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TWO STAGES OF ECONOMIC DEVELOPMENT

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Abstract

This study suggests that the development process of a less-developed country can be divided into two stages, which demonstrate significantly different properties in areas such as structural endowments, production modes, income distribution, and the forces that drive economic growth. The two stages of economic development have been indicated in the growth theory of macroeconomics and in the various "turning point" theories in development economics, including Lewis's dual economy theory, Kuznets curve, and the middle-income trap. A dynamic macroeconomic model is constructed to simulate the development process that reveals these two stages. Using the two-stage theory of economic development, we find that the People's Republic of China's economy is currently at the intersection between the first and second stages. This is the definition of "new normal" in the current Chinese economy.

JEL Classification: O11, E10, C61, C62

Contents

1. INTRODUCTION

In May 2014, President Xi used the term “new normal” to describe the current Chinese economy. The phrase has become commonplace in Chinese media and official documents, and as a concept is a logical starting point when designing Chinese economic policy. Chinese economists have attempted to explain the “new normal” through many different perspectives.

The concept of the “middle-income trap” was proposed by the World Bank in 2007 and has been tentatively accepted by economists since 2007. The concept appears to be supported by empirical evidence but has not been academically proven or supported by rigid economic analysis. [1] This lack of theoretical support has led some economists to doubt its viability.[2]

This study proposes a theory known as the “two stages of economic development” (TSED), which can explain both Xi’s “new normal” and the World Bank’s “middle-income trap,” while being consistent with the growth theories in macroeconomics and the turning point theory in development economics, put forward by Lewis (1954) and Kuznets (1955).

In Section 2, we explain the TSED using the growth theory of macroeconomics and the turning point theories of development economics. This explanation does not rely on a theoretical model. In Section 3, we construct a model that enables us to derive the behavior function of investment, which is a key variable in the development process. This concept is used in Section 4, where a theoretical model is constructed to explain the development process of a less-developed economy. Section 5 analyzes the dynamics of technology, which captures the properties of technological progress in a less-developed economy. This dynamic is autonomous and thus independent from the rest of the model. Section 6 provides an analysis of the model, enabling us to examine the aforementioned development features. Finally, in Section 7, we argue that the current Chinese economy has reached an intersection between the first and second stages of economic development, and provide a perspective on Xi’s “new normal.” The mathematical proof of the proposition is given in the Appendix.

2. TWO STAGES OF ECONOMIC DEVELOPMENT: INDICATIONS FROM GROWTH THEORY AND DEVELOPMENT ECONOMICS

The TSED theory was first proposed by Gong (2008, 2012) and later roughly modeled (Gong 2013). It is constructed on solid economic foundations, following the logic of growth theory in macroeconomics and the “turning point” theories in development economics.

[1] See evidence provided by the World Bank (2007) and Eichengreen, Park, and Shin (2012, 2013), among others.

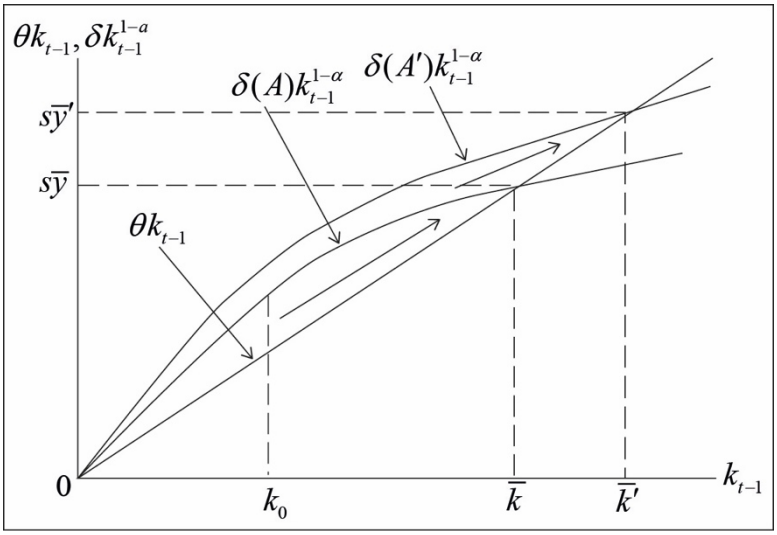
[2] See, for example, Han and Wei (2015).

2.1 Growth Theory in Macroeconomics

Differences in gross domestic product (GDP), or output, per capita are the main indicators of the extent to which developing countries lag behind developed countries. According to the growth theory of macroeconomics, two scenarios may lead to lower GDP per capita in less-developed economies: lower capital per capita, which indicates that the economy as a whole mainly involves labor-intensive modes of production; and underdeveloped technology, indicating that total factor productivity is low in the production function.[3] Given these two scenarios, improvements of per capita output can therefore be realized by adopting either capital- or knowledge-intensive production modes.

In reality, growth in capital per capita is often due to urbanization. The labor force leaves agricultural production in increasing numbers and moves to work with methods of capital production (machines, facilities, etc.). Cities can attract labor from rural areas through continuous investment in facilities. Thus, for the economy as a whole, the movement of labor from rural to urban areas is also a process of transformation of the production mode from being labor intensive to capital intensive, improving the country's capital per capita.

Figure 1: The Increase in Capital per Capita and Gross Domestic Product per Capita in a Growth Model



Source: Author.

Growth theory states that there is a limit to the increase in capital per capita and thus in GDP per capita at a given level of technology. Figure 1 shows that if the level of technology is kept at A and the initial economy is at k_0, the capital per capita will eventually reach a steady state at $\bar{k}$.[4] This process can be regarded as the transformation of the production mode from being labor intensive, such as at k_0 to being more capital intensive, such as at $\bar{k}$. GDP per capita is improves to $\bar{y}$, as

[3] See Solow (1956) or any intermediate textbook.

[4] Given the production function $Y_t = (A_t L_t)^{\alpha} K_{t-1}^{1-\alpha}$ and the capital accumulation as $K_t = (1-d)K_{t-1} + sY_t$, where Y_t, A_t, L_t and K_t are the output, technology, labor supply, and capital stock, respectively; and *d*, *s*, and *l* are the depreciation rate, the saving ratio, and the labor supply growth rate, respectively, it is not difficult to prove that $\theta = (d+l)/(1+l)$ and $\delta(A) = sA^{\alpha}/(1+l)^{1-\alpha}$.

indicated in the figure. It is clear that $(\bar{k}, \bar{y})$ is indeed the limit in capital per capita and output per capita if technology is kept at A.

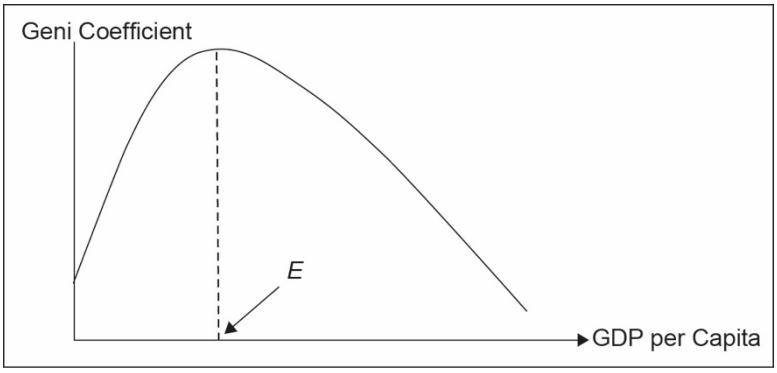
Once the limit has been achieved, only an improvement in technology can lead to further improvement in capital per capita and thus in GDP per capita. Figure 1 shows that if technology is improved to A', capital per capita will increase to $\bar{k}'$, and thus GDP per capital will increase to $\bar{y}'$. Technological improvements indicate that the economy's production mode leans towards being knowledge intensive, which is of course the mode that brings further increases in capital per capita and thus output per capita.

2.2 Turning Point Theories in Development Economics

There are numerous turning point theories in development economics. Lewis's (1954) was proposed in his work on the dual economy. In his view, a less-developed country has a large or even infinite amount of surplus labor in the early stages of economic development, and society is separated into a relatively modern industrial (or urban) sector and a traditional agriculture (or rural) sector. A large amount of surplus labor remains stuck in the rural area, but economic development enables it to be gradually absorbed into the modern industrial sector. The Lewis turning point refers to the stage in the development process when the labor surplus no longer exists, which means further growth will accelerate wage increases.

Kuznets curve (1955), another turning point theory, suggests that income distribution in the development process (measured by GDP per capita) of a country only improves after an initial deterioration. From this, the "turning point" can be found (see point *E* in Figure 2) with regard to income distribution in the development process.

Figure 2: Kuznets Curve



GDP = gross domestic product.

Source: Author.

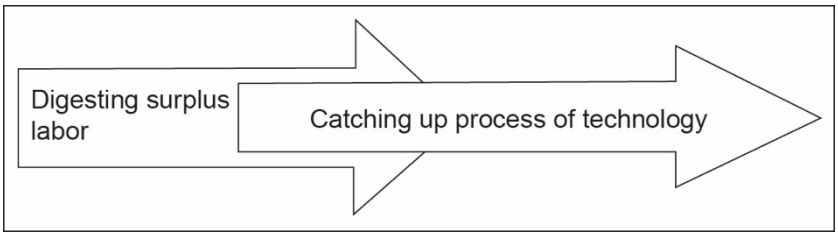
The "middle-income trap" proposed by the World Bank (2007) can also be regarded as a turning point theory. This term refers to the situation when a country's per capita GDP reaches the middle-income range, resulting in the driving force of economic growth becoming inadequate, with the risk that per capita GDP growth could stagnate. A considerable number of developing countries achieved rapid economic growth in the earlier stage of their economic development, but are still caught in the middle-income trap. In the middle-income trap situation, the turning point in the Kuznets curve is not easy to identify, and it is difficult for per capita GDP to grow further after reaching the middle level, *E* (Figure 2).

2.3 The Two Stages of Economic Development

Economic development is a process in which GDP per capita continuously improves. It is often accompanied by the steady evolution of the endowment structure and the production mode, which among other features forms the different stages of economic development.[5] As discussed, we can divide the development process of a country into two stages (the two stages may of course partially overlap: see Figure 3):

- Stage one: digesting surplus labor. The Lewis turning point has not appeared and the economy stays on the left side of the Kuznets curve. The country is classified as a lower-income country and the production mode is transforming from being labor intensive to being capital intensive.
- Stage two: the catching-up process of technology. The Lewis turning point has appeared and the economy is on the right side of the Kuznets curve. The country has escaped the middle-income trap and the production mode is transforming from being capital intensive to being knowledge intensive.

Figure 3: Two Stages in the Development Process



Source: Author.

As digesting surplus labor often occurs through the process of more rural labor moving into the city—that is, an increasing number of workers leaving agricultural land and becoming incorporated into capital production (machines, facilities, etc.)—then, for the country as a whole, digesting surplus labor is also a process of transforming the production mode from being labor intensive to being capital intensive, steadily improving capital per capita. Second, the driving force of economic growth in the first stage is not only the input of a large amount of surplus labor brought about by capital investment, but also the inputs of technology. This combination is adequate for economic growth. In the second stage, the surplus labor is digested, so the source of per capita GDP growth is only due to the inputs of technology. This indicates that the driving force of economic growth is relatively inadequate, compared with that of the first stage. Table 1 compares these two stages.

Table 1: Comparing the Two Stages of Economic Development

	First Stage	Second Stage
Basic property	Digesting surplus labor	Catching up technology
Production mode	From labor intensive to capital intensive	From capital intensive to knowledge intensive
Lewis inflection point	Has not appeared	Has appeared
Kuznets curve	On the left side	On the right side
Driving forces of growth	Adequate	Inadequate

[5] This view of economic development is consistent with the New Structuralist approach to economic development as proposed by Lin (2009, 2012) and Ju, Lin, and Wang (2015).

Development level	From low to middle level	From middle to high level

Source: Author.

2.4 Why the Middle-Income Trap?

We have proposed that a less-developed country experiences two stages in its development process, but why is there a middle-income trap between these two stages?

First, as discussed, when surplus labor is exhausted in the second stage, the only driving force of economic growth is through technological progress or improvement in total factor productivity. Therefore, the rate of economic growth decreases when the economy enters the second stage.

Second, the pattern of technological progress is also transformed and the rate of improvement in total factor productivity is slower in the second stage than in the first. The difference in the level of development means that a developing economy can simply import and imitate technologies from developed economies in the first stage. Therefore, technological progress does not need to rely on domestic research and development (R&D). In the second stage, there is less of a difference in the level of development, and importing technology is often restricted, so technological progress must rely on domestic R&D. [6] This, therefore, makes technological progress in middle-income countries more difficult, and we believe that this is the principal reason for the middle-income trap despite the other possible reasons as discussed in literature.

The remainder of this paper is devoted to modeling the two stages of economic development, as proposed.

3. INVESTMENT BEHAVIOR

Investment is a key economic variable in the development process as it accumulates the capital stock that attracts more labor. As mentioned, this is a main feature of development, particularly in the first stage. In addition to creating capital stock (or capacity) that the labor can work with, investment also creates demand. In this section, we demonstrate that under certain conditions, there exists a time-invariant optimum capacity utilization, and investment can be understood as an adjustment to that optimum.

3.1 Technology

For a typical firm $j \in [0,1]$ in period *t*, the production technology (or the input–output relation) is assumed to be in the form of Cobb-Douglas described as follows:

$$Y_{j,t} = a(A_{j,t}L_{j,t})^{\alpha} K_{j,t-1}^{1-\alpha} \quad (1)$$

where, $Y_{j,t}$ is the output produced for *j* at *t*; $K_{j,t-1}$ is the capital stock specific to *j* measured at the end of period *t*−1, so it provides the production facility in period *t*; $L_{j,t}$

[6] See discussions of technological progress in developing countries compared with developed countries in Acemoglu, Aghion, and Zilibotti (2006); Benhabib and Spiegel (1994); Vandenbussche, Aghion, and Meghir (2006); and Gong (2015), among others. We will return to this issue in more detail later in the paper.

is the labor employed by *j*; $A_{j,t}$ is a measure of labor efficiency (or technology) in which its dynamics reflect technical progress; and a is a time-invariant parameter.

This production function is very common in the literature. Here, we use $K_{j,t-1}$ rather than $K_{j,t}$ for the production function to simply emphasize that the capital stock in period *t* is fixed. Given the fixed stock, the capacity utilization can naturally be derived as follows.

3.2 Capacity Utilization

The input–output relation expressed in (1) implies that

$$A_{j,t}L_{j,t} = \left(\frac{Y_{j,t}}{aK_{j,t-1}^{1-\alpha}}\right)^{1/\alpha}$$

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$$= Y_{j,t}\left(\frac{Y_{j,t}}{K_{j,t-1}}\right)^{\frac{1-\alpha}{\alpha}} \frac{1}{a^{1/\alpha}}$$

Define $B^{(1-\alpha)/\alpha} \equiv a^{1/\alpha}$. Thus, we find from the above that

$$L_{j,t} = \frac{Y_{j,t}}{A_{j,t}}\left(\frac{Y_{j,t}}{BK_{j,t-1}}\right)^{\frac{1-\alpha}{\alpha}} \tag{2}$$

Equation (2) can be understood as the demand function for labor given the firm's output $Y_{j,t}$, the capital stock $K_{j,t-1}$, and the labor efficiency $A_{j,t}$.[7]

Next we define the capacity utilization $U_{j,t}$ as

$$U_{j,t} \equiv \frac{Y_{j,t}}{BK_{j,t-1}} \tag{3}$$

Further expressions may be needed here to capture the economic meaning of capacity utilization $U_{j,t}$ under the Cobb-Douglas production function that allows substitution between capital and labor. Suppose that in period *t*, the production facility is represented by $K_{j,t-1}$. The production activity can thus be understood as employing workers to run the facility: the longer the facility runs, the larger the output produced. Therefore, we can define the capacity utilization $U_{j,t}$ as in equation (3), which roughly reflects the proportion of time the facility runs in period *t* (generating output $Y_{j,t}$) over the normal working time available in a period (which generates the potential output, or capacity $BK_{j,t-1}$).[8]

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[7] Note that this approach of demand for labor is different from the standard neoclassical approach in which the demand for labor is derived from the condition of the marginal product of labor equal to wage. It is, therefore, a more Keynesian approach to demand for labor.

[8] For instance, we can assume that when the capacity utilization $U_{j,t}$ is equal to 1, the facility runs for 40 hours a week. Given this capacity utilization, and the capital stock $K_{j,t-1}$ and the output $Y_{j,t}$ produced in 40 hours, we can always find a value of *B*, the capital coefficient, that makes $Y_{j,t}/(BK_{j,t-1})$ equal 1. It should be noted that the capacity utilization used here is different from the "capital utilization"

Substituting (3) into (2), we obtain

$$L_{j,t} = \frac{Y_{j,t}}{A_{j,t}}(U_{j,t})^{\frac{1-\alpha}{\alpha}} \quad (4)$$

Equation (4) simply states that the demand for labor, $L_{j,t}$, is used to run the facility to produce output $Y_{j,t}$. Thus, $L_{j,t}$ is positively determined by output $Y_{j,t}$, and negatively by labor efficiency $A_{j,t}$, and is adjusted by capacity utilization $U_{j,t}$. There is no doubt that the longer the facility runs in a given period (or the higher the capacity utilization), the more labor is needed to produce additional output.

3.3 Cost Function

Given the definition of capacity utilization $U_{j,t}$ as expressed in equation (3), we find that the production cost can also be understood as a function of $U_{j,t}$. Let $C_{j,t}$ denote the total cost in real terms for firm *j* at *t*, and let $W_{j,t}$ denote the real wage rate paid by the firm. Ignoring other intermediate inputs (such as raw materials), the total cost of the firm can be written as

$$C_{j,t} = L_{j,t}W_{j,t} + \upsilon K_{j,t-1}$$

where the labor cost $L_{j,t}W_{j,t}$ can be regarded as a variable cost (as shown below, it varies with the produced output), and $\upsilon K_{j,t-1}$ is a fixed cost, which does not vary with output $Y_{j,t}$ but with the capital stock $K_{j,t-1}$. Now, expressing $L_{j,t}$ in terms of (4), we obtain

$$C_{j,t} = \frac{W_{j,t}Y_{j,t}}{A_{j,t}}(U_{j,t})^{\frac{1-\alpha}{\alpha}} + \upsilon K_{j,t-1} \quad (5)$$

Suppose $W_{j,t} / A_{j,t} = \omega$, that is, the real wage $W_{j,t}$ increases at the same rate as the labor efficiency $A_{j,t}$. Thus, given the total cost as in equation (5), the marginal cost $C'_{j,t} \equiv \partial C_{j,t} / \partial Y_{j,t}$ and the average cost $c_{j,t} \equiv C_{j,t} / Y_{j,t}$ can be written as

$$C'_{j,t} = \frac{\omega}{\alpha}(U_{j,t})^{\frac{1-\alpha}{\alpha}} \quad (6)$$

$$c_{j,t} = \omega(U_{j,t})^{\frac{1-\alpha}{\alpha}} + \frac{\upsilon}{B}(U_{j,t})^{-1} \quad (7)$$

Above, $\omega(U_{j,t})^{(1-\alpha)/\alpha}$ can be regarded as the average variable cost (AVC) and $(\upsilon / B)(U_{j,t})^{-1}$ as the average fixed cost (AFC).

that is used in other literature. The “capital utilization” is not related to capacity or to capacity utilization, which is a key concept that helps us derive the optimum investment as in this paper.

3.4 Optimum Capacity Utilization when Capital Stock is Not Adjustable

It is useful to derive the level of capacity utilization that minimizes the average cost. From equation (7), the first-order condition for this minimization problem can be written as

$$\frac{1-\alpha}{\alpha}\omega(U_{j,t})^{\frac{1-\alpha}{\alpha}-1}-\frac{\upsilon}{B}(U_{j,t})^{-2}=0$$

Solving the equation for $U_{j,t}$, we obtain

$$U_{j,t}^{*}=\left[\frac{a\upsilon}{\omega(1-a)B}\right]^{a} \tag{8}$$

This can be regarded as the optimum capacity utilization when the capital stock is given (or not adjustable). We find that it is indeed time invariant.

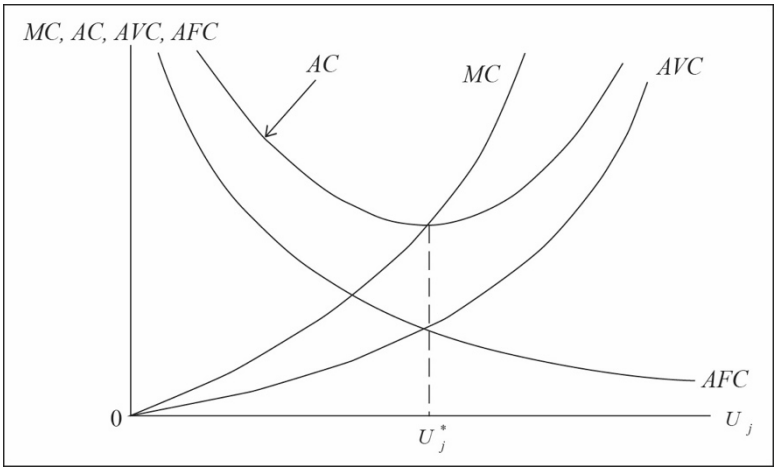
Let $C'_{j,t}=c_{j,t}$. From equations (6) and (7), we find that

$$\frac{\omega}{\alpha}(U_{j,t})^{\frac{1-\alpha}{\alpha}}=\omega(U_{j,t})^{\frac{1-\alpha}{\alpha}}+\frac{\upsilon}{B}(U_{j,t})^{-1}$$

Solving the above equation for $U_{j,t}$, we again obtain (8). Therefore, the level of capacity utilization that minimizes the average cost, as expressed in (8), is also the level at which the marginal cost cuts the average cost. Figure 4 provides the different costs as functions of capacity utilization

The above discussion appears to suggest that the standard firm theory in microeconomics with regard to the cost function still holds in terms of capacity utilization. In particular, given the capital stock, $K_{j,t-1}$, the marginal cost and the various average costs can all be expressed as a function of capacity utilization $U_{j,t}$. If $W_{j,t}/A_{j,t}=\omega$, the functions are also time invariant. This also indicates that the optimum level of capacity utilization that minimizes the average cost can be a constant (see equation (8)).

Figure 4: Marginal Cost, Average Cost, Average Variable Cost, and Average Fixed Cost as Functions of Capacity Utilization



AC = average cost, AFC = average fixed cost, AVC = average variable cost, MC = marginal cost.

Source: Author.

3.5 Optimum Capacity Utilization when Capital Stock is Adjustable

The capacity utilization expressed in (8) can be understood as the optimum capacity utilization when capital stock is given (not adjustable). Suppose now that the average cost is too high due to high capacity utilization. Investment is then needed to increase the capacity and reduce the average cost.

Investment is constructed for future capacity, so we assume that the firm has been given a sequence of expected demands, $E\{Y_{j,t+k}\}_{k=0}^{\infty}$, a sequence of technologies $E\{A_{j,t+k}\}_{k=0}^{\infty}$, and a sequence of real wages $E\{W_{j,t+k}\}_{k=0}^{\infty}$ when making an investment decision in period *t*. The investment decision problem can thus be expressed as the choice of a sequence of investments, $I_{j,t+k}$, so that

$$\max_{\{I_{j,t+k}\}_{k=0}^{\infty}} E\sum_{k=0}^{\infty}\beta^k[P_{j,t+k}Y_{j,t+k} - P_{t+k}c_{j,t+k}Y_{j,t+k} - (1+r)P_{t+k}I_{j,t+k}] \tag{9}$$

subject to

$$K_{j,t+k} = (1-d_j)K_{j,t+k-1} + I_{j,t+k} \tag{10}$$

where *E* is the expectation operator; β is the discount factor; *r* can be regarded as the interest rate that reflects the firm's opportunity cost of investment; $P_{j,t+k}$ is the price of the product produced by firm *j*; P_{t+k} is the aggregate price level; $c_{j,t+k}$ is the average cost expressed in equation (7); and d_j is the depreciation rate. Equation (10) can be regarded as the process of capital accumulation.

Proposition 1 provides the solution to this optimization problem.

Proposition 1 *Suppose* $E[W_{j,t+k} / A_{j,t+k}] = \omega$ *and* $E[P_{t+k} / P_{t+k-1}] = \pi$. Then, the problem in equations (9) and (10) with $c_{j,t+k}$ given by (7) allows us to obtain

$$U^*_{j,t+k} = U^*_j = \left(\frac{\alpha[(1+r)/(\beta\pi) - (1+r)(1-d_j) + \upsilon]}{(1-\alpha)B\omega} \right)^{\alpha} \tag{11}$$

where k =1,2,3,….

The proof of this proposition is given in the Appendix.

Equation (11) is similar to equation (8) when the capital stock is given (or not adjustable). Consider β = 1, r = 0, π = 1, and $d_j = 0$, so that we return to a one-period decision. In this case, equations (11) and (8) coincide.

3.6 Investment without Financial Constraint

Given the optimum level of capacity utilization, U^*_j, as expressed in (11), we now consider how the investment should be made. The investment carried out in period *t* creates capital stock $K_{j,t}$, which serves the capacity for period *t*+1. The optimum investment, denoted as $I^*_{j,t}$, should satisfy

$$\frac{EY_{j,t+1}}{B[(1-d_j)K_{j,t-1} + I^*_{j,t}]} = U^*_j$$

where the left side of the equation can be understood as the expected capacity utilization for period *t*+1. Resolving this equation for $I^*_{j,t}$, we obtain

$$I^*_{j,t} = \frac{E[Y_{j,t+1}]}{BU^*_j} - (1-d_j)K_{j,t-1} \tag{12}$$

Dividing both sides by $K_{j,t-1}$, we obtain

$$\begin{aligned} \frac{I^*_{j,t}}{K_{j,t-1}} &= \frac{E[y_{j,t+1}]}{BU^*_j K_{j,t-1}} - (1-d_j) \\ &= -(1-d_j) + \frac{E[y_{j,t+1}]}{U^*_j} U_{j,t} \end{aligned} \tag{13}$$

where $E[y_{j,t+1}]$ is the expected gross growth rate of product *j*.

We assume that at the very beginning of period *t*, when the investment decision is made, the firm may not observe its market demand in *t* and therefore the capacity utilization $U_{j,t}$. Here, $U_{j,t}$ in equation (13) can simply be regarded as the expected capacity utilization of $U_{j,t}$ given the information in $U_{j,t-1}$. Suppose that $E[U_{j,t}] = U_{j,t-1}$ and $E[y_{j,t+1}] = y_j$. We find that equation (13) can be rewritten as

$$\frac{I^*_{j,t}}{K_{j,t-1}} = -(1-d_j) + \frac{y_j}{U^*_j} U_{j,t-1} \tag{14}$$

This equation indicates that the investment rate $I^*_{j,t} / K_{j,t-1}$ depends on the observed capacity utilization: the higher the capacity utilization, the higher the investment rate.

This discussion enables us to obtain the following interpretation of the investment decision:

Suppose the investment is divisible. Whatever the level of expected demand $E[Y_{j,t+1}]$, the purpose of investment $I_{j,t}$ is simply to adjust the capital stock $K_{j,t}$ to the level at which $E[Y_{j,t+1}]/(BK_{j,t})$ is equal to U_j^*, as in equation (11), which minimizes the average cost of production.

3.7 Investment with Financial Constraint

Next, we consider the effect of monetary policy on the economy as this will affect investment. The investment considered in equation (14) is optimum for firm *j* if there is no financial restriction. A financial restriction may affect the investment through the interest rate and the credit supply. Consistent with the money supply rule used in this study (see equation (28) later in the paper), we consider the credit supply.

Suppose that our representative firm *j* is able to acquire a loan from a commercial bank (in real terms) of up to $\Delta M_{j,t-1}$ for its investment.[9] The firm's investment under the credit constraint can therefore be written as

$$I_{j,t} = \begin{cases} I_{j,t}^* & I_{j,t}^* < \Delta M_{j,t-1} \\ \Delta M_{j,t-1} & otherwise \end{cases} \tag{15}$$

Let ΔM_{t-1} denote the total additional money (or credit) from the commercial bank system in period *t*−1. This money supply is targeted by the monetary authority, and is the amount that a commercial bank can lend to finance the investment. Given ΔM_{t-1}, we write $\Delta M_{j,t-1}$ as

$$\Delta M_{j,t-1} = l_j \Delta M_{t-1} \tag{16}$$

where $l_j \in [0,1)$ is the proportion of total credit allocated to *j*.

Under this credit plan, the firm makes its investment decisions according to equation (15). Summing all $I_{j,t}$, we get the aggregate investment I_t:

$$I_t = \int_0^1 I_{j,t} dj \tag{17}$$

Depending on the credit ratio, l_j, assigned to the firm, we find that for some *j*, investments are bounded, that is, $I_{t-1}^* = \Delta M_{j,t-1}$; whereas for others, investments are at the optimum, that is, $I_{j,t} = I_{j,t}^*$, where $I_{j,t}^*$ is given by equation (14). Rearranging the index of the firms so the first n_1 proportion of the firms are bounded, we can write equation (17) as

$$I_t = \phi \Delta M_{t-1} + \int_{n_1}^1 \left[(1-d_j) K_{j,t-1} + \left(\frac{1+y_j}{U_j^*} \right) U_{j,t-1} K_{j,t-1} \right] dj \tag{18}$$

[9] We assume that the money supply (or credit) in period *t*−1 is used for financing the investment in period *t*.

where $\phi = \int_0^{n_1} I_j dj$. Under the identical assumption of a representative agent, the above equation can be rewritten as

$$I_t = \phi \Delta M_{t-1} + (1-d_j)\int_{n_1}^{1} K_{j,t-1} dj + \left(\frac{1+y_j}{U_j^*}\right) U_{j,t-1} \int_{n_1}^{1} K_{j,t-1} dj$$

Dividing both sides of the above equation by K_{t-1}, the aggregate capital stock, we obtain from the above

$$\frac{I_t}{K_{t-1}} = \phi \frac{\Delta M_{t-1}}{K_{t-1}} - (1-d) n_k + \frac{(1+y) n_k}{U^*} U_{t-1} \tag{19}$$

where n_k can be regarded as the proportions of capital stock from unrestricted firms:

$$n_k \equiv \frac{\int_{n_1}^{1} K_{j,t-1} dj}{K_{t-1}}$$

Here, we assume this proportion to be time invariant.

To ensure our analysis is tractable, we assume a linear relationship between the aggregate money supply and capital stock $K_{t-1} = \eta M_{t-2}$. As we are working with aggregate variables, the rationality of this linear relationship is considered more from a statistical viewpoint.[10]

Given this linear proposition, we can now rewrite our aggregate investment function (19) as

$$\frac{I_t}{K_{t-1}} = -\xi_i + \xi_u U_{t-1} + \xi_m (m_{t-1} - p_{t-1}) \tag{20}$$

where $m_{t-1} - p_{t-1} \approx M_{t-1} / M_{t-2}$ is the approximate gross growth rate of the credit supply in real terms. Note that here, m_t is the nominal growth rate of the credit supply, which is targeted by the monetary authority. The parameters ξ_i, ξ_u, and ξ_m are given by

$$\xi_i = (1-d) n_k, \ \xi_u = \frac{(1+y) n_k}{U^*}, \quad \xi_m = \phi / \eta$$

4. THE MODEL

From the investment function described in the previous section, we build our model reflecting the development process of a less-developed dual economy.

[10] Gong and Lin (2008) and Gong (2013) estimate the investment function using data from the People's Republic of China and show that the estimation is statistically significant.

4.1 The Structural Form of the Model

The model includes the following equations:

$$Y_t = I_t + C_t \tag{21}$$

$$C_t = (1-s)Y_t \tag{22}$$

$$U_t = \frac{Y_t}{BK_{t-1}} \tag{23}$$

$$K_t = (1-d)K_{t-1} + I_t \tag{24}$$

$$L_t = \frac{Y_t}{A_t}(U_t)^{\frac{1-\alpha}{\alpha}} \tag{25}$$

$$L_t^s = (1+l)L_{t-1}^s \tag{26}$$

$$N_t = \frac{L_t}{L_t^s} \tag{27}$$

$$m_t = \kappa(p^* - p_{t-1}) - m_{t-1} \tag{28}$$

$$\frac{I_t}{K_{t-1}} = -\xi_i + \xi_u U_{t-1} + \xi_m(m_{t-1} - p_{t-1}) \tag{29}$$

$$p_t = \beta_p + \beta_w w_t + \beta_a U_{t-1}, \quad \beta_p, \beta_w, \beta_a > 0 \tag{30}$$

$$w_t = \alpha_w + \alpha_p p_t + \alpha_{n,t} N_{t-1} + \alpha_x x_t, \quad \alpha_p, \alpha_{n,t}, \alpha_x > 0 \tag{31}$$

$$a_{n,t} = \begin{cases} 0 & 0 \le N_{t-1} \le N^b \\ -a + bN_{t-1} & N^b \le N < N^*, N^b = a/b \\ c & N^* \le N_{t-1} < +\infty \end{cases} \tag{32}$$

$$A_t - A_{t-1} \begin{cases} \theta_f [A_{t-1}^f (1-\varepsilon) - A_{t-1}] + \theta_a A_{t-1}, & if \ A_{t-1}^f (1-\varepsilon) > A_{t-1} \\ \theta_a A_{t-1} & otherwise \end{cases} \tag{33}$$

$$A_t^f = (1+x^f)A_{t-1}^f \tag{34}$$

Above, Y_t refers to the output, which is made up of consumption C_t and investment I_t; U_t is defined as the capacity utilization; K_t is the capital stock; L_t is employment; A_t is technology; L_t^s is the labor supply with growth rate *l*; N_t is the employment rate; m_t is the growth rate of the money supply; p_t is the inflation rate; I_t / K_{t-1} is the

investment rate; w_t is the growth rate of the nominal wage; $x_t \equiv (A_t - A_{t-1}) / A_{t-1}$ is the growth rate of technology; $\alpha_{n,t}$ is a time-varied parameter; and A^f_{t-1} is the technology in a frontier country used for comparison. All variables are assumed to be measured at the aggregate level.

Equation (21) is the definition of the aggregate output in a closed and simple economy; (22) is a consumption function with *s* the savings ratio, as usual; (23) is a definition of capacity utilization, the economic definition of which has been given in the previous section; (24) reflects the accumulation of capital stock; (25) is the demand for labor, also discussed in the previous section (see equation [4]); (26) reflects the dynamics of the labor supply, which follows a constant growth rate *l*; (27) is the definition of employment rate; (28) reflects the behavior of monetary policy targeted only at the inflation rate p^*; (29) is the behavior function of the investment rate as derived in the previous section; (30) and (31) are the dual Philips curves as discussed by Flaschel, Gong, and Semmler (2001, 2002), Fair (2000), and others. The key difference here is that we have assumed a time-varied parameter, $\alpha_{n,t}$ with behavior as given in (32), which reflects the development process of a less-developed dual economy. Finally, (33) and (34) reflect the dynamics of technology in the domestic economy and the frontier economy.

A frontier developed country must rely on its own R&D for its technological progress. Standard endogenous growth theory (or new growth theory) such as that of Romer (1990) and Lucas (1988), among others, are therefore often applied to frontier developed economies. Technological progress via R&D is often difficult as it requires both a large and relatively high-risk investment in R&D and high-quality human resources. More importantly, it also requires an institutional environment that can provide incentives and protection for R&D. Less-developed economies may find it difficult to meet such requirements, but they can take advantage of their huge distance from frontier economies in technological terms, and import technology. This type of technological progress is more economic and less risky, though only technologies no longer suitable for the frontier economy can be imported rather than the most advanced. Therefore, in equation (33), we classify the technological progress, $A_t - A_{t-1}$, of our less-developed economy into two types: one from importing technology, $\theta_f[A^f_{t-1}(1-\varepsilon) - A_{t-1}]$, the other from domestic R&D, $\theta_a A_{t-1}$. In particular, ε can be regarded as the blocking rate, indicating that as long as the distance $A^f_t - A_{t-1}$ is less than a certain proportion, εA^f_t, there will be no importing of technology; θ_f can be regarded as the import coefficient, while θ_a is the growth rate of technology from domestic independent R&D. There is no doubt that both θ_f and θ_a depend on the institutional and human capital level in the domestic economy.[11]

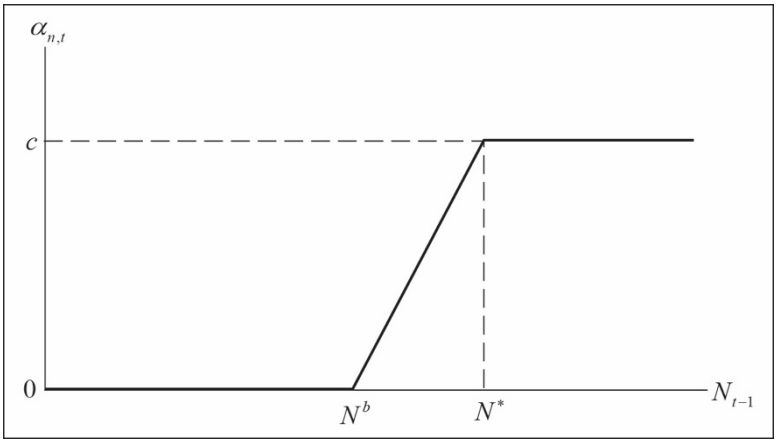
Second, in our dual Philips curves (30) and (31), we have assumed that the dynamics of price and wage are based on fairly symmetric assumptions about the causes of price and wage inflation. Both are driven by the demand pressure components U_{t-1} or N_{t-1}, and by the cost push terms measured by p_t, w_t, and x_t on the right side of (30) and (31). These price and wage dynamics are consistent with the recent New Keynesian concept of sticky pricing in the sense that they respond to market pressure, but do not necessarily clear the markets at every period.[12] The non-clearing market state appears to be more evident in developing than developed economies.

[11] See Benhabib and Spiegel (1994); Acemoglu, Aghion, and Zilibotti (2006); Vandenbussche, Aghion, and Meghir (2006); and Gong (2015), among others.

[12] We borrow the term “sticky” as the price equation we define here is similar to the New Keynesian Phillips curve, which from various sources (e.g., Gali 2008) we know can be defined as

$\pi_t = \beta E[\pi_{t+1}] + \kappa \tilde{y}_t$, where π_t is the gross inflation rate, and $\tilde{y}_t$ is the output gap, similar to our capacity utilization. If we assume that $E[\pi_{t+1}]$ is a linear projection from w_t , with w_t as the wage inflation, this will be the same as the price equation in our dual Philips curves.

Figure 5: The Curve of $\alpha_{n,t}$



Source: Author.

The economic meaning of equation (32) can be expressed as follows. In a dual economy, there is a large amount of surplus labor in rural areas. In a one-sector model, this can be regarded as a low employment rate (lower than N^b), and it is reasonable to assume that the dynamics of the wage rate, w_t, do not respond to the labor market status reflected by N_t, that is, $\alpha_{n,t} = 0$. With economic development, more surplus labor is absorbed and utilized in cities. This indicates that N_t is increasing, and when it increases to the threshold N^b, the wage rate becomes sensitive to the labor market status N_t, and thus $\alpha_{n,t} > 0$. However, we shall assume that there is an upper limit for $\alpha_{n,t}$ of *c*, when the economy is close to full employment. Figure 5 illustrates the function of $\alpha_{n,t}$.[13]

4.2 The Intensive Form of the Model

The following proposition addresses the intensive form of the model.

Proposition 2 *Let* $i_t \equiv I_t / K_{t-1}$, $h_t \equiv A_t^f / A_t$, $k_t \equiv (K_t - K_{t-1}) / K_{t-1}$ *and* $y_t \equiv (Y_t - Y_{t-1}) / Y_{t-1}$. *Then, the structural form of the model* (21)–(34) *can be reduced to the following intensive form.*

$$m_t = \kappa(p^* - p_{t-1}) - m_{t-1} \tag{35}$$

$$i_t = \xi_1 + \frac{\xi_u}{sB} i_{t-1} + \xi_m (m_{t-1} - p_{t-1}) \tag{36}$$

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 a_{n,t} N_{t-1} + \beta_3 x_t \tag{37}$$

$$N_t = \frac{1 - d + i_{t-1}}{(1 + x_t)(1 + l)} \left(\frac{i_t}{i_{t-1}} \right)^{\frac{1}{\alpha}} N_{t-1} \tag{38}$$

$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 \alpha_{n,t} N_{t-1} + \alpha_3 x_t \tag{39}$$

[13] A similar nonlinearity has been posited by Fair (2000).

$$k_t = -d + i_t \tag{40}$$

$$y_t = \frac{i_t(1-d+i_{t-1})}{i_{t-1}} - 1 \tag{41}$$

$$x_t = \begin{cases} \theta_f(1-\varepsilon)h_{t-1} + (\theta_a - \theta_f), & if\ h_{t-1} > \dfrac{1}{1-\varepsilon} \\ \theta_a & otherwise \end{cases} \tag{42}$$

$$h_t = \begin{cases} \dfrac{(1+x^f)h_{t-1}}{1+\theta_f(1-\varepsilon)h_{t-1} + (\theta_a - \theta_f)}, & if\ h_{t-1} > \dfrac{1}{1-\varepsilon} \\ \dfrac{(1+x^f)}{1+\theta_a}h_{t-1}, & otherwise \end{cases} \tag{43}$$

where $\alpha_{n,t}$ in (39) is governed by (32) and

$$\beta_0 = \frac{\beta_p + \beta_w\alpha_w}{1-\beta_w\alpha_p}, \qquad \beta_0 = \frac{\beta_u}{sB(1-\beta_w\alpha_p)}$$

$$\beta_2 = \frac{\beta_w}{1-\beta_w\alpha_p}, \qquad \beta_0 = \frac{\beta_w\alpha_x}{1-\beta_w\alpha_p}$$

$$\alpha_0 = \frac{\alpha_w + \alpha_p\beta_p}{1-\alpha_p\beta_w}, \qquad \alpha_0 = \frac{\alpha_p\beta_u}{(1-\alpha_p\beta_w)sB'}$$

$$\alpha_2 = \frac{\alpha_w + \alpha_p\beta_p}{1-\alpha_p\beta_w}, \qquad \alpha_3 = \frac{\alpha_x}{1-\alpha_p\beta_w}$$

The proof of this proposition is given in the Appendix.

The intensive form of the model appears to be highly dimensional, but the variable h_t is autonomously determined via (43). Given the determination of h_t, x_t is determined via (42), and thus enters (37), (38), and (39) as exogenous variables. The system is also recursive, in the sense that the variables such as w_t, k_t, and y_t can be derived given the dynamics in (m_t, i_t, p_t, N_t), indicating the system is four-dimensional in (m_t, i_t, p_t, N_t).

Our model involves three subsystems, which may include different dynamics and steady states depending on the labor market status N_t. If N_t is in the range of $[0, N^b)$, the system is three-dimensional in the space of (m_t, i_t, p_t), with the equations including (35), (36), and

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_3 x_t \tag{44}$$

The dynamics of (m_t, i_t, p_t), N_t, k_t, and y_t are determined by (38), (40), and (41), respectively, while

$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_3 x_t \tag{45}$$

We call this subsystem system 1.

When N_t is in the range of $[N^b, N^*)$, the system becomes four-dimensional in the space of (m_t, i_t, p_t, N_t) with the equations including (35), (36), (38), and

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 a N_{t-1} + \beta_2 b N_{t-1}^2 + \beta_3 x_t \tag{46}$$

Given the dynamics of (m_t, i_t, p_t, N_t), k_t and y_t can again be derived via (40) and (41), respectively, while w_t is determined via

$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 a N_{t-1} + \alpha_2 b N_{t-1}^2 + \alpha_3 x_t \tag{47}$$

We call this subsystem system 2.

System 3 refers to the trajectory of N_t moving in the range of $[N^*, +\infty)$. It is similar to system 2 except that (46) and (47) are replaced by

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 c N_{t-1} + \beta_3 x_t$$

$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 c N_{t-1} + \alpha_3 x_t$$

4.3 The Steady States

The derivation of the steady states of x_t and h_t are given in Section 5. Here, we simply assume that x_t has a steady state denoted as $\bar{x}$. Given $\bar{x}$, the steady states of the model in (35)–(41) are given in the following proposition.

Proposition 3 *Let* $(\bar{m}, \bar{i}, \bar{p}, \bar{N}, \bar{k}, \bar{y})$ *denote the steady states of* $(m_t, i_t, p_t, N_t, k_t, y_t)$, *respectively. Given that the steady state of* x_t *is equal to* $\bar{x}$, *we find that the system composed of* (35)–(41) *has the following economically meaningful steady states:*

1. For system 1

$$\bar{p} = p^* \tag{48}$$

$$\bar{i} = \frac{p^* - \beta_0 - \beta_3 \bar{x}}{\beta_1} \tag{49}$$

$$\bar{m} = \frac{1}{\xi_m}\left[\left(1 - \frac{\xi_u}{sB}\bar{i}\right) - \xi_i + \xi_m p^*\right] \tag{50}$$

$$\bar{w} = \alpha_0 + \alpha_1 \bar{i} + \alpha_3 \bar{x} \tag{51}$$

$$\bar{k} = \bar{y} = -d + \bar{i} \tag{52}$$

while N_t moves at the steady state according to

$$N_t = \frac{1-d+\bar{i}}{(1+\bar{x})(1+l)} N_{t-1} \tag{53}$$

2. For system 2, the steady state can be written as

$$\bar{i} = \bar{x} + l + d + \bar{x}l \tag{54}$$

$$\bar{N} = \frac{1}{2\beta_2 b}\left(a\beta_2 + \sqrt{(-a\beta_2)^2 - 4\beta_2 b(\beta_0 + \beta_3\bar{x} + \beta_1\bar{i} - p^*)}\right) \tag{55}$$

$$\bar{w} = \alpha_0 + \alpha_1\bar{i} + \alpha_2 a\bar{N} + \alpha_2 b\bar{N} + \alpha_3\bar{x} \tag{56}$$

while $\bar{p}$, $\bar{m}$, $\bar{k}$, and $\bar{y}$ are the same as in system 1.

3. For system 3, the steady state can be written as

$$\bar{N} = -\frac{\beta_0 + \beta_3\bar{x}}{\beta_2 c} - \frac{\beta_1}{\beta_2 c}\bar{i} + \frac{1}{\beta_2 c}\bar{p}$$

$$\bar{w} = \alpha_0 + \alpha_1\bar{i} + \alpha_2 c\bar{N} + \alpha_3\bar{x}$$

while $\bar{i}$, $\bar{p}$, $\bar{m}$, $\bar{k}$, and $\bar{y}$ are the same as in system 2.

The proof of this proposition is simple and thus not provided in this paper.

Several remarks should be made with regard to the proposition.

First, for system 1, we find that N_t has no steady state. It moves according to (53) when other variables, such as i_t , are at their steady states. This indicates that as long as

$$\frac{1-d+\bar{i}}{(1+\bar{x})(1+l)} > 1 \tag{57}$$

N_t will increase and thus it will eventually switch to system 2. Substituting (49) into (57), we find that condition (57) will be satisfied if

$$\frac{p^* - \beta_0 - \beta_3\bar{x}}{\beta_1} - d > \bar{x} + l + \bar{x}l$$

This indicates that the government can attempt to shift the economy into system 2 by setting a higher target of inflation rate, p^* .

Second, from (52), we have found that the gross growth rate of K_t and Y_t are equal to $1-d+\bar{i}$. Using (54) to express $\bar{i}$, we find that the gross growth rate in systems 2 and 3 is equal to $(1+\bar{x})(1+l)$. In other words,

$$1+\overline{y}=1+\overline{k}=(1+\overline{x})(1+l). \tag{58}$$

The economy should therefore eventually grow at the natural rate, which is approximately equal to the sum of the technology and the labor supply growth rates.[14]

Third, from (52) and condition (57), it is also clear that for system 1 to switch to systems 2 and 3, the growth rate of K_t and Y_t must be larger than the natural rate of growth.

Our steady state analysis has therefore shown that economic growth is gradually reduced from a higher level (higher than the natural rate of growth) to a lower level (equal to the natural rate of growth) during the development process (from system 1 to 2 and 3).

5. THE DYNAMICS OF TECHNOLOGY

In this section, we analyze the dynamics of x_t and h_t using (42) and (43). As stated earlier, the dynamics of x_t and h_t are autonomous to the whole system (35)–(43).

5.1 The Steady States of h_t

Proposition 4 *For the autonomous dynamic system (43), the steady state of h_t can be expressed as follows:*

1. *When $\theta_a < x_f$, there are two possible steady states denoted as $(\overline{h}_1, \overline{h}_2)$ with $\overline{h}_1 = 0$ and*

$$\overline{h}_2 = \frac{1}{1-\varepsilon} + \frac{x_f - \theta_a}{\theta_f(1-\varepsilon)} \tag{59}$$

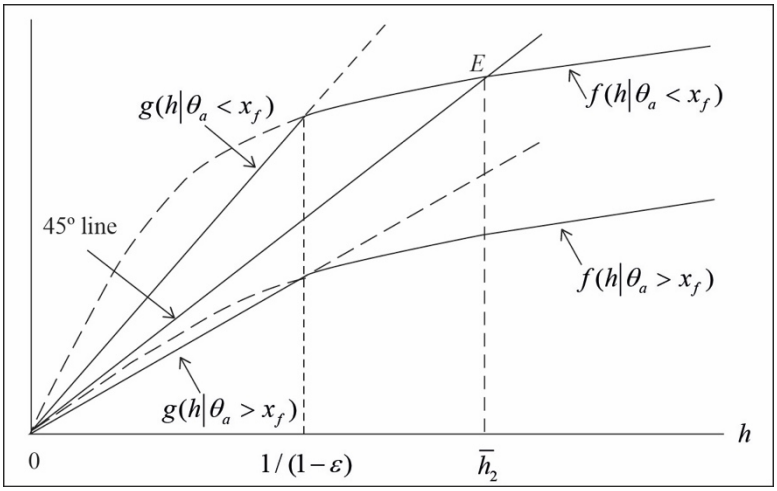
   where $\overline{h}_2$ lies in the range $(1/(1-\varepsilon), +\infty)$

2. *When $\theta_a > x_f$, the system has only one steady state, which is 0.*

The proof of this proposition is given in the Appendix, and its meaning can be expressed as in Figure 6.

[14] The definition of the natural rate of growth was first put forward by Harrod (1939).

Figure 6: The Steady State of h_t



Source: Author.

Note that the two functions *f*(*h*) and *g*(*h*) in Figure 6 are defined as

$$f(h)=\frac{(1+x_f)h}{1+\theta_f(1-\varepsilon)h+(\theta_a-\theta_f)}, \quad g(h)=\frac{1+x_f}{1+\theta_a}h$$

Therefore, equation (43) can be written as

$$h_t=\begin{cases} f(h_{t-1}), & if\ h_{t-1}>\dfrac{1}{1-\varepsilon} \\ g(h_{t-1}), & otherwise \end{cases}$$

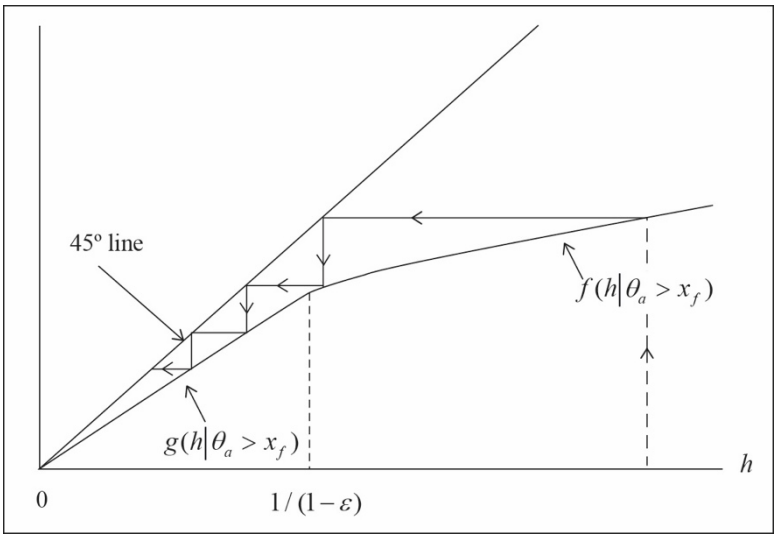
Thus, h_t, given h_{t-1}, can be completely described by two separate functions, $f(\cdot)$ and $g(\cdot)$, which intersect at $1/(1-\varepsilon)$. The dashed lines in the figure are the extensions of $f(\cdot)$ and $g(\cdot)$, but this does not reflect h_t. Depending on whether $\theta_a < x_f$, two curves of $f(\cdot)$ and two lines of $g(\cdot)$ are drawn from the bottom up, corresponding to the two situations $\theta_a > x_f$ and $\theta_a < x_f$. Thus, when $\theta_a < x_f$, there are two steady states $(\bar{h}_1, \bar{h}_2)$ as in the proposition; when $\theta_a > x_f$, there is only one steady state, which is 0, as in the proposition. Also, note that as $\varepsilon \in (0,1)$, $\bar{h}_1 > 1$ when $\theta_a < x_f$.

5.2 The Dynamics of h_t

Figures 7 and 8 provide the trajectories of h_t for the two cases of $\theta_a < x_f$ and $\theta_a > x_f$, respectively. Due to the relatively undeveloped technology, it is reasonable to assume that the initial condition of h_t, denoted as h_0, is higher than $1/(1-\varepsilon)$, as in the figures. If $\theta_a > x_f$, h_t will move to 0 as it is the only steady state (see Figure 7). If $\theta_a < x_f$, the trajectory of h_t will move to $\bar{h}_2$ (Figure 8).

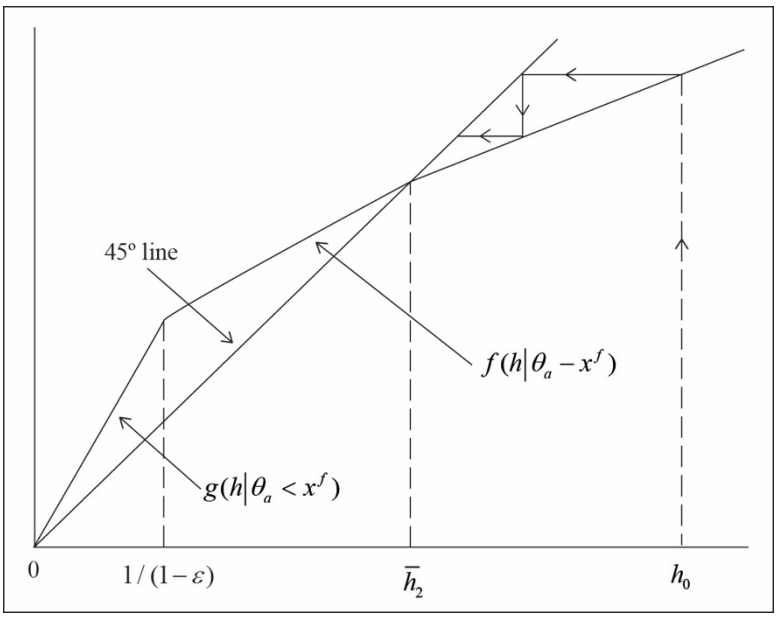
By definition, h_t is the proportion of frontier technology over domestic technology, that is, A_t^f / A_t. Thus $h_t \to 0$ indicates that the domestic technology h_t will gradually overtake the frontier technology A_t^f and continue upward. From proposition 3, however, $\bar{h}_2$ is significantly larger than 1. Thus, $h_t \to \bar{h}_2$ indicates that the domestic technology A_t is significantly lower than the frontier technology.

Figure 7: Dynamics of h_t, $\theta_a > x_f$



Source: Author.

Figure 8: Dynamics of h_t, $\theta_a > x_f$



Source: Author.

Note that whether $h_t \to 0$ or $h_t \to \overline{h}_2$ purely depends on whether $\theta_a > x_f$, or whether the growth rate of technology due to domestic R&D is larger than the growth rate of the frontier technology. We therefore find that if $\theta_a > x_f$, or the growth rate of technology due to the domestic R&D is larger than the frontier's growth rate, the less-developed country will eventually catch up with the frontier country in its technology level, or $h_t \to 0$. Otherwise, h_t will stagnate at $\overline{h}_2$, indicating that the domestic country will never catch up with the frontier country.

5.3 The Dynamics of x_t

Given the discussion of the dynamics of h_t, we are now able to examine the dynamics of x_t by relying on equation (42). We already know that if $\theta_a < x_f$, $h_t \to \overline{h}_2$. Equation (42) also means that $x_t \to \theta_f(1-\varepsilon)\overline{h}_2 - \theta_f + \theta_a$. Using (59) in Proposition 4 to explain $\overline{h}_2$, we find that $x_t \to x_f$. However, if $\theta_a > x_f$, we find from (42) that $x_t \to \theta_f$. This enables us to write the steady state of x_t as follows:

$$\overline{x} = \begin{cases} \theta_a, & if\ \theta_a > x_f \\ x_f, & otherwise \end{cases} \tag{60}$$

The economic meaning of (60) can be expressed as follows. First, for $\theta_a > x_f$, domestic technology will eventually overtake the frontier country's technology even if no technology is imported. Indeed, the importing technology will become zero as long as the domestic technology reaches a certain proportion of the frontier technology, that is, $A_t / A_{f,t} > (1-\varepsilon)$. The domestic technology will then grow at θ_a.

Next, we consider $\theta_a < x_f$. First, $\overline{x}$ cannot be higher than x_f. Consider if $\overline{x} > x_f$. In this case, domestic technology A_t will eventually surpass the frontier technology $A_{f,t}$, but when A_t surpasses $A_{f,t}$, the importing technology will equal 0, and thus x_t will be reduced to θ_a, which is less than x_f. We therefore find that $\overline{x}$ cannot be higher than x_f in the steady state. Second, $\overline{x}$ also cannot be lower than x_f. Consider if $\overline{x} < x_f$. Here, the technological gap becomes increasingly larger, so the importing technology will increase, and x_t will then increase. We thus find that $\overline{x}$ cannot be lower than x_f. All these indicate that at the steady state, $\overline{x}$ must be equal to x_f. In particular, the equilibrium condition must be the coexistence of two types of technological progress: from domestic R&D, which is θ_a, and from importing technology, which is equal to $\theta_a - x_f$.

From this and the previous discussion, we find that if $\theta_a < x_f$, that is, if the growth rate of technology from domestic R&D is less than that of frontier technology, the less-developed country will never be able to catch up with the frontier country in terms of technology, even if the growth rate is the same as the frontier country.

6. THE DEVELOPMENT PROCESS

The determination of x_t enables us to analyze the model.

6.1 The Parameters

The model has high dimensions, (4 dimensions for systems 2 and 3), so a formal mathematical analysis of the dynamic property of our model seems intractable. Therefore, we mainly rely on a numerical method to detect the dynamic property of our model. For this, we first specify the parameters used in our numerical analysis (Table 2).

Table 2: Parameters Used in the Simulation (Benchmark Case)

ξ_i	0.01	β_u	0.345417	p^*	0.03	N^b	0.7	θ_a	0.015
ξ_u	0.229063	α_w	−0.001	B	0.655967	N^*	0.9	θ_f	0.001300
ξ_m	0.060529	α_p	0.748611	s	0.4	c	0.118000	ε	0.3
β_p	−0.2	α_x	0.433629	l	0.01	α	0.66		
β_w	0.584509	d	0.8	κ	0.4	x_f	0.02		

Source: Author.

Most of the parameters used in this study are derived from Gong (2013), where the behavior functions of (29), (30), and (31) are estimated from annual Chinese data. A slight adjustment is applied to make the derived steady state more suitable. The other parameters in the table are either estimated using method of moments or simply specified when corresponding data is not available. It should be noted that although we provide some estimates (which are rather unsophisticated due to the lack of data sources in the PRC), the followed analytical result should not be sensitive to the possible bias of our estimated parameters if they are in an economically meaningful region. The benchmark case $\theta_a < x_f$ also indicates that the growth rate of technology from domestic R&D is less than that of frontier technology.

From the parameters in Table 2, the steady states are given in Table 3.

Table 3: Steady States of the Subsystems (Benchmark Case)

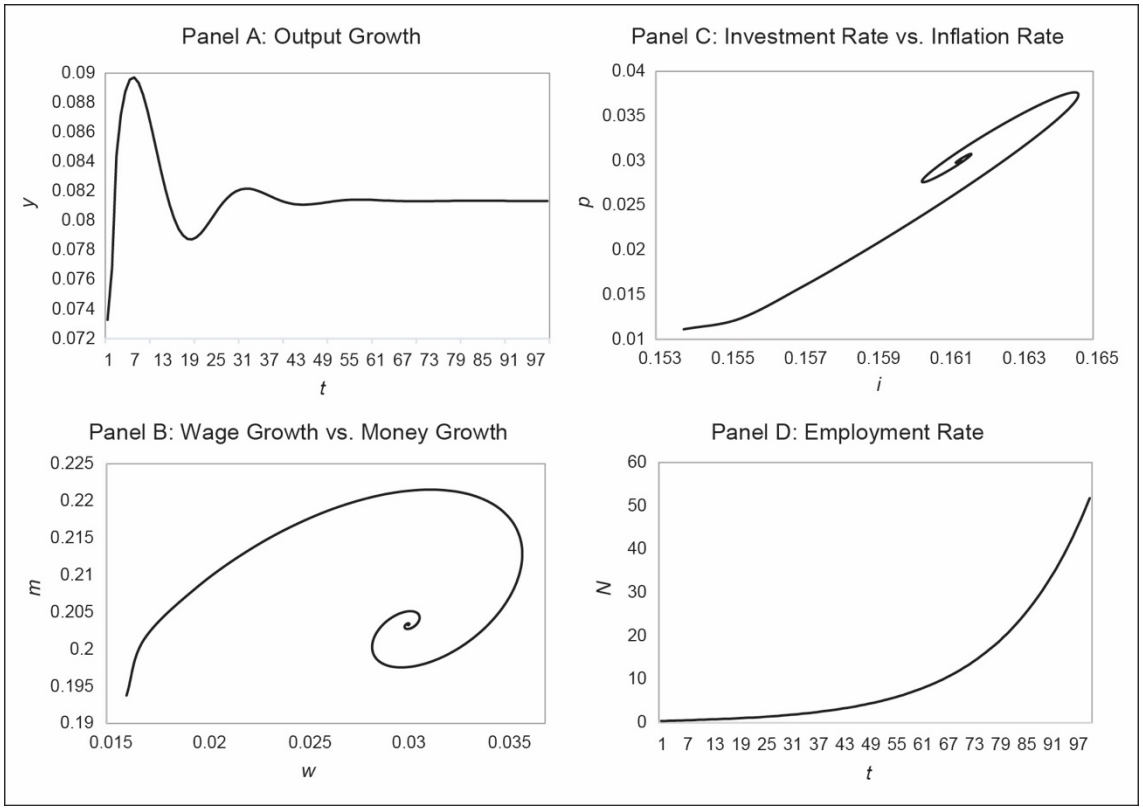
System	$\bar{i}$	$\bar{p}$	$\bar{w}$	$\bar{m}$	$\bar{N}$	$\bar{k}, \bar{y}$	$\bar{h}$	$\bar{x}$
1	0.1613	0.0300	0.0094	0.2033	N.A.	0.0813		
2	0.1102	0.0300	N.A.	0.0960	N.A.	0.0302		
3	0.1102	0.0300	0.1453	0.0960	0.976	0.0302	4.8951	0.02

Source: Author.

6.2 Simulating the Development Process

As an exercise, we first provide a simulation to system 1 when N_t is assumed to have no feedback effect on price and wage, that is, when p_t and w_t follow (44) and (45), respectively. We also assume that the growth rate of technology is fixed at its steady state, so we focus only on the issue of switching out of system 1. The initial condition is at 5% less than the corresponding steady states, while N_0 is set at 0.40. Figure 9 shows that though the system is asymptotically stable in the space of $(i_t, p_t, m_t, w_t, y_t)$, employment N_t continually increases. Therefore, the economy will eventually switch out of system 1.

Figure 9: The Dual Economy in System 1

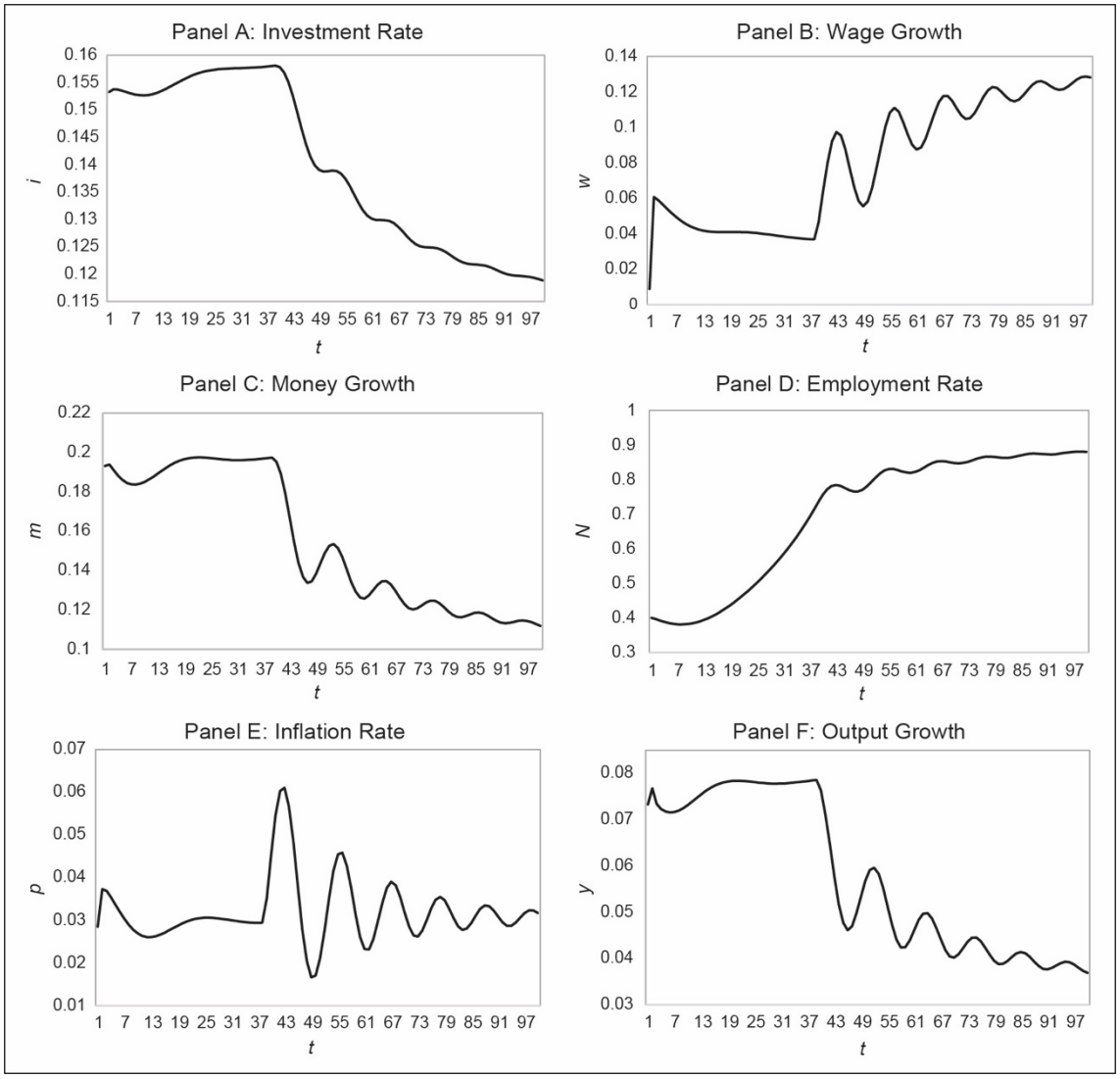


Source: Author.

Figure 10 provides the simulation for the development process, with N_t going through different ranges. The initial conditions are expressed as follows. First, we set the initial condition N_0 at 0.4, indicating that the economy begins as a less-developed dual economy. Second, we set the initial condition h_0 at 50, which represents roughly the ratio of technology in a frontier country (such as the US) to that in the PRC, when the economy moves into the convergence path (due to economic reform, for example). Given this h_0, the initial condition x_0 is set at about 0.08 by following (43). All other initial conditions are as in Figure 9.

We can see that over about 40 years, significant changes do occur in the structure of the economy: investment, money, and output growth experience a sharp decline (Panels A, C, and F, respectively); wages start to cyclically increase (Panel B); the employment increase slows down (Panel D); and the inflation rate fluctuates more (Panel E). This suggests that the second stage of economic development begins after about 40 years.

Figure 10: The Development Process



Source: Author.

6.3 Understanding the Mechanism of the Development Process

In a market economy, there are two distinctive forces that determine output, demand and supply. Given the supply level (or production capacity as defined in our model), output is determined by demand. However, if the output as determined by demand is too close to the given supply, inflation may arise, which indicates that supply only provides the possibility of economic growth, while actual growth must be realized through demand.

Taking this into consideration, we find that investment is key to the development process. As previously discussed, investment creates both demand via the multiplier and production capacity through the accumulation of capital stock (i.e., building new production lines, new plants, and new facilities). For plants and facilities to function fully, more labor must be attracted into cities, while the existence of huge levels of surplus labor in the earlier stage of economic development (i.e., in system 1) means that the labor supply does not pose a constraint. Thus, the high growth of output y_t in

the earlier stage of economic development is primarily driven by the high growth of investment i_t.

However, economic development means that increasingly more surplus labor is absorbed by cities, and therefore employment increases. When employment is above N^b, which is 0.7 in this study, it has a feedback effect on wage w_t and therefore on inflation p_t. The higher inflation rate of p_t will prompt the monetary authority to shrink the money supply, m_t, and thus have a negative effect on investment (see the investment function (36)).

Therefore, the development process can be viewed as a gradual increase in the employment rate N_t (see Panel D in Figure 10), and also as an increase in the growth rate of wages, w_t, (Panel B) and a decrease in the growth rate of the money supply, m_t (Panel C). The inflation rate, p_t, can be kept at around the target level, p^*, through monetary policy (see Panel E). These outcomes express the process of the investment rate i_t and the growth rate of output y_t from higher to lower levels, particularly after N^b is passed (see Panels A and F, respectively).

6.4 Kuznets Curve

An important and frequently discussed property of the development processes is reflected in the Kuznets curve. Kuznets (1955) discovered that in the development process of an economy, income distribution will first worsen and then gradually improve. The whole process is likely to result in a U-shaped curve. We demonstrate that our model generates a similar curve in the development process of our model economy.

Assume that we can use the wage share as a measurement of income distribution: income distribution gets worse if the wage share of the total income decreases.[15] Let ω_t denote the wage share over total income. Thus, by definition,

$$\omega_t = \frac{W_t L_t}{P_t Y_t}$$

where W_t and P_t are the wage and price levels in nominal terms. Using (25) to express L_t in the above, we obtain

$$\omega_t = \frac{W_t (U_t)^{(1-\alpha)/\alpha}}{A_t P_t}$$

Note that in (21)–(23), $U_t = i_t / (sB)$, so the above equation can further be written as

$$\omega_t = \frac{1+w_t}{(1+x_t)(1+p_t)} \left(\frac{i_t}{i_{t-1}} \right)^{(1-\alpha)/\alpha} \omega_{t-1} \tag{61}$$

Clearly, whether the wage share ω_t increases or decreases depends on how fast wages increase relative to the increase in price.

[15] This method of measuring income distribution is similar to the classical methods of Ricardo and Marx, among others. It is often called the functional income distribution.

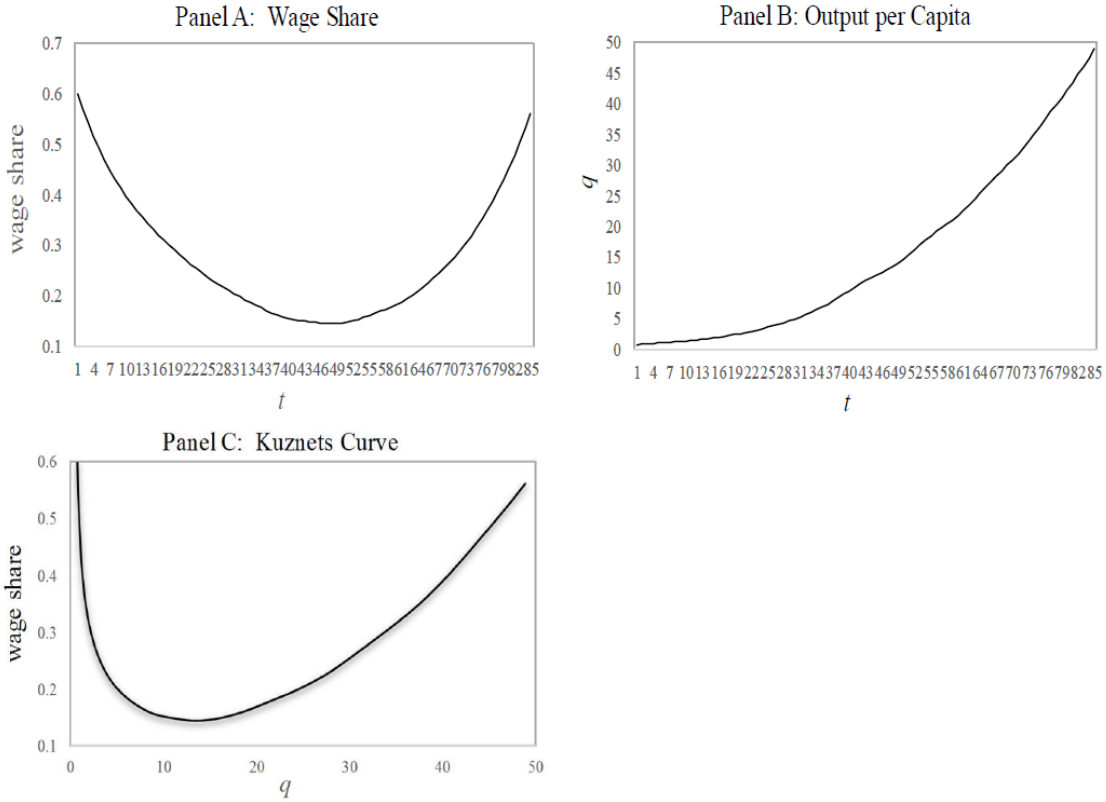
Let q_t denote the per capita output so that $q_t \equiv Y_t / L_t$. It is clear that

$$q_t = \frac{1+y_t}{1+l} q_{t-1} \tag{62}$$

where y_t is given by (41).

Figure 11 illustrates the trajectories of the wage share ω_t and per capita output q_t, as given in (61) and (62), respectively, and the Kuznets curve, with the wage share ω_t in response to per capita output q_t in the development process. The initial conditions of this exercise are the same as in Figure 10. We also find a U-shaped curve, as proposed by Kuznets (1955).

Figure 11: Kuznets Curve



Source: Author.

6.5 The Middle-Income Trap

To examine the middle-income trap, we assume that the frontier economy grows at a steady state, so that its per capita GDP q_t^f grows at x_f :

$$q_t^f = (1+x_f)q_{t-1}^f \tag{63}$$

The per capita GDP in the domestic country, q_t, is governed by (62). We define $H_t \equiv q_t^f / q_t$ so that H_t is the ratio of per capita GDP in the frontier to the developing economy. We thus can therefore obtain the mathematical definition of the middle-income trap as follows:

Definition 5 *The middle-income trap refers to the situation in which a less-developed country cannot continue to grow faster after experiencing a period of rapid growth. This makes its distance from frontier economies, in terms of GDP per capita, stop shrinking. Mathematically, this can be written as*

$$\lim_{t\to+0} H_t = \bar{H}$$

where $\bar{H}$ is significantly larger than 1.

This definition of the middle-income trap can be referred to as a relative definition.[16] Using (62) and (63) to express q_t and q_t^f in the definition, we find that

$$H_t = \frac{(1+x_f)(1+l)}{1+y_t} H_{t-1}$$

From (58), we find that if the economy successfully progresses from system 1 (the first stage of economic development), the economy will eventually grow at a natural rate, so in the long run, the dynamics of H_t follow

$$H_t = \frac{1+x_f}{1+\bar{x}} H_{t-1}$$

Applying (60) to the above equation, we obtain

$$\bar{H} = \begin{cases} 0, & if\ \theta_a > x_f \\ H^* & otherwise \end{cases}$$

Therefore in our benchmark case, $\theta_a < x_f$ so that $\bar{x} = x_f$ (see equation (60)), the trajectory H_t will rest at a certain point, H^*, which can be larger than 1. The value of H^* depends on the initial condition H_0. As H_0 represents the ratio of the per capita GDP of the frontier to the domestic economy, it is almost certain that H_0 will be high enough for the trajectory of H_t to rest at an H^* that is significantly larger than 1.[17]

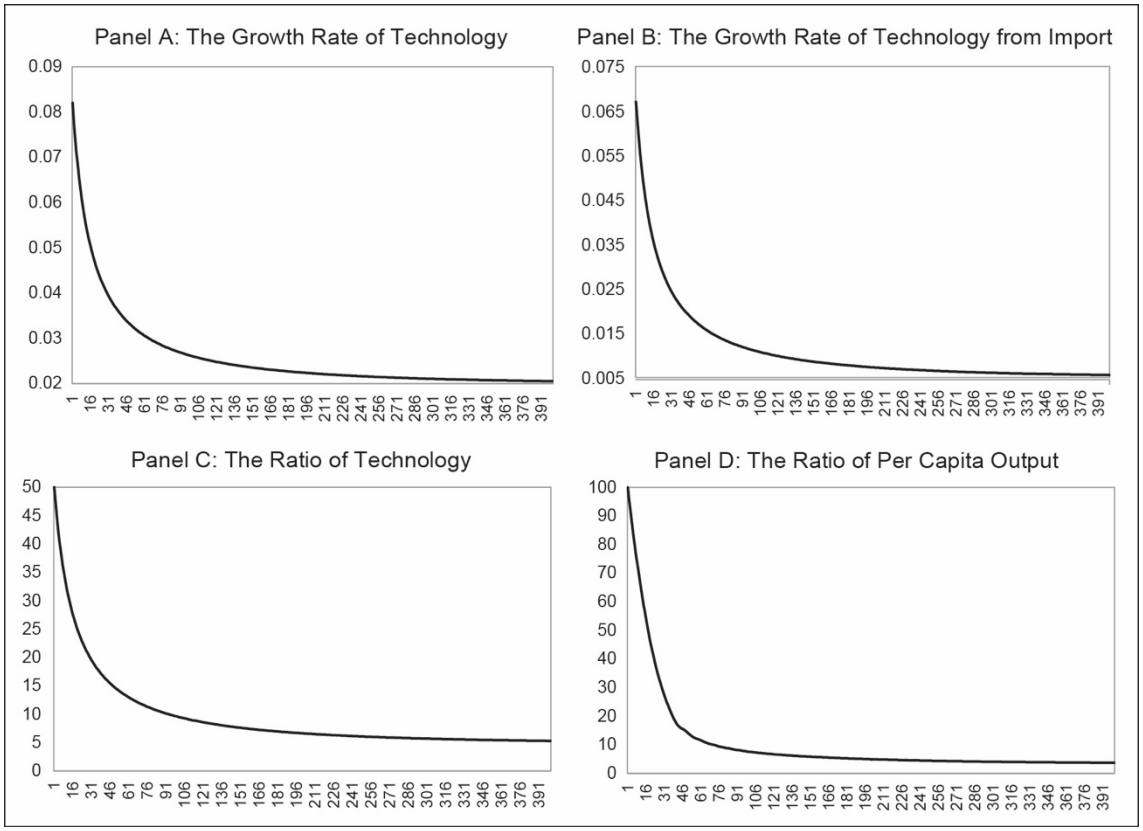
[16] Han and Wei (2015) pointed out that no absolute middle-income trap exists if the per capita GDP growth rate is positive in the long run.

[17] It is possible for the rest point to be less than 1 if the initial H_0 is less than 1 in our dynamic system.

Figure 12 illustrates the dynamics of H_t. The initial condition of this figure is the same as in Figure 10.

As Panel A of the figure shows, the initial growth rate of technology, x_t, is 8.2%, which gradually declines to its steady state of 2%, which is x_f, the growth rate of the frontier technology. The growth rate of importing technology gradually declines from its original 6.7% to 0.05% (Panel B). Therefore, the steady state $\bar{x}$ equal to 2% consists of θ_a (1.5%), plus 0.05% from importing technology. This growth pattern in technology results in the ratio of technology h_t, stopping its decline at the steady state $\bar{h}_2$, which is 4.8951 (Table 3). Given this distance in terms of technology, the ratio of per capita GDP, H_t, stops declining at 3.7. Therefore, the economy enters the middle-income trap at 3.7, relative to the frontier economy.

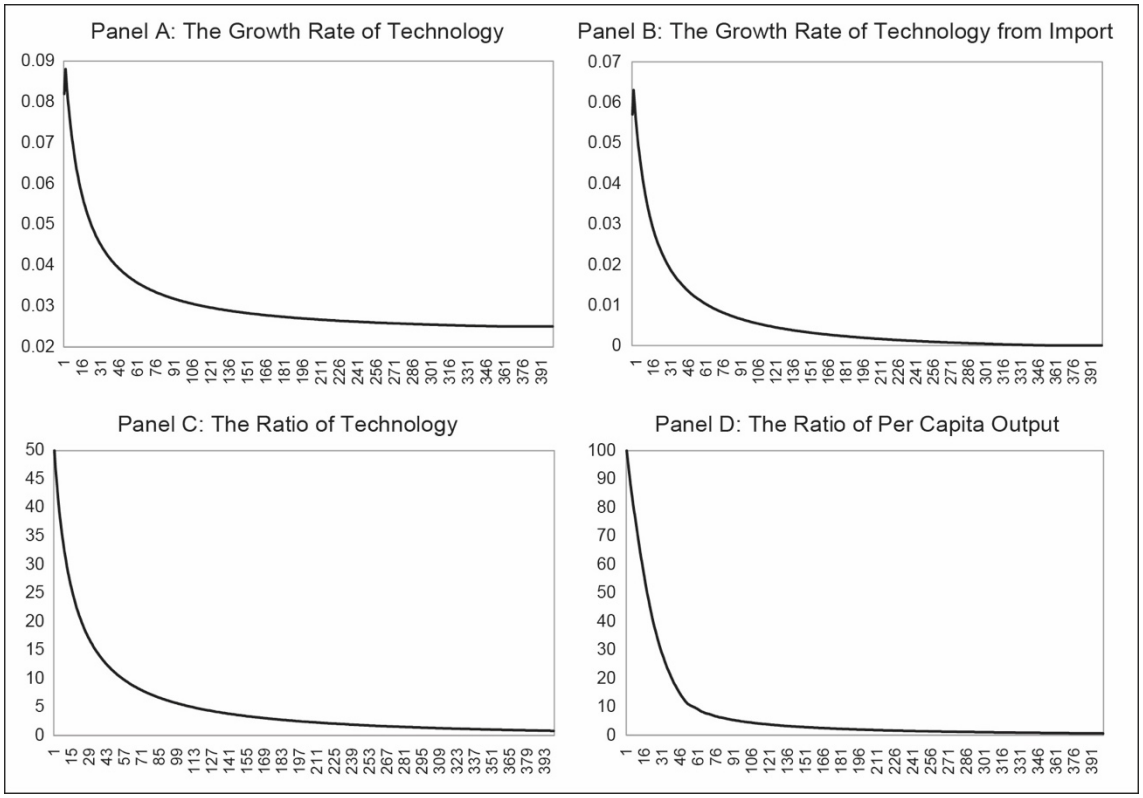
Figure 12: Middle-Income Trap (Benchmark Case)



Source: Author.

Note that this benchmark case assumes that $\theta_a = 0.015$, so it is less than x_f. If we consider $\theta_a = 0.025$, then $\theta_a > x_f$. Figure 13 illustrates the dynamics of H_f. The initial condition is the same as in Figure 12. The growth rate of technology, x_t, can be seen to stop declining at its steady state of 2.5%, which is greater than x_f (Panel A) while importing technology eventually reaches 0 (Panel B). The ratio of technology h_t and the ratio of GDP per capita, H_t, both tend to 0. Therefore, the economy moves on from the frontier state, and gets out of the middle-income trap.

Figure 13: Escaping from the Middle-Income Trap



Source: Author.

7. DISCUSSION: THE "NEW NORMAL" IN THE CHINESE ECONOMY

This study suggests that the development process of a less-developed country can be divided into two stages of economic development with significantly different properties in areas such as structural endowments, the production model, income distribution, level of development, and the forces driving economic growth. These stages have been identified in the growth theory of macroeconomics and in the various "turning point" theories in development economics, including Lewis's dual economy theory, the Kuznets curve, and the middle-income trap. We also construct a model of the development process that reveals these two stages.

It is generally assumed that the Chinese economy has now entered a "new normal" stage. Can this "new normal" be taken as a sign that the economy has completed (or almost completed) its first stage of economic development?

First, GDP per capita in the PRC reached $6,700 in 2013, which according to international standards means the PRC has already become a middle-income country.

Second, after 30 years of high growth, large-scale surplus labor in the PRC no longer exists. Recruitment difficulties have in fact occurred in the eastern coastal area. Surplus labor in the PRC has dropped from 98.00 million in 1990 to 42.67 million in 2012.[18] Although this is still a huge number compared with most other countries, the result is an overall unemployment rate of about 6.6%, lower than the rate of European countries such as France.

Third, the shortage of surplus labor has led to a faster rise in wages. Urban private sector wages have increased at a higher rate since 2011 than GDP growth, which has led to a reversal of the ratio of wage income to GDP. The PRC's income distribution has therefore begun to improve, and the turning point in its Kuznets curve has emerged.

Fourth, as a developing country, the PRC's economy has been largely characterized by capital-intensive industries. For instance, the steel industry is a typical capital-intensive industry. Chinese steel production has been the highest in the world for over 10 years, while the excess capacity is also concentrated in the steel industry.

In summary, the Chinese economy has entered the second stage of its economic development, so if we describe it as the "new normal," the term must refer to the second stage. However, moving into the second stage out of the first stage does not necessarily mean the PRC will escape the middle-income trap. As we have demonstrated, this change will depend on the PRC's rate of technological progress. It is increasingly difficult for the country to import technology, so its source of technological progress must be its own research and development. For the PRC to catch up with developed countries in terms of GDP per capita, its own technology growth rate must become higher than those of developed countries. This is the biggest challenge the PRC faces in its economic future.

[18] See Xu (2015).

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APPENDIX 1: PROOF OF PROPOSITION 1

Expressing $I_{j,t+k}$ in (9) in terms of (10) and $c_{j,t+k}$ in terms of (7), we find that the problem (9) can be rewritten as

$$\max E\sum_{k=0}^{\infty}\beta^k\left\{P_{j,t+k}Y_{j,t+k}-P_{t+k}c(U_{j,t+k})Y_{j,t+k}-(1+r)P_{t+k}[K_{j,t+k}-(1-d_j)K_{j,t+k-1}]\right\}$$

where $c(U_{j,t+k})$ denotes the average cost as expressed by (7). Note that from (3) we find that $U_{j,t+k}$ is also a function of $K_{j,t+k-1}$. In particular,

$$\frac{\partial U_{j,t+k}}{\partial K_{j,t+k-1}}=-\frac{Y_{j,t+k}}{B(K_{j,t+k-1})}=-\frac{B(U_{j,t+k})^2}{Y_{j,t+k}}$$

Therefore, the problem becomes the choice of the sequence $\{K_{j,t+k}\}_{k=0}^{\infty}$. The Euler equation for this problem can be written as

$$E\beta^k P_{t+k}\left[\frac{1-\alpha}{\alpha}\omega(U_{j,t+k})^{\frac{1-\alpha}{\alpha}}B(U_{j,t+k})^2-\upsilon(U_{j,t+k})^{-2}(U_{j,t+k})^2+(1+r)(1-d_j)\right]$$
$$-P_{t+k-1}(1+r)\beta^{k-1}=0$$

which can further be simplified as

$$E\beta\pi\left[\frac{1-\alpha}{\alpha}(U_{j,t+k})^{\frac{1}{\alpha}}B-\upsilon+(1+r)(1+d_j)\right]=1+r$$

This equation allows us to obtain

$$(U_{j,t+k})^{\frac{1}{\alpha}}=\frac{(1+r)/(\beta\pi)-(1+r)(1-d_j)+\upsilon}{\frac{1-\alpha}{\alpha}B\omega}$$

We therefore prove the proposition.

APPENDIX 2: PROOF OF PROPOSITION 2

First, equation (35) in the proposition is simply the same as (28). From (21) and (22), we first obtain

$$Y_t = \frac{1}{s} I_t. \tag{64}$$

Substituting (64) into (23) while recognizing that $i_t \equiv I_t / K_{t-1}$, we obtain

$$U_t = \frac{1}{sB} i_t. \tag{65}$$

The investment function (29) can thus be rewritten as (36) as in the proposition. Next, expressing w_t and p_t in (30) and (31) in terms of (31) and (30) while using (65) to express U_t, we obtain

$$p_t = \beta_p + \beta_w(\alpha_w + \alpha_p p_t + \alpha_{n,t} N_{t-1} + \alpha_x x) + \frac{\beta_u}{sB} i_{t-1}$$

$$w_t = \alpha_w + \alpha_p \beta_p + \beta_w w_t + \frac{\beta_u}{sB} i_{t-1} + \alpha_{n,t} N_{t-1} + \alpha_x x$$

Reorganizing the above equations, we obtain (37) and (39) as in the proposition.

Next, we derive (38) in the proposition. Expressing L_t in (27) in term of (25), we obtain

$$N_t = \frac{Y_t}{A_t L_t^s} (U_t)^{\frac{1-\alpha}{\alpha}} = \frac{(1+y_t) Y_{t-1}}{(1+\bar{x})(1+l) A_{t-1} L_{t-1}^s} (U_t)^{\frac{1-\alpha}{\alpha}}$$

Dividing both sides by N_{t-1} while expressing N_{t-1} in the right side in terms of

$$\frac{Y_{t-1}}{A_{t-1} L_{t-1}^s} (U_{t-1})^{\frac{1-\alpha}{\alpha}}$$

we find that

$$\frac{N_t}{N_{t\text{-}1}} = \frac{(1+y_t) Y_{t-1} A_{t-1} L_{t-1}^s}{(1+\bar{x})(1+l) A_{t-1} L_{t-1}^s Y_{t-1}} \left(\frac{U_t}{U_{t-1}}\right)^{\frac{1-\alpha}{\alpha}} = \frac{(1+y_t)}{(1+\bar{x})(1+l)} \left(\frac{U_t}{U_{t-1}}\right)^{\frac{1-\alpha}{\alpha}}$$

Error! Bookmark not defined. Substituting (65) into the above, we obtain

$$\frac{N_t}{N_{t\text{-}1}} = \frac{(1+y_t)}{(1+\bar{x})(1+l)} \left(\frac{i_t}{i_{t-1}}\right)^{\frac{1-\alpha}{\alpha}}. \tag{66}$$

Next, we need to derive y_t. For this, we first divide both sides of (64) by Y_{t-1}. This gives us

$$1 + y_t = \frac{I_t}{sY_{t-1}}$$

Note that in the above equation, we can express I_t in terms of $i_t K_{t-1}$. Therefore,

$$1 + y_t = \frac{i_t K_{t-1} / K_{t-2}}{BsY_{t-1} / (BK_{t-2})} = \frac{i_t (1 + k_{t-1})}{BsU_{-1}}$$

where $k_t \equiv (K_t - K_{t-1}) / K_{t-1}$ is the growth rate of the capital stock. From (24), we find that

$$k_t = -d + i_t.$$

This is the equation (40) as in the proposition. Expressing k_{t-1} in terms of $-d + i_t$ and U_{t-1} in terms of (65), we obtain

$$1 + y_t = \frac{i_t (1 - d + i_{t-1})}{i_{t-1}} \tag{67}$$

This is indeed the equation (41) as in the proposition. Substituting (67) into (66) and reorganizing, we obtain (38) as in the proposition.

Finally, we derive (42) and (43) as in the proposition. By definition, $h_t \equiv A_t^f / A_t$. Thus when $A_{f,t-1}(1-\varepsilon) - A_t$, $h_t > 1/(1-\varepsilon)$. Therefore, the two conditions in (33) and in (42)–(43) are equivalent. Using (34) and the first half of (33) to express h_t , we obtain

$$\begin{aligned} h_t &= \frac{(1 + x_f) A_{t-1}^f}{\theta_f [A_{t-1}^f (1-\varepsilon) - A_{t-1}] + (1 + \theta_a) A_{t-1}} \\ &= \frac{(1 + x_f)\, \mathrm{h}_{t-1}}{\theta_f [A_{t-1}^f (1-\varepsilon) - 1] + (1 + \theta_a)} \end{aligned}$$

This is the first half of equation (43) in the proposition. The second half of equation (43) comes directly from the second half of (33) and (34). Similarly, the first half of (33) allows us to obtain

$$x_t = \theta_f [h_t (1-\varepsilon) - 1] + \theta_a A_{t-1}$$

This is the first half of (42) in the proposition. The second half of (42) in the proposition directly comes from the second half of (33). We thus prove the proposition.

APPENDIX 3: PROOF OF PROPOSITION 3

We first consider the range $h_t > 1/(1-\varepsilon)$ so we can use the first half of (43) to derive the steady state. Assume $h_t = h_{t-1} = \bar{h}$. Thus, from (43), we obtain

$$1 = \frac{1+x^f}{1+\theta_f(1-\varepsilon)\bar{h} + (\theta_a - \theta_f)}$$

Solving the above equation for $\bar{h}$, we obtain

$$h = \frac{1}{1-\varepsilon} + \frac{x^f - \theta_a}{\theta_f(1-\varepsilon)}$$

This is indeed $\bar{h}_2$, which is within the range $(1/(1-\varepsilon), +\infty)$ if $\theta_a < x^f$.

Next, we consider the range $h_t \le 1/(1-\varepsilon)$. In this range, the dynamics of h_t are governed by the second half of (43). It is clear that the only steady state is 0 if $\theta_a \neq x^f$. Otherwise, any number in the range $[0, 1/(1-\varepsilon)]$ can be a steady state at which $h_t = h_{t-1}$ is satisfied.