i.e., not as much as in the efficient equilibrium, since this would imply an expected return of less than R . But if consumers invest less, then the slope of aggregate investment $X(s)$ in \bar{s} cannot be equal to $(\alpha/R)^{1/(1-\alpha)}$ as in the efficient equilibrium. That is, investment cannot scale up one-to-one with future aggregate demand. Consequently, there exists at least one state where aggregate investment is inefficient, which contradicts the earlier assumption that an efficient equilibrium in which funds invest exists.

3.3.4 Equilibrium if all consumers can invest

As benchmark, we again consider the case where all consumers have wealth $w_i \geq (\alpha/R)^{1/(1-\alpha)}$. In this case, the equilibrium of section 2 persists after adding investment funds: All consumers with type $\theta_i = 1$ invest, which is efficient and gives an investment return of R in each state (Proposition 2). Given this investment strategy by crowdinvestors, it does not pay for funds to participate; they do not buy market research and do not invest.

Proposition 11. *If all consumers have wealth $w_i \geq (\alpha/R)^{1/(1-\alpha)}$, then there exists an equilibrium where the consumer investment strategies are the same investment strategies as in Proposition 2 ($\hat{x}_i = \theta_i(\alpha/R)^{1/(1-\alpha)}$), and investment funds neither acquire information nor invest. This equilibrium is efficient.*

Proof. Suppose all consumers with $\theta_i = 1$ invest $\hat{x}_i = (\alpha/R)^{1/(1-\alpha)}$.

Investment stage: The profit of one of the N corporate investors when using investment strategy f_j with opportunity cost R and information set \mathcal{I}_j , given the investment strategies \hat{x}_i of all consumers, is

$$\mathbb{E}_s[\pi_j(f_j, f_{-j}, \hat{x})|\mathcal{I}_j] = f_j(\mathbb{E}_s[p(f, \hat{x})|\mathcal{I}_j] - R).$$

The first order condition of Cournot competition with respect to f_j , taking investment strategies of all other players as given, is

$$0 = \mathbb{E}_s[p'(f, \hat{x})|\mathcal{I}_j]f_j + \mathbb{E}_s[p(f, \hat{x})|\mathcal{I}_j] - R \iff \mathbb{E}_s[p(f, \hat{x})|\mathcal{I}_j] = R - \mathbb{E}_s[p'(f, \hat{x})|\mathcal{I}_j]f_j, \quad (13)$$

hence funds aim to realize a price $p > R$, since $p' < 0$. However, the investments of the consumers are enough to realize a price $p = R$ in all states. Hence, first order condition (13) cannot be fulfilled with equality for any positive f_j , and the optimal choice is a corner solution $f_j = 0$ for all j .

Acquisition stage: Since investment funds do not invest, buying market research is strictly dominated for $p_m > 0$. \square

3.3.5 Equilibrium if one group of consumers cannot invest

If a group of consumers is poor and cannot invest (imbalanced wealth case), then there may be investment opportunities for the financial sector. If the poor are interested in the novel good and the wealthy are not, then future demand for the novel good will be large but investment by the wealthy and consequently supply will be small. Hence, the price of the novel good p —which is also the per unit return of an investment in the novel good—is larger than R . In this state it would pay for the financial sector to swoop in and arbitrage away (part of) the excess return on investment, because wealthy investors underestimate future demand for the novel good.

However, as a consequence of Proposition 3 and Proposition 10, there will be some inefficiency in capital allocation whenever there is a group of consumers that does not have enough wealth to invest. Throughout this section, we assume that all consumers of group 2 (the poor) have no wealth, i.e., $\int_{0.5}^1 w_i di = 0$.

Corollary 1. *Suppose w_i is constant within each group of crowdinvestors. There exists no equilibrium with an efficient state-dependent capital allocation if $w_i < (\alpha/R)^{1/(1-\alpha)}$ in any of the consumer groups.*

Proof. *If investment funds do not invest, then the equilibrium cannot be efficient.* This follows from Proposition 3.

If investment funds invest, then the equilibrium cannot be efficient. This follows from Proposition 10. \square

In order to see why an efficient outcome is impossible if some consumer groups cannot invest, we describe the frictions involved in more detail. One obstacle to efficiency is the market power of investment funds if $N < \infty$. Efficient investment implies that all investors make zero profits compared to the outside option at rate R , but if the fund sector is not perfectly competitive, then funds will withhold some investment to drive up prices (and therefore investment returns). This can be directly seen in the first order condition (13) of the fund investment problem. Thus, even if funds were perfectly informed about the state of consumer preferences s , they would not want to remove all inefficiency, as this would imply zero profits (or in fact a loss, since becoming informed is costly).

If the fund sector is competitive ($N \rightarrow \infty$), then an efficient equilibrium is still not possible. To understand why, consider the following proposition, which establishes that, if the investment fund sector is competitive, then aggregate investment will not be affected by market research in equilibrium.

Proposition 12. *Suppose the investment fund sector is competitive ($N \rightarrow \infty$), so that $X = \int \hat{x}_i di + \int f_j dj$. Then there exists no equilibrium where a positive mass of funds buys market research for $p_m > 0$.*

Proof. Suppose there is an equilibrium with a positive mass of funds buying market research and investing in the novel good using the superior information. Because a single investment fund j is small and its investment does not influence p , j can deviate by not buying research, keep investing, and making the same investment return as before, yet saving cost $p_m > 0$. \square

Proposition 12 shows that information acquisition is subject to a free-rider problem in a continuum of investment funds. As soon as aggregate investment reacts to market research information—which can only be the case if a positive probability mass of funds acquire it—then it pays to deviate for informed funds to not buying market research, and free-ride

on the information incorporated in the aggregate investment by others.¹⁵ Consequently, even if there is a continuum of investment funds, no or only finitely many funds will become informed in equilibrium, but their impact on aggregate investment is negligible.¹⁶

Thus, with a competitive fund sector, the market for information breaks down. This result has a similar flavor as the one in [Grossman and Stiglitz \(1980\)](#) for financial markets, who show that there is no fully revealing equilibrium with costly information acquisition, because uninformed traders can free-ride on the information of informed traders.

Finally, even if a competitive fund sector somehow got hold of the market research signal for free, this would still not lead to efficient investment, unless market research was noiseless ($\gamma = 1$). That is, a noisy signal ($\gamma < 1$) prevents efficiency, because a wrong market research signal—which occurs with positive probability—leads to an inefficiently high or low investment.

Thus, an efficient equilibrium if not all groups of consumers can invest the efficient amount exists only if $\gamma = 1$, $N \rightarrow \infty$, and market research is costlessly available ($p_m = 0$). But this is equivalent to a situation where the consumer preference distribution realization is common knowledge, which is not realistic.

Our results show that financial intermediaries cannot fully correct the inefficiency that arises when wealth and income distribution do not match. However, they may still play a useful role in increasing social welfare in such situations. To see why this is so, consider as a simple example the case where no consumer holds any wealth. Then the addition of intermediaries is unambiguously welfare improving—even without the possibility to purchase market research.

4 Conclusion

Equity crowdfunding allows entrepreneurs to obtain funding by selling a share of their firm to the public. Unlike in classical venture capital financing, the firm funding decision may therefore be made by potential customers of the firm. For this reason, we find that equity crowdfunding can efficiently allocate capital to new firms if all potential consumer groups are wealthy enough to invest. However, wealth is more concentrated than income in most western nations, so the wealthy may have to invest on behalf of consumers on the lower end of the wealth distribution. In this case, firms with products favored by the wealthy will attract the most funding, but these are not necessarily the firms that meet the highest demand and need the most funding.

¹⁵The same argument would apply to crowdinvestors if they were allowed to buy market research. Hence, assuming that consumers may also buy market research would not change our results.

¹⁶Moreover, independent market research signals cannot yield efficient investment either. Although a law of large numbers guarantees that many independent market research draws m_j , $j = 1, \dots, N$ reveal the state as $N \rightarrow \infty$ perfectly even for $\gamma < 1$, the market for information would break down, because it does not pay for funds to become informed (Proposition 12).

In an extension, we allow intermediaries to compete with crowdinvestors and endogenously determine the extent to which funding relies on intermediaries. Financial intermediaries cannot completely fix the capital misallocation that arises with a mismatch of wealth and income distribution. Even if the intermediaries were perfectly informed, they would not want to fully fund the new product, since the efficient investment implies zero profits.

Our analysis generates several empirical predictions. First, consumers invest in firms whose products they like. This behavior should not (only) be driven by a sympathy for a brand name or the firm, but by the favorable information that the own preference for a product contains. Second, wealth constraints among potential consumers limits the scope of direct financing mechanisms such as crowdfunding (compared to intermediated finance). The (mis-)match of income and wealth distribution among the consumers of the product, and not the population, is crucial: The capital allocation for luxury products aimed at wealthy consumers may work even with very unequal wealth distributions in the population, because all potential consumers have sufficient wealth and income to invest and buy. But information aggregation may not work with products aimed at less fortunate consumers, who consume but cannot invest. Third, and relatedly, funding outcomes are on average more efficient when the wealth distribution of consumers better matches the income distribution. This could either be tested across countries, or alternatively, within a country by comparing product success after different crowdfunding campaigns that target consumers from different wealth and income groups.

Recent technological advances and the widespread use of the internet made it possible to match a large amount of investors with projects or firms seeking funding at substantially lower cost. Thus, firms and projects that were previously too small to offer equity directly to the public, and therefore had to rely on financial intermediaries, now have access to the money and wisdom of crowds. Our results show that the improved access to financing from crowdinvestors increases the efficiency of capital allocation for those small firms, if the mismatch of wealth and income distribution of consumers is not too large. Hence, our paper shows that crowdfinancing may be a valuable financial innovation which can improve social welfare.

A Proofs

Proof of Proposition 1. We go backwards from the consumption stage. Since u_i is continuous, Berge's maximum theorem guarantees that a solution to the consumer demand problem (4), $x_i(p, \tilde{y})$, exists. In the consumption stage, a market clearing price p determined by equating aggregate supply (determined by the candidate investment strategy profile) and demand $\int_0^1 x_i(R, w_i R + y_i) di = \int_0^1 x_i(p, w_i p + y_i) di$ is $p = R$ in all states s .

In the investment stage, every investor is indifferent between investing any $\hat{x}_i \in [0, w_i]$ given everybody else plays the equilibrium candidate. This is because the return on investment equals the market clearing price R , which atomistic individual investors cannot change by changing their own investment \hat{x}_i , and since R is also the return on the riskless asset. Thus, there is no profitable deviation at the investment stage, and $\hat{x}_i = x_i(R, w_i R + y_i)$ is an equilibrium. It is furthermore an ex post equilibrium, since any change in the θ -profile that induces a change in the aggregate demand function is exactly matched by a change in the aggregate supply given the equilibrium candidate strategy profile. Hence $p = R$ for all realizations of the θ -profile so that $\hat{x}_i = x_i(R, w_i R + y_i)$ is always optimal.

We show Pareto efficiency by analogy: A Walrasian equilibrium allocation with the same production technology for profit maximizing and price taking firms (who know consumer preferences) is exactly the same as the ex post equilibrium allocation in our model. Hence, by the first welfare theorem and non-satiation of preferences, the ex post equilibrium is Pareto-efficient.

In the first welfare theorem setup, suppose there is one competitive profit maximizing firm with a production technology that turns R units of c into one unit of x . This exactly reproduces the opportunity cost of production and thus feasible production levels from our model. Then, given this production technology, a Walrasian market price for good x is $p = R$. Suppose not, so that $p > R$ is the market price. Then the profit for each produced unit x is $p - R > 0$ (buying R units of c at price 1 and selling one unit of x at price $p > R$). But given the constant returns to scale production technology, there is no profit maximizing production level, so this cannot be an equilibrium. Next, suppose $p = q < R$, then each produced unit of x implies a loss of $q - R < 0$, so that the firm does not produce x . Now we distinguish two subcases. First, if utility functions are such that aggregate demand at $p = q$ is positive, then this contradicts market clearing and $p = q < R$ cannot be a Walrasian equilibrium. Second, if utility functions are such that aggregate demand at $p = q$ is zero, then since u is weakly increasing, aggregate demand will also be zero at $p > q$. In this case, any $p \geq q$, and in particular $p = R > q$, can be a Walrasian equilibrium price.

Now at a price of $p = R$ with budget $\tilde{y} = w_i R + y_i$ (which is the same budget as in the ex post equilibrium), the demand correspondence that maximizes utility for each i is $x_i(p = R, \tilde{y} = w_i R + y_i; \theta_i)$ by definition. Aggregate demand is therefore $\int_0^1 x_i(p = R, \tilde{y} = w_i R + y_i; \theta_i) di$, and since firms have a constant returns to scale production technology,

they can satisfy aggregate demand with equality at zero profits given $p = R$. Hence, the allocation $(c_i, x_i) = (\tilde{y} - Rx_i(p = R, \tilde{y} = w_i R + y_i; \theta_i), x_i(p = R, \tilde{y} = w_i R + y_i; \theta_i))$ for all $i \in [0, 1]$ with $p = R$ is a Walrasian equilibrium, and it is the same allocation as in the ex post equilibrium. By the first welfare theorem, this allocation is Pareto-efficient. \square

Proof of Lemma 1. Concavity of the utility function (7) in consumption x_i implies that x_i must be equal for all $\theta_i = 1$ types in the social optimum, and equal zero for all $\theta_i = 0$ types. No waste and feasibility requires that per capita production equals per capita consumption for $\theta_i = 1$ types, i.e., $x_i = \hat{x}_i$. Thus, the social planner determines a constant per capita investment $\hat{x}_i = \hat{x}$ for all $\theta = 1$ types.

Because an investment \hat{x} has opportunity cost of R units of c_i consumption, the budget constraint of the economy is

$$\int (w_i - c_i - \theta_i R \hat{x}) di = 0 \iff \int c_i di = \int (w_i - \theta_i R \hat{x}) di.$$

The planner's problem determines \hat{x} to maximize total welfare,

$$\max_{\hat{x}} \int (\theta_i x_i^\alpha + c_i) di \text{ s.t. } \int c_i di = \int (w_i - \theta_i R \hat{x}) di.$$

Substituting from the budget constraint and using $x_i = \hat{x}$, this is equivalent to the unconstrained problem

$$\max_{\hat{x}} \int (\theta_i \hat{x}^\alpha + w_i - \theta_i R \hat{x}) di.$$

The first order necessary and sufficient condition of the concave objective is

$$0 = \int (\alpha \theta_i \hat{x}^{\alpha-1} - \theta_i R) di \iff \hat{x} = \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}},$$

where $x_i = \hat{x} = (\alpha/R)^{\frac{1}{1-\alpha}}$ is also the socially optimal per capita consumption for $\theta_i = 1$ types. The corresponding efficient aggregate investment is $X = \bar{s} \hat{x}$.

In a market equilibrium, consumption choices $x_i(p) = (\alpha/p)^{\frac{1}{1-\alpha}}$ for $\theta_i = 1$ types depend on market clearing price p , and correspond to the planner's solution if and only if aggregate investment is such that $p = R$ in every state. \square

Proof of Proposition 2.

- i. This is a corollary of Proposition 1.
- ii. From the market clearing condition (9), the market price in a state s with \bar{s} equals R if and only if aggregate investment is $X = \bar{s} \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$. Clearly, larger X leads to lower p and smaller X to larger p (see (9)). Suppose there was an ex post equilibrium with aggregate investment function $X(s)$ so that $p \neq R$ for some state s . If $p < R$, then the investment return is less than R and it would be ex post optimal for some i to decrease

his investment and receive the safe return R instead. If $p > R$, then it would be ex post optimal for some i to increase investment to $\hat{x}_i = w_i$, which is feasible for some i by assumption of $w_i \geq (\frac{\alpha}{R})^{\frac{1}{1-\alpha}}$ for all i . Thus, in any ex post equilibrium, the aggregate investment function must be $X(s) = \bar{s} (\frac{\alpha}{R})^{\frac{1}{1-\alpha}}$.

- iii. (ii.) established that every ex post equilibrium has $p(s) = R$ for all s . Thus, Lemma 1 implies that every ex post equilibrium is efficient. In the other direction, if an equilibrium is efficient, then it has $p(s) = R$ for all s . Thus, every i is ex post indifferent between investing and the safe return R , so an efficient equilibrium is an ex post equilibrium.
- iv. In a first step, we will show that consumers with type $\theta_i = 0$ do not invest in equilibrium if $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ for all i . In a second step, we will show that crowdinvestors with $\theta_i = 1$ in group 1 have a larger expected investment return if and only if the average investment strategy in case of $\theta_i = 1$ in their group is smaller (the symmetric case for group 2 holds as well). In a third step, we show that the average equilibrium investment strategy in case of $\theta_i = 1$ must be the same in both groups, and must be the one that leads to a return of R in every state.

Throughout the proof, we will use the following posterior probabilities about the state. When a consumer from the first group ($g = 1$) receives signal $\theta_i = 1$, he receives information regarding the aggregate preference distribution in his own group, but still relies on his prior to estimate demand in the other group. Hence, he attaches the following posterior probabilities to the vector of states (s_1, s_2) :

(s_1, s_2)	(β, β)	$(1 - \beta, 1 - \beta)$	$(1 - \beta, \beta)$	$(\beta, 1 - \beta)$
$\Pr((s_1, s_2) \theta_i = 1, g = 1)$	$\frac{\beta}{2}$	$\frac{1-\beta}{2}$	$\frac{1-\beta}{2}$	$\frac{\beta}{2}$

In case a consumer from group 1 receives signal $\theta_i = 0$, the posterior probabilities are

(s_1, s_2)	(β, β)	$(1 - \beta, 1 - \beta)$	$(1 - \beta, \beta)$	$(\beta, 1 - \beta)$
$\Pr((s_1, s_2) \theta_i = 0, g = 1)$	$\frac{1-\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{1-\beta}{2}$

First step: consumers with $\theta_i = 0$ do not invest in equilibrium. Denote the price in state $s = (\beta, \beta)$ by p_{11} , the price in state $s = (\beta, 1 - \beta)$ by p_{10} and so on. Then we can write the expected returns of crowdinvestors of type $\theta_i = 1$ in group 1 and 2 as

$$\begin{aligned}\mathbb{E}_s[p | \theta_i = 1, g = 1] &= \frac{\beta}{2}p_{11} + \frac{\beta}{2}p_{10} + \frac{1-\beta}{2}p_{00} + \frac{1-\beta}{2}p_{01}, \\ \mathbb{E}_s[p | \theta_i = 1, g = 2] &= \frac{\beta}{2}p_{11} + \frac{1-\beta}{2}p_{10} + \frac{1-\beta}{2}p_{00} + \frac{\beta}{2}p_{01}.\end{aligned}$$

Similarly, the expected returns of consumers with $\theta_i = 0$ are

$$\begin{aligned}\mathbb{E}_s[p|\theta_i = 0, g = 1] &= \frac{1-\beta}{2}p_{11} + \frac{1-\beta}{2}p_{10} + \frac{\beta}{2}p_{00} + \frac{\beta}{2}p_{01}, \\ \mathbb{E}_s[p|\theta_i = 0, g = 2] &= \frac{1-\beta}{2}p_{11} + \frac{\beta}{2}p_{10} + \frac{\beta}{2}p_{00} + \frac{1-\beta}{2}p_{01}.\end{aligned}\tag{14}$$

We want to show that consumers with $\theta_i = 0$ always expect a weakly lower investment return compared to consumers with $\theta_i = 1$. Thus, comparing $\mathbb{E}_s[p|\theta_i = 1, g = 2]$ with $\mathbb{E}_s[p|\theta_i = 1, g = 1]$,

$$\begin{aligned}\mathbb{E}_s[p|\theta_i = 1, g = 1] &\geq \mathbb{E}_s[p|\theta_i = 0, g = 2] \\ \iff \frac{2\beta-1}{2}p_{11} &\geq \frac{2\beta-1}{2}p_{00} \iff p_{11} \geq p_{00}.\end{aligned}\tag{15}$$

Denote the investment amount of investor i if $\theta_i = 1$ by $\hat{x}_i(\theta_i = 1)$ and if $\theta_i = 0$ by $\hat{x}_i(\theta_i = 0)$. Recall that group 1 are all consumers $i \in [0, 0.5]$ and group 2 are all consumers $i \in (0.5, 1]$. Now we can rewrite condition (15) in terms of investment strategies. After simplifying, (15) is equivalent to

$$\int_0^{0.5} \hat{x}_i(\theta_i = 0)di + \int_{0.5}^1 \hat{x}_i(\theta_i = 0)di \geq 0,\tag{16}$$

which always holds true. Moreover, any positive aggregate investment by consumers with $\theta_i = 0$ from either group leads to $\mathbb{E}_s[p|\theta_i = 1, g = 1] > \mathbb{E}_s[p|\theta_i = 0, g = 2]$. Using the same reasoning, we also get $\mathbb{E}_s[p|\theta_i = 1, g = 2] \geq \mathbb{E}_s[p|\theta_i = 0, g = 1]$, and if (16) holds with strict inequality, then $\mathbb{E}_s[p|\theta_i = 1, g = 2] > \mathbb{E}_s[p|\theta_i = 0, g = 1]$.

Note that consumers only invest if their expected return is equal to or exceeds R , otherwise investing at the riskless rate R is a profitable deviation. Therefore, whenever consumers with $\theta_i = 0$ from either group invest, i.e., (16) holds with strict inequality, then consumers with $\theta_i = 1$ expect a return exceeding R . However, this cannot occur in equilibrium if $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ for all i . Suppose (16) holds with strict inequality, then it is optimal for all consumers with $\theta_i = 1$ to increase their investment $\hat{x}_i(\theta_i = 1)$ until their expected return equals R . For at least one consumer this deviation must be feasible, since the wealth endowment w_i is sufficient for all consumers with $\theta_i = 1$ to invest $\hat{x}_i(\theta_i = 1) = (\alpha/R)^{1/(1-\alpha)}$, which guarantees a return of R or less. But if consumers with $\theta_i = 1$ expect a return of R , then by (15), consumers with $\theta_i = 0$ expect a return below R , which contradicts that (16) holds with strict inequality. Consequently, no consumer of type $\theta_i = 0$ invests in equilibrium.

Second step: The expected return from investing if $\theta_i = 1$ differs between both groups whenever the average investment strategy in case of $\theta_i = 1$ differs between the groups. Suppose that there is an equilibrium where some consumers with type $\theta_i = 1$ from group 1 (without loss of generality) invest in x , which implies $\mathbb{E}_s[p|\theta_i = 1, g = 1] \geq R$,

otherwise not investing would be a profitable deviation. Now suppose that investors from group 2 do not invest in this case, which implies they expect a weakly lower return from investing,

$$\begin{aligned} \mathbb{E}_s[p|\theta_i = 1, g = 1] &\geq \mathbb{E}_s[p|\theta_i = 1, g = 2] \\ \iff \frac{2\beta - 1}{2}p_{10} &\geq \frac{2\beta - 1}{2}p_{01} \iff p_{10} > p_{01}. \end{aligned} \quad (17)$$

Since the aggregate demand is the same in state $s = (\beta, 1 - \beta)$ and $s = (1 - \beta, \beta)$, price differences between these two states must be due to differences in aggregate investment. Rewriting condition (17) in terms of investment strategies gives

$$\begin{aligned} &\int_0^{0.5} [\beta \hat{x}_i(\theta_i = 1) + (1 - \beta) \hat{x}_i(\theta_i = 0)] di + \int_{0.5}^1 [(1 - \beta) \hat{x}_i(\theta_i = 1) + \beta \hat{x}_i(\theta_i = 0)] di \\ &\leq \int_0^{0.5} [(1 - \beta) \hat{x}_i(\theta_i = 1) + \beta \hat{x}_i(\theta_i = 0)] di + \int_{0.5}^1 [\beta \hat{x}_i(\theta_i = 1) + (1 - \beta) \hat{x}_i(\theta_i = 0)] di \\ &\iff \int_0^{0.5} [\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)] di \leq \int_{0.5}^1 [\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)] di \\ &\iff \int_0^{0.5} \hat{x}_i(\theta_i = 1) di \leq \int_{0.5}^1 \hat{x}_i(\theta_i = 1) di, \end{aligned} \quad (18)$$

where the last line follows from the fact that consumers with $\theta_i = 0$ do not invest in equilibrium (see first step). Thus, if a positive mass of consumers from group 1 with $\theta_i = 1$ invest (i.e., average investment strategy in case of $\theta_i = 1$ is $\int_0^{0.5} \hat{x}_i(\theta_i = 1) di > 0$), then consumers from group 2 with $\theta_i = 1$ must also invest ($\int_{0.5}^1 \hat{x}_i(\theta_i = 1) di > 0$). The same reasoning holds in the opposite direction as well: If consumers from group 2 with $\theta_i = 1$ invest, then so must those from group 1. Moreover, the expected returns of both groups are the same if and only if (18) holds with equality, i.e., both groups have the same aggregate investment strategy in case of $\theta_i = 1$.

Third step: Both groups have the same aggregate investment strategy in case of $\theta_i = 1$ that leads to $p = R$ for all s . From step 2, whenever the average investment strategies of the two groups in case of $\theta_i = 1$ diverge, then so do the expected returns, which means that crowdinvestors in at least one group have a profitable deviation to either invest more or less. Thus, in equilibrium, we must have a profile of investment strategies $\{\hat{x}_i(\theta_i = 1)\}_i$ such that

$$\int_0^{0.5} \hat{x}_i(\theta_i = 1) di = \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}, \quad \int_{0.5}^1 \hat{x}_i(\theta_i = 1) di = \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}, \quad (19)$$

which leads to an aggregate investment by group 1 of $\frac{s_1}{2}(\alpha/R)^{1/(1-\alpha)}$ and by group 2 of $\frac{s_2}{2}(\alpha/R)^{1/(1-\alpha)}$, which implies $p = R$ in all states and is efficient (Lemma 1).

To show (19), suppose by contradiction that $\int_0^{0.5} \hat{x}_i(\theta_i = 1)di > \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$ and $\int_{0.5}^1 \hat{x}_i(\theta_i = 1)di > \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$, then $\mathbb{E}_s[p|\theta_i = 1, g = 1] < R$ and $\mathbb{E}_s[p|\theta_i = 1, g = 2] < R$, and not investing is a profitable deviation. Suppose $\int_0^{0.5} \hat{x}_i(\theta_i = 1)di < \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$ and $\int_{0.5}^1 \hat{x}_i(\theta_i = 1)di < \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$, then $\mathbb{E}_s[p|\theta_i = 1, g = 1] > R$ and $\mathbb{E}_s[p|\theta_i = 1, g = 2] > R$, and investing more is profitable and feasible. Suppose $\int_0^{0.5} \hat{x}_i(\theta_i = 1)di > \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$ and $\int_{0.5}^1 \hat{x}_i(\theta_i = 1)di < \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$, then from step 2 either $\mathbb{E}_s[p|\theta_i = 1, g = 1] < R$ or $\mathbb{E}_s[p|\theta_i = 1, g = 2] > R$, so that investing more or less is a profitable deviation. Finally, suppose $\int_0^{0.5} \hat{x}_i(\theta_i = 1)di < \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$ and $\int_{0.5}^1 \hat{x}_i(\theta_i = 1)di > \frac{1}{2}(\alpha/R)^{1/(1-\alpha)}$, then by step 2 either $\mathbb{E}_s[p|\theta_i = 1, g = 1] > R$ or $\mathbb{E}_s[p|\theta_i = 1, g = 2] < R$. \square

Proof of Proposition 3. Sufficiency: If all consumers have $w_i \geq (\alpha/R)^{1/(1-\alpha)}$, then Proposition 2 proves that an efficient equilibrium exists.

Necessity: To be shown: If an efficient equilibrium exists, then $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ for investors of both groups. In an efficient equilibrium, aggregate investment scales up linearly with the share of interested consumers in each group (Lemma 1):

$$X(s) = \frac{s_1 + s_2}{2}(\alpha/R)^{1/(1-\alpha)}. \quad (20)$$

Denote the investment amount of investor i if $\theta_i = 1$ by $\hat{x}_i(\theta_i = 1)$ and if $\theta_i = 0$ by $\hat{x}_i(\theta_i = 0)$. Recall that group 1 are all consumers $i \in [0, 0.5]$ and group 2 are all consumers $i \in (0.5, 1]$. Now we can write aggregate investment X in terms of investment strategies of all consumers,

$$\begin{aligned} X &= \int_0^{0.5} [s_1 \hat{x}_i(\theta_i = 1) + (1 - s_1) \hat{x}_i(\theta_i = 0)]di + \int_{0.5}^1 [s_2 \hat{x}_i(\theta_i = 1) + (1 - s_2) \hat{x}_i(\theta_i = 0)]di \\ &= \int_0^{0.5} [s_1(\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)) + \hat{x}_i(\theta_i = 0)]di \\ &\quad + \int_{0.5}^1 [s_2(\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)) + \hat{x}_i(\theta_i = 0)]di. \end{aligned} \quad (21)$$

Since by assumption an efficient equilibrium exists, both (20) and (21) have to hold for all realizations of (s_1, s_2) . This is only possible if $\int_0^1 \hat{x}_i(\theta_i = 0)di = 0$, i.e., if consumers of type $\theta_i = 0$ do not invest. Hence, simplifying (21) and equating aggregate investment with efficient aggregate investment (20), the following conditions hold in any efficient equilibrium:

$$\begin{aligned} 2 \cdot \int_0^{0.5} \hat{x}_i(\theta_i = 1)di &= (\alpha/R)^{1/(1-\alpha)} \text{ and} \\ 2 \cdot \int_{0.5}^1 \hat{x}_i(\theta_i = 1)di &= (\alpha/R)^{1/(1-\alpha)}. \end{aligned} \quad (22)$$

The investment budget constraint requires $\hat{x}_i(\theta_i = 1) \leq w_i$ for all i . Thus, (22) implies

$$2 \cdot \int_0^{0.5} w_i di \geq (\alpha/R)^{1/(1-\alpha)} \text{ and } 2 \cdot \int_{0.5}^1 w_i di \geq (\alpha/R)^{1/(1-\alpha)},$$

and hence (since w_i is constant in each group)

$$w_i \geq (\alpha/R)^{1/(1-\alpha)} \text{ for } i \in [0, 0.5] \text{ and } w_i \geq (\alpha/R)^{1/(1-\alpha)} \text{ for } i \in (0.5, 1]. \quad \square$$

Proof of Proposition 5. Sufficiency: To show: 1. and 2. imply existence of an efficient equilibrium. We show this by construction. In this equilibrium, both wealthy and poor consumer groups reveal their type distribution by making type dependent investments at $t = 0$, and then react to this revelation at $t = 1$.

Consider an equilibrium with the following $t = 0$ investment strategy profile with $\varepsilon > 0$ small so that $s_1\varepsilon + s_2\underline{w}$ is invertible in $(s_1, s_2) \in \{1 - \beta, \beta\}^2$:

$$\begin{aligned} \hat{x}_i^{t=0}(\theta_i = 1) &= \underline{w} \quad \forall i \in (0.5, 1], \\ \hat{x}_i^{t=0}(\theta_i = 0) &= 0 \quad \forall i \in (0.5, 1], \\ \hat{x}_i^{t=0}(\theta_i = 1) &= \varepsilon \quad \forall i \in [0, 0.5], \\ \hat{x}_i^{t=0}(\theta_i = 0) &= 0 \quad \forall i \in [0, 0.5]. \end{aligned}$$

This investment strategy profile results in aggregate investment $X_0(s_1, s_2) = \varepsilon s_1/2 + \underline{w}s_2/2$, which reveals the tuple (s_1, s_2) by construction. Consequently, on the equilibrium path of this candidate, the state is observable before making investments at $t = 1$. Now any equilibrium strategy profile at $t = 1$ which results in aggregate investment implying a return of R is incentive compatible. In state (β, β) with largest demand, the required aggregate investment is $\beta(\alpha/R)^{1/(1-\alpha)}$, which is exactly feasible by adding the aggregate wealth of both groups (determined from 1. and 2.):

$$\begin{aligned} &\underline{w}/2 + ((\alpha/R)^{1/(1-\alpha)} - \min\{(\alpha/R)^{1/(1-\alpha)}, \underline{w}\})\beta/2 + \beta(\alpha/R)^{1/(1-\alpha)}/2 \\ &= (\alpha/R)^{1/(1-\alpha)}\beta/2 + \beta(\alpha/R)^{1/(1-\alpha)}/2 = \beta(\alpha/R)^{1/(1-\alpha)}. \end{aligned}$$

Given the implied return of R on investment, all investors are indifferent between individually investing earlier or later, or investing in the safe asset, since no unilateral deviation will change X_0 due to the zero mass of all i . Hence, the described investment strategy profiles constitute a Bayesian Nash equilibrium of the dynamic investment game. Since the return is R in every state, it is also an ex post equilibrium.

Necessity: To show: An efficient equilibrium implies 1. and 2. First, if 1. does not hold so that $\underline{w} = 0$, then aggregate investment cannot depend on s_2 , but this contradicts efficiency (Lemma 1). Second, if 2. does not hold, then there cannot be a return equal to R in every state. This is because the state $(s_1, s_2) = (\beta, \beta)$ requires an aggregate investment

of $\beta(\alpha/R)^{1/(1-\alpha)}$, which is just feasible for 2. given \underline{w} and infeasible otherwise. But a return unequal R contradicts this being an efficient allocation. Hence, 1. and 2. are necessary for the existence of an efficient equilibrium in the sequential model. \square

Proof of Proposition 7.

- i. In an equilibrium where only types $\theta_i = 1$ invest, we must have that the expected return from investing in the company equals R for $\theta_i = 1$ and weakly less than R for types $\theta_i = 0$ (otherwise they would prefer to invest, which is feasible by assumption of sufficient wealth). Denote the return in state (β, β) by r_{11} , in state $(1 - \beta, 1 - \beta)$ by r_{00} etc. Consequently,

$$R = \mathbb{E}_s[r|\theta_i = 1] = \frac{1 - \beta}{2}r_{00} + \frac{\beta}{2}r_{11} + \frac{1}{2}r_{10} \geq \mathbb{E}[r|\theta_i = 0] = \frac{\beta}{2}r_{00} + \frac{1 - \beta}{2}r_{11} + \frac{1}{2}r_{10} \\ \iff r_{11} \geq r_{00}, \quad (23)$$

since $r_{10} = r_{01}$ in a symmetric equilibrium. Now, using the explicit expressions for r_{11} and r_{00} depending on the investment strategy \hat{x}_1 of $\theta_i = 1$ types on (23), we get

$$\alpha\beta^{1-\alpha}(\beta\hat{x}_1)^{\alpha\lambda-1} \geq \alpha(1 - \beta)^{1-\alpha}((1 - \beta)\hat{x}_1)^{\alpha\lambda-1},$$

which—in the range $\lambda < 1/\alpha$ we look at—is equivalent to

$$\beta^{\frac{1-\alpha}{\alpha\lambda-1}+1} \leq (1 - \beta)^{\frac{1-\alpha}{\alpha\lambda-1}+1}. \quad (24)$$

(24) holds iff $\frac{1-\alpha}{\alpha\lambda-1} + 1 = \frac{\alpha\lambda-\alpha}{\alpha\lambda-1} \leq 0$, which in turn holds iff $1/\alpha \geq \lambda \geq 1$. Thus, combining both, the return expectation of type $\theta_i = 1$ weakly exceeds the expectation of type $\theta_i = 0$ iff $\lambda \geq 1$.

Now, \hat{x}_1 is determined by equating the return expectation of type $\theta_i = 1$ with R , i.e., by solving the equation in (23). Since the expected return is strictly decreasing in \hat{x}_1 for $\lambda < 1/\alpha$, there exists a unique solution and hence a unique symmetric equilibrium.

- ii. Assuming sufficient wealth, if both types invest, then both types must have a return-expectation of R from investing in the company. If either type had an expectation less than R , then it should not invest; if either type had an expectation exceeding R , then it should invest more, which is feasible by assumption of sufficient wealth. Hence, the expected return from investing in the company for type $\theta_i = 1$ fulfills

$$R = \frac{1 - \beta}{2}r_{00} + \frac{\beta}{2}r_{11} + \frac{1}{2}r_{10}, \quad (25)$$

where $r_{10} = r_{01}$ in a symmetric equilibrium. The expected return from investing for

type $\theta_i = 0$ similarly fulfills

$$R = \frac{\beta}{2}r_{00} + \frac{1-\beta}{2}r_{11} + \frac{1}{2}r_{10}. \quad (26)$$

Thus, combining (25) and (26), we get

$$\begin{aligned} \frac{1-\beta}{2}r_{00} + \frac{\beta}{2}r_{11} + \frac{1}{2}r_{10} &= \frac{\beta}{2}r_{00} + \frac{1-\beta}{2}r_{11} + \frac{1}{2}r_{10} \\ &\iff r_{11} = r_{00}. \end{aligned}$$

Consequently, the return in both the state (β, β) and $(1-\beta, 1-\beta)$ must be equal if both types have the same return expectation. Using the explicit expressions for r_{11} and r_{00} , we get

$$\begin{aligned} & r_{11} = r_{00} \\ \iff & \alpha \left(\frac{\beta}{(\beta\hat{x}_1 + (1-\beta)\hat{x}_0)^\lambda} \right)^{1-\alpha} \frac{(\beta\hat{x}_1 + (1-\beta)\hat{x}_0)^\lambda}{\beta\hat{x}_1 + (1-\beta)\hat{x}_0} \\ &= \alpha \left(\frac{1-\beta}{((1-\beta)\hat{x}_1 + \beta\hat{x}_0)^\lambda} \right)^{1-\alpha} \frac{((1-\beta)\hat{x}_1 + \beta\hat{x}_0)^\lambda}{(1-\beta)\hat{x}_1 + \beta\hat{x}_0} \quad (27) \\ \iff & \beta^{1-\alpha}(\beta\hat{x}_1 + (1-\beta)\hat{x}_0)^{\alpha\lambda-1} = (1-\beta)^{1-\alpha}((1-\beta)\hat{x}_1 + \beta\hat{x}_0)^{\alpha\lambda-1} \\ \iff & \beta^{\frac{1-\alpha}{\alpha\lambda-1}}(\beta\hat{x}_1 + (1-\beta)\hat{x}_0) = (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}}((1-\beta)\hat{x}_1 + \beta\hat{x}_0) \\ \iff & \hat{x}_0 = -\hat{x}_1 \left[\frac{\beta^{\frac{1-\alpha}{\alpha\lambda-1}+1} - (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}+1}}{\beta^{\frac{1-\alpha}{\alpha\lambda-1}}(1-\beta) - (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}}\beta} \right] \end{aligned}$$

Thus, we have the investments of type $\theta_i = 0$ as function of the investments of type $\theta_i = 1$. For an equilibrium where both types invest, we need that the coefficient of \hat{x}_1 in (27) is positive, so that $\hat{x}_0 > 0$ implies $\hat{x}_1 > 0$ and vice versa. Now,

$$- \left[\frac{\beta^{\frac{1-\alpha}{\alpha\lambda-1}+1} - (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}+1}}{\beta^{\frac{1-\alpha}{\alpha\lambda-1}}(1-\beta) - (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}}\beta} \right] > 0$$

if and only if numerator and denominator are both positive or both negative. First, consider the case where both are positive. For the numerator we have the condition

$$\beta^{\frac{1-\alpha}{\alpha\lambda-1}+1} < (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}+1},$$

which holds iff $\frac{1-\alpha}{\alpha\lambda-1} + 1 = \frac{\alpha\lambda-\alpha}{\alpha\lambda-1} < 0$, which in turn holds iff $1/\alpha > \lambda > 1$. Next, the condition for the denominator is

$$\beta^{\frac{1-\alpha}{\alpha\lambda-1}}(1-\beta) > (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}}\beta \iff \beta^{\frac{1-\alpha}{\alpha\lambda-1}-1} > (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}-1}$$

which holds iff $\frac{1-\alpha}{\alpha\lambda-1} - 1 = \frac{2-\alpha-\alpha\lambda}{\alpha\lambda-1} > 0$, which in turn holds iff $\frac{2-\alpha}{\alpha} > \lambda > 1/\alpha$. Since

both of these conditions cannot hold simultaneously, there is no equilibrium where both the denominator and numerator is positive. We still need to check the case where both are negative. For the numerator, the condition is

$$\beta^{\frac{1-\alpha}{\alpha\lambda-1}+1} > (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}+1},$$

which holds iff $\frac{1-\alpha}{\alpha\lambda-1} + 1 = \frac{\alpha\lambda-\alpha}{\alpha\lambda-1} > 0$, which in turn holds iff $1/\alpha < \lambda$ or $\lambda < 1$. The condition that the denominator is negative is

$$\iff \beta^{\frac{1-\alpha}{\alpha\lambda-1}-1} < (1-\beta)^{\frac{1-\alpha}{\alpha\lambda-1}-1},$$

which holds iff $\frac{1-\alpha}{\alpha\lambda-1} - 1 = \frac{2-\alpha-\alpha\lambda}{\alpha\lambda-1} < 0$, which in turn holds iff $\lambda < 1$ or $\lambda > \frac{2-\alpha}{\alpha}$. The latter is ruled out by our restriction of $\lambda < 1/\alpha$. Thus, the condition $r_{11} = r_{00}$ implies $\hat{x}_0 > 0 \implies \hat{x}_1 > 0$ (and vice versa) if and only if $\lambda < 1$.

Now, the expression for \hat{x}_0 in (27) can be substituted into (25) or (26) to obtain \hat{x}_1 . Clearly, the right hand sides of both equations are strictly decreasing in the aggregate investment for $\lambda < 1/\alpha$, hence there is a unique solution with $\hat{x}_0 > 0, \hat{x}_1 > 0$ if $\lambda < 1$.

- iii. If only type $\theta_i = 1$ in group 1 invests, then the expected return of this type must equal R , and the expected return of type $\theta_i = 0$ in group 1 must be $\leq R$. Thus,

$$\begin{aligned} R &= \frac{\beta}{2}r_{11} + \frac{1-\beta}{2}r_{00} + \frac{\beta}{2}r_{10} + \frac{1-\beta}{2}r_{01} \geq \frac{1-\beta}{2}r_{11} + \frac{\beta}{2}r_{00} + \frac{1-\beta}{2}r_{10} + \frac{\beta}{2}r_{01} \\ &\iff r_{11} + r_{10} \geq r_{00} + r_{01}. \end{aligned} \quad (28)$$

Using the explicit expressions for these state returns with \hat{x}_1^1 as investments of $\theta_i = 1$ types in group 1, and simplifying somewhat, we obtain

$$\begin{aligned} &\iff \beta^{1-\alpha}(\beta\hat{x}_1^1/2)^{\alpha\lambda-1} + (1/2)^{1-\alpha}(\beta\hat{x}_1^1/2)^{\alpha\lambda-1} \\ &\geq (1-\beta)^{1-\alpha}((1-\beta)\hat{x}_1^1/2)^{\alpha\lambda-1} + (1/2)^{1-\alpha}((1-\beta)\hat{x}_1^1/2)^{\alpha\lambda-1} \\ &\iff \beta^{1-\alpha}(\beta/2)^{\alpha\lambda-1} + (1/2)^{1-\alpha}(\beta/2)^{\alpha\lambda-1} \\ &\geq (1-\beta)^{1-\alpha}((1-\beta)/2)^{\alpha\lambda-1} + (1/2)^{1-\alpha}((1-\beta)/2)^{\alpha\lambda-1} \\ &\iff \frac{\beta^{\alpha\lambda-1}}{(1-\beta)^{\alpha\lambda-1}} \geq \frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}} \\ &\iff (\alpha\lambda - 1) \cdot \log\left(\frac{\beta}{1-\beta}\right) \geq \log\left(\frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}}\right) \\ &\iff \lambda \geq \frac{\log\left(\frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}}\right)}{\log\left(\frac{\beta}{1-\beta}\right)\alpha} + 1/\alpha. \end{aligned} \quad (29)$$

Thus, the expected return of type $\theta_i = 0$ in group 1 is weakly lower than the return expectation of type $\theta_i = 1$ in group 1 if and only if (30) holds. Now we still need to

show that the right hand side of (30) is greater than 1. To see this, set $\lambda = 1$ in (29) to obtain

$$\frac{(1-\beta)^{1-\alpha}}{\beta^{1-\alpha}} \geq \frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}},$$

which is false since

$$\frac{(1-\beta)^{1-\alpha}}{\beta^{1-\alpha}} < \frac{(1-\beta)^{1-\alpha} + \varepsilon}{\beta^{1-\alpha} + \varepsilon} < 1$$

for any $\varepsilon > 0$. Thus, the λ that fulfills (29) is greater than 1, as the LHS is increasing in λ . Moreover, the threshold is smaller than $1/\alpha$, since $\log\left(\frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}}\right) < 0$ and $\log\left(\frac{\beta}{1-\beta}\right) > 0$ due to $\beta > 1/2$. Finally, the equilibrium investment \hat{x}_1^1 is the solution to the equality in (28), which exists and is unique for $\lambda < 1/\alpha$.

- iv. If both types invest, then both types must have a return-expectation of R from investing in the company. If either type had an expectation less than R , then it should not invest; if either type had an expectation exceeding R , then it should invest more, which is feasible by assumption of sufficient wealth in group 1. Thus, equating the expected returns, in equilibrium we must have

$$R = \frac{1-\beta}{2}r_{00} + \frac{\beta}{2}r_{11} + \frac{1-\beta}{2}r_{01} + \frac{\beta}{2}r_{10} = \frac{\beta}{2}r_{00} + \frac{1-\beta}{2}r_{11} + \frac{\beta}{2}r_{01} + \frac{1-\beta}{2}r_{10} \quad (31)$$

$$\iff r_{11} + r_{10} = r_{00} + r_{01}.$$

Now we use the explicit expressions for these state returns and simplify somewhat to get:

$$\begin{aligned} & \beta^{1-\alpha}(\beta\hat{x}_1^1/2 + (1-\beta)\hat{x}_0^1/2)^{\alpha\lambda-1} + (1/2)^{1-\alpha}(\beta\hat{x}_1^1/2 + (1-\beta)\hat{x}_0^1/2)^{\alpha\lambda-1} \\ &= (1-\beta)^{1-\alpha}((1-\beta)\hat{x}_1^1/2 + \beta\hat{x}_0^1/2)^{\alpha\lambda-1} + (1/2)^{1-\alpha}((1-\beta)\hat{x}_1^1/2 + \beta\hat{x}_0^1/2)^{\alpha\lambda-1} \\ & \iff (\beta\hat{x}_1^1 + (1-\beta)\hat{x}_0^1) [\beta^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} \\ &= ((1-\beta)\hat{x}_1^1 + \beta\hat{x}_0^1) [(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} \\ & \hat{x}_1^1 = \hat{x}_0^1 \frac{\beta [(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} - (1-\beta) [\beta^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}}}{\beta [\beta^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} - (1-\beta) [(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}}}. \end{aligned} \quad (32)$$

Thus, $\hat{x}_1^1 > 0 \iff \hat{x}_0^1 > 0$ in equilibrium iff the fraction is positive, which is the case iff both numerator and denominator are positive or both are negative. First, the numerator is positive iff

$$\begin{aligned} & \beta [(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} > (1-\beta) [\beta^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} \\ & \iff \frac{\beta}{1-\beta} > \left(\frac{\beta^{1-\alpha} + (1/2)^{1-\alpha}}{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}} \right)^{\frac{1}{\alpha\lambda-1}}. \end{aligned} \quad (33)$$

Clearly, (33) holds for $\frac{1}{\alpha\lambda-1}$ sufficiently small. $\frac{1}{\alpha\lambda-1}$ is discontinuous at $\lambda = 1/\alpha$; $\frac{1}{\alpha\lambda-1} \rightarrow -\infty$ as $\lambda \nearrow 1/\alpha$. For the range $\lambda \in (0, 1/\alpha)$ it is maximal as $\lambda \rightarrow 0$ with $\frac{1}{\alpha\lambda-1} \rightarrow -1$, where the RHS is less than 1. Thus, the condition (33) holds for $\lambda \in (0, 1/\alpha)$.

Next, the denominator is positive iff

$$\begin{aligned} \beta [\beta^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} &> (1-\beta) [(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}]^{\frac{1}{\alpha\lambda-1}} \\ \iff \frac{\beta}{1-\beta} &> \left(\frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}} \right)^{\frac{1}{\alpha\lambda-1}}. \end{aligned} \quad (34)$$

The condition does not hold for $\frac{1}{\alpha\lambda-1}$ sufficiently negative. $\frac{1}{\alpha\lambda-1} \rightarrow -\infty$ as $\lambda \nearrow 1/\alpha$, and it decreases in λ in $\lambda \in (-\infty, 1/\alpha)$. The condition holds at $\lambda = 1$, since for $\beta > 1/2$,

$$\begin{aligned} \frac{\beta}{1-\beta} &> \left(\frac{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}{\beta^{1-\alpha} + (1/2)^{1-\alpha}} \right)^{\frac{1}{\alpha-1}} \\ \iff \frac{\beta^{1-\alpha}}{(1-\beta)^{1-\alpha}} &> \frac{\beta^{1-\alpha} + (1/2)^{1-\alpha}}{(1-\beta)^{1-\alpha} + (1/2)^{1-\alpha}}. \end{aligned}$$

Consequently, the denominator is positive except for $\lambda \in [\underline{t}, 1/\alpha]$, where $\underline{t} > 1$ equals λ such that (34) holds with equality; see (12) for the explicit solution.

Thus, the numerator and denominator have the same sign in the range $\lambda \in (0, \underline{t})$. Now, as a final step, we can substitute \hat{x}_1^1 as a function of \hat{x}_0^1 in (32) into (31), where the RHS is strictly decreasing in \hat{x}_0^1 . Thus, since \hat{x}_0^1 and \hat{x}_1^1 have the same sign for all $\lambda \in (0, \underline{t})$, there exists a unique solution $\hat{x}_0^1 > 0$ and $\hat{x}_1^1 > 0$ such that (31) holds. \square

Proof of Proposition 8. The conditions that determine the market investments for balanced wealth (23) and imbalanced wealth (28) are continuous in the investment amounts. There is a switch in the equilibrium type as $\lambda \rightarrow 1$ from below, but (27) shows that the investment of $\theta_i = 0$ types (\hat{x}_0) converges to zero as $\lambda \rightarrow 1$, hence the equilibrium strategy \hat{x}_0 is continuous in λ . Moreover, the market clearing price (10) is continuous in aggregate investment X and so is utility (7) and the budget constraint. Thus, since Pareto-efficiency for quasi-linear utility (7) implies a utilitarian welfare maximum, and since the allocation is Pareto-efficient at $\lambda = 1$ for the balanced wealth case but not the imbalanced wealth case (Proposition 3), the welfare must be larger for the balanced wealth case in a neighborhood of $\lambda = 1$. \square

Proof of Proposition 9. We shall confirm that all equilibrium requirements of definition 2 can be fulfilled.

A unique market clearing price p exists for all aggregate investment levels X and all realizations of preferences (s_1, s_2) . In the consumption stage, consumers use the demand function (8), which by construction maximizes utility.

Every price p_m set by the market research firm induces a Bayesian investment game at

the acquisition and investment stage. In this investment game, all crowdinvestors i choose $\hat{x}_i \in [0, w_i]$ for each $\theta_i \in \{0, 1\}$ and all funds choose $(a_j, f_j) \in \{0, 1\} \times [0, W_j]$ for each $p_m \in \mathbb{R}_+$ and $m_j \in \{\{0, 1\}^2, \emptyset\}$, where $w_i \in [0, \infty)$ and $W_i \in [0, \infty)$.

Consider first a reduced game, where the strategy space for funds is $f_j \in [0, W_j]$ and information acquisition decisions (a_1, a_2, \dots, a_N) are exogenous. Then strategy spaces of all investors are compact and convex, and strategy f_j is concave and continuous in the expected payoff π_j for a given strategy profile (f_{-j}, \hat{x}) of all other investors, where

$$\mathbb{E}[\pi_j(f_j, f_{-j}, \hat{x}) | \mathcal{I}_j(a_j)] = f_j(\mathbb{E}[p(f_j, f_{-j}, \hat{x}) | \mathcal{I}_j(a_j)] - R),$$

and \hat{x}_i is quasi-concave and continuous for crowdinvestors i . Thus, the Debreu-Glicksberg-Fan theorem (e.g., Theorem 1 in [Reny, 2008](#)) guarantees the existence of a pure strategy equilibrium for any exogenous profile (a_1, a_2, \dots, a_N) .

Going back to the actual game with fund strategy space $(a_j, f_j) \in \{0, 1\} \times [0, W_j]$, which is not convex, every information acquisition profile (a_1, a_2, \dots, a_N) induces a reduced game for which we just showed a pure strategy equilibrium exists. By allowing mixed strategies in a_j , we can convexify the strategy space to $[0, 1] \times [0, W_j]$, and the expected payoffs from the mixed strategies are just linear combinations of the payoffs of the reduced game. Since a linear combination is quasi-concave and continuous, the Debreu-Glicksberg-Fan theorem guarantees existence of an equilibrium of the investment game, with possible mixing in a_j and corresponding f_j for all j and pure strategies for crowdinvestors \hat{x}_i .

We still have to show that, given the outcomes of the investment game for every p_m , there exists a profit maximizing price p_m for the MR firm. For a given p_m , all funds j determine the information acquisition decision by solving the problem

$$\max_{a_j \in [0, 1]} \mathbb{E}[\pi_j | a_j, a_{-j}, f, \hat{x}] - a_j p_m,$$

where the set of mixed strategies $[0, 1]$ is compact, and $\mathbb{E}[\pi_j | a_j, a_{-j}, f, \hat{x}] - a_j p_m$ is continuous in p_m . Berge's maximum theorem implies that $a_j(p_m)$ —the expected demand for market research by fund j —is upper hemi-continuous (uhc) in p_m . Aggregate expected demand for market research is $\sum_j a_j(p_m)$. The profit function for the market research firm is given by

$$\pi_{MR}(p_m) = p_m \sum_j a_j(p_m) - \mathbf{1} \left\{ \sum_j a_j(p_m) > 0 \right\} c.$$

Since summation and integration preserves upper hemi-continuity, $\sum_j a_j(p_m)$ is uhc. Moreover, the product of two non-negative uhc correspondences p_m and $\sum_j a_j(p_m)$ is uhc. The negative of the last term $\mathbf{1} \left\{ \sum_j a_j(p_m) > 0 \right\} c$ is lower hemi-continuous, since the indicator function $\mathbf{1} \{x \in X\}$ is lower hemi-continuous if and only if X is an open set. Consequently, $-\mathbf{1} \left\{ \sum_j a_j(p_m) > 0 \right\} c$ is uhc, and thus $\pi_{MR}(p_m)$ is uhc.

We can find an upper bound for a profit maximizing p_m , since no fund will buy market research if p_m is larger than the maximally possible earnings in the capital market, which are bounded. Denote such a bound by $0 < P < \infty$. Then, the market research firm chooses $p_m \in [0, P]$, which is a compact set, hence the Weierstrass extreme value theorem implies there exists a p_m which maximizes $\pi_{MR}(p_m)$. \square

Proof of Proposition 10. Suppose an equilibrium with efficient investment exists in which some investment funds invest, which implies that the return on investment is R in every state (Lemma 1). In this case it does not pay for funds to buy market research at any price $p_m > 0$, as funds can by assumption obtain an investment return R by investing elsewhere without paying p_m . Consequently, investment funds must be uninformed in any efficient equilibrium, and invest a state independent amount $F := \sum_j f_j > 0$ in every state.

In any equilibrium, each consumer can condition his investment plan \hat{x}_i on θ_i . Consequently, aggregate investment by consumers depending on the preference realization can be written as

$$\int_0^1 \hat{x}_i di = \int_0^{0.5} [s_1 \hat{x}_i(\theta_i = 1) + (1 - s_1) \hat{x}_i(\theta_i = 0)] di + \int_{0.5}^1 [s_2 \hat{x}_i(\theta_i = 1) + (1 - s_2) \hat{x}_i(\theta_i = 0)] di.$$

Efficiency requires that the price in each state equals R . In particular,

$$R = \alpha \left(\frac{\beta}{F + \int [\beta \hat{x}_i(\theta_i = 1) + (1 - \beta) \hat{x}_i(\theta_i = 0)] di} \right)^{1-\alpha} \quad \text{if } s = \beta, \quad (35)$$

$$R = \alpha \left(\frac{1 - \beta}{F + \int [(1 - \beta) \hat{x}_i(\theta_i = 1) + \beta \hat{x}_i(\theta_i = 0)] di} \right)^{1-\alpha} \quad \text{if } s = 1 - \beta, \quad (36)$$

and combining (35) and (36) implies

$$(2\beta - 1)F = (1 - 2\beta) \int \hat{x}_i(\theta_i = 0) di.$$

This condition is fulfilled with $F = \int \hat{x}_i(\theta_i = 0) di = 0$, which contradicts the assumption that investment funds invest. For $F > 0$ it implies $\int \hat{x}_i(\theta_i = 0) di < 0$, but negative investments are impossible, thus contradicting feasibility. \square

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