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**Working Paper**

## Welfare optimal reliability and reserve provision in electricity markets with increasing shares of renewable energy sources

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May 2017

UNIVERSITÄT  
DUISBURG  
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# Welfare optimal reliability and reserve provision in electricity markets with increasing shares of renewable energy sources

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## Abstract

We develop an analytical model to derive the competitive market equilibrium for electricity spot and reserve markets under stochastic demand and uncertain renewable electricity generation. We then derive the welfare-optimal provision of reserves. At first-best, cost of reserve capacity is balanced against expected cost of outages. The first-best market equilibrium of the model implies an increase of reserve provision with a growing share of renewable generation. Furthermore, a growing share of renewable generation decreases the level of reliability as measured in energy not served. Additionally, required reserves to balance higher expected deviations will be more expensive, resulting in a trade-off between higher reserve costs and costs of energy not served.

Keywords: Renewable Energy Sources, Electricity Reserves, Reliability, Electricity Market, TSO. JEL codes: D81, L52, Q41, Q42

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# 1 Introduction

Power systems with increasing shares of fluctuating renewable in-feed experience higher variability and uncertainty of generation than electricity systems based on conventional generation alone. This increased variability challenges the reliability of electric power systems. One option to balance demand and supply comes from reserve power markets (Dany 2001; Weber 2010), which allow addressing different sources of uncertainty. However, provision of reserves comes at a cost and therefore there is a need to identify the welfare-optimal amount of reserve provision in the presence of fluctuating renewables. The methodology for answering this need is the focus of our paper.

One of the main tasks of the Transmission System Operator (TSO) is to maintain the balance of supply and demand in the system at all times. As TSOs do not own generation capacity, they need to acquire the necessary reserves externally. There are three main models used for that: mandatory provision of reserve capacity by generators (with or without compensation), bilateral contracts between generators and TSOs or one-sided procurement auctions often called reserve power or reserve capacity market (Just and Weber 2008). In Europe the trend is currently to procure reserve capacity in organized reserves markets (Mott MacDonald 2013).

Reserve power markets are closely linked with electricity spot markets, and therefore the two markets should be analyzed jointly. In this paper, we therefore consider an electricity spot market where part of the electricity supply is intermittent due to generation with renewable energy sources (RES), such as wind or solar, and a reserve power market on which the TSO commissions reserve power from conventional power suppliers. We develop a stylized analytical model with the aim to derive the competitive market equilibrium for the electricity spot market and the reserve power market under stochastic demand and uncertain renewable electricity generation. From this equilibrium we then derive the welfare-optimal provision of reserves. To illustrate the model and provide some more intuition for the results we present and discuss a numerical example based on the German electricity system.

The purpose of the paper is to develop a methodology to determine the level of reserves that maximizes social welfare. We derive the first-best quantity of reserves to be commissioned by the TSO acting in lieu of a benevolent social planner. This is neither given naturally nor perfectly implemented by existing regulatory schemes. As a consequence, optimal reserves according to the model do not necessarily predict real-world outcomes yielded by current TSO behavior, which may or may not coincide with the social optimum. Rather than providing estimates for the actual reserve need this approach is meant to illustrate the trade-off between cost of reserves and

cost of service interruptions as a consequence of insufficient reserve provision. The approach can be used, e.g. by TSOs and regulators, in the development of appropriate models for reserve provision. Therefore, it complements empirical approaches on estimating reserve needs such as Bucksteeg et al. (2016) and Bruninx and Delarue (2015).

The paper is organized as follows: Section ?? describes the functioning of the German reserve market design and provides a review of the relevant literature. Section ?? introduces the model with the market equilibrium conditions, real-time outcomes, welfare-optimal reserve provision and comparative statistics in subsequent subsections. The simulation results are described in Section ?? and Section ?? concludes. The appendix provides proofs for propositions (??).

## 2 Context

Reserve power markets differ from electricity spot markets in several aspects. Certain design issues are crucial for deriving the welfare-optimal reserve provision. An example is the two-part compensation scheme rewarding both capacity provision and actual delivery. This section presents relevant aspects of reserve markets using as an example the German case on which the numerical illustration is based. Recently, reserve markets have received attention in the economic literature. Section ?? introduces the associated field of research and identifies the context for this work.

### 2.1 German reserve market design

In Germany as well as in other continental European electricity markets<sup>1</sup> three qualities of reserves are differentiated with respect to technical and economic characteristics. These are primary, secondary and tertiary (or minute) reserves and comprise positive and negative reserve power each.<sup>2</sup> Technically, these qualities differ according to activation and response speed. Primary and secondary reserve power is activated automatically and immediately after a disturbance in the system occurs and is obliged to be fully available after 30 seconds and 5 minutes, respectively. This short response speed necessitates primary and secondary reserve power to be provided by spinning reserves, i.e. power plants which are online. In contrast,

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<sup>1</sup> The continental European countries are organized in one (formerly Union for the Co-ordination of Transmission of Electricity, UCTE) of the five regional groups forming the European Network of Transmission System Operators for Electricity (ENTSO-E).

<sup>2</sup> Positive reserves means being able to ramp up whereas negative reserves means to ramp down electricity generation in case of occurring imbalances.

tertiary reserves have to reach full capacity after 15 minutes and, therefore, may also be provided from non-spinning power plants.

Primary reserves are scheduled in order to stabilize system frequency and are relieved by secondary reserves when being fully deployed within 5 minutes. The main objective of secondary reserves is to balance fast-changing deviations such as fluctuation from RES and load, schedule shifts and power plant failures. As the more economic alternative, tertiary reserves are intended to counter larger, longer lasting imbalances, such as power plant failures and forecast errors, and serve to support and restore secondary reserves.

The dimensioning of reserves also differs between qualities. Primary reserve capacity is jointly sized for the regional group of the continental European synchronously interconnected system (3,000 MW) and is symmetric with respect to positive and negative reserves - it is also procured as a joint product for positive and negative reserve. It is dimensioned to securely control two simultaneously occurring reference incidents being described as the largest expected power imbalance due to a single cause. The determined capacity is allocated to the countries in relation to their proportional electricity generation. For Germany the primary reserve amount is relatively constant around 600 MW. Since ENTSO-E is less stringent for secondary and tertiary reserve, practical dimensioning differs significantly among the European TSOs. The German TSOs apply a probabilistic method, the so-called Graf-Haubrich method, which dimensions control reserves to be sufficient to completely balance the system in all but a few hours a year; for more details see Just (2015) and Bucksteeg et al. (2016). TSOs base the dimensioning on the corresponding quarters of the previous four years to account for seasonal dependencies. Exemplary reserve quantities are 1,973 MW positive and 1,904 MW negative secondary reserves and 2,779 MW positive and 2,006 MW negative tertiary reserves as tendered in the second quarter of 2016 (German TSOs 2016). The dimensioning methodology is exclusively based on electricity quantities of potential imbalances and does not consider any economic factors such as costs associated with reserve procurement or energy not served (ENS).

TSOs procure the five different products of reserves (primary, positive and negative secondary and positive and negative tertiary reserves) in separate auction markets by competitive tendering. Primary and secondary reserves are tendered on a weekly basis whereas tertiary reserves are tendered daily. The products further differ with respect to delivery periods. Primary reserves have to be provided for a weekly period whereas secondary reserves are separated into two time slices per week, and tertiary reserves even into six time slices per day. For secondary reserves,

peak (Monday to Friday from 8 a.m. to 8 p.m.) and off-peak periods are differentiated. Tertiary reserves are provided for six different periods a day of four hours each; for details see Swider 2006; Swider 2007.

Bids for providing primary reserve power are composed of a quantity and a reservation price whereas the actual use is not rewarded. This is different for secondary and tertiary reserves, where bids specify a quantity, a reservation (or capacity) price and an energy price for deployment. Bidders are selected for providing reserves by the merit order of reservation price only and paid according to their individual reservation price bids (pay-as-bid). The selection of reserve deployment considers the merit order of energy prices in a similar manner.

Further information on the (German) control power market is provided in Con-sentec (2014) and Just (2015).

## 2.2 Literature review

The determination of adequate reserves needed to balance the demand and supply of electricity is an important topic and has attracted a lot of attention and research. Indeed, there is a vast body of literature discussing increased levels of intermittent energy and their effect on the reserve power market. Among others, Holttinen et al. (2012) and De Vos et al. (2011) discuss methods used in wind integration studies to indicate emerging trends (Holttinen) and review examples of operating practice in Belgium (De Vos et al.); for the Nordic countries see Gebrekiros et al. (2015) and for Germany Just (2015).

For the procurement of reserves reserve power markets are increasingly important (Dany 2001; Weber 2010). However, reserve power markets are closely linked with electricity spot markets, and therefore should not be investigated in isolation. An individual plant owner can choose between producing into the spot market or keeping the capacity available for reserves. Thus, the plant owner faces opportunity costs when bidding into the reserve power market. The resulting indifference condition between the spot and reserve market is used by Just and Weber (2008) who set up a model for secondary reserve capacity and spot electricity markets and derive the price of reserves under equilibrium conditions. Just (2011) develops the Just and Weber (2011) model further and focuses on the implications of changing delivery periods for primary and secondary reserves in Germany.

In this paper, we develop a model that uses a similar indifference condition as in Just and Weber (2008) and Just (2011) where we note that the marginal unit in the spot market does not face opportunity costs from providing capacity for the reserve market. A similar intuition is offered by Müsgens et al. (2014) who distinguish infra-

and extramarginal power plants and compare the reserve costs they face with the spot price depending on the group to which suppliers belong. In market simulation models based on cost minimization employed by Bucksteeg et al. (2016), similar mechanisms of price formation may be observed.

The engineering literature, studying different unit commitment (UC) models, considers the issue of adequate reserve dimensioning, often introducing probabilistic reserve sizing techniques as more adequate to capture the stochastic nature of RES (Liu and Tomsovic 2012; Bruninx and Delarue 2015). Ortega-Vazquez and Kirschen (2007) include levels of optimal spinning reserves, determined with the use of cost-benefit analysis, as constraints in reserve-constrained UC. The uncertainty of wind power is subsequently also included in their model Ortega-Vazquez and Kirschen (2009). Jost et al. (2015) and Bucksteeg et al. (2016) propose dynamic reserve sizing approaches in order to ensure a steady reliability level through time-adaptive reserves.

With respect to reserve power market design, various aspects have also been discussed in the literature. A strand of literature investigates coordinated bidding in the reserve and spot markets under market structures characterized by different degrees of competition. Wen and David (2002) and Attaviriyanupap et al. (2005) discuss the case of competitive suppliers bidding separately into day-ahead and reserve markets. Swider (2007) also analyzes competitive spot markets but assumes strategically behaving players, who are interested in profit maximization, bidding into reserve markets. He analyzes simultaneous bidding in the day-ahead and reserve auctions and we also follow this approach in our model.

Some authors have focused primarily on the adequacy of the German reserve market design. Just (2011) investigates the contract duration and concludes that the design of the market, where the contract duration has been shortened to one month, is still inefficient and recommends even shorter durations of contracts in order to improve market results. The design of the reserve market is also considered by Müsgens et al. (2014) who use the two-part bids model for electricity markets developed by Chao and Wilson (2002) and adapt it for an investigation of the scoring and settling rules. They conclude that the current pay-as-bid system should be changed and replaced by uniform pricing as “pay-as-bid is not the preferred choice for balancing power markets”. They, however, recommend that the rule used on the German market for determining the “winning bids” (“the scoring rule”) based on the capacity price is kept as it leads to market efficiency. Wieschhaus and Weigt (2008) are also interested in the comparison of the impact of different pricing regimes on the German reserve market (discriminatory vs. uniform). They set up two



market equilibrium models: a Cournot model of spot and reserve markets, both with uniform pricing, and a Bayesian approach for the sequential market clearing process with discriminatory pricing on the reserve market. They find that more competitive balancing markets result in a drop in spot market prices.

Despite this considerable body of literature, there is to our knowledge no publication deriving the optimal sizing of reserves within an analytical approach of welfare maximization. This is consequently the main purpose of the subsequent analysis.

### 3 Methodology

We develop our methodology in several steps. First, we set up the two-stage analytical problem consisting of welfare maximization with respect to the level of reserves subject to generation cost minimization (Section ??). Via backward induction, we then derive the equilibrium spot and reserve power prices resulting from solving the cost minimization (Section ??). In the subsequent section (Section ??), we calculate generation costs for different cases of realized uncertainties as input for solving the welfare-optimal reserve provision (Section ??) and conduct comparative statics in order to investigate the influence of different parameters on optimal reserve levels.

#### 3.1 General model formulation

We consider a spot market for electricity and a reserve power market for the procurement of reserve capacity. On the spot market, electricity is traded whereas on the reserve market the provision of capacity is bid. Part of the electricity delivered on the spot market is generated from RES.

The agents in our model are consumers, RES and conventional producers as well as the transmission system operator (TSO). We assume perfect competition of suppliers in all markets and price-inelastic demand on the spot market. On the reserve power market, the TSO is in the position of a monopsonist, yet behaves like a benevolent social planner who aims to maximize expected social surplus. Hence, the objective is to derive the optimal amount of positive and negative reserves the TSO shall procure from a social-welfare perspective, i.e. to size the optimal reserve requirements.

In addition to RES, there exists a conventional generation park in which generation capacity is characterized by marginal generation costs. Overall generation capacity (both from RES and conventional generation plants) is ordered by increasing marginal generation costs to form a so-called merit order or supply stack.<sup>3</sup> We

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<sup>3</sup> The regulation of some countries, e.g. Germany, prioritizes in-feed from RES. However,

describe this conventional generation capacity as a continuous cost function assigning marginal generation costs to generation capacity  $K$  depending on the position within the merit order. This marginal cost function is known to all market participants. Generation capacity is assumed to be perfectly reliable and available at all times, yet production is limited by the natural conditions regarding RES capacity. In-feed from renewables  $k_{RES}$  is subject to uncertainty, forecasted (or expected) in-feed is denoted by  $K_{RES}$ .

Similar to generation from RES, demand is forecasted; the forecast is denoted by  $H$ . Both forecasts are subject to errors  $\varepsilon_h$  and  $\varepsilon_{RES}$  which have to be settled in real time. These errors are assumed to be statistically independent, with zero mean, so  $E[h] = H$ , and  $E[k_{RES}] = K_{RES}$ . Forecasted residual demand - i.e. demand after RES supply has been netted out - is denoted by  $D = H - K_{RES}$ . Realized residual demand  $d$  is the difference between realized demand  $h$  and realized RES in-feed  $k_{RES}$ :

$$d = h - k_{RES} \quad (3.1)$$

where  $h = H + \varepsilon_h$  and  $k_{RES} = K_{RES} + \varepsilon_{RES}$ . Clearly,  $E[d] = D$ . The cumulative distribution function of  $d$  is denoted by  $F_d$ . Its probability density,  $f_d = F'_d$  is assumed to be continuous and only strictly positive for positive values of  $d$ .<sup>4</sup> It follows that  $F_d$  is positive and increasing on the positive real axis.

In reality, in addition to load and RES generation forecast errors, more uncertainties and resulting deviations from scheduled quantities are present, such as power plant failures, schedule shifts, and noise from RES generation and load. These are also considered in the current practice of reserve dimensioning (cf. Section ??). Yet the primary objective of this paper is to develop a novel methodology to determine the socially optimal reserve requirement rather than reproducing actual market outcomes. Therefore, for the sake of analytical (and also numerical, cf. Section ??) feasibility and for enabling fundamental insights, the uncertainties explicitly considered in our model are limited to those associated with RES and demand.

The TSO is responsible for secure and adequate system operation. In order to ensure system reliability while facing uncertainties from electricity generation and demand, the TSO procures reserve energy in advance. In this model we only allow spinning reserves, which means that reserve energy can exclusively be provided

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marginal generation costs close to zero should ensure utilization of renewable energy before committing conventional generation, which has positive marginal costs, by market forces.

<sup>4</sup> I.e., there is zero probability of negative residual demand values. This assumption simplifies the analysis, but could easily be relaxed.

from online generation units.<sup>5</sup> In principle, every conventional generation unit can provide reserve capacity which is limited to a maximum share resulting from up and down ramping constraints. While we model ramping constraints, ramping costs are neglected. Furthermore, conventional plants have a minimum stable operation limit.

We assume that provision of reserves is procured at the same time as the spot market is cleared. Conventional electricity producers can act both on the spot market and on the reserve power market as long as reserves are provided from capacity online. The spot market clears at a uniform price which, due to the assumption of perfect competition, corresponds to the marginal generation cost. As regards reserves procurement, power plants are selected according to their reservation price bids in increasing order.<sup>6</sup>

This setting enables the investigation of the interrelations between the electricity spot and the reserve power market. More specifically, the simultaneous market clearing of both markets and the fact that conventional capacity (potentially) utilized in these markets is supplied by the same group of producers constitute an appropriate basis in order to understand principles of these interrelated markets and to gain insights from a social welfare perspective.

The amount of positive/negative reserves procured is denoted by  $R^{+/-}$ . Positive reserves will be utilized when  $d \geq D$ ; since  $d - D = \varepsilon_h - \varepsilon_{RES}$  this is equivalent to  $\varepsilon_h - \varepsilon_{RES} \geq 0$ .<sup>7</sup> Conversely, negative reserves will be utilized when  $d < D$  or  $\varepsilon_h - \varepsilon_{RES} < 0$ . The quantity of energy produced out of positive reserves is given by  $r^+ = \min[R^+, \varepsilon_h - \varepsilon_{RES}]$  and the quantity of energy saved out of negative reserves is given by  $r^- = \min[R^-, \varepsilon_{RES} - \varepsilon_h]$ . Energy not supplied (ENS)  $\delta$  is positive when positive reserves are not sufficient to cover positive residual demand:

$$\delta = \max[0, \varepsilon_h - \varepsilon_{RES} - R^+]. \quad (3.2)$$

On the other hand, dumping (curtailment) of renewable production  $\rho$  occurs when the forecast error for renewables minus the demand forecast error exceeds the quantity of negative reserve available:

$$\rho = \max\{0; \varepsilon_{RES} - \varepsilon_h - R^-\}. \quad (3.3)$$

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<sup>5</sup> This condition is valid for primary and secondary reserves in the German reserve market design. For providing tertiary reserves, generation units can be online or offline. We abstract from differences of these reserves with respect to activation and response speed as existent e.g. in the German reserve power market (cf. Section ??).

<sup>6</sup> Whether the reserve market uses pay-as-bid (as currently in Germany) or uniform price clearing, makes thereby neither a difference in the bid selection nor in the marginal cost.

<sup>7</sup> For convenience we include here the event  $d = D$  where no reserves are needed; note, however, that this event has zero probability.

Social welfare is composed of surpluses of the different stakeholders, consumers, RES producers, conventional producers and the TSO.

First, consumers derive utility from being supplied with the demanded electricity  $h$  and disutility from ENS  $\delta$ . Additionally, they have to pay for the supplied electricity at spot price  $p_S$  as well as energy produced out of positive (negative) reserves due to demand in excess of (below) forecast  $\varepsilon_h$  at price  $p_{A+}$  ( $p_{A-}$ ). We measure consumers' utility from received electricity with the value of lost load (VoLL)  $v$  which is defined as the amount of money a consumer is willing to pay for avoiding that another unit of electricity is not being supplied. Thus, consumer surplus is given by

$$S_C = v(h - \delta) - p_S H - p_{A+} 1_{\{\varepsilon_h - \varepsilon_{RES} \geq 0\}}(\varepsilon_h - \delta) - p_{A-} 1_{\{\varepsilon_h - \varepsilon_{RES} < 0\}}\varepsilon_h. \quad (3.4)$$

The VoLL depends on several factors such as consumer group or temporal context. In this model, for simplicity,  $v$  is assumed to be a constant value, representing both consumers' valuation of electricity consumption and of electricity that is not supplied (see Kjolle et al. 2008). We assume  $v$  to be larger than marginal generation cost  $c(K)$  for all values of  $K$  considered. Moreover,  $\delta$  (ENS) results from a positive deviation of realized residual demand from the expected demand - which can stem from a shortfall of RES in-feed or from excess realized demand or a combination of both - and lack of positive reserve provision.

Second, producers of electricity from RES earn revenues by selling their scheduled production on the spot market. They are not able to provide reserves because of the unpredictable nature of RES but are compensated by the TSO or have to compensate the TSO according to the deviation between planned and realized production at price  $p_A$ ; this is the case under most market designs. We assume zero marginal costs for RES producers, so their surplus (profit) is given by

$$S_{RES} = p_S K_{RES} + p_{A+} 1_{\{\varepsilon_h - \varepsilon_{RES} \geq 0\}}\varepsilon_{RES} + p_{A-} 1_{\{\varepsilon_h - \varepsilon_{RES} < 0\}}(\varepsilon_{RES} - \rho). \quad (3.5)$$

Third, the conventional producers of electricity have the opportunity to generate revenues both on the spot market, where they meet the planned residual consumer demand  $D$  and on the reserve power market, where they satisfy positive and negative reserve requirements through the provision of capacity  $R^{+/-}$  for the corresponding reserve capacity price  $p_{R+/-}$ . In case imbalances in the system lead to an activation of reserve energy  $r^{+/-}$ , conventional producers receive additional payments according to the reserve energy price  $p_{A+/-}$ . Since they face positive marginal costs in contrast to RES producers, generation costs  $G(d)$  have to be deducted from their revenues.

Hence, the surplus (profit) of conventional producers is given by

$$S_c = p_S D + p_{R^+} R^+ + p_{R^-} R^- + p_{A^+} r^+ - p_{A^-} r^- - G(d). \quad (3.6)$$

Finally, the TSO balances RES and load forecast errors and pays or receives payments from RES producers as well as load serving entities depending on the sign of the forecast error. These payments cancel out in the one-price reserve energy system described here. Moreover, the TSO is in charge of the reserve markets which incur transfer payments to the conventional electricity producers providing the services. Last but not least, the TSO has to step in for consumers paying the conventional producers the spot price  $p_S \delta$  in case of ENS.

$$S_T = -p_{R^+} R^+ - p_{R^-} R^- - p_S \delta. \quad (3.7)$$

This last transfer, which corresponds to the last term in (??) is owed to the fact that consumers pay for realized demand whereas conventional producers receive payments for scheduled (residual) demand. In this formulation, depending on the sign of the forecast errors, the TSO surplus is negative. Depending on the regulation in place, in reality costs can be passed through to consumers. This is not modeled here due to different regulatory regimes and because the terms cancel out in overall social surplus (??). Also, it takes time to be reflected in tariffs and so this will not affect marginal income.

Adding the different stakeholders' surpluses, (??), (??), (??) and (??), results in the following expression for social welfare  $S$  which depends on realized residual demand,  $d$ ,

$$S(d) = v(h - \delta) - G(d). \quad (3.8)$$

Payments for reserve capacity and reserve generation drop out of social surplus; all that remains in the end is consumers' utility from demand less consumers' disutility from unserved demand and costs of conventional generation. Costs of conventional generation depend on RES in-feed, realized demand and producers' bidding strategies, i.e. their decision of how much conventional generation capacity to bid into either market.

The TSO decides on reserve provision under uncertainty with respect to realized residual demand. So the objective function to be maximized under the assumption of a welfare-maximizing, i.e. perfectly regulated TSO is given by expected social surplus

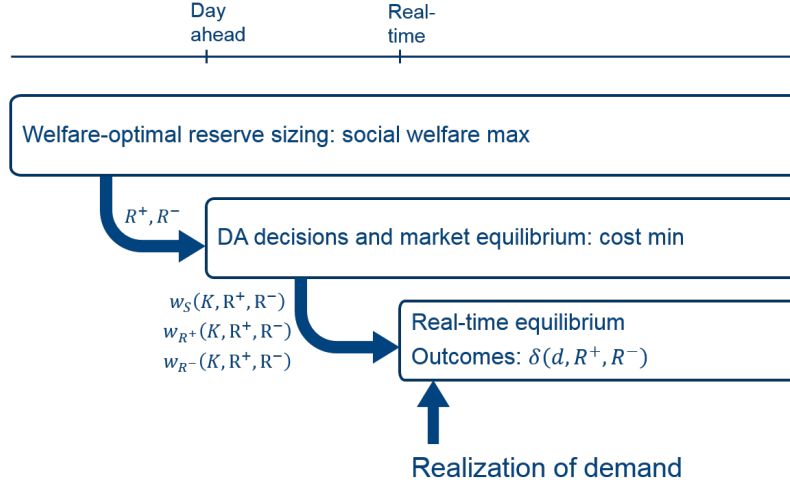


Figure 3.1: Problem Structure

$$E[S] = vH - v \int_{K_m}^{\infty} (d - K_m) dF_d(d) - E[G(d)] \quad (3.9)$$

with  $K_m$  as the marginal generation unit utilized.

Following economic intuition, the events in this model take place in three stages. In the first stage, the TSO (or a social planner) maximizes total welfare with the aim of determining the optimal reserve provision (cf. figure ??). In the second stage, conventional power generators need to decide how much capacity to commit to the spot market and whether and how much to the reserve market. Thus, they face a problem of generation cost minimization. Finally, the realization of residual demand determines the real-time equilibrium with ENS (and the activation of reserve energy) as outcome.

As noted before, producers of conventional generation capacity are allowed to bid on the three markets, the spot market and the markets for positive and negative reserves. A producer makes his decision by comparing his variable costs with the expected spot market price and the prices for providing reserves. Under the assumption of perfect competition and perfect information, producers' strategies correspond to the first-best solution of cost minimization of generation with respect to the share of generation capacity dedicated to the different markets,  $w_S$ ,  $w_{R+}$ ,  $w_{R-}$

$$C = \int_0^K w_S(K, R^+, R^-) c(k) dk. \quad (3.10)$$

Cost minimization is subject to several constraints. Energy  $w_S$  and positive

reserve provision  $w_{R+}$  are jointly produced as shares of capacity online  $K$  with

$$w_S(K, R^+, R^-) + w_{R+}(K, R^+, R^-) \leq 1. \quad (3.11)$$

Furthermore, the share of conventional generation capacity dedicated to the energy spot market must be high enough as to exceed or equal a minimum stable operation limit  $\gamma$  even when providing negative reserves  $w_{R-}$

$$w_S(K, R^+, R^-) - w_{R-}(K, R^+, R^-) \geq \gamma. \quad (3.12)$$

Combining ?? and ?? yields the following condition

$$w_{R+}(K, R^+, R^-) + w_{R-}(K, R^+, R^-) \leq 1 - \gamma, \quad (3.13)$$

stating that shares for positive and negative reserve provision must not exceed the capacity left after considering the minimum stable operation limit.

Both positive and negative reserve provision are limited by technical restrictions such as ramping capabilities to a maximum share of  $\alpha$  and  $\beta$  respectively.<sup>8</sup>

$$w_{R+} \leq \alpha \quad (3.14)$$

$$w_{R-} \leq \beta \quad (3.15)$$

Given that the equations (??) and (??) hold and assuming that  $\alpha + \beta \leq 1 - \gamma$ , the condition (??) is satisfied.

Furthermore, in market equilibrium, total (conventional) energy production  $y_S(K)$  equals scheduled (residual) energy demand. Additionally, positive and negative reserve supply,  $y_{R+}(K)$  and  $y_{R-}(K)$ , respectively, meet the respective reserve requirements.

$$y_S(K) = \int_0^K w_S(x) dx = D \quad (3.16)$$

$$y_{R+}(K) = \int_0^K w_{R+}(x) dx = R^+ \quad (3.17)$$

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<sup>8</sup> Ramping constraints are technically similar both for positive and negative reserves. In order to allow for exceptions from the rule and to identify separately the effects on positive and negative reserve provision,  $\alpha$  and  $\beta$  are allowed to differ.

$$y_{R^-}(K) = \int_0^K w_{R^-}(x) dx = R^- \quad (3.18)$$

### 3.2 Market equilibrium

We solve the model by backward induction starting with cost minimization, which yields the optimal shares of generation capacities dedicated to the spot and reserve market depending on the forecasted residual demand and given positive and negative reserve requirements (cf. Just, Weber 2008, Weber 2016).

$$w_{R^+}^*(K, R^+, R^-) = \begin{cases} 0 & K < K_0^+ \vee K > K_m \\ \alpha & K_0^+ \leq K \leq K_m \end{cases} \quad (3.19)$$

$$w_{R^-}^*(K, R^+, R^-) = \begin{cases} 0 & K < K_0^- \vee K > K_m \\ \beta & K_0^- \leq K \leq K_m \end{cases} \quad (3.20)$$

$$w_S^*(K, R^+, R^-) = \begin{cases} 1 - 1_{\{K \geq K_0^+\}}\alpha - 1_{\{K \geq K_0^-\}}\beta & K \leq K_m \\ 0 & K > K_m \end{cases} \quad (3.21)$$

These are the economically efficient shares of capacity allocated to the different markets. Thereby  $K_m$  stands for the capacity online, i.e. the marginal unit activated. Compared to a situation without the explicit consideration of uncertainty, where electricity generation from RES and demand can be perfectly predicted and hence there is no need for reserves, the optimal provision of reserves for a given level of reserve requirements is the following. In equilibrium, the marginal generation unit, and those units close to it, shall provide reserves since the marginal unit does not face opportunity costs arising from the spot market. This can be reasoned from economic intuition and is in line with Just and Weber (2008), Just (2011) and Müsgens (2014) (cf. Section ??). An efficient spot market without uncertainties results in a market equilibrium where units commit to generation in order of their marginal costs. With the need for positive reserves induced by uncertainty, overall capacity needed increases.

In our model, the spot and reserve markets clear simultaneously and overall capacity needed determines capacity online and the marginal generation unit. Since only spinning reserves are considered, the ramping constraints preclude that only extramarginal generation units provide reserves. Therefore, providing reserves from



inframarginal power plants reduces generation from these plants on the spot market which has to be replaced by power plants with variable costs above the spot price. The minimum operation limit constraint requires reserve-providing extramarginal plants to generate electricity for the spot market. Since providing positive reserves reduces the revenues obtained on the spot market, only units from a lower bound  $K_0^+$  onwards are used to provide positive reserves. Similarly, negative reserves are provided by units starting from the lower bound  $K_0^-$  onwards so that activation of negative reserves leads to maximum cost savings.

From the social welfare perspective, it is clearly efficient that under the given technical restrictions, of all generation capacity online, those units with the highest marginal generation costs reserve a share for positive reserves. It is furthermore efficient that these units provide positive reserves with the highest technically feasible share. In case no positive reserve is activated in real time, this minimizes overall generation cost. The order of activation would start from generation capacity withheld for reserves with the lowest marginal generation cost. Similarly, for the provision of negative reserves, it is economically intuitive that generation capacity with the highest marginal generation cost is scheduled with their technically maximum share. The reasoning is analogous to the positive counterpart whereas the order of activation is reversed. In case of activation, capacity scheduled for negative reserve provision with the highest marginal generation costs shall be called first from a social welfare point of view in order to yield the highest savings from avoided generation. It follows that the optimal shares of positive and negative reserve provision are determined by the maximum level of reserves a generation unit is able to provide, i.e. the ramping restrictions.

As a consequence of the simultaneous determination of the overall capacity and the spinning reserve requirement, needed reserves can only be provided from the capacity online. In combination with the ramping constraints, which limit reserve provision to a certain share of each unit, this implicitly precludes that producers are scheduled exclusively for the provision of positive reserves. This guarantees that all reserves procured are provided by capacity online.

Since reserves are not activated frequently (e.g. only about 8 % on average in the German system), we only take into account revenues from reserve capacity provision; expected revenues (payments for energy delivered) from utilized reserves are not explicitly modeled in the second stage of our model. This allows deriving the market equilibrium without taking into account uncertainty for a given level of reserve requirements. Two equilibrium conditions must then hold: a zero-profit condition for the conventional producer providing the last marginal unit of reserves

and an indifference condition for the first conventional producer that both produces energy for the spot market and provides capacity to the reserve market.

A producer that serves both the spot and reserves markets earns revenues from positive and negative reserve provision at a price of  $p_{R+}$  and  $p_{R-}$ , respectively, as well as from selling energy on the spot market at the market price,  $p_S$ . In the market equilibrium under the assumption of perfect competition, the marginal producer earns zero profit, leading to the zero-profit condition:

$$\alpha p_{R+} + \beta p_{R-} + (1 - \alpha)(p_S - c_m) = 0. \quad (3.22)$$

Moreover, the first producer that serves both energy to the spot market and capacity to the positive reserve market is indifferent between these two options. This leads to an indifference condition, which relates the price of positive reserve provision  $p_{R+}$  to the spot price of electricity and marginal generation cost at the first producer to sell positive reserves  $c_0^+$

$$\alpha p_{R+} = \alpha (p_S - c_0^+). \quad (3.23)$$

No opportunity costs arise for the provision of negative reserves, since a producer can still sell his entire capacity as energy to the spot market. Hence the indifference condition for the provision of negative reserves is simply

$$\beta p_{R-} = 0. \quad (3.24)$$

Solving these equilibrium conditions, yields the competitive market equilibrium prices

$$p_S = c_m - \alpha (c_m - c_0^+) = (1 - \alpha) c_m + \alpha c_0^+ \quad (3.25)$$

$$p_{R+} = (1 - \alpha) (c_m - c_0^+) \quad (3.26)$$

$$p_{R-} = 0 \quad (3.27)$$

In equilibrium,  $p_S$  is a weighted average of costs of the marginal producer including reserves,  $K_m$ , and the first provider of positive reserves,  $K_0^+$ , where the weight of the former is the share of capacity supplied to the spot market and the weight of the latter is the share of the capacity committed to reserves. On the other hand,  $p_{R+}$  is the difference between the two marginal costs weighted by the share of supply

to the spot market. If the marginal generation unit with idle capacity could provide positive reserves to exclusively meet reserve demand, the price of positive reserve provision should be equal to zero ( $c_m = c_0^+$ ). For all producers with lower marginal generation costs than the marginal unit online, this price is positive since they do face opportunity costs. Providers of negative reserves are not faced with any opportunity costs. They are able to use capacity both for the production of energy for the spot market and the provision of negative reserves. Therefore, the price  $p_{R-}$  should be equal to zero.

In reality, positive prices for the provision of negative reserves are observed which is not reflected by our model due to the chosen simplifying assumptions. To start with, independent of the respective type, in reality reserves are procured for a certain period of time, e.g. for one week only distinguished in peak and off-peak periods in case of secondary reserves in Germany (cf. ??).<sup>9</sup> Hence, the producer commits himself to produce for the spot market within this period of time during which prices are unknown in advance. Consequently, he might incur costs from periods in which he has to sell his generation at a price below his marginal generation costs; these are his so-called must-run costs (Just 2011). As a consequence, the contract duration of reserve provision increases prices. This logic holds for both, positive and negative reserves. A time lag between reserve procurement and the time of provision adds to this uncertainty about prices. In addition, technical constraints like minimum operation times or start-up costs may induce a different bidding behaviour in reality than assumed here. E.g. power plants will frequently continue operation during night hours in order to avoid start-up costs the next morning. They may then be willing to incur losses when running at the minimum stable operation limit yet will require compensation if they increase their output above that level in order to be able to provide negative reserve. In addition to these technical reasons for why negative reserves may fetch a positive price, there is the possibility of market power potentially distorting market prices. This is ruled out in our model by the assumption of perfect competition.

### 3.3 Real-time outcomes

In real time, the previously uncertain residual demand is revealed. With regard to consequences for social welfare, four cases can be differentiated. These are characterized by the direction and extent of the deviation of realized residual demand in relation to expected residual demand, i.e. the relevance of forecast errors for demand as well as for RES generation. Furthermore a distinction is made whether

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<sup>9</sup> This period has already been shortened from one month, as was the case prior to 2007.

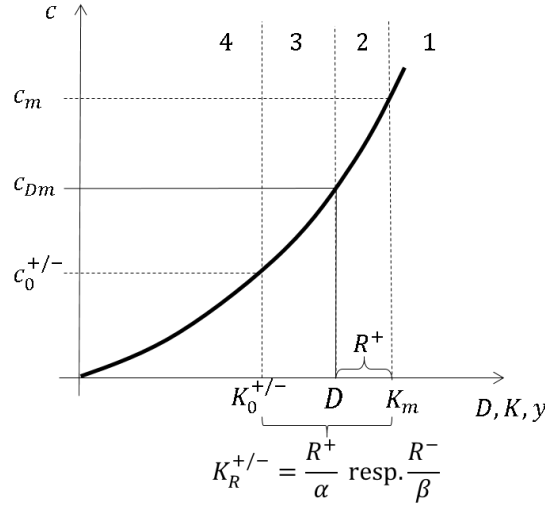


Figure 3.2: Marginal Cost Function

or not procured reserves are sufficient to cover the deviation. For each case, generation costs are calculated, a necessary step for the maximization of (expected) social welfare. Figure ?? depicts a stylized merit-order curve including these four cases:

1. Realized residual demand exceeds expected residual demand to the extent that positive reserves do not suffice to cover the shortfall in electricity generation, i.e.  $d > D + R^+$ . As a consequence, load is shed in the amount of residual demand exceeding generation plus positive reserve,  $\delta = d - D - R^+$ . In this case, reserves are fully utilized so that the marginal unit of reserves employed is the same as the marginal unit of the scheduled conventional generation capacity, i.e.

$$K_m = D + R^+ \quad (3.28)$$

with (conventional) generation costs

$$G = C(K_m). \quad (3.29)$$

2. Realized residual demand exceeds expected demand so that positive reserves are employed, but these are sufficient in this case to cover the shortfall, i.e.  $D \leq d \leq D + R^+$ . Denote the marginal unit of positive reserves employed by  $k_r^+$ . Generation capacity is utilized fully up to  $k_r^+$ ; to the right of this point on the supply curve, only the fraction  $1 - \alpha$  is utilized, the remainder are idle (positive) reserves. This implies  $d = k_r^+ + (1 - \alpha)(K_m - k_r^+) = (1 - \alpha)K_m + \alpha k_r^+$  and

solving for  $k_r^+$  gives

$$k_r^+ = \frac{1}{\alpha} (d - (1 - \alpha) K_m). \quad (3.30)$$

Costs accrue fully up to the marginal utilized reserve unit but are a fraction,  $1 - \alpha$ , for capacity beyond that unit, so for  $D \leq d \leq K_m$

$$\begin{aligned} G &= C(k_r^+) + (1 - \alpha) (C(K_m) - C(k_r^+)) \\ &= \alpha C(k_r^+) + (1 - \alpha) C(K_m). \end{aligned} \quad (3.31)$$

3. Residual demand is lower than expected. In this case negative reserves are utilized, yet not beyond the limits of procurement. Here we assume supply of plants providing negative reserves will be reduced by the fraction  $\beta$  in real time according to the merit order, i.e. starting with the unit that has the highest marginal cost (cf. Section ??). By assumption, residual demand exceeds the first capacity unit of reserves commissioned, i.e.  $K_0^- \leq d < D$  in this case. Denote the marginal unit of negative reserve capacity employed by  $k_r^-$ . Taking idle reserves into account, we must have  $d = (1 - \alpha - \beta)K_m + \alpha K_0^+ + \beta k_r^-$  so<sup>10</sup>

$$k_r^- = \frac{1}{\beta} [d - (1 - \alpha - \beta) K_m - \alpha K_0^+]. \quad (3.32)$$

Costs accrue fully up to either  $K_0^+$  or  $k_r^-$  depending on which is smaller. Only a fraction,  $1 - \alpha$  or  $1 - \beta$ , of capacity beyond that unit incurs costs. Beyond the maximum of  $K_0^+$  and  $k_r^-$ , only the fraction  $1 - \alpha - \beta$  is producing (and thus incurring cost). Denote the level of residual demand where negative reserves have been exhausted by  $\underline{D} = \alpha K_0^+ + \beta K_0^- + (1 - \alpha - \beta) K_m = D - R^-$ . It is easy to show that for  $\underline{D} \leq d \leq D$ ,

$$G = \alpha C(K_0^+) + \beta C(k_r^-) + (1 - \alpha - \beta) C(K_m). \quad (3.33)$$

4. Residual demand is lower than expected and scheduled negative reserves do not suffice to compensate this shortfall in demand (which is lower than the first capacity unit of reserves commissioned), i.e.  $d < D - R^-$ . Consequently, further generation capacity, in addition to the fully utilized conventional negative reserves, has to be reduced. We assume that this reduction is provided

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<sup>10</sup> Depending on where  $k_r^-$  is located, there are three different cases that need to be considered but they result in the same formula, i.e. (??).

as additional negative reserves by RES<sup>11</sup>  $k_{rw}^-$

$$d = (1 - \alpha - \beta)K_m + \alpha K_0^+ + \beta K_0^- - k_w^-. \quad (3.34)$$

For  $-\infty < d < \underline{D}$  generation costs correspond to the lower bound of those in case 3 since RES reduction is cost neutral. Hence

$$G = \alpha C(K_0^+) + \beta C(K_0^-) + (1 - \alpha - \beta)C(K_m). \quad (3.35)$$

Costs identified for all cases are used for the welfare maximization in the following section.

### 3.4 Welfare-optimal reserve provision

The welfare-optimal reserve capacity is found by maximizing expected welfare with respect to commissioned reserves, i.e. the TSO (in lieu of a social planner) needs to solve  $\max_{R^+, R^-} E[S(d)]$  where  $S(d)$  is given by ???. Taking expectations in (??) and using (??), (??), (??), and (??), we get an expression of expected social surplus

$$\begin{aligned} E[S] &= v(H - E[\delta]) - E[G(d)] \\ &= vH - v \int_{K_m}^{\infty} [d - K_m] dF_d - \int_{K_m}^{\infty} C(K_m) dF_d \\ &\quad - \int_D^{K_m} [\alpha C(k_r^+) + (1 - \alpha) C(K_m)] dF_d \\ &\quad - \int_{\underline{D}}^D [\alpha C(K_0^+) + \beta C(k_r^-) + (1 - \alpha - \beta) C(K_m)] dF_d \\ &\quad - \int_{-\infty}^{\underline{D}} [\alpha C(K_0^+) + \beta C(K_0^-) + (1 - \alpha - \beta) C(K_m)] dF_d \end{aligned} \quad (3.36)$$

where the first two terms represent the consumer surplus due to electricity consumed, i.e. the amount demanded less ENS.

We now proceed to deriving conditions for the optimal provision of reserves. In those derivations we rely heavily on a version of the Fundamental Theorem of

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<sup>11</sup> Calling negative reserves from RES in contrast to using conventional negative reserves does not save variable generation costs such as fuel costs. An alternative shutdown of conventional capacity incurs further costs. Hence, the applied calling order is economically rational if allowed under the respective regime. Furthermore, reducing wind generation RES simultaneously increases residual demand, i.e. decreasing the need for additional negative reserves.

Calculus, which states that

$$\frac{d}{dy} \int_a^{b+y} g(x, y) dF(x) = \int_a^{b+y} \frac{\partial}{\partial y} g(x, y) dF(x) + g(b+y, y) f(b+y) \quad (3.37)$$

for any smooth function  $g(x, y)$  and c.d.f.  $F$  with density function  $f = F'$ .

First, consider negative reserves. Deriving the expected welfare function with respect to  $R^-$  and applying (??) to the integral terms yields:

$$\frac{\partial E[S]}{\partial R^-} = c(K_0^-) F(\underline{D}). \quad (3.38)$$

Since  $c(K_0^-) F(\underline{D})$  is positive for all values of  $R^-$  it is obvious that in order to maximize welfare  $R^-$  should be chosen as large as possible. In what follows, we assume that such a choice has been made, i.e. that  $R^-$  is set at a maximum technical value.

In order to derive the first-order condition for the optimal positive reserve capacity, we proceed to take the derivative of  $E[S]$  with respect to  $R^+$ . Noting that  $k_r^+|_{d=K_m} = K_m$ ,  $\frac{dk_r^+}{dR^+} = -\frac{1-\alpha}{\alpha}$ ,  $k_r^-|_{d=K_0^-} = K_0^-$  and  $\frac{dk_r^-}{dR^+} = 1$  we get

$$\begin{aligned} \frac{\partial E[S]}{\partial R^+} &= [v - c(K_m)] Pr\{d > K_m\} \\ &- (1 - \alpha) \int_D^{K_m} [c(K_m) - c(k_r^+)] dF_d \\ &- \int_{\underline{D}}^D [-(1 - \alpha)c(K_0^+) + \beta c(k_r^-) + (1 - \alpha - \beta)c(K_m)] dF_d \\ &- [-(1 - \alpha)c(K_0^+) + \beta c(K_0^-) + (1 - \alpha - \beta)c(K_m)] Pr\{d < \underline{D}\}. \end{aligned} \quad (3.39)$$

Assuming an internal solution, the first-order condition for the optimal value of positive reserves is

$$\frac{\partial E[S]}{\partial R^+} = 0. \quad (3.40)$$

A similar calculation as that leading to (??) shows that the second derivative of expected social surplus is negative so a solution to the first-order condition will provide the welfare-optimal level of positive reserves (see the proof of Proposition ??).

We collect and formalize our findings in the following proposition:

**Proposition 1.** *a) Negative reserves should be set at the maximum technical value.  
b) The second derivative  $\frac{\partial^2 E[S]}{(\partial R^+)^2}$  is negative.*

b) Assume  $\left. \frac{\partial E[S]}{\partial R^+} \right|_{R^+=R_{max}^+} < 0$ . Then there is a unique internal solution  $\hat{R}^+$  to the first-order condition (??), which maximizes the social welfare expression (??) and thus provides the socially optimal level of positive reserves.

c) If  $\left. \frac{\partial E[S]}{\partial R^+} \right|_{R^+=R_{max}^+} \geq 0$  the optimal level of positive reserves is given by the technical maximum  $R_{max}^+$ .

*Proof.* See Appendix (??). □

It is also of interest to consider the implications of changes in individual parameters for the level of optimal reserves. The qualitative effects are summarized in the following proposition.

**Proposition 2.** Assume  $\left. \frac{\partial E[S]}{\partial R^+} \right|_{R^+=R_{max}^+} < 0$  so that the welfare maximizing level of positive reserves  $\hat{R}^+$  is given by the internal solution to the first-order condition (??). It then holds that:

- a) An increase in  $v$  leads to an increase in  $\hat{R}^+$ .
- b) An increase in  $\alpha$  leads to an increase in  $\hat{R}^+$ .
- c) An increase in  $\beta$  leads to an increase in  $\hat{R}^+$  provided  $c$  is strictly convex; for a linear  $c$ , a change in  $\beta$  has no effect on  $\hat{R}^+$ .
- d) A mean-preserving increase in the variance of  $d$  leads to an increase in  $\hat{R}^+$ .
- e) An upwards shift of  $c$  leads to a decrease in  $\hat{R}^+$ .
- f) An upwards multiplicative shift in  $c$  leads to a decrease in  $\hat{R}^+$ .

*Proof.* See Appendix (??). □

Most of the comparative statics are intuitive: an increase in the value of lost load reflects higher valuation by consumers of energy not served and hence the optimal level of reserves is higher (a); an increase in the positive ramping parameter makes it less costly to allocate capacity to reserves so the optimal reserves level rises (b); increased variability of residual demand leads to a higher incidence of service interruptions for a given level of reserves and so the optimal level rises (d); higher generation costs raise the spot price of electricity so the opportunity cost of reserves rises and the optimal level falls (e and f). The least intuitive result is the one on the negative reserve share parameter  $\beta$ , but essentially it is related to the cost saving when negative reserves are activated - only with convex marginal costs, the cost savings increase with higher  $\beta$ . Then providing positive reserves is effectively less costly, since the costs are partly compensated by higher savings in those cases negative reserves are needed.



## 4 Application

As an illustration of the practical implications of the model, we determine optimal reserves in a numerical application scaled to a typical hour in the German electricity market in the year 2013. Since more and more RES generation will be needed in order to reach climate goals set by the German government (Federal Ministry for the Environment, Nature Conservation, Building and Nuclear Safety 2015), we focus on imbalances driven by photovoltaic (PV) and wind generation, namely the forecast error of residual load and PV and wind noise. Note that we do not strive for an explicit comparison with historic values like e.g. Bucksteeg et al. (2016), but only aim at illustrative calculations of how the approach can be applied.

In 2013, the average hourly electricity consumption is 64.4 GW (International Energy Agency 2015; European Network of Transmission System Operators for Electricity 2015) with renewables supplying 8.7 GW (German TSOs 2015) on average. This leaves an average residual demand of 55.7 GW. We therefore assume that residual demand  $d$  is normally distributed with mean  $D = 55.7$  GW. For the standard deviation we distinguish two cases. First, we set it to the standard deviation of the one-hour forecast error of residual demand which corresponds to  $\sigma = 0.83$  GW. The one-hour forecast error is derived by scaling the readily available day-ahead forecast error with a factor taken from (Deutsche Energie-Agentur 2010). Second, we use the standard deviation of the noise of PV and wind generation which equals 0.43. This noise results from the 15-minute contracts on the spot market and describes the deviations of RES production around the mean of the scheduled interval. We obtain the noise by investigating the differences in RES generation between the quarter-hourly intervals. The actual dimensioning of reserves in Germany takes both factors into account, but is more complicated (cf. Bucksteeg et al. 2016).

The marginal cost curve is approximated by the relationship between the spot price and the residual load. We fit and implement two different functional forms: A linear function with slope  $m$  equal to 1 €/MWh/GW (adjusted  $R^2 = 0.77$ ) which yields a spot price of 55.7 €/MWh and a polynomial of degree three (poly3) (adjusted  $R^2 = 0.78$ ). These two specifications of the marginal cost curve together with the two assumptions about the standard deviation result in four different cases to be analyzed. For all four cases, we assume a value of lost load,  $v$ , of 10,000 €/MWh and allow 20 per cent of conventional generation capacity to be used for positive and negative reserves respectively, so that  $\alpha = \beta = 0.2$  (Hundt et al. 2009; VDMA PowerSystems 2013; Ziems et al. 2012).

The baseline results for the four cases are displayed in table ?? . Interestingly, there is virtually no difference between the outcomes when only the functional form

Table 1: Baseline results in GW

uncertainty / cost function	$\hat{R}^+$	$\hat{R}^-$	$P\{d > K_m\}$	$E[\delta]$
forecast error / linear	2.60	11.66	0.09%	0.20
forecast error / poly3	2.58	11.66	0.09%	0.21
RES noise / linear	1.42	11.43	0.05%	0.06
RES noise / poly 3	1.42	11.43	0.05%	0.06

of the marginal cost curve is altered. The reason for this is that the slopes at the level of residual demand in question are almost the same. However, we do see considerable difference in the values for different standard deviations. Optimal positive reserve requirements  $\hat{R}^+$  increase roughly proportionately with  $\sigma$  from 1.4 to 2.6 GW. These results are in the same order of magnitude as the German values (cf. Section ??). The optimal negative reserve capacity  $\hat{R}^-$  delivered by our model is less interpretable because we assume the provision of negative reserves to be costless and only limited by technical restrictions. Therefore, the optimal level of negative reserves is set to its highest possible technical value (cf. Section ??). The probability of reserves being insufficient  $P\{d > K_m\}$  and the expected ENS  $E[\delta]$  are also given in table ?? and are raised significantly by an increase in the standard deviation.

We have performed sensitivity analyses on the above results. These are presented in table ?. Changes in the optimal amount of negative reserves are omitted as those are determined by technical restrictions alone and therefore the interpretation is of limited interest. All four cases show the same tendencies and reflect the outcomes of the analytical results of Proposition (?). The optimal level of reserve capacity is increasing in the ramping parameter for positive reserves  $\alpha$ . The basic intuition is that when the ramping constraint is eased conventional power plants become more flexible and thus the provision of reserves becomes less costly. Consequently, at the optimum more reserves are commissioned. Yet the effect is not very pronounced. Put differently: the TSO does not commit a large error in the dimensioning of the reserve, if he does not exactly know the reserve providing capabilities of the available generators. The next parameter which we vary is the slope of the marginal cost curve  $m$  in case of the linear functional form. A steeper cost curve leads to lower optimal reserve levels because electricity generation becomes more costly implying higher opportunity costs for reserve provision. But again, there are only moderate effects. By contrast, the impact of the uncertainty as characterized by the standard deviation  $\sigma$  is much stronger. As already pointed out in the baseline results above, a higher uncertainty raises the optimal reserve level considerably, although the stronger variations clearly show, that the change in reserves is not strictly proportional to the variation in  $\sigma$ . Finally, a higher value of lost load  $v$

Table 2: Sensitivity Analyses in GW

	$\hat{R}^+$			
Param value	forec. error/linear	forec. error/poly3	RES noise/linear	RES noise/poly 3
$\alpha = 0.1$	2.41	2.37	1.33	1.32
$\alpha = 0.3$	2.72	2.71	1.48	1.48
$m = 0.0005$	2.75	-	1.50	-
$m = 0.0020$	2.44	-	1.34	-
$\sigma = 0.1$	0.37	0.36	0.37	0.36
$\sigma = 2$	5.78	5.67	5.78	5.67
$v = 5000$	2.44	2.43	1.34	1.34
$v = 20000$	2.75	2.73	1.50	1.49

also leads to an increase in the optimal reserve capacity since ENS is more heavily weighted. But again the sensitivity is limited. The probability of reserves being insufficient and the expected ENS change in accordance with the variations in the optimal level of positive reserves.

## 5 Conclusions

We have developed a consistent methodology to derive the welfare-optimal level of spinning reserve provision. The model makes use of an opportunity cost approach for the provision of reserves instead of selling energy to the spot market. Although the approach is stylized and neglects some technical details and market imperfections, it provides a benchmark for real-world models with strong technical focus. The comparative statics of our model confirm that salient features of reserve dimensioning are replicated by the model. The stylized application however suggests that some sensitivities, e.g. with respect to the value of lost load, are rather low. By contrast, the reserve dimensioning is strongly influenced by the level of uncertainty - hence increasing shares of renewables will lead to higher reserve needs. Yet the scaling is not linear in the residual demand uncertainty as would typically be the case with engineering approaches.

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## Appendix: Proofs

### Proposition ??

*Proof.* Evaluating the derivative (??) at the lowest possible value of positive reserves, i.e. zero, we get:

$$\begin{aligned} \left. \frac{\partial E[S]}{\partial R^+} \right|_{R^+=0} &= (v - c(D))(1 - F_d(D)) \\ &+ \beta \int_{\underline{D}}^D [c(D) - c(k_r^-)] dF_d + \beta [c(D) - c(K_0^-)] F_d(\underline{D}) > 0 \end{aligned} \quad (.1)$$

where the inequality follows from  $v > c(D)$ ,  $c' > 0$ ,  $k_r^- \leq D$  and  $K_0^- \leq D$ .

Evaluating the derivative at the largest possible value of positive reserves,<sup>12</sup>  $R_{max}^+ = \frac{\alpha}{1-\alpha}D$  we get

$$\begin{aligned} \left. \frac{\partial E[S]}{\partial R^+} \right|_{R^+=R_{max}^+} &= \left( v - c\left(\frac{D}{1-\alpha}\right) \right) \left( 1 - F_d\left(\frac{D}{1-\alpha}\right) \right) \\ &- (1-\alpha) \int_D^{\frac{D}{1-\alpha}} \left[ c\left(\frac{D}{1-\alpha}\right) - c(k_r^+) \right] dF_d \\ &- \int_{\underline{D}}^D \left[ (1-\alpha-\beta) c\left(\frac{D}{1-\alpha}\right) + \beta c(k_r^-) - (1-\alpha) c(0) \right] dF_d \\ &- \left[ (1-\alpha-\beta) c\left(\frac{D}{1-\alpha}\right) + \beta c(D - R^-) - (1-\alpha) c(0) \right] F_d(\underline{D}). \end{aligned} \quad (.2)$$

The last three terms are easily seen to be negative, since  $c$  is increasing. The first term is positive, but for reasonable distributions of  $d$  it is very small. In what follows we assume  $\left. \frac{\partial E[S]}{\partial R^+} \right|_{R^+=R_{max}^+} < 0$ , so that there is at least one solution to the first-order condition (??).

As for the second-order condition for maximum expected welfare, since  $R^-$  is set and fixed at its maximum value, it suffices to show that the second derivative with

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<sup>12</sup> That is, where all conventional generation capacity is provisioned for reserves, subject to the ramping constraint.

respect to  $R^+$  is negative. After some simplification we get (using (??))

$$\begin{aligned}
\frac{\partial^2 E[S]}{(\partial R^+)^2} &= -c'(K_m)(1 - F(K_m)) - (v - c(K_m))f(K_m) \\
&- (1 - \alpha) \int_D^{K_m} \left[ c'(K_m) + \frac{1 - \alpha}{\alpha} c'(k_r^+) \right] dF \\
&- \int_{\underline{D}}^D \left[ (1 - \alpha - \beta) c'(K_m) + \beta c'(k_r^-) + \frac{(1 - \alpha)^2}{\alpha} c'(K_0^+) \right] dF \quad (.3) \\
&- \left[ (1 - \alpha - \beta) c'(K_m) + \beta c'(K_0^-) + \frac{(1 - \alpha)^2}{\alpha} c'(K_0^+) \right] F(\underline{D}),
\end{aligned}$$

where we omit the subscript  $d$  on the distribution function  $F_d$  and probability density  $f_d$  in order to reduce clutter. Since  $c' > 0$ ,  $v > c(K_m)$  and  $1 - \alpha - \beta > 0$ , we get  $\frac{\partial^2 E[S]}{(\partial R^+)^2} < 0$ .  $\square$

## .1 Proposition ??

*Proof.* First, if  $\tau$  is a parameter (e.g.,  $v$ ,  $\alpha$ ,  $\beta$ ), then, differentiating through the first-order condition (??) with respect to that parameter and rearranging yields:

$$\frac{d\hat{R}^+}{d\tau} = \frac{\frac{\partial^2 E[S]}{\partial \tau \partial R^+}}{-\frac{\partial^2 E[S]}{(\partial R^+)^2}}, \quad (.4)$$

where the right-hand side is evaluated at  $\hat{R}^+$ . Since the denominator is positive (by virtue of the second-order condition for maximum), the sign of the numerator will also provide the sign of the derivative of  $\hat{R}^+$  with respect to the parameter.

In the remainder of this proof, to reduce clutter, we shall omit the ‘hat’ on the optimal value of positive reserves and simply write  $R^+$  for the optimal value.

First, differentiating (??) with respect to  $v$  we get

$$\frac{\partial^2 E[S]}{\partial v \partial R^+} = 1 - F(K_m) > 0, \quad (.5)$$

which establishes a).

To prove b), we differentiate (??) with respect to  $\alpha$  and get



$$\begin{aligned}
\frac{\partial^2 E[S]}{\partial \alpha \partial R^+} &= - \int_D^{K_m} \left[ -c(K_m) + c(k_r^+) - c'(k_r^+) \left( -\frac{1}{\alpha^2} \{d - K_m\} \right) \right] dF \quad (.6) \\
&- \int_{\underline{D}}^D \left[ -c(K_m) + c(K_0^+) - (1 - \alpha) c'(K_0^+) \frac{1}{\alpha^2} R^+ \right] dF \\
&- \left[ -c(K_m) + c(K_0^+) - (1 - \alpha) c'(K_0^+) \frac{1}{\alpha^2} R^+ \right] F(\underline{D}),
\end{aligned}$$

where we have used  $\frac{dk_r^+}{d\alpha} = -\frac{1}{\alpha^2} [d - K_m]$ ,  $\frac{dK_0^+}{d\alpha} = \frac{1}{\alpha^2} R^+$ ,  $\frac{dk_r^-}{d\alpha} = 0$  and  $\frac{dK_0^-}{d\alpha} = 0$ . Since  $c' > 0$ ,  $K_m \geq k_r^+$ ,  $K_m \geq K_0^+$  all the integrands are found to be negative. Thus, the right-hand side as a whole is positive and b) is confirmed.

To see that c) holds we differentiate (??) with respect to  $\beta$  and get

$$\begin{aligned}
\frac{\partial^2 E[S]}{\partial \beta \partial R^+} &= \int_{\underline{D}}^D \left[ c(K_m) - c(k_r^-) - c'(k_r^-) \frac{1}{\beta} (D - d) \right] dF \quad (.7) \\
&+ \left[ c(K_m) - c'(K_0^-) - c'(K_0^-) \frac{1}{\beta} R^- \right] F(\underline{D}),
\end{aligned}$$

where we have used  $\frac{dk_r^+}{d\beta} = 0$ ,  $\frac{dk_r^-}{d\beta} = \frac{1}{\beta} [D - d]$  and  $\frac{dK_0^-}{d\beta} = \frac{1}{\beta} R^-$ . Note that both expressions inside the square brackets are of the form

$$\{c(d_1) - c(d_0)\} - c'(d_0) \{d_1 - d_0\} \quad (.8)$$

with  $d_0 < d_1$ . This expression is strictly positive if  $c$  is strictly convex, zero if  $c$  is linear, and negative if  $c$  is concave. Hence, c) is established.

As for d), assume, for simplification of presentation, but without loss of generality, that the variance of  $d$  is equal to 1. Introduce a mean-preserving perturbation of the variance by introducing a new distribution, viz.

$$F_\sigma(d) = F_d \left( \frac{d - D}{\sigma} + D \right) \quad (.9)$$

Here it is crucial that we assume  $D$  to be an unbiased prediction of  $d$  so  $E[d] = D$  under the original distribution  $F_d$ .

Under  $F_\sigma$  the expectation of  $d$  is still  $D$ , but its variance is  $\sigma^2$ . Obviously, the probability density of  $F_\sigma$  is  $f_\sigma(d) = \frac{1}{\sigma} f_d \left( \frac{d - D}{\sigma} + D \right)$  and its derivative is  $f'_\sigma(d) = \frac{1}{\sigma^2} f'_d \left( \frac{d - D}{\sigma} + D \right)$ .

The derivatives of  $F_\sigma$  and  $f_\sigma$  with respect to  $\sigma$  are given by

$$\frac{\partial}{\partial \sigma} F_\sigma(d) = -\frac{d-D}{\sigma} f_\sigma(d) \quad (.10)$$

$$\frac{\partial}{\partial \sigma} f_\sigma(d) = -\frac{1}{\sigma} f_\sigma(d) - \frac{d-D}{\sigma} f'_\sigma(d). \quad (.11)$$

A lengthy calculation, employing integration by parts of integrals involving  $f'_\sigma(d)$ , leads, after much simplification, to the expression

$$\begin{aligned} \frac{\partial^2 E[S]}{\partial \sigma \partial R^+} &= [v - c(K_m)] \frac{R^+}{\sigma} f_\sigma(K_m) \\ &+ \frac{1-\alpha}{\alpha \sigma} \int_D^{K_m} c'(k_r^+) (d-D) dF_\sigma \\ &+ \frac{1}{\sigma} \int_{\underline{D}}^D c'(k_r^-) (D-d) dF_\sigma. \end{aligned} \quad (.12)$$

All three terms of the right-hand side are positive and d) follows.

To see e) introduce a new marginal cost function

$$c_a = a + c(d) \quad (.13)$$

where  $a$  is a non-negative constant. Clearly,  $c_a$  satisfies all conditions imposed on  $c$ . Moreover,  $a$  drops out of all but the first term of (??) so differentiating with respect to  $a$  yields

$$\frac{\partial^2 E[S]}{\partial a \partial R^+} = -[1 - F(K_m)] < 0, \quad (.14)$$

which implies that e) holds.

Finally, introduce a multiplicative shift of  $c$  by defining

$$c_b(d) = c(0) + b[c(d) - c(0)]. \quad (.15)$$

Employing  $c_b$  as a new marginal cost function, differentiating (??) with respect to  $b$  and using the first-order condition (??) for the optimal value of positive reserves (this is the only place in this proof that we rely on the fact that we evaluate the derivatives calculated at that value) we get

$$\frac{\partial^2 E[S]}{\partial b \partial R^+} = -v[1 - F(K_m)] < 0, \quad (.16)$$

which establishes f). □

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