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Wealth Taxation, Non-listed Firms, and the Risk of Entrepreneurial Investment

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Wealth Taxation, Non-listed Firms, and the Risk of Entrepreneurial Investment

Abstract

How to incorporate hard-to-measure assets into the wealth tax? We analyze the effect of an optimal wealth tax on risk-taking behavior and welfare when investors do not only have the standard portfolio choice with a well-diversified market portfolio, but can alternatively choose to invest all their wealth into a non-diversifiable, indivisible project. The latter is interpreted as entrepreneurial investment into a small, non-listed firm for which the actual value is hard to measure and non-verifiable. For such firms, real-world wealth tax systems base the wealth tax on deterministic book values. We show that this tax treatment does not distort the choice of projects if the tax is set optimally with an imputed interest rate on book values, actually larger than the risk-free market rate of return. The market equilibrium and a proportional tax on the market portfolio will ensure an efficient risk allocation between private and public consumption and across projects. Failing to apply an imputed inflation of book values, instead, gives rise to an implicit subsidy on entrepreneurial activity and distorts investment. Our findings also have implications for taxation of hard-to-measure assets under capital-gains and inheritance taxation.

JEL-Codes: H210, D140, G110.

Keywords: wealth taxation, portfolio choice, non-listed firms, risk diversification, hard-to-measure assets.

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1 Introduction

Until the beginning of this decade, it looked like a comprehensive wealth tax will soon be of academic interest only, if any at all. Actually, among OECD countries, only France, Norway, and Switzerland still raise such a wealth tax; India as another major economy abolished its wealth tax in 2016. But in the wake of the financial crisis, and particular driven by the publication of Thomas Piketty's book 'Capital in the 21st century' in 2014, wealth taxation became a hotly discussed issue again.[1] The momentum was fueled by tax researchers who pointed out that the famous Atkinson-Stiglitz theorem of optimally not taxing capital does not apply in reality. They documented that a wealth tax is part of the optimal tax mix and voiced confidence that the costs and problems, traditionally attached to a wealth tax, can be overcome (see Piketty et al., 2013, for a summary).

The proponents of a wealth tax remain silent, however, on how to solve the valuation issue for non-traded assets (e.g., Boadway et al., 2010, p. 784), and in particular, for non-listed (private) firms and closely-held companies. With almost 40%, such private business accounts for the largest wealth category of the wealthiest top 1% in the U.S., and its valuation is seen as key for a wealth tax (Frank, 2012).

As a practical solution for such hard-to-measure assets, e.g., in Norway, wealth tax falling on ownership in small, non-listed firms is based on pre-period (historical) book value. This approach neglects both that hard-to-measure assets usually come with substantial risk and that there are significant returns during the tax year. Adjusting Auerbach (1991) and adopting his view on systematic risk, one does not need to care about the risk effects, but should invoke an imputed return at the market interest rate.[2] This view, however, neglects both risk balancing on private and public consumption and the fact that deterministic, fixed tax payments are regressive with stochastic returns. Therefore, end of 2014, Johnson and Lensberg (2014) attacked the practical Norwegian solution by arguing that such a wealth tax does not participate in entrepreneurs' risk. In a CAPM analysis, the tax increases risk premia and induces less activity in the sector of non-listed firms. The authors' calculations, showing an excess burden of 250% of underlying tax revenue, fueled the Norwegian wealth tax debate. The right-wing Norwegian government used this new argument in order to push for an abolishment of the wealth tax, and a heated debate among economists, politicians, and the broad public ensued.[3]

The major part of non-listed firms is constituted by small and medium-sized enter-

[1] Germany and Denmark, for example, abolished their wealth taxes in 1997, the remaining Nordic countries followed in 2006/07, except for Norway. In 2010, however, Iceland and Spain temporarily reintroduced a wealth tax and there are debates, for example in Germany, Japan and the U.S., on (re-) introducing comprehensive wealth taxation. See, e.g., NOU2014:13 and KPMG (2012).

[2] In Auerbach's (1991) proposal for retrospective capital gains taxation, the imputed return needs to be imposed retrospectively in order to ensure holding period neutrality. Imposing imputed returns on historical values will still trigger lock-in effects in a multi-period model.

[3] See, e.g., Johnson and Lensberg on 'En dyr skatt' in Dagens Næringsliv from October 22, 2014 vs. Bjerksund, Møen, and Schjelderup on 'Feil skatteanalyse' in Dagens Næringsliv from November 12, 2014.

prises, including small start-up firms that drive the 'new information economy'. Because these enterprises are the backbone of all OECD economies, the overall welfare loss from such a tax effect and from distortions in entrepreneurial investment would be enormous.[4] But, does a wealth tax really deter investment in small and new businesses? Should there be some risk balance? More generally, how should (and can) assets – and their capital gains – be taxed, both effectively and efficiently, if their market value cannot be easily inferred from observable prices?

This paper shows that the optimal wealth tax rate on personal investors' level is positive, balances systematic risk across private and public consumption, and does not harm non-listed firms via an increased exposure to risk. Using inflated historical book values for such non-diversifiable investment both avoids valuation problems for non-traded assets and ensures an efficient allocation of resources and risk. A deterministic wealth tax on non-listed firm investment has only second-order effects on entrepreneurial activity, but allows to tax infra-marginal rents. But thereto, book values need to be inflated by an imputed return that is larger than the risk-free rate or return. These findings suggest that current wealth-tax systems, if expanded for deterministic imputed rents, are an efficient second-best solution to tax hard-to-measure risky assets, for which only the initial value, but no sales price or actual value is known.

In order to derive these findings, we apply a standard portfolio-choice model (e.g., Mossin, 1968) with exogenous wealth (i.e., there will be no savings distortion) and amend it by an investment possibility that requires to have one's entire wealth invested in this project so that there is no possibility to diversify one's portfolio. This is the most extreme case of small, non-listed firms facing prohibitive borrowing constraints. The non-diversifiable investment also corresponds to a small start-up firm that requires the full wealth of its owner as equity investment in order to avoid adverse moral hazard effects. The wealth tax is raised on personal investor level only. Following real-world wealth tax codes such as the Norwegian one, the tax base for ownership in non-listed firms (i.e., the non-diversifiable investment) is based on historical book value, not on actual, end-of-period value. Investors have to decide whether to become entrepreneur and to invest all their wealth into their non-listed firm or to hold a fully diversified ('market') portfolio with stocks of listed firms. We then apply a Ramsey-like approach with ex-ante homogenous, but ex-post heterogeneous investors in order to determine the optimal tax structure given the available instruments.

In this model, the classic Domar-Musgrave effect applies and diversified investors can avoid any tax effect falling on their risky part of investment. They do not experience

[4]Both in the U.S. and in Europe, 99% of all firms are non-listed (private) firms, that for example in the U.S., account for 59% of total sales (Forbes, 2013; Claessens and Tziumois, 2006). Though there are some very large non-listed firms such as Dell Inc., the major part falls into the category of small and medium-sized enterprises. Using this segment as a conservative estimate, non-listed firms accounted for 67% of total employment and 57.5% of gross value added in the EU28 in 2012 (Eurostat, 2016, Table 1).

any (private-consumption) insurance effect from taxation either, and effectively face a deterministic wealth tax on their inflated initial wealth. Capital market equilibrium enforces a risk balancing between investors in non-listed firms and holders of a diversified portfolio; the wealth tax ensures an efficient balancing of systematic risk on public and private consumption. In addition, distortions in the entrepreneurial decision are avoided if the historical book value of non-listed firms is inflated by an imputed rent. This imputation needs to be larger than the risk-free rate of return in order to tax infra-marginal rents generated by non-listed firms. In such a system, the optimal wealth tax rate (combined with some lump-sum transfer and the provision of a public good) is clearly positive and no preferential tax treatment is given to hard-to-measure assets.

Our findings contribute in several ways to the literature and the wealth tax debate. First, the analyzed wealth-tax rule for non-listed firms has some similarity to retrospective capital gains taxation. Auerbach (1991) proposed the latter system to tax hard-to-measure investments. He showed that if the current value (sales value) of an asset and its holding period is known, retrospectively attaching an imputed rate of return equal to the risk-free rate of return and taxing this return in each period ensures holding period neutrality and avoids distortions because it does not affect ex-ante decision making. Such a system, however, does not work if legal restrictions require annual taxation and current wealth is not known because there is no market transaction (e.g., gifts between family members) or if the asset is never sold (e.g., a family firm that is bequeathed, but never listed). Our analysis shows that although the 'imputed interest on initial wealth' method violates holding-period neutrality (Auerbach, 1991, pp. 172f), it allows taxing hard-to-measure assets without distortions in investment behavior, whenever initial values (historical book values), but not current values are known. Besides non-listed firms under wealth taxation, another straightforward application for this is a gift- and inheritance tax, the latter being seen as an important complement to optimal capital income taxation (Piketty et al., 2013, section 4).

Second, an Auerbach-like retrospective tax system lacks the welfare-improving ability to diversify and insure systematic (i.e., macroeconomic) risk by balancing it on private consumption and public spending.[5] Mutatis mutandis, our findings imply that retrospective taxation should only be applied to those assets where a mark-to-market and annual taxation at accrual is not possible; it should not be used for all assets. Our model shows that a standard capital tax on actual returns in listed assets allows for an efficient diversification of systematic market risk on private and public consumption, while hard-to-measure assets can be taxed retrospectively via imputed returns without distorting

[5]Though Auerbach (1991, p. 171) comments on this and claims that a tax on excess returns has a value of zero, this view misses the fact that welfare increases if the government uses stochastic tax revenue to balance systematic risk on private and public consumption. This holds for any capital income tax (Christiansen, 1993) and particularly for a tax on risky excess returns, see Schindler (2008) and more recently Spiritus and Boadway (2017).

portfolio choice (entrepreneurial investment) and overall risk allocation. It might require a higher than the normal rate of return imputation and could challenge holding period neutrality somewhat, however.

Third, arguments such as wealth taxation (on investors' level) deters investment in non-listed firms relative to listed firms, because the tax base for the former is based on historic values and does not incorporate current business income risk, do not apply when a full-fledged model, market equilibrium forces, and an optimal tax structure are taken into account. Focusing on a CAPM model, and looking at the increase in systematic risk for the owner of the non-listed firm only, misses important effects. First, the tax falling on the compensation for risk (i.e., the realized risk premium) of the diversified portfolio equilibrium does not affect the risk-income position after tax of investors. Their remaining tax burden is comparable to the wealth tax on imputed returns, paid by non-diversified investors. Second, equilibrium forces in the capital market will ensure a risk-return balance between (indivisible) investments into non-listed firms and holding a portfolio of listed firms. The effect of an optimal wealth tax on this trade-off is of second order so that the decision to become entrepreneur in a non-listed firm remains undistorted (as long as the wealth tax does not worsen credit constraints). Lastly, infra-marginal rents from decreasing returns to scale in non-listed firm investment constitute a legitimate part of the wealth tax base. Evaluated for an optimal tax structure, real-world wealth tax systems such as the Norwegian one that do not inflate historical book values tax-favor ('subsidize') non-listed firms and in fact rather induce *higher* activity in the sector of non-listed firms than optimal.

Fourth, it follows from our analysis that there is a difference between occupational-choice and portfolio-choice set-ups. Contrary to Hansson (2008), wealth taxation will not affect the investment decision in non-listed (or start-up) firms vs. holding a diversified portfolio of listed firms as long as all assets are taxed at the same tax rate and entrepreneurs face a tax on imputed return on initial wealth (i.e., on historical book values). Instead, Hansson (2008) assumes that entrepreneurs can avoid the wealth tax by becoming employees which leads to less entrepreneurial investment. A main difference between our set-up and her model, however, is that there is no minimum capital requirement for investments into non-listed firms in our model. Hence, the wealth tax does not worsen credit constraints, while these constraints seem to drive the empirical findings in Hansson (2008).[6]

The remainder of the paper is as follows. In the next section, some relevant literature is reviewed. The model is presented in Section 3, while in Section 4, the properties of the market equilibrium are discussed. Finally, the welfare effects as well as the optimal tax structure are derived in Section 5. The last section offers some conclusions.

[6]As Hansson (2008, p. 153) stresses, it is difficult to find good natural experiments for the analyzed effect and her empirical findings also need to be interpreted with care, because of few observations, potentially confounded control groups, and potential endogeneity problems.

2 Brief Literature Review

From standard portfolio choice models with given initial wealth, it is well understood that a proportional tax on final wealth combines a Domar-Musgrave substitution effect on risky investment, leaving expected utility constant all else equal, with a deterministic lump-sum tax on the normal gross rate of return on initial wealth. Such a tax will not affect total investment in the risky component if the investor features constant relative risk aversion, and in general, affects investors' expected utility only via a pure income effect (see, e.g., Sandmo, 1985, section 8).[7] But, this tax literature on portfolio choice focuses on diversified portfolios and neglects non-diversifiable investment into non-listed (or start-up) firms.

Empirical studies on the composition of savings are reviewed in Poterba (2002). He reports somewhat ambiguous findings, but concludes, that overall, taxation has an important impact on portfolio choice. In particular, asset-specific capital income taxes will trigger substitution effects in households' portfolios. Studies on the impact of a comprehensive wealth tax, falling on all assets and (real) property seemed to be lacking, however. Closest to a wealth tax comes the real estate tax for which some unclear portfolio evidence is reported (Poterba, 2002, section 4.5).

Because of new insights on optimal capital taxation, showing that the Atkinson-Stiglitz non-capital tax result does not hold, and the fact that income and consumption for top wealth holders are difficult to disentangle, there is a (new) case for welfare-improving wealth taxation, see Piketty et al. (2013) for a summary. Wealth taxation can help to improve redistribution, and it is particularly helpful for taxing super-wealthy people who hardly report taxable income and shield their income streams within foundation vehicles and similar institutions. The traditional criticism of wealth taxation (e.g., Boadway et al., 2010), pointing to very high administrative and compliance costs, is seen to be solved by increasing information exchange between banks and tax authorities on (cross-border) financial assets, more international cooperation, and improved fiscal technology (e.g., pre-filled wealth declarations). Granting substantial allowances would avoid severe distortions in savings behavior and exempt most tax payers from declaration. Nevertheless, such a wealth tax can raise substantial revenue, as a simulation for Germany shows. Bach and Thiemann (2016) estimate the revenue potential of the 2012-proposal by some socialdemocrat-led German States (Bundesländer) to reintroduce the wealth tax to be about 10 billion Euro (i.e., 0.4% of 2011 GDP), taking tax avoidance reactions into account already. Total tax collection costs (excl. excess burden) are estimated to be 9% of tax revenue. None of the recent proponents, however, offers a solution for the valuation issue of non-traded assets, in particular non-listed (private) and closely-held companies.

[7] Of course, in a general setting with endogenous savings, the wealth tax should also create distortions in intertemporal consumption choice. Neither Seim (2016) nor Zoutman (2014), however, find any (strong) wealth-tax effect on real savings and wealth accumulations, see the discussion below.

Following up on the urge in Piketty et al. (2013, p. 6) for more empirical research on relevant wealth-tax related elasticities, the most recent literature on wealth taxation tries to estimate the elasticity of the wealth tax base. Contrary to the equivalent elasticity for income taxes, little is yet known on the distortive effect of wealth taxation. Seim (2016) relies on kinks in the former Swedish wealth tax system and administrative Swedish data. Estimating the elasticities with respect to wealth after tax, he reports a value of about 0.2 for the unitary Swedish system and his findings are mainly driven by evasion and avoidance motives. In fact, he concludes that there are no significant real effects (on savings) from wealth taxation, and based on additional calculations, identifies the wealth tax as a good instrument for redistribution. Brülhart et al. (2016) use unique Swiss data, exploiting variation of tax rates across cantons as well as within cantons. They report a wealth tax base elasticity that is much larger than the one for taxable income and find that most responses happen by adjustments in reporting, not by actual mobility within or across cantons.

Closest to our portfolio choice setting is Zoutman (2014). In 2001, a capital income tax reform in The Netherlands increased the wealth tax on financial wealth, but abolished the wealth-tax burden on housing, and left taxation of ownership in small firms and closely-held companies unchanged. Zoutman has information on households' financial and housing wealth and estimates the wealth-tax effects both on aggregate savings and the portfolio composition.[8] This set-up is particularly interesting for our study. Just as non-listed firms in our model, housing constitutes an indivisible and hardly diversifiable investment that receives a special wealth tax treatment. The empirical results for the Dutch reform confirm the Swedish findings by Seim (2006) that overall savings (or 'initial wealth') hardly react on wealth taxation, estimating a tax-rate elasticity of 0.036. Also the portfolio effects are modest. The portfolio share of financial wealth decreases by 0.033% if the tax rate on financial wealth increases by 1%. Though statistically significant, the effect of a wealth tax on housing is economically negligible.

An area and cost aspect that has been neglected by and large, however, is the impact of wealth taxation on non-listed firms that are hard to value, and particularly on small and medium-sized enterprises and entrepreneurial activity. Hansson (2008) predicts in an occupational-choice model that wealth taxation will reduce entrepreneurial activity. The tax reduces the incentive to become entrepreneur, instead of employee, and reduces the available capital so that credit-constraint entrepreneurs are less able raise the necessary funds to set up their firms. Using data for 22 OECD countries between 1980 and 2003, she documents a modest, but significant increase in the share of self-employed by about 0.2 to 0.5 percentage points in response to an abolishment of a wealth tax in a country.

Johnson and Lensberg (2014) look at systematic risk in non-listed firms and capital

[8]He does not have information on small firms and closely-held companies and filters out households owning closely-held companies or being self-employed.

cost effects of the wealth tax. They start from the observation that non-listed firms are taxed on historical (pre-period) book values that do not relate to firm's performance during the year. Hence, the tax base of non-listed firms is deterministic and the effective wealth tax rate, relative to profits, becomes regressive. Relative high tax payments in bad times increase systematic risk and the costs of equity. None of these effects applies to listed firms and diversified portfolios that are taxed based on market value. Applying a standard CAPM approach, Johnson and Lensberg predict that wealth taxation triggers less risky investment behavior and less activity in non-listed firms and fewer non-listed firms (entrepreneurs), the latter also interpreted as less technological progress and innovation. Using data on Norway, these authors do back-of-the-envelope calculations to derive the non-listed firms' risk measure beta after tax and calibrate the resulting loss in activity and output which turns out as 2.5 times the generated tax revenue.

Optimal diversification of systematic risk via taxation is analyzed for capital income taxes only. Based on an early contribution by Gordon (1985), some literature focuses on a setting in which the stochastic capital tax revenue is returned as state-contingent lump-sum transfer to the individuals so that the insurance effect from taxation is nullified. Thus, it is concluded that capital taxation cannot insure against systematic risk and in particular that a tax on excess returns does not provide any welfare gain.[9] As pointed out by Christiansen (1993), however, stochastic tax revenue can optimally be used to finance a state-contingent provision of a public good. Such a diversification of systematic risk on private and public consumption increases welfare. Schindler (2008) extends the latter analysis and shows that a tax on pure excess returns allows for a very efficient risk diversification between consumption types, but does not distort the resource allocation.

Recently, Spiritus and Boadway (2017) embedded the approach by Schindler (2008) in a Mirrlees setting with endogenous labor supply and non-linear labor taxation. They generalize the findings of preferable risk-diversification effects of a tax on risky excess returns and the separability between excess taxation and the tax burden on risk-free capital income. The analysis in Spiritus and Boadway is closest and complementary to our work, because they also embed a closely-held asset ('private investment opportunity') into their model and document that the risk-less component of its return should be taxed if the investment opportunity features decreasing returns to scale. But, they assume that the return of the closely-held asset is verifiable (i.e., can be taxed), that it features idiosyncratic risk only, and that it is part of a continuous investment portfolio decision. Hence, the authors cannot analyze, how a hard-to-measure asset should be incorporated into a (wealth) tax system, how taxation affects the discrete decision to become entrepreneur, and how the existence of a non-taxable risky asset affects optimal risk diversification on private and public consumption.

[9]This view also explains why Auerbach (1991) argues that risky returns will not affect his analysis of a retrospective capital gains tax system.

3 The Model

In order to focus on the portfolio decision and the question whether the wealth tax will harm investment in non-listed firms, we apply a standard portfolio-choice model, as it is usually used to analyze risk-taking and taxation (see, e.g., Mossin, 1968; Stiglitz, 1969). This implies, that for simplicity, we will treat initial wealth as exogenous and neglect distortions in wealth accumulation (i.e., in the savings decision). This per-se hard assumption is supported by recent empirical evidence (Seim, 2016; Zoutman, 2014).

In such portfolio-choice models of a small open economy with a perfectly integrated world capital market, risk-averse investors have initial wealth W_0 and can decide on investing the amount a into a risky asset that delivers a risky rate of return $\tilde{x}$.[10] The remaining amount $W_0 - a$ is invested into a riskless asset that delivers the riskless (normal) rate of return $r > 0$. The risky asset can be interpreted as investing in the market portfolio in the stock market (e.g., Sandmo, 1977) that delivers a positive expected risk premium, i.e., $\mathrm{E}[\tilde{x} - r] > 0$. If doing these investments, before-tax wealth in period 1 is given by

$$\tilde{W}_1^d = (1 + \tilde{x})a + (1 + r)(W_0 - a) = (\tilde{x} - r)a + (1 + r)W_0. \tag{1}$$

We extend the standard model by an alternative investment opportunity that is not part of the market portfolio. Ex ante homogenous investors can choose to become entrepreneur and set-up a small, non-listed firm (an 'n-type investor'). This investment is indivisible and requires to invest one's entire wealth W_0 so that n-type investors cannot diversify their investment. This features conditions in small family-owned firms well. For simplicity, we also assume that the non-listed firms suffer from prohibitive borrowing constraints. The latter would fit to a start-up company in which the entrepreneur needs to invest all its savings (and might need some additional venture capital). In sum, these assumptions provide the strongest possible case for tax distortions that are created by asymmetric treatment, amplification of systematic risk exposure and non-diversifiability in (small) non-listed firms.

The investment in a non-listed firm delivers a risky return $\tilde{x}_n = \tilde{x}_n(\gamma)$ that negatively depends on the share of entrepreneurs γ that are active in the non-listed sector. That is $\frac{\partial \tilde{x}_n}{\partial \gamma} < 0$. The basic idea behind this is that more competition in the sector of non-listed firms will drive down the (still risky) supernormal profits in that sector. Furthermore, investors in non-listed firms face an idiosyncratic risk component ε with expected value $\mathrm{E}[\varepsilon] = 0$, because they cannot diversify their investment. Hence, the total rate of return on investment in a non-listed firm is given by $\tilde{x}_n(\gamma) + \varepsilon$ and before-tax wealth in period 1 can be written as

$$\tilde{W}_1^n = (1 + \tilde{x}_n(\gamma) + \varepsilon)W_0. \tag{2}$$

[10] Variables with a " $\sim$ " will indicate stochastic variables throughout the paper.

Suppose now that the government raises a wealth tax with tax rate t that falls on the investors.[11] For wealth hold in listed firms, i.e., for the market-portfolio investment, the tax base is the market value of investment which is equal to the wealth in period 1 ($\tilde{W}_1^d$) so that the tax payment is equal to $t\tilde{W}_1^d$. For small, non-listed firms, it is often considered that determining the actual market value is impossible or too expensive. Therefore, the tax base is usually not market but book value, a point in case is the Norwegian wealth tax. We assume that book value is equal to the original investment at the beginning of the period, that is W_0. Furthermore, we assume that this book value is inflated by a deterministic imputed (notional) interest rate δ, implying a tax payment of $t(1+\delta)W_0$ which is deterministic, as well.[12] Additionally, the government can raise a state-independent lump-sum tax T that needs to be paid by each investor at the end of the period, no matter in which sector the investor invested and no matter what its return is. Effectively, this lump-sum tax, if negative, corresponds to the lump-sum transfer in traditional income-tax settings.

Normalizing the mass of investors to one, the stochastic tax revenue is given by

$$\begin{aligned} \tilde{R} &= \gamma(1+\delta)tW_0 + (1-\gamma)t[(1+r)W_0 + (\tilde{x}-r)a] + T, \\ &= \gamma T_n + (1-\gamma)T_d + (1-\gamma)t^e(\tilde{x}-r)a, \end{aligned} \tag{3}$$

where we used the fact that initial wealth is exogenous to simplify the tax structure and analysis. Effectively, the government has three tax instruments at its disposal: (i) a participation (or extensive-margin) tax falling on entrepreneurs (n-type investors) with a tax payment equal to $T_n = t(1+\delta)W_0 + T$; (ii) a participation tax falling on diversified investors with a tax payment equal to $T_d = t(1+r)W_0 + T$; and (iii) a proportional tax rate t^e on excess returns $(\tilde{x}-r)a$ in the diversified market portfolio, which is identical to the wealth tax rate, i.e., $t^e = t$. Note that the participation taxes only depend on the extensive-margin decision to become entrepreneur and diversified investor, respectively. The government will use its tax revenue $\tilde{R}$ to finance a public consumption good G so that the provision of public consumption is stochastic as well, $\tilde{G} = \tilde{R}$.

Following the main line in the literature on risk and taxation, we assume an expected utility function that is additively separable in private and public consumption, featuring risk aversion in both components. Assuming that all investors are infinitesimally small relative to society, all investors will treat the public good (and its risk distribution) as independent of their decisions. Private consumption is given by after-tax wealth, and

[11] Some countries such as Germany applied the wealth tax both on corporate and individual level. We abstract from such double taxation and follow the Norwegian example where only individuals are liable to wealth tax.

[12] Hence, the Norwegian wealth tax system would be received by assuming $\delta = 0$.

investors maximize a concave expected utility function

$$\Sigma = EU(\tilde{W}_1^i) + EV(\tilde{G}), \quad i = d, n, \tag{4}$$

with $U'(\cdot), V'(\tilde{G}) > 0$ and $U''(\cdot), V''(\tilde{G}) < 0$. The utility function is the same for investments in the market portfolio and in non-listed firms; the only difference is the after-tax wealth and its risk distribution. Applying the definition of tax instruments in equation (3), we have expected utility from private consumption according to $EU(\tilde{W}_1^d) = \mathrm{E}[U((1-t^e)(\tilde{x}-r)a + (1+r)W_0 - T_d)]$ for diversified investors and to $EU(\tilde{W}_1^n) = \mathrm{E}[U((1+\tilde{x}_n(\gamma)+\varepsilon)W_0 - T_n)]$ for an entrepreneur in a non-listed firm.

The optimal investment for an investor in the market portfolio is found by

$$\max_a EU(\tilde{W}_1^d) = \mathrm{E}[U((1-t^e)(\tilde{x}-r)a - T_d)] \tag{5}$$

that leads to the standard first-order condition

$$\mathrm{E}[U'(d)(\tilde{x}-r)] = 0, \tag{6}$$

where $\mathrm{E}[U'(d)] = \mathrm{E}[U'(\tilde{W}_1^d)]$ represents expected marginal utility of a diversified investor.

The wealth tax does not distort the risk-structure choice of investors, and the risk-adjusted (i.e., utility-adjusted) return on the risky investment is equal to the riskless return in the optimum,

$$\frac{\mathrm{E}[U'(d)\cdot\tilde{x}]}{\mathrm{E}[U'(d)]} = r. \tag{7}$$

By applying Steiner's Rule on covariances, it follows that the market risk premium $\mathrm{E}[\tilde{x}-r]$ equals the utility-based risk premium π_d of the investor:

$$\mathrm{E}[\tilde{x}-r] = -\frac{cov(\tilde{x}, U'(d))}{\mathrm{E}[U'(d)]} = \pi_d. \tag{8}$$

Nevertheless, the excess-return tax affects total risk taking of investors by a standard Musgrave substitution effect (cf. Mossin, 1968, p. 76f). Increased risky investment allows for offsetting the impact of the tax and to restore the former risk-return position. To see this, totally differentiate the FOC (6) for the risky investment a and the tax rate t. After collecting terms, we find

$$\frac{da}{dt} = \frac{a}{1-t^e}. \tag{9}$$

The participation tax has a pure income effect on risky investment a, and we receive

$$\frac{da}{dT^d} = \frac{\mathrm{E}[U''(d)(\tilde{x}-r)]}{\mathrm{E}[U''(d)(\tilde{x}-r)^2]}, \tag{10}$$

which is negative whenever absolute risk aversion is decreasing (i.e., the wealth elasticity

of risk taking is positive, $\eta_{aW_0} = \frac{da}{dW_0}\frac{W_0}{a} > 0$).

4 Equilibrium Activity in Non-listed Firms

For optimal investment behavior a^*, expected utility of a diversified investor is

$$EU(\tilde{W}_1^d) = \mathrm{E}[U((1-t^e)(\tilde{x}-r)a^* + (1+r)W_0 - T_d)], \tag{11}$$

while an 'n-type investor', investing all its wealth in a non-listed firm, can expect utility

$$EU(\tilde{W}_1^n) = \mathrm{E}[U([1+\tilde{x}_n(\gamma)+\epsilon]W_0 - T_n)] = \mathrm{E}[U(\{\tilde{x}_n(\gamma^*)+\epsilon-r)\}W_0 + (1+r)W_0 - T_n)]. \tag{12}$$

Any investor has the choice between two alternatives. By investing its wealth W_0 in one of them, the investor sells deterministic consumption today at the deterministic price $(1+r)$. This is represented by the second wealth terms on the far right hand sides in equations (11) and (12), and it is identical for both alternatives. In addition, the investor acquires a 'bad,' namely risk, and is compensated for that by an excess return after tax.[13] If the investor decides to become entrepreneur in a non-listed firm, it faces a risk premium (i.e., the expected excess return) that is larger than for the market portfolio, because the return is not taxed and is riskier because there is no diversification.

As long as $EU(\tilde{W}_1^n)$ is larger than $EU(\tilde{W}_1^d)$, there is an incentive for some investors to enter the sector of non-listed firms, even though this implies to have a non-diversified exposure to risk. For reaching an equilibrium, an increasing share γ of 'n-type investors' in total investors must drive down the (risky) return in non-listed firms, until expected utilities equalize. Consequently, the equilibrium condition gives a share γ^* that ensures

$$EU_n = \mathrm{E}[U([1+\tilde{x}_n(\gamma^*)+\epsilon]W_0 - T_n)] = \mathrm{E}[U((1-t^e)(\tilde{x}-r)a^* + (1+r)W_0 - T_d)] = EU_d. \tag{13}$$

A first insight follows immediately from the equilibrium condition (13). Any marginal change in the activity γ in the sector of non-listed firms does not have any effect on welfare from private consumption (all else equal). Since expected utilities must be equal, a marginal decrease (or increase) in 'n-type investors' is fully compensated.[14]

How does the tax system affect entrepreneurial activity, that is, the equilibrium share of investors active in non-listed firms? Totally differentiating the equilibrium condition

[13] This interpretation follows the interpretation of risky assets as tradeable commodities in Sandmo (1977) and the further development in Schindler (2008) of handling risk as a 'bad' that can be sold at price of the excess return. Note that acquiring some risk is always attractive due to the expected excess returns (i.e., the market risk premia) being positive, $\mathrm{E}[\tilde{x}(\gamma)+\epsilon-r] > 0$ and $\mathrm{E}[\tilde{x}-r] > 0$.

[14] See Kanbur (1980) for an equivalent concept in case of occupational choice under uncertainty. Note that the result holds even if investors are modelled as being heterogeneous. The marginal investor must still be indifferent between becoming an 'n-type investor' and investing in the market portfolio.

(13) for the share γ and the three tax instruments, leads to

$$\begin{aligned} & \mathrm{E}\left[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}\right] W_0 d\gamma - \mathrm{E}[U'(n)]dT_n \qquad (14) \\ = \; & \underbrace{\mathrm{E}[U'(d)(\tilde{x}-r)]}_{\overset{(6)}{=}0}\left(-a+(1-t^e)\frac{da}{dt^e}\right)dt^e - \Big\{\underbrace{\mathrm{E}[U'(d)(\tilde{x}-r)]}_{\overset{(6)}{=}0}\frac{da}{dT_d} + \mathrm{E}[U'(d)]\Big\}dT_d, \end{aligned}$$

where we can make use of $\mathrm{E}[U'(d)(\tilde{x}-r)]=0$ from the first-order condition (6) so that

$$\frac{d\gamma}{dt^e} = 0. \qquad (15)$$

The part of the wealth tax falling on the excess return for diversified investors does not affect entrepreneurial activity at all. Entrepreneurs do not face an excess-return tax, while diversified investors use the Musgrave effect to undo all tax effects. Hence, in both sectors utility remains unchanged and so does the investment in non-listed firms.

From equation (14) also follows that an increase in a participation tax will foster investment in the other sector,

$$\frac{d\gamma}{dT_n} = \frac{\mathrm{E}[U'(n)]}{\mathrm{E}[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}]W_0} < 0 \quad \text{and} \quad \frac{d\gamma}{dT_d} = -\frac{\mathrm{E}[U'(d)]}{\mathrm{E}[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}]W_0} > 0 \qquad (16)$$

because $\mathrm{E}[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}] < 0$.

Put together, a real-world wealth tax with rate t only affects the deterministic part of wealth and whether it will hamper investment in non-listed firms or even foster entrepreneurial activity depends on the exact setting of the tax instruments and on the difference in equilibrium expected marginal utility:

$$\frac{d\gamma}{dT_n}\frac{\partial T_n}{\partial t} + \frac{d\gamma}{dT_d}\frac{\partial T_d}{\partial t} = \frac{(1+\delta)\mathrm{E}[U'(n)] - (1+r)\mathrm{E}[U'(d)]}{\mathrm{E}[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}]} \gtrless 0. \qquad (17)$$

All else equal, imputing less than the risk-free rate of return on book values in non-listed firms, in particular not inflating them at all, works like a subsidy on entrepreneurial investment and rather fosters investment in non-listed firms. Any general statement on the impact of wealth taxation on entrepreneurs' investment and the desirability of wealth taxation, however, requires to take the full tax system and its exact welfare effects into account.

5 Welfare Effects of the Wealth Tax

The social welfare function of a benevolent government is given as the sum of expected utility of n-type investors (i.e., entrepreneurs) and diversified investors plus their expected

utility from public consumption. Because the marginal entrepreneur has the same expected utility as a diversified investor, we can follow the approach in Kanbur (1980) and simply maximize expected utility of diversified investors. But, we need to take into account the government budget constraint (3) in which entrepreneurs' tax payments enter, of course.

Consequently, the welfare function can be summarized as

$$\begin{aligned}\Omega &= \gamma EU(\tilde{W}_1^n) + (1-\gamma)EU(\tilde{W}_1^d) + EV(\tilde{G}) = EU(\tilde{W}_1^d) + EV(\tilde{G}) \qquad (18)\\ &= \mathrm{E}[U((1-t^e)(\tilde{x}-r)a^* + (1+r)W_0 - T_d) + \mathrm{E}[V(\gamma T_n + (1-\gamma)\{T_d + t^e(\tilde{x}-r)a\})],\end{aligned}$$

where we used $\tilde{G} = \tilde{R}$ from the revenue constraint (3).

5.1 Deriving the optimal tax structure

Differentiating the revenue constraint (3) for changes in entrepreneurial activity γ leads to the tax wedge on investment into non-listed firms and results in

$$\Delta_\gamma = \frac{\partial \tilde{G}}{\partial \gamma} = T_n - T_d - t^e(\tilde{x}-r)a. \qquad (19)$$

The first-order conditions to the optimization problem $\max_{t^e,T_n,T_d}\Omega$ for maximizing welfare by choosing the tax rate t^e on excess returns in the market portfolio and the two participation taxes, T_n and T_d, then read

$$\begin{aligned}\frac{\partial\Omega}{\partial t^e} &= -\mathrm{E}[U'(d)(\tilde{x}-r)]a + (1-\gamma)\mathrm{E}[V'(\tilde{G})(\tilde{x}-r)]\left(a + t^e\frac{\partial a}{\partial t^e}\right) + \mathrm{E}[V'(\tilde{G})\Delta_\gamma]\frac{\partial\gamma}{\partial t^e}\\ &= (1-\gamma)\mathrm{E}[V'(\tilde{G})(\tilde{x}-r)]\left(a + t^e\frac{\partial a}{\partial t^e}\right) = 0, \qquad (20)\\ \frac{\partial\Omega}{\partial T_d} &= -\mathrm{E}[U'(d)] + (1-\gamma)\left\{\mathrm{E}[V'(\tilde{G})] + \mathrm{E}[V'(\tilde{G})(\tilde{x}-r)]t^e\frac{\partial a}{\partial T_d}\right\}\\ &+ \mathrm{E}[V'(\tilde{G})\Delta_\gamma]\frac{\partial\gamma}{\partial T_d} = 0, \qquad (21)\\ \frac{\partial\Omega}{\partial T_n} &= \gamma\mathrm{E}[V'(\tilde{G})] + (1-\gamma)\mathrm{E}[V'(\tilde{G})(\tilde{x}-r)]t^e\frac{\partial a}{\partial T_n} + \mathrm{E}[V'(\tilde{G})\Delta_\gamma]\frac{\partial\gamma}{\partial T_n} = 0, \qquad (22)\end{aligned}$$

where we used optimal investors' behavior (6) and the fact that $\frac{\partial\gamma}{\partial t^e} = 0$ from equation (15) to simplify equation (20).

From this first-order condition (20) follows

$$\mathrm{E}[V'(\tilde{G})(\tilde{x}-r)] = 0, \qquad (23)$$

as $a + t^e\frac{\partial a}{\partial t^e}$ and $1-\gamma > 0$. Consequently, the tax rate t^e should optimally be set such that the marginal expected value of adding one more unit of risk to public consumption is zero.

This condition is the equivalent to risk-taking in private consumption, see equation (6).

When we apply the result (23) in the first-order condition for the tax payment T_n, equation (22), we receive after straightforward simplifications

$$\gamma + (T_n - T_d)\frac{\partial \gamma}{\partial T_n} = 0 \quad \Longleftrightarrow \quad T_n - T_d = -\frac{\gamma}{\frac{\partial \gamma}{\partial T_n}} > 0, \tag{24}$$

as $\frac{\partial \gamma}{\partial T_n} < 0$. Hence, the deterministic wealth tax payment for holding non-listed firms is always higher than the deterministic part of wealth tax payments on portfolio investment, $T_n > T_d$. At the outset, deterring the marginal n-type investor does not affect utility from private consumption; thus, this distortion does not reduce welfare. But, a higher tax on remaining entrepreneurs increases tax revenue.

The underlying intuition is based on a standard Ramsey argument. Because of decreasing returns in the entrepreneurial sector, $\frac{\partial \tilde{x}_n(\gamma)}{\partial \gamma} < 0$, entrepreneurs earn infra-marginal rents. The government wants to tax these (stochastic) profits and can do so by charging a higher participation tax for entrepreneurs as long as the decision to become entrepreneur is not infinitively elastic.[15] This finding corresponds to Spiritus and Boadway (2017, Proposition 1) who analyze a setting that fulfills the standard conditions for the Atkinson-Stiglitz theorem of optimally not taxing capital income as long as all assets feature constant returns to scale. The authors show that the optimal tax on risk-free capital returns is positive as soon as one asset has decreasing returns to scale and capital owners earn infra-marginal rents.[16]

Finally, applying result (23) in the first-order conditions (21) and (22) and adding these two equations establishes

$$\frac{\mathrm{E}[U'(d)] - \mathrm{E}[V'(\tilde{G})]}{\mathrm{E}[V'(\tilde{G})]} = (T_n - T_d)\left(\frac{\partial \gamma}{\partial T_n} + \frac{\partial \gamma}{\partial T_n}\right) = (T_n - T_d)\frac{\mathrm{E}[U'(n)] - \mathrm{E}[U'(d)]}{\mathrm{E}[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}]W_0}. \tag{25}$$

Optimally, for diversified investors, the relative difference between expected marginal utilities of private and public consumption equals the tax revenue effect of participation taxes that is generated by inducing investors to change sectors.

Applying the results above to the optimal risk taking of diversified investors, equation (6), and of the government, equation (23), as well as utilizing the risk balancing that the optimal wealth tax enforces in the economy allow after lengthy rearrangements for

[15] Indeed, it can be shown that the tax burden on deterministic returns in both the market portfolio and non-listed firms is identical (i.e., the imputed rent equals the risk-free market rate of return) if investors can invest some fraction of their wealth into non-listed firms, remain fully diversified and only earn constant returns to scale (i.e., $\tilde{x}_n(\gamma) = \tilde{x}_n \ \forall \gamma$).

[16] A main difference is that all capital income is verifiable in Spiritus and Boadway (2017), but specific assets cannot be observed. In our setting, the asset with decreasing returns to scale can be identified (e.g., the non-listed firm), but its end-of-year value and its return are non-verifiable to the tax authorities.

establishing that

$$\mathrm{E}[U'(d)] = \mathrm{E}[V'(\tilde{G})]. \tag{26}$$

See Appendix A for the exact derivations.

Thus, the optimal wealth tax balances expected marginal utilities of consumption for a diversified investor. From equation (25) and the fact that $T_n > T_d$, this implies that also the expected marginal utilities of private consumption need to be equalized for diversified investors and entrepreneurs,

$$\frac{\partial \gamma}{\partial T_n} + \frac{\partial \gamma}{\partial T_d} = 0 \quad \Longleftrightarrow \quad \mathrm{E}[U'(n)] = \mathrm{E}[U'(d)]. \tag{27}$$

While equation (24) determines the optimal *differentiation* between the participation taxes (in order to tax inframarginal returns of entrepreneurs), equation (27) determines the optimal *level* of the participation taxes so that income and risk-aversion effects ensure equated expected marginal utilities and a non-distorted entrepreneur decision.

To summarize, by taxing the excess returns, the optimal wealth tax structure ensures an efficient market risk balancing on private and public consumption for diversified investors

$$\mathrm{E}[U'(d)(\tilde{x} - r)] = 0 = \mathrm{E}[V'(\tilde{G})(\tilde{x} - r)] \quad \Longleftrightarrow \quad cov(U'(d), \tilde{x}) = cov(V'(\tilde{G}), \tilde{x}), \tag{28}$$

see equations (6) and (23) and apply Steiner's Rule as well as $\mathrm{E}[U'(d)] = \mathrm{E}[V'(\tilde{G})]$ to establish the equality of the covariances. Moreover, via sector choice, market equilibrium implies some risk balancing between normal investors and entrepreneurs that is not distorted by the wealth tax, $\frac{\partial \gamma}{\partial T_n} + \frac{\partial \gamma}{\partial T_d} = 0$.

In addition, optimal wealth taxation enforces an ex-ante efficient resource allocation on market portfolio, entrepreneurial activity, and public consumption so that expected marginal utilities are equalized,

$$\mathrm{E}[U'(n)] = \mathrm{E}[U'(d)] = \mathrm{E}[V'(\tilde{G})]. \tag{29}$$

The risk-diversification properties of a tax on excess returns are well-known from Schindler (2008). Note that this risk diversification is generally not first best, however, because it can only ensure a linear ex-post resource reshuffling between private and public consumption.[17] Recently, Spiritus and Boadway (2017) have shown that the properties of the excess-return tax carry over to an Atkinson-Stiglitz setting with endogenous labor supply, ex-ante heterogeneous individuals, and optimal labor taxation. If the tax system creates distortions in labor supply (or savings), expected marginal utilities will not be

[17] Such a linear risk balancing is first best if the risk tolerance is linear so that state-specific marginal utilities are linearly dependent. This only holds for utility functions featuring hyperbolic absolute risk aversion, see Gollier (2001), pp. 313 and Proposition 80.

balanced, however, and the efficient risk balance is traded off against these distortions.

Mutatis mutandis, these insights also apply to retrospective capital gains taxation à la Auerbach (1991). Efficient risk balancing requires to tax the excess returns of risky assets upon accrual. Under an annual wealth tax, this happens automatically for assets where mark to market is possible. Only hard-to-measure assets whose market value is unknown until they are sold and for which it is not clear at which point in time capital gains occurred, should be taxed according to a retrospective imputation of the (market) rate of return. We conclude

Proposition 1 *Market (macroeconomic) risk can still be efficiently diversified on private and public consumption, even if some assets are hard to measure so that their current values or the exact emergence of capital gains over time cannot be determined. To achieve such diversification, mark-to-market assets should be taxed upon accrual at their actual value, only hard-to-measure assets should be taxed with imputed returns and retrospective capital taxation, respectively.*

Importantly, even for linear risk tolerance, the risk diversification in our setting is still not first best. It would be desirable to redistribute between ex-post heterogeneous entrepreneurs and to insure them against their idiosyncratic risk. Such redistributive taxation is impossible, however, because we assumed that the end-of-period value of entrepreneurial investment cannot be verified by the tax authorities. Therefore, they rely on a uniform imputed rent δ on book values only, but cannot tax the stochastic value of the non-listed firm $\tilde{W}_1^n$.

5.2 Mapping the optimal structure into tax instruments

How can the optimal wealth tax structure, derived in the previous subsection, be mapped into practical, real-world tax instruments? The optimal tax rate t^* of the wealth tax follows directly from optimal risk balancing (28) and the fact that $t^e = t$. As long as individuals are risk averse in both private and public consumption, there needs to be some risky component in both consumption types and for the optimal tax rate holds $0 < t^* < 1$. Some wealth tax is always beneficial. In particular, a wealth tax is the preferable instrument to balance such macroeconomic risk, because it automatically provides full loss offset, i.e., it also shares losses in bad states of nature. This is a key difference to real-world capital taxes that are asymmetric in the sense that they do not reduce losses or only allow for loss carry forwards without interest, while they fully tax gains. This asymmetry prevents capital taxes from providing insurance against largely negative income shocks.

If we assume that the returns in the market portfolio are multivariate normally distributed and define global absolute risk aversion as $GARA = -\frac{\mathrm{E}[U''(C)]}{\mathrm{E}[U'(C)]} > 0$, the optimal

wealth tax rate becomes

$$\frac{t^*}{1-t^*} = \frac{GARA(\tilde{W}_1^d)}{GARA(\tilde{G})} > 0. \tag{30}$$

See Corollary 1 and its proof in Schindler (2008). The higher risk aversion in private consumption relative to public consumption is, the larger is the optimal tax rate. Following up on the view in Seim (2016), the wealth tax is not only a good instrument to redistribute wealth between ex-ante heterogeneous individuals, it is also a good instrument to balance systematic risk and provide insurance against macroeconomic shocks as our analysis shows.

The optimal tax rate can be large, even if absolute risk aversion in consumption is low. What matters for the optimal tax rate (and the optimal risk balance) is the ratio between global risk aversion in private and public consumption. At the outset, if risk aversion would be uniform across consumption types, the optimal tax rate would be 50%. With an average tax to GDP ratio of roughly one third in the OECD, private consumption is at least twice as large as public consumption, and with decreasing absolute risk aversion, individuals should be more risk averse in public consumption. But, in order to reach a wealth tax rate of 5% that Piketty (2014) proposes on the superrich, global risk aversion in public consumption needs to be 20 times larger than the one in private consumption. Existing wealth tax systems with a tax rate of about 1% would require a 100 times larger risk aversion in public consumption to be optimal under risk-balancing considerations.

In a next step, we can identify the optimal imputed rent δ^* on book values in non-listed firms. We know from equation (24) that the participation tax on entrepreneurs is larger than the one on diversified investors, $T_n > T_d$, and using the definitions of the participation taxes, $T_n = t(1+\delta)W_0 + T$ and $T_d = t(1+r)W_0 + T$ in (24), we receive

$$\delta^* = r - \frac{\gamma}{t^* W_0 \frac{\partial \gamma}{\partial T_n}} > r > 0 \quad \text{and} \quad \frac{(\delta^* - r)t^* W_0}{(1+\delta^*)t^* W_0 + T} = \frac{1}{\varepsilon_{\gamma T_n}} > 0, \tag{31}$$

where $\varepsilon_{\gamma T_n} = -\frac{\partial \gamma}{\partial T_n}\frac{T_n}{\gamma} > 0$ is the wealth elasticity for entering the sector of non-listed firms.

Catching up on the interpretation of equation (24), the imputed rent on non-listed firms needs to be higher than the normal, risk-free rate of return in order to tax the economic rents generated in the entrepreneurial sector. Effectively, part of the expected risk premium is capitalized and taxed. The marginal distortion on entrepreneurial activity is of second order. This implies that any wealth tax system that only imputes the risk-free return or, as the Norwegian system, does not impute any return and only taxes historical book values in non-listed undercharges entrepreneurs and subsidizes holding non-listed firms. Evaluated at an optimal wealth tax system, this would induce too much activity in non-listed firms. If evaluated outside the welfare maximum, anything goes if $\delta < r$, of course.

Importantly, notice that the wealth tax burden on non-diversifiable risky assets that are hard to measure, such as non-listed firms, could be abolished by setting an imputed return $\delta = -1$. As we have shown, this is never optimal and the fact that entrepreneurs cannot diversify their risk does not justify preferential tax treatment. On the contrary, even extreme distortions in entrepreneurial activity (i.e., $\varepsilon_{\gamma T_N} \to \infty$) will still trigger an imputed return equal to the risk-free market rate of return r. Thus, we conclude

Proposition 2 *Non-diversifiable risk in hard-to-measure assets and potential distortions in entrepreneurial investment do not justify a preferential treatment under wealth taxation. The wealth tax base should include these assets at initial wealth (i.e., historical book values) plus some imputed rent. The imputed rent on initial wealth is larger than the risk-free market rate of return whenever a hard-to-measure asset features decreasing returns to scale and earns infra-marginal rents.*

Finally, the general lump-sum tax (or lump-sum transfer) follows as $T^* = T_d - t^*(1 + r)W_0$ and $T^* = T_n - t^*(1 + \delta^*)W_0$, respectively. Its level is set such that the generated income and risk-aversion effects balance all expected marginal utilities of consumption and eliminate distortions in the decision to become entrepreneur in a non-listed firm (as $\frac{\partial \gamma}{\partial T_n} + \frac{\partial \gamma}{\partial T_d} = 0$).

6 Conclusions

Real-world wealth taxes usually tax hard-to-measure assets such as non-listed firms upon their historical book values and do not share income risk for these investments. Does such a wealth tax cause (tremendous) excess burden on owners of small, non-listed firms who cannot diversify their investment? And how should hard-to-measure assets be taxed in general? We applied a standard portfolio-choice approach and extended it by an indivisible project, giving rise to a sector with fully diversified investors and a sector with non-listed firms, owned by fully undiversified investors.

In such a model, the optimal wealth tax rate is strictly positive. By implicitly taxing excess returns of mark-to-market assets, the wealth tax features a Domar-Musgrave substitution effect and efficiently diversifies market risk on private and public consumption. The tax burden falling on deterministic returns and initial wealth in both a market portfolio and non-listed firms can be coordinated such that market equilibrium forces and a lump-sum transfer ensure an efficient resource allocation from an ex-ante (investment) point of view. In a model without distortions in savings and labor effort, the decision to hold non-listed firms (i.e., entrepreneurial activity) will not be distorted either. But, this requires to tax an imputed return on book values of non-listed firms. In order to tax infra-marginal rents that are created by decreasing returns to scale in investment into the sector of non-listed firms, this imputation needs to be larger than the risk-free market

rate of return. Evaluated against this optimal structure, real-world wealth tax systems, such as the Norwegian one, grant a subsidy for non-listed firms and rather induce a too high entrepreneurial activity.

The important insights in this paper are, first, that the risk valuation of non-diversifiable investment does not affect the welfare analysis of taxes, because market forces will ensure an efficient allocation in equilibrium and a marginal distortion of entrepreneurial activity is of second order to social welfare. Second, special tax rules that apply imputed returns should be used for hard-to-measure assets only. Otherwise, an efficient diversification of macroeconomic risk on private and public consumption is not possible. Third, tax payments for investors in non-listed firms on deterministic book values in order to avoid problems with valuation of non-traded assets do not create distortions, but require to be taxed upon an imputed return that is larger than the risk-free market rate of return.

A popular argument against wealth taxation is that it worsens financial constraints, particularly in non-listed firms with limited liquidity. Just as for inheritance taxation, this problem can be avoided if the tax authorities turn the tax payment into a loan that is granted at the normal market rate of return.

Nevertheless, two remarks of caution are appropriate. This paper does not state that the wealth tax does not have any (traditional) excess burden. Endogenizing the savings decision will immediately introduce welfare-relevant distortions into the model (empirically, these distortions seem to be very modest for a wealth tax, however; see Seim, 2016; Zoutman, 2014). That a positive wealth tax (and positive capital taxation) is still optimal in presence of such distortions has been shown in a string of recent papers (see, e.g., Saez and Piketty, 2013, and for some summary, Cremer, 2010). What can be taken from our results is that the findings in these papers continue to hold even if these models are extended for risk-taking, portfolio choice and indivisible investment (e.g., small, non-listed firms). Second, we neglected entrepreneurial effort, moral hazard, and financing constraints. It is well-known that capital-gains taxation can trigger substantial moral-hazard distortions in start-up firms that need to rely on venture capital from external sources (e.g., Keuschnigg and Nielsen, 2004). How such financing constraints and moral hazard interact with a wealth tax in a portfolio choice model needs to be left for further research, however.

A Expected Marginal Utility of Consumption

The result (24) can be rearranged to

$$\mathrm{E}[U'(n)\frac{\partial \tilde{x}_n}{\partial \gamma}]W_0 = -\frac{(T_n - T_d)\mathrm{E}[U'(n)]}{\gamma}. \tag{A.1}$$

Inserting equation (23) into the first-order condition (21) and using equation (A.1) to replace the denominator of $\frac{\partial \gamma}{\partial T_d}$, we can express marginal utility of a diversified investor as

$$\mathrm{E}[U'(d)] = \frac{(1-\gamma)\mathrm{E}[V'(\tilde{G})]\mathrm{E}[U'(n)]}{\mathrm{E}[U'(n)] - \gamma \mathrm{E}[V'(\tilde{G})]}. \tag{A.2}$$

Inserting this expression into the condition (8) for optimally diversified portfolio investment and collecting terms leads to

$$\gamma \mathrm{E}[V'(\tilde{G})] = \left[1 - (1-\gamma)\frac{cov(V'(G), \tilde{x})}{cov(U'(d), \tilde{x})}\right] \mathrm{E}[U'(n)], \tag{A.3}$$

where we made use of $\mathrm{E}[V'(\tilde{G})]\mathrm{E}[\tilde{x} - r] = -cov(V'(\tilde{G}), \tilde{x})$ from optimal risk balancing in public consumption, equation (23).

When we use equations (23) and (A.1) in the first-order condition (21) to isolate marginal utility of public consumption instead, we receive

$$\mathrm{E}[V'(\tilde{G})] = \frac{\mathrm{E}[U'(d)]\mathrm{E}[U'(n)]}{(1-\gamma)\mathrm{E}[U'(n)] + \gamma \mathrm{E}[U'(d)]}. \tag{A.4}$$

Applying Steiner's Rule on covariances to the optimal risk balancing in public consumption (23) and inserting equation (A.4) there, delivers

$$\gamma \mathrm{E}[U'(d)] = -\left[(1-\gamma) - \frac{cov(U'(d), \tilde{x})}{cov(V'(\tilde{G}), \tilde{x})}\right] \mathrm{E}[U'(n)]. \tag{A.5}$$

Optimal risk balancing in the economy implies from equations (6) and (23) that

$$\mathrm{E}[U'(d)(\tilde{x} - r)] = 0 = \mathrm{E}[V'(\tilde{G})(\tilde{x} - r)] \Leftrightarrow cov(U'(d), \tilde{x}) = cov(V'(\tilde{G}), \tilde{x}) \frac{\mathrm{E}[U'(d)]}{\mathrm{E}[V'(\tilde{G})]}. \tag{A.6}$$

Now, subtract equation (A.3) from equation (A.5) and apply equation (A.6) to substitute the ratio of covariances so that

$$\begin{aligned}
\mathrm{E}[U'(d)] - \mathrm{E}[V'(\tilde{G})] &= -\frac{\mathrm{E}[U'(n)]}{\gamma}\left\{(1-\gamma) - \frac{cov(U'(d), \tilde{x})}{cov(V'(\tilde{G}), \tilde{x})} + 1 - (1-\gamma)\frac{cov(V'(G), \tilde{x})}{cov(U'(d), \tilde{x})}\right\} \\
&= -\frac{\mathrm{E}[U'(n)]}{\gamma}\left\{(1-\gamma) - \frac{\mathrm{E}[U'(d)]}{\mathrm{E}[V'(\tilde{G})]} + 1 - (1-\gamma)\frac{\mathrm{E}[V'(\tilde{G})]}{\mathrm{E}[U'(d)]}\right\} \\
&= -\frac{\mathrm{E}[U'(n)]\left\{(1-\gamma)\mathrm{E}[V'(\tilde{G})] - \mathrm{E}[U'(d)]\right\}}{\gamma \mathrm{E}[U'(d)]\mathrm{E}[V'(\tilde{G})]}\left\{\mathrm{E}[U'(d)] - \mathrm{E}[V'(\tilde{G})]\right\}.
\end{aligned}$$

Hence, we have

$$\Big\{\mathrm{E}[U'(d)] - \mathrm{E}[V'(\tilde{G})]\Big\}\left(1 + \frac{\mathrm{E}[U'(n)]\Big\{(1-\gamma)\mathrm{E}[V'(\tilde{G})] - \mathrm{E}[U'(d)]\Big\}}{\gamma \mathrm{E}[U'(d)]\mathrm{E}[V'(\tilde{G})]}\right) = 0. \qquad \text{(A.7)}$$

From equation (A.7) follows that expected marginal utility of private and public consumption needs to be equalized for diversified investors. This proves the result in equation (26). Equation (25) together with $T_n > T_d$ then implies

$$\mathrm{E}[U'(n)] = \mathrm{E}[U'(d)] = \mathrm{E}[V'(\tilde{G})]. \qquad \text{(A.8)}$$

For this combination of expected marginal utilities, the second term in equation (A.7) also becomes equal to zero.

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