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Frequency aspects of information transmission in a network of three Western equity markets

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**FREQUENCY ASPECTS OF INFORMATION
TRANSMISSION IN A NETWORK OF THREE
WESTERN EQUITY MARKETS**

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Frequency aspects of information transmission in a network of three Western equity markets

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Abstract

Cycles in the behavior of stock markets have been widely documented. There is an increasing body of literature on whether stock markets anticipate business cycles or its turning points. Several recent studies assert that financial integration impacts positively on business cycle comovements of economies.

We consider three Western equity markets, represented by their respective stock indices: DJIA (USA), FTSE 100 (UK), and Euro Stoxx 50 (euro area). Connecting these three markets together via vector autoregressive processes in index returns, we construct “propagation values” to measure and trace, on a daily basis, the relative importance of a market as a volatility creator within the network, where volatility is due to a return shock in a market.

A cross-wavelet analysis reveals the joint frequency structure of pairs of the propagation value series, in particular whether or not two series tend to move in the same direction at a given frequency. Our main findings are: (i) From 2001 onwards, the daily propagation values of markets have been fluctuating much less than before, and high frequencies have become less pronounced; (ii) the European markets are in phase at business cycle frequency, while the US market is not in phase with either European market; (iii) in 2008, the euro area has taken over the leading role. This approach not only provides new insight into the time-dependent interplay of equity markets, but it can also replicate certain findings of traditional business cycle research, and it has the advantage of using only readily available stock market data.

Keywords: Equity market network; propagation value; cycle; synchronization; wavelet analysis; phase difference

JEL classification: C32, C58, E32, F44, G15

1 Introduction

Efforts to understand cyclical behavior of economic time series go back to as early as the 19th century, with researchers aiming to forecast the future of economies. Among the first to identify economic cycles and their synchronicity was Juglar [35] in 1862. He proposed 7–11 year cycles of fixed capital investments which were more or less synchronous for France, the UK and the US. Beginning with the 20th century, several other cycles have been identified: (i) the Kitchin [36] cycles with 3–5 years of periodicity arising from fluctuations of inventories, (ii) the Kuznets [40] swings of 15–25 years associated with infrastructure investments and (iii) the Kondratieff [37]

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“Long Waves” of 40–60 years along with smaller cycles of 3–4 and 7–10 years. The beginning of the modern analysis could be attributed to the formalization of the notion of business cycle by Burns and Mitchell [11], who define it as “... a type of fluctuation found in aggregate economic activity”.

With its focus on GDP, the prime target of classical business cycle analysis is to investigate domestic aspects of an economy, see Altug [4], but business cycle research has recently also been undertaken to study cross-country differences and similarities of macroeconomic fluctuations. Various factors that could be affecting business cycle synchronization have been proposed in the literature, including trade relations, specialization, monetary integration, financial relations and fiscal policy, see de Haan et al. [19] for an extended survey of studies on business cycle synchronization in the euro area. The channels of synchronization, common dynamic properties of business cycles across countries and evolution of the degree of global cyclical interdependence are also investigated in Imbs [33] and Kose et al. [38, 39], respectively. Only recently, Aguiar-Conraria et al. [1] have suggested using cross-wavelet methodology to identify the joint frequency content of multivariate macroeconomic data, pointing out that wavelet analysis, unlike Fourier analysis, is able to detect structural changes across time. Aguiar-Conraria and Soares [2] and Crowley and Mayes [18] apply wavelet-based methods to monthly, respectively quarterly, macroeconomic data to study the cyclical behavior of economies within the euro area.

The cyclical behavior of stock markets has also been widely documented. The patterns identified are, among other things: (i) the “Halloween effect”, also known as “Sell in May and Go Away”, which refers to returns during winter (November–April) exceeding those during summer (May–October), see e.g. Bouman and Jacobson [10], Dichtl and Drobetz [20]; (ii) the “January effect” that leads to abnormally large returns on stocks in January of most years, see e.g. Gu [30], Haug and Hirschey [32]; and (iii) the four-year US Presidential Election Cycle that implies US stock prices are tracking US presidential elections, see e.g. Wong and McAleer [57], Booth and Booth [9].

Early opinions on whether stock markets help predict business cycles span a large spectrum ranging from Fischer and Merton [25] who find that “stock price changes are the best single variable predictor of the business cycle” to Stock and Watson’s [54] verdict that “although there is some evidence that the stock market improves forecasting performance, this improvement is slight”. More recent studies have suggested that the stock market anticipates business cycle turning points, based on the conception that the investors’ behavior reflects their assessment of the state of the economy; see Chauvet [16], Chauvet et al. [17] and Andreou et al. [5]. Franz [27] concludes that “the S&P 500 is highly consistent in anticipating contractions and expansions in economic activity, as measured by near-term changes in real GDP”. Similarly, Angelidis et al. [6] provide evidence that stock market return dispersion has significant predictive power for the business cycles across a data set of G7 economies.

Financial integration often goes along with the opening of potential transmission channels between economies, and indeed Imbs [33, 34] and Kose et al. [38] show that financial integration impacts positively on business cycle comovements. Olivero and Madak [46] find that financial integration within Europe does matter for the international transmission of business cycles, and “as the European economies become more integrated among themselves, European business cycles start to decouple from those in the US”. As for cycle length, Fidrmuc et al. [24] find that, for European countries, “financial integration is insignificant at short-term frequencies and peaks at business-cycle frequencies for business cycles”. The results of the latter two studies, although obtained with different data and a different set of methods, are in line with our results.

The cross-country analysis of business cycles is conceptually close to the study of “contagion” and “interconnectedness” of economies; see, for example, Forbes [26]. In a similar vein, the idea of information transmission in networks of equity markets, in particular: shocks creating

volatility across the network, lends itself to studying “importance cycles” in such a network. Many stochastic models assume that shocks (news; “residuals” after the model has been fitted) drive stock market returns. Certain network models of equity markets allow for shock spillovers such that a return shock in one market creates volatility elsewhere in the network. Based on such models, it can be shown that, on a given day, not all markets in a network are equally important with respect to network volatility creation via return shocks, or equivalently, with respect to information transmission: a shock in a “central” market may spark massive repercussions throughout the network, while a shock in an isolated market may go almost unnoticed. This concept is used in Schmidbauer et al. [53] to investigate the impact of sanctions during the Ukrainian crisis on the Russian stock market. In the present study, our focus is not on the assessment of singular events, but rather on investigating periodic patterns of importance in a network of stock markets: Are there cycles in the relative importance of a stock market as news (or shock) propagator? Which cycles (or periods) are dominating at a given point in time? When the relative importance of one stock market increases, what can be said about the others?

The present study is thus an effort to investigate frequency aspects with respect to information transmission, focusing on a network consisting of three Western stock markets, each represented by a stock index: DJIA (US), FTSE 100 (UK) and Euro Stoxx 50 (proxy for the euro area). To that end, departing from the Diebold-Yilmaz [21, 22, 23] connectedness framework, and extensions detailed in Schmidbauer et al. [52, 53], we undertake the following steps:

- Compute propagation values, which measure the relative importance of an asset market as a news propagator within a network of asset markets (see Schmidbauer et al. [53]). Each market is represented by a stock index, and markets are connected together via a vector autoregressive model, as suggested by Diebold and Yilmaz [21, 22, 23]. “News” in a market on a given day means a shock to the return of the corresponding stock index, and “propagation” means that this shock creates volatility across the network. Each market has its own propagation value, updated on a daily basis. A shock in an important market (that is, with a relatively high propagation value) will create more network volatility than a shock in a less important market (which has a relatively low propagation value).
- For pairs selected from the three stock markets under consideration, identify jointly significant and powerful periods (or frequencies) in their propagation value series, using cross-wavelet analysis. “Powerful” means that this period will be selected with high priority when reconstructing a propagation value series on the basis of wavelets; it is thus a substantial constituent of both propagation value series involved. Statistical significance is assessed by comparison with simulated white noise.
- Investigate whether, at a given powerful period, the propagation value series of two markets are in phase or out of phase at a given point in time. If they are in phase, their propagation values will tend to move, at a certain pace determined by the period, together from trough to peak (for example). If they are out of phase, one market will become more important in the sense explained above, again at the pace determined by the period, at the expense of the other market’s importance as a news spreader.
- Similarly, investigate which of two propagation value series is leading at a given powerful period and a given point in time. The leading one will be the first to increase (or decrease) in importance at a certain pace, and the other one will follow suit.

Our approach to investigating these frequency aspects has the advantage that it is based exclusively on readily available stock market data, which are usually not subject to revision,

unlike macroeconomic data, and allow a finer frequency analysis than monthly or quarterly data. Our working hypotheses in this study are threefold: (i) we expect to observe cycles similar to those observed in traditional macroeconomic research; (ii) the increasing financial integration has contributed to European markets moving in phase, and (iii) in line with decreasing costs and holding periods of stocks and increasing transaction speeds, we expect to find diminished relevance of low periods.

Tracking business cycles as well as financial market cycles and their synchronicity across borders has potential benefits to enterprise managers, investors, and policy makers. Typically, enterprise managers act in response to their anticipation of business cycles. Investors can use cycle information to time their portfolio rebalancing decisions. Identifying cycles precisely in almost real-time may provide positive return opportunities on their portfolios, see Candelon et al. [12]. On the other hand, the absence of periodic structure may give investors a hint about where not to seek investment opportunities. Furthermore, policy makers and regulators may use knowledge about stock market cycles to infer information about business cycles, and stimulate the economy when recession nears or slow it down when it is too heated.

This paper is organized as follows. Section 2 specifies the data on which this study is based. The methodology, as far as relevant to obtain a market’s series of propagation values, is expounded in Section 3. Section 4 introduces some concepts of wavelet and cross-wavelet analysis. Empirical results are presented in Section 5, followed by a discussion in Section 6. Section 7 summarizes and concludes the paper. — All computations were carried out with scripts written in R [49]; wavelet computations and plots are accomplished with R package *WaveletComp* [50].

2 Data

The empirical starting point of the present study consists of daily closing quotations of three Western equity market indices: DJIA (New York Stock Exchange, in the following called *dji*), FTSE 100 (London Stock Exchange, *ftse*) and Euro Stoxx 50 (proxy for euro area equity markets, *sx5e*) in the time period from August 1987 through May 2016 (7443 observations). Data from 1987 are used for model initialization, but the wavelet analysis below is carried out using data from January 1988 onwards in order to avoid a 1987 “Black Monday” (1987-10-19) crash bias. The time series of daily simple returns in percent (beginning with January 1988) are plotted in Figure 1.

A visual inspection of the return series in Figure 1 suggests a simultaneous occurrence of periods of high volatility in the three markets considered, and the impression that returns are somehow “connected”. Our approach of how to assess, in this network of markets, relative importance with respect to information transmission will be briefly described next.

3 Propagation values

The goal of the present study is to analyze, in the frequency domain, the time pattern of the relative importance of an equity market as a news disseminator in a network of equity markets. The dissemination of news implies that volatility is created across the network. The relative importance of a market in this respect is quantified in a daily series of propagation values.¹ This procedure will be outlined briefly in the following (for a more comprehensive discussion, see Schmidbauer et al. [52, 53]).

¹Similar concepts were developed in population science under the name *reproductive value* in a Leslie-model framework, and under the name *eigenvector centrality* in social network analysis; see Caswell [14] and Bonacich [8].

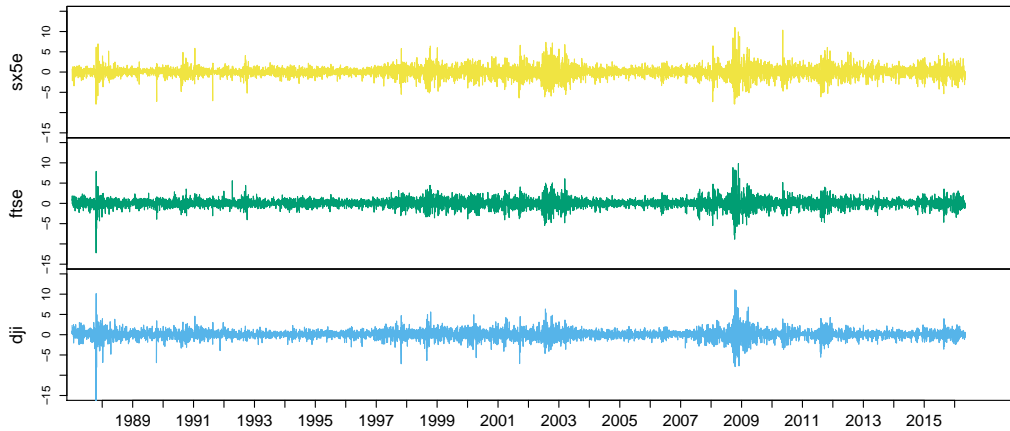


Figure 1: Daily return series

The starting point is to fit a VAR (vector autoregressive) model to N series $(x_{lt})_t$, $l = 1, \dots, N$, of daily returns on stock indices, using the past 100 days. In our case, $N = 3$. In order to remove the dependence of an imposed hierarchy of markets, we follow Diebold and Yilmaz [21] and use an approach suggested by Pesaran and Shin [48], namely: in order to identify the impulse response function of a component, give the highest priority to that component. The method to assess return-to-volatility spillovers is then to decompose the forecast error variance for each market with respect to its origin (the decomposition is acceptably settled when forecasting 5 steps ahead), and the resulting shares of forecast variability in x_l due to shocks in x_k can be arranged in a spillover table (or matrix) \mathbf{M} .² An example of \mathbf{M} with $N = 3$ return series is:

$$\mathbf{M} = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}, \quad (1)$$

which means that 60% (20% each) of forecast error variance of x_1 is due to return shocks in x_1 itself (shocks in x_2 and x_3 , respectively). Each *row* of \mathbf{M} thus sums up to 1 (or 100%) and provides a breakdown of the forecast error variance of the corresponding stock index return with respect to its origin; forecast error variance quantifies the market volatility on a given day. The *columns* of \mathbf{M} provide the key to analyzing the propagation of a shock: An initial shock to market k can be represented by a unit vector n_0 with 1 in its k -th component. The subsequent repercussions of this shock across markets can then be modeled by the transmission equation

$$\mathbf{n}_{s+1} = \mathbf{M} \cdot \mathbf{n}_s, \quad s = 0, 1, 2, \dots \quad (2)$$

The ultimate (as $s \rightarrow \infty$) relative impact level of a shock to market k is given by the k -th component of the (normed) left eigenvector \mathbf{v} of \mathbf{M} , that is, the vector satisfying $\mathbf{v}' = \mathbf{v}' \cdot \mathbf{M}$. We call this the *propagation value* of market k on the day in question. For \mathbf{M} as in Equation (1), $\mathbf{v}' = (0.2, 0.4, 0.4)$, which means that a shock in market 2 is twice as powerful ($0.4/0.2 = 2$) as a shock in market 1 in terms of creating network volatility.

These steps will then be repeated for each day, with a rolling window of 100 days, resulting in N daily series of propagation values reflecting the markets' daily relative importance as news disseminators.

²Diebold and Yilmaz [21, 22, 23] developed the *spillover index* to summarize the spillover matrix.

4 Wavelets, cross-wavelets and phase differences

With the time series of daily propagation values at hand, cyclical phenomena and synchronization among the three markets with respect to their importance as network volatility creators can be studied in the next step. Cycles of different frequencies and of limited duration may overlap, necessitating a decomposition of the time series which exceeds the capabilities of Fourier transforms, namely in the time and frequency domains simultaneously. Wavelets are utilized to tackle the arising time and frequency resolution dilemma (resulting from the Heisenberg uncertainty principle), and provide appropriate tools for (i) comparing the frequency content of the series across time for pairs of the three markets, (ii) drawing conclusions about the series synchronicity at certain periods and across certain ranges of time. We will briefly outline the concepts of wavelet analysis as far as relevant for this study.

4.1 Morlet wavelets

The subsequent analysis is based on the functionality of the R package *WaveletComp* [50].³ We thus adopt the continuous Morlet wavelet transform, which yields a finer resolution than discrete wavelet transforms, and is complex-valued and therefore highly redundant and information-preserving with any careful selection of parameters. It provides information on both amplitude and phase. The latter is a prerequisite for the analysis of two time series with respect to synchronicity at a given period. It also provides a method to reconstruct the original series.⁴

The “mother” Morlet wavelet, defined by

$$\psi(\eta) = \pi^{-1/4} e^{i6\eta} e^{-\eta^2/2} \quad (3)$$

(with a particular choice of six oscillations per one revolution of 2π (radians), which is the preferred value for computational purposes in literature since it makes the Morlet wavelet approximately analytic), is depicted in Figure 2.

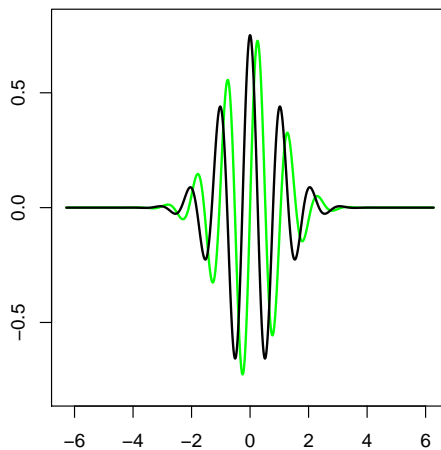


Figure 2: The Morlet mother wavelet — real part (black) and imaginary part (green)

³In the following, we use the notation of *WaveletComp* [50].

⁴The Morlet wavelet dates back to the early 1980s, cf. Morlet et al. [44, 45]. It builds on a Gaussian-windowed sinusoid, the Gabor transform, which was introduced in 1946 by Gabor [28] to decompose a signal into its frequency and phase contents as time evolves. Unlike the Gabor transform, the Morlet wavelet keeps its shape in frequency shifts, thus providing a “reasonable” separation of contributions from different frequency bands “without excessive loss” in time resolution (Goupillaud et al. [29]; see also Aguiar-Conraria et al. [1]).

Characteristics of any wavelet are its compact support across time and zero area underneath the curve (so as to make its energy equally distributed, and the multiplication by a constant signal, without periodic fluctuations, results in an area of zero). Intuitively, wavelet transformation decomposes the time series at hand into a set of base functions, called the “wavelet daughters”, which are generated from the mother wavelet by translation in time and by scaling (compressing and stretching). Accordingly, an increase in scale, which corresponds to a decrease in frequency, is at the expense of time resolution, while a decrease in scale and increase in frequency sacrifices on frequency resolution. The transformation results in a matrix of (complex-valued) coefficients, namely the (complex) wavelet transform of the time series (x_t) as a function of translation τ and scale s :

$$\text{Wave}(\tau, s) = \sum_t x_t \frac{1}{\sqrt{s}} \psi^* \left(\frac{t - \tau}{s} \right) \quad (4)$$

with \star denoting the complex conjugate. The shift of the wavelet daughters’ translation is determined by an increment dt , the sampling resolution in the time domain. The scale is usually set to a fractional power of 2, a “voice” in an “octave” (according to octaves in music denoting intervals between pitches with half or double of each other’s frequency). An increment of dj determines the sampling resolution in the frequency domain, and $1/dj$ is the number of voices per octave. In our study, adopting the Morlet wavelet with 6 oscillations, the Fourier factor $2\pi/6$ is used to convert scales to periods, and, for ease of interpretation, it is the period which is set to a fractional power of 2, entailing the corresponding values of scale (this is the default setting in *WaveletComp* [50]); sampling resolutions in our study are set to $dt = 1$ and $dj = 1/100$ (i.e. 100 voices per octave in scale direction).

The local amplitude of any periodic component of (x_t) and how it evolves with time can then be retrieved from the modulus of its wavelet transform.⁵ Its square⁶ has an interpretation as wavelet energy density, which is called the wavelet power spectrum and is usually displayed as a “heat map” in the time-scale (or, correspondingly: time-frequency or time-period) domain (c.f. Carmona et al. [13]):

$$\text{Power}(\tau, s) = \text{Ampl}(\tau, s)^2 = \frac{1}{s} \cdot |\text{Wave}(\tau, s)|^2 \quad (5)$$

In case of a white noise process, its expectation at each time and scale, disregarding the proportionality factor $1/s$, corresponds to the process variance. Therefore, in applications of wavelet methodology, it is conventional to standardize the time series at hand, after detrending it,⁷ to obtain a measure of the wavelet power which is relative to unit-variance white noise and directly comparable to results of other time series.

The complex nature of the Morlet wavelet bears information about the local wavelet phase, that is: $\text{phase}(\tau, s) = \text{Arg}(\text{Wave}(\tau, s))$, which is an angle in the interval $[-\pi, \pi]$ measuring displacements of any periodic component of (x_t) relative to a localized origin in the time domain; it will be utilized in our study of time series synchronicity.

⁵Liu et al. [42] showed that a rectification by the factor $1/s^{1/2}$ is necessary.

⁶In wavelet applications, it is common to raise the squared local amplitude to a further power in order to accentuate contrast in the corresponding heat map; see, e.g. Percival and Walden [47].

⁷The detrending of an input time series by local polynomial regression is another optional feature of *WaveletComp* [50]. In our application to series of propagation values, i.e. per se normed values, detrending was not necessary.

4.2 Cross-wavelets and phase differences

The concepts of cross-wavelet analysis are appropriate for a comparison of the frequency contents of two time series, and conclusions about their synchronicity.⁸ The cross-wavelet transform of two time series, say (x_t) and (y_t) , with respective wavelet transforms Wave.x and Wave.y , decomposes the Fourier co- and quadrature-spectra in the time and frequency domains simultaneously:⁹

$$\text{Wave.xy}(\tau, s) = \frac{1}{s} \cdot \text{Wave.x}(\tau, s) \cdot \text{Wave.y}^*(\tau, s), \quad (6)$$

with translation parameter τ and scale parameter s . Its modulus has the interpretation as cross-wavelet power (sometimes called cross-wavelet energy) and lends itself to an assessment of the similarity of the two series' wavelet power with respect to any periodic component and how it evolves with time:

$$\text{Power.xy}(\tau, s) = |\text{Wave.xy}(\tau, s)| \quad (7)$$

Again, a heat map is the usual way to visualize the cross-wavelet power spectrum. Power averages illustrate the prominence of certain periodic components across time.

In a geometric sense, the cross-wavelet power is the analog of the covariance, and like the latter, it depends on the unit of measurement of the series involved and may not be ready for interpretation with regard to the degree of association of the two series. The concept of wavelet coherency, which is analogous to correlation, may remedy this. However, it requires smoothing both the cross-wavelet spectrum and the normalizing individual wavelet power spectra (without smoothing, its absolute value would be identically 1; see Liu [41]), and generates more or less blurred results depending on the particular smoothing filter installed. For the purpose of this study, we abstain from developing this step any further, since the time series in our application are normed values, and conclusions drawn from the cross-wavelet power spectrum are supported by comparisons with the power spectra in the univariate case.

It is crucial for our purposes in the present study that the cross-wavelet transform carries information about the synchronicity of the two series in terms of the local phase advance of any periodic component of the one series with respect to the corresponding component of the other:

$$\text{Angle.xy}(\tau, s) = \text{Arg}(\text{Wave.xy}(\tau, s)) = \text{phase.x}(\tau, s) - \text{phase.y}(\tau, s) \quad (8)$$

This so-called phase difference of x over y at each time and scale equals the difference of individual local phase displacements (relative to a localized origin) when converted into an angle in the interval $[-\pi, \pi]$. An absolute value less (or larger) than $\pi/2$ indicates that the two series move in phase (anti-phase, respectively) at the scale (or, equivalently: frequency, period) in question, while the sign of the phase difference shows which one is the leading series in this relationship. Figure 3 (in the style of a diagram by Aguiar-Conraria and Soares [2]) illustrates the range of possible phase differences and their interpretation. Information on phase differences at certain periods can be retrieved and analyzed separately in WaveletComp [50].

For practical purposes, it is more appropriate to use a cross-wavelet transform in which smoothing in time as well as in scale directions is performed by convolution with appropriate windows, see e.g. Cazelles et al. [15], Aguiar-Conraria and Soares [3]. Proceeding in this way, cross-wavelet results become less noisy. To this effect, we apply Bartlett windows of length 5 days in time and 25 days in scale direction.

⁸See, e.g. Aguiar-Conraria et al. [1] for a more comprehensive display of cross-wavelet methodology and an application to macroeconomic data.

⁹Here, again, a rectification factor was found necessary; c.f. Velela et al. [56].

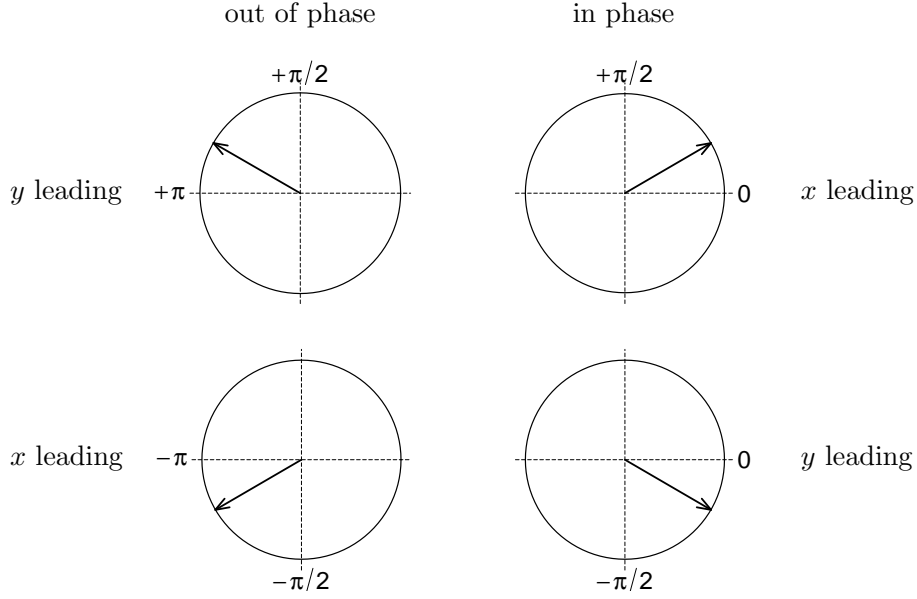


Figure 3: Phase differences and their interpretation

4.3 Assessing the statistical significance of periodic components

The statistical significance of the patterns emerging from (cross-) wavelet transformation is assessed by comparison with simulated white noise (500 surrogates for each of the three time series of propagation values). In our context, the null hypothesis of white noise to be tested reflects two agnostic statements: (i) there is no periodicity in either series of propagation values, and (ii) neither market is prominent with respect to its propagation value at any time. Localized p-values in the time and scale domains are derived from simulated shares of exceedances of power levels attained by the time series to be tested, following the approach by Aguiar-Conraria and Soares [3]. In addition, selective time series reconstruction tools support the identification of “powerful”, i.e. substantial, periodic constituents of the series.

5 Empirical results

The following three subsections report the empirical findings from the study of daily propagation values (Subsection 5.1), periodicity (Subsection 5.2) and phase differences (Subsection 5.3) in the network of three Western markets represented by their respective stock indices: DJIA (dji; US), FTSE 100 (ftse; UK) and Euro Stoxx 50 (sx5e; euro area).

5.1 Propagation values

Figure 4 shows a stacked plot of the three series of propagation values — one for each stock index under consideration. They were obtained along the steps outlined in Section 3. For a given day, the propagation values reflect the relative importance of asset markets as network volatility creators due to return shocks. Many characteristics of the series can be related to economic and geopolitical events; for example: The two most distinct peaks of the dji propagation value series coincide with the “Friday the 13th mini-crash”¹⁰ of October 1989, and with the culmination and

¹⁰Major U.S. indices had seen all-time highs on Monday when they plunged on Friday 13th 1989, just minutes after the announcement of a deal’s failure for UAL, the parent company of United Airlines, resulting in a 6.91 percent drop for DJIA. According to Shiller [51] the news event was “a ‘story’ that enhanced the feedback from

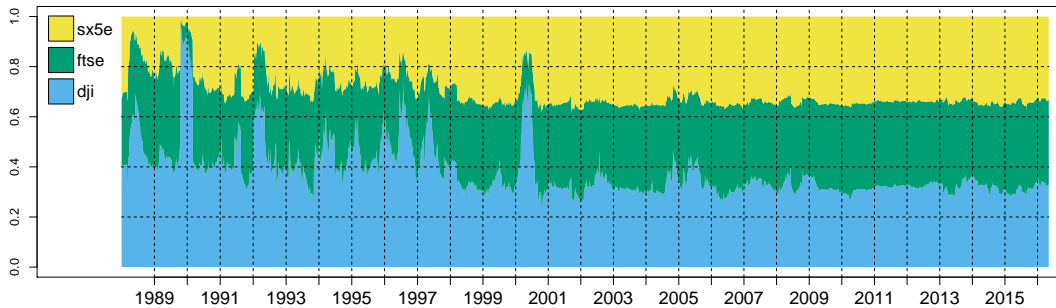


Figure 4: Daily propagation values

crash of the “dot-com bubble”¹¹ in 2000.

However, the prominence of dji as return shock disseminator in the network has diminished in recent years. From about 2001 onwards, each series has been fluctuating around $1/3$, with less pronounced bulges and spikes. When all three propagation values exactly equal $1/3$, it does not matter, for the creation of volatility in the network, from which market a shock is coming.

5.2 Cross-wavelet analysis

Figure 5 shows heat maps of the wavelet transform powers of the three pairs of propagation value series. The power spectrum gives information on the relative power of a wavelet component at a certain period length (the vertical axis) and at a certain location in time (the horizontal axis). The period ranges from 32 to 2048 days; about $21^{2/3}$ days represent a month. The white contour lines delineate the time-period domain of joint significance at the 10% level with respect to deviations from the null hypothesis of white noise. Actual powers were taken to the exponent of 0.5 for the plot, which is a common practice in wavelet analysis; this affects the color rendering of the heat map, but not the significant area.¹² The arrows within the significant area indicate the phase difference, at a given time and period, between a pair of propagation value series, according to the scheme in Figure 3.

The power spectrum plots of Figure 5 reveal that the range of significant frequencies has become narrower from the late 1980s through 2016. For example, a period of 256 (corresponding to approximately one year) was persistently significant until 2005, but only sporadically after 2005. These plots also show that the power of the cross-wavelet transforms has diminished; this is in line with the smoother character of the propagation value series (Figure 4).

At period 1024 (corresponding to approximately four years), turbulences occurred during the 1990s, leaving dji and ftse out of phase after 1995 (arrows pointing to the left), after they had been in phase (arrows pointing to the right) around 1990. Overall, however, the heat maps in Figure 5 can only give a rough impression of phase differences; see Section 5.3 for a finer analysis.

For further investigation of the joint behavior of the series, it is useful to identify those periods which are powerful across time. Figure 6 (top) shows a plot of average power, taken over the entire time interval 1988–2016, by period. It has essentially local peaks at periods 195,

stock price drops to further stock price drops, thereby preserving the feedback effect for a longer period than would otherwise have been the case. Yet it was unlikely to have been the cause of the crash.”

¹¹ “... The technology-heavy Nasdaq reached its pinnacle of 5,048.62 on March 10, [2000]. Then the Internet bubble burst and the index plummeted nearly 40 percent, dropping below 3,000 in December [2000] in its worst annual loss”, The New York Times, 2012-03-13; available at <http://www.nytimes.com/2012/03/14/business/stocks-rally-to-pre-2008-heights.html>. Retrieved 2016-11-29.

¹² See, e.g. Percival and Walden [47].

period length	260 days	600 days	1040 days
full cycle ($\hat{=2\pi}$)	12 months	27.7 months	48 months
half-cycle ($\hat{=\pi}$)	6 months	13.8 months	24 months
quarter-cycle ($\hat{=\pi/2}$)	3 months	6.9 months	12 months

Table 1: Period lengths and corresponding actual calendar time durations

292, 639, 1046, and 1652. The heat maps in Figure 5 might raise the suspicion that the powerful periods in the early years after 1988 are responsible for this pattern. This is, however, not the case: Analogous plots of average powers, computed over shorter time intervals (for example, 2001–2016, see the bottom of Figure 6) have a very similar pattern. In the following, we shall therefore analyze phase differences of propagation value series at periods 260, 600 and 1040.

5.3 Phase differences at selected periods

The cross-wavelet transformation of two time series results in a decomposition of both series into periodic components, and it also yields the phase difference between the components of the same period at any point in time. For periods 260, 600 and 1040, the time series of phase differences are plotted in Figure 7. Significant (insignificant) parts are represented by solid (dashed, respectively) lines. The value of each series gives, together with the scheme shown in Figure 3, the angles of the arrows of the heat maps in Figure 5 belonging to the period in question. The values in Table 1 may help to put the angle time series in perspective: In very good approximation, 260 stock exchange days correspond to one year; consequently, a half-cycle at period 1040 days corresponds to 24 months. For example, the phase difference between the propagation values of ftse and sx5e at period 1040 at the beginning of 2016 is approximately $-\pi/6$ (the bottom plot in Figure 7), which corresponds to about four months: At this period (and at this time), sx5e reaches its peak importance four months before ftse. The results¹³ can be described as follows (mentioning a stock index name in the following means that we speak of its propagation value series):

- period 260: The pair dji and ftse is out of phase (except in short, or insignificant, time intervals), and so is the pair dji and sx5e. The pair ftse and sx5e is mostly in phase. The leading index is alternating. There is no persistent significance at period 260: From about 2005 onwards, in all three cases, there are only short significant time intervals at this period.
- period 600: Except for the two time intervals 1997–2001 and 2007–2009, the following can be observed:
 - The pair dji and ftse is out of phase, with dji leading.
 - The pair dji and sx5e is out of phase, with sx5e leading.
 - The pair ftse and sx5e is in phase, with sx5e leading.
- period 1040: The relation between the stock markets changed during a time interval 1992–1993. Before 1992:
 - The pair dji and ftse is in phase, with ftse leading.

¹³The robustness of the results can be assessed by varying the periods slightly, for example, compare angle series at periods 250/260/270. These plots are not shown here. It turns out that the following conclusions still hold for neighboring periods.

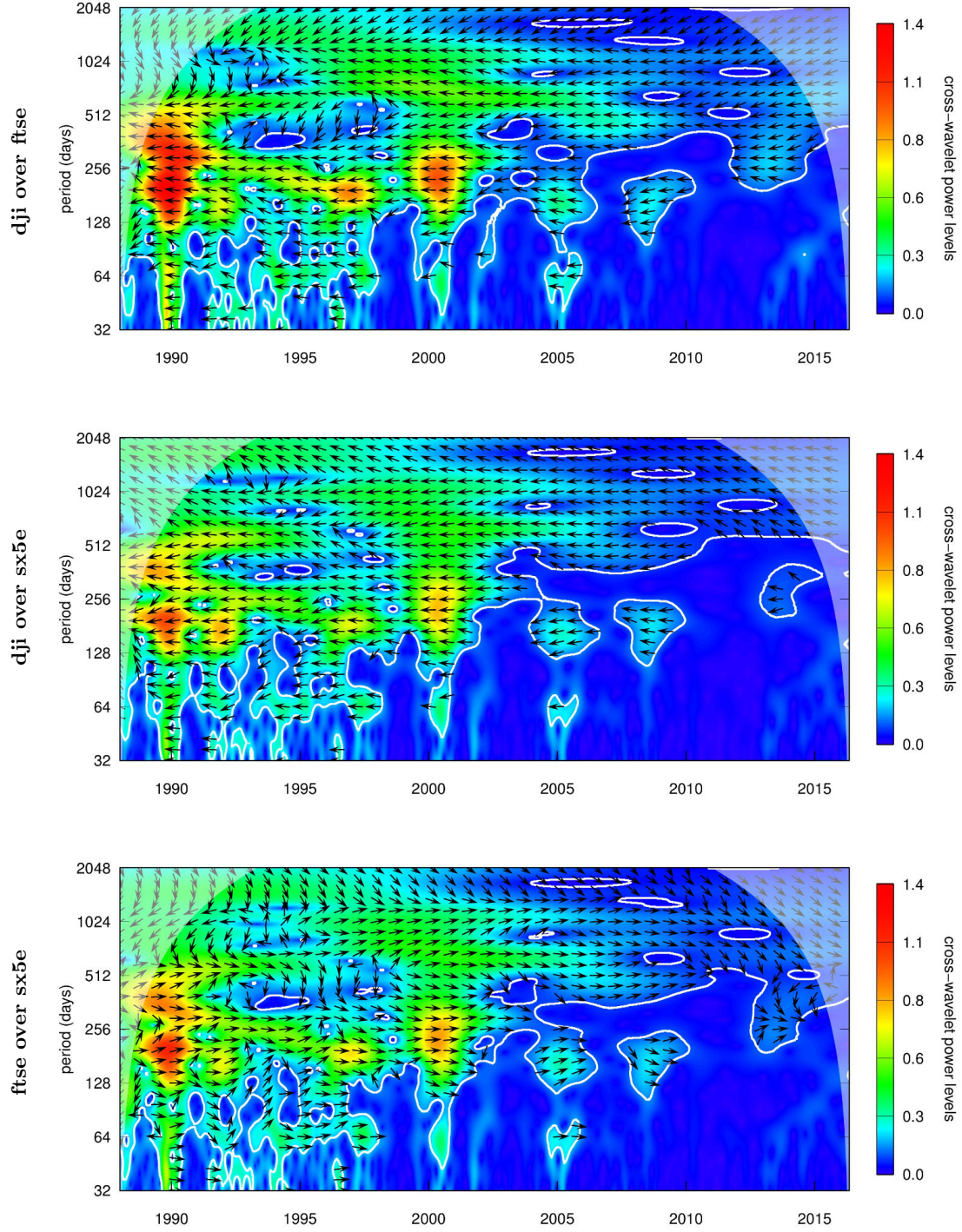


Figure 5: Cross-wavelet power spectra

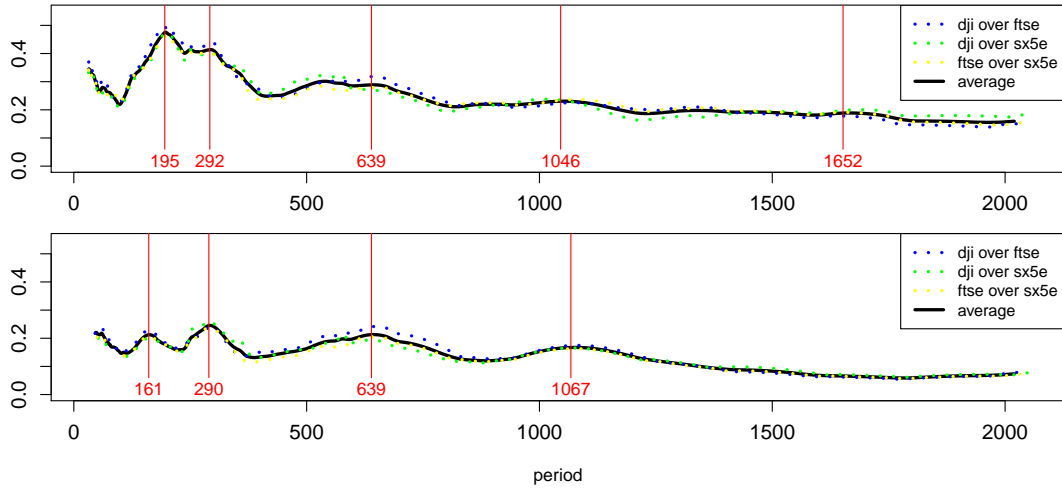


Figure 6: Average power of cross-wavelet transform, starting 1988 (top), 2001 (bottom)

- The pair dji and sx5e is out of phase, with sx5e leading.
- The pair ftse and sx5e is out of phase, with ftse leading.

From 1994 onwards:

- The pair dji and ftse is out of phase, with ftse leading until 2007; dji has been leading from 2008 onwards.
- The pair dji and sx5e is out of phase, with dji leading until 2007; sx5e has been leading from 2008 onwards.
- The pair ftse and sx5e is in phase, with ftse leading until 2007; sx5e has been leading from 2008 onwards.

These results can be further illustrated by means of schematic reconstructions of the propagation value series at periods 260, 600 and 1040, based on sine waves and using phase differences measured at the beginning of years 1990, 1999, 2008, and 2015. This is shown in Figure 8. Dashed lines again indicate insignificance. The three series shown in each plot of Figure 8 have a constant sum at any point in time; it is thus assumed that the only relevant constituent of the propagation value series is the period in question. For example, the plot “period 260, year 1990” shows that when the propagation value of dji moves from a trough to a peak in 130 days (this pace is determined by the period of 260), it will be accompanied by propagation values of ftse and sx5e *diminishing together* at the same pace, where ftse (the leading index) precedes sx5e.

The European markets represented by ftse and sx5e are in phase in nine of the 12 plots in Figure 8; they are out of phase, with a significant pattern, in 1990 at period 1040. The plots further illustrate that, considering the recent process of comovement among the European markets, ftse was leading at periods 600 and 1040 in the years 1999 and 2008, but sx5e held the leading role in 2015; this transition had started after 2008.

6 Discussion

Our findings suggest that the dynamics in the network of the three equity markets in our study, namely, the US (represented by dji), UK (ftse) and euro area (sx5e), have undergone a substantial change from 2001 onwards. Concerning network volatility creation, the series of propagation

values reveal that the relative importance of markets as news spreaders has been fluctuating much less during the last 15 years than it has before.

Another finding concerns the frequency content of the three series of propagation values: The band of significant frequencies, which potentially carries information about the structure of risk dissemination in the network, has become narrower from 2001 onwards.

The diminishing range of significant frequencies may have an explanation in terms of the intensity of intraday information exchange; intraday phenomena are not captured by the approach pursued in the present paper, which is based on daily spillovers of return shocks. Information flows easily today. Decreasing holding times of stocks point in the same direction.¹⁴ Two decades ago, the frequency structure of information transmission was richer and more telltale for investors waiting for similar patterns to occur repeatedly, while they rather tend to act immediately today and are less inclined to bet on frequency aspects. With a daily perspective in mind, the short-frequency structure of information transmission has become too unpredictable to build one’s portfolio upon.

Another finding, in line with relevant literature,¹⁵ is the joint powerful cyclicity at periods 260 (even if no longer significant), 600, and 1040 days. Among these, the 260-day period corresponds to the average number of trading days in a year. Similarly, the 1040-day period reflects a four-year period in the stock markets under scrutiny. These periods correspond to cycles as in the “January effect” and the “four-year US Presidential Election Cycle” or the “Kitchin Cycle”, respectively.

With respect to the synchronicity of information transmission, a pattern can be detected which has persisted since the mid-1990s: the US market is in anti-phase with the European markets, while the European markets are in phase with each other. Moreover, European markets are highly synchronous (in the sense of a small phase difference) news propagators today when compared to what they were 25 years ago, which, on the other hand, emphasizes the consistently special role of the US market in the network. Today, the phasing of information propagation has evolved into a distinct pattern, for example: we found evidence that among the European markets the euro area has taken over the leading role from the UK in 2008 as news propagator. In addition, there is no more evidence of a systematic and persistent “informational divide” between the US and European markets with respect to their potential of information propagation.

In their study of long-run comovements in the stock market for the G7 countries between 1973 and early 2009, Menezes and Dionisio [43] find an overall long-run relationship governing the process of market integration and conclude that “the US market leads the G7 market space and the UK emerges as a regional attractor within the European context, which, in turn, is strongly affected by the North-American markets.” Taking on a time-frequency perspective, our findings point to similar conclusions for the overlapping time interval.

Recently, a different network model setup, with nodes representing countries and edges representing the degree of similarity of the countries’ GDP cyclical components, has been suggested by Antonakakis et al. [7]. The authors study the evolution of international business cycle synchronization during fundamental periods of globalization between 1875 and 2013, including up to 27 developed and developing countries over time. Among their findings is that the most

¹⁴The average holding period of stocks has dropped secularly in all markets studied over our analysis period. Haldane [31] reports that the mean duration for the US equity holdings has dropped from around 7 years (in 1940) to around 7 months (in 2007), for the UK market, the similar trend is observed with average holding period of stocks around 5 years (in mid-1960s) to 7.5 months (in 2007). Furthermore, at the international level this trend is also confirmed for the major equity markets; for the Shanghai stock index, the mean duration is close to 6 months. Decreasing transaction costs as well as advances in High-Frequency-Trading (HFT) technology, allowing transactions in milli- or micro-seconds, are believed to have an impact on the decreasing average holding periods (see Haldane [31]).

¹⁵E.g. Gu [30], Haug and Hirschey [32]; Wong and McAleer [57], Booth and Booth [9].

recent period (1973–2013) was associated with the historically highest degree of business cycle synchronization; they could not, however, generally confirm Kose et al.’s [39] convergence hypothesis for financially integrated economies. Our study, although obtained with different data and a different methodology, contributes to this strand of discussion, and is in line with findings by Olivero and Madak [46] and Fidrmuc et al. [24]. We found evidence that as they became more integrated from the mid-1990s onward, the European markets started to decouple from the US market at business cycle frequency, while low periods are of diminishing relevance.

7 Summary and Conclusions

The methodological framework used in this study permits tracing an aspect of centrality within networks of equity markets on a daily basis: a market’s propagation value measures its relative importance in creating volatility across the network when a return shock hits that market. The concept of propagation values lends itself to studying “importance cycles” in such a network. As the frequency content of propagation values may change over time, we use the cross-wavelet methodology. The present study is thus an effort to investigate frequency aspects across time with respect to daily return-to-volatility transmission in a network of three Western equity markets, each represented by a stock index: DJIA (US), FTSE 100 (UK) and Euro Stoxx 50 (euro area); the data used covers the time period from August 1987 through May 2016.

Our findings reveal a diminishing range of frequencies from 2001 onwards: a loss of short-frequency phenomena, which could be related to investors’ increasing preference for ad-hoc trades and immediate decisions. This loss goes along with an evolution of the three markets to be almost on a par with respect to information transmission.

Nevertheless, and in line with literature on cycles in stock markets, a joint powerful cyclicity at periods 260, 600 and 1040 days can still be observed. The detected four-year cycle corresponds to the “Kitchin Cycle”, suggesting that the study of “importance cycles” in networks of equity markets could contribute to the determination of business cycles, although (or maybe precisely because) our findings are obtained with different, namely daily and easily available, data. Furthermore, we found evidence that, at the business cycle length, the markets’ synchronization in terms of information transmission has witnessed a substantial change: Since the mid-1990s, the US market has been in anti-phase with the European markets, while the latter have been in phase; before that, it was the US and UK that had been in phase. This is in line with literature on financial integration and their impact on business cycle synchronization. Among European markets, another change of pattern has evolved recently: In 2008, the euro area has taken over the leading role at business cycle frequency.

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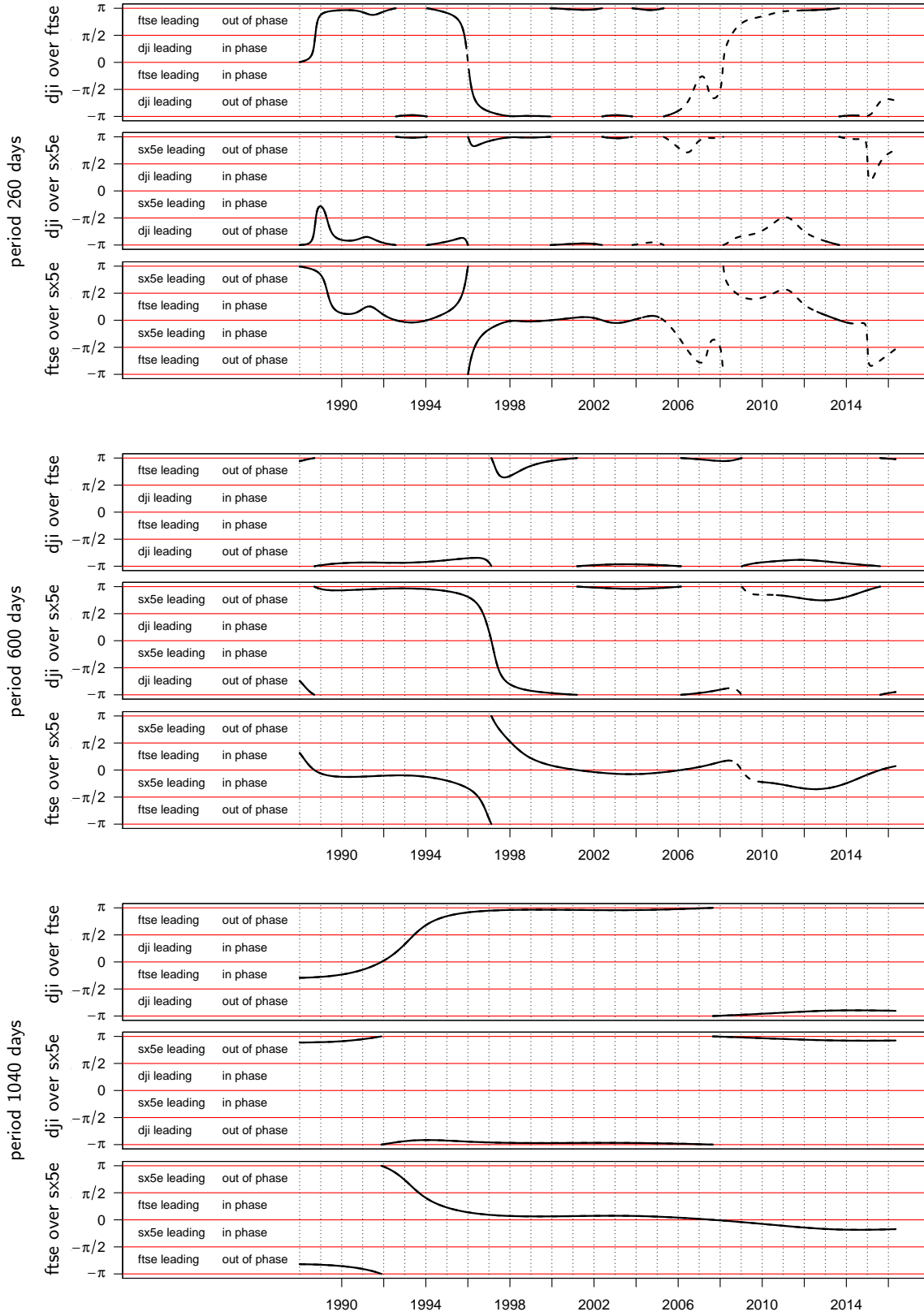


Figure 7: Phase differences, selected periods

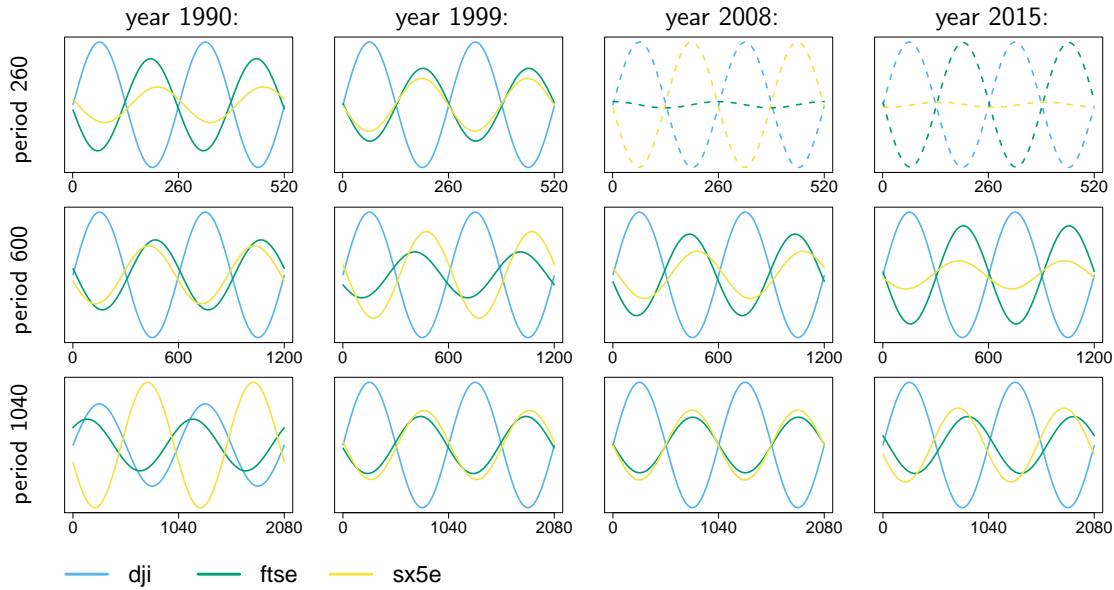


Figure 8: Schematic reconstruction of propagation values using selected periods

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