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Interest Rates Under Falling Stars

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Interest Rates Under Falling Stars

Abstract

Theory predicts that the equilibrium real interest rate, r^*_t , and the perceived trend in inflation, π^*_t , are key determinants of the term structure of interest rates. However, term structure analyses generally assume that these endpoints are constant. Instead, we show that allowing for time variation in both r^*_t and π^*_t is crucial for understanding the empirical dynamics of U.S. Treasury yields and risk pricing. Our evidence reveals that accounting for fluctuations in both r^*_t and π^*_t substantially increases the accuracy of long-range interest rate forecasts, helps predict excess bond returns, improves estimates of the term premium in long-term interest rates, and captures a substantial share of interest rate variability at low frequencies.

JEL-Codes: E430, E440, E470.

Keywords: yield curve, macro-finance, inflation trend, equilibrium real interest rate, shifting endpoints, bond risk premia.

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1 Introduction

The level of the risk-free real rate of return that would prevail in the absence of transitory disturbances has long been of interest to researchers, policymakers, and financial investors. However, recent structural changes in the global economy have brought questions about the variation and influence of this equilibrium real rate to the forefront (Clarida, 2014; Yellen, 2015). Various underlying fundamental economic forces, such as lower productivity growth and an aging population, appear to have slowly altered global saving and investment and, in turn, pushed down the steady-state real interest rate. Accordingly, much new research has examined potential changes in the level of the equilibrium real short-term interest rate, which we denote as r_t^* .¹ Remarkably, thus far, there has been very little work addressing the effects of changes in r_t^* on the dynamics of the term structure of interest rates. We fill this gap and analyze how changes in the equilibrium real rate will alter yield curves and bond risk pricing.

A useful illustration of the potential importance of accounting for the equilibrium real rate is provided in Figure 1. The secular decline in the 10-year Treasury yield since the early 1980s reflects a gradual downtrend in the general level of U.S. interest rates. Early on, a key driver of this decline was the reduction in the long-run expected level of inflation, which is often referred to as trend inflation and denoted as π_t^* . Figure 1 displays a measure of U.S. trend inflation based largely on long-horizon inflation projections from a survey of professional forecasters (details of this measure are described in the data section below). While the link between trend inflation and interest rates has been highlighted by some, e.g., Cieslak and Povala (2015), this macroeconomic trend clearly cannot account for all of the decline in yields on its own. For the past two decades, π_t^* has stabilized close to 2 percent while longer-term interest rates have continued to drift lower. Instead, much of the recent downtrend in yields seems to reflect a lower equilibrium real short-term interest rate, as is evident in the simple composite measure of r_t^* shown in Figure 1 (details of this measure are also described below). According to this measure, the equilibrium real short rate remained little changed in a range between 2 and 3 percent from 1970 to around 2000 when it started to gradually fall to below 1 percent currently. We argue that accounting for this recent downtrend in r_t^* as well as the earlier decline in π_t^* —that is, an environment of falling stars—is critical for understanding the dynamics of interest rates and for assessing bond risk and expected returns.

Long-term nominal interest rates reflect expectations of future inflation and real short rates—including their trend components—subject to a risk adjustment. We first demonstrate

¹ Discussions of the decline in r^* include Hamilton et al. (2016), Rachel and Smith (2015), Laubach and Williams (2003), Johannsen and Mertens (2016), Kiley (2015), Lubik and Matthes (2015), Christensen and Rudebusch (2017), and Holston et al. (2017) among many others. In the macroeconomics literature, r_t^* is often labeled the neutral or natural rate of interest.

the theoretical significance of such longer-run components in an affine no-arbitrage term structure model that allows for stochastic trends in inflation and the real short rate— π_t^* and r_t^* . The model shows that π_t^* and r_t^* act as level factors for the nominal yield curve, as suggested by Figure 1. In addition, the model describes how the cyclical components of inflation and the real rate disproportionately influence the shorter end of the yield curve and how a risk-premium factor affects long rates more than short rates. Our model leads to testable implications about the importance of both r_t^* and π_t^* for interest rates, which we take to the data. Our empirical examination considers five tests of whether the trends in inflation and the real rate are quantitatively important for the determination of interest rates, using common empirical proxies of these macroeconomic trends.

As a first test, we document that time variation in both π_t^* and r_t^* is responsible for most of the persistence in yields. While interest rates themselves are extremely persistent, the difference between long-term interest rates and the equilibrium nominal short rate, which is denoted $i_t^* = \pi_t^* + r_t^*$, exhibits quick mean reversion. Furthermore, regressions of long-term yields on the macroeconomic trends recover the unit coefficients predicted by our theoretical model. Our findings generalize the results of Cieslak and Povala (2015), henceforth CP, who regress nominal yields on a proxy for trend inflation. While we confirm their finding that π_t^* is an important persistent component of interest rates, we show that it is also crucial to include r_t^* in order to fully capture the trend component. Furthermore, after accounting for shifts in r_t^* , we uncover strong evidence for a long-run Fisher effect, which has found at best lukewarm support in previous studies that have focused only on a bivariate relationship between yields and inflation.² Instead, from our broader perspective that allows for fluctuations in the equilibrium real rate, we find that the trend components of inflation and the real rate are related to interest rates exactly as standard finance theory predicts.

Second, we show that accounting for both macroeconomic trends leads to substantial improvements in interest rate forecast accuracy at medium and long forecast horizons relative to the usual martingale forecasting benchmark. Our theoretical model shows that such forecast improvements are a natural consequence of the time series properties of inflation and the real short rate. We document that, relative to the random walk alternative, the improvements in forecast accuracy from including the trend components are both economically and statistically significant. In addition, as predicted by our model, to achieve these better forecasts, it is crucial to incorporate r_t^* in the shifting endpoint of nominal interest rates—including only π_t^* is insufficient.

Third, accounting for a time-varying r_t^* is important for understanding return predictabil-

²Prominent examples include Mishkin (1992), Wallace and Warner (1993), and Evans and Lewis (1995).

ity and estimating bond risk premia. Recently, CP used the connection between trend inflation and interest rates to decompose the nominal yield curve into risk premiums and the expectations hypothesis (EH) component (i.e., a term that equals the average short-term interest rate that investors expect to prevail during the life of a bond). They present strong evidence for the predictive power of the inflation trend for excess bond returns, which suggests that incorporating the inflation trend is important to understand the risk premium/EH decomposition. Since r_t^* plays the same role of a persistent level factor for nominal interest rates as does π_t^* , our theoretical analysis then suggests that r_t^* , like π_t^* , should have predictive power for bond returns. Hence, we extend the results of CP by introducing shifts in the equilibrium real interest rate into the prediction of risk premiums, and we find substantial improvements from incorporating r_t^* in predicting excess bond returns.

Fourth, these bond return results suggest that measures of the bond risk premium should account for movements in both r_t^* and π_t^* . Therefore, we provide an empirical decomposition of long-term yields into expectations and term premium components that takes into account the macroeconomic trends. We contrast our estimated decomposition with a conventional one based on a stationary VAR of yield-curve factors. The conventional decomposition implies an implausibly stable expectations component and attributes most of the secular decline in interest rates to the residual term premium, as discussed in critiques by [Kim and Orphanides \(2012\)](#) and [Bauer et al. \(2014\)](#). Our decomposition, which allows the mean of nominal yields to shift with i_t^* , leads to a very different interpretation of the historical evolution of long-term yields: the majority of their secular decline is attributed to the decrease in i_t^* . Consequently, the term premium, instead of exhibiting a dubious secular downtrend, behaves in a predominantly cyclical fashion like other risk premia in asset prices ([Campbell and Cochrane, 1999](#)).

As a final avenue of empirical examination, we compare the variance of changes in the trend components of inflation and real rates to the variance of interest rate changes at different frequencies. [Duffee \(2016\)](#) proposes using the ratio of the variance of inflation news to the variance of innovations in nominal interest rates as a useful metric to assess the importance of inflation in the determination of interest rates. He documents that for one-quarter innovations, this ratio is small for U.S. Treasury yields. We generalize his measure to consider variance ratios for longer h -period innovations, which allows us to compare the size of unexpected changes, over, say, a span of five years, in inflation and in nominal bond yields. For one-quarter changes, we replicate the small inflation variance ratio reported by Duffee. But in line with our theoretical predictions, the inflation variance ratio increases substantially with the horizon. We also generalize Duffee's measure to incorporate fluctuations in r_t^* and i_t^* . Although confidence intervals are unavoidably wide, our estimates suggest that during the

postwar U.S. sample, a large share of the interest rate variability faced by investors over longer holding periods was due to changes in the macroeconomic trend components of nominal yields.

Taken altogether, these five complementary strands of evidence provide the key contribution of our paper: strong empirical support for the significance of macroeconomic trends in understanding the nominal yield curve. Variations in both the inflation trend and the equilibrium real interest rate are shown to be quantitatively important, and while the former has been increasingly recognized in yield curve analysis, a time-varying r_t^* has been essentially ignored—a substantial oversight in the literature. Our empirical evidence linking inflation and real rate trends to the yield curve and risk pricing has far-reaching implications for modeling of interest rates and bond risk premia. In order to accurately capture interest rate dynamics, both structural and reduced-form models of the yield curve should allow for slow-moving trend components. The common approach of specifying a stationary system for the term structure of interest rates with constant means for the risk factors is problematic, because it counterfactually rules out any structural, long-run changes of economic variables and asset prices. Accounting for such long-run changes, i.e., for shifting trends, is crucial for researchers and practitioners assessing the term premium in long-term interest rates, forecasting bond yields and returns, and understanding the drivers and historical evolution of the term structure of interest rates.

Our paper relates to several strands of literature. A large literature documents the importance of a trend component in the inflation rate, notably [Kozicki and Tinsley \(2001\)](#), [Stock and Watson \(2007\)](#), [Faust and Wright \(2013\)](#), and [Clark and Doh \(2014\)](#). The persistence in the real interest rate is considered early on by [Rose \(1988\)](#) and was surveyed by [Neely and Rapach \(2008\)](#). A related literature investigates the long-run Fisher effect (see the references in footnote 2). Much recent literature uses macroeconomic models and data to estimate the equilibrium real interest rate and to understand its structural drivers (see the references in footnote 1). Our paper sheds a new light on these related strands of literature by showing that the apparent trends in inflation and real interest rates are not only consistent with the observed behavior of the yield curve, but are in fact critically important to understanding that behavior. Within the macro-finance literature on the term structure of interest rates, some models have been proposed that allow for changes in trend inflation; see [Hördahl et al. \(2006\)](#), [Rudebusch and Wu \(2008\)](#), and [Bekaert et al. \(2010\)](#). However these models, like essentially all equilibrium models (including [Wachter, 2006](#); [Bansal and Shaliastovich, 2013](#)) and reduced-form no-arbitrage models (such as [Chernov and Mueller, 2012](#); [Joslin et al., 2014](#)) of the term structure of interest rates, generally assume a constant equilibrium real rate, in contrast to

the findings of the literature cited above.³ Finally, we contribute to the literature on interest rate forecasting. The work in this area most closely related to ours is [Dijk et al. \(2014\)](#), who documented improvements in forecast relative to a random walk by including shifting endpoints that were linked to proxies for trend inflation. We demonstrate that substantial further gains are possible by forecasting that interest rates gradually revert to the endpoint i_t^* which includes the equilibrium real rate

The paper is structured as follows: Section 2 introduces a simple no-arbitrage model of the yield curve that allows for trend components in inflation and the real rates, and derives testable implications. Section 3 describes our empirical proxies for π_t^* and r_t^* . In Section 4, we investigate sources for the persistence of interest rates. In Section 5, we show substantial improvements in forecast accuracy are possible after accounting for macroeconomic trends in interest rates. Section 6 documents the evidence for the incremental predictive power of π_t^* and r_t^* for future excess bond returns. In Section 7, we present a novel estimate of the term premium in long-term interest rates that accounts for shifting endpoints. Section 8 provides new variance ratios for assessing the importance of the inflation and real-rate trends for interest rate changes at different frequencies. Section 9 concludes.

2 A no-arbitrage model with macro trends

Our theoretical framework is a stylized affine term structure model for real and nominal yields that demonstrates how, under absence of arbitrage, changes in π_t^* and r_t^* —the inflation trend and the equilibrium real short rate—affect interest rates. Although our framework is quite parsimonious—with few risk factors, no stochastic volatility, and strong risk pricing restrictions—it is sufficient to provide some important testable predictions about fundamental aspects of the relationship between the trend in inflation, the equilibrium real rate, and the yield curve.⁴

We model inflation, π_t , as the sum of trend, cycle, and noise components:

$$\pi_{t+1} = \pi_t^* + c_t + e_{t+1}, \quad \pi_t^* = \pi_{t-1}^* + \xi_t, \quad c_t = \phi_c c_{t-1} + u_t,$$

where the innovations ξ_t and u_t and the noise component e_t are mutually independent *iid*

³The only exceptions we are aware of are [Ang et al. \(2008\)](#), who allow regime switching in the mean of the real short rate, and the macro-finance models of Hans Dewachter and co-authors, including [Dewachter and Lyrjo \(2006\)](#) where r_t^* is deterministically linked to π_t^* and [Dewachter and Iania \(2011\)](#) where r_t^* is tied to trend output growth.

⁴This model generalizes the one in CP to allow for time variation in r_t^* and flexible trend-cycle dynamic specifications.

normal random variables with standard deviations σ_ξ , σ_u and σ_e . Inflation expectations at various horizons are given by $E_t\pi_{t+h+1} = \pi_t^* + \phi_c^h c_t$. In the limit, expectations converge to the time-varying inflation endpoint, $\lim_{h \rightarrow \infty} E_t\pi_{t+h} = \pi_t^*$, which can be viewed as the perceived inflation target of the central bank. This specification of inflation dynamics is in line with the modern empirical macro literature, which allows for a trend component in inflation (e.g., [Stock and Watson, 2007](#)) and is similar to the one in [Duffee \(2016\)](#). We assume that the shocks ξ_t and u_t affect only expectations of future inflation but not current inflation, which slightly simplifies the bond pricing formulas but has no fundamental significance.

The one-period real rate, r_t , also has trend and stationary components, respectively, the equilibrium real rate, r_t^* , and the cyclical real-rate gap, g_t :

$$r_t = r_t^* + g_t, \quad r_t^* = r_{t-1}^* + \eta_t, \quad g_t = \phi_g g_{t-1} + v_t.$$

The shocks are again mutually uncorrelated and *iid* normal, with standard deviations σ_η and σ_v . Since $\lim_{h \rightarrow \infty} E_t r_{t+h} = r_t^*$, the equilibrium real rate can be understood as the real rate that prevails in the economy after all shocks have died out.

Formally, of course, this specification includes an exact unit root in inflation and the real rate. We view this assumption merely as a convenient way to model very persistent processes. The use of unit roots simplifies the exposition of our model and the arguments regarding trend components, but it is not crucial. Taken literally, a unit root specification is implausible because the forecast error variances of inflation and real rates do not in fact increase linearly with the forecast horizon as predicted by a unit root. Instead, both variables have always remained within certain bounds. However, in finite samples, a stationary process can always be approximated arbitrarily well by a unit root process, and it is well-known that doing so can often be beneficial for forecasting (e.g., [Campbell and Perron, 1991](#)). Thus, π_t^* and r_t^* can be viewed simply as highly persistent components of π_t and r_t that capture expectations at the long horizons relevant for investors, even if infinite-horizon expectations are constant. In practice, these relevant time horizons are often in the 5- to 10-year range when cyclical shocks have largely dissipated, as noted by [Laubach and Williams \(2003\)](#) and [Summers \(2015\)](#).

The final state variable determining interest rates is a risk price factor x_t , which follows an independent autoregressive process:⁵

$$x_t = \mu_x + \phi_x x_{t-1} + w_t,$$

⁵[Cochrane and Piazzesi \(2005\)](#) provide evidence supporting a stationary single factor driving bond risk premiums. Also, see the discussion in CP.

where w_t is *iid* normal with standard deviation σ_w . We collect the state variables as $Z_t = (\pi_t^*, c_t, r_t^*, g_t, x_t)'$, so their dynamics can be compactly written as a first-order vector autoregression, a VAR(1):

$$Z_t = \mu + \phi Z_{t-1} + \Sigma \varepsilon_t, \quad (1)$$

where $\mu = (0, 0, 0, 0, \mu_x)'$, $\phi = \text{diag}(1, \phi_c, 1, \phi_g, \phi_x)$, $\Sigma = \text{diag}(\sigma_\xi, \sigma_u, \sigma_\eta, \sigma_v, \sigma_w)$, and ε_t is a (5×1) *iid* standard normal vector process.

The model is completed by a specification for the log real stochastic discount factor (SDF), m_{t+1}^r , for which we choose the usual essentially affine form of [Duffee \(2002\)](#):

$$m_{t+1}^r = -r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}, \quad \lambda_t = \Sigma^{-1}(\lambda_0 + \lambda_1 Z_t).$$

We only allow x_t to affect the price of risk, so that the first four columns of λ_1 are zero. Furthermore, shocks to x_t are not priced, so that the last element of λ_0 and the last row of λ_1 are zero. The non-zero elements of λ_0 are denoted by $\lambda_{0\pi^*}$, λ_{0c} , λ_{0r^*} , and λ_{0g} , and those of λ_1 by λ_{π^*x} , λ_{cx} , λ_{r^*x} , and λ_{gx} .

The log nominal SDF is $m_{t+1}^n = m_{t+1}^r - \pi_{t+1}$, and the nominal short-term interest rate is

$$i_t = -E_t m_{t+1}^n - \frac{1}{2} \text{Var}_t(m_{t+1}^n) = r_t^* + g_t + \pi_t^* + c_t - \frac{1}{2} \sigma_e^2.$$

Due to our timing assumption for the inflation process, and because the noise shocks e_t are not priced, there is no inflation risk premium in the nominal short rate.⁶ Prices of zero-coupon bonds with maturity n , denoted by $P_t^{(n)}$, are easily verified to be exponentially affine, i.e., $\log(P_t^{(n)}) = A_n + B_n' Z_t$, using the pricing equation $P_t^{(n+1)} = E_t(\exp(m_{t+1}^n) P_{t+1}^{(n)})$. The coefficients follow the usual recursions of affine term structure models (e.g., [Ang and Piazzesi, 2003](#)):

$$A_{n+1} = A_n + B_n'(\mu - \lambda_0) + C_n, \quad C_n := \frac{1}{2}(\sigma_e^2 + B_n' \Sigma \Sigma' B_n), \quad B_{n+1} = -(1, 1, 1, 1, 0)' + (\phi - \lambda_1)' B_n,$$

where C_n captures the convexity in bond prices. The initial conditions are $A_0 = 0, B_0 = (0, 0, 0, 0, 0)'$. For the individual loadings of bond prices on the risk factors we have

$$B_{n+1}^{\pi^*} = B_n^{\pi^*} - 1, \quad B_{n+1}^c = \phi_c B_n^c - 1, \quad B_{n+1}^{r^*} = B_n^{r^*} - 1, \quad B_{n+1}^g = \phi_g B_n^g - 1,$$

⁶That is, $\text{Cov}_t(m_{t+1}^r, \pi_{t+1}) = 0$. CP make the same assumption, which is justified because estimates of this short-run inflation risk premium are typically very small. Note that the nominal short rate equation can be related to the popular Taylor rule for monetary policy in which $r_t^* + \pi_t^*$ represents the level of the policy-neutral nominal short rate and $g_t + c_t$ captures the cyclical response of the central bank.

$$B_{n+1}^x = -\lambda_{\pi^*x} B_n^{\pi^*} - \lambda_{cx} B_n^c - \lambda_{r^*x} B_n^{r^*} - \lambda_{gx} B_n^g + \phi_x B_n^x,$$

and the explicit solutions are

$$B_n^{\pi^*} = -n, \quad B_n^c = \frac{\phi_c^n - 1}{1 - \phi_c}, \quad B_n^{r^*} = -n, \quad B_n^g = \frac{\phi_g^n - 1}{1 - \phi_g},$$

$$B_n^x = \frac{\lambda_{\pi^*x} + \lambda_{r^*x}}{1 - \phi_x} \left(n - \frac{1 - \phi_x^n}{1 - \phi_x} \right) + \frac{\lambda_{cx}}{1 - \phi_c} \left(\frac{1 - \phi_x^n}{1 - \phi_x} - \frac{\phi_x^n - \phi_c^n}{\phi_x - \phi_c} \right) + \frac{\lambda_{gx}}{1 - \phi_g} \left(\frac{1 - \phi_x^n}{1 - \phi_x} - \frac{\phi_x^n - \phi_g^n}{\phi_x - \phi_g} \right).$$

For nominal bond yields, the model implies the following decomposition:

$$\begin{aligned} y_t^{(n)} &= -\log(P_t^{(n)})/n = -A_n/n - B'_n Z_t/n \\ &= \underbrace{\pi_t^* + \frac{1 - \phi_c^n}{n(1 - \phi_c)} c_t}_{\sum_{i=1}^n E_t \pi_{t+i}/n} + \underbrace{r_t^* + \frac{1 - \phi_g^n}{n(1 - \phi_g)} g_t}_{\sum_{i=0}^{n-1} E_t r_{t+i}/n} + \underbrace{-A_n/n - B_n^x x_t/n}_{\text{convexity and yield term premium}}. \end{aligned} \quad (2)$$

This equation shows the effect of fluctuations in the trend and cyclical components of inflation and real rates and the risk-premium factor on interest rates of different maturities. The trend components π_t^* and r_t^* naturally act as level factors by affecting yields of all maturities equally.⁷ The cyclical components c_t and g_t are slope factors as they affect short-term yields more strongly than long-term yields, and their loadings approach zero for large n . The risk-premium factor affects long-term yields more strongly than short-term yields: The loadings of yields on x_t start at zero and tend to $\lim_{n \rightarrow \infty} -B_n^x/n = -(\lambda_{\pi^*x} + \lambda_{r^*x})/(1 - \phi_x)$. Thus, long-term yields are mostly driven by the trend components π_t^* and r_t^* , as well as by the risk-premium factor x_t . Appendix A provides additional details and results for this affine model, including expressions for forward rates.

The model has several predictions about the relationship between interest rates and the two macroeconomic trends. A first straightforward prediction is that for any yield maturity n , the detrended yield $y_t^{(n)} - \pi_t^* - r_t^*$ will be much less persistent than the yield itself. The difference in persistence is particularly pronounced for long-term yields or forward rates, which place a heavy weight on the trend components. Strictly speaking, under the unit root specification, $y_t^{(n)}$ is I(1) while $y_t^{(n)} - \pi_t^* - r_t^*$ is I(0). Furthermore, a regression of $y_t^{(n)}$ on π_t^* and r_t^* is a cointegrating regression that should recover unit coefficients, since the cointegrating vector of $(y_t^{(n)}, \pi_t^*, r_t^*)$ is $(1, -1, -1)$. The model also predicts that a partial detrending with only

⁷The constant loadings of yields on the trend variables shows that unit roots are also present under the risk-neutral measure. Strictly speaking, this is inconsistent with absence of arbitrage—see [Dybvig et al. \(1996\)](#) and [Campbell et al. \(1997, p. 433\)](#)—because the convexity in $-A_n/n$ diverges to minus infinity. However, as in the case of the real-world measure, our predictions are unaffected if the largest roots under the risk-neutral measure are taken to be very close to but below one.

the inflation trend will be much less successful. Namely, $y_t^{(n)} - \pi_t^*$ will exhibit substantial persistence. Similarly, a regression of $y_t^{(n)}$ on π_t^* is a spurious regression that will not recover a unit coefficient and will produce residuals that remain highly persistent. We will investigate these predictions empirically in Section 4.

The fact that interest rates do not have a constant mean but contain the stochastic trend $i_t^* = \pi_t^* + r_t^*$ also has important implications for yield forecasts. From (1) and (2) we have

$$\lim_{h \rightarrow \infty} E_t y_{t+h}^{(n)} = k^{(n)} + \pi_t^* + r_t^*,$$

where the constant $k^{(n)}$ captures convexity and the unconditional mean term premium. Hence, interest rates mean-revert to a “shifting endpoint” (Kozicki and Tinsley, 2001) that is driven by both macroeconomic trends. For our empirical investigation we will ignore the constant $k^{(n)}$, which is of second-order concern for our purposes here. The model predicts that forecasts of long-horizon interest rates that incorporate knowledge of i_t^* are more accurate than forecasts that ignore it. In particular, long-range forecasts that account for i_t^* should improve upon the common forecast benchmark of a random walk. We will examine these potential forecast improvements empirically in Section 5.

A third area in which macro trends may be empirically relevant is for the prediction of excess bond returns. CP report substantial gains in such prediction from including their measure of trend inflation. In our model, as in CP’s model, excess bond returns, $rx_{t+1}^{(n)}$, are driven only by the risk premium factor x_t :

$$rx_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} = -\frac{1}{2}B'_{n-1}\Sigma\Sigma'B_{n-1} + B'_{n-1}(\lambda_0 + \lambda_1\iota_5)x_t + B'_{n-1}\Sigma\varepsilon_{t+1},$$

where $p_t^{(n)}$ denotes the log-price of a zero-coupon bond with maturity of n quarters and ι_5 is a (5×1) -vector of ones. The better performance found by CP in predicting excess bond returns with trend inflation suggests that accounting for the trend component can uncover the information about expected returns (i.e., about x_t) that is contained in observed interest rates. In other words, detrending interest rates with inflation helps estimate bond risk premia. However, according to our model, the equilibrium real rate can drive nominal yields in the same fashion as the trend component of inflation. Therefore, our model’s prediction is that including not only π_t^* but also r_t^* should improve the predictive power of excess return regressions and the estimation of bond risk premia. We assess the roles of both trend components for return predictability in Section 6. Another measure of the bond risk premium is the term premium in long-term yields, and in Section 7, we contrast term premium estimates that account for shifts in π_t^* and r_t^* with conventional estimates that do not.

Finally, the model predicts that macroeconomic trends should play a particularly important role in accounting for interest rate changes at low frequencies. The trend components are highly persistent, so while their influence can be obscured by transitory volatility at higher frequencies, they should be key drivers of interest rate changes over longer intervals (e.g., over several years or more). This pattern implies that variance ratios of changes in the trend components relative to changes in interest rates should exhibit a pronounced tendency to rise with the span of the changes. In Section 8, we will use such variance ratios to estimate the importance of the trend components at different frequencies.

In sum, our stylized theoretical model provides clear predictions about five aspects of the link between interest rates and macroeconomic trends. Of course, these predictions will be evident in the data only to the extent that there is material variation in the trend components. Furthermore, an empirical assessment requires accurate estimates of π_t^* and r_t^* . Thus, our empirical analysis should be viewed as tests of the joint hypothesis that (a) the behavior of the trend components and their links to the yield curve conform to our no-arbitrage model, (b) variation in the trend components is quantitatively important, and (c) our empirical proxies accurately capture the evolution of the trend components.

3 Data and trend estimates

We now describe the data and the estimates of the macroeconomic trends that we will use in testing the model’s predictions. Our data set is quarterly and extends from 1971:Q4 to 2015:Q4. The interest rate data are end-of-quarter zero-coupon Treasury yields from [Gürkaynak et al. \(2007\)](#) with maturities from one to 15 years. We augment these data with three- and six-month Treasury bill rates from the Federal Reserve’s H.15 data. In our empirical analysis, we mainly focus on long-term (five-year and ten-year) yields as well as long-term (five-to-ten-year) forward rates to exhibit the importance of π_t^* and r_t^* , and these are the relevant horizons for our trend measures as well.

For measuring the macroeconomic trends, we simply take existing estimates from the macroeconomics literature. Our goal is to assess whether such off-the-shelf measures can provide evidence linking the inflation and real rate trends to the yield curve and risk pricing. An alternative strategy would be to estimate time-varying r_t^* and π_t^* within a no-arbitrage term structure model. Our approach, which conditions on existing estimates, is more conservative, because our macro estimates of r_t^* and π_t^* have not been fine-tuned to incorporate no-arbitrage restrictions or information in long-term yields. We do not include trend estimates that are based on information in the yield curve, such as estimates of π_t^* by [Christensen et al. \(2010\)](#)

or estimates of r_t^* by [Johannsen and Mertens \(2016\)](#), [Christensen and Rudebusch \(2017\)](#), or [Del Negro et al. \(2017\)](#). All our empirical trend proxies are based only on information in macroeconomic variables, short-term interest rates and surveys, in order to provide the cleanest evidence of the empirical links between macro trends and the yield curve.

Several different empirical proxies for trend inflation, π_t^* , are available including surveys, statistical models, or a combination of the two—see, for example, [Stock and Watson \(2016\)](#) and the references therein. We employ a well-known survey-based measure, namely, the Federal Reserve’s research series on the perceived inflation target rate, denoted PTR. It measures long-run expectations of inflation in the price index of personal consumption expenditure (PCE), and is often used in empirical work—see, for example, [Clark and McCracken \(2013\)](#). PTR is based exclusively on survey expectations since 1979 (i.e., for most of our sample).⁸ [Figure 1](#) shows that from the beginning of our sample to the late 1990s, this estimate mostly mirrored the increase and decrease in the ten-year yield. Since then, however, it has been essentially flat at two percent, which is the level of the longer-run inflation goal of the Federal Reserve, which was first publicly designated as such in 2012. Other survey expectations of inflation over the longer run, such as the long-range forecasts in the Blue Chip survey, exhibit a similar pattern. The inflation trend that CP use is a simple weighted moving average of past core inflation, which, as they note, co-moves closely with PTR.

The recent literature on macro-based estimation of the equilibrium real interest rate has grown rapidly. These estimates almost invariably rely on models with macroeconomic structural underpinnings and macroeconomic data.⁹ A popular example is the model of [Laubach and Williams \(2003\)](#), in which the unobserved natural rate is inferred from macroeconomic data using a simple structural specification and the Kalman filter. In [Figure 2](#), we plot the (filtered) Laubach-Williams estimate of r_t^* . In addition, the figure also includes the estimates of [Lubik and Matthes \(2015\)](#), who employ a time-varying parameter VAR model, and [Kiley \(2015\)](#), who augments the Laubach-Williams model with credit spreads. [Figure 2](#) shows that since the early 1980s, these three macro-based estimates have evolved in a broadly similar fashion. A straightforward method to aggregate and smooth the information from these three specific modeling strategies is to take their average. In the 1970s, 80s, and 90s, this average

⁸Since 1979, PTR corresponds to long-run inflation expectations from the Survey of Professional Forecasters. Before 1979, PTR is based on estimates from the learning model for expected inflation of [Kozicki and Tinsley \(2001\)](#). For details on the construction of PTR, see [Brayton and Tinsley \(1996\)](#). PTR can be downloaded with the updates of the Federal Reserve’s FRB/US large-scale macroeconomic model at <https://www.federalreserve.gov/econresdata/frbus/us-models-package.htm>.

⁹Alternative survey-based estimates of r_t^* would require long-run expectations of both nominal interest rates and inflation. Unfortunately, the available time span for the former is quite limited (the earliest is a biannual Blue Chip Financial Forecasts series that starts in 1986) and they have also been found to be less accurate than forecasts of inflation (see, for example, [Dijk et al., 2014](#)).

fluctuated modestly between 2 and 3 percent, which is consistent with the common view of that era that viewed the equilibrium real rate as effectively constant. However, from 2000 to 2015, all of the measures fell, with an average decline of 2.5 percentage points, consistent with the shifts in global saving and investment that have been identified.¹⁰

Ideally, our trend estimates should reflect information that was available contemporaneously to investors. Having a reasonable alignment of r_t^* and π_t^* to the real-time evolution of investors' information sets is particularly relevant for properly assessing the value of macro trends in predicting future yields and bond returns and determining the term premium in long-term yields (as in Sections 5–7). Since 1979, our survey-based estimate of π_t^* has been available to bond investors at the end of each quarter, when our yields are sampled. Real-time concerns have been more acute for estimates of r_t^* (Clark and Kozicki, 2005). To construct r_t^* , we use filtered (i.e., one-sided) estimates of the equilibrium real rate from the three macroeconomic models cited above. That is, these estimates only use data up to quarter t to infer the unobserved value of r_t^* . While the estimated model parameters are based on the full sample of final revised data, Laubach and Williams (2016) show that truly real-time estimation of their model delivers an estimated series of r_t^* that is close to their final revised estimate over the period that both are available. This suggests that an alternative real-time estimation with real-time empirical trend proxies would be unlikely to overturn our results.

Intuitively, our empirical measures of π_t^* and r_t^* are consistent with a compelling narrative about the evolution of long-term nominal interest rates, as shown in Figure 1. Starting with the Volcker disinflation of the 1980s, interest rates and inflation trended down together. Around the turn of the millennium, long-run inflation expectations stabilized near 2 percent. However, long-term interest rates continued to decline in part because structural changes in the global economy started pushing down the equilibrium real rate. These structural changes likely included slowdowns in trend growth in various countries, increases in desired saving due to global demographic forces and strong precautionary saving flows from emerging market economies, as well as declines in desired investment spending partly reflecting a fall in the relative price of capital goods (Rachel and Smith, 2015; Carvalho et al., 2016). The following analysis investigates whether the link between macro trends and the yield curve that underlies this narrative is supported by the empirical evidence.

¹⁰Laubach and Williams (2003) and Kiley (2015) define r_t^* as the neutral real rate at which monetary policy is neither expansionary nor contractionary. This differs from our definition (and that in Lubik and Matthes (2015)) of r_t^* as the trend (or long-run) component of the real rate. However, in the models of Laubach and Williams (2003) and Kiley (2015), r_t^* is assumed to follow a random walk, so the two definitions in fact coincide.

4 Persistence in long-term yields from r_t^* and π_t^*

If the trend components of inflation and the real interest rate play an important role in driving interest rates, then these trends should exhibit a close statistical relationship with long-term interest rates and account, in particular, for most of the persistence of these rates. A natural starting point for assessing this relationship is to consider simple regressions of yields on the trend components. Table 1 reports the results for such regressions with three different dependent variables: bond yields with 5 and 10 years maturity, and the 5-to-10-year forward rate. For each maturity, we estimate two versions of the regressions (with standard errors calculated using the Newey-West estimator with six lags). The first version has only a constant and π_t^* as regressors, which is the same regression that CP estimated using their simple moving-average estimate of the inflation trend (see their table 1). These regression results show high R^2 's at all maturities and π_t^* coefficients that are just above one and highly significant.¹¹ CP interpret these results as indicating that trend inflation drives the level of yield curve. However, the results for the second regression specification show that incorporating the real rate trend is also important. Indeed, with the addition of r_t^* to the regressions, both the inflation and real rate trends coefficients are highly significant, and the regression R^2 's increase a further 7 to 12 percentage points.

Taken at face value, these estimates suggest that changes in r_t^* along with fluctuations in π_t^* , are key sources of variation in long-term interest rates. The interpretation of these results is complicated by the fact that all of the variables in the regressions are highly persistent. Conventional asymptotic arguments, which justify inference based on the Newey-West standard errors and R^2 's in Table 1, are not valid if some of the variables contain autoregressive roots close to or equal to one. For example, under the assumptions of the model in Section 2, these roots are equal to one, and the static regressions in Table 1 estimate cointegrating relationships between long-term interest rates, inflation, and real rates.¹² If there is indeed such a cointegrating relationship, then the regression provides super-consistent estimates of the cointegrating vector, the R^2 converges to one, and conventional hypothesis tests are likely invalid for inference about the coefficients (e.g., Hamilton, 1994, Chapter 19). Consequently, the regression results in Table 1—although suggestive of the joint importance of r_t^* and π_t^* for the determination of yields—cannot provide definitive answers.

¹¹Our estimated coefficients on π_t^* are somewhat higher than in CP because our measure of the inflation trend is less variable, though when r_t^* is added, the estimated coefficients for π_t^* decrease toward one.

¹²Much empirical work, for example, King et al. (1991), has documented the substantial persistence in nominal interest rates, inflation, and real interest rates. The main difference between our static regressions and the usual cointegration regressions in this context (as in Rose, 1988, for example) is that we use directly observable proxies for the trend components of π_t and r_t .

To provide more compelling statistical evidence that our measures of π_t^* and r_t^* capture the persistent variation in interest rates, we examine the time series properties of detrended long-term interest rates with one or both of the trend components subtracted out. For the same three interest rates considered above, Table 2 reports the standard deviation and two measures of persistence: the estimated first-order autocorrelation coefficient, $\hat{\rho}$, and the half-life, which indicates the number of quarters until half of a given shock has died out and is calculated as $\ln(0.5)/\ln(\hat{\rho})$. For each interest rate, the table reports these statistics for five different series: the level of the interest rate, the rate detrended by simply subtracting out π_t^* or i_t^* (assuming unit coefficients on these trends), and the detrended rate calculated as the residuals from one of the two static detrending regressions in Table 1. Several findings stand out: First, detrending with r_t^* as well as π_t^* removes substantially more persistence, typically reducing the half-life by about 40-50%. That is, π_t^* is not the only important driver of interest rate persistence. Second, the detrended series are substantially less variable and less persistent than the original interest rate series. For example, shocks to the ten-year yield have a half-life of about 5-1/2 years, whereas shocks to the difference between the ten-year yield and i_t^* have a half-life of just under one year. Finally, there is little difference in the statistical properties between series that are detrended using unit coefficients and those that are detrended using estimated coefficients, notably for the forward rate.

Unit root tests provide further evidence supporting detrending with *both* r_t^* and π_t^* . The last two columns of Table 2 provide parametric Augmented Dickey-Fuller (ADF) t -statistics and non-parametric Phillips-Perron (PP) Z_α statistics, which examine the persistence properties of these series by testing the null hypothesis of a unit autoregressive root.¹³ These tests show strong evidence against the unit root null for the series that are detrended with both π_t^* and r_t^* . By contrast, the unit root null is never rejected at the 5 percent level for the original interest rate series or for series that are detrended with just π_t^* . This evidence strongly supports the use of $i_t^* = r_t^* + \pi_t^*$ when accounting for interest rate persistence and understanding interest rate dynamics. Finally, when detrending with both π_t^* and r_t^* , the rejections are at least as strong for the series that are detrended using unit coefficients as for the residuals from the static regressions, consistent with the prediction of equation (2).

These results have important implications for the debate about the long-run Fisher effect,

¹³For the ADF test, we include a constant and k lagged difference in the test regression, where k is determined using the general-to-specific procedure suggested by Ng and Perron (1995). We start with $k = 4$ quarterly lags and reduce the number of lags until the coefficient on the last lag is significant at the ten percent level. For the PP test, we use a Newey-West estimator of the long-run variance with four lags. When the series under consideration is a residual from an estimated cointegration regression, we don't include intercepts in the ADF or PP regression equations and use the critical values provided by Phillips and Ouliaris (1990), which depend on the number of regressors in the cointegration equation.

which posits a common trend for inflation and interest rates with a unit coefficient (leaving aside tax considerations). A sizeable literature has tested this hypothesis with mixed results, often estimating an inflation coefficient that is larger than one (see the references in footnote 2). We have provided evidence that long-term interest rates contain two trend components but that the series $y_t - \pi_t^* - r_t^*$ is stationary, consistent with the prediction of our model. Our results thus provide strong support for a long-run Fisher effect if shifts in the equilibrium real rate are taken into account. The importance of time variation in r_t^* can explain why past research has generally been unable to find a stable relationship between nominal interest rates and inflation, going back to [Rose \(1988\)](#) and recently summarized by [Neely and Rapach \(2008\)](#). Under the unit root assumption, ignoring r_t^* as a trend component, by regressing interest rates only on π_t^* (or π_t), leads to a misspecified cointegration relationship as the residual contains the trend r_t^* . In such a situation, it will be very difficult to uncover the Fisher effect. By contrast, including both r_t^* and π_t^* , we obtain clear, unambiguous results: the trend components of inflation and the real rate are related to interest rates exactly as standard finance theory predicts with a long-run Fisher effect and the two trends together capture a large share of the persistence in long-term interest rates.

5 Forecasting interest rates with r_t^* and π_t^*

The previous section illustrated the strong persistence of interest rates and the close link between that persistence and measures of the inflation trend, π_t^* , and the equilibrium real rate, r_t^* . Here, we provide complementary evidence based on interest rate forecasts.

We forecast the five- and ten-year yields and the five-to-ten-year forward rate. At each point in time, starting in 1976:Q1 (at $t = 20$) when five years of data are available, we forecast each interest rate, y_t , at horizons from $h = 1$ to $h = 40$ quarters. The benchmark model in the literature is a driftless random walk, i.e., the forecast $\hat{y}_{t+h} = y_t$ for all h . This benchmark is denoted in the results as Method 1. The two alternative forecast specifications assume that interest rates mean-revert to a time-varying endpoint \hat{i}_t . Of these, Method 2 assumes that this endpoint is only driven by the inflation trend, i.e., that \hat{i}_t equals π_t^* plus a constant. We estimate this constant recursively as the mean of $y_\tau - \pi_\tau^*$, using observations from $\tau = 1$ to $\tau = t$.¹⁴ Method 3 instead assumes that the endpoint is $\hat{i}_t = i_t^* = \pi_t^* + r_t^*$. For both shifting endpoint forecast methods, the speed of mean-reversion is recursively estimated as the first-order autocorrelation coefficient of $y_t - \hat{i}_t$. Denoting this coefficient as $\hat{\rho}_t$, the forecasts for Methods 2 and 3 are constructed as $\hat{y}_{t+h} = \hat{\rho}_t^h y_t + (1 - \hat{\rho}_t^h) \hat{i}_t$. The exact value of $\hat{\rho}_t$, or even

¹⁴We have found in additional, unreported results that forecasts which assume that this constant is zero perform much worse than this method.

the methodology for determining it, affects short-horizon forecasts but was inconsequential for our main results.

Table 3 reports the results of this forecasting exercise in terms of root-mean-squared errors (RMSEs) and mean-absolute errors (MAEs), in percentage points. We also calculate p -values for tests of equal finite-sample forecast accuracy using the approach of Diebold and Mariano (1995) (DM).¹⁵ We calculate these DM p -values, using standard normal critical values, for one-sided tests of whether forecasts using the endpoint i_t^* (Method 3) improve upon random walk forecasts (Method 1) and upon forecasts that use only π_t^* for the endpoint (Method 2). We find that Method 3 leads to substantial and highly significant gains in forecast accuracy at long horizons. Such gains are evident for both RMSEs and MAEs, but are larger and more strongly significant for absolute-error loss. For example, when forecasting the ten-year yield five years ahead, using the information in i_t^* lowers the RMSE by over 25% relative to the random walk forecast, an improvement that is significant at the five-percent level, while the MAE drops by more than 40% and is significant at the one percent level. Using i_t^* for forecasts also improves upon Method 2, which does not use r_t^* , by a magnitude that is typically large and statistically significant.

This long-horizon forecast exercise thus confirms that interest rates exhibit reversion to the time-varying endpoint i_t^* , as predicted by the theory of Section 2.¹⁶ Earlier research, such as Dijk et al. (2014), has found that when forecasting interest rates it is beneficial to link long-run projections of interest rates to long-run expectations of inflation. Our new result shows that including r_t^* along with π_t^* improves on long-horizon interest rate forecasting to an even greater degree. That is, accounting only for the time variation in π_t^* is insufficient, as it is important to include r_t^* in understanding yield curve dynamics. Furthermore, while earlier authors ran regressions of the level of interest rates on long-run inflation expectations to scale π_t^* for forecasting, our results show that no such scaling or estimation is necessary for accurate long-range forecasts if the endpoint is simply taken as i_t^* .¹⁷

Many studies have documented that it is difficult to beat the random walk model when

¹⁵Following common use, we construct the DM test with a rectangular window for the long-run variance and the small-sample adjustment of Harvey et al. (1997). Monte Carlo evidence in Clark and McCracken (2013) indicates that this test has good size in finite samples. However, for very long forecast horizons there are of course only few non-overlapping observations in our sample, so the p -values are subject to substantial uncertainty.

¹⁶The model in Section 2 implies that the endpoint equals a maturity-specific constant plus i_t^* , but our results show that it is not necessary to try to estimate and include this constant in the endpoint to obtain accurate forecasts.

¹⁷Our approach could easily be extended to jointly forecast the whole yield curve as in Diebold and Li (2006) and Dijk et al. (2014), by simply forecasting the Nelson-Siegel level factor in the same fashion that we have forecast each individual interest rate above.

forecasting bond yields, which reflects the extreme persistence of interest rates.¹⁸ But our results show that one can obtain better forecasts if one accounts for the source of this persistence. Our use of empirical proxies for the macroeconomic trends inherent in interest rates leads to substantially better long-term forecasts than the random walk model.

6 Predicting bond returns with r_t^* and π_t^*

We now turn to the role of macroeconomic trends in predicting bond returns. In the analysis of CP, proxies for the inflation trend have significant predictive power for annual excess bond returns, above and beyond the information contained in the yield curve itself. Since r_t^* also is an important trend driving long-term interest rates, we address the natural question of what role it plays in such predictive regressions.

We predict excess returns for holding periods of one quarter and four quarters. The excess return for a holding period of h quarters for a bond with maturity n is calculated as

$$rx_{t,t+h}^{(n)} = p_{t+h}^{(n-h)} - p_t^{(n)} - hy_t^{(h)} = -(n-h)y_{t+h}^{(n-h)} + ny_t^{(n)} - hy_t^{(h)}.$$

Our long-term bond yields are available only at annual maturities, so we calculate one-quarter returns with the usual approximation $y_{t+1}^{(n-1)} \approx y_{t+1}^{(n)}$. Our dependent variable is the average excess return for all bonds with two to 15 years maturity.¹⁹ We begin with three specifications of the predictive regressions: The first includes a constant and the first three principal components (PCs) of yields. This common baseline regression is motivated by work going back to [Fama and Bliss \(1987\)](#) and [Campbell and Shiller \(1991\)](#) that showed there is information in the yield curve itself, and in particular in its slope (PC2), about expected bond returns. The second specification adds π_t^* and is closely related to the specification with an inflation trend that was estimated by CP. The third specification also includes r_t^* in order to simultaneously capture the effects of both macroeconomic trends in the regression.

The top panel of [Table 4](#) reports the estimation results for the full sample. We calculate White’s heteroskedasticity-robust standard errors for the case of one-quarter returns and Newey-West standard errors with six lags for the four-quarter returns (since the overlap introduces serial correlations in the error term). Our results replicate CP’s main finding, namely that inclusion of the inflation trend raises the predictive power quite substantially compared

¹⁸Prominent examples include [Duffee \(2002\)](#) and [Diebold and Li \(2006\)](#); see [Duffee \(2013\)](#) for a review of this literature.

¹⁹CP focused on annual holding periods and used a weighted average that downweights longer-term bonds. Our use of a simple average made no material difference to the results and is more common in this literature. Our sample period is similar to the one considered in CP but ends in 2015 instead of 2011.

to a regression that only includes yield-curve information, and that both the inflation trend and the level of yields (PC1) appear highly significant. However, adding r_t^* to the regressions leads to further impressive gains in predictive power. For both one-quarter and four-quarter returns, the R^2 increases substantially, the coefficients and t -statistics for π_t^* and PC1 rise, and the coefficient on r_t^* itself is large and highly significant. Interestingly, the coefficient on r_t^* is about as large as the coefficient on π_t^* , which confirms the prediction from our theoretical model that these two trends play similar roles in determining interest rates.

It is well known that these kinds of predictive regressions for bond returns raise some knotty econometric issues. [Bauer and Hamilton \(2016\)](#) show that the small-sample inference in such cases is particularly problematic when the predictors are highly persistent, like interest rates and our macro trends. They propose a parametric bootstrap procedure to carry out robust inference in such cases. It tests the null hypothesis that only the information in the yield curve is useful for predicting excess returns, and it can accurately gauge the statistical significance of additional proposed predictors. Following their recommendation, we calculate bootstrap small-sample p -values for the coefficients on π_t^* and r_t^* . Our bootstrap simulates yields from a simple VAR(1) factor model and the additional predictors from a separate VAR(1) model, so that the null hypothesis—that macro trends are unimportant—holds by construction.²⁰ The predictive regression is estimated in each bootstrap sample and the t -statistics for the additional predictors are recorded. With this small-sample distribution of the test statistics in hand, p -values are calculated as the fraction of simulated samples in which the t -statistic is at least as large (in absolute value) as the t -statistic in the actual data. These p -values, reported in squared brackets in [Table 4](#), indicate that both of our trends are statistically significant even when we account for small-sample econometric concerns.

In the subsample starting in 1985, the inflation trend is not statistically significant when included on its own according to the small-sample p -values.²¹ Only when we add our measure of the equilibrium real rate do both trends matter for bond risk premia; the coefficients on π_t^* and PC1 more than double, the R^2 increases substantially, and the coefficients on π_t^* and r_t^* are statistically significant. These results confirm our intuition from [Figure 1](#) that the real-rate trend has gained in importance over time relative to the trend in inflation. Therefore, empirical analysis of long-term interest rates implies that the trend in the real interest rate is as important as, and recently more important than, the trend in inflation.²²

²⁰Estimates of the VAR coefficients are bias-corrected to more accurately capture the high persistence of interest rates and the trend components.

²¹This result is consistent with [Bauer and Hamilton \(2016\)](#) who also found that in a subsample starting in 1985 the inflation trend is only marginally significant.

²²In additional, unreported results we have found that the predictive gains from including r_t^* are particularly large during the early 2000s when both r_t^* and long-term interest rates decreased while long-run inflation

In the presence of persistent predictors, it is generally difficult to interpret the magnitude of R^2 as a measure of predictive accuracy, because even predictors that are irrelevant in population can substantially increase R^2 in small samples (Bauer and Hamilton, 2016). By using the bootstrap to avoid this pitfall, we generate small-sample distributions of R^2 under the null hypothesis that only yields have predictive power, and interpret the statistics obtained in the actual data by comparing them to the quantiles of these distributions. Table 5 reports this comparison for predictive regressions of annual excess returns for the three specifications we have considered so far, as well as for two additional ones that will be discussed below. Adding π_t^* to the regression increases R^2 by 20 percentage points, but this is only barely higher than the upper end of the 95%-bootstrap interval, which suggests that under the null hypothesis it would not be too uncommon to observe an increase in R^2 of up to 19 percentage points. In contrast, adding r_t^* increases R^2 to 54%, and the increase relative to the yields-only specification of 31 percentage points is much higher than what is plausible under the null hypothesis. In the post-1985 subsample, the increase in R^2 from only adding π_t^* is not statistically significant, whereas the increase of 29 percentage points from adding both trends is strongly significant.

CP defined “interest rate cycles” as the de-trended component in interest rates, which they denoted as $c_t^{(n)}$, and showed that these cycles captured the predictive power in interest rates and the inflation trend for future bond returns. They estimated these interest rate cycles as the residuals of regressions of interest rates on their measure of the inflation trend. Our results here and in the previous sections indicate that a better estimate of the cycle can be obtained by incorporating both an inflation trend and a real rate trend. Furthermore, our evidence in Section 4 supporting a cointegration vector between interest rates and the macro trends of $(1, -1, -1)$ suggests that interest cycles ($c_t^{(n)}$ in CP’s notation) should be calculated simply as $y_t^{(n)} - \pi_t^* - r_t^*$, as prescribed by our simple no-arbitrage model in Section 2. In addition we also consider cycles that are calculated as yields detrended by only π_t^* , i.e., $y_t^{(n)} - \pi_t^*$, in order to understand the separate importance of both trend components. Figure 3 compares these two measures of interest rate cycles by plotting for each case the average cycle across yields of maturities from two to 15 years (as in CP). The cycle that is calculated by detrending only with π_t^* still contains an important trend component, as evident in the substantial decline of about four percentage points from the level prevailing in the 1990s to the end of the sample. The cycle measure that also accounts for changes in the equilibrium real interest rate does not exhibit this trend. Instead, it exhibits cyclical behavior with clear mean reversion.

Furthermore, in the spirit of CP, we investigate the use of interest rate cycles, i.e., detrended expectations where anchored close to two percent.

yields, in predictive regressions for bond returns. The first three rows of Table 5 summarize our earlier results from the regression of excess returns on yield PCs alone, PCs jointly with π_t^* , and PCs with both trends. The bottom two rows provide results for predictions using interest rate cycles. In this case the predictors are the three PCs of the detrended yield curve. Table 5 shows that using PCs of yields that are detrended only by π_t^* increases R^2 by 10 percentage points relative to the baseline regression with three PCs of yields that are not detrended. But using PCs of yields that are detrended by both macro trends increases R^2 by 20 percentage points. The difference is even more striking in the later subsample that starts in 1985: R^2 increases only seven percentage points when detrending with only π_t^* but 28 percentage points when detrending with both π_t^* and r_t^* . We again calculate small-sample distributions of these test statistics under the null hypothesis by running the exact same regressions in bootstrapped samples. For the later subsample, the increase in R^2 is insignificant for π_t^* -detrending but strongly significant when we use the correct detrending using both macro trends as prescribed by no-arbitrage theory.

An intriguing final issue is why the information in π_t^* and r_t^* is not spanned by the yield curve. Both reduced-form and structural/equilibrium models of the term structure of interest rates generally imply that the yield curve contains all (i.e., spans) the relevant information for predicting future yields and bond returns (Joslin et al., 2013; Duffee, 2013). The apparent conflict of this prediction with the empirical evidence of unspanned macro risks in bond returns is an important current issue in macro-finance (Ludvigson and Ng, 2009; Joslin et al., 2014; Cieslak and Povala, 2015). CP and Bauer and Rudebusch (2017) suggest that measurement error, which breaks spanning, can reconcile the models with the data. Bauer and Hamilton (2016) demonstrate that much of the apparent evidence for unspanned macro risks is substantially weaker and partly spurious once small-sample problems are accounted for, as the presence of trending, persistent predictors renders conventional tests of the spanning hypothesis unreliable. While these econometric issues suggest that the evidence on the predictive power of macroeconomic trends should be taken with a grain of salt, there are possible complementary explanations for this phenomenon. We use observable proxies for trends and have the benefit of hindsight, but estimation of the trend components of time series is fraught with a large amount of uncertainty, and it is difficult to learn about trends in real time (Clark and Kozicki, 2005). Because forecasters and investors are bound to learn slowly about changes in trends, and because they may disagree about the trend, bond prices may not fully incorporate the evolution of macroeconomic trends. A related issue is the rational expectations assumption that underlies all hypothesis tests about bond returns using time series data. The wedge between subjective (e.g., survey-based) and objective (statistical) expectations of in-

terest rates and bond returns—documented by Piazzesi et al. (2015), among others—could cause statistical findings of unspanned risks that were not in fact a feature of subjective risk premia of investors at the time. More work is needed to better understand how these issues relate to apparent failures of the spanning hypothesis and the incremental predictive power of macro trend estimates for bond returns that we document here.

In sum, accounting for the persistent components of yields is important for understanding return predictability. We find that r_t^* has strong incremental predictive power for bond returns, about on par with the importance of π_t^* as a predictor. This suggests that for estimation of bond risk premia it is crucial to account for not only the inflation trend but also for the equilibrium real rate.

7 Calculating the term premium with r_t^* and π_t^*

Our evidence supports the view that for modeling the yield curve, forecasting interest rates, or predicting bond returns, it is important to account for movements in the trend components r_t^* and π_t^* . Accordingly, these trends should also be integral for estimation of the term premium in long-term interest rates, which we denote as $TP_t^{(n)}$. This alternative measure of bond risk is defined as the difference between holding an n -month bond to maturity or facing a sequence of 1-period rates over the same period:

$$TP_t^{(n)} = y_t^{(n)} - \frac{1}{n} \sum_{j=0}^{n-1} E_t y_{t+j}^{(1)}.$$

Our earlier results show that the expected path of future short rates should not be assumed to return to a constant mean but to a shifting endpoint $i_t^* = r_t^* + \pi_t^*$. Here, we introduce a methodology to incorporate information about r_t^* and π_t^* in the estimation of the term premium and compare the results to a conventional estimate.

Decompositions of long-term interest rates into short-rate expectations and term premia are commonly obtained from a variety of dynamic models ranging from simple factor models to no-arbitrage term structure models. We employ the former in this section, by estimating a time-series process for yield curve PCs and fitting the cross section of yields using the PC loadings.²³ Short-rate expectations are then calculated from the VAR forecasts of the yield

²³The results from such a factor model will be essentially identical to those from a model that imposes maximally-flexible no-arbitrage restrictions on the loadings of yields on risk factors (Duffee, 2011; Joslin et al., 2013). Still, Cochrane and Piazzesi (2008) and Bauer (2017) emphasized that restricting the prices of risk can let the cross-sectional behavior of yields help pin down the time-series specification for yields, which could be relevant for developing no-arbitrage models that incorporate our findings.

PCs and the PC loadings of the short rate. Our model uses three PCs calculated from 15 Treasury yields with maturities from one year to 15 years over an estimation sample from 1971:Q4 to 2007:Q4. We omit the period of near-zero short-rates beginning in 2008, since the lower bound on nominal interest rates poses problems for linear factor models (Bauer and Rudebusch, 2016).

To provide a conventional baseline measure of the term premium, we estimate a first-order annual VAR(1) in quarterly observations using the first three PCs of interest rates.²⁴ In line with the underlying structure of the majority of no-arbitrage models, this is a stationary VAR. For example, Cochrane and Piazzesi (2008) estimated an annual VAR(1) for five interest rates in monthly observations, and our results closely parallel theirs. In the top-left panel of Figure 4, we plot the model-implied expectations of the one-year yield at different horizons, as well as the observed one-year yield. As emphasized by Cochrane and Piazzesi (2008), stationary VARs for the levels of interest rates imply forecasts that quickly revert to the unconditional mean of the short rate (which is 6.5 percent). The top-right panel of Figure 4 shows the five-to-ten-year forward rate with its expectations and term premium components. (Results for the ten-year yield are qualitatively similar.) Not surprisingly, in light of the behavior of model-implied forecasts, the expectations component is very stable, hovering around the short-rate mean. Therefore, the term premium, as the residual component, has to account for the trend in the long-term interest rate since the 1980s. As argued by Kim and Orphanides (2012) and Bauer et al. (2014), such behavior of expectations and term premium components appears at odds with observed trends in survey-based expectations (Kozicki and Tinsley, 2001) and the cyclical behavior of risk premia in asset prices (Campbell and Cochrane, 1999).

To incorporate a trend component into forecasts of the yield curve, we instead model detrended yields as a stationary VAR. That is, we subtract i_t^* from each yield as called for by the evidence in Sections 4–6, calculate three PCs of detrended yields, and again estimate an annual VAR(1). We calculate the term premium by forecasting the detrended interest rates and adding the trend back in to obtain the expected path of future short rates. The bottom two panels of Figure 4 show the implications of this model for expectations and the term premium. As the forecast horizon increases, short-rate expectations approach the trend component i_t^* instead of the unconditional mean of the short rate. Consequently, the expectations component reflects the movements in i_t^* and accounts for the low-frequency movements in the long-term forward rate. The term premium, by contrast, behaves in a cyclical fashion with no discernible trend. The drop in the forward rate from its average during 1980–1982 to its average during

²⁴That is, the PCs are regressed on an intercept and the one-year lagged values of the PCs, a specification motivated by the evidence of Cochrane and Piazzesi (2005) that the dynamics of risk premia are more evident at the annual frequency (see also Cochrane and Piazzesi, 2008).

2005-2007 was about 7.5 percentage points. In the conventional VAR of interest rates, the estimated term premium accounts for over 80% of this decline. In contrast, the VAR with detrended yields attributes only a quarter of this decline to the term premium. This stark difference demonstrates how accounting for the slow-moving trend component in interest rates fundamentally alters our understanding of the driving forces of long-term interest rates.

This model—a VAR for interest rate cycles conditional on estimates of the trend i_t^* —provides a simple but effective way to account for a common trend (i.e., shifting endpoints) in bond yields. In addition to estimation of the term premium, it could be useful in related applications, such as in forecasting the entire yield curve jointly, or for calculating of expected returns for any bond and holding period. While it does not impose no-arbitrage, the representation is consistent with the key predictions of our no-arbitrage model in Section 2 regarding the role of time-variation in i_t^* for bond yields.

8 Variance contributions of r_t^* and π_t^*

Finally, we compare the size of fluctuations in r_t^* and π_t^* with those of long-term nominal bond yields. Duffee (2016) finds that news about future inflation is generally small relative to the innovations in nominal yields. We interpret his results in the context of our theoretical model and extend them empirically to consider movements in both r_t^* and π_t^* and at horizons greater than just one quarter.

The variance ratio used by Duffee divides the variance of quarterly innovations to average inflation expectations over n periods by the variance of innovations to the bond yield for maturity n :

$$VR_1^{(n)} = \frac{\text{Var}((E_t - E_{t-1})n^{-1} \sum_{i=1}^n \pi_{t+i})}{\text{Var}((E_t - E_{t-1})y_t^{(n)})}.$$

We generalize this measure to allow for a longer time span to calculate the change in expectations,

$$VR_h^{(n)} = \frac{\text{Var}((E_t - E_{t-h})n^{-1} \sum_{i=1}^n \pi_{t+i})}{\text{Var}((E_t - E_{t-h})y_t^{(n)})},$$

so the variances are calculated for the h -period innovation (where a period will remain a quarter). Our theoretical model from Section 2 implies analytical expressions for these variance

ratios. For one-period and h -period innovations, respectively, they are

$$\begin{aligned}
VR_1^{(n)} &= \frac{\sigma_\xi^2 + a_c(n)\sigma_u^2}{\sigma_\xi^2 + a_c(n)\sigma_u^2 + \sigma_\eta^2 + a_g(n)\sigma_v^2 + \left(\frac{B_n^x}{n}\right)^2 \sigma_w^2}, \\
VR_h^{(n)} &= \frac{h\sigma_\xi^2 + a_c(n)b_c(h)\sigma_u^2}{h\sigma_\xi^2 + a_c(n)b_c(h)\sigma_u^2 + h\sigma_\eta^2 + a_g(n)b_g(h)\sigma_v^2 + \left(\frac{B_n^x}{n}\right)^2 b_x(h)\sigma_w^2}, \\
a_i(n) &= \left(\frac{1 - \phi_i^n}{n(1 - \phi_i)}\right)^2, \quad b_i(h) = \frac{1 - \phi_i^{2h}}{1 - \phi_i^2}, \quad i = c, g, x.
\end{aligned}$$

These expressions help elucidate the factors determining the variance ratios.

An important result of Duffee's analysis is that even for long-term bonds, $VR_1^{(n)}$ appears to be surprisingly small. That is, one-period changes in expected average future inflation are much less variable than one-period surprises in long-term bond yields. This result can be understood by noting that through the lens of our model

$$\lim_{n \rightarrow \infty} VR_1^{(n)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\eta^2 + \left(\frac{\lambda_{\pi^*x} + \lambda_{r^*x}}{1 - \phi_x}\right)^2 \sigma_w^2}.$$

This ratio will be small if shocks to the equilibrium real rate (η_t) and to the risk-premium factor (w_t) make more important contributions to yield innovations than shocks to the inflation trend (ξ_t). One plausible interpretation of Duffee's finding is that at a quarterly frequency the trend in inflation moves much less than the term premium component of long-term interest rates. The term premium is a catch-all component of price movements due to a variety of factors including changes in risk-sentiment and "animal spirits" not attributable to changes in real-rate and inflation expectations. This interpretation is consistent with additional evidence in Duffee's paper on the role of the term premium and with a large body of evidence on excess volatility of interest rates, going back to [Shiller \(1979\)](#).

Another way to understand the link between inflation expectations and bond yields is to consider longer horizons h . In the limit, this generalization of the variance ratio appears promising because

$$\lim_{h \rightarrow \infty} VR_h^{(n)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\eta^2},$$

so asymptotically, this inflation variance ratio is only affected by changes in the trend components. Changes in term premia become irrelevant at very low frequencies if, as in our model, the risk premium factor x_t is stationary. Of course, empirical variance ratios for large h do have the drawback that as h increases, so does the overlap of observations; therefore, we quickly

lose degrees of freedom and precision in our estimates. However, the profile of variance ratios across horizons should give us additional insights about interest rate dynamics, even if the maximum horizon is severely limited in practice by data availability.

Estimation of these variance ratios requires expectations of inflation and interest rates. Like Duffee we consider survey-based inflation expectations (in our measure of π_t^*) and martingale interest rate forecasts. But instead of modeling the inflation process we approximate inflation news by the change in π_t^* . This allows us to calculate the simplified variance ratio

$$\widetilde{VR}_h^{(n)} = \frac{Var(\Delta_h \pi_t^*)}{Var(\Delta_h y_t^{(n)})}, \quad \Delta_h z_t = z_t - z_{t-h}.$$

For any given n and h , $\widetilde{VR}_h^{(n)}$ differs from $VR_h^{(n)}$ because the variances in the simplified ratio include movements that were anticipated at $t - h$. However, in many cases, these differences are likely to be small, in particular for longer time spans of the innovations/changes, since in the limit

$$\lim_{h \rightarrow \infty} \widetilde{VR}_h^{(n)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\eta^2} = \lim_{h \rightarrow \infty} VR_h^{(n)}.$$

Thus, for low-frequency movements, the two inflation variance ratios are asymptotically identical.

Figure 5 shows estimates of $\widetilde{VR}_h^{(n)}$ for changes from one quarter to $h = 40$ quarters in the five-year yield, the ten-year yield, and the 5-to-10-year forward rate. This shows the relative importance of fluctuations in long-run inflation expectations. Similarly, we also calculate these variance ratios for the contribution of changes in the real-rate trend, r_t^* , and in the overall trend component $i_t^* = \pi_t^* + r_t^*$, simply by replacing the variance in the numerator of $\widetilde{VR}_h^{(n)}$.

For the inflation trend, we find that one-quarter variance ratios are around 0.1. This result is consistent with Duffee's findings and suggests that changes in the inflation trend play a small role for variation in yields at the quarterly frequency. At lower frequencies, however, the relative variability of the inflation trend increases. The point estimates of the π_t^* -variance ratio quickly rise with the horizon, and for $h = 40$ reach a magnitude of around 0.3. This shows that inflation expectations are of substantial importance for movements in bond yields once we shift the focus from month-to-month or quarter-to-quarter variation and look at lower frequencies changes over several years, which are arguably more relevant for long-term bond investors.

The variance ratio for changes in the real-rate trend, i.e., $Var(\Delta_h r_t^*)/Var(\Delta_h y_t)$, is much lower than for inflation, remaining below 0.1 even at long horizons. This is unsurprising in light of Figure 1, which shows that over the full sample the movements in r_t^* were substantially less

pronounced than movements in long-term interest rates and the inflation trend.²⁵ Of course, this perspective of unconditional variances should not be taken to conclude that changes in the equilibrium real rate are unimportant for interest rate dynamics, given the ample evidence in Sections 4–6 of the crucial role of r_t^* for modeling and forecasting interest rates. Furthermore, in more recent subsamples, variance ratios should suggest that r_t^* has become a more important driver of changes in long-term interest rates.

To assess the overall importance of the trend components in interest rates, we consider variance ratios for i_t^* , i.e., $Var(\Delta_h i_t^*)/Var(\Delta_h y_t)$. Confidence intervals are obtained using the asymptotic distribution of the sample variances and the delta method. To account for persistence in conditional variances, Newey–West estimates of long-run variances are used.²⁶ Figure 5 shows that while the sampling uncertainty around the variance ratios for changes in i_t^* is substantial we can be reasonably confident that these variance ratios increase from below 0.15 to a range of around 0.25 to 0.4, depending on the maturity of the interest rate. The highest levels are reached for the 5-to-10-year forward rate—the confidence interval at $h = 40$ extends from about 0.3 to 0.4—which is consistent with the notion that distant forward rates are more strongly affected by the trend components.

While our theoretical model implies that the variance ratios for i_t^* approach one for sufficiently large h , our estimates top out around 0.4. One possible explanation is that we are simply not capturing the trend component with sufficient accuracy. Although we have provided ample evidence that our trend proxies are closely linked to the low-frequency movements in interest rates, the true trend components might be more (or less) variable than our measures. Additionally, we may not be able to consider large enough h given the available data sample. Finally, it may be that the risk-premium factor itself is sufficiently persistent that it drives interest rate variation at such low frequencies.

To shed further light on this topic, it is useful to consider not only the direct contribution of changes in the trend to changes in interest rates, but also indirect contributions, due to covariance terms. In Table 6, we report the variances of changes in interest rates, in the trend component i_t^* , and in the cycle components $y_t - i_t^*$. The variance of yield changes can be decomposed as follows:

$$Var(\Delta_h y_t) = Var(\Delta_h i_t^*) + Var(\Delta_h y_t - \Delta_h i_t^*) + 2Cov(\Delta_h i_t^*, \Delta_h y_t - \Delta_h i_t^*).$$

²⁵This finding may partly reflect our use of an average of different estimates for r_t^* —some of the individual estimates exhibit distinctly more volatility.

²⁶We use 12 quarterly lags for all long-run variance estimates as indicated by the automatic lag selection procedure of Newey and West. These confidence intervals may understate the true sampling variability due to the small number of non-overlapping observations, which decreases the reliability of the asymptotic approximations.

The first term captures the direct contribution of trends, whereas the last term captures the their indirect contribution to movements in yields. Table 6 reports all three components. In the data, the contribution of the covariance is small at short horizons, but substantial at long horizons. The two last rightmost columns of Table 6 report the same variance ratio shown in Figure 5 along with a ratio that also includes the covariance contribution—the indirect effects—in the numerator. This second ratio rises to over 0.7 with horizon, which suggests that the cycle component accounts for less than 30% of the variance of interest rate changes at low frequencies.²⁷ These estimates confirm that after accounting for both direct and indirect effects, changes in macro trends play an important role in low-frequency movements in interest rates.

9 Conclusion

In this paper, we have provided new evidence that interest rates and risk pricing are substantially driven by time variation in the trend in inflation and the equilibrium real rate of interest. Our empirical approach employed existing estimates of these macroeconomic trends to investigate this relationship from a variety of perspectives. We showed that accounting for these trend components can help understand, model, and forecast long-term interest rates and bond returns. Our results confirm the predictions of no-arbitrage theory for the links between macroeconomic trends and the yield curve, and they demonstrate that these links are quantitatively important.

Our findings open up several avenues for future research. Most importantly, they demonstrate the advantages of yield curve models that allow for slow-moving changes in the long-run mean of inflation and the real rate. This applies to both reduced-form no-arbitrage models as well as to more structural, equilibrium models of the yield curve. The purpose of specifying and estimating such models is typically to understand bond risk premia/term premia and risk compensation. Calculating the required long-run expectations without accounting for changes in the underlying trends is likely to give misleading results for risk premia. To avoid this pitfall we recommend that shifting macro trends should be incorporated in future models of the term structure of interest rates.

Our analysis should be viewed as a first step in the integration of macroeconomic trends into yield curve modeling. We took estimates of macroeconomic trends from surveys and models and treated them as data. While this is a useful starting point, future research will have to investigate the role of model and estimation uncertainty for the links between macro

²⁷Note that our theoretical model, which assumed that shocks to trend and cycle components are uncorrelated, would need to be generalized to provide a model-based interpretation of these results.

trends and the yield curve. Another important additional dimension to consider is potential change over time in the relative importance of the macroeconomic trends in accounting for long-run interest rates. For example, it is well known in empirical macroeconomics (e.g., [Stock and Watson, 2007](#)) that the trend component of inflation was much more variable in the 1970s and 1980s than in more recent decades. This raises the question how our (mostly unconditional) results are affected by taking a conditional perspective. Furthermore, this suggests that incorporating not only macroeconomic trends but also stochastic volatility in these trend components will be useful for term structure modeling.

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Appendix

A Additional details for affine term structure model

Here we provide further details and additional results for the affine term structure model of Section 2. First, we consider prices and yields of real (i.e., inflation-indexed) bonds. Just like prices of nominal bonds, prices of real bonds are exponentially affine in the risk factors, $\log(\hat{P}_t^{(n)}) = \hat{A}_n + \hat{B}'_n Z_t$. Hats denote variables pertaining to real bonds. The loadings are determined by the recursions

$$\hat{A}_{n+1} = \hat{A}_n + \hat{B}'_n(\mu - \lambda_0) + \hat{C}_n, \quad \hat{C}_n := \frac{1}{2} \hat{B}'_n \Sigma \Sigma' \hat{B}_n, \quad \hat{B}_{n+1} = -(0, 0, 1, 1, 0)' + (\phi - \lambda_1)' \hat{B}_n,$$

\hat{C}_n captures the convexity in real bonds, and the initial conditions are $\hat{A}_0 = 0, \hat{B}_0 = (0, 0, 0, 0, 0)'$. Specifically,

$$\begin{aligned} \hat{B}_n^{r^*} &= 0, & \hat{B}_n^c &= 0, & \hat{B}_{n+1}^{r^*} &= \hat{B}_n^{r^*} - 1, & \hat{B}_{n+1}^g &= \phi_g \hat{B}_n^g - 1, \\ \hat{B}_{n+1}^x &= -\lambda_{r^*x} \hat{B}_n^{r^*} - \lambda_{gx} \hat{B}_n^g + \phi_x \hat{B}_n^x. \end{aligned}$$

Real yields, $\hat{y}_t^{(n)} = -\log(\hat{P}_t^{(n)})/n$, are affine in the risk factors. It is instructive to consider real forward rates for inflation-indexed borrowing from n to $n+1$, for which we have

$$\begin{aligned} \hat{f}_t^{(n)} &= \log(\hat{P}_t^{(n)}) - \log(\hat{P}_t^{(n+1)}) = \hat{A}_n - \hat{A}_{n+1} + (\hat{B}_n - \hat{B}_{n+1})' Z_t \\ &= -\hat{B}'_n(\mu - \lambda_0) - \hat{C}_n + r_t^* + \phi_g^n g_t + (\hat{B}_n^x - \hat{B}_{n+1}^x) x_t \\ &= -\hat{C}_n + E_t(r_{t+n}) + \hat{f}tp_t^{(n)}. \end{aligned}$$

Therefore, changes in r_t^* affect all real forward rates equally and hence act as a level factor. Changes in the real-rate gap g_t affect short-term real rates more strongly than long-term rates, and therefore affect the slope. The last row clarifies that real forward rates can be decomposed into convexity, an expectations component, $E_t(r_{t+n}) = r_t^* + \phi_g^n g_t$, and a real forward term premium, $\hat{f}tp_t^{(n)} = -\hat{B}_n^x \mu_x + \hat{B}_n' \lambda_0 + (\hat{B}_n^x - \hat{B}_{n+1}^x) x_t$.

For real yields we have

$$\begin{aligned} \hat{y}_t^{(n)} &= -\log(\hat{P}_t^{(n)})/n = -\hat{A}_n/n - \hat{B}'_n Z_t/n \\ &= \underbrace{r_t^* + \frac{1 - \phi_g^n}{n(1 - \phi_g)} g_t}_{\sum_{i=0}^{n-1} E_t r_{t+i}/n} \quad \underbrace{-\hat{A}_n/n - \hat{B}_n^x x_t/n}_{\text{convexity and real yield term premium}}. \end{aligned}$$

which shows that the equilibrium real rate r_t^* acts as a level factor for the real yield curve, and that the impact of the real-rate gap g_t diminishes with the yield maturity.

To understand the real term premium it is helpful to first consider the term premium in

the one-period-ahead real forward rate, which is

$$\hat{f}tp_t^{(1)} = Cov_t(m_{t+1}^r, r_{t+1}) = -[\lambda_{0r^*} + \lambda_{0g} + (\lambda_{r^*x} + \lambda_{gx})x_t].$$

If the real SDF positively correlates with the real rate, then real bonds are risky in the sense that their payoffs are low in times of high marginal utility. In this case, the real term premium is positive to compensate investors for this risk.²⁸

Nominal forward rates from n to $n + 1$ are:

$$\begin{aligned} f_t^{(n)} &= \log(P_t^{(n)}) - \log(P_t^{(n+1)}) = A_n - A_{n+1} + (B_n - B_{n+1})'Z_t \\ &= \underbrace{-c_n}_{\text{convexity}} + \underbrace{\pi_t^* + \phi_c^n c_t}_{E_t(\pi_{t+n+1})} + \underbrace{r_t^* + \phi_g^n g_t}_{E_t(r_{t+n})} + \underbrace{-B_n^x \mu_x + B_n' \lambda_0 + (B_n^x - B_{n+1}^x)x_t}_{\text{forward term premium}} \end{aligned}$$

Naturally, nominal forward rates reflect expectations of future inflation and real rates. Changes in the trend components π_t^* and r_t^* parallel-shift the entire path of these expectations, and therefore affect forward rates at all maturities equally. Distant forward rates are, on the other hand, only minimally affected by changes in c_t and g_t . The loading of forward rates on x_t can be shown to approach $-(\lambda_{\pi^*x} + \lambda_{r^*x})/(1 - \phi_x)$ for large n , meaning that x_t affects distant forward rates due to its effect on the prices of risk of π_t^* and r_t^* .

In our empirical analysis we will consider the five-to-ten-year forward rate, i.e.,

$$f_t^{(n_1, n_2)} = (n_2 - n_1)^{-1} \sum_{n=n_1}^{n_2-1} f_t^{(n)}, \quad n_1 = 20, \quad n_2 = 40.$$

Our model implies that this interest rate is even less affected by the cyclical components c_t and g_t and should exhibit a particularly close relationship with the trend components π_t^* and r_t^* .

Although our empirical analysis does not focus on term premia, it is worth noting the intuition for term premia, which is particularly simple within our model. The term premium in nominal forward rates, $f_t p_t^{(n)} = -B_n^x \mu_x + B_n' \lambda_0 + (B_n^x - B_{n+1}^x)x_t$, is composed of the real forward term premium, $\hat{f}tp_t^{(n)}$, and a forward inflation risk premium, $firp_t^{(n)}$. The intuition is again easiest for $n = 1$:

$$firp_t^{(1)} = Cov_t(m_{t+1}^r, E_{t+1}(\pi_{t+2})) = -[\lambda_{0\pi^*} + \lambda_{0c} + (\lambda_{\pi^*x} + \lambda_{cx})x_t].$$

If shocks to inflation expectations are positively correlated with the real SDF, then nominal bonds are more risky than real bonds and require a higher risk premium, i.e., a positive inflation risk premium. Like the real term premium, the inflation risk premium in this model is driven only by changes in x_t .

²⁸Variation in the real term premium is driven by changes in the risk-premium factor x_t , which affects prices of risk; quantities of risk are constant due to homoskedasticity of the state variables.

Table 1: Regressions of long-term interest rates on macroeconomic trends

	$y_t^{(5y)}$		$y_t^{(10y)}$		$f_t^{(5y,10y)}$	
	(1)	(2)	(3)	(4)	(5)	(6)
constant	-0.14 (0.68)	-1.69 (0.31)	1.03 (0.57)	-0.24 (0.30)	2.20 (0.50)	1.21 (0.37)
π_t^*	1.77 (0.15)	1.26 (0.14)	1.58 (0.14)	1.17 (0.13)	1.40 (0.13)	1.08 (0.13)
r_t^*		1.58 (0.15)		1.29 (0.14)		1.01 (0.17)
R^2	0.79	0.91	0.80	0.90	0.78	0.85

Regressions of long-term Treasury yields and forward rates on measures of long-run inflation expectations and the equilibrium real rate, which are described in the text. Numbers in parentheses are Newey-West standard errors with six lags. The data are quarterly from 1971:Q4. to 2015:Q4.

Table 2: Persistence of interest rates and detrended interest rates

Series	SD	$\hat{\rho}$	Half-life	ADF	PP
$y_t^{(5y)}$	3.18	0.97	23.1	-1.14	-3.27
$y_t^{(5y)} - \pi_t^*$	1.90	0.93	9.1	-1.83	-9.56
$y_t^{(5y)} - \pi_t^* - r_t^*$	1.26	0.86	4.5	-3.66***	-23.67***
$y_t^{(5y)} - 1.77\pi_t^*$	1.45	0.87	5.0	-2.64	-19.71*
$y_t^{(5y)} - 1.26\pi_t^* - 1.58r_t^*$	0.95	0.75	2.4	-4.54***	-44.91***
$y_t^{(10y)}$	2.84	0.97	22.5	-1.05	-3.02
$y_t^{(10y)} - \pi_t^*$	1.58	0.92	7.9	-2.36	-10.57
$y_t^{(10y)} - \pi_t^* - r_t^*$	1.02	0.83	3.7	-4.01***	-29.05***
$y_t^{(10y)} - 1.58\pi_t^*$	1.27	0.87	5.0	-2.66	-19.23*
$y_t^{(10y)} - 1.17\pi_t^* - 1.29r_t^*$	0.90	0.78	2.8	-4.69***	-38.45***
$f_t^{(5y,10y)}$	2.55	0.96	19.2	-1.12	-3.47
$f_t^{(5y,10y)} - \pi_t^*$	1.37	0.90	6.4	-2.62*	-13.79*
$f_t^{(5y,10y)} - \pi_t^* - r_t^*$	1.00	0.83	3.8	-4.02***	-30.02***
$f_t^{(5y,10y)} - 1.40\pi_t^*$	1.21	0.87	5.0	-2.87	-19.68*
$f_t^{(5y,10y)} - 1.08\pi_t^* - 1.01r_t^*$	0.99	0.83	3.7	-4.09**	-30.73**

Standard deviation (SD); first-order autocorrelation coefficient ($\hat{\rho}$); half-life, calculated as $\ln(0.5)/\ln(\hat{\rho})$; Augmented Dickey-Fuller (ADF) and Phillips-Perron unit root test statistics, for (detrended) interest rates, with *, ** and *** indicating significance at 10%, 5%, and 1% level. Detail on the unit root tests are in the main text. The data are quarterly from 1971:Q4 to 2015:Q4.

Table 3: Forecasting long-term interest rates

Horizon h (quarters):	RMSE					MAE				
	4	10	20	30	40	4	10	20	30	40
<i>Five-year yield</i>										
(1) Random walk	1.40	2.07	2.72	2.92	3.25	1.09	1.61	2.08	2.50	2.78
(2) Endpoint $\pi_t^* + \hat{\mu}_t$	1.51	2.18	2.72	2.82	3.05	1.19	1.74	2.31	2.54	2.78
(3) Endpoint $\pi_t^* + r_t^*$	1.35	1.75	2.08	2.03	2.18	1.00	1.30	1.48	1.67	1.92
DM p -value (3) vs. (1)	(0.39)	(0.07)	(0.05)	(0.04)	(0.07)	(0.23)	(0.03)	(0.04)	(0.02)	(0.02)
DM p -value (3) vs. (2)	(0.03)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.00)	(0.00)	(0.02)
<i>Ten-year yield</i>										
(1) Random walk	1.26	1.77	2.44	2.59	2.91	0.97	1.27	1.87	2.22	2.54
(2) Endpoint $\pi_t^* + \hat{\mu}_t$	1.33	1.86	2.32	2.36	2.51	1.03	1.46	1.92	2.11	2.34
(3) Endpoint $\pi_t^* + r_t^*$	1.25	1.56	1.79	1.55	1.50	0.89	1.05	1.08	1.09	1.30
DM p -value (3) vs. (1)	(0.49)	(0.17)	(0.04)	(0.01)	(0.01)	(0.26)	(0.09)	(0.00)	(0.00)	(0.00)
DM p -value (3) vs. (2)	(0.22)	(0.13)	(0.08)	(0.03)	(0.02)	(0.09)	(0.08)	(0.01)	(0.00)	(0.02)
<i>5-to-10-year forward rate</i>										
(1) Random walk	1.23	1.66	2.31	2.36	2.64	0.95	1.13	1.77	1.97	2.30
(2) Endpoint $\pi_t^* + \hat{\mu}_t$	1.24	1.64	2.02	1.99	2.05	0.93	1.23	1.55	1.69	1.90
(3) Endpoint $\pi_t^* + r_t^*$	1.30	1.64	1.80	1.43	1.11	0.99	1.23	1.19	0.90	0.88
DM p -value (3) vs. (1)	(0.65)	(0.48)	(0.09)	(0.02)	(0.01)	(0.65)	(0.75)	(0.02)	(0.00)	(0.00)
DM p -value (3) vs. (2)	(0.74)	(0.50)	(0.24)	(0.07)	(0.01)	(0.72)	(0.51)	(0.10)	(0.00)	(0.01)

Accuracy of different forecasts for long-term interest rates over horizons from 4 to 40 quarters, measured by the root-mean-squared error (RMSE) and the mean-absolute error (MAE) in percentage points. Method (1) is a driftless random walk. Methods (2) and (3) predict a smooth path from the current interest rate to the endpoint, using a recursively estimated mean-reversion parameter (the first-order autocorrelation coefficient of the detrended interest rate). For method (2) the endpoint is the sum of π_t^* and a recursively estimated mean $\hat{\mu}_t$. For method (3) the endpoint is $\pi_t^* + r_t^*$. The data are quarterly from 1971:Q4 to 2015:Q4. The first forecast is made at $t = 20$ (1976:Q3). The last two rows in each panel report one-sided p -values for testing the null hypothesis of equal forecast accuracy against the alternative that method (3) is more accurate, using the method of [Diebold and Mariano \(1995\)](#).

Table 4: Predicting excess returns

Holding period:	One quarter			Four quarters		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Full sample: 1971:Q4–2015:Q4</i>						
PC1	0.02 (0.04)	0.25 (0.07)	0.48 (0.12)	0.00 (0.10)	0.71 (0.19)	1.29 (0.19)
PC2	0.61 (0.25)	0.70 (0.24)	0.67 (0.23)	2.28 (0.55)	2.58 (0.42)	2.46 (0.40)
PC3	-1.84 (1.05)	-1.21 (1.01)	-1.61 (0.92)	-2.83 (1.60)	-0.72 (1.36)	-2.03 (1.28)
π_t^*		-2.01 (0.45) [0.00]	-2.80 (0.54) [0.00]		-6.08 (1.27) [0.02]	-7.94 (1.24) [0.00]
r_t^*			-2.68 (0.88) [0.02]			-6.93 (1.44) [0.01]
R^2	0.09	0.16	0.21	0.24	0.44	0.54
<i>Subsample: 1985:Q1–2015:Q4</i>						
PC1	0.08 (0.04)	0.18 (0.06)	0.55 (0.12)	0.15 (0.11)	0.55 (0.16)	1.65 (0.25)
PC2	0.50 (0.20)	0.67 (0.21)	0.72 (0.20)	1.71 (0.55)	2.44 (0.49)	2.56 (0.39)
PC3	-0.77 (0.97)	-0.41 (0.96)	0.46 (1.04)	-2.05 (1.89)	-0.06 (2.09)	2.43 (1.71)
π_t^*		-1.16 (0.73) [0.35]	-2.33 (0.75) [0.05]		-4.59 (1.48) [0.16]	-8.03 (1.24) [0.01]
r_t^*			-3.17 (0.98) [0.02]			-9.53 (2.03) [0.01]
R^2	0.08	0.10	0.16	0.22	0.31	0.51

Predictive regressions for quarterly and annual excess bond returns, averaged across two- to 15-year maturities. The predictors are three principal components (PCs) of yields and measures of long-run inflation expectations (π_t^*) and the equilibrium real rate (r_t^*), which are described in the text. Numbers in parentheses are White standard errors for quarterly (non-overlapping) returns, and Newey-West standard errors with 6 lags for annual (overlapping) returns. Numbers in squared brackets are small-sample p -values obtained with the bootstrap method of [Bauer and Hamilton \(2016\)](#). The data are quarterly from 1971:Q4 to 2015:Q4.

Table 5: Goodness-of-fit of regressions for annual excess returns

Predictors	Full sample: 1971–2015		Subsample: 1985–2015	
	R^2	ΔR^2	R^2	ΔR^2
Yields only	0.24 (0.05, 0.40)		0.22 (0.08, 0.52)	
Yields and π_t^*	0.44 (0.10, 0.44)	0.20 (0.00, 0.19)	0.31 (0.14, 0.55)	0.09 (0.00, 0.19)
Yields, π_t^* and r_t^*	0.54 (0.13, 0.47)	0.30 (0.00, 0.23)	0.51 (0.17, 0.58)	0.29 (0.00, 0.25)
Yields detrended by π_t^*	0.34 (0.05, 0.40)	0.10 (-0.06, 0.07)	0.28 (0.11, 0.52)	0.07 (-0.12, 0.11)
Yields detrended by π_t^* and r_t^*	0.44 (0.06, 0.40)	0.20 (-0.08, 0.11)	0.50 (0.10, 0.51)	0.28 (-0.18, 0.16)

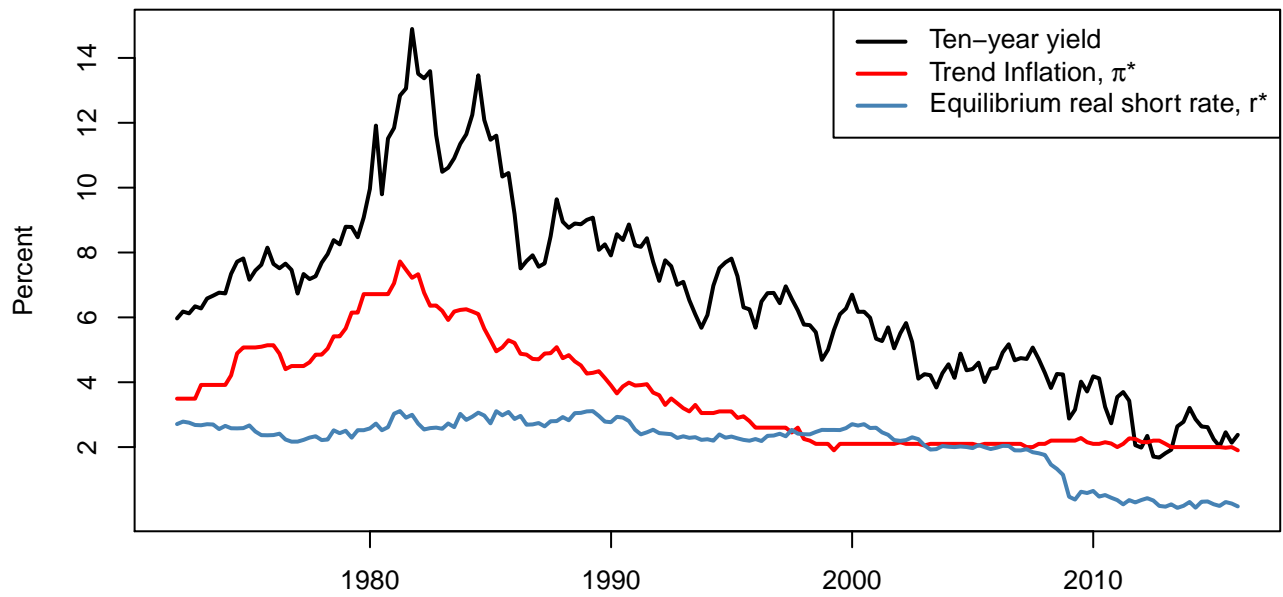
Goodness-of-fit, measured by R^2 , of regressions for annual excess bond returns, averaged across two- to 15-year maturities. The predictors are three principal components (PCs) of yields and measures of long-run inflation expectations (π_t^*) and the equilibrium real rate (r_t^*), which are described in the text. The last two specifications use three PCs of detrended yields, i.e., of interest rate cycles, which are constructed as either $y_t^{(n)} - \pi_t^*$ or $y_t^{(n)} - \pi_t^* - r_t^*$. Increase in R^2 (ΔR^2) is reported relative to the first specification with only PCs of yields. Numbers in parentheses are 95%-bootstrap intervals obtained by running the same regression in 5,000 bootstrap data sets generated under the spanning hypothesis, i.e., the null hypothesis that only yields have predictive power for bond returns.

Table 6: Variance ratios

h	Variances and covariances				Variance ratios	
	$\Delta_h y_t$ (1)	$\Delta_h i_t^*$ (2)	$\Delta_h (y_t - i_t^*)$ (3)	$2 \cdot Cov$ (4)	(2)/(1)	[(2)+(4)]/(1)
<i>Five-year yield</i>						
1	0.49	0.05	0.45	-0.00	0.10	0.09
5	2.33	0.36	1.56	0.42	0.15	0.33
10	3.89	0.80	2.01	1.08	0.21	0.48
20	7.04	1.69	2.77	2.58	0.24	0.61
30	9.29	2.80	2.96	3.53	0.30	0.68
40	9.10	2.86	2.75	3.50	0.31	0.70
<i>Ten-year yield</i>						
1	0.38	0.05	0.34	-0.01	0.13	0.10
5	1.85	0.36	1.23	0.26	0.19	0.33
10	2.84	0.80	1.35	0.69	0.28	0.52
20	5.84	1.69	2.09	2.05	0.29	0.64
30	8.03	2.80	2.16	3.07	0.35	0.73
40	8.19	2.86	2.16	3.17	0.35	0.74
<i>5-to-10-year forward rate</i>						
1	0.36	0.05	0.32	-0.01	0.14	0.10
5	1.72	0.36	1.26	0.10	0.21	0.27
10	2.50	0.80	1.40	0.30	0.32	0.44
20	5.33	1.69	2.11	1.53	0.32	0.60
30	7.28	2.80	1.87	2.62	0.38	0.74
40	7.73	2.86	2.02	2.84	0.37	0.74

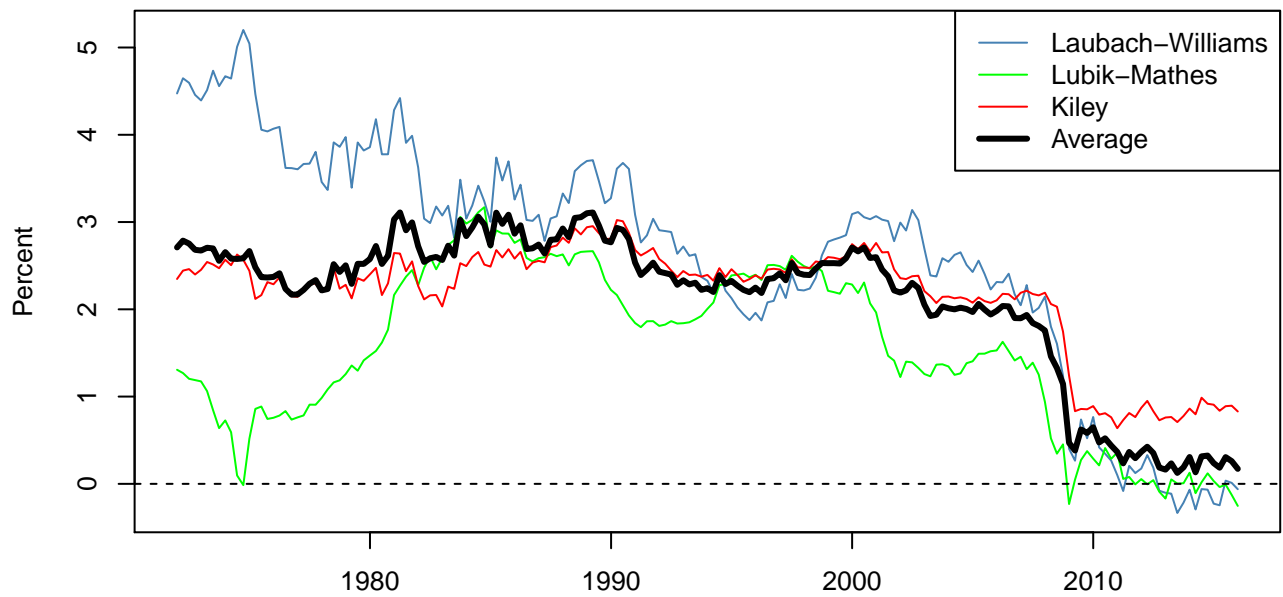
Variances, covariances, and variance ratios for changes in long-term interest rates and the trend component $i_t^* = \pi_t^* + r_t^*$. The first three columns report sample variances for h -quarter changes in the interest rate y_t , the trend component i_t^* , and the cycle component $y_t - i_t^*$. The fourth column reports twice the covariance between changes in the trend component and the cycle component. The last two columns report two different ratios: The first is the ratio of the variance of changes in the trend component relative to the variance of interest rate changes. The second includes the twice the covariance between changes in the trend and cycle components in the numerator, and equals one minus the variance ratio for the cycle component. The data are quarterly from 1971:Q4 to 2015:Q4.

Figure 1: Ten-year yield and macroeconomic trends



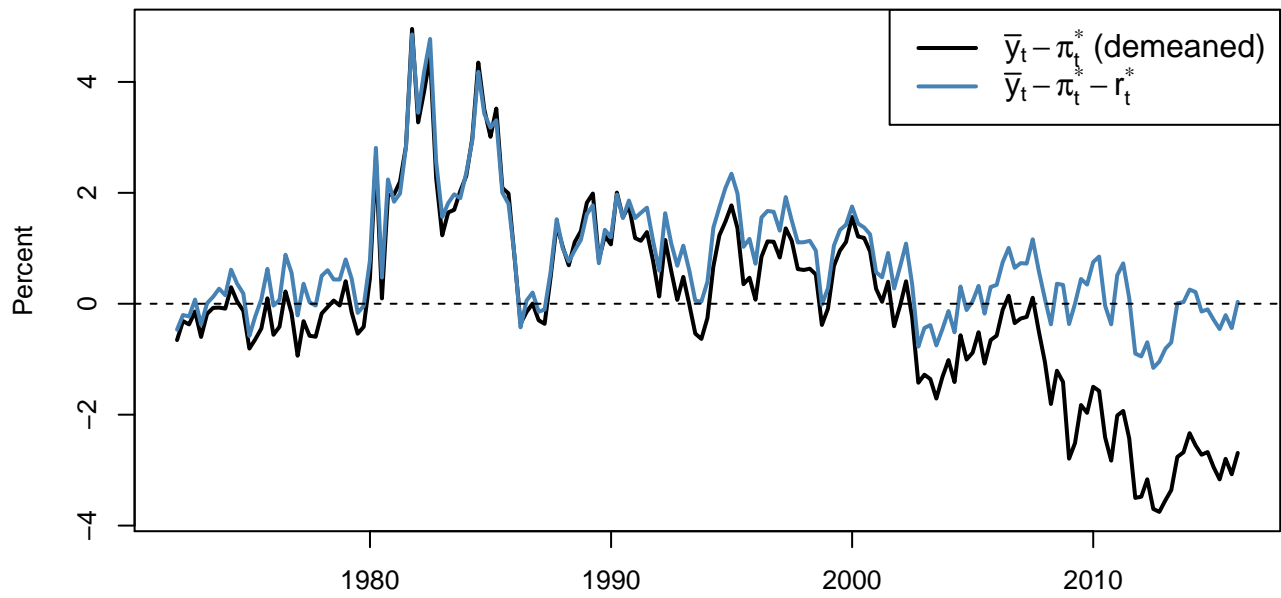
Ten-year Treasury yield and estimates of trend inflation, π^* (the mostly survey-based PTR measure from FRB/US), and of the equilibrium real rate, r^* (the average of the estimates in Figure 2). The data are quarterly from 1971:Q4 to 2015:Q4.

Figure 2: Measures of the equilibrium real interest rate



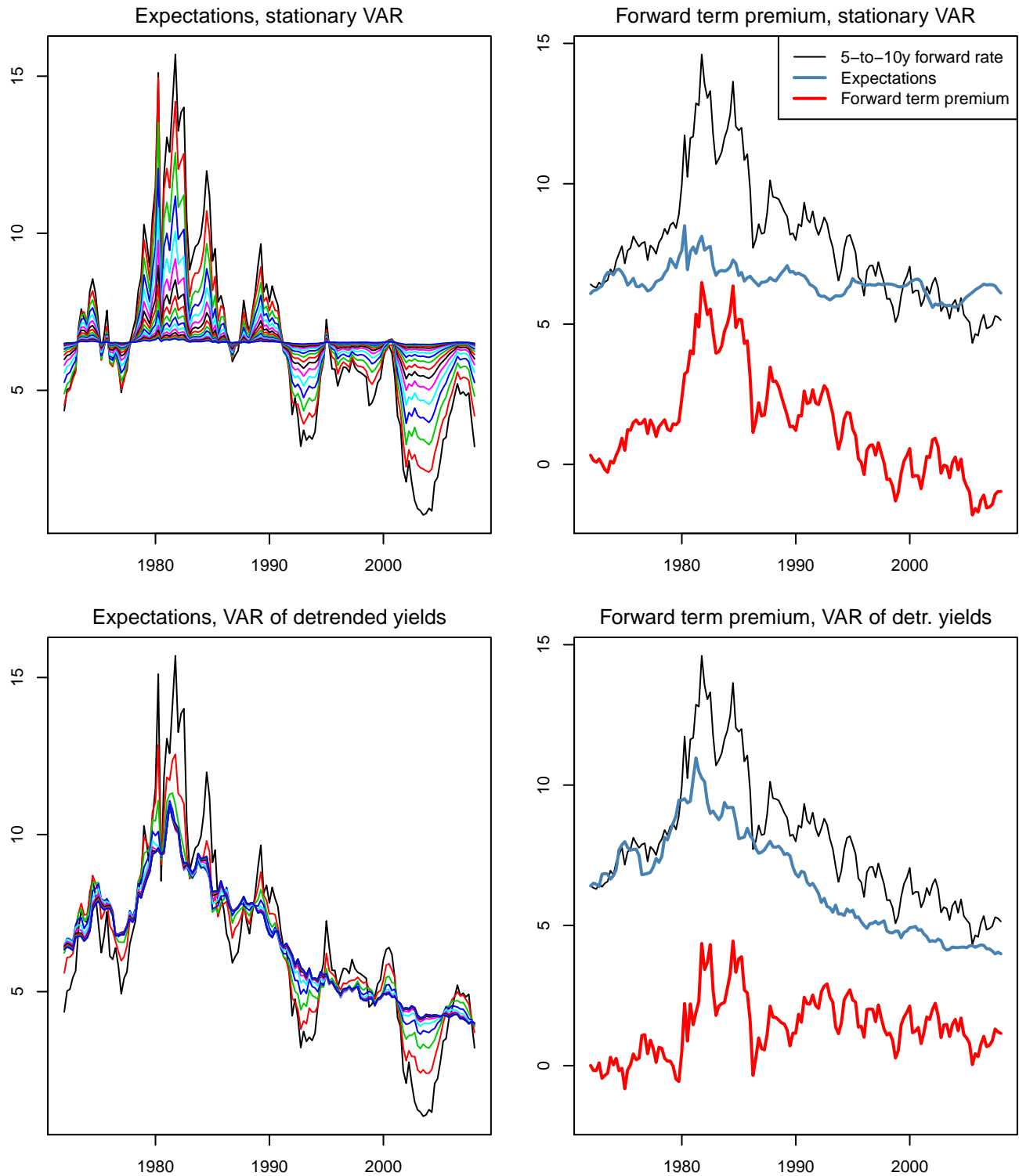
Three macroeconomic estimates of r^* from [Laubach and Williams \(2003\)](#), [Lubik and Matthes \(2015\)](#), and [Kiley \(2015\)](#), as well as the average of these measures. The data are quarterly from 1971:Q4 to 2015:Q4.

Figure 3: Measures of the interest rate cycle



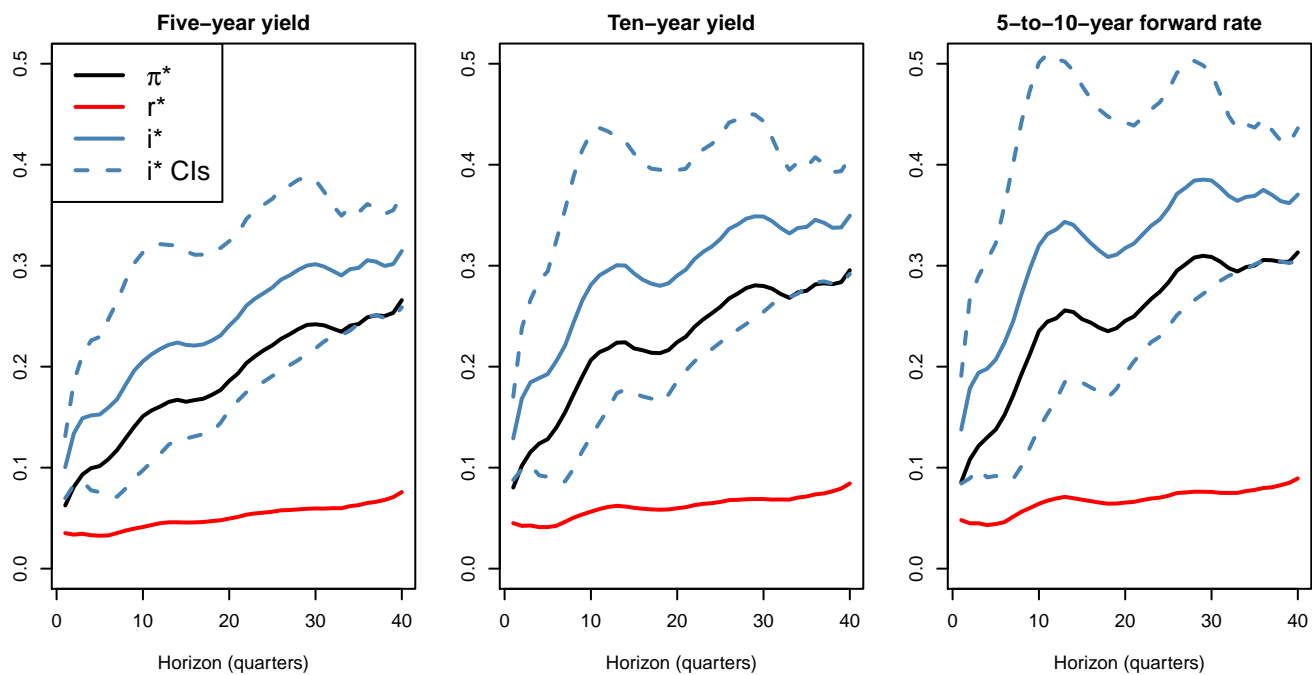
Estimates of the cycle in the level of interest rates. Here the level, \bar{y}_t , is the average yield across one- through 15-year maturities. The black line is the demeaned difference of \bar{y}_t and the estimated inflation trend π_t^* . The blue line is the difference of \bar{y}_t and $i_t^* = \pi_t^* + r_t^*$. The data are quarterly from 1971:Q4 to 2015:Q4.

Figure 4: Short-rate expectations and term premium



Left panels: current one-year yield (black line) and expectations of the future one-year yield at horizons from two to 14 years (colored lines) for a stationary VAR (top panel) and based on shifting macro trends and a VAR of detrended yields (bottom panel). Right panels: five-to-ten-year forward rate with estimated expectations and term premium components. The data are quarterly from 1971:Q4 to 2007:Q4.

Figure 5: Variance ratios



Variance of h -quarter changes in π_t^* , r_t^* , and $i_t^* = \pi_t^* + r_t^*$ relative to variance of h -quarter changes in long-term interest rate. The dashed lines show 95%-confidence intervals for the i_t^* -variance ratio, constructed as described in the text. The data are quarterly from 1971:Q4 to 2015:Q4.