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The effect of fiscal policy and forward guidance with preferences over wealth

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Abstract

I examine how the effects of fiscal policy and forward guidance are shaped by preferences over wealth calibrated based on microeconomic evidence on household saving behavior and individual discount rates in a stylized and a medium scale DSGE model. This assumption effectively limits the horizon of unconstrained households. Therefore, the contractionary effect of a permanent cut in government expenditure implemented during a period when monetary policy is constrained by the zero lower bound becomes larger due to smaller crowding in of unconstrained household consumption. Furthermore, the assumption much reduces the effect of forward guidance on the future policy interest rate.

JEL Codes: E52; E62; E32. Keywords: Forward guidance, Fiscal consolidation, finite horizon, zero lower bound.

1 Introduction

The manner in which households and firms respond to the future stance of fiscal and monetary policy is at the heart of recent macroeconomic debates. More specifically, in standard models, a fiscal contraction enacted during a period when monetary policy is constrained by the zero lower bound has very different effects on private consumption depending whether it is anticipated to be temporary or permanent (e.g. Woodford (2011), Denes et al. (2013)). A temporary fiscal contraction lasting as long as the period of constrained monetary policy reduces consumption by lowering inflation and thus increasing the real interest rate. By contrast, a permanent fiscal contraction tends to crowd in the consumption of forward looking households, as it entails the promise of a period of looser-than-otherwise monetary policy after the exit from the zero lower bound and a permanently lower tax burden in the new steady state the economy will ultimately converge to. Another topical example illustrating

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the role of expectations is the effect of forward guidance by the central Bank regarding the path of the future policy interest rate. Credible announcements to keep the policy rate “lower for longer” can overcome the zero lower bound constraint (e.g. Eggertsson and Woodford (2003)). As shown by Del Negro et al. (2015) and Carlstrom et al. (2015), the effects of such policies in estimated DSGE models are very strong and bigger than suggested by the available empirical evidence. A related finding is that the impact of forward guidance increases exponentially in the length of the forward guidance episode. A key factor behind these results is the consumption response of forward looking households.

However, the standard infinite horizon model of the consumer embedded in most DSGE models is at odds with the micro evidence on the inter-temporal choices of high income households, who would seem to be the natural real world counterpart of “Riccardian” households with their unconstrained consumption smoothing opportunities. Firstly, as has been shown by Carroll (2000), the standard model under predicts the saving of high income households relative to their permanent income. More specifically, the marginal propensity to save out of an increase in their permanent income is zero in the model, but estimated by Dynant et al. (2004) and Kumhof et al. (2014) to lie between 0.25 and 0.5 depending on the specific income bracket considered. Secondly, the micro evidence on individual discount rates typically estimates them to be substantially higher than market interest rates relevant for the inter temporal choice under examination by the researcher, even for income rich and highly educated individuals (e.g. Harrison et al. (2002) and Warner and Pleeter (2001)). This discrepancy cannot be accommodated by the standard model, where the individual discount rate applied to future nominal income streams always equals the inverse of the (gross) nominal interest rate, implying that financial market exactly compensate the household’s impatience.

This paper examines the macroeconomic effects of fiscal policy and forward guidance at the zero lower bound in a framework where consumption smoothing households derive utility from accumulating wealth in the form of safe assets. Carroll (2000), Francis (2009) and Kumhof et al. (2014) show that such a “capitalist spirit” assumption allows the infinite horizon model to replicate the saving behavior of rich households, while Fisher (2015) appeals to liquidity preference as an additional motivation. Furthermore, by generating a marginal benefit of saving over and above increasing future consumption, this assumptions also allows the equilibrium market interest rate to fall short of the individual discount rate households apply to future income streams, thus introducing discounting into the linearized Euler equation. I first embed these preferences in a linear fashion in the small, analytically tractable New Keynesian model used by Woodford (2011) and calibrate them based on the aforementioned microevidence. As a result, the effect of temporary and permanent fiscal contractions in the presence of the zero lower bound become much more similar than in the absence of preferences over wealth. A permanent contraction in government consumption causes a smaller crowding in of private consumption as households discount the decline in their future tax burden more heavily. Furthermore with preferences over wealth, announcements of the Central Bank regarding the path of the future policy rate have much smaller and for reasonable calibrations increases linearly in the length of the forward guidance episode.

I then embed these preferences into a medium scale DSGE model with capital

accumulation, sticky prices and wages and credit constrained households and firms. In this richer model I also allow for curvature in the utility derived from wealth, which I calibrate such that the marginal propensity to save out of an increase in their permanent matches the aforementioned micro evidence. Regarding the impact of fiscal policy, the main results of the simple model carry over to the medium scale model. An additional result absent from the simple model is that with preferences over wealth, adding credit constraints has a much bigger impact on the contractionary effect of permanent government spending shocks. Without preferences over wealth, adding credit constrained households and firms causes an even bigger crowding in of unconstrained household consumption than without constrained agents, triggered by a stronger monetary loosening during the transitional phase following the economy's exit from the ZLB. This offsetting behavior by unconstrained households considerably dilutes the effect of adding the credit constraints in the first place. By contrast, with preferences over wealth, this offsetting movement of unconstrained household consumption is essentially absent as unconstrained households are less sensitive to real interest rate movements in the distant future.

Furthermore, as in the simple model, adding preferences over wealth substantially reduces the impact of forward guidance regarding the future path of the policy rate. With curvature in the preferences over wealth, the model even comes close to matching the empirical evidence of Del Negro et al. (2015). Curvature matters on top of discounting in the Euler equation because forward guidance is associated with a decline in unconstrained household real wealth, which with curvature implies an increase in the marginal benefit of saving and thus dampens the consumption increase associated with the forward guidance policy. Hence it seems that adding preferences over wealth are a possible solution to the "Forward Guidance puzzle".

To my knowledge, this paper is the first to analyze the effect of permanent fiscal policy changes at the zero lower bound in a model where the horizon of unconstrained forward looking household is effectively limited by assuming preferences over wealth informed by micro evidence on the intertemporal choices of households. This modification of household preferences also represents an at least partially novel approach to addressing the forward guidance puzzle. While Fisher (2015) and Campbell et al. (2016) also employ the preference specification I use to introduce discounting to the linearized consumption Euler equation, they find that this assumption makes only a minor difference for their simulated effect of forward guidance. Part of the explanation might be that their calibration of household preferences over wealth is based on matching targets obtained from the national accounts rather than micro evidence, implying less discounting in the Euler equation than my calibration. Furthermore, in their model unconstrained household wealth and thus the marginal utility of saving is constant as there are no public or private sector borrowers, implying that curvature in household preferences over wealth is irrelevant for macroeconomic dynamics. At the same time, their estimated wage and price markup coefficients are very small, the estimated degree of habit formation is high and the wealth effect on labor supply is largely absent, all of which would tend to reduce the effect of forward guidance even in the absence of preferences over wealth. Other contributions attempting to solve the forward guidance puzzle by introducing discounting into the linearized consumption Euler equation comprise Del Negro et al. (2015), who replace the infinite horizon assumption with a Blanchard-Yaari type perpetual youth structure, and McKay et

al. (2015), who drop the complete markets assumption.

Finally, Lemoine and Linde (2016) examine the effect of permanent government spending cuts in a monetary union under imperfect credibility regarding the spending cut’s duration. This approach represents an alternative mechanism limiting the crowding in of unconstrained household consumption and thus increasing the cost of permanent spending cuts if monetary policy is constrained.

In the next section, I develop a stylized analytically tractable New Keynesian model with linear preferences over wealth and credit constrained households, discuss the micro evidence on individual discount rates I use to parameterize the weight on wealth in the utility function, and thus the extend of discounting in the linearized consumption Euler equation, and analyze the effects of changes in government consumption and forward guidance policies at the zero lower bound. In section 3, I address the same questions in a richer medium scale model, which on top of many standard features of quantitative models allows for curvature in preferences over wealth.

2 A stylized model with preferences over wealth and constrained households

2.1 Households

The economy features a fraction $(1 - \omega)$ of households who participate in credit markets and are thus able to smooth consumption inter-temporally by trading a safe bond B_t , and a fraction ω which consumes only their disposable income. Household j derives utility from consumption and the possession of safe government bonds, and disutility from supplying labor. Her objective is given by

$$\sum_{i=0}^{\infty} \beta^i \left[\frac{C_{j,t+i}^{1-1/\sigma_H}}{1-1/\sigma_H} - \chi N_{j,t+i}^{1+\eta} + \phi_B \left(\frac{B_{j,t+i}}{P_{t+i}} \right)^{1-\sigma_B} \right] \quad (1)$$

The budget constraint of unconstrained households is given by

$$B_t + P_t C_{O,t} = R_{t-1} B_{t-1} + W_t N_{O,t} - P_t T_{O,t} + P_t \Xi_t \quad (2)$$

where P_t , $C_{O,t}$, R_t , W_t , $N_{O,t}$, $T_{O,t}$ and Ξ_t denote the price level, the consumption basket, the nominal interest rate on bonds (which is also the policy rate set by the central bank), the nominal wage, labor supply, real lump sum taxes and real profits of firms, respectively, while the subscript O refers to unconstrained (“optimizing”) households. Throughout the paper I adopt the convention that only period t decision variables are indexed with t, implying that B_t denotes the stock of safe bonds at the end of period t. This convention also allows to drop the expectations operator on period $t + 1 + j$ variables. Preferences over wealth have been found useful, or indeed necessary, to explain a range of phenomena, the most conventional example being liquidity preference used to explain the presence of (usually non-interest bearing) money in agents portfolios. Krishnamurthy and Vissing Jorgenson (2012) argue that liquidity preference may extend to assets with a positive yield if they have money-like qualities, and argue that preferences over such assets explain the demand curve for

US treasuries. Following KVJ, Fisher (2015) argues that the “risk premium shock” in the estimated DSGE model of Smets and Wouters (2007) can be interpreted as shocks to the demand for safe and liquid assets (i.e. shocks to ϕ_B). Carroll (2000) argues that “capitalist spirit” type preferences over wealth are necessary to explain the saving behavior of rich households in US data. Specifically, the standard life-cycle model substantially under predicts the amount of wealth rich households hold relative to (estimates of) their permanent income and their propensity to save out of permanent income changes, as is also argued by Dynant et al. (2004). Francis (2009) shows that preferences over wealth can replicate this phenomenon. Kumhof et al. (2014) build on this literature in order to link the increase in US inequality, household leverage and financial fragility in the decades before the 2008-2009 financial crisis.

The FOCs of unconstrained households with respect to consumption and bonds are given by

$$\Lambda_{O,t} = \beta \frac{R_t}{\hat{\Pi}_{t+1}} \Lambda_{O,t+1} + \phi_B \left(\frac{B_{O,t}}{P_t} \right)^{-\sigma_B} \quad (3)$$

$$\Lambda_{O,t} = C_{O,t}^{-1/\sigma_H} \quad (4)$$

where $\Lambda_{O,t}$ denotes the real marginal utility of consumption. Note that (3) differs from the standard consumption Euler equation due to the presence of $\phi_B \left(\frac{B_{O,t}}{P_t} \right)^{-\sigma_B}$. Linearizing and combining equations (3) and (4) yields

$$\hat{C}_{O,t} = -\theta \sigma_H \left[\hat{R}_t - \hat{\Pi}_{t+1} \right] + \theta \hat{C}_{O,t+1} + (1 - \theta) \sigma_H \sigma_B \hat{b}_{O,t} \quad (5)$$

where $b_{O,t} = \frac{B_{O,t}}{P_t}$ a hat on top of a variable denotes the percentage deviation of that variable from the non-stochastic steady state, with $\theta = \beta \frac{R}{\Pi}$, i.e. the product of the steady-state household discount factor and the real interest rate. For the remainder of this section, I will assume linear preferences over wealth ($\sigma_B = 0$), in order to keep the model analytically tractable. Iterating (5) until period $t+n$ yields

$$\hat{C}_{O,t} = \sum_{i=0}^n -\theta^i \sigma_H \left[\hat{R}_{t+i} - \hat{\Pi}_{t+1+i} \right] + \theta^n \hat{C}_{O,t+1+n} \quad (6)$$

θ may be interpreted as the equilibrium weight the household attaches to period $t+1$ consumption, i.e. the net effect of utility discounting and the (steady state) market real interest rate. In the absence of preferences over wealth, $\theta = 1$, implying that the equilibrium weight of an additional unit of future consumption -no matter how far removed from the present- equals one. Hence a permanent increase in government consumption will reduce (increase) current consumption (the current marginal utility of consumption) of optimizing households to the same extent that it reduces (increases) their future (marginal utility of) consumption, no matter how distant the point in time when the new steady state will be reached. Future changes in the real interest rate are equally powerful no matter how far away from period t they are located. By contrast, with preferences over wealth ($\theta < 1$), the equilibrium weight the

the household attaches to future periods declines as they are pushed further into the future. Thus future events will have a smaller effect on the choices of the household in period t , the more so the further away in time they are located. Even though the household maximizes over an infinite number of periods, her effective horizon is thus finite.

Constrained households are assumed to have identical preferences over consumption and labor but preference over wealth, and are unable to save or borrow, implying that their consumption is given by

$$C_{ROT,t} = w_t N_{ROT,t} - T_{ROT,t} \quad (7)$$

where we use the subscript ROT (=Rule of Thumb) to denote constrained households. Their real marginal utility of consumption is given by

$$\Lambda_{ROT,t} = C_{ROT,t}^{-1/\sigma_H} \quad (8)$$

2.2 Wage setting

I assume that there is a continuum of unions which recruit the labor of both household types according to their share in the population, and transforms their labor into a specific variety i , which forms part of a CES labor basket employed by firms. The union operates under monopolistic competition and sets a joint wage for both household types. The demand curve for labor variety i is given by

$$N(i)_t = \left(\frac{w_t(i)}{w_t} \right)^{-e_w} N_t$$

where $w_t = \frac{W_t}{P_t}$ and $e_w > 1$ denotes the demand elasticity for type i labor. Due to the assumption of identical preferences over consumption and labor across household types, we can write the union's problem as maximizing

$$w_t(i) \Lambda_t N(i)_t + \chi \frac{N(i)_t^{1+\eta}}{1+\eta}$$

subject to

$$N(i)_t = \left(\frac{w_t(i)}{w_t} \right)^{-e_w} N_t$$

where

$$\Lambda_t = \omega \Lambda_{ROT,t} + (1 - \omega) \Lambda_{O,t} \quad (9)$$

The union's FOC is thus given by

$$w_t \Lambda_t = \chi N_t^\eta \mu_w \quad (10)$$

where $\mu_w > 1$ denotes the markup of the real wage over the aggregate marginal rate of substitution of both household types.

2.3 Retailers

There is a continuum of monopolistically competitive firms owned by unconstrained households which each produce a variety j from a CES basket of goods. They set prices subject to nominal rigidities in the form of Calvo (1983) pricing, i.e. a fraction ξ_p of firms is unable to reset their price in a given period. The production technology of firm j is given by

$$Y_t(j) = N_t^\alpha(j) \quad (11)$$

with $\alpha \leq 1$, implying that their real marginal cost mc_t is denoted as

$$mc_t = \frac{w_t}{N_t^{\alpha-1}} \quad (12)$$

These assumptions imply that up to first order, inflation evolves according to the familiar New Keynesian Phillips Curve

$$\hat{\Pi}_t = \frac{(1 - \xi_p)(1 - \beta\xi_p)}{\xi_p} \hat{m}c_t + \beta\hat{\Pi}_{t+1} \quad (13)$$

2.4 Government

The government levies taxes and buys goods from retailers. Its budget constraint is given by

$$B_t = (1 + R_t) B_{t-1} + P_t G_t - P_t ((1 - \omega) T_{O,t} + \omega T_{ROT,t}) \quad (14)$$

For simplicity, I assume that in the steady state, government debt is zero, taxes on unconstrained households $T_{O,t}$ exactly equal the profits of firms, and taxes on constrained households are zero. This assumption requires that the share of government expenditure in GDP equals the profit share.¹ As a consequence, the steady state consumption level is identical across household types. Furthermore, I assume that government solvency is guaranteed by varying taxes on unconstrained households only, implying that the evolution of government debt does not matter for macroeconomic dynamics.

However, I assume that an increase in government consumption may be partially funded by taxing credit constrained households, i.e.

$$\phi_{ROT,S} dG_t = dT_{ROT,t} \quad (15)$$

where $\phi_{ROT,t}$ denotes the per-capita increase in taxes on credit constrained households per unit of dG_t . $\phi_{ROT,t}$ has a t index since I will below allow $\phi_{ROT,t}$ to vary exogenously.

Monetary policy is described by a simple rule where the Central Bank responds to inflation and the deviation of output from its flexible price level $\Gamma \hat{G}_t$, which for a given actual output level depends positively on government consumption as a consequence of the wealth effect on labor supply.

¹Hence $\frac{G}{Y} = \left(1 - \frac{1}{\mu_P}\right)$, where μ_P denotes the price markup.

$$\hat{R}_t = \max \left(\phi_\pi \hat{\Pi}_t + \phi_y \left(\hat{Y}_t - \Gamma \hat{G}_t \right), -i \right) \quad (16)$$

where i denotes the steady state nominal interest rate.

2.5 Equilibrium

Total consumption is the sum of constrained and unconstrained household consumption:

$$C_t = (1 - \omega)C_{O,t} + \omega C_{ROT,t} \quad (17)$$

GDP is the sum of household and government consumption

$$Y_t = C_t + G_t \quad (18)$$

2.6 Linearized equations

Linearizing and combining the above equations allows to express the model in three equations (for details see the appendix):

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \theta \tilde{\sigma} \left(\hat{R}_t - \hat{\Pi}_{t+1} - r_t^{net} \right) + \sigma_{g,t} \hat{G}_t - \theta \sigma_{g,t+1} \hat{G}_{t+1} \quad (19)$$

$$\hat{\Pi}_t = \kappa (\hat{Y}_t - \Gamma \hat{G}_t) + \beta \hat{\Pi}_{t+1} \quad (20)$$

$$\hat{R}_t = \max \left(\phi_\pi \hat{\Pi}_t + \phi_y \left(\hat{Y}_t - \Gamma \hat{G}_t \right), \hat{R}_L \right) \quad (21)$$

The meaning of the reduced form coefficients can be obtained from table 1. In the absence of constrained households ($\omega = 0$), and without preferences over wealth ($\theta = 1$), the model collapses to the familiar New Keynesian three equation model (i.e. $\tilde{\sigma} = \sigma$ and $\sigma_{g,t} = 1$). By contrast, for $\omega > 0$ and debt financing or taxation of unconstrained households only ($\phi_{ROT,t} = 0$), the model features a “Keynesian multiplier effect” of the type observed in the simple Keynesian income-expenditure model ($\sigma_{g,t} > 1$). For a given real interest rate $\hat{R}_t - \hat{\Pi}_{t+1}$, an increase in government spending \hat{G}_t increases the consumption of constrained households due to higher employment and wages, implying that the overall increase in GDP exceeds the increase in government consumption. Furthermore, for $\omega > 0$, GDP also becomes more sensitive to the real interest rate. Intuitively, for $\omega > 0$ a decline in the real interest raises not only unconstrained household consumption but also constrained household employment and disposable income. As constrained household spend all of their disposable income, the total increase in consumption and GDP is typically higher than in their absence.²

The presence of \hat{G}_t in the monetary policy rule reflects the fact that in the flexible price economy, an increase in government expenditure crowds out private consumption, which increases labor supply. Finally, r_t^{net} denotes a consumption preference shock, i.e. a shock to the natural rate of interest.

²For $\tilde{\sigma} > \sigma$ to hold, one needs $\frac{C}{Y} \left(\frac{1}{\sigma_H} + \frac{1+\eta}{\alpha} \right) > 1$. For $\sigma_H \leq 1$ and reasonable calibrations for $\frac{C}{Y}$, this condition will generally met for permissible values of the other parameters.

Table 1: Reduced form coefficients of the stylized model

σ	$\tilde{\sigma}$	$\sigma_{g,t}$	β	Γ	κ	η_v
$\sigma_H \frac{C}{Y}$	$\frac{\sigma(1-\omega)}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$	$\frac{\left(1-\frac{\omega C}{\sigma Y}\right)-\omega\phi_{ROT,t}}{\left(1-\frac{\omega C}{\sigma Y}-\frac{C}{Y}\frac{\omega(1+\eta)}{\alpha}\right)}$	$\frac{\theta}{R/\Pi}$	$\frac{\frac{1}{\sigma}}{\frac{1}{\sigma}+\eta_v}$	$\frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \left(\frac{1}{\sigma} + \eta_v\right)$	$\frac{1-\alpha+\eta}{\alpha}$

2.7 Calibration

Regarding the standard parameters (Table 2), I follow Woodford (2011). The assumed target for $\frac{R}{\Pi}$ implies that for $\theta = 1$, β is as in Woodford (2011). For the share of constrained households, I consider two values, namely 0 and 0.15. I assume that monetary policy outside the ZLB is described by the standard Taylor rule coefficients.

Table 2: Parameters in the stylized model

σ	η	$\frac{R}{\Pi}$	α	Γ	ω	$\frac{C}{Y}$	ϕ_y	ϕ_π	μ	θ
0.8621	1.6	1/0.997	1	0.4203	0; 0.15	0.8	0.5/4	1.5	0.875	0.96 – 1.0

For θ , I consider values between 0.96 and 1. This calibration is based on a large literature attempting to estimate the personal discount rate from micro data. Note that the bond Euler equation may be rearranged as

$$1 - \frac{\phi_B \left(\frac{B_{O,t}}{P_t}\right)^{-\sigma_B}}{\Lambda_{O,t}} = R_t \beta \frac{\Lambda_{O,t+1}}{\hat{\Pi}_{t+1} \Lambda_{O,t}}$$

or

$$1 - \frac{\phi_B \left(\frac{B_{O,t}}{P_t}\right)^{-\sigma_B}}{\Lambda_{O,t}} = \frac{1 + i_t}{1 + d_t} \quad (22)$$

where $i_t = R_t - 1$ and $1 + d_t = \beta \frac{\Lambda_{O,t+1}}{\hat{\Pi}_{t+1} \Lambda_{O,t}}$ denotes the -time varying- stochastic discount factor the household applies to nominal $t + 1$ income. In the steady state, we have $1 - \frac{\phi_B}{\Lambda_O} = \beta \frac{R}{\Pi} = \theta$. However, for θ close 1, $\frac{1+i_t}{1+d_t}$ represents a good approximation of θ even outside the steady state.³ Therefore, even for large deviation of consumption or bonds from their respective steady states, we have

$$\theta \approx \frac{1 + i_t}{1 + d_t} \quad (23)$$

Hence θ may be estimated using an estimate of the personal discount rate and an appropriate market interest rate. Economists have attempted to estimate the personal discount rate at least since Friedman's (1957) seminal tests of the permanent income hypotheses by studying economic agents behavior when faced with a variety

³A linear approximation of the left hand side of equation (22) yields $\frac{\phi_B \left(\frac{B_{O,t}}{P_t}\right)^{-\sigma_B}}{\Lambda_O} \left(\hat{\Lambda}_{O,t} + \sigma_B \hat{b}_{O,t}\right) = (1 - \theta) \left(\sigma_B \hat{b}_{O,t} - \frac{1}{\sigma_H} \sigma_H \hat{C}_{O,t}\right)$. Hence for $1 - \theta$ close to zero and reasonable calibrations of σ_H and σ_B even large deviations of $\hat{C}_{O,t}$ and $\hat{b}_{O,t}$ would only slightly bias the estimate of θ .

of inter temporal trade-offs (see Table 3). These range from trading off the energy efficiency and price of household appliances (Ruderman et al. (1984)) to the effects of paying bonuses (Cylke et al. (1982)) or severance packages (Warner and Pleeter (2001)) as a lump sum instead of installments, as well as field experiments where probants choose between a payment and a higher deferred payment (Harrison et al. (2002)). As can be obtained from Table 3, the elicited discount rates are quite high, although typically below the estimate of 33% of Friedman (1962,1957). What is more, they also typically exceed safe market interest rates on safe investments with a comparable maturity observed at the time the discount rates were elicited, implying that the implied value of θ is smaller than one, sometimes substantially so. Since I interpret the optimizing households as “rich” households, the contributions of Harrison et al. (2002) and Warner and Pleeter (2001) are of particular relevance. Harrison et al. (2002) report estimates for (income-) rich households, while Warner and Pleeter’s (2002) elicit discount rate of officers of the United States armed forces choosing between two severance packages during the 1992-1995 military draw-down.⁴ My calibration for θ is thus clearly at the upper end of what is implied by the available evidence.

⁴The authors report that virtually all of the officers in their sample have a college degree, while according to the Current Population survey the same was true for only 24.5% of individuals in the same age group.

2.8 Determinacy conditions and fiscal multipliers

As shown in the appendix, as long as monetary policy is not constrained by the zero lower bound and is not expected to be so in the future, the necessary and sufficient condition for a unique and stable equilibrium is given by the following proposition:

Proposition 1: Consider an economy described by equations (19) to (21). Then the necessary and sufficient condition for a unique and stable equilibrium is given by $\frac{\phi_y(1-\beta)}{\kappa} + \phi_\pi > 1 - \frac{(1-\beta)(1-\theta)}{\theta\kappa\tilde{\sigma}}$. Proof: see Appendix 8.2.

For $\theta = 1$, the condition collapses to the familiar requirement that to ruling out sunspot equilibria requires an active monetary policy, i.e. the real interest rate has to rise if inflation increases (e.g. Woodford (2003)). For $\theta < 1$, the economy becomes less prone to expectation driven fluctuations, as the right hand side of the equation becomes smaller as θ and thus β decline, as expectations of future real interest rates now have an exponentially decreasing effect on aggregate demand (see equation 19). However, I restrict the discussion below to calibrations of the monetary policy rule which yield a unique and stable equilibrium for all values of θ considered.

Assuming that the share of the increase in government expenditure funded via taxes on credit constrained households is fixed, the fiscal multiplier is determined as follows:

Proposition 2: Consider an economy described by equations (19) to (21), with government spending described by $\hat{G}_t = \rho\hat{G}_{t-1}$, with $\rho < 1$. Assume also that the tax burden of ROT households per unit of $d\hat{G}_t$ remains fixed at $\phi_{ROT,t} = \phi_{ROT} \geq 0$. Assume also that the determinacy condition given of Proposition 1 holds. The minimum state variable solutions for output and inflation are given by

$$\hat{Y}_t = \gamma_y \hat{G}_t$$

$$\hat{\Pi}_t = \gamma_\pi \hat{G}_t$$

with

$$\gamma_y = \frac{\sigma_g(1 - \rho\theta) + \Gamma\tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)}\right)}{(1 - \rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)}\right)}$$

$$\gamma_\pi = \frac{\kappa(1 - \rho\theta)(\sigma_g - \Gamma)}{(1 - \rho\theta) + \tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)}\right)}$$

Proof: See Appendix 8.3.1.

In the absence of constrained households ($\sigma_g=1$), $\gamma_y < 1$ as an increase in government spending increases flexible price output by less than one for one ($\Gamma < 1$). Furthermore, the fiscal multiplier decreases in the persistence of the government spending shock ρ , the slope of the Phillips curve κ , the hawkishness of monetary policy (ϕ_y and ϕ_π) and θ . For a permanent government spending increase and no preferences over wealth ($\rho = 1$ and $\theta = 1$), $\gamma_y = \Gamma$, i.e. the increase in GDP corresponds exactly to the increase in flexible price output caused by the wealth

Table 3: Empirical evidence on θ

Sample period	d (APR)	i_t (APR)	Implied θ	Source of D; R used for comparison	Estimate of D based on
1929-1948	33.0*	0.8*	0.82	Friedman (1962,1957); real treasury maturity ≥ 10 years	Tests of PIH
1960	19.6*	2.0*	0.96	Heckman (1976); real 10 year treasury	Estimated life cycle earnings model
1979	27.4	9.5	0.96	Cylke et al. (1982); 5 year treasury	US Military reenlistment decisions
1972; 1978; 1980	54.7; 64.0; 72.1*	2.9; 1.9; 2.8*	0.9; 0.89; 0.88	Ruderman et al. (1984), Median; 10 year real treasury	Price of household appliances
1982-1989	18.3	8.6	0.98	Ausubel (1991); one month certificate of deposit	US credit card interest rates
1992-1993	18.7	6.3	0.97	Warner and Pleter (2001), 20 year treasury	US officers severance package choices
1996	22.5	4.2	0.96	Harrison et al. (2002); 1 year money market rate	Experiment, income rich households

Notes: If information on the horizon of the choice of the agent under observation was available, i_t is the safe (e.g. government) interest rate with a maturity as close as possible to this horizon during the year the decision was made. In most other cases, I use the 10 year government bond yield. Numbers marked with a * are estimates of the *real* personal discount rate. The corresponding R I use to compute θ is therefore a measure of the real interest rate, where expected inflation is assumed to equal the average CPI inflation rate over the preceding 10 years. In case of Friedman (1962, 1957), I calculated the relevant i_t as the difference between the average interest rate on long term government bonds (maturity 10 years or more, the only long term government bond series for this period I am aware of) over this period, and the average PCE deflator inflation rate. Finally, Ausubel's (1991) investigation of the US market for credit cards is frequently cited as evidence in favor of high personal discount rates. In his sample, more than three quarters of customers holding credit cards incur finance charges on substantial outstanding balances in spite of credit card interest rates ranging between 18 and 19%, and cites industry publications saying that about 90% of an issuers outstanding balance accrue interest.

effect on labor supply, implying that inflation remains is ($\gamma_\pi = 0$). Intuitively, for a given household income, the increase in government consumption reduces private consumption one-for one, which however expands labor supply and GDP, implying that in general equilibrium private consumption declines by less. With $\theta < 1$, $\gamma_y > \Gamma$ and $\gamma_\pi > 0$, households discount the higher marginal utility associated with a unit of future consumption more heavily compared to the market interest rate, implying that even for a given expected path of their income, their consumption declines less than one for one. As a result, the real interest rate increases permanently.

I now assume that an exogenous decline in the natural rate of interest hits the economy, i.e. r_t^{net} takes a negative value r^L , with the superscript L denoting the “low state” of the economy, following Eggertsson (2008) and Woodford (2011). This decline in the natural rate is sufficiently big to reduce the policy rate to its lower bound (if that bound is zero, $\hat{R}_L = \frac{1-R}{R}$). Furthermore, with probability μ , r_t^{net} will continue to equal r^L in the following quarter, while with probability $(1 - \mu)$, it will return to zero, and is expected to remain there forever. If the determinacy condition in proposition 1 is met, the condition for determinacy in the low state is given by:

Proposition 3: Consider an economy described by equations (19) to (21), and assume that r_t^{net} takes a value r^L sufficiently negative for $\hat{R}_L = -i$. Assume further that $r_{t+1}^{net} = r^L$ with probability μ , and $r_{t+1}^{net} = 0$ with probability $(1 - \mu)$, and that once $r_{t+i}^{net} = 0$, it is expected to remain there forever after. Then the necessary and sufficient condition for a unique and stable equilibrium is given by $(1 - \mu\beta)(1 - \theta\mu) > \theta\tilde{\sigma}\kappa\mu$. Proof: see Appendix 9.1.

For $\theta = 1$, this condition is identical to the one given in Woodford (2011). For $\theta < 1$, the economy becomes less prone to expectation driven fluctuations, as the left hand side of the equation increases and the right hand side declines as θ and thus β decline.

I now assume that during the low state, the government may increase spending by a percentage \hat{G}_L of steady state output. After the end of the low state, the government will revert its spending to zero with a probability $1 - \lambda$ and keep spending at \hat{G}_L with probability λ . I denote this transitional state in which the economy has exited the ZLB but government spending remains elevated as S . Furthermore, I assume that the share of \hat{G}_L funded by taxes on constrained households remains fixed within the two states, i.e. $\phi_{ROT,t} = \phi_{ROT,L}$ and $\phi_{ROT,t} = \phi_{ROT,S}$ during the low state and the transitional state, respectively.

Hence the economy in the low state is described by

$$\hat{Y}_L - \sigma_{g,L}\hat{G}_L = \theta(\mu\hat{Y}_L - \mu\sigma_{g,L}\hat{G}_L + \lambda(1-\mu)\hat{Y}_S - \lambda(1-\mu)\sigma_{g,S}\hat{G}_L) - \tilde{\sigma}\theta \left(\hat{R}_L - \mu\hat{\Pi}_L - (1-\mu)\lambda\hat{\Pi}_S - r_L \right) \quad (24)$$

$$\hat{\Pi}_L = \kappa(\hat{Y}_L - \Gamma\hat{G}_L) + \beta \left(\mu\hat{\Pi}_L + (1-\mu)\lambda\hat{\Pi}_S \right) \quad (25)$$

The effect of a change in government expenditure on GDP in the low state is thus determined as follows:

Proposition 4: Take the assumptions from proposition 3 and add that during the low state L $\hat{G}_t = \hat{G}_L$, while after the end of state L , $\hat{G}_t = \hat{G}_L$ with probability λ and $\hat{G}_t = 0$ with probability $(1 - \lambda)$. Furthermore, during state L $\phi_{ROT,t} = \phi_{ROT,L} \geq 0$

, and during state S , $\phi_{ROT,t} = \phi_{ROT,S} \geq 0$ respectively. Then GDP and inflation during the low state are described by

$$\hat{Y}_L = \gamma_{G_L Y_L} \hat{G}_L + \gamma_{r_L Y_L} (r_L - \hat{R}_L)$$

$$\hat{\Pi}_L = \gamma_{G_L \Pi_L} \hat{G}_L + \gamma_{r_L \Pi_L} (r_L - \hat{R}_L)$$

with

$$\gamma_{G_L Y_L} = \frac{(1 - \theta\mu)(1 - \beta\mu)\sigma_{g,L} - \kappa\theta\tilde{\sigma}\mu\Gamma + \tilde{\sigma}\theta(1 - \mu)\lambda\gamma_\pi + \theta\lambda(1 - \mu)(1 - \beta\mu)(\gamma_y - \sigma_{g,S})}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu}$$

$$\gamma_{\Pi_L} = \kappa \frac{(1 - \theta\mu)(\sigma_{g,L} - \Gamma) + \frac{\tilde{\sigma}\theta(1 - \mu)\lambda\gamma_\pi}{1 - \beta\mu} + \theta\lambda(1 - \mu)(\gamma_y - \sigma_{g,S})}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu} + \frac{\beta(1 - \mu)\lambda\gamma_\pi}{1 - \beta\mu}$$

with

$$\gamma_{r_L Y_L} = \frac{\tilde{\sigma}\theta(1 - \beta\mu)}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu}$$

$$\gamma_{r_L \Pi_L} = \frac{\kappa\tilde{\sigma}\theta}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu}$$

with γ_y and γ_π as determined in proposition 2 and $\rho = \lambda$. Proof: See Appendix 9.2.

I now examine the effect of an increase in government consumption on GDP in the low state under the calibrations given in Table 3. Throughout, I will assume that during the low state, taxes on constrained households remain fixed at zero ($\phi_{ROT,L} = 0$).

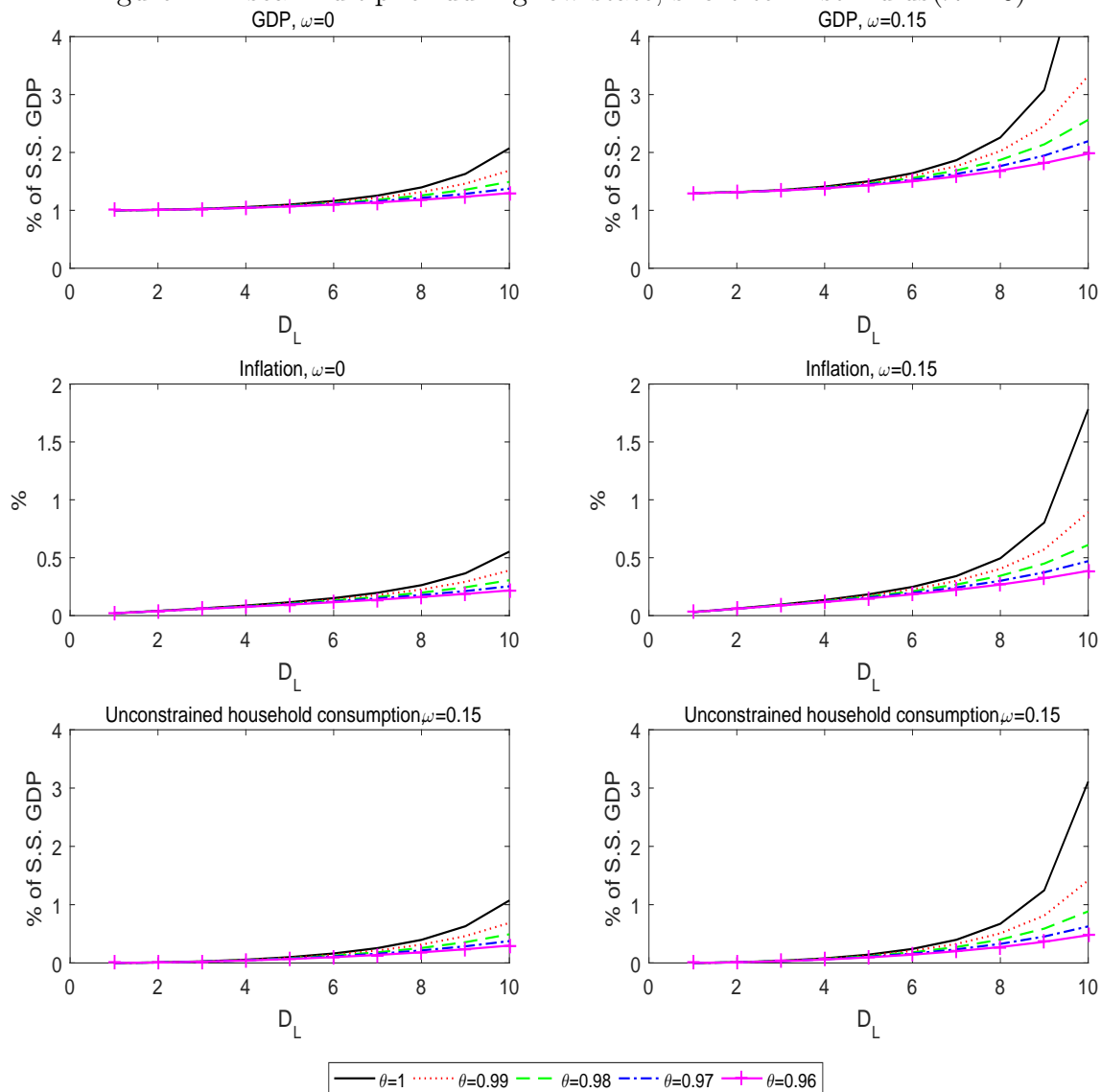
In case of a stimulus expected to last only as long as the economy's low state ($\lambda = 0$), no credit constraint households ($\omega = 0$ and thus $\sigma_g = 1$), zero probability of the low state persisting into the next period ($\mu = 0$) and no stimulus during the high state ($\lambda = 0$), the multiplier equals exactly 1. Intuitively, in the absence of credit constrained households, perfectly timed fiscal stimulus affects private consumption only via its effect on expected (as opposed to current) inflation and thus the real interest rate, but this effect is zero if the low state ends with certainty in the next period. If the low state is expected to persist with some probability ($\mu > 0$), $\gamma_{G_L Y_L} \geq 1$ (as $\Gamma < 1$), expected inflation increases and the (sum of future) real interest rate declines, which crowds in private consumption. As can be obtained from Figure 1, the multiplier monotonously increases in the expected duration of the low state D_L (with $D_L = \frac{1}{1 - \mu}$), exponentially so for high values of D_L . However, the increase in the multiplier is smaller for lower values of θ . For instance, for an expected duration of the low state of 10 quarters, and no constrained households ($\omega = 0$), the multiplier equals 2.1 without preferences over wealth, but only 1.3 for $\theta = 0.96$. Lower values of θ reduce importance of future real interest rates for current consumption of forward looking households (see equation (6)), and also reduces the effect of expected inflation rates in periods $t + 2$ and higher on period $t + 1$ inflation, as lowering θ also lower β for a given steady state real interest rate (see equation 20).

Adding credit constraint households to the model ($\omega = 0.15$) increases the fiscal multiplier for all values of μ and θ . Even if the economy exits the low state

in the next period with certainty ($\mu = 0$), a fiscal stimulus will crowd in private consumption by increasing the disposable income of credit constraint households via higher employment and a higher real wage, implying a multiplier larger than 1 (as $\sigma_{g,L} > 1$). If the low state is expected to persist with some probability ($\mu > 0$), the stronger stimulative effect of government spending thus translates into a stronger increase in expected inflation than without credit constrained households and thus a bigger decline in the real interest rate. Furthermore, in the presence of constrained households, GDP becomes more sensitive to the real interest rate, i.e. $\theta\tilde{\sigma}$ in equation (25) increases due to a larger $\tilde{\sigma}$.

As without constrained households, the effect of μ on the multiplier is smaller for $\theta < 1$. The observed attenuation of the relationship between μ and the multiplier is however much bigger. The larger interest rate sensitivity in the presence of constrained households $\theta\tilde{\sigma}$ due to the higher value of $\tilde{\sigma}$ implies that $\theta\tilde{\sigma}$ also declines more in θ than in the absence of constrained households. Furthermore, in the presence of constrained households, a given reduction in the real interest rate sensitivity due to a lower θ multiplies with a larger reduction in the real interest rate due to a bigger increase in inflation and output, as $\sigma_g > 1$.

Figure 1: Fiscal multiplier during low state, short term stimulus ($\lambda = 0$)



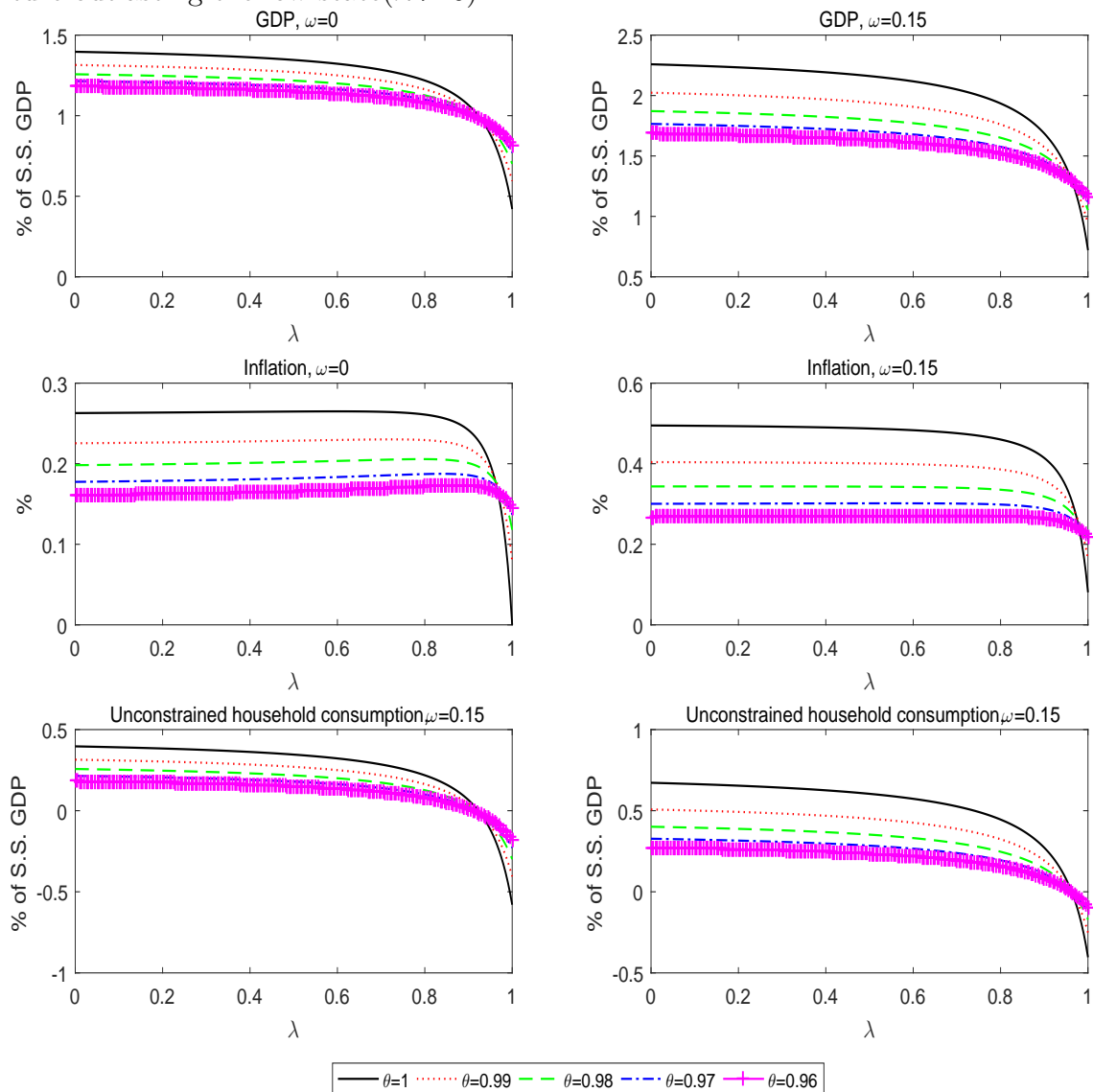
Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP during the low state only ($\lambda = 0$). GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. Taxes on credit constrained households are fixed at zero ($\phi_{ROT,L} = 0$). The horizontal axis depicts the expected duration of the low state $D_L = \frac{1}{1-\mu}$. All other parameters are as in Table 2.

I now examine the case where the increase in government expenditure is expected to persist beyond the duration of the zero lower bound (i.e. $\lambda > 0$). I assume that during this transitional state, the share of the government consumption increase funded via taxes on credit constrained households corresponds to their population share ($\phi_{ROT,S} = 1$). Extending the increase in government expenditure beyond the length of the zero lower bound lowers the fiscal multiplier (see Figure 2) for all values of θ considered, especially so once $\lambda > 0.8$. However, the negative effect of increasing λ is strongest in the absence of preferences over wealth ($\theta = 1$), as can be obtained

from the black solid line. For instance, in the absence of constrained households ($\omega = 0$) and $\lambda = 0, 0.8$ and 1 , the multiplier equals $1.4, 1.2$ and 0.4 , respectively. Intuitively, outside the zero lower bound, an increase in government expenditure causes an increase in the real interest rate via the central bank's reaction function, which lowers current consumption of unconstrained households, the more so the longer the increase in government spending and thus the period of high real interest rates is expected to last (see equation (6)). For a permanent increase in government expenditure ($\lambda = 1$), the consumption of forward looking households moves to a new lower steady state once the economy exits the zero lower bound, which for $\theta = 1$ affects current consumption one for one. By contrast, for $\theta < 1$, the multiplier is much less sensitive to increasing λ . For instance, for $\theta = 0.96$, values of $\lambda = 0, 0.8$ and 1 correspond to multipliers of $1.2, 1.1$ and 0.8 . The period of elevated interest rates outside the zero lower bound has a smaller effect on current household consumption, and a permanent increase in government spending lowers current consumption less than one-for-one.

In the presence of constrained households ($\omega = 0.15$), allowing for preferences over wealth attenuates the effect of λ on the multiplier even more. While for $\theta = 1$, the multiplier decreases from 2.3 to 0.7 as λ moves from zero to one, for $\theta = 0.96$, it decreases from 1.7 to 1.2 .

Figure 2: Fiscal multiplier during low state with the change in government expenditure outlasting the low state ($\lambda > 0$)



Note: Effects on output, unconstrained household consumption and annualized inflation of increasing government spending by 1% of GDP. GDP and unconstrained household consumption are expressed as a percentage of steady state GDP. The horizontal axis displays the probability λ that the stimulus persists after the economy's exit from the low state and thus the ZLB. The expected length of the low state is 8 quarters (i.e. $\mu = 0.875$). During the low state, taxes on credit constrained households are fixed at zero ($\phi_{ROT,L} = 0$). During the transitional state, the share of the government consumption increase funded via taxes on credit constrained households corresponds to their share in the population ($\phi_{ROT,S} = 1$). All other parameters are as in Table 2.

2.9 Forward guidance

I now turn to examine the effects of forward guidance regarding the path of the short term interest rate in the stylized model. Specifically, I assume that once the economy

exits the ZLB, the central bank nevertheless fixes the interest rate at a level \hat{R}_f . Furthermore, it will keep the interest rate at \hat{R}_f in subsequent periods with probability μ_f , and with probability $1 - \mu_f$ it will revert to its interest feedback rule. The effect of forward guidance on output and inflation in the low state is then given by

Proposition 4: Take the assumptions from proposition 3 and add that once the economy exits the low state, the central bank, with probability λ_f fixes the interest rate at a level \hat{R}_f . Conditional on fixing the interest rate in the first post-ZLB quarter, it will keep the quarterly interest rate at \hat{R}_f in subsequent periods with probability μ_f . With probability $1 - \mu_f$ it will revert to its interest feedback rule (Equation 21), and is expected to stick to this rule forever after. Output and inflation in the low state are then given by

$$\hat{Y}_L = \gamma_{R_f Y_L} \hat{R}_f + \gamma_{r_L Y_L} (r_L - \hat{R}_L)$$

$$\hat{\Pi}_L = \gamma_{R_f \Pi_L} \hat{R}_f + \gamma_{r_L \Pi_L} (r_L - \hat{R}_L)$$

with

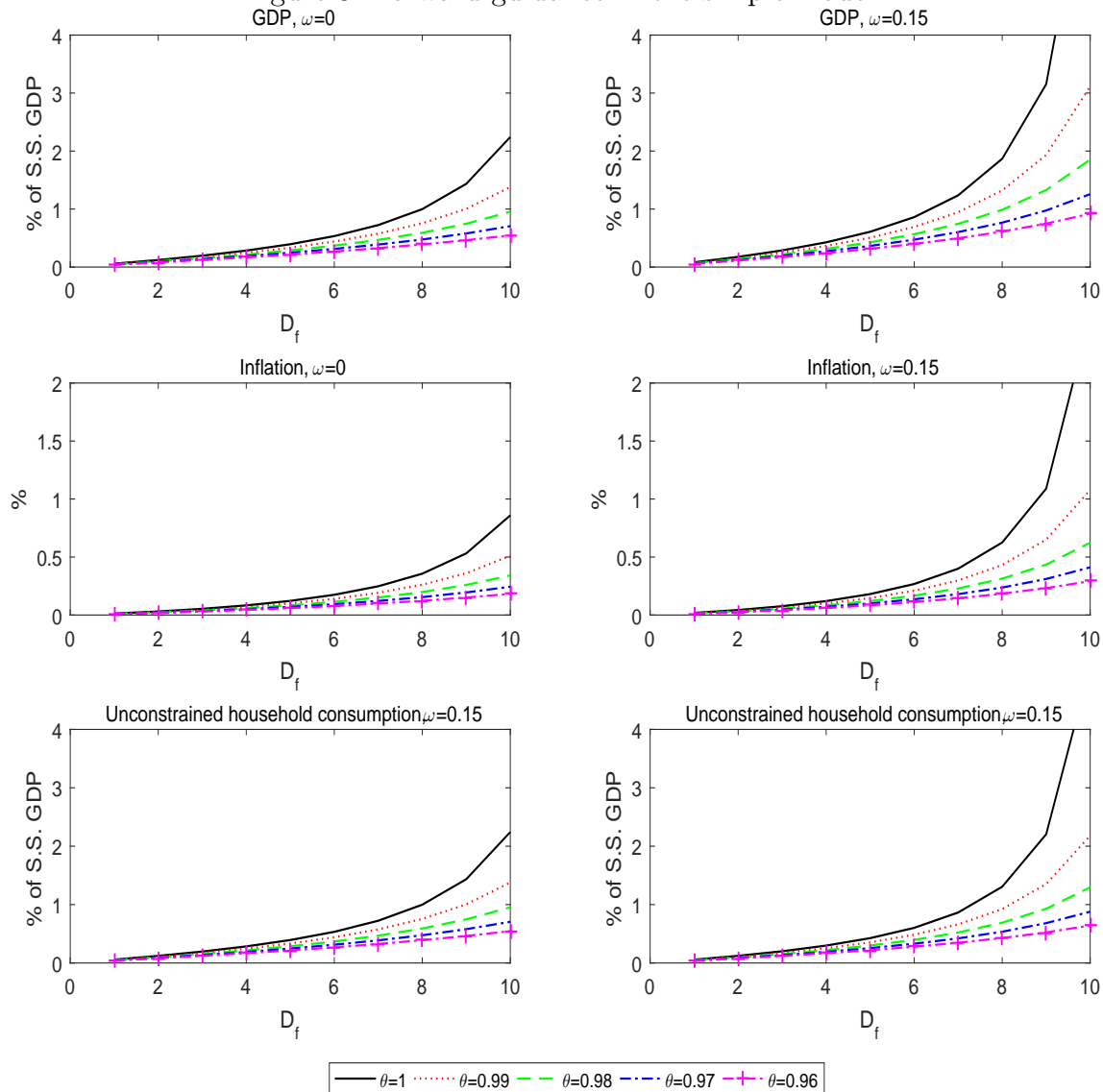
$$\gamma_{R_f Y_L} = - \frac{\tilde{\sigma} \theta^2 \lambda_f (1 - \mu) (\kappa \tilde{\sigma} + (1 - \beta \mu) (1 - \beta \mu_f))}{((1 - \theta \mu) (1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma}) ((1 - \theta \mu_f) (1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)}$$

$$\gamma_{R_f \Pi_L} = - \frac{(1 - \mu) \lambda_f \kappa \tilde{\sigma} \theta \left[\theta \left(\frac{\kappa \tilde{\sigma}}{1 - \beta \mu} + 1 - \beta \mu_f \right) + \beta \left(1 - \theta \mu - \frac{\mu \kappa \theta \tilde{\sigma}}{1 - \beta \mu} \right) \right]}{((1 - \theta \mu) (1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma}) ((1 - \theta \mu_f) (1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)}$$

To investigate the effect of increasing expected length of the forward guidance episode D_f (where $D_f = \frac{1}{1 - \mu_f}$), I assume an expected length of the low state D_L of 6 quarters and set $\lambda_f = 1$. These assumptions are consistent with the evidence provided by Del Negro et al. (2015) on financial market expectations regarding the timing of the exit of the Federal Funds rate from the ZLB prior to the forward guidance announcements of the US Federal Reserve in September 2011, January 2012 and September 2013, as well as their evidence on the announcement's effect on private sector forecasts of three month and 10 year treasury bonds. Furthermore, I assume that the central bank sets \hat{R}_f to an annualized value of -0.2%, which is also in line with their evidence.

As can be obtained from Figure 3, with no preferences over wealth, GDP and inflation in the low state increase exponentially in the expected length of the forward guidance episode D_f , especially so once there are some credit constrained households. This result mirrors the finding of Carlstrom et al. (2015), who analyze a deterministic forward guidance policy. However, in the presence of preferences over wealth ($\theta < 1$), the increase in GDP is much lower. The dampening of the GDP effect is especially large in the presence of credit constrained households. For $\theta = 0.96$, the GDP effect becomes approximately linear in the expected length of the forward guidance episode. With preferences over wealth and the implied effectively finite horizon, the low interest rates promised by forward guidance for the recovery period have a lower effect on current consumption than with an infinite horizon (see equation 6). Preferences over wealth thus seem to have the potential to alleviate the so called "Forward guidance puzzle" documented by Del Negro et al. (2015) and Carlstrom et al. (2015).

Figure 3: Forward guidance in the simple model



Note: Effects on output and annualized inflation in the low state of fixing the annualized interest rate at 0.2% below its steady state level once the economy has exited the low state. The horizontal axis depicts the expected length of the fixed interest rate policy D_f (where $D_f = \frac{1}{1-\mu_f}$). The expected length of the low state is 6 quarters (i.e. $\mu = 0.83$), and the Central Bank embarks on its fixed interest rate policy after the end of the low state with certainty ($\lambda_f = 1$). All other parameters are as detailed in Table 2.

3 The impact of preferences over wealth in a medium scale model

I now investigate whether the impact of preferences over wealth on the effects of fiscal policy and forward guidance carry over to a richer, quantitative model. Like the simple model of the previous section, it features constrained and rule of thumb

(or credit constrained) households. Unconstrained households save in the form of government bonds and safe deposits, and have preferences over wealth, where I examine both linear preferences and preferences with curvature. Both household types supply labor to retailers via a labor union, which sets a common wage for both. An entrepreneurial sector owns and accumulates the capital stock, who obtains external funds in the form of one period loans from a financial intermediary. The cost of external finance of entrepreneurs increases in their leverage due to a costly state verification (CSV) problem as in Bernanke et al. (1999). Entrepreneurs rent their capital stock to retailers. Retailers produce the homogeneous output good employing physical capital and labor in a Cobb Douglas technology. Perfectly competitive investment good producers owned by unconstrained households convert the output good into new capital goods and sell them to entrepreneurs. Investment goods producers are subject to investment adjustment costs, while retailers and unions are subject to nominal rigidities in the form of Calvo contracts. The full set of linearized equations can be obtained from Appendix 11. I estimate key parameters of the model by matching the impulse response functions of a VAR to a monetary policy and a government spending shock.

3.1 Financial accelerator

We will consider a version of the model with a financial accelerator along the lines of Bernanke et al. (1999). Risk neutral entrepreneurs accumulate the physical capital stock \widehat{K}_t and rent it to retailers. After the collection of rental income, the liquidate their capital stock. Their period t return on capital \widehat{R}_t^K is thus given by

$$\widehat{R}_t^K = \widehat{\Pi}_t + \frac{\Pi \left(dr_t^k (1 - \tau_K) + \widehat{Q}_t (1 - \delta) \right)}{R^k} - \widehat{Q}_{t-1} \quad (26)$$

where dr_t^k and \widehat{Q}_t denote the deviation of the rental rate on physical capital and the price of a unit of capital from their respective steady states, respectively. Entrepreneurs fund their capital stock using their own net worth \widehat{N}_t and a loan from a financial intermediary. As a consequence of idiosyncratic shocks to the return on capital, some entrepreneurs default on their debt in period $t + 1$. In case of default, the bank seizes a fraction $(1 - \mu)$ of the assets of the entrepreneur, while the remainder represents a monitoring costs. Banks pass the costs of bankruptcy to entrepreneurs in the form of a state contingent debt contract, implying that they always earn the same interest rate R_t . As a result, the entrepreneurs first order conditions require a positive relationship between the spread of the entrepreneurs expected return on capital $E_t \widehat{R}_{t+1}^K$ over the risk free rate \widehat{R}_t and entrepreneurial leverage:

$$E_t \widehat{R}_{t+1}^K - \widehat{R}_t = \chi^{\phi^e} \left(\widehat{Q}_t + \widehat{K}_t - \widehat{N}_t \right) \quad (27)$$

with $\chi^{\phi^e} > 0$ if bankruptcy is costly (i.e. if $\mu > 0$). $E_t \widehat{R}_{t+1}^K - \widehat{R}_t$ is typically referred to as the cost of external finance. Furthermore, each period a small fraction $1 - \gamma$ of entrepreneurs dies each and consumes its net worth, and are replaced with a fraction of newly born entrepreneurs. This assumption assures that entrepreneurs never become fully self-financing. Entrepreneurs supply one unit of labor to retailers at wage

$w_{e,t}$, which allows newly borne entrepreneurs to start their business. Entrepreneurial net worth thus depends positively on on past net worth and the real return on capital net of bankruptcy costs $(1 - \mu G(\bar{\omega})) (\widehat{R}_t^K - \widehat{\Pi}_t)$ as well as negatively on the real save interest rate $\widehat{R}_{t-1} - \widehat{\Pi}_t$

$$\widehat{N}_t = \gamma \left[\begin{aligned} & \frac{R}{\Pi} \widehat{N}_{t-1} + \phi^e \frac{R^K}{\Pi} (1 - \mu G(\bar{\omega})) (\widehat{R}_t^K - \widehat{\Pi}_t) + \phi^e \left(\frac{R^k}{\Pi} (1 - \mu G(\bar{\omega})) - \frac{R}{\Pi} \right) (\widehat{Q}_{t-1} + \widehat{K}_{t-1}) \\ & - \frac{R}{\Pi} (\phi^e - 1) (\widehat{R}_{t-1} - \widehat{\Pi}_t) - \phi^e \frac{R^K}{\Pi} \mu G'(\bar{\omega}) \varpi \widehat{\omega}_t \end{aligned} \right] + (1 - \gamma) \widehat{N}_{t-1} \quad (28)$$

where $\phi^e = \frac{K}{N}$, $\widehat{\omega}_t$, $G(\bar{\omega})$ and γ denote steady state leverage, the bankruptcy threshold, the steady-state probability weighted expected value of the idiosyncratic shock ω (conditional on $\omega < \bar{\omega}$) and the survival probability of entrepreneurs, respectively. The financial accelerator works as follows: Any shock causing a jump in \widehat{Q}_t by say lowering the monetary policy rate \widehat{R}_t and/ or increasing expected future output and thus dr_t^k (see equations (26) and (27)) will increase not only investment, but also \widehat{R}_t^K and thus period t net worth (see equation (28)). The financial accelerator amplifies the jump in \widehat{Q}_t and thus in investment by letting the associated decline in entrepreneurial leverage $(\widehat{Q}_t + \widehat{K}_t - \widehat{N}_t)$ reduce the external finance premium $E_t \widehat{R}_{t+1}^K - \widehat{R}_t$. Furthermore, for a given nominal interest rate, an increase in inflation $\widehat{\Pi}_t$ lowers entrepreneurial leverage and thus the cost of external finance by lowering their real debt burden.

Finally, entrepreneurial consumption \widehat{C}_t^e is given by

$$\widehat{C}_t^e = \frac{(1 - \gamma) N}{\gamma V} \widehat{N}_t \quad (29)$$

Financial intermediaries fund their loans by collecting deposits from unconstrained households. Hence the safe assets $B_{O,t}$ entering their utility function now consist of both bank deposits and government bonds.

3.2 Government

Government revenue consists of labor taxes borne by the employer and the employee, profit, lump sum and consumption taxes. In order to ensure the stationarity of government debt in the long run, the government adjusts the consumption tax according to the following fiscal rule:

$$\widehat{\tau}_{C,t} = (1 - d_{\tau_C,t}) \left[(1 - \rho_\tau) \phi_\tau \widehat{b}_t + \rho_\tau \widehat{\tau}_{C,t-1} \right] \quad (30)$$

where $\widehat{\tau}_{C,t}$ and \widehat{b}_t denote the percentage deviation of the employees labor tax and real government debt from their respective steady states, while $d_{\tau_C,t}$ denotes a dummy variable with a value of zero unless otherwise mentioned. Using consumption taxes as the fiscal instrument in the fiscal rule simplifies the analysis of the permanent cut in government consumption considered below by ensuring that the long run percentage increase of unconstrained households, constrained households and entrepreneurial consumption is almost identical. Furthermore, the long run decline

in the consumption tax offsets the decline in labor supply associated with higher household disposable income, implying that hours and GDP change only marginally in the long run.

For the IRF matching exercise to be conducted below, I assume that government spending follows a simple auto-regressive process

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + e_{G,t} \quad (31)$$

where $e_{G,t}$ denotes an i.i.d. government spending shock.

Monetary policy is described by a rule relating the policy rate to its own lag as well as inflation and the deviation of real marginal cost from its steady state $\widehat{m}c_t$, which serves a proxy for the output gap

$$\widehat{R}_t R = (1 - d_{R,t}) (1 - \rho_R) \left[\psi_\pi \widehat{\Pi}_t + \psi_y (\widehat{m}c_t) \right] + \rho_R \widehat{R}_{t-1} + e_{R,t} \quad (32)$$

where d_R denotes a dummy variable with a value of zero unless otherwise mentioned.

3.3 Calibration

I calibrate the parameters to Euro Area data where possible and divide them into two groups. Parameters in the first group are set to standard values in the literature, empirical estimates (see Table 5) or indirectly calibrated by setting targets for the steady state values of some variables (see Table 4). I set the share of unconstrained households $1 - \omega$ to 75%, in line with the Euro Area estimates of Coenen and Straub (2005). Regarding household preferences over consumption, I assume log utility ($\sigma_H = 1$), set the degree of habit formation to the median estimate reported by Havranek et al. (2015) for European countries and assume an inverse Frisch elasticity of labor supply η of 2. I consider three specifications for preferences over wealth, namely no preferences over wealth ($\theta = 1$), linear preferences over wealth ($\sigma_B = 0$) with $\theta = 0.96$, and finally the case of $\theta = 0.96$ and curvature of the utility from wealth of $\sigma_B = 0.5$. Following Kumhof et al. (2014), I calibrate σ_B such that the unconstrained households marginal propensity to save (MPS) out of an increase in permanent income matches the micro evidence on the saving behavior of high income households provided by Dynant et al. (2004) and Kumhof et al. (2014). For the top 5% income group, Kumhof et al. (2014) report an MPS of 0.4 based on their own estimates and an upper bound of 0.5 based on the estimates of Dynant (2004). I use an MPS of 0.5 as a target for the calibration σ_B because wealth is likely to be concentrated among a subset of those households forming the empirical counterpart of my unconstrained household group. However, my main results become even stronger if I target a lower value of the MPS. Details of the partial equilibrium calibration exercise are provided in appendix 5. I assume a wage markup of 1/3 and an elasticity of production with respect to physical capital of 1/3.

I set the elasticity of the external finance premium with respect to entrepreneurial leverage χ^e and the survival probability of entrepreneurs γ equal to the Euro Area estimates of Gelain (2010). Given the choices of χ^e , I set the degree of idiosyncratic uncertainty σ , the bankruptcy cost parameter μ and the transfer to entrepreneurs such that steady state entrepreneurial leverage and the entrepreneurial bankruptcy rate equal the values reported by Christiano et al. (2010) and Gelain (2010). Given

the respective value of θ , I calibrated the household discount factor β , the steady state level of government expenditure G , the markup charged by retailers and the depreciation rate such that the steady state real interest rate as well as the shares of government expenditure, private investment and the compensation of employees are close to their respective averages over the 1972Q1-2008Q4 period.

Finally, I assume long run responses of the policy rate to inflation and the output gap of 2.0 and 0.05, respectively, close to the Euro Area estimates of Gaddatsch et al. (2016). I obtain the steady state tax rates from the ECBs New Area Wide Model (Coenen et al. (2008)) and calibrate the fiscal rule such that in the long run such that the consumption tax rate moves very gradually in response to deviations of government debt from its steady state, while the response is still sufficiently strong to guarantee long-run debt stationarity.

Table 4: Values of steady state targets and empirical counterparts/ sources

Variable	Model	AWM database or other source
$\frac{R}{\Pi}$, APR	2.2	2.2
$\frac{I}{GDP}$	22.7	22.3
$\frac{G}{GDP}$	19.5	19.5
$\frac{W(1+\tau)w_{R,t}l}{GDP}$	50.1	52.6
$\frac{K}{N}$	2.1	As in Christiano et al. (2010) and Gelain (2010)
$F(\bar{\omega})$	0.0075	Gelain (2010)
MPS	0.5	MPS out of an increase in permanent income, estimates of Dynant et al. (2004) and Kumhof et al. (2014).

Note: Unless otherwise mentioned, the data is taken from the Area Wide Model database (see Fagan et al. (2001)). Averages from the AWM where calculated over the 1972Q1-2008Q4.

Table 5: Medium scale model: calibrated Parameters

Parameter	Name	Value
χ^e	Elasticity of EFP w.r.t. leverage	0.038, in line with Gelain (2010)
μ	Bankruptcy cost	0.1, in line with Gelain (2010)
σ	Idiosyncratic uncertainty of capital return	0.25*
γ	Survival probability of entrepreneurs	0.9797 as in Gelain (2010)
τ_{W_h}	Labor tax borne by employee	0.24, as in Coenen et al. (2008)
τ_{W_f}	Labor tax borne by employer	0.219, as in Coenen et al. (2008)
τ_C	Consumption tax	0.183, as in Coenen et al. (2008)
τ_K	Capital income tax	0.184, as in Coenen et al. (2008)
α	Labor share	0.33, as in Coenen et al. (2008)
ϕ_τ	Long run response to debt level	0.2
ρ_τ	Response to lagged tax rate	0.98
σ_H	Intertemporal elasticity of substitution	1.0
h	Degree of habit formation	0.6, as estimated by Havranek et al. (2015)
σ_B	Curvature preferences over wealth	0/0.5, evidence on MPS of Dynant (2004) and Kumhof et al. (2014)
θ	$\beta \frac{R}{\Pi}$	0.96, 1
δ	Depreciation rate physical capital	0.03*
ϵ_p	Goods market, elasticity of substitution	11*
ϵ_W	Labor market, elasticity of substitution	4
ψ_π	Interest feedback rule: Inflation response	2.0, as in Christoffel et al. (2008)
ψ_Y	Interest feedback rule: output gap response	0.05, as estimated by Gaddatsch et al. (2016)

I estimate the second group of parameters by matching the impulse response functions of GDP, private consumption, private Investment, government expenditure, the nominal wage, inflation (measured by the change in the GDP deflator) and the policy rate to a government spending shock and a monetary policy from a structural VAR model estimated for this purpose by Blanchard et al. (2015) over the 1972Q1-2008Q4 period using the Area Wide Model (AWM) database of Fagan et al. (2002). On top of the aforementioned series, the VAR also includes the effective real exchange rate as an additional endogenous variable (which is however absent from the model), as well as US GDP, the Federal Funds rate, commodity prices and quadratic time trend as exogenous variables. The two shocks are identified by assuming (1) that government spending does not respond contemporaneously to any of the endogenous variables (following Blanchard and Perotti (2002)) and (2) that none of the endogenous variables except for the real exchange rate respond contemporaneously to policy rate innovations (following Christiano et al. (1999)).⁵ These assumptions are also implemented in the model when computing impulse response functions to the two shocks.⁶ I set the standard deviations of the government spend-

⁵I would like to thank Jesper Linde for sharing their code for the estimation of the VAR as well as valuable hints on how to implement the restrictions of the VAR in the model.

⁶Specifically, I assume that financial variables (e.g. Tobin's Q and the net worth of entrepreneurs) respond to the policy rate innovation contemporaneously, but that the information set of households and firms does not include a contemporaneous policy innovation nor its effect on financial variables. For instance, unconstrained households act as if the (expected) value of current and future real interest rates is the same as in the absence of the policy rate innovation, while investment good

Table 6: Medium scale model: Estimated Parameters

Parameter name	Parameter	Estimation results for various versions of the model		
		$\theta = 1$	$\theta = 0.96, \sigma_B = 0$	$\theta = 0.96, \sigma_B = 1$
Investment adjustment cost	ϕ_I	7.9	9.9	9.8
Price markup coefficient	κ_p	0.007	0.013	0.015
Wage markup coefficient	κ_w	0.001	0.0014	0.0016
Persistence gov. spending shock	ρ_g	0.90	0.90	0.90
Interest rate smoothing	ρ_R	0.73	0.76	0.77
Sd. gov. spending shock	σ_g	0.29	0.29	0.29
Sd. monetary policy shock	σ_R	$\frac{0.37}{4}$	$\frac{0.37}{4}$	$\frac{0.37}{4}$

ing and monetary policy shocks to their VAR estimates and collect the remaining estimated parameters in the vector $\Omega = \{\phi_I, \kappa_p, \kappa_w, \rho_g, \rho_R\}$, and choose Ω in order to minimize the criterion function

$$\left(\hat{\Psi} - \Psi(\Omega)\right)' F^{-1} \left(\hat{\Psi} - \Psi(\Omega)\right)$$

where $\hat{\Psi}$ and $\Psi(\Omega)$ denote vectors stacking all VAR and model impulse response functions (for a given value of Ω) on top of each other (excluding the contemporaneous response to the monetary policy shock), respectively. F denotes a diagonal weighting matrix, where each element equals the estimated variance of the corresponding IRF element from the VAR.⁷ I match the first 20 periods of each impulse response, and estimate a model variant without preferences over wealth ($\theta = 1$), with linear preferences over wealth ($\theta = 0.96, \sigma_B = 0$) and with curvature in preferences over wealth ($\theta = 0.96, \sigma_B = 0.5$).

As can be obtained from Figure 4, the three models respond in a virtually identical fashion to the monetary policy shock (see the dashed and circled lines). They perform well at tracking the empirical impulse response of GDP, as well as the stronger decline of private investment relative to private consumption. All three models understate the persistence of the consumption decline. Furthermore, the models are unable to match the initial increase in inflation (usually referred to as the “price puzzle”). Both models somewhat understate the initial response of GDP to the government spending shock and are unable to generate the observed increase in private investment. The estimated price and wage markup coefficients are low (see table 6), although, Linde et al. (2016) and Brave et al. (2012) estimate wage and price markup coefficients of a similarly small order of magnitude.⁸ They are higher in the presence than in the absence of preferences over wealth ($\theta = 0.96$), although in absolute terms they are very close. The reason why price and wages are estimated to be more flexible for $\theta < 1$ is that reducing θ implicitly reduces β , implying that expectations of future variables (e.g. future marginal cost and the future wage markup) now have a smaller effect on price and wage setting. Matching

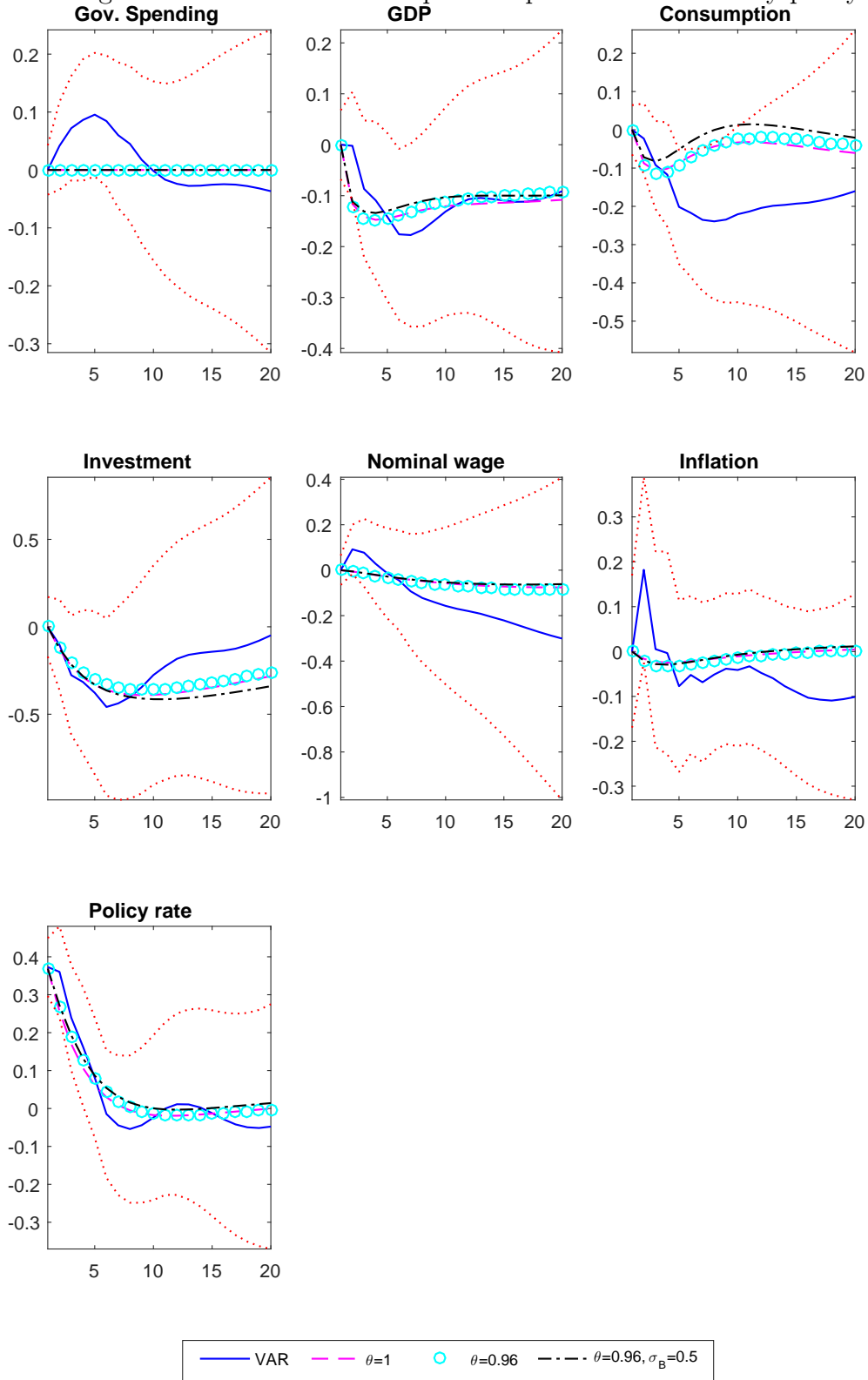
producers ignore the effect of the innovation on Tobin’s Q .

⁷Hence with the length of each IRF denoted as T , $\hat{\Psi}$ and $\Psi(\Omega)$ each comprise $n \times T(2T-1)$ elements, while the dimension of F is $n \times n$.

⁸For instance, their wage and price markup coefficient in their model with a financial accelerator equal 0.004 and 0.007, respectively.

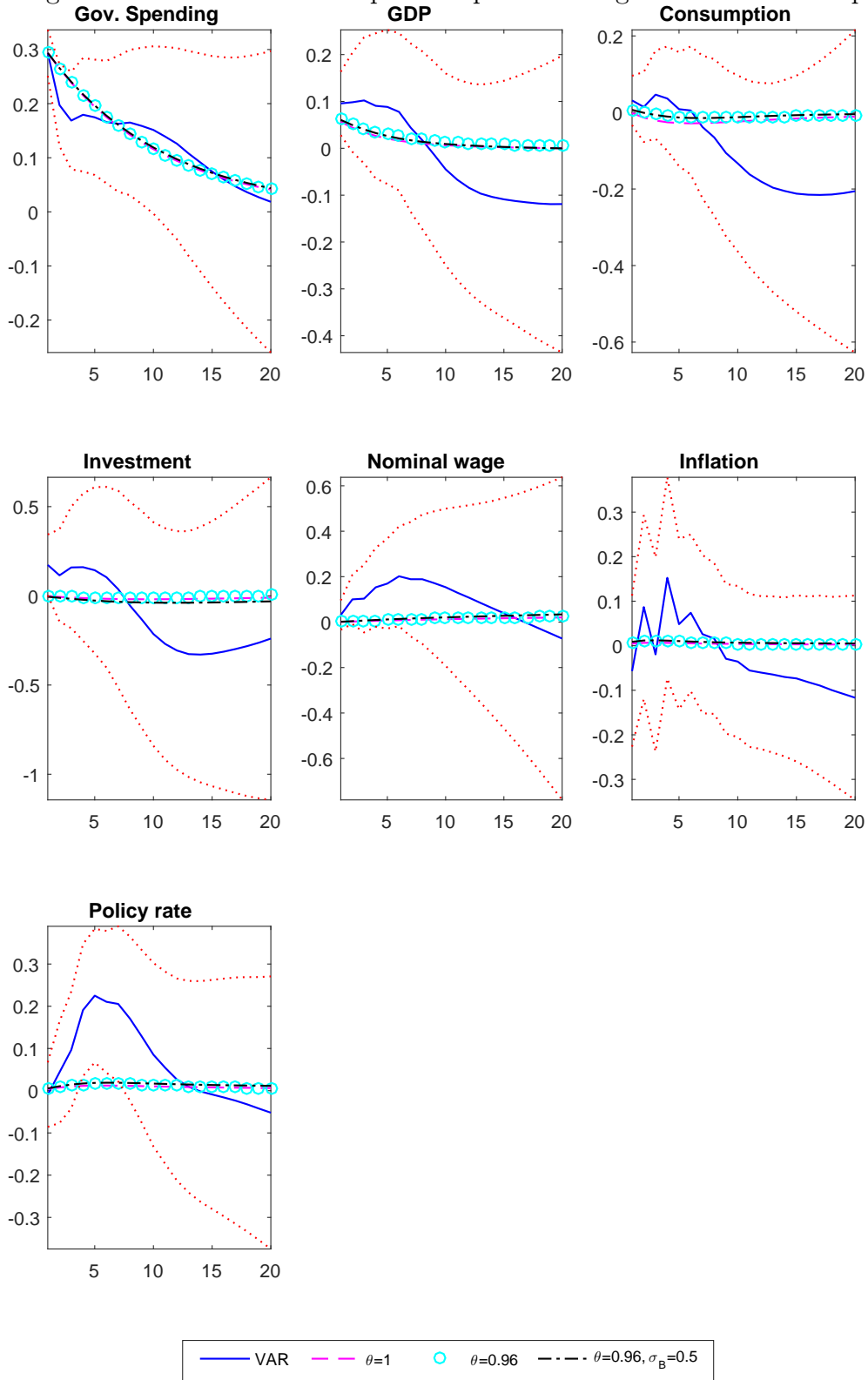
the response of inflation and nominal wages requires a bigger role for current economic activity. By contrast, adding curvature ($\theta = 0.96, \sigma_B = 1$) has only a minor effect on the parameter estimates, as as due to their temporary nature, both shocks have only a moderate effect on the wealth of unconstrained households and thus on the marginal utility of wealth relative to consumption.

Figure 4: VAR and model impulse responses to a monetary policy shock



Notes: The graph plots the impulse response function of the estimated VAR as well as each model variant to a one standard deviation policy rate shock. Dotted lines denote 95% confidence intervals.

Figure 5: VAR and model impulse responses to a government consumption shock



Notes: The graph plots the impulse response function of the estimated VAR as well as each model variant to a one standard deviation government consumption shock. Dotted lines denote 95% confidence intervals.

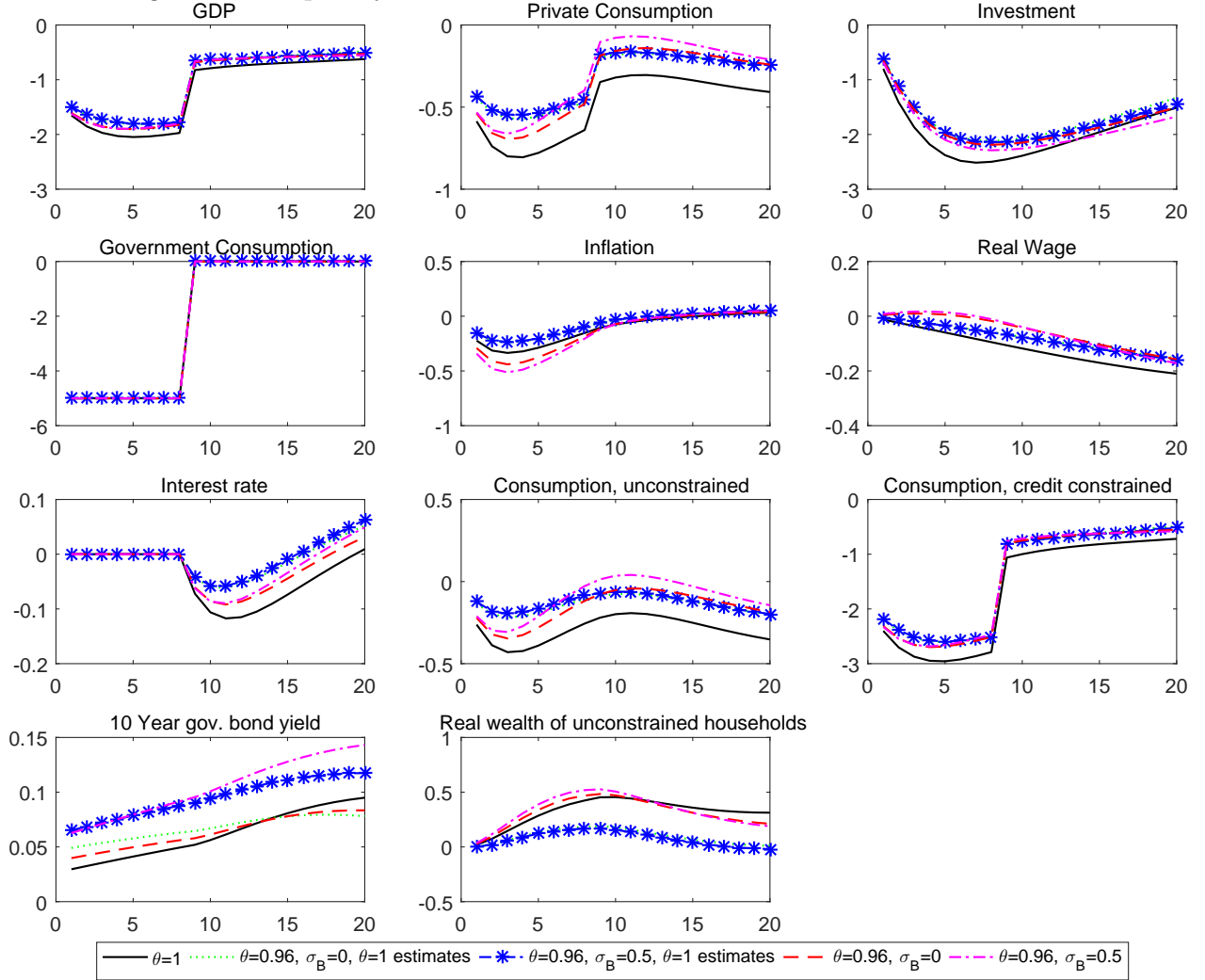
3.4 Temporary fiscal consolidation in the medium scale model

I first examine the effects of contraction of government consumption of 1% of GDP lasting eight quarters, 10 years and infinitely long, respectively. As a proxy for the ZLB, I assume that the central bank's response to the inflation and GDP effect of the consolidation given by equation (32) as well as the fiscal rule (equation (30)) are switched off for 2 years by setting $d_{R,t} = d_{\tau_C,t} = 1$ and during this period. I consider the three different model versions estimated in the previous section, as well as a model with linear preferences over wealth ($\theta = 0.96$) but all other parameters as estimated for $\theta = 1$. Doing so serves to highlight the impact of the higher nominal flexibility estimated for the model with preferences over wealth.

As can be obtained from Figure 6, the effect of temporary contraction are similar across the different values of θ . In all cases the decline in government consumption also reduces private consumption by lowering the disposable income of constrained households as well as inflation, which increases the real interest rate and thus lowers the consumption of unconstrained households. Furthermore, the increase in real interest rates and the decline in demand lower Tobin's Q and thus investment. As a result of the adverse effect on private expenditure, the multiplier exceeds one (see Table 7). These mechanisms are well documented by literature on fiscal policy changes when monetary policy is constrained (e.g. Eggertsson and Krugman (2012) Freedman et al. (2012), Carillo and Poilly (2013)). Lowering θ below one renders unconstrained households less sensitive to the effect of lower future real interest rates (see equation 6). It therefore somewhat attenuates their consumption decline, as can be obtained by comparing the model without preferences over wealth (black solid line in Figure 6) with the model with linear preferences over wealth but otherwise identical parameters (the green dotted line which is covered by the blue starred line), even after accounting for the larger decline of inflation and thus stronger real interest rate increase observed with preferences over wealth (red dashed line) as a consequence higher estimated nominal price and wage flexibility. However, for a two year fiscal contraction, the effect of lowering θ is too small to have a big effect on the multiplier.

Finally, the impact higher estimated nominal price and wage flexibility with preferences over wealth is largely restricted to the paths of inflation and the real wealth of unconstrained households.

Figure 6: Temporary fiscal contraction in the medium scale model



Note: The figure displays the effect of a contraction of government consumption of 1% of GDP lasting 8 quarters ($T_G = 8$) in the three model variants. Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points. See the note below Table 7 for further details.

3.5 Permanent fiscal contraction in the medium scale model

I now turn to the effect of a permanent fiscal contraction under varying assumptions regarding preferences over wealth. Figure 7 displays the effect in the absence credit constraints in the household and firm sector. As in the simple model, without preferences over wealth and thus an effective infinite horizon ($\theta = 1$), unconstrained household consumption works as a powerful stabilizer against a permanent decline in government consumption and the associated decline in constrained household consumption. Unconstrained household consumption increases as a consequence of the monetary loosening after the economy's exit from the ZLB and the fact that in the new steady state, the share of government consumption in GDP is lower and the share

Table 7: Fiscal contraction in the medium scale model, cumulative multipliers

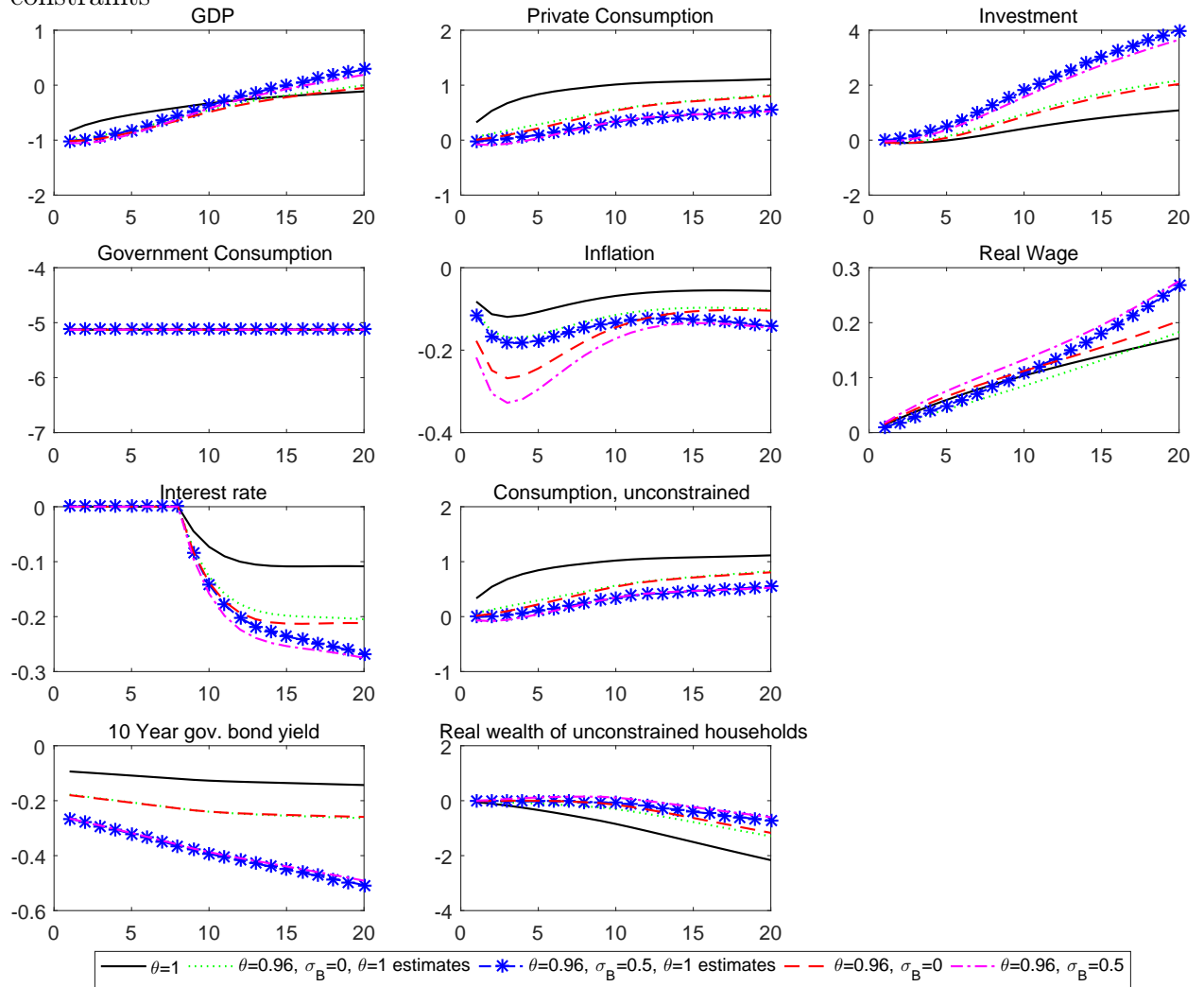
Baseline model			
	$T_G = 8$	$T_G = 40$	$T_G = \infty$
$\theta = 1$	2.0	0.6	1.0
$\theta = 0.96, \sigma_B = 0$, estimates for $\theta = 1$	1.8	1.6	1.5
$\theta = 0.96, \sigma_B = 0.5$, estimates for $\theta = 1$	1.8	1.6	1.6
$\theta = 0.96, \sigma_B = 0$	1.9	1.6	1.6
$\theta = 0.96, \sigma_B = 0.5$	1.9	1.6	1.7
No credit constraints ($\omega = \mu = 0$)			
	$T_G = 8$	$T_G = 40$	$T_G = \infty$
$\theta = 1$	1.2	0.3	0.6
$\theta = 0.96, \sigma_B = 0$, estimates for $\theta = 1$	1.2	0.7	0.8
$\theta = 0.96, \sigma_B = 0.5$, estimates for $\theta = 1$	1.2	0.7	0.8
$\theta = 0.96, \sigma_B = 0$	1.2	0.8	0.9
$\theta = 0.96, \sigma_B = 0.5$	1.2	0.8	0.9

Note: The tables display the cumulative multiplier of increasing government spending for T_G quarters. In all scenarios, I assume that $d_{R,t} = 1$ for $t = 1 : D_L$ in equation (32), with $D_L = 8$, to proxy a zero lower bound duration of 8 quarters. The cumulative multiplier is calculated as $m_T = \frac{\sum_{t=1}^{D_L} \widehat{GDP}_t}{\sum_{t=1}^{D_L} \widehat{G}_t \frac{G}{\overline{GDP}}}$. The calibrated parameters are as displayed in Table 5. σ_B is zero unless otherwise mentioned. The estimated parameters are displayed in Table 6 unless otherwise mentioned. In the row labeled “ $\theta = 0.96, \sigma_B = x$, estimates for $\theta = 1$ ”, I set $\theta = 0.96$ but use the parameters estimated for $\theta = 1$. In the model without credit constraints, I assume that there is no entrepreneurial consumption. Instead, entrepreneurs return their wealth to unconstrained households once they die.

of private consumption higher. With preferences over wealth ($\theta = 0.96$), household consumption increases much less than without ($\theta = 1$).

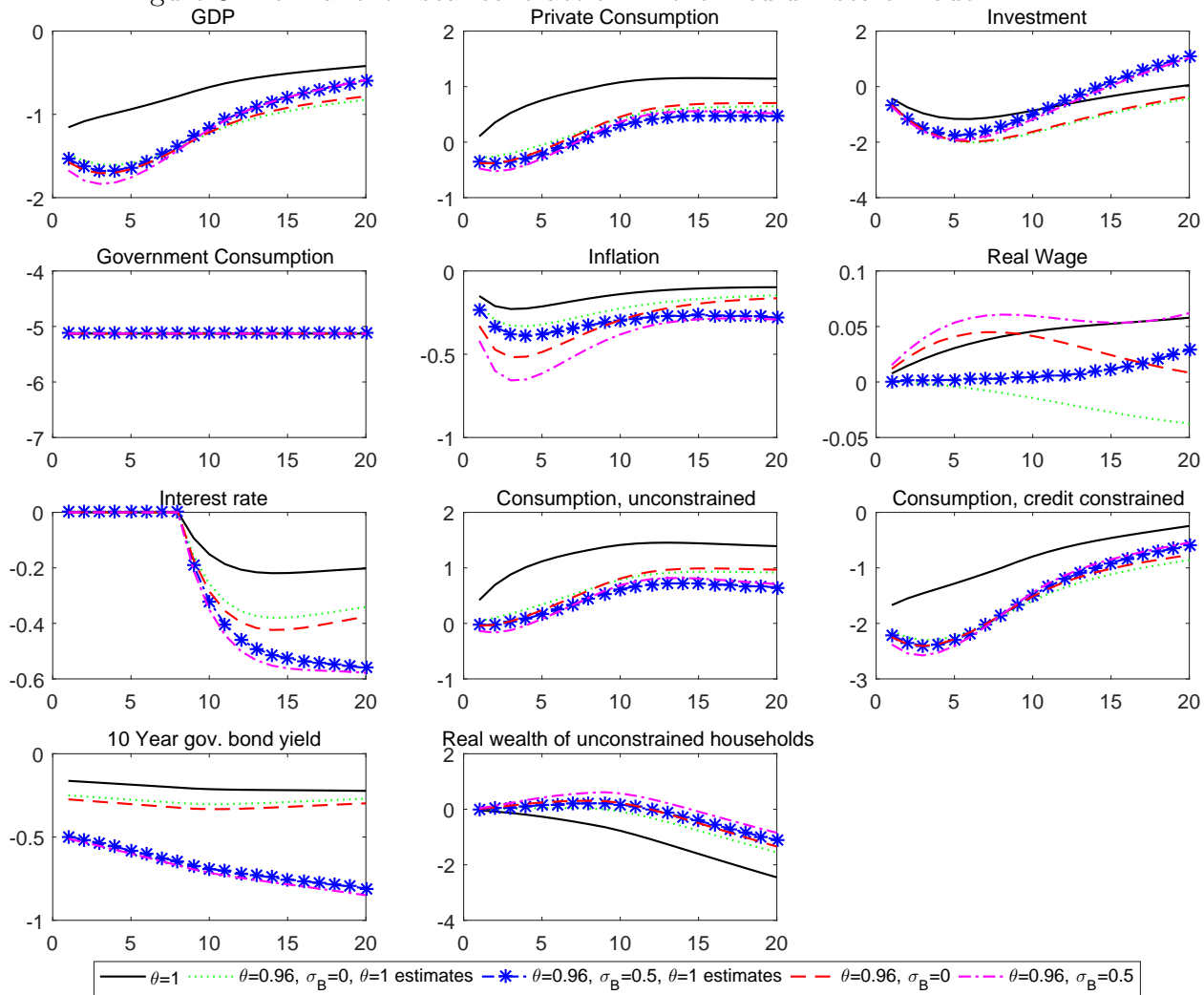
In the presence of credit constrained households and firms, the effect of the fiscal contraction increases (see Figure 8 and Table 7). The decline in employment directly lowers the consumption of credit constrained households. Lower inflation tends to increase the real debt burden of entrepreneurs and thus their cost of external finance, implying that investment now declines. However, the amplification provided by adding credit constraints to the model is substantially bigger with preferences over wealth than without. For $\theta = 1$, the increase in unconstrained household consumption is even larger than in the absence of constrained households and firms, implying that the path of private consumption is virtually unaffected by the introduction of credit constraints. With credit constrained households and firms, monetary policy is loosened more after the economy's exit from the ZLB, which causes a bigger crowding in of unconstrained household consumption if $\theta = 1$. With preferences over wealth ($\theta = 0.96$), the introduction of credit constraint households and firms does not cause such a stabilizing upward shift in the consumption trajectory of unconstrained households following a permanent government spending cut, as they are less sensitive to real interest rate movements, and thus the GDP effect of the decline in constrained household consumption and investment is diluted to a much smaller extent.

Figure 7: Permanent fiscal contraction in the medium scale model without credit constraints



Note: The figure displays the effect of a permanent contraction of government consumption of 1% of GDP ($T_G = \infty$) in the model without credit constraints. See the note below Table 7 for further details. Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points.

Figure 8: Permanent fiscal contraction in the medium scale model



Note: The figure displays the effect of a permanent contraction of government consumption of 1% of GDP ($T_G = \infty$). in the baseline model. Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points. See the note below Table 7 for further details.

3.6 Forward guidance

I now examine the effect of forward guidance on the path of the short term interest rate in the medium scale model. Analogously to the simple model, I assume that the Central Bank is initially constrained by the zero lower bound for 6 quarters, which I proxy by assuming $d_{R,t} = 1$ in equation (32) during this period. Furthermore, I assume that starting in quarter seven, the Central Bank voluntarily fixes \hat{R}_f at an annualized value of 0.2% below its steady state and keeps it at this level for D_f quarters, i.e. $d_{R,t} = 1$ and $e_{R,t} = \frac{-0.2\%}{4}$ during quarters 7 to $6 + D_f$. This setup is broadly consistent with the evidence Del Negro et al. (2015) provide on financial market expectations regarding the timing of the exit of the Federal Funds rate from the ZLB prior to the forward guidance announcements of the US Federal Reserve in

Table 8: Peak GDP effect of forward guidance for varying duration of fixed interest rate period

Baseline model			
	$D_f = 6$	$D_f = 8$	$D_f = 10$
$\theta = 1$	1.3	1.9	2.2
$\theta = 0.96, \sigma_B = 0$, estimates for $\theta = 1$	0.8	1.0	1.2
$\theta = 0.96, \sigma_B = 0.5$, estimates for $\theta = 1$	0.7	0.8	0.9
$\theta = 0.96, \sigma_B = 0$	1.0	1.4	1.7
$\theta = 0.96, \sigma_B = 1$	0.8	1.0	1.1

Note: This table reports the peak GDP effect of forward guidance policies of the indicated duration T_f . In all cases, I assume that:

- for $t = 1 : 6$, $d_{R,t} = 1$ in equation (32) ,
- for $t = 7$ to $7 + D_f$, $d_{R,t} = 1$ and $e_{R,t} = \frac{-0.2\%}{4}$.

the central bank then fixes the nominal interest rate at an annualized 0.2% below its steady state level, and keeps it there for the number of quarters indicated in the table. The calibrated parameters are as displayed in Table 5. σ_B is zero unless otherwise mentioned. The estimated parameters are displayed in Table 6 unless otherwise mentioned. In the row labeled “ $\theta = 0.96$, estimates for $\theta = 1$ ”, I set $\theta = 0.96$ but use the parameters estimated for $\theta = 1$.

September 2011, January 2012 and September 2013, as well as their evidence on the effect of the announcement on private sector forecasts of three month and 10 year treasury bonds one to four quarters ahead, which gradually decline by 0.15 and 0.2 percentage points, respectively. The magnitude of D_f is less clear. I allow D_f to vary between 6 quarters, which closely follows Del Negro et al.’s (2015) simulations, and 10 quarters.

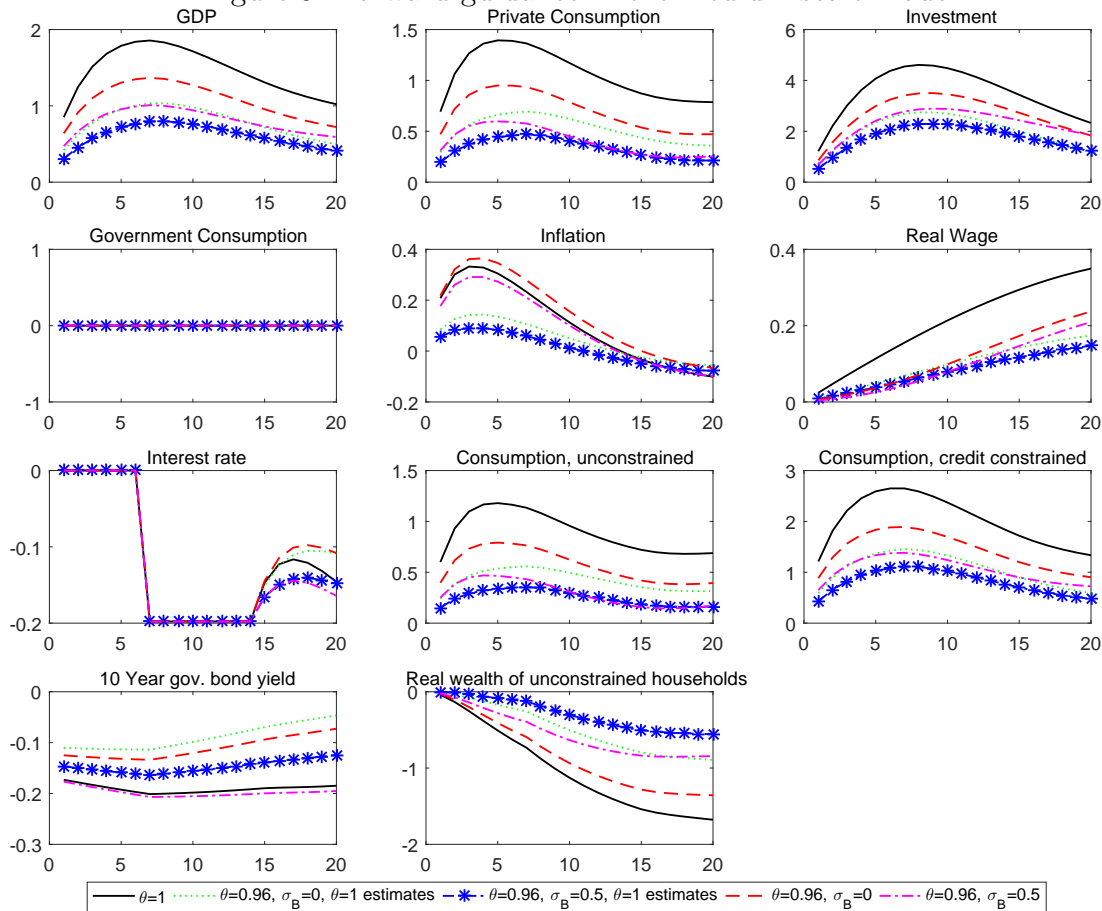
Figure 9 displays the effects of forward guidance for the $D_f = 8$ quarter case. The forward guidance policy stimulates private investment and consumption by lowering the expected real interest rate, increasing the value of the collateral of entrepreneurs, future expected demand and the disposable income of constrained households. While the policy is a powerful tool for all values of θ , just as in the simple model, the peak GDP response is lower with preferences over wealth ($\theta < 1$). Reducing θ to 0.96 but keeping all other parameters as in the model without preferences over wealth, the peak GDP effect is almost halved (0.8% with vs. 1.3% without preferences over wealth, Table 8/ green dotted line and black solid line in Figure 9). The expected decline in the future real interest rate has a smaller effect on unconstrained household consumption, and thus employment, constrained households consumption and investment. Once I use the parameters estimated for the $\theta = 0.96$, part of this attenuation is compensated by the higher price and wage flexibility estimated for the model with preferences over wealth, which implies a stronger decline in the real interest rate. Even after taking this compensating effect into account, the peak GDP effect still equals only two thirds of its value without preferences over wealth (see Table 8/ green dotted line vs. red dashed line in Figure 9) .

Once I allow for curvature in the preferences over wealth, the increase in GDP in response to the policy is attenuated even further (compare the blue starred line with the green dotted and the black solid line). The increase in unconstrained household consumption and inflation reduces their real financial wealth. With curvature ($\theta = 0.96, \sigma_B = 0.5$), the decline in their real wealth increases the marginal benefit of

wealth accumulation relative to the marginal utility of consumption, which dampens their consumption increase (see equation 6). Increasing the period during which the central bank voluntarily reduces the interest rate D_f increases the peak GDP effect in all model variants, but considerably less so with preferences over wealth (see Table 8). With curvature in preferences over wealth, the marginal effect of lengthening the forward guidance episode actually declines. Allowing for the higher nominal flexibility estimated for the model with preferences over wealth implies only a marginally smaller attenuation effect.

Preferences over wealth parameterized based on micro evidence on personal discount rates and the saving behavior of high income households thus considerably alleviate the “Forward Guidance Puzzle” (Del Negro et al. (2015)), especially once curvature in the preferences over wealth is allowed for. For the $D_f = 8$ case, the simulated decline in the 10 year government bond yield by quarter four equals the empirical reduced form estimates of Del Negro et al. (2015) of 0.2 percentage points in both the model without preferences over wealth and curvature in preferences over wealth. However, with preferences over wealth and curvature, the cumulative increase in GDP in the fourth quarter after following the announcement of the policy is with 0.9% much closer to Del Negro et al.’s (2015) finding of 0.6% than it is in the model without preferences over wealth (1.7%).

Figure 9: Forward guidance in the medium scale model



Note: The figure displays the effects of forward guidance regarding the short term interest rate in the medium scale model ($D_f = 8$). Vertical axes display percentage deviations of the respective variable from its steady state, unless the respective variable is naturally expressed in percentage points. See the note below Table 8 for further details.

4 Conclusion

This paper examines how the effects of fiscal policy and forward guidance are shaped by preferences over wealth calibrated based on microeconomic evidence on household saving behavior and individual discount rates in a stylized and a medium scale DSGE model. This assumption effectively limits the horizon of unconstrained households, as the intrinsic benefit of wealth over and above allowing more future consumption implies that the equilibrium real interest rate is smaller than the individual discount factor of the household. Therefore, the contractionary effect of a permanent cut in government expenditure implemented during a period when monetary policy is constrained by the zero lower bound becomes larger due to smaller crowding in of unconstrained household consumption.

Furthermore, preferences over wealth much reduce the effect of forward guidance on the future policy interest rate. Once I allow for curvature in wealth, the attenuation of the effect of forward guidance becomes even bigger as forward guidance

policies imply a decline in unconstrained household real wealth, which motivates them to save more.

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5 Calibration of the curvature of household preferences over wealth

Following Kumhof et al. (2014), I calibrate the curvature of preferences over wealth σ_B by targeting an empirical estimate of the marginal propensity to save out of an increase in permanent income. I assume that constrained household disposable income $Y_{O,t}$ is exogenous, implying that its budget constraint is given by

$$B_{O,t} + P_t C_{O,t} = R_{t-1} B_{O,t-1} + Y_{O,t} \quad (33)$$

The linearized partial equilibrium economy is described by the following equations:

$$\hat{\Lambda}_{O,t} = \frac{-\sigma_H (\hat{C}_{O,t} - h\hat{C}_{O,t-1})}{1 - h} \quad (34)$$

$$\hat{\Lambda}_{O,t} = \theta \left(\hat{\Lambda}_{O,t} + \hat{R}_t - \hat{\Pi}_{t+1} \right) - (1 - \theta) \sigma_B \hat{b}_{O,t} \quad (35)$$

$$\hat{b}_{O,t} = \frac{R}{\Pi} \left(\hat{b}_{O,t-1} + \hat{R}_t - \hat{\Pi}_t \right) + \frac{Y \hat{Y}_{o,t} - C_O \hat{C}_{O,t}}{b_O} \quad (36)$$

Following Kumhof et al. (2014). I then assume that $\hat{Y}_{O,t}$ increases permanently in period one to a level $\hat{Y}_{O,P}$ and calculate the MPS over a six year horizon, holding inflation and the nominal interest rate constant. The reason for the six year horizon is that the empirical estimates of the MPS of Kumhof et al. (2014) and Dynant et al. (2004) uses data on saving rates which is six years apart. Hence the model counterpart of their empirical estimate of the MPS is given by

$$MPS = \frac{b_O \hat{b}_{O,24}}{24 \hat{Y}_{O,P} Y_O} \quad (37)$$

6 Derivation of Woodford model with ROT households

6.1 Households

Households seek to maximize the objective

$$\sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-1/\sigma_H}}{1-1/\sigma_H} - \chi N_{t+i}^{1+\eta} + \phi_B \frac{B_{t+i}}{P_{t+i}} \right] \quad (38)$$

A fraction of households $(1 - \omega)$ has unconstrained access to financial markets and thus can vary its holding of safe bonds B_t . The remaining fraction ω of households is restricted to $B_t = 0$. A common wage is set for both households groups. Wage setting is discussed in the next section.

The budget constrained of optimizing households is (denoted with the subscript O) is given by

$$B_t + P_t C_{O,t} = R_{t-1} B_{t-1} + W_t N_{O,t} - P_t T_{O,t} \quad (39)$$

Maximizing (38) subject to (39) yields

$$\Lambda_{O,t} = \beta \frac{R_t}{\hat{\Pi}_{t+1}} \Lambda_{O,t+1} + \phi_B \quad (40)$$

$$\Lambda_{O,t} = C_{O,t}^{-1/\sigma_H} \quad (41)$$

while for ROT households we have

$$C_{ROT,t} = w_t N_{ROT,t} - T_{ROT,t} \quad (42)$$

$$\Lambda_{ROT,t} = C_{ROT,t}^{-1/\sigma_H} \quad (43)$$

Aggregate consumption is given by

$$C_t = (1 - \omega)C_{O,t} + \omega C_{ROT,t} \quad (44)$$

7 Firms: As in Woodford (2011)

There is a continuum of monopolistically competitive firms who set price subject to nominal rigidities in the form of Calvo pricing. The production technology of firm j is given by

$$Y_t(j) = N_t^\alpha(j) \quad (45)$$

Real marginal cost mc_t is given by

$$mc_t = \frac{w_t}{N_t^{\alpha-1}} \quad (46)$$

$$\pi_t = \kappa \hat{m}_t + \beta \hat{\Pi}_{t+1} \quad (47)$$

7.1 Wage setting

Assume that there is a union maximizing the weighted utilities of ROT and optimizing households. Assume identical preferences. Union supplies labor variety i to labor packer, using labor from both household types. objective is given by

$$w_t(i) \left(\omega \Lambda_{ROT,t} N(i)_{ROT,t} + (1 - \omega) \Lambda_{O,t} N(i)_{O,t} \right) + \omega \chi \frac{N(i)_{ROT,t}^{1+\eta}}{1 + \eta} + (1 - \omega) \chi \frac{N(i)_{O,t}^{1+\eta}}{1 + \eta}$$

where $w_t(i) = \frac{W(i)_t}{P_t}$ Assumption: Firms higher household types on proportion to their population share, hence $N_{O,t} = N_{ROT,t} = N_t$. Hence

$$w_t(i) \Lambda_t N(i)_t + \chi \frac{N(i)_t^{1+\eta}}{1 + \eta}$$

with $r_{\{L\}}$

$$\Lambda_t = \omega \Lambda_{ROT,t} + (1 - \omega) \Lambda_{O,t} \quad (48)$$

and

$$N(i)_t = \left(\frac{w_t(i)}{w_t} \right)^{-e_w}$$

The FOC is then given by (taking into account that all unions set the same wage)

$$w_t \Lambda_t = \chi N_t^\eta \mu_w \quad (49)$$

with $\mu > 1$ denoting the wage markup.

7.2 Log-linearization: Aggregate demand

Linearizing (40) yields

$$\begin{aligned} T_t \hat{\Lambda}_{O,t} &= \theta \left[\hat{R}_t - \hat{\Pi}_{t+1} + \hat{\Lambda}_{O,t} \right] \\ \hat{C}_{O,t} &= -\theta \sigma_H \left[\hat{R}_t - \hat{\Pi}_{t+1} \right] + \theta \hat{C}_{O,t+1} \end{aligned} \quad (50)$$

where a hat denotes the percentage deviation of a variable from its steady state, with the exception of government spending, where it is expressed as a percentage of steady state GDP.

GDP is given by

$$Y_t = C_t + G_t$$

Substituting (44) and linearizing yields

$$\hat{Y}_t Y = (1 - \omega) C_O \hat{C}_{O,t} + \omega C_{ROT} \hat{C}_{ROT,t} + Y \hat{G}_t \quad (51)$$

Linearizing (42) and (45) as well as using $N_{O,t} = N_{ROT,t} = N_t$ yields

$$\hat{C}_{ROT,t} = \hat{w}_t + \frac{1}{\alpha} \hat{Y}_t - \hat{T}_{ROT,t} \quad (52)$$

Linearizing (48), (41) and (43) and using $C_O = C_{ROT}$ and $\Lambda_{ROT} = \Lambda_O$ yields

$$\hat{\Lambda}_t = -\frac{1}{\sigma_H} \hat{C}_t \quad (53)$$

Linearizing (49) and (45) and combining the result with (53) yields $\hat{w}_t - \frac{1}{\sigma_H} \hat{C}_t = \eta \hat{N}_t$

$$\hat{w}_t = \frac{\eta}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \hat{C}_t$$

or, using $\hat{C}_t = \frac{Y}{C} (\hat{Y}_t - \hat{G}_t)$

$$\hat{w}_t = \frac{\eta}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \frac{Y}{C} (\hat{Y}_t - \hat{G}_t) \quad (54)$$

Substituting this equation into (52) yields

$$\hat{C}_{ROT,t} = \frac{\eta + 1}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \frac{Y}{C} (\hat{Y}_t - \hat{G}_t) - \hat{T}_{ROT,t} \quad (55)$$

Combining (51) and (55) yields $(1 - \omega) \hat{C}_{O,t} = (\hat{Y}_t - \hat{G}_t) \frac{Y}{C} - \omega \left(\frac{\eta + 1}{\alpha} \hat{Y}_t + \frac{1}{\sigma_H} \frac{Y}{C} (\hat{Y}_t - \hat{G}_t) - \hat{T}_{ROT,t} \right)$, or

$$(1 - \omega) \hat{C}_{O,t} = (\hat{Y}_t - \hat{G}_t) \frac{Y}{C} \left(1 - \omega \frac{1}{\sigma_H} \right) - \omega \frac{\eta + 1}{\alpha} \hat{Y}_t + \omega \hat{T}_{ROT,t} \quad (56)$$

Combining (56) and (50) yields

$$\left(\hat{Y}_t - \hat{G}_t \right) \left(1 - \omega \frac{1}{\sigma_H} \right) - \frac{C}{Y} \omega \frac{\eta + 1}{\alpha} \hat{Y}_t + \omega \frac{C}{Y} \hat{T}_{ROT,t} = -(1 - \omega) \theta \sigma_H \frac{C}{Y} \left[\hat{R}_t - \hat{\Pi}_{t+1} \right] + \theta \left((\hat{Y}_{t+1} - \hat{G}_{t+1}) \left(1 - \omega \frac{1}{\sigma_H} \right) - \frac{C}{Y} \omega \frac{\eta + 1}{\alpha} \hat{Y}_{t+1} + \omega \frac{C}{Y} \hat{T}_{ROT,t+1} \right)$$

or

$$\hat{Y}_t \left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right) - \hat{G}_t \left(1 - \omega \frac{1}{\sigma_H} \right) + \omega \frac{C}{Y} \hat{T}_{ROT,t} = -(1-\omega)\theta\sigma_H \frac{C}{Y} [\hat{R}_t - \hat{\Pi}_{t+1}] + \theta \left(\hat{Y}_{t+1} \left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right) - \hat{G}_{t+1} \left(1 - \omega \frac{1}{\sigma_H} \right) + \omega \frac{C}{Y} \hat{T}_{ROT,t+1} \right)$$

or

$$\hat{Y}_t - \frac{\left(1 - \omega \frac{1}{\sigma_H} \right) \hat{G}_t - \omega \frac{C}{Y} \hat{T}_{ROT,t}}{\left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right)} = - \frac{(1-\omega)\theta\sigma_H \frac{C}{Y}}{\left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right)} [\hat{R}_t - \hat{\Pi}_{t+1}] + \theta \left(\hat{Y}_{t+1} - \frac{\left(1 - \omega \frac{1}{\sigma_H} \right) \hat{G}_{t+1} - \omega \frac{C}{Y} \hat{T}_{ROT,t+1}}{\left(1 - \omega \frac{1}{\sigma_H} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right)} \right)$$

Using $dT_{ROT,t} = \phi_{ROT}dG_t$ and $C_{ROT} = C$, we can write $\hat{T}_{ROT,t} = \frac{Y}{C}\phi_{ROT}\hat{G}_t$, with $\phi_{ROT} = 1$ if credit constrained households contribute to the increase in government expenditure according to their population share. I can then write $\sigma = \sigma_H \frac{C}{Y}$, $\tilde{\sigma} =$

$$\frac{\sigma(1-\omega)}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right)}, \sigma_g = \frac{\left(1 - \frac{\omega C}{\sigma Y} \right) - \omega \phi_{ROT}}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha} \right)}.$$

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \theta \tilde{\sigma} \left(i_t - \hat{\Pi}_{t+1} - \bar{r} \right) + \sigma_g \left(\hat{G}_t - \theta \hat{G}_{t+1} \right)$$

In the absence of ROT households ($\omega = 0$), $\tilde{\sigma} = \sigma$ and $\sigma_g = 1$, and thus the equation reduces to equation (3.12) in Woodford (2011).

7.3 Phillips Curve

Linearizing and combining equations (46) and (45) yields $\hat{m}c_t = \hat{w}_t + \frac{1-\alpha}{\alpha} \hat{Y}_t$. Substituting (54) yields

$$\hat{m}c_t = \frac{1}{\sigma} (\hat{Y}_t - \hat{G}_t) + \frac{1-\alpha+\eta}{\alpha} \hat{Y}_t = \left(\eta_v + \frac{1}{\sigma} \right) \hat{Y}_t - \frac{1}{\sigma} \hat{G}_t = \left(\eta_v + \frac{1}{\sigma} \right) (\hat{Y}_t - \Gamma \hat{G}_t)$$

This equation is identical to the marginal cost schedule in Woodford (2011), equation (3.7), implying that the Phillips curve is identical as well, with $\sigma = \frac{u'(C_0)}{u''(C_0)Y}$, $\eta_v = \frac{1-\alpha+\eta}{\alpha}$ and $\Gamma = \frac{\frac{1}{\sigma}}{\frac{1}{\sigma} + \eta_v}$.

Furthermore, note that for $\hat{m}c_t = 0$, we have $\hat{Y}_t = \hat{Y}_t^*$, implying that

$$\hat{Y}_t^* = \Gamma \hat{G}_t \tag{57}$$

8 Collected Model equations and solution of model off ZLB

8.1 Model equations

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \theta \tilde{\sigma} \left(i_t - \hat{\Pi}_{t+1} - \bar{r} \right) + \sigma_g \left(\hat{G}_t - \theta \hat{G}_{t+1} \right) \tag{58}$$

$$\hat{\Pi}_t = \kappa (\hat{Y}_t - \Gamma \hat{G}_t) + \beta \hat{\Pi}_{t+1} \tag{59}$$

$$i_t = \bar{r} + \phi_\pi \hat{\Pi}_t + \phi_y (\hat{Y}_t - \Gamma \hat{G}_t) \tag{60}$$

$$\theta = \beta \frac{R}{\Pi} \iff \beta = \frac{\theta}{\frac{R}{\Pi}}$$

8.2 Determinacy

Eliminate all exogenous variables and bring into canonical form:

From (58)\sigma

$$\begin{aligned}\hat{\Pi}_t &= \kappa \hat{Y}_t + \beta E_t \hat{\Pi}_{t+1} \\ \Leftrightarrow E_t \hat{\Pi}_{t+1} &= -\frac{\kappa}{\beta} \hat{Y}_t + \frac{\hat{\Pi}_t}{\beta}\end{aligned}$$

$$\left(1 - \frac{\omega C}{\sigma Y} - \frac{\omega(1+\eta)}{\alpha}\right) \hat{Y}_t = \theta \left[\left(1 - \frac{\omega C}{\sigma Y} - \frac{\omega(1+\eta)}{\alpha}\right) \hat{Y}_{t+1} \right] - \sigma \theta (1-\omega) \left(i_t - \hat{\Pi}_{t+1} - \bar{r}\right)$$

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \frac{\sigma \theta (1-\omega)}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{\omega(1+\eta)}{\alpha}\right)} \left(i_t - \hat{\Pi}_{t+1} - \bar{r}\right)$$

$$\hat{Y}_t = \theta (E_t \hat{Y}_{t+1}) + \tilde{\sigma} \theta \left(E_t \hat{\Pi}_{t+1} - (\phi_\pi \hat{\Pi}_t + \phi_y \hat{Y}_t)\right)$$

$$\Leftrightarrow \hat{Y}_t (1 + \tilde{\sigma} \theta \phi_y) + \tilde{\sigma} \theta \phi_\pi \hat{\Pi}_t = \theta (E_t \hat{Y}_{t+1}) + \tilde{\sigma} \theta \left(\frac{\hat{\Pi}_t}{\beta} - \frac{\kappa}{\beta} \hat{Y}_t\right)$$

$$\Leftrightarrow \hat{Y}_t \left(1 + \frac{\tilde{\sigma} \kappa \theta}{\beta} + \tilde{\sigma} \theta \phi_y\right) + \left(\tilde{\sigma} \theta \phi_\pi - \frac{\tilde{\sigma} \theta}{\beta}\right) \hat{\Pi}_t = \theta E_t \hat{Y}_{t+1}$$

$$\Leftrightarrow E_t \hat{Y}_{t+1} = \hat{Y}_t \left(\frac{1}{\theta} + \frac{\tilde{\sigma} \kappa \theta}{\theta \beta} + \tilde{\sigma} \phi_y\right) + \left(\tilde{\sigma} \phi_\pi - \frac{\tilde{\sigma}}{\beta}\right) \hat{\Pi}_t$$

$$E_t \hat{Y}_{t+1} = \hat{Y}_t \left(\frac{1}{\theta} + \frac{\tilde{\sigma} \kappa}{\beta} + \tilde{\sigma} \phi_y\right) + \left(\tilde{\sigma} \phi_\pi - \frac{\tilde{\sigma}}{\beta}\right) \hat{\Pi}_t$$

$$E_t \hat{\Pi}_{t+1} = -\frac{\kappa}{\beta} \hat{Y}_t + \frac{\hat{\Pi}_t}{\beta}$$

System in Matrix form

$$\begin{pmatrix} E_t \hat{Y}_{t+1} \\ E_t \hat{\Pi}_{t+1} \end{pmatrix} = A \begin{pmatrix} \hat{Y}_t \\ \hat{\Pi}_t \end{pmatrix} \quad (61)$$

with $A = \begin{pmatrix} \frac{1}{\theta} + \frac{\tilde{\sigma} \kappa}{\beta} + \tilde{\sigma} \phi_y & \tilde{\sigma} \phi_\pi - \frac{\tilde{\sigma}}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}$

Calculate determinant and trace:

$$|A| = \left(\frac{1}{\theta} + \frac{\tilde{\sigma} \kappa}{\beta} + \tilde{\sigma} \phi_y\right) \frac{1}{\beta} - \frac{\kappa}{\beta} \left(\frac{\tilde{\sigma}}{\beta} - \tilde{\sigma} \phi_\pi\right) = \frac{1}{\theta \beta} + \frac{\tilde{\sigma} \phi_y}{\beta} + \frac{\kappa \tilde{\sigma} \phi_\pi}{\beta}$$

,

$$t\sigma r - (1-\mu) \lambda_f \kappa \tilde{\sigma} \theta \left[\frac{\theta \left(\frac{\kappa \tilde{\sigma}}{1-\beta \mu} + 1 - \beta \mu_f\right) + \beta \left(1 - \theta \mu - \frac{\mu \kappa \theta \tilde{\sigma}}{1-\beta \mu}\right)}{\left((1-\theta \mu)(1-\beta \mu) - \mu \kappa \theta \tilde{\sigma}\right) \left((1-\theta \mu_f)(1-\beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f\right)} \right] (A) = \frac{1}{\theta} + \frac{\tilde{\sigma} \kappa}{\beta} + \frac{\tilde{\sigma} \phi_y}{1} +$$

Case I Woodford (2003): $\det(A) > 1$; $\det(A) - \text{tr}(A) > -1$; $\det(A) + \text{tr}(A) > -1$

$$|A| = \frac{1}{\theta\beta} + \frac{\tilde{\sigma}\phi_y}{\beta} + \frac{\kappa\tilde{\sigma}\phi_\pi}{\beta} > 1$$

Always true!

$$|A| - \text{tr}(A) > -1$$

$$\Leftrightarrow \frac{1}{\theta\beta} + \frac{\tilde{\sigma}\phi_y}{\beta} + \frac{\kappa\tilde{\sigma}\phi_\pi}{\beta} - \frac{1}{\theta} - \frac{\tilde{\sigma}\kappa}{\beta} - \tilde{\sigma}\phi_y - \frac{1}{\beta} > -1$$

$$\Leftrightarrow 1 + \tilde{\sigma}\theta\phi_y + \theta\kappa\tilde{\sigma}\phi_\pi - \frac{\beta}{1} - \frac{\theta\tilde{\sigma}\kappa}{1} - \theta\beta\tilde{\sigma}\phi_y - \theta > -\theta\beta$$

$$\Leftrightarrow \tilde{\sigma}\theta\phi_y(1 - \beta) + \theta\kappa\tilde{\sigma}\phi_\pi - \frac{\theta\tilde{\sigma}\kappa}{1} > \theta - 1 + \beta(1 - \theta) = -(1 - \beta)(1 - \theta)$$

$$\Leftrightarrow \frac{\theta\phi_y(1 - \beta)}{\kappa} + \theta\phi_\pi > \theta - \frac{(1 - \beta)(1 - \theta)}{\kappa\tilde{\sigma}}$$

$$-(1 - \beta)(1 - \theta) = -(1 - \theta - \beta + \beta\theta) = \theta - 1 + \beta - \beta\theta$$

$$\Leftrightarrow \frac{\phi_y(1 - \beta)}{\kappa} + \phi_\pi > 1 - \frac{(1 - \beta)(1 - \theta)}{\theta\kappa\tilde{\sigma}}$$

For $\theta = 1$, condition collapses to Woodford (2003). For $\theta < 1$: Determinacy achieved even if $\text{LHS} < 1$.

8.3 MSV solution

Proposed solution:

$$\hat{Y}_t = \gamma_y \hat{G}_t + i_y e_{i,t} \tag{62}$$

$$\hat{\Pi}_t = \gamma_\pi \hat{G}_t + i_\pi e_{i,t} \tag{63}$$

$$i_t = \bar{r} + \gamma_i \hat{G}_t + i_i e_{i,t} \tag{64}$$

Determine $\gamma_y, \gamma_\pi, \gamma_i$ and i_y, i_π, i_i .

8.3.1 Solution with Monetary Policy response to deviation of output from flexible price level

Euler equation:

$$\hat{Y}_t = \theta \hat{Y}_{t+1} - \theta \tilde{\sigma} \left(i_t - \hat{\Pi}_{t+1} - \bar{r} \right) + \sigma_g \left(\hat{G}_t - \theta \hat{G}_{t+1} \right)$$

$$\gamma_y \hat{G}_t + i_y e_{i,t} = \theta \left(\rho \gamma_y \hat{G}_t + \rho_i e_{i,t} \right) - \theta \tilde{\sigma} \left(\gamma_i \hat{G}_t + i_i e_{i,t} - \rho \gamma_\pi \hat{G}_t - \rho_i i_i e_{i,t} \right) + \sigma_g \left(\hat{G}_t - \theta \rho \hat{G}_t \right)$$

$$[\gamma_y (1 - \rho \theta) + \tilde{\sigma} \theta \gamma_i - \tilde{\sigma} \theta \rho \gamma_\pi] \hat{G}_t + [i_y (1 - \rho_i \theta) + \tilde{\sigma} \theta i_i - \tilde{\sigma} \theta \rho_i i_\pi] e_{i,t} = \sigma_g (1 - \theta \rho) \hat{G}_t \quad (65)$$

Phillips curve:

$$\begin{aligned} \gamma_\pi \hat{G}_t + i_\pi e_{i,t} &= \kappa (\gamma_y \hat{G}_t - \Gamma \hat{G}_t + i_y e_{i,t}) + \beta \left(\gamma_\pi \rho \hat{G}_t + i_\pi \rho_i e_{i,t} \right) \\ (-\kappa \gamma_y + \gamma_\pi (1 - \beta \rho)) \hat{G}_t + (-\kappa i_y + i_\pi (1 - \beta \rho_i)) e_{i,t} &= -\kappa \Gamma \hat{G}_t \end{aligned} \quad (66)$$

MP rule:

$$\begin{aligned} \gamma_i \hat{G}_t &= \phi_\pi \gamma_\pi \hat{G}_t + \phi_y (\gamma_y \hat{G}_t - \Gamma \hat{G}_t) + e_{i,t} \\ (-\gamma_y \phi_y + \gamma_i - \phi_\pi \gamma_\pi) \hat{G}_t + (-i_y \phi_y + i_i - \phi_\pi i_\pi) e_{i,t} &= -\phi_y \Gamma \hat{G}_t + e_{i,t} \end{aligned} \quad (67)$$

Equation system to solve for $\gamma_y, \gamma_\pi, \gamma_i$ using Kramer's rule (collecting equations (65) to (67))

$$A_G \begin{pmatrix} \gamma_y \\ \gamma_i \\ \gamma_\pi \end{pmatrix} = \begin{pmatrix} \sigma_g (1 - \rho \theta) \\ -\kappa \Gamma \\ -\phi_y \Gamma \end{pmatrix} \quad (68)$$

with

$$A_G = \begin{pmatrix} (1 - \rho \theta) & \tilde{\sigma} \theta & -\tilde{\sigma} \theta \rho \\ -\kappa & 0 & (1 - \beta \rho) \\ -\phi_y & 1 & -\phi_\pi \end{pmatrix}$$

Solution

$$\begin{aligned} |A_G| &= (1 - \rho \theta) \begin{vmatrix} 0 & (1 - \beta \rho) \\ 1 & -\phi_\pi \end{vmatrix} - \tilde{\sigma} \theta \begin{vmatrix} -\kappa & (1 - \beta \rho) \\ -\phi_y & -\phi_\pi \end{vmatrix} - \tilde{\sigma} \theta \rho \begin{vmatrix} -\kappa & 0 \\ -\gamma_y & 1 \end{vmatrix} \\ &= -(1 - \rho \theta)(1 - \beta \rho) - \tilde{\sigma} \theta [\kappa \phi_\pi + \phi_y (1 - \beta \rho)] + \tilde{\sigma} \theta \rho \kappa \\ &= -(1 - \beta \rho) \left[(1 - \rho \theta) + \tilde{\sigma} \theta \frac{\kappa \phi_\pi}{(1 - \beta \rho)} - \frac{\tilde{\sigma} \theta \rho \kappa}{(1 - \beta \rho)} + \tilde{\sigma} \theta \phi_y \right] \end{aligned}$$

$$\begin{aligned}
&= -(1 - \beta\rho) \left[(1 - \rho\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right) \right] \\
|A_{G,Y}| &= \begin{vmatrix} \sigma_g(1 - \rho\theta) & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho \\ -\kappa\Gamma & 0 & (1 - \beta\rho) \\ -\phi_y\Gamma & 1 & -\phi_\pi \end{vmatrix} \\
&= -\sigma_g(1 - \rho\theta)(1 - \beta\rho) - \tilde{\sigma}\theta \begin{vmatrix} -\kappa\Gamma & (1 - \beta\rho) \\ -\phi_y\Gamma & -\phi_\pi \end{vmatrix} - \tilde{\sigma}\theta\rho \begin{vmatrix} -\kappa\Gamma & 0 \\ -\phi_y\Gamma & 1 \end{vmatrix} \\
&= -\sigma_g(1 - \rho\theta)(1 - \beta\rho) - \tilde{\sigma}\theta(\kappa\Gamma\phi_\pi + \phi_y\Gamma(1 - \beta\rho)) + \tilde{\sigma}\theta\rho\kappa\Gamma \\
&= -(1 - \beta\rho) \left[\sigma_g(1 - \rho\theta) + \tilde{\sigma}\theta\phi_y\Gamma + \frac{\Gamma\tilde{\sigma}\kappa\phi_\pi}{(1 - \beta\rho)} - \frac{\Gamma\kappa\tilde{\sigma}\theta\rho}{(1 - \beta\rho)} \right] \\
&= -(1 - \beta\rho) \left[\sigma_g(1 - \rho\theta) + \Gamma\tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right) \right] \\
|A_{G,\pi}| &= \begin{vmatrix} (1 - \rho\theta) & \tilde{\sigma}\theta & \sigma_g(1 - \rho\theta) \\ -\kappa & 0 & -\kappa\Gamma \\ -\phi_y & 1 & -\phi_y\Gamma \end{vmatrix} \\
&= \kappa(-\tilde{\sigma}\theta\phi_y\Gamma - \sigma_g(1 - \rho\theta)) + \kappa\Gamma((1 - \rho\theta) + \tilde{\sigma}\theta\phi_y) = -\kappa\sigma_g(1 - \rho\theta) + \kappa\Gamma(1 - \rho\theta) = -\kappa(1 - \rho\theta)(\sigma_g - \Gamma) \\
\gamma_y &= \frac{|A_{G,Y}|}{|A|} = \frac{\sigma_g(1 - \rho\theta) + \Gamma\tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right)}{(1 - \rho\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right)} \\
\gamma_\pi &= \frac{|A_{G,\pi}|}{|A|} = \frac{\kappa(1 - \rho\theta)(\sigma_g - \Gamma)}{(1 - \rho\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right)}
\end{aligned}$$

Equation system to solve for i_y, i_π, i_i using Kramer's rule (collecting equations (65) to (67))

$$A_e \begin{pmatrix} \gamma_y \\ \gamma_i \\ \gamma_\pi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (69)$$

with

$$A_e = \begin{pmatrix} (1 - \rho_i\theta) & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho_i \\ -\kappa & 0 & (1 - \beta\rho_i) \\ -\phi_y & 1 & -\phi_\pi \end{pmatrix}$$

Solution: $|A_e|$ is calculated analogously to $|A_G|$:

$$|A_e| = -(1 - \beta\rho_i) \left[(1 - \rho_i\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho_i)}{(1 - \beta\rho_i)} \right) \right]$$

$$|A_{e,y}| = \begin{vmatrix} 0 & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho_i \\ 0 & 0 & (1 - \beta\rho_i) \\ 1 & 1 & -\phi_\pi \end{vmatrix} = \begin{vmatrix} \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho_i \\ 0 & (1 - \beta\rho_i) \end{vmatrix} = \tilde{\sigma}\theta(1 - \beta\rho_i)$$

$$|A_{e,\pi}| = \begin{vmatrix} (1 - \rho_i\theta) & \tilde{\sigma}\theta & 0 \\ -\kappa & 0 & 0 \\ -\phi_y & 1 & 1 \end{vmatrix} = \begin{vmatrix} (1 - \rho_i\theta) & \tilde{\sigma}\theta \\ -\kappa & 0 \end{vmatrix} = \kappa\tilde{\sigma}\theta$$

Hence

$$i_y = \frac{|A_{e,y}|}{|A_e|} = \frac{-\tilde{\sigma}\theta}{\left[(1 - \rho_i\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho_i)}{(1 - \beta\rho_i)} \right) \right]}$$

$$i_\pi = \frac{|A_{e,\pi}|}{|A_e|} = \frac{-\kappa\tilde{\sigma}\theta}{(1 - \beta\rho_i) \left[(1 - \rho_i\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho_i)}{(1 - \beta\rho_i)} \right) \right]}$$

8.3.2 Response of interest rate to deviation from steady state output (rather than flex-price output)

Monetary Policy rule:

$$\hat{i}_t = \bar{r} + \phi_\pi \hat{\Pi}_t + \phi_y \hat{Y}_t$$

$$\begin{pmatrix} (1 - \rho\theta) & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho \\ -\kappa & 0 & (1 - \beta\rho) \\ -\phi_y & 1 & -\phi_\pi \end{pmatrix} \begin{pmatrix} \gamma_y \\ \gamma_i \\ \gamma_\pi \end{pmatrix} = \begin{pmatrix} \sigma_g(1 - \rho\theta) \\ -\kappa\Gamma \\ 0 \end{pmatrix}$$

In this case, $|A|$ remains unchanged, and

$$\sigma |A_{G,Y}| = \begin{vmatrix} \sigma_g(1 - \rho\theta) & \tilde{\sigma}\theta & -\tilde{\sigma}\theta\rho \\ -\kappa\Gamma & 0 & (1 - \beta\rho) \\ 0 & 1 & -\phi_\pi \end{vmatrix}$$

$$= -\sigma_g(1 - \rho\theta)(1 - \beta\rho) - \tilde{\sigma}\theta \begin{vmatrix} -\kappa\Gamma & (1 - \beta\rho) \\ 0 & -\phi_\pi \end{vmatrix} - \tilde{\sigma}\theta\rho \begin{vmatrix} -\kappa\Gamma & 0 \\ 0 & 1 \end{vmatrix}$$

$$= -(1 - \beta\rho) \left[\sigma_g(1 - \rho\theta) + \frac{\Gamma\tilde{\sigma}\kappa\phi_\pi}{(1 - \beta\rho)} - \frac{\Gamma\kappa\tilde{\sigma}\theta\rho}{(1 - \beta\rho)} \right]$$

$$= -(1 - \beta\rho) \left[\sigma_g(1 - \rho\theta) + \Gamma\tilde{\sigma}\theta \left(\frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right) \right]$$

$$\gamma_y = \frac{|A_{G,Y}|}{|A|} = \frac{\sigma_g(1 - \rho\theta) + \Gamma\tilde{\sigma}\theta \left(\frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right)}{(1 - \rho\theta) + \tilde{\sigma}\theta \left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1 - \beta\rho)} \right)}$$

$$\begin{aligned}
|A_{G,\pi}| &= \begin{vmatrix} (1-\rho\theta) & \tilde{\sigma}\theta & \sigma_g(1-\rho\theta) \\ -\kappa & 0 & -\kappa\Gamma \\ -\phi_y & 1 & 0 \end{vmatrix} \\
&= \kappa \begin{vmatrix} \tilde{\sigma}\theta & \sigma_g(1-\rho\theta) \\ 1 & 0 \end{vmatrix} + \kappa\Gamma \begin{vmatrix} (1-\rho\theta) & \tilde{\sigma}\theta \\ -\phi_y & 1 \end{vmatrix} \\
&= \kappa(-\sigma_g(1-\rho\theta)) + \kappa\Gamma((1-\rho\theta) + \phi_y\tilde{\sigma}\theta) \\
&= -\kappa(1-\rho\theta)(\sigma_g - \Gamma) + \kappa\Gamma\phi_y\tilde{\sigma}\theta \\
\gamma_\pi = \frac{|A_{G,\pi}|}{|A|} &= \frac{\kappa(1-\rho\theta)(\sigma_g - \Gamma) - \kappa\Gamma\phi_y\tilde{\sigma}\theta}{(1-\beta\rho)\left[(1-\rho\theta) + \Gamma\tilde{\sigma}\theta\left(\phi_y + \frac{\kappa(\phi_\pi - \rho)}{(1-\beta\rho)}\right)\right]}
\end{aligned}$$

9 Model equations and solution of model in the presence of ZLB

Model with riskiness shock Δ_t

$$\sigma\hat{Y}_t - \sigma_g\hat{G}_t = \theta(E_t\hat{Y}_{t+1} - \sigma_g E_t\hat{G}_{t+1}) - \tilde{\sigma}\theta\left(i_t - E_t\hat{\Pi}_{t+1} - r_t^{net}\right) \quad (70)$$

$$\hat{\Pi}_t = \kappa(\hat{Y}_t - \Gamma\hat{G}_t) + \beta E_t\hat{\Pi}_{t+1} E_t\hat{R}_{t+1}^K - \hat{R}_t = \chi^{\phi^e}\hat{\phi}_t^e \quad (71)$$

$$i_t = \max\left\{\bar{r} + \phi_\pi\hat{\Pi}_t + \phi_y\hat{Y}_t, 0\right\} \quad (72)$$

Shock to Δ_t sufficiently strong to lower inflation below ZLB. Stays at level r_L with probability μ , otherwise disappears. Government spending at \hat{G}_L for as long as $r_L < 0$. The associated lump sum tax is borne by unconstrained households. Once $r_L = 0$, in each period, with probability λ , $\hat{G}_t = \hat{G}_L$, while with probability $1 - \lambda$, it returns to zero. Furthermore, after the end of the low state, unconstrained households contribute to the increase in government expenditure to the increase in government expenditure as well, with ϕ_{ROT} denoting the per-capita amount of government spending:

$$\begin{aligned}
\sigma_{g,L} &= \frac{\left(1 - \frac{\omega C}{\sigma Y}\right)}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha}\right)} \\
\sigma_{g,S} &= \frac{\left(1 - \frac{\omega C}{\sigma Y}\right) - \omega\phi_{ROT}}{\left(1 - \frac{\omega C}{\sigma Y} - \frac{C}{Y} \frac{\omega(1+\eta)}{\alpha}\right)}
\end{aligned}$$

9.1 Determinacy

From (71)

$$\hat{\Pi}_t = \kappa \hat{Y}_t + \mu \beta E_t \hat{\Pi}_{t+1} \Leftrightarrow E_t \hat{\Pi}_{t+1} = -\frac{\kappa}{\mu \beta} \hat{Y}_t + \frac{\hat{\Pi}_t}{\mu \beta}$$

From (70)

$$\hat{Y}_t = \theta(\mu E_t \hat{Y}_{t+1}) - \tilde{\sigma} \theta \left(-\mu E_t \hat{\Pi}_{t+1} \right)$$

$$\hat{Y}_t = \theta(\mu E_t \hat{Y}_{t+1}) + \tilde{\sigma} \theta \left(\mu E_t \hat{\Pi}_{t+1} \right)$$

$$\Leftrightarrow \hat{Y}_t (1 + \tilde{\sigma} \theta \phi_y) + \tilde{\sigma} \theta \phi_\pi \hat{\Pi}_t = \theta(\mu E_t \hat{Y}_{t+1}) + \tilde{\sigma} \theta \left(\frac{\hat{\Pi}_t}{\beta} - \frac{\kappa}{\beta} \hat{Y}_t \right)$$

$$\Leftrightarrow \hat{Y}_t \left(1 + \frac{\tilde{\sigma} \kappa \theta}{\beta} + \tilde{\sigma} \theta \phi_y \right) + \left(\tilde{\sigma} \theta \phi_\pi - \frac{\tilde{\sigma} \theta}{\beta} \right) \hat{\Pi}_t = \theta \mu E_t \hat{Y}_{t+1}$$

$$\Leftrightarrow E_t \hat{Y}_{t+1} = \hat{Y}_t \left(\frac{1}{\theta \mu} + \frac{\tilde{\sigma} \kappa}{\mu \beta} + \frac{\tilde{\sigma} \phi_y}{\mu} \right) + \left(\frac{\tilde{\sigma} \phi_\pi}{\mu} - \frac{\tilde{\sigma}}{\mu \beta} \right) \hat{\Pi}_t$$

System to solve

$$\begin{pmatrix} E_t \hat{Y}_{t+1} \\ E_t \hat{\Pi}_{t+1} \end{pmatrix} = A \begin{pmatrix} \hat{Y}_t \\ \hat{\Pi}_t \end{pmatrix}$$

with

$$A = \begin{pmatrix} \frac{1}{\theta \mu} + \frac{\tilde{\sigma} \kappa}{\mu \beta} & -\frac{\tilde{\sigma}}{\mu \beta} \\ -\frac{\kappa}{\mu \beta} & \frac{1}{\mu \beta} \end{pmatrix}$$

$$|A| = \left(\frac{1}{\theta \mu} + \frac{\tilde{\sigma} \kappa}{\mu \beta} \right) \frac{1}{\mu \beta} - \frac{\kappa}{\mu \beta} \frac{\tilde{\sigma}}{\mu \beta} = \frac{1}{\theta \mu^2 \beta}$$

$$tr(A) = \frac{1}{\theta \mu} + \frac{\tilde{\sigma} \kappa}{\mu \beta} + \frac{1}{\mu \beta}$$

Case I Woodford (2003): $\det(A) > 1$; $\det(A) - tr(A) > -1$; $\det(A) + tr(A) > -1$

$$|A| = \frac{1}{\theta \mu^2 \beta} > 1$$

Always true!!

$$\beta = \frac{\theta}{\bar{\pi}}$$

$$E_t \hat{R}_{t+1}^K - \hat{R}_t = \chi^{\phi^e} \hat{\phi}_t^e$$

$$|A| - tr(A) > -1 \Leftrightarrow \frac{1}{\theta \mu^2 \beta} - \frac{1}{\theta \mu} - \frac{\tilde{\sigma} \kappa}{\beta \mu} - \frac{1}{\mu \beta} > -1 \Leftrightarrow 1 - \frac{\mu \beta}{1} - \frac{\theta \tilde{\sigma} \kappa \mu}{1} + -\theta \mu > -\theta \mu^2 \beta$$

$$\Leftrightarrow 1 - \mu\beta - \theta\mu + \theta\mu^2\beta > \theta\tilde{\sigma}\kappa\mu$$

$$\Leftrightarrow (1 - \mu\beta)(1 - \theta\mu) > \theta\tilde{\sigma}\kappa\mu$$

9.2 GDP and inflation in the low state with fiscal stimulus

Euler equation (from (70))

$$\hat{Y}_L - \sigma_{g,L}\hat{G}_L = \theta(\mu\hat{Y}_L - \mu\sigma_{g,L}\hat{G}_L + \lambda(1-\mu)\hat{Y}_S - \lambda(1-\mu)\sigma_{g,S}\hat{G}_S) - \tilde{\sigma}\theta \left(\hat{R}_L - \mu\hat{\Pi}_L - (1-\mu)\lambda\hat{\Pi}_S - r_L \right)$$

$$\left(\hat{Y}_L - \sigma_{g,L}\hat{G}_L \right) (1 - \theta\mu) = \tilde{\sigma}\theta \left(\mu\hat{\Pi}_L + (1-\mu)\lambda\hat{\Pi}_S + r_L - \hat{R}_L \right) + \theta\lambda(1-\mu) \left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S \right) \quad (73)$$

Phillips Curve (from (71))

$$\hat{\Pi}_L = \kappa(\hat{Y}_L - \Gamma\hat{G}_L) + \beta \left(\mu\hat{\Pi}_L + (1-\mu)\lambda\hat{\Pi}_S \right)$$

$$\hat{\Pi}_L = \frac{\kappa}{1-\beta\mu}(\hat{Y}_L - \Gamma\hat{G}_L) + \frac{\beta(1-\mu)\lambda\hat{\Pi}_S}{1-\beta\mu} \quad (74)$$

Combining (73) and (74):

$$\begin{aligned} \left(\hat{Y}_L - \sigma_{g,L}\hat{G}_L \right) (1 - \theta\mu) &= \tilde{\sigma}\theta \left(-\hat{R}_L + \mu\frac{\kappa}{1-\beta\mu}(\hat{Y}_L - \Gamma\hat{G}_L) + r_L + (1-\mu)\lambda\hat{\Pi}_S + \frac{\mu\beta(1-\mu)\lambda\hat{\Pi}_S}{1-\beta\mu} \right) \\ &\quad + \theta\lambda(1-\mu) \left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S \right) \end{aligned}$$

$$\hat{Y}_L \left((1 - \theta\mu) - \mu\frac{\kappa\tilde{\sigma}\theta}{1-\beta\mu} \right) = \left((1 - \theta\mu)\sigma_{g,L} - \tilde{\sigma}\theta \left(\mu\frac{\kappa}{1-\beta\mu}\Gamma \right) \right) \hat{G}_L + \tilde{\sigma}\theta \left(r_L - \hat{R}_L + \frac{1-\beta\mu+\beta\mu}{1-\beta\mu} (1 - \theta\mu) \right)$$

$$\hat{Y}_L = \frac{\left((1 - \theta\mu)\sigma_g - \tilde{\sigma}\theta \left(\mu\frac{\kappa}{1-\beta\mu}\Gamma \right) \right) \hat{G}_L + \tilde{\sigma}\theta \left(r_L - \hat{R}_L + \frac{1}{1-\beta\mu} (1 - \mu) \lambda\hat{\Pi}_S \right)}{\left((1 - \theta\mu) - \mu\frac{\kappa\tilde{\sigma}\theta}{1-\beta\mu} \right)}$$

$$+ \frac{\theta\lambda(1-\mu) \left(\hat{Y}_S - \sigma_{g,S}\hat{G}_S \right)}{\left((1 - \theta\mu) - \mu\frac{\kappa\tilde{\sigma}\theta}{1-\beta\mu} \right)}$$

$$\hat{Y}_L = \frac{((1 - \beta\mu)(1 - \theta\mu)\sigma_g - \tilde{\sigma}\theta\mu\kappa\Gamma)\hat{G}_L + \tilde{\sigma}\theta(1 - \beta\mu)\left(r_L - \hat{R}_L + \frac{1}{1 - \beta\mu}\lambda(1 - \mu)\hat{\Pi}_S\right)}{((1 - \theta\mu)(1 - \beta\mu) - \mu\kappa\theta\tilde{\sigma})}$$

$$+ \frac{\theta(1 - \mu)\lambda(1 - \beta\mu)(\hat{Y}_S - \sigma_{g,S}\hat{G}_S)}{((1 - \theta\mu)(1 - \beta\mu) - \mu\kappa\theta\tilde{\sigma})}$$

With $\hat{G}_S = \hat{G}_L$, we have $\hat{Y}_S = \gamma_y\hat{G}_S + i_y e_{i,S}$ and $\hat{\Pi}_S = \gamma_\pi\hat{G}_S + i_\pi e_{i,S}$
Solution for

$$\hat{Y}_L = \frac{(1 - \theta\mu)(1 - \beta\mu)\sigma_g - \kappa\theta\tilde{\sigma}\mu\Gamma + \tilde{\sigma}\theta(1 - \mu)\lambda\gamma_\pi + \theta\lambda(1 - \mu)(1 - \beta\mu)(\gamma_y - \sigma_{g,S})}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu}\hat{G}_L \quad (75)$$

$$+ \frac{\tilde{\sigma}\theta(1 - \beta\mu)}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu}\left(r_L - \hat{R}_L\right) + \frac{\tilde{\sigma}\theta(1 - \mu)\lambda i_\pi + \theta\lambda(1 - \mu)(1 - \beta\mu)i_y}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu}e_{i,S}$$

$$\frac{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu\Gamma}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu} > 1 \text{ since } \Gamma < 1$$

Solution for $\hat{\Pi}_L$

$$\hat{\Pi}_L = \left(\kappa \frac{(1 - \theta\mu)(\sigma_g - \Gamma) + \frac{\tilde{\sigma}\theta(1 - \mu)\lambda\gamma_\pi}{1 - \beta\mu} + \theta\lambda(1 - \mu)(\gamma_y - \sigma_{g,S})}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu} + \frac{\beta(1 - \mu)\lambda\gamma_\pi}{1 - \beta\mu} \right) \hat{G}_L + \frac{\kappa\tilde{\sigma}\theta(1 - \mu)\lambda i_\pi + \theta\lambda(1 - \mu)(1 - \beta\mu)i_y}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu} e_{i,S}$$

$$+ \left[\kappa \left(\frac{\frac{\tilde{\sigma}\theta(1 - \mu)\lambda}{1 - \beta\mu} i_\pi + \theta\lambda(1 - \mu)i_y}{(1 - \theta\mu)(1 - \beta\mu) - \kappa\theta\tilde{\sigma}\mu} \right) + \frac{\beta(1 - \mu)\lambda i_\pi}{1 - \beta\mu} \right] e_{i,S}$$

10 Forward guidance: Extension of fixed interest rate policy until after $r = 0$

After r returns to zero, with probability λ_f , the Central bank keeps the nominal interest rate fixed at level \hat{R}_f . It will keep it at this level with probability μ_f , and will return it to zero with probability $1 - \mu_f$. Hence GDP in the low state is now given by

$$\hat{Y}_L = \frac{\tilde{\sigma}\theta(1 - \beta\mu)\left(r_L - \hat{R}_L + \frac{1}{1 - \beta\mu}\lambda_f(1 - \mu)\hat{\Pi}_f\right)}{((1 - \theta\mu)(1 - \beta\mu) - \mu\kappa\theta\tilde{\sigma})} + \frac{\theta(1 - \mu)\lambda_f(1 - \beta\mu)\hat{Y}_f}{((1 - \theta\mu)(1 - \beta\mu) - \mu\kappa\theta\tilde{\sigma})}$$

where $\hat{\Pi}_f$ and \hat{Y}_f refer to inflation and GDP once r is at zero but the policy rate is still fixed. $\hat{\Pi}_f$ and \hat{Y}_f are given by

$$\hat{Y}_f = -\frac{\tilde{\sigma}\theta(1 - \beta\mu_f)}{(1 - \theta\mu_f)(1 - \beta\mu_f) - \kappa\theta\tilde{\sigma}\mu_f}\left(\hat{R}_f\right) \quad (76)$$

$$\hat{\Pi}_f = - \frac{\kappa \tilde{\theta}}{(1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f} (\hat{R}_f)$$

Hence

$$\begin{aligned} \hat{Y}_L &= - \left[\frac{\tilde{\theta} \lambda_f (1 - \mu) \kappa \tilde{\theta}}{((1 - \theta \mu)(1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma})((1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)} + \frac{\theta(1 - \mu) \lambda_f (1 - \beta \mu)}{((1 - \theta \mu)(1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma})((1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)} \right] \\ &= - \left[\frac{\tilde{\theta}^2 \lambda_f (1 - \mu) (\kappa \tilde{\theta} + (1 - \beta \mu)(1 - \beta \mu_f))}{((1 - \theta \mu)(1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma})((1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)} \right] \hat{R}_f + \frac{\tilde{\theta} \theta (1 - \beta \mu)}{(1 - \theta \mu)(1 - \beta \mu) - \kappa \theta \tilde{\sigma} \mu_f} (r_L) \end{aligned}$$

and

$$\begin{aligned} \hat{\Pi}_L &= \frac{\kappa}{1 - \beta \mu} \hat{Y}_L + \frac{\beta(1 - \mu) \lambda_f \hat{\Pi}_f}{1 - \beta \mu} \\ &= - \left[\frac{\kappa \tilde{\theta}^2 \lambda_f (1 - \mu) \left(\frac{\kappa \tilde{\theta}}{1 - \beta \mu} + (1 - \beta \mu_f) \right)}{((1 - \theta \mu)(1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma})((1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)} \right] \hat{R}_f + \frac{\tilde{\theta} \theta (1 - \beta \mu)}{(1 - \theta \mu)(1 - \beta \mu) - \kappa \theta \tilde{\sigma} \mu_f} (r_L) \\ &= - \left[\frac{\kappa \tilde{\theta}^2 \lambda_f (1 - \mu) \left(\frac{\kappa \tilde{\theta}}{1 - \beta \mu} + (1 - \beta \mu_f) \right)}{((1 - \theta \mu)(1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma})((1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)} \right] \hat{R}_f + \frac{\tilde{\theta} \theta (1 - \beta \mu)}{(1 - \theta \mu)(1 - \beta \mu) - \kappa \theta \tilde{\sigma} \mu_f} (r_L) \\ &= - (1 - \mu) \lambda_f \kappa \tilde{\theta} \left[\frac{\theta \left(\frac{\kappa \tilde{\theta}}{1 - \beta \mu} + 1 - \beta \mu_f \right) + \beta \left(1 - \theta \mu - \frac{\mu \kappa \theta \tilde{\sigma}}{1 - \beta \mu} \right)}{((1 - \theta \mu)(1 - \beta \mu) - \mu \kappa \theta \tilde{\sigma})((1 - \theta \mu_f)(1 - \beta \mu_f) - \kappa \theta \tilde{\sigma} \mu_f)} \right] \hat{R}_f + \frac{\tilde{\theta} \theta (1 - \beta \mu)}{(1 - \theta \mu)(1 - \beta \mu) - \kappa \theta \tilde{\sigma} \mu_f} (r_L) \end{aligned}$$

11 Medium scale model equations

$$\hat{\lambda}_{ROT,t} = - \frac{\hat{C}_{ROT,t} - h \hat{C}_{ROT,t-1}}{\sigma_{ROT} (1 - h)} - \frac{\hat{\tau}_{C,t}}{1 - \tau_C}$$

$$(1 + \tau_C) C_{ROT} \hat{C}_{ROT,t} + C_{ROT} \hat{R}_t R = (1 - \rho_i) \left[\psi_\pi \hat{\Pi}_t + \psi_y \left(\widehat{GDP}_t - \widehat{GDP}_{t-1} \right) \right] + \rho_i \hat{R}_{t-1} + e_t^i \hat{\tau}_{C,t} = w_t$$

$$\hat{\lambda}_{O,t} = - \frac{\hat{C}_{O,t} - h \hat{C}_{O,t-1}}{\sigma_O (1 - h)} - \frac{\hat{\tau}_{C,t}}{1 - \tau_C}$$

$$\hat{\lambda}_{O,t} = E_t \left\{ \hat{\lambda}_{O,t+1} + \hat{R}_t - \hat{\Pi}_{t+1} \right\}$$

$$\lambda \hat{\lambda}_t = (1 - \omega) \lambda_O \hat{\lambda}_{O,t} + \omega \lambda_{ROT} \hat{\lambda}_{ROT,t}$$

$$\hat{l}_{ROT,t} = \hat{l}_{O,t}$$

$$\hat{l}_t = \hat{l}_{ROT,t}$$

$$\hat{w}_t = \frac{1}{1+\beta} \left[\begin{array}{l} \beta E_t \hat{w}_{t+1} + \hat{w}_{t-1} + \beta E_t \hat{\Pi}_{t+1} - (1 + \beta \gamma_w) \hat{\Pi}_t \\ + \gamma_w \hat{\Pi}_{t-1} - \frac{(1-\beta\xi^w)(1-\xi^w)}{\xi^w(1+\varepsilon^w\varphi)} \left[\hat{w}_t - \frac{\hat{\tau}_{W_h,t}}{1-\tau_{W_h}} + \hat{\lambda}_t - \varphi \hat{l}_t \right] \end{array} \right] \text{Wage setting}$$

$$\hat{\Pi}_t = \frac{1}{1+\beta\gamma_P} \left[\beta E_t \hat{\Pi}_{t+1} + \gamma_P \hat{\Pi}_{t-1} + \frac{(1-\beta\xi^P)(1-\xi^P)}{\xi^P} \widehat{mc}_t \right] \text{Price setting}$$

$$\hat{w}_t + \frac{\hat{\tau}_{W_f,t}}{1-\tau_{W_f}} + \frac{\psi_l R \hat{R}_t}{1+\psi_l(R-1)} = \widehat{mc}_t + \hat{Y}_t - \hat{l}_t$$

$$\frac{dr_t^k}{r^k} + \frac{\psi_K R \hat{R}_t}{1+\psi_K(R-1)} = \widehat{mc}_t + \hat{Y}_t - \hat{K}_{t-1} - \hat{U}_t$$

$$L^r \hat{L}_t^r = \psi_l w l (\hat{w}_t + \hat{l}_t) + \psi_K r^k K \left(\frac{dr_t^k}{r^k} + \hat{U}_t + \hat{K}_{t-1} \right)$$

$$\hat{K}_t = (1-\delta) \hat{K}_{t-1} + \delta \hat{I}_t$$

$$\hat{I}_t = \frac{1}{1+\beta} \left[\hat{I}_{t-1} + \beta E_t \hat{I}_{t+1} + \frac{\hat{Q}_t}{\phi_I} \right]$$

$$\hat{R}_t^K = \hat{\Pi}_t + \frac{\Pi \left(dr_t^k (1-\tau_K) + \hat{Q}_t (1-\delta) \right)}{R^k} - \hat{Q}_{t-1}$$

$$dr_t^k = c^U r^k \hat{U}_t$$

$$\hat{\phi}_t^e = \hat{Q}_t + \hat{K}_t - \hat{N}_t$$

$$E_t \hat{R}_{t+1}^K - \hat{R}_t = \chi^{\phi^e} \hat{\phi}_t^e$$

$$\widehat{\varpi}_t + \widehat{R}_t^K = \widehat{R}_t^L + \frac{1}{\phi^e - 1} \widehat{\phi}_{t-1}^e$$

$$R \frac{\widehat{\phi}_{t-1}^e}{\phi^e} = e_t^{R^b} R^K Expr1 + \widehat{R}_t^K R^K Expr1 + R^K Expr2 \varpi \widehat{\varpi}_t - R \frac{\phi^e - 1}{\phi^e} \widehat{R}_{t-1}$$

$$\widehat{N}_t = \gamma \frac{V}{N} \widehat{V}_t$$

$$\widehat{V}_t = \widehat{N}_{t-1} + \widehat{R}_t^K - \widehat{\Pi}_t + \widehat{\phi}_{t-1}^e - \frac{\Gamma'(\bar{\omega}) \bar{\omega}}{1 - \Gamma(\bar{\omega})} \widehat{\varpi}_t + e_t^N$$

$$\widehat{L}_t^e = \widehat{N}_t + \frac{\phi^e}{\phi^e - 1} \widehat{\phi}_t^e$$

$$\widehat{C}_t^e = \widehat{V}_t$$

$$\widehat{R}_t R = (1 - \rho_i) \left[\psi_\pi \widehat{\Pi}_t + \psi_y \left(\widehat{GDP}_t - \widehat{GDP}_{t-1} \right) \right] + \rho_i \widehat{R}_{t-1} + e_t^i$$

$$\widehat{Y}_t = \alpha \left(\widehat{U}_t + \widehat{K}_{t-1} \right) + (1 - \alpha) \left(\widehat{a}_t + \widehat{l}_t \right)$$

$$\widehat{b}_t b = b \frac{R}{\Pi} \left(\widehat{b}_{t-1} + \widehat{R}_{t-1} - \widehat{\Pi}_t \right) + Gov \widehat{Gov}_t + TRT \widehat{R}_t - \left(\widehat{C}_t^P \tau_C + \tau_C C^P + (\tau_{W_h,t} + \tau_{W_f,t}) w l + \tau_K r^k K \widehat{K}_{t-1} + T \widehat{T}_t \right)$$

$$\widehat{\tau}_{W_h,t} = (1 - \rho_\tau) \phi_\tau \widehat{b}_t + \rho_\tau \widehat{\tau}_{W_h,t-1}$$

$$\widehat{Y}_t = \frac{I}{Y} \widehat{I}_t + \frac{C^P}{Y} \widehat{C}_t^P + \frac{Gov}{Y} \widehat{Gov}_t + \frac{R^K}{\Pi} \frac{K}{Y} \mu G(\varpi) \left(\widehat{R}_t^K - \widehat{\Pi}_t + \widehat{Q}_{t-1} + \widehat{K}_{t-1} + \frac{G'(\varpi)}{G(\varpi)} \varpi \widehat{\varpi}_t \right) + r^k \frac{K}{Y} \widehat{U}_t$$

$$C \widehat{C}_t = \omega C_{ROT} \widehat{C}_{ROT,t} + (1 - \omega) C_O \widehat{C}_{O,t}$$

$$\widehat{C}_t^P = \frac{C}{C^P} \widehat{C}_t + \frac{C^e}{C^P} \widehat{C}_t^e$$

$$\widehat{GDP}_t = \frac{I}{GDP} \widehat{I}_t + \frac{C^P}{GDP} \widehat{C}_t^P + \frac{Gov}{GDP} \widehat{g}_t$$

$$Expr1 = [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]$$

$$Expr2 = [\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]$$

$$R \frac{\widehat{Q}_{t-1} + \widehat{K}_{t-1} - \widehat{N}_{t-1}}{\phi^e} = e_t^{R^b} R^K Expr1 + \widehat{R}_t^K R^K Expr1 + R^K Expr2 \bar{\omega} \widehat{\omega}_t - R \frac{\phi^e - 1}{\phi^e} \widehat{R}_{t-1}$$

$$\widehat{N}_t = \gamma \frac{V}{N} \widehat{V}_t$$

$$\widehat{V}_t = \widehat{Q}_{t-1} + \widehat{K}_{t-1} + \widehat{R}_t^K - \widehat{\Pi}_t - \frac{\Gamma'(\bar{\omega}) \bar{\omega}}{1 - \Gamma(\bar{\omega})} \widehat{\omega}_t$$