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Monetary policy and cross-border interbank market fragmentation: lessons from the crisis

Online Appendices for "Monetary policy and cross-border interbank market fragmentation: lessons from the crisis"

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Appendix A Interbank Credit Contract Solution

This appendix details the solution to the contract problem faced by savings banks, in which savings banks choose volumes and lending rates for extended domestic and international interbank credit. Dropping bank indexes for convenience, lending banks balance sheet

$$N_t + IB_t^H + IB_t^F + = CR_t$$

Net worth evolves according to

$$N_t = \omega R_t^{CR} CR_{t-1} - R_{t-1}^{IB,H} IB_{t-1}^H - R_{t-1}^{IB,F} IB_{t-1}^F$$

where R_t^{CR} is the ex post return across all lending. Default threshold

$$\bar{\omega}_t \equiv \frac{R_{t-1}^{IB,H} IB_{t-1}^H + R_{t-1}^{IB,F} IB_{t-1}^F}{R_t^{CR} CR_{t-1}}$$

Banks face CES preference toward domestic and foreign interbank borrowing, so choose IB_t^H and IB_t^F to maximise

$$IB_t = \left[\tau^{IB\,1/\theta_{IB}} IB_t^{H\,\frac{\theta_{IB}-1}{\theta_{IB}}} + \left(1-\tau^{IB}\right)^{1/\theta_{IB}} IB_t^{F\,\frac{\theta_{IB}-1}{\theta_{IB}}} \right]^{\frac{\theta_{IB}}{\theta_{IB}-1}} \tag{A.1}$$

subject to

$$R_t^{IB} IB_t = R_t^{IB,H} IB_t^H + R_t^{IB,F} IB_t^F. \tag{A.2}$$

This leads to demand schedules

$$IB_t^F = \left(1-\tau^{IB}\right)\left(\frac{R_t^{IB,F}}{R_t^{IB}}\right)^{-\theta_{IB}} IB_t \tag{A.3}$$

$$IB_t^H = \tau^{IB}\left(\frac{R_t^{IB,H}}{R_t^{IB}}\right)^{-\theta_{IB}} IB_t \tag{A.4}$$

and interest rate

$$R_t^{IB} = \left[\tau^{IB} R_t^{IB,H^{1-\theta_{IB}}} + \left(1-\tau^{IB}\right) R_t^{IB,F^{1-\theta_{IB}}}\right]^{\frac{1}{1-\theta_{IB}}} \tag{A.5}$$

We also find

$$\begin{aligned} IB_t^H + IB_t^F &= IB_t \frac{\tau^{IB} R_t^{IB,H^{-\theta_{IB}}} + \left(1-\tau^{IB}\right) R_t^{IB,F^{-\theta_{IB}}}}{R_{IB,t}{}^{-\theta_{IB}}} && \text{(A.6)} \\ &= IB_t \Upsilon\left(R_t^{IB,H}, R_t^{IB,F}\right) && \text{(A.7)} \end{aligned}$$

Let

$$\begin{aligned} G_{t+1} &\equiv G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \equiv \int_0^{\bar{\omega}_t} \omega dF(\omega, \sigma_t) && \text{(A.8)} \\ F_{t+1} &\equiv F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \equiv \int_0^{\bar{\omega}} f\left(\omega; \frac{-\sigma^2}{2}, \sigma^2\right) d\omega && \text{(A.9)} \end{aligned}$$

The ex post return on interbank lending for the savings banks can then by given by

$$\tilde{R}_t^{IB,H} = (1-F_t) R_{t-1}^{IB,H} + (1-\mu) G_t R_t^{CR} \frac{CR_{t-1}}{IB_{t-1}^H + IB_{t-1}^F} \tag{A.10}$$

We assume there is a cost in lending internationally that depends on the net foreign asset position of the destination country. The ex post return on international lending (for foreign banks) is given by

$$\begin{aligned} \tilde{R}_t^{IB,F} IB_{t-1}^F = \left[1-\Gamma_{IB,t}^*\right](1-F_t) R_{t-1}^{IB,F} IB_{t-1}^F \\ + (1-\mu) G_t R_{t+1}^{CR} CR_{t-1} \frac{IB_{t-1}^F}{IB_{t-1}^H + IB_{t-1}^F} && \text{(A.11)} \end{aligned}$$

Objective

$$\max_{\substack{CR_t,\bar{\omega}_{t+1},R_t^{IB,H},\\ IB_t^H,R_t^{IB,F},IB_t^F}} \mathbb{E}_t\left[\left(\left(1-G_{t+1}\right)R_t^{CR}CR_t-\left(1-F_{t+1}\right)R_t^{IB}IB_t\right)\tfrac{\Lambda_{t,t+1}}{P_{t+1}^C}\right]$$

$$\begin{aligned}\text{s.t.}\quad & \bar{\omega}_{t+1}=\frac{R_t^{IB}IB_t}{R_{t+1}^{CR}CR_t}\\ & \mathbb{E}_t\left[\tfrac{\Lambda_{t,t+1}}{P_t^C}\tilde{R}_t^{IB,H}IB_{t-1}^H\right]=\mathbb{E}_t\left[\tfrac{\Lambda_{t,t+1}}{P_t^C}R_t^S IB_{t-1}^H\right]\\ & \mathbb{E}_t\left[\tfrac{\Lambda_{t,t+1}^*}{P_t^{C^*}}\tilde{R}_t^{IB,F}IB_{t-1}^F\right]=\mathbb{E}_t\left[\tfrac{\Lambda_{t,t+1}^*}{P_t^{C^*}}R_t^{S^*}IB_{t-1}^F\right]\\ & N_t+IB_t\Upsilon\left(R_t^{IB,H},R_t^{IB,F}\right)=CR_t\end{aligned}$$

or, suppressing arguments of G_{t+1}, F_{t+1}, R_t^{IB}, Υ_t for convenience, CR_t,$\bar{\omega}_{t+1}$,$R_t^{IB,H}$,$R_t^{IB,F}$ chosen to maximise Lagrangian

$$\mathcal{L}_t=\mathbb{E}_t\left[\begin{array}{c}\tfrac{\Lambda_{t,t+1}}{P_{t+1}^C}\left(1-G_{t+1}-\left(1-F_{t+1}\right)\bar{\omega}_{t+1}\right)R_t^{CR}CR_t\\ +\varrho_t\left(\bar{\omega}_{t+1}R_{t+1}^{CR}CR_t-\left(CR_t-N_t\right)\tfrac{R_t^{IB}}{\Upsilon_t}\right)\\ +\lambda_t^{IB,H}\begin{pmatrix}\left(1-F_{t+1}\right)R_t^{IB,H}(CR_t-N_t)\\ +(1-\mu)\left[G_{t+1}R_{t+1}^{CR}CR_t\right]\\ -R_t^S(CR_t-N_t)\end{pmatrix}\tfrac{\Lambda_{t,t+1}}{P_{t+1}^C}\\ +\lambda_t^{IB,F}\begin{pmatrix}\left(1-\Gamma_{IB,t}^*\right)\left(1-F_{t+1}\right)R_t^{IB,F}(CR_t-N_t)\\ +(1-\mu)\left[G_{t+1}R_{t+1}^{CR}CR_t\right]\\ -R_t^{S*}(CR_t-N_t)\end{pmatrix}\tfrac{\Lambda_{t,t+1}^*}{P_{t+1}^{C}{}^*}\end{array}\right]$$

So with

$$\Upsilon\left(R_t^{IB,H},R_t^{IB,F}\right)=\frac{\tau^{IB}R_t^{IB,H^{-\theta_{IB}}}+\left(1-\tau^{IB}\right)R_t^{IB,F^{-\theta_{IB}}}}{R_{IB,t}{}^{-\theta_{IB}}}\tag{A.12}$$

$$R_t^{IB}=\left[\tau^{IB}R_t^{IB,H^{1-\theta_{IB}}}+\left(1-\tau^{IB}\right)R_t^{IB,F^{1-\theta_{IB}}}\right]^{\frac{1}{1-\theta_{IB}}}\tag{A.13}$$

$$\frac{R_t^{IB}}{\Upsilon\left(R_t^{IB,H},R_t^{IB,F}\right)}=\frac{\left(\tau^{IB}R_t^{IB,H^{1-\theta_{IB}}}+\left(1-\tau^{IB}\right)R_t^{IB,F^{1-\theta_{IB}}}\right)}{\left(\tau^{IB}R_t^{IB,H^{-\theta_{IB}}}+\left(1-\tau^{IB}\right)R_t^{IB,F^{-\theta_{IB}}}\right)}\tag{A.14}$$

$$\begin{aligned}\frac{\partial\frac{R_t^{IB}}{\Upsilon\left(R_t^{IB,H},R_t^{IB,F}\right)}}{\partial R_t^{IB,H}}&=\left(1-\left[1-\theta_{IB}\left(\frac{R_{IB}^H}{R_{IB,t}}\right)^{-1}\frac{1}{\Upsilon_t}\right]\right)\frac{IB_t^H}{IB_t^H+IB_t^F}\\ &\equiv\Xi_t^{IB,H}\end{aligned}\tag{A.15}$$

$$
\begin{aligned}
\frac{\partial \frac{R_t^{IB}}{\Upsilon\left(R_t^{IB,H},R_t^{IB,F}\right)}}{\partial R_t^{IB,F}} &= \left(1-\left[1-\theta_{IB}\left(\frac{R_t^{IB,F}}{R_{IB,t}}\right)^{-1}\frac{1}{\Upsilon_t}\right]\right)\frac{IB_t^F}{IB_t^H+IB_t^F} \\
&\equiv \Xi_t^{IB,F}
\end{aligned}
\tag{A.16}
$$

we find

$$
\begin{aligned}
&\mathbb{E}_t\left[-G'\left(\bar{\omega}_{t+1}\right)+F'\left(\bar{\omega}_{t+1}\right)\bar{\omega}_{t+1}-\left(1-F\left(\bar{\omega}_{t+1}\right)\right)R_{t+1}^{CR}CR_t\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\right] \\
&+\varrho_t\left(R_{t+1}^{CR}CR_t\right) \\
&+\lambda_t^{IB,H}\left(\mathbb{E}_t\left[\begin{array}{l}-F'\left(\bar{\omega}_{t+1}\right)R_t^{IB,H}(CR_t-N_t)\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\\+(1-\mu)\left[G'\left(\bar{\omega}_{t+1}\right)R_{t+1}^{CR}CR_t\right]\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\end{array}\right]\right) \\
&+\lambda_t^{IB,F}\left(\mathbb{E}_t\left[\begin{array}{c}-\left(1-\Gamma_{IB,t}^*\right)F'\left(\bar{\omega}_{t+1}\right)R_t^{IB,F}(CR_t-N_t)\frac{\Lambda_{t,t+1}^*}{{P_{t+1}^C}^*}\\+(1-\mu)\left[G'\left(\bar{\omega}_{t+1}\right)R_{t+1}^{CR}CR_t\right]\frac{\Lambda_{t,t+1}^*}{{P_{t+1}^C}^*}\end{array}\right]\right) \\
&=0
\end{aligned}
\tag{A.17}
$$

$$
\begin{aligned}
&\mathbb{E}_t\left[\left(1-G_{t+1}-\left(1-F_{t+1}\right)\bar{\omega}_{t+1}\right)R_{t+1}^{CR}\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\right] \\
&+\varrho_t\left(\bar{\omega}_tR_{t+1}^{CR}-\frac{\bar{R}_t^{IB}}{\Upsilon_t}\right) \\
&+\lambda_t^{IB,H}\left(\mathbb{E}_t\left[\begin{array}{l}\left(1-F_{t+1}\right)R_t^{IB,H}\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\\+(1-\mu)G_{t+1}R_{t+1}^{CR}\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\end{array}\right]-R_t^S\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\right) \\
&+\lambda_t^{IB,F}\left(\mathbb{E}_t\left[\begin{array}{c}\left(1-\Gamma_{IB,t}^*\right)\left(1-F_{t+1}\right)R_t^{IB,F}\frac{\Lambda_{t,t+1}^*}{{P_{t+1}^C}^*}\\+(1-\mu)G_{t+1}R_{t+1}^{CR}\frac{\Lambda_{t,t+1}^*}{{P_{t+1}^C}^*}\end{array}\right]-R_t^{S*}\frac{\Lambda_{t,t+1}^*}{{P_{t+1}^C}^*}\right) \\
&=0
\end{aligned}
\tag{A.18}
$$

$$
\begin{aligned}
&\varrho_t\left(-\left(CR_t-N_t\right)\Xi_t^{IB,H}\right) \\
&\quad+\lambda_t^{IB,H}\left(\mathbb{E}_t\left[\left(1-F_{t+1}\right)\left(CR_t-N_t\right)\frac{\Lambda_{t,t+1}}{P_{t+1}^C}\right]\right)=0
\end{aligned}
\tag{A.19}
$$

$$
\begin{aligned}
&\varrho_t\left(-\left(CR_t-N_t\right)\Xi_t^{IB,F}\right) \\
&\quad+\lambda_t^{IB,F}\left(\mathbb{E}_t\left[\left(1-\Gamma_{IB,t}^*\right)\left(1-F_{t+1}\right)\left(CR_t-N_t\right)\frac{\Lambda_{t,t+1}^*}{{P_{t+1}^C}^*}\right]\right)=0
\end{aligned}
\tag{A.20}
$$

Then

$$
\mathbb{E}_t\left[R_{t+1}^{CR}\right]
$$

$$= \mathbb{E}_t \left[\begin{pmatrix} \lambda_t^{IB,H} R_t^{IB,H} \frac{\Lambda_{t,t+1}}{P_{t+1}^C} \\ \lambda_t^{IB,F} \left(1 - \Gamma_{IB,t}^*\right) R_t^{IB,F} \Upsilon_t \frac{\Lambda_{t,t+1}^*}{P_{t+1}^{C\ *}} \\ -(1-\mu) R_t^{IB} \left(\lambda_t^{IB,H} \frac{\Lambda_{t,t+1}}{P_{t+1}^C} + \lambda_t^{IB,F} \frac{\Lambda_{t,t+1}^*}{P_{t+1}^{C\ *}} \right) \\ + \left(1 - F_{t+1}\right) R_{t+1}^{CR} \frac{CR_t}{IB_t} \frac{\Lambda_{t,t+1}}{P_{t+1}^C} \end{pmatrix} \frac{F'(\bar{\omega}_{t+1})}{\varrho_t} \frac{IB_t}{CR_t} \right]$$

where

$$\begin{aligned} &\varrho_t \\ &= \frac{\lambda_t^{IB,H} + \lambda_t^{IB,F} - \mathbb{E}_t \left[\begin{pmatrix} \left[1 - G_{t+1} - \left(1 - F_{t+1}\right) \bar{\omega}_{t+1}\right] \frac{\Lambda_{t,t+1}}{P_{t+1}^C} \\ + \lambda_t^{IB,H} (1-\mu) G_{t+1} \frac{\Lambda_{t,t+1}}{P_{t+1}^C} \\ + \lambda_t^{IB,F} (1-\mu) G_{t+1} \frac{\Lambda_{t,t+1}^*}{P_{t+1}^{C\ *}} \end{pmatrix} R_{t+1}^{CR} \right]}{\bar{\omega}_{t+1} R_{t+1}^{CR} - \frac{R_t^{IB}}{\Upsilon_t} + \Xi_t^{IB,H} R_t^{IB,H} + \Xi_t^{IB,F} R_t^{IB,F}} \qquad \text{(A.21)} \\ &\lambda_t^{IB,H} = \frac{\varrho_t}{\mathbb{E}_t \left[\left(1 - F_{t+1}\right) \frac{\Lambda_{t,t+1}}{P_{t+1}^C} \right]} \Xi_t^{IB,H} \\ &\lambda_t^{IB,F} = \frac{\varrho_t}{\mathbb{E}_t \left[\left(1 - F_{t+1}\right) \frac{\Lambda_{t,t+1}^*}{P_{t+1}^{C\ *}} \right]} \frac{\Xi_t^{IB,F}}{1 - \Gamma_{IB,t}^*} \end{aligned}$$

These conditions imply that the expected return on credit, $\mathbb{E}_t \left[R_{t+1}^{CR}\right]$, depends on the rates and the leverage ratio $\frac{IB_t}{CR_t}$. Given that perfect capital markets imply $\mathbb{E}_t \left[R_{t+1}^{CR}\right]$ will be the same for all banks, ω_t only depends on the leverage, and so the conditions imply the same rates and leverage ratio for all banks of any N_t.