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## Beyond sorting: A more powerful test for cross-sectional anomalies

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# **Beyond Sorting: A More Powerful Test for Cross-Sectional Anomalies**

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# Beyond Sorting: A More Powerful Test for Cross-Sectional Anomalies\*

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## Abstract

Many researchers seek factors that predict the cross-section of stock returns. The standard methodology sorts stocks according to their factor scores into quantiles and forms a corresponding long-short portfolio. Such a course of action ignores any information on the covariance matrix of stock returns. Historically, it has been difficult to estimate the covariance matrix for a large universe of stocks. We demonstrate that using the recent DCC-NL estimator of [Engle et al. \(2016\)](#) substantially enhances the power of tests for cross-sectional anomalies: On average, ‘Student’  $t$ -statistics more than double.

KEY WORDS: Cross-section of returns, dynamic conditional correlations, GARCH, Markowitz portfolio selection, nonlinear shrinkage.

JEL CLASSIFICATION NOS: C13, C58, G11.

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# 1 Introduction

The search for factors that predict the cross-section of stock returns generates an abundant literature. Instead of “factors”, some authors may use alternative terms such as signals, predictors, characteristics, anomalies, cross-sectional patterns, forecasting variables, etc. What we mean specifically is a function of historical data that can explain the cross-section of subsequent stock returns: discriminate between the stocks that will tend to outperform their peers and the ones that will tend to underperform their peers. Both [Green et al. \(2013\)](#) and [Harvey et al. \(2015\)](#) find more than 300 articles and factors in this strand of literature.

At least since [Fama and French \(1992\)](#), the preferred method for establishing the validity of factors has been to sort stocks into portfolios. For example, one can form a long-short dollar-neutral portfolio by going long the stocks that are in the top quintile according to their factor scores, and short the bottom quintile. Instead of quintiles, some authors may prefer terciles, deciles, etc. The long-short portfolio is then held for a certain period of time, at which point it is rebalanced according to freshly updated factor data. This procedure generates a time series of portfolio returns. The factor is deemed successful if the portfolio return exceeds some benchmark, generally zero percent, at the usual level of statistical significance. Thus, the central quantity is the ‘Student’  $t$ -statistic of the long-short portfolio return. This test is called *predictive* in the sense that, at any point in time, portfolio formation rules involve only data that was acquired earlier. Such investment strategies are realistic and can be implemented by a quantitative fund manager.

This *status quo* poses a conundrum: How come we have a quantitative investment strategy that does not employ the covariance matrix of asset returns? Indeed, the historical foundation of finance as a mathematically rigorous discipline can be traced back to the discovery of [Markowitz \(1952\)](#) portfolio selection. He proved that optimal portfolio weights depend not only on (a factor that proxies for) the first moment of returns, but also on the second moment: the covariance matrix — or, to be precise, its inverse. A more powerful test for cross-sectional anomalies can be designed by replacing the traditional sorting procedure with a portfolio formation rule that incorporates the (inverse) covariance matrix, at least in theory.

From theory to practice there is a gap: The true covariance matrix is unobservable, therefore it needs to be estimated somehow. At the epoch when the standard procedure for testing factors crystallized around sorting, there was no covariance matrix estimator that could cope with inversion in large dimensions. Indeed, [Michaud \(1989\)](#) described portfolio optimization as an “error-maximization procedure”. [Ledoit and Wolf \(2004\)](#) show that the textbook estimator, the sample covariance matrix, is ill-conditioned when the dimension is not negligible with respect to sample size: inverting it amplifies any estimation error. This unfortunate behavior is pushed to a numerical extreme when the number of stocks exceeds the number of time series observations, at which point the supposedly optimal portfolio weights blow up to plus or minus infinity for no reason whatsoever — which violates economic sense. Even with two years of daily data at hand, this systemic breakdown happens as soon as we consider the universe of S&P 500 constituents.



for cross-sectional anomaly detection, on average, by a factor of more than *two* relative to sorting. Therefore, it is everybody’s interest to move beyond the theoretically and empirically underpowered procedure of sorting.

The power boost from using DCC-NL is significant because it enables factor candidates that have a short history to get a chance at getting detected. Multiplying the  $t$ -statistic by two is equivalent to multiplying the number of years in the dataset by approximately four. Thus, if a given factor requires 40 years of historical data to achieve statistical significance with sorting, with DCC-NL the same factor can attain the same level of statistical significance in only ten years. This is especially relevant for all factors that are extracted from traffic on social networks, as these have only been active on a massive scale for a relatively small number of years. Given the explosion in data collection driven by the precipitous fall in storage cost per petabyte in recent years, this is just the tip of the iceberg: Big data is young data.

On a separate but equally important note, given that [Harvey et al. \(2015\)](#) claim that the significance threshold for  $t$ -statistics should be raised from two to three due to multiple-testing issues, it will be much harder for subsequent authors to meet this hurdle. Any candidate needs all the power boost he or she can get. Having a more accurate telescope to detect elusive objects always constitutes scientific progress.

The methodology we use in this paper — that is, harnessing a wide variety of cross-sectional anomalies to shed new light on an important problem in financial econometrics — is very much in tune with recent developments in other strands of the literature that are unrelated to covariance matrix estimation. For example, [Hou et al. \(2015\)](#) argue that the usefulness of a parsimonious model of expected stock returns should be judged against its ability to explain away a large number of cross-sectional anomalies. [McLean and Pontiff \(2016\)](#) measure the speed of convergence of financial markets towards informational efficiency by computing the decay rate of a large number of cross-sectional anomalies subsequent to academic publication.

Just as the merit for inventing DCC-NL does not belong to the present paper, the burden of proving that it is better than the multitude of covariance matrix estimators that have been proposed by countless authors does not fall on the present paper either. DCC-NL is the default choice at this juncture because it is the only one that addresses concomitantly the two major issues in the estimation of the covariance matrix of stock returns, namely conditional heteroskedasticity and the curse of dimensionality. Our point is only to establish that DCC-NL, as representative of best practices in covariance matrix estimation, has enough accuracy to reinstate the covariance matrix in its rightful place at the center of the [Markowitz \(1952\)](#) program and empirical asset pricing: The time has come to abandon the practice of sorting.

The paper is organized as follows. Section 2 gives a brief presentation of the DCC-NL covariance matrix estimator. Section 3 describes the empirical methodology for comparing test power with and without DCC-NL. Section 4 contains the empirical results. Section 5 concludes. Appendix A contains all figures and tables; Appendix B details the technique of ‘Winsorization’ that is applied to cross-sectional vectors of factors in our empirical work; and Appendix C details the set of factors we consider and how these factors are constructed in practice.

## 2 The DCC-NL Estimator of the Covariance Matrix

This brief recapitulation is only intended to make the present paper self-contained. The interested reader is referred to [Engle et al. \(2016\)](#) for the original exposition.

### 2.1 Time Variation in the Second Moments

The modelling and estimation of time-varying variances, covariances, and correlations requires aggregating the contributions from three different ideas.

#### 2.1.1 Dynamic Conditional Correlation (DCC)

A key idea promoted by [Engle \(2002\)](#) is that modelling conditional heteroskedasticity is easy and successful in the univariate case, so we should take care of that *prior* to looking at the covariance matrix as a whole. Thus, for every asset  $i = 1, \dots, N$ , we fit a GARCH(1,1) or similar model to the series  $i$  individually. Dividing the raw returns by the corresponding conditional standard deviations yields *devolatilized* returns that have unit variance. Call  $\mathbf{s}_t$  the  $N$ -dimensional column vector of devolatilized residuals at time  $t \in \{1, 2, \dots, T\}$ . Then the dynamics of the pseudo-correlation matrix  $Q_t$  can be specified as:

$$Q_t = \Theta + \alpha \mathbf{s}_{t-1} \mathbf{s}_{t-1}' + \beta Q_{t-1} , \quad (2.1)$$

where  $\alpha$  and  $\beta$  are non-negative scalars satisfying  $\alpha + \beta < 1$  that govern the dynamics, and  $\Theta$  is an  $N$ -dimensional symmetric positive definite matrix.  $Q_t$  is called a *pseudo*-correlation matrix because its diagonal terms are close, but not exactly equal, to one. Therefore the following adjustment is needed to recover the proper correlation matrix  $R_t$ :

$$R_t := \text{Diag}(Q_t)^{-1/2} Q_t \text{Diag}(Q_t)^{-1/2} , \quad (2.2)$$

where  $\text{Diag}(\cdot)$  denotes the function that sets to zero all the off-diagonal elements of a matrix.

#### 2.1.2 Volatility Targeting

The second ingredient is the notion of “variance targeting” introduced by [Engle and Mezrich \(1996\)](#). Although originally invented in a univariate context, the extension to the multivariate case of interest here is straightforward ([Engle, 2002](#), Eq. (11)). The basic idea is that a suitable rescaling of the matrix  $\Theta$  in equation (2.1) can be interpreted as the unconditional covariance matrix. Therefore, it can be estimated using standard techniques that ignore time series effects, separately from the other parameters. This approach yields the reparametrized model

$$Q_t = \Gamma(1 - \alpha - \beta) + \alpha \mathbf{s}_{t-1} \mathbf{s}_{t-1}' + \beta Q_{t-1} , \quad (2.3)$$

where  $\Gamma$  is the long-run covariance matrix of the devolatilized returns  $\mathbf{s}_t$  for  $t = 1, \dots, T$ .<sup>1</sup>

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<sup>1</sup>Since the devolatilized returns all have unit variance,  $\Gamma$  is actually a proper correlation matrix, that is, its diagonal elements are all equal to one.

### 2.1.3 Composite Likelihood

After having dealt with the conditional variances and partialled out the problem of estimating the unconditional covariance matrix, the only remaining task is to estimate the dynamic correlation parameters  $\alpha$  and  $\beta$ . These two scalars play the same role as their counterparts in the more familiar ARMA(1,1) and GARCH(1,1) models, but for conditional correlation matrices.

When the matrix dimension is large, say  $N = 1000$ , the standard likelihood maximization technique would require inverting  $T$  matrices of dimension  $1000 \times 1000$  at every iteration, which is numerically challenging. [Pakel et al. \(2014\)](#) found a more efficient solution called the 2MSCLE method: combine the individual likelihoods generated by  $2 \times 2$  blocks of contiguous variables. Maximizing this *composite likelihood* yields asymptotically consistent estimators for  $\alpha$  and  $\beta$ , as long as the DCC model is well-specified. The intuition is that every individual correlation coefficient shows traces of the dynamic parameters  $\alpha$  and  $\beta$  in its own time series evolution, so a sufficiently large subset of individual correlations will reveal (a consistent approximation of) the true parameters. The advantage of this procedure is that it is numerically stable and fast in high dimensions; for example, [Engle et al. \(2016\)](#) manage to take it to a large universe of  $N = 1000$  stocks.

### 2.1.4 DCC Estimation Procedure

To summarize, the estimation of the DCC model unfolds in three steps:

1. Fit a univariate GARCH(1,1) model to every stock return series individually, and divide the raw returns by their conditional standard deviations to devolatilize them.
2. Estimate the unconditional covariance matrix of devolatilized returns somehow.
3. Maximize the 2MSCLE composite likelihood to obtain consistent estimators of the two parameters of correlation dynamics in a numerically stable and efficient way.

At this juncture, it becomes apparent from step 2 that we need an estimator of the unconditional covariance matrix of devolatilized returns that performs well when the dimension is large.<sup>2</sup>

## 2.2 Estimation of Large-Dimensional Unconditional Covariance Matrices

The reader is invited to peruse [Ledoit and Wolf \(2012, 2015\)](#) for a more detailed treatment.

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<sup>2</sup>Note that in practice the devolatilized returns have to be based on estimated univariate GARCH models rather than the ‘true’, unobservable univariate GARCH models.



### 2.2.1 Spectral Decomposition

The textbook estimator of  $\Gamma$  is the sample covariance matrix  $C := \sum_{t=1}^T \mathbf{s}_t \mathbf{s}_t' / T$ . Both matrices admit spectral decompositions:

$$C = \sum_{i=1}^N \lambda_i \cdot \mathbf{u}_i \mathbf{u}_i' \quad \text{and} \quad \Gamma = \sum_{i=1}^N \tau_i \cdot \mathbf{v}_i \mathbf{v}_i', \quad (2.4)$$

where  $(\lambda_1, \dots, \lambda_N; \mathbf{u}_1, \dots, \mathbf{u}_N)$  denotes a system of eigenvalues and eigenvectors of the sample covariance matrix  $C$ , and  $(\tau_1, \dots, \tau_N; \mathbf{v}_1, \dots, \mathbf{v}_N)$  denotes a system of eigenvalues and eigenvectors of the population covariance matrix  $\Gamma$ . Eigenvalues are indexed in ascending order without loss of generality.

In the traditional asymptotic framework, where the sample size  $T$  goes to infinity, while the number of assets  $N$  remains constant, the sample eigenvalue  $\lambda_i$  is a consistent estimator of its population counterpart  $\tau_i$ , and the sample eigenvector  $\mathbf{u}_i$  is a consistent estimator of its population counterpart  $\mathbf{v}_i$ , for  $i = 1, \dots, N$ . However, this asymptotic framework is not robust against the curse of dimensionality. When  $N$  is no longer negligible with respect to  $T$ , the sample spectrum is far from its population counterpart.

This is why it is necessary to turn to another asymptotic framework that offers a different family of analytical solutions. Unlike the formulas from traditional asymptotics, they work also if  $N$  is not negligible with respect to  $T$ , and even if  $N$  is greater than  $T$ . The key assumption is that the ratio  $N/T$  converges to some limit  $c \in [0, +\infty)$  called the *concentration (ratio)*. This framework is called *large-dimensional* asymptotics, and it includes traditional (fixed-dimensional) asymptotics as a special case when the concentration  $c$  is equal to zero. Thus, it is a generalization of traditional asymptotics that is able to cope with the curse of dimensionality by making necessary corrections (whose intensity increases in  $c$ ) to the standard formulas.

### 2.2.2 Portfolio Selection

Stein (1986) argued that, in the absence of *a priori* knowledge about the structure of the eigenvectors of the (unobservable) population covariance matrix  $\Gamma$ , estimators should preserve the sample covariance matrix eigenvectors  $(\mathbf{u}_1, \dots, \mathbf{u}_N)$ , and correct the sample eigenvalues only. This framework is called *rotation-equivariant* because the economic outcome is immune to repackaging the  $N$  original stocks into a collection of  $N$  funds investing in these stocks, as long as the funds span the same investment universe as the stocks.

It is easy to show that, among rotation-equivariant estimators of the covariance matrix, the one that performs the best across all possible linear constraints for the purpose of portfolio selection in terms of minimizing out-of-sample variance is:

$$\tilde{C} := \sum_{i=1}^N \underbrace{(\mathbf{u}_i' \Gamma \mathbf{u}_i)}_{\phi_i} \cdot \mathbf{u}_i \mathbf{u}_i'. \quad (2.5)$$

This makes economic sense because  $\mathbf{u}_i' \Gamma \mathbf{u}_i$  is the out-of-sample variance of the portfolio whose weights are given by the  $i$ th sample eigenvector  $\mathbf{u}_i$ . Thus we notice the emergence of a third quantity, after the sample eigenvalue  $\lambda_i = \mathbf{u}_i' C \mathbf{u}_i$ , and the population eigenvalue  $\tau_i = \mathbf{v}_i' \Gamma \mathbf{v}_i$ : the hybrid  $\phi_i := \mathbf{u}_i' \Gamma \mathbf{u}_i$ , which represents the best we can do with the sample eigenvectors.

The key is that, under large-dimensional asymptotics, the vectors  $\boldsymbol{\lambda} := (\lambda_i)_{i=1,\dots,N}$ ,  $\boldsymbol{\tau} := (\tau_i)_{i=1,\dots,N}$ , and  $\boldsymbol{\phi} := (\phi_i)_{i=1,\dots,N}$  are all far apart from one another. It is only as the concentration  $c$  goes to zero, that is, as we approach standard (fixed-dimension) asymptotics, that their mutual differences vanish. When  $c > 0$ , which is the case when the investment universe is large, appropriate corrections must be applied to go from  $\boldsymbol{\lambda}$  to  $\boldsymbol{\tau}$  to  $\boldsymbol{\phi}$ .<sup>3</sup> Qualitatively,  $\boldsymbol{\lambda}$ ,  $\boldsymbol{\tau}$ , and  $\boldsymbol{\phi}$  have the same cross-sectional average, but  $\boldsymbol{\lambda}$  is more dispersed than  $\boldsymbol{\tau}$ , which in turn is more dispersed than  $\boldsymbol{\phi}$ .

### 2.2.3 NonLinear (NL) Shrinkage Estimator of the Covariance Matrix

The ideal would be to have two deterministic functions  $\boldsymbol{\Lambda}^{N,T}$  and  $\boldsymbol{\Phi}^{N,T}$  from  $[0, +\infty)^N$  to  $[0, +\infty)^N$  mapping out the two important expectations:

$$\begin{aligned}\boldsymbol{\tau} &\mapsto \boldsymbol{\Lambda}^{N,T}(\boldsymbol{\tau}) := (\Lambda_1^{N,T}(\boldsymbol{\tau}), \dots, \Lambda_N^{N,T}(\boldsymbol{\tau})) = (\mathbb{E}[\lambda_1], \dots, \mathbb{E}[\lambda_N]) = (\mathbb{E}[\mathbf{u}_1' C \mathbf{u}_1], \dots, \mathbb{E}[\mathbf{u}_N' C \mathbf{u}_N]) \\ \boldsymbol{\tau} &\mapsto \boldsymbol{\Phi}^{N,T}(\boldsymbol{\tau}) := (\Phi_1^{N,T}(\boldsymbol{\tau}), \dots, \Phi_N^{N,T}(\boldsymbol{\tau})) = (\mathbb{E}[\phi_1], \dots, \mathbb{E}[\phi_N]) = (\mathbb{E}[\mathbf{u}_1' \Gamma \mathbf{u}_1], \dots, \mathbb{E}[\mathbf{u}_N' \Gamma \mathbf{u}_N])\end{aligned}$$

Then we would use the observed eigenvalues of the sample covariance matrix,  $\boldsymbol{\lambda}$ , to reverse-engineer an estimator of the population eigenvalues by solving the optimization problem

$$\hat{\boldsymbol{\tau}} := \underset{\mathbf{t} \in [0, +\infty)^N}{\operatorname{argmin}} \frac{1}{N} \sum_{i=1}^N \left( \Lambda_i^{N,T}(\mathbf{t}) - \lambda_i \right)^2, \quad (2.6)$$

and the nonlinear shrinkage estimator of the covariance matrix would follow as

$$\hat{C} := \sum_{i=1}^N \Phi_i^{N,T}(\hat{\boldsymbol{\tau}}) \cdot \mathbf{u}_i \mathbf{u}_i'. \quad (2.7)$$

Due to tractability issues, however, we only know approximations to the functions  $\boldsymbol{\Lambda}^{N,T}$  and  $\boldsymbol{\Phi}^{N,T}$  that are valid asymptotically as the universe dimension  $N$  goes to infinity along with the sample size  $T$ , with their ratio  $N/T$  converging to the concentration  $c$ . [Ledoit and Wolf \(2012, 2015\)](#) show that replacing the true expectation functions with their approximations can be done at no loss asymptotically. Therefore, this procedure yields a nonlinear shrinkage estimator of the covariance matrix that is optimal in the large-dimensional asymptotic limit.

Qualitatively speaking, the effect of composing  $\boldsymbol{\Phi}^{N,T}$  with the inverse of  $\boldsymbol{\Lambda}^{N,T}$  (or approximations thereof) moves the sample eigenvalues closer to one another, while preserving their cross-sectional average. The effect is increasing in  $N/T$  and highly nonlinear; for example, isolated eigenvalues that lie near the bulk of the other eigenvalues move in the direction of the bulk more than those distant from the bulk.

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<sup>3</sup>Correcting these relationships when the ratio of variables to observations is significant is analogous to correcting Newtonian relationships when the ratio of velocity to speed of light is significant ([Einstein, 1905](#)).

### 2.3 DCC-NL Model

In summary, the estimation of the DCC-NL model of [Engle et al. \(2016\)](#) proceeds as follows:

1. Fit univariate GARCH models to devolatilize returns.
2. Compute the sample covariance matrix of devolatilized returns.
3. Decompose it into eigenvalues and eigenvectors.
4. Invert an approximation of the function  $\mathbf{A}^{N,T}$  to estimate population eigenvalues.
5. Apply an approximation of the function  $\Phi^{N,T}$  to shrink eigenvalues nonlinearly.
6. Recompose with the sample eigenvectors to estimate the unconditional covariance matrix  $\Gamma$  in (2.3).
7. Transform the resulting estimator of  $\Gamma$  from a covariance matrix to a proper correlation matrix.<sup>4</sup>
8. Maximize the 2MSCLE composite likelihood to estimate the correlation dynamics.
9. Recombine the estimated conditional correlation matrix with the estimated univariate GARCH processes to obtain an estimated conditional covariance matrix.

The outside steps (1–2 and 7–9) compose the DCC part, while the inside steps (3–6) compose the NL part of the DCC-NL estimation procedure. The final product is a time-series of  $N$ -dimensional conditional covariance matrix estimates, which we call  $(H_t)_{t=1,\dots,T}$ . More explicit formulas are in [Engle et al. \(2016\)](#).

## 3 Empirical Methodology

The goal is to construct long-short portfolios exposed to a given factor. The size of the investment universe is denoted by  $N$ , and stocks in this universe are indexed by  $i$ . Days on which investment and trading takes place are indexed by  $t$ . The cross-sectional vector of factor scores observable at the beginning of day  $t$  is denoted by  $\mathbf{m}_t := (m_{t,1}, \dots, m_{t,N})'$ . A long-short portfolio is defined by a weight vector  $\mathbf{w}_t := (w_{t,1}, \dots, w_{t,N})'$  that satisfies

$$\sum_{w_{t,i} < 0} |w_{t,i}| = \sum_{w_{t,i} > 0} |w_{t,i}| = 1 . \quad (3.1)$$

Note that the weights of such a long-short portfolio necessarily sum to zero.

### 3.1 Portfolios Based on Sorting

Let  $B$  be the number of quantiles considered; for example,  $B = 3$  for terciles,  $B = 5$  for quintiles, and  $B = 10$  for deciles. Let  $d$  be the largest integer that is smaller than or equal to  $N/B$ . Finally, let  $\{(1), (2), \dots, (N)\}$  be permutation of  $\{1, 2, \dots, N\}$  that results in ordered factor scores (from smallest to largest):

$$m_{t,(1)} \leq m_{t,(2)} \leq \dots \leq m_{t,(N)} . \quad (3.2)$$

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<sup>4</sup>Doing so is motivated by the fact that  $\Gamma$  itself is a proper correlation matrix, as pointed out previously.

Then the long-short portfolio based on sorting is given by the weight vector  $\mathbf{w}_t^{\text{So}}$  with

$$w_{t,(1)}^{\text{So}} = \dots = w_{t,(d)}^{\text{So}} := -1/d , \quad (3.3)$$

$$w_{t,(d+1)}^{\text{So}} = \dots = w_{t,(N-d)}^{\text{So}} := 0 , \text{ and} \quad (3.4)$$

$$w_{t,(N-d+1)}^{\text{So}} = \dots = w_{t,(N)}^{\text{So}} := 1/d . \quad (3.5)$$

The resulting portfolio return is denoted by  $r_t^{\text{So}} := \mathbf{x}_t' \mathbf{w}_t^{\text{So}}$ , where  $\mathbf{x}_t$  is the  $N \times 1$  vector of returns at date  $t$ .

### 3.2 Markowitz Portfolios

The Markowitz investment problem is formulated as

$$\min_{\mathbf{w}} \mathbf{w}' H_t \mathbf{w} \quad (3.6)$$

$$\text{subject to } \mathbf{m}_t' \mathbf{w} = \mathbf{m}_t' \mathbf{w}_t^{\text{So}} , \text{ and} \quad (3.7)$$

$$\sum_{w_i < 0} |w_i| = \sum_{w_i > 0} |w_i| = 1 , \quad (3.8)$$

where  $H_t$  is the DCC-NL estimate of the covariance matrix of  $\mathbf{x}_t$ . Denote a solution of this investment problem by  $\mathbf{w}_t^{\text{Ma}}$ . The resulting portfolio return is denoted by  $r_t^{\text{Ma}} := \mathbf{x}_t' \mathbf{w}_t^{\text{Ma}}$ .

The motivation here is that we want to construct a portfolio that has the same expected return as the portfolio based on sorting (according to the vector of factors  $\mathbf{m}_t$ ) because of (3.7) but has a smaller variance because of (3.6). If this goal is accomplished, then the resulting portfolio returns will generally result in a larger (in magnitude) ‘Student’  $t$ -statistic (3.10) below, since the smaller variance of the returns will result in a smaller standard error in the denominator of the  $t$ -statistic whereas the sample average in numerator will be roughly the same. It is key to have an accurate estimate of the covariance matrix of  $\mathbf{x}_t$  in order to achieve this goal: this where the DCC-NL model comes in.

### 3.3 Tests for Predictive Ability

The ability of a factor to forecast the cross-section of stock returns is judged by whether a long-short portfolio exploiting the factor can deliver returns with a positive expected value. In particular, we consider the hypothesis testing problem

$$H_0 : \mathbb{E}(r_t^{\text{St}}) = 0 \quad \text{vs.} \quad H_1 : \mathbb{E}(r_t^{\text{St}}) \neq 0 , \quad (3.9)$$

where  $\text{St} \in \{\text{So}, \text{Ma}\}$  stands for one of the two strategies, sorting or Markowitz. The testing problem (3.9) is two-sided, since it generally cannot be ruled out *a priori* that the factor works in the opposite way as intended, that is, that the long-short portfolio delivers returns with a negative expected value.

The test is based on observed strategy returns  $r_t^{\text{St}}, t = 1, \dots, T$ . The ‘Student’  $t$ -statistic of the test is given by

$$t^{\text{St}} := \frac{\bar{r}^{\text{St}}}{\text{SE}(\bar{r}^{\text{St}})} \quad \text{with} \quad \bar{r}^{\text{St}} := \frac{1}{T} \sum_{t=1}^T r_t^{\text{St}} , \quad (3.10)$$

where  $\text{SE}(\bar{r}^{\text{St}})$  denotes a standard error of  $\bar{r}^{\text{St}}$ . The common choice in the literature for such a standard error is the ‘naïve’ standard error based on an assumption of independent and identically distributed (i.i.d.) returns. Specifically, it is given by  $s^{\text{St}}/\sqrt{T}$ , where  $s^{\text{St}}$  denotes the sample standard deviation of the observed returns  $r_t^{\text{St}}, t = 1, \dots, T$ .

Instead, we consider it important to use a HAC standard error that is robust against heteroskedasticity and serial correlation in the returns. In particular, we use the standard error based on the quadratic spectral (QS) kernel with automatic choice of bandwidth as detailed in [Andrews \(1991\)](#).

The common critical value in the literature is two: If the  $t$ -statistic is larger than two, the factor is deemed successful. On the other hand, [Harvey et al. \(2015\)](#) call for a more demanding critical value of three due to multiple-testing issues.

## 4 Empirical Analysis

### 4.1 Data and General Portfolio-Formation Rules

We download daily stock return data from the Center for Research in Security Prices (CRSP) starting in 01/01/1980 and ending in 12/31/2015. We restrict attention to stocks from the NYSE and NASDAQ stock exchanges. For simplicity, we adopt the common convention that 21 consecutive trading days constitute one ‘month’. The out-of-sample period ranges from 01/08/1986 through 12/31/2015, resulting in a total of 360 ‘months’ (or 7560 days). All portfolios are updated ‘monthly’.<sup>5</sup> We denote the investment dates by  $h = 1, \dots, 360$ . At any investment date  $h$ , the Markowitz portfolio (3.6)–(3.8) uses the DCC-NL estimate  $H_t$  of the covariance matrix based on the most recent 1250 daily returns, which roughly corresponds to using to five years of past data. The portfolio based on sorting uses quintiles, which seems to be the most common choice in the literature.

We consider the following portfolio sizes:  $N \in \{100, 500, 1000\}$ . For a given combination  $(h, N)$ , the investment universe is obtained as follows. We find the set of stocks that have a complete return history over the most recent  $T = 1250$  days as well as a complete return ‘future’ over the next 21 days.<sup>6</sup> We then look for possible pairs of highly correlated stocks, that is, pairs of stocks that have returns with a sample correlation exceeding 0.95 over the past 1250 days. With such pairs, if they should exist, we remove the stock with the lower volume of the two on investment date  $h$ .<sup>7</sup> Of the remaining set of stocks, we then pick the largest  $N$  stocks (as measured by their market volume on investment date  $h$ ) as our investment universe. In

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<sup>5</sup>‘Monthly’ updating is common practice to avoid an unreasonable amount of turnover and thus transaction costs. During a ‘month’, from one day to the next, we hold number of shares fixed rather than portfolio weights; in this way, there are no transactions at all during a ‘month’.

<sup>6</sup>The latter, ‘forward-looking’ restriction is not a feasible one in real life but is commonly applied in the related finance literature on the out-of-sample evaluation of portfolios.

<sup>7</sup>The reason is that we do not include highly similar stocks, or even the same stock, listed under two different permanent issue identification numbers (PERMNOs) in the CRSP database. In the early years, there are no such pairs; in the most recent years, there are never more than three such pairs.

this way, the investment universe changes slowly from one investment date to the next.

We consider a total of 61 factors taken from [Green et al. \(2013\)](#) and [Hou et al. \(2015\)](#); the corresponding data are downloaded from the merged CRSP/Compustat database. Table 1 lists the factors and Appendix C contains a detailed description of how the factor scores are computed. Note that for  $N = 1000$ , there are not sufficient data available for factors 23, 32, 37, and 51–56. We apply ‘Winsorization’ to any cross-sectional vector of factor scores  $\mathbf{m}_t$  in order to mitigate potential problems with ‘outlying’ scores that are unusually large in magnitude; see Appendix B for the corresponding details.

## 4.2 Results

The individual  $t$ -statistics are detailed in Table 2. Not surprisingly, in some cases the  $t$ -statistic based on sorting is negative (though generally not significantly so). It can be assumed that the corresponding factors will be discarded immediately by a researcher, since they can never be established as successful based on a negative  $t$ -statistic. For each universe size  $N \in \{100, 500, 1000\}$ , we therefore restrict attention to factors for which sorting yields a positive  $t$ -statistic. For such factors, we also present the value of the ratio of the two  $t$ -statistics: the one based on Markowitz divided by the one based on sorting.

Table 3 presents the average ratio for each universe size  $N \in \{100, 500, 1000\}$ . The average ratio is always larger than two, meaning that, on average, the  $t$ -statistic more than doubles when a researcher upgrades from Sorting to Markowitz.

It is natural to ask whether these averages might be influenced by a few ‘outlying’ ratios which can occur when the  $t$ -statistic based on sorting (which appears in denominator) is close to zero. For example, take the case of factor 33 with a universe size  $N = 100$ . In this case, the  $t$ -statistic based on sorting equals 0.020 whereas the  $t$ -statistic based on Markowitz equals 1.048, resulting in a ratio of 52.4. Consequently, we also compute averages only for cases where the  $t$ -statistic based on sorting is bounded away from zero. First, we only consider cases where the  $t$ -statistic based on sorting is larger than 0.5; second, we only consider cases where the  $t$ -statistic based on sorting is larger than 1.0. The corresponding averages are also found in Table 3. It can be seen that the averages decrease as the lower bound increases (from 0 to 0.5 to 1.0), especially for  $N = 100$ . But when the lower bound is 0.5, the averages for  $N = 500, 1000$  still exceed two; and when the lower bound is 1.0, the averages for  $N = 500, 1000$  are still close to two (if less than two now). Therefore, the impressive power gains of Markowitz over sorting are not driven by a few  $t$ -statistics based on sorting that are close to zero.

Arguably, it is of main interest how much the number (and proportion) of significant factors increase when a researcher upgrades from sorting to Markowitz. The common critical value in the literature for the value of a  $t$ -statistic is two. On the other hand, [Harvey et al. \(2015\)](#) argue that a critical value of three should be used instead due to multiple-testing issues. We consider both critical values, two and three, in Table 4. One can see that for both strategies, So(rting) and Ma(rkowitz), the number (and proportion) of significant factors increase in  $N$ ; therefore, it is in the best interest of researchers to use as large an investment universe as

possible. One can further see that the number (and proportion) of significant factors are much bigger for Markowitz compared to sorting. In particular, when a critical value of three is used, the number (and proportion) of significant factors more than double for all universe sizes.

## 5 Conclusion

This paper demonstrates that, in accordance with the theory of [Markowitz \(1952\)](#), the portfolio selection rule in predictive tests of cross-sectional anomalies should incorporate a suitable estimator of the covariance matrix of stock returns. When a researcher upgrades from simplistic sorting to Markowitz portfolio optimization based on the DCC-NL covariance matrix estimator of [Engle et al. \(2016\)](#), ‘Student’  $t$ -statistics, on average, more than *double* — across a large panel of return-predictive signals (or “factors”) — when the investment universe is large. This power boost is especially needed because multiple-testing issues may justify raising the  $t$ -statistic significance threshold from its usual level of two to a more demanding level of three, as proposed by [Harvey et al. \(2015\)](#). The power boost also cures the inherent handicap of short-history datasets by multiplying the effective number of years by approximately four in large dimensions. Cross-sectional testing methodologies that do not use a suitable estimator of the covariance matrix, such as DCC-NL, are underpowered and their use should be abandoned.

Directions for further research include (i) using more flexible univariate models than the straightforward GARCH(1,1) to devolatilize individual return series in the first step of the procedure (such as models that incorporate asymmetric responses) and (ii) pre-conditioning the cross-section of stock returns by a low-dimensional model with exogenous risk factors.

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## A Tables

Table 1: List of factors

Number	Name
1	11-month momentum, 11-MM
2	1-month momentum (reversal), 1-MM
3	6-month momentum, 6-MM
4	Maximum daily return in prior month (reversal), Mxret
5	Change in 6-month momentum (reversal), $\Delta 6$ -MM
6	Cumulative abnormal stock returns around earnings announcement, Abr
7	Dollar trading volume from month $t - 2$ (reversal), Dvol
8	Firm size (reversal), ME
9	Book-to-market, B/M
10	Asset growth, Agr
11	Earnings-to-price, E/P
12	Change in long-term debt (reversal), $\Delta lgr$
13	Change in common shareholder equity, $\Delta ceq$
14	Cash flow from operation, Cflow
15	Cash-to-price (reversal), Cash
16	Dividend yield, D/P
17	Payout yield, O/P
18	Net payout yield, NO/P
19	Sales growth, SG
20	Market leverage, A/ME
21	Abnormal volume in earnings announcement month, Aevol
22	Earnings surprise, Sue
23	Change in order backlog, OB
24	Working capital accrual (reversal), Acc
25	Capital expenditures and inventory (reversal), $\Delta capx$
26	Changes in inventory (reversal), Cii
27	Abnormal corporate investment (reversal), Aci
28	Net stock issues (reversal), Nsi
29	Net operating assets (reversal), Noa
30	Investment growth (reversal), IG
31	Net external financing (reversal), Nxf
32	Composite issuance (reversal), Cei
33	Total accruals (reversal), TA/A
34	Inventory growth (reversal), Ivg
35	Percent operating accruals (reversal), Poa

Table 1 continued

Number	Name
36	Percent total accruals (reversal), Pta
37	Change in deferred revenues, $\Delta drev$
38	F-score
39	Change in profit margin, $\Delta PM$
40	Asset turnover, Ato
41	Change in tax expense, $\Delta tax$
42	Return on assets, Roa
43	Gross profits-to-assets, Gma
44	Return on invested capital, Roic
45	Return on equity, Roe
46	Return on net operating assets, Rna
47	Taxable income-to-book income, TI/BI
48	Capital turnover, Cto
49	O-score
50	Employee growth rate (reversal), Egr
51	Change in advertising expense, $\Delta ade$
52	R&D increase, Rdi
53	Advertisement expense-to-market, Ad/M
54	R&D-to-sales, RD/S
55	R&D-to- market, RD/M
56	R&D capital-to-assets, Rc/A
57	Operating leverage, OL
58	Turn (reversal)
59	Total Volatility (reversal), Tvol
60	Accrual Volatility (reversal), Avol
61	Cash flow volatility (reversal), Cvol

Table 2: Results. The columns labeled So contain the  $t$ -statistics (3.10) based on sorting; the columns labeled Ma contain the test statistics (3.10) based on Markowitz; the columns labeled Ma/So contain the corresponding ratios Ma/So for the cases when So is positive. NaN denotes missing values due to lack of sufficient data. NoI stands for “Not of Interest” and corresponds to cases when So is negative.

Number	$N = 100$			$N = 500$			$N = 1000$		
	So	Ma	Ma/So	So	Ma	Ma/So	So	Ma	Ma/So
1	1.441	2.611	1.81	1.398	1.758	1.26	1.402	1.671	1.19
2	0.115	2.664	23.18	0.816	4.862	5.96	1.074	5.064	4.71
3	-0.296	0.051	NoI	0.312	-0.397	1.27	0.560	-0.595	1.06
4	0.483	0.710	1.47	-0.156	-0.820	NoI	-0.498	-2.812	NoI
5	0.217	0.623	2.88	1.198	2.422	2.02	1.342	2.433	1.81
6	1.259	1.698	1.35	2.658	3.066	1.15	3.219	4.460	1.39
7	-0.529	0.222	NoI	1.612	4.202	2.61	2.998	4.100	1.37
8	0.219	0.766	3.50	1.147	3.801	3.31	2.323	3.038	1.31
9	-0.123	-0.550	NoI	0.640	1.116	1.74	1.047	1.736	1.66
10	-0.056	0.249	NoI	0.016	0.235	14.62	0.573	1.050	1.83
11	2.411	4.672	1.94	4.544	11.085	2.44	5.345	15.716	2.94
12	0.517	1.854	3.58	0.849	2.875	3.39	2.056	4.257	2.07
13	1.006	0.717	0.71	2.671	4.075	1.53	3.308	7.862	2.38
14	5.306	6.088	1.15	6.713	9.825	1.46	7.108	16.031	2.26
15	1.820	3.361	1.85	2.807	5.667	2.02	3.864	6.434	1.67
16	-0.417	1.201	NoI	-0.291	0.995	NoI	-1.160	0.399	NoI
17	0.857	1.519	1.77	0.892	2.888	3.24	0.726	3.418	4.71
18	0.729	1.467	2.01	0.503	3.373	6.70	0.536	4.981	9.29
19	0.282	1.066	3.78	1.533	4.779	3.12	2.752	7.416	2.69
20	-0.661	-0.922	NoI	0.133	0.388	2.91	0.451	0.884	1.96
21	0.889	1.028	1.16	2.212	1.976	0.89	2.259	4.263	1.89
22	2.417	3.260	1.35	4.854	10.062	2.07	8.116	16.914	2.08
23	-0.064	-0.180	NoI	-0.300	1.683	NoI	NaN	NaN	NaN
24	3.046	4.703	1.54	5.102	7.006	1.37	7.363	10.803	1.47
25	0.631	1.883	2.98	1.579	3.964	2.51	3.287	5.050	1.54
26	1.340	2.221	1.66	1.886	3.748	1.99	2.715	4.589	1.69
27	1.406	2.581	1.84	3.346	3.975	1.19	3.760	5.354	1.42
28	-0.382	1.507	NoI	1.531	2.718	1.78	1.411	3.437	2.44
29	2.741	1.823	0.67	3.697	4.012	1.086	3.486	5.296	1.52
30	0.929	1.499	1.61	2.461	4.305	1.75	2.759	4.033	1.46
31	2.309	1.548	0.67	2.595	2.766	1.07	2.726	5.671	2.08
32	1.136	1.089	0.96	1.647	3.756	2.28	NaN	NaN	NaN

Table 2 continued

Number	$N = 100$			$N = 500$			$N = 1000$		
	So	Ma	Ma/So	So	Ma	Ma/So	So	Ma	Ma/So
33	0.020	1.048	52.40	1.857	3.354	1.81	3.068	3.422	1.12
34	1.516	2.138	1.41	1.874	4.336	2.31	2.945	4.801	1.63
35	1.736	2.975	1.71	2.174	3.461	1.59	4.229	6.919	1.64
36	1.397	1.711	1.22	1.555	3.418	2.20	2.450	3.249	1.33
37	2.257	1.069	0.47	3.491	4.098	1.17	NaN	NaN	NaN
38	0.541	1.304	2.41	1.505	3.097	2.06	1.368	4.478	3.27
39	2.012	2.500	1.24	3.482	7.704	2.21	5.778	11.764	2.04
40	1.427	2.452	1.72	2.339	3.259	1.39	2.802	4.576	1.63
41	1.761	2.924	1.66	4.557	8.957	1.97	6.968	15.678	2.25
42	2.302	3.641	1.58	3.538	6.453	1.82	4.459	10.142	2.27
43	1.963	3.798	1.93	2.424	4.320	1.78	2.964	6.265	2.11
44	2.435	4.010	1.65	3.105	5.941	1.91	4.165	9.310	2.24
45	2.537	3.207	1.26	4.297	7.340	1.71	4.975	11.897	2.39
46	3.243	4.532	1.40	3.869	5.956	1.54	4.506	9.812	2.18
47	1.414	2.424	1.71	1.031	2.626	2.55	0.752	3.208	4.26
48	1.822	1.964	1.08	1.605	2.543	1.58	2.435	3.876	1.59
49	-2.158	-0.915	NoI	-1.474	-0.620	NoI	0.532	-1.696	-3.19
50	0.573	0.350	0.61	0.802	2.028	2.53	1.261	2.701	2.14
51	0.919	0.320	0.35	-0.011	0.657	NoI	NaN	NaN	NaN
52	-0.638	0.174	NoI	-0.614	-0.320	NoI	NaN	NaN	NaN
53	0.213	1.200	5.63	2.018	1.110	0.55	NaN	NaN	NaN
54	0.719	1.204	1.67	1.550	3.333	2.15	NaN	NaN	NaN
55	1.553	1.312	0.84	3.348	4.737	1.42	NaN	NaN	NaN
56	1.132	1.762	1.56	1.960	5.521	2.82	NaN	NaN	NaN
57	1.463	1.899	1.30	1.718	2.370	1.38	2.675	3.376	1.26
58	-0.211	-0.882	NoI	-0.347	0.213	NoI	-0.175	0.691	NoI
59	0.114	1.244	10.95	-0.251	0.408	NoI	-0.548	-1.158	NoI
60	1.653	1.172	0.71	1.138	1.087	0.95	0.602	1.955	3.25
61	2.486	1.738	0.70	2.758	2.257	0.82	2.862	2.999	1.05

$N$	So > 0	So > 0.5	So > 1.0
100	3.29	1.45	1.33
500	2.24	2.04	1.79
1000	2.05	2.05	1.93

Table 3: Averages based on the columns labeled Ma/So in Table 2. The second column reports averages when the  $t$ -statistic based on sorting is positive; the third column reports averages when the  $t$ -statistic based on sorting is greater than 0.5; and the fourth column reports averages when the  $t$ -statistic based on sorting is greater than 1.0.

Critical value = 2					Critical value = 3				
$N$	So	Ma	So	Ma	$N$	So	Ma	So	Ma
100	13	20	0.21	0.33	100	3	10	0.05	0.16
500	25	45	0.41	0.74	500	14	35	0.23	0.57
1000	33	41	0.63	0.79	1000	18	38	0.35	0.73

Table 4: Number (columns two and three) and proportion (columns four and five) of the  $t$ -statistics in Table 2 whose value exceed two (left panel) and three (right panel), respectively.

## B Winsorization of Factor Scores

‘Outlying’ factor scores that are unusually large in magnitude can have undesirable impacts when used as input in Markowitz optimization. We mitigate this potential problem by properly truncating very small and very large values in any cross-sectional vector of factor scores  $\mathbf{m}_t$ . Such truncation is commonly referred to as ‘Winsorization’, a method that is widely used by quantitative portfolio managers; for example, see (Chincarini and Kim, 2006, Appendix 5B).

Consider a generic vector  $\mathbf{a} := (a_1, \dots, a_N)'$ . We first compute a robust measure of location that is not (heavily) affected by potential outliers. To this end, we use the trimmed mean of the data with trimming fraction  $\eta \in (0, 0.5)$  on the left and on the right. This number is simply the mean of the middle  $(1 - 2\eta) \cdot 100\%$  of the data. More specifically, denote by

$$a_{(1)} \leq a_{(2)} \leq \dots \leq a_{(N)} \quad (\text{B.1})$$

the ordered data (from smallest to largest) and denote by

$$M := \lfloor \eta \cdot N \rfloor \quad (\text{B.2})$$

the smallest integer less than or equal to  $\eta \cdot N$ . Then the trimmed mean with trimming fraction  $\eta$  is defined as

$$\bar{a}_\eta := \frac{1}{N - 2M} \sum_{i=M+1}^{N-M} a_{(i)} . \quad (\text{B.3})$$

We employ the value of  $\eta = 0.1$  in practice.

We next compute a robust measure of spread. To this end, we use the mean absolute deviation (MAD) given by

$$\text{MAD}(\mathbf{a}) := \frac{1}{N} \sum_{i=1}^N |a_i - \text{med}(\mathbf{a})| , \quad (\text{B.4})$$

where  $\text{med}(\mathbf{a})$  denotes the sample median of  $a_1, \dots, a_N$ .

We next compute upper and lower bounds defined by

$$a_{\text{lo}} := \bar{a}_{0.1} - 5 \cdot \text{MAD}(\mathbf{a}) \quad \text{and} \quad a_{\text{up}} := \bar{a}_{0.1} + 5 \cdot \text{MAD}(\mathbf{a}) . \quad (\text{B.5})$$

The motivation here is that for a normally distributed sample, it will hold that  $\bar{a} \approx \bar{a}_{0.1}$  and  $s(\mathbf{a}) \approx 1.5 \cdot \text{MAD}(\mathbf{a})$ , where  $\bar{a}$  and  $s(\mathbf{a})$  denote the sample mean and the sample standard deviation of  $a_1, \dots, a_N$ , respectively. As a result, for a ‘well-behaved’ sample, there will usually be no points below  $a_{\text{lo}}$  or above  $a_{\text{up}}$ . Our final truncation rule is that any data point  $a_i$  below  $a_{\text{lo}}$  will be changed to  $a_{\text{lo}}$  and any data point  $a_i$  above  $a_{\text{up}}$  will be changed to  $a_{\text{up}}$ .

We then apply this truncation rule to the cross-sectional vector of factor scores  $\mathbf{m}_t$  in place of the generic vector  $\mathbf{a}$ .

## C Description of Factors

Daily data used are from the Center for Research in Security Prices (CRSP), including holding period returns (item ret), return without dividends (item retx), prices (item prc), number of shares traded (item vol), number of shares outstanding (item csho), factor to adjust shares (item ajex), and value-weighted return (item vwret). The other data are from the Compustat Annual and Quarterly Fundamental Files. For each factor, we describe how the factor scores are computed at a generic investment date  $h = 1, \dots, 360$ .

### C.1 Momentum

#### C.1.1 11-MM

Following [Fama and French \(1996\)](#), we calculate 11-month momentum (11-MM) as the average return over the previous 12 months but excluding the most recent month. That is, we compute the average return from day  $h - 252$  through day  $h - 22$ .

#### C.1.2 1-MM

Following [Jegadeesh and Titman \(1993\)](#), we calculate 1-month momentum (1-MM) as the average return from day  $h - 21$  through day  $h - 1$ . Reversal of 1-MM (that is, the negative of 1-MM) is used as the actual factor.

#### C.1.3 6-MM

Following [Jegadeesh and Titman \(1993\)](#), we calculate 6-month momentum (6-MM) as the average return over the previous seven months but excluding the most recent month. That is, for any investment date  $h$ , we compute the average return from day  $h - 147$  through day  $h - 22$ .

#### C.1.4 Mxret

Following [Bali et al. \(2011\)](#), Mxret is the maximum daily return from day  $h - 21$  through day  $h - 1$ . Reversal of Mxret is used as the actual factor.

#### C.1.5 $\Delta 6$ -MM

Following [Gettleman and Marks \(2006\)](#), change in 6 month momentum ( $\Delta 6$ -MM) is calculated as current 6-MM minus previous 6-MM (that is, 6-MM at investment date  $h - 1$ ). Reversal of  $\Delta 6$ -MM is used as the actual factor.



### C.1.6 Abr

Following [Chan et al. \(1996\)](#), we measure cumulative abnormal stock return (Abr) around the latest quarterly earnings announcement date as

$$\text{Abr}_i := \sum_{d=-2}^1 (r_{id} - r_{md}) , \quad (\text{C.1})$$

where  $r_{id}$  and  $r_{md}$  are, respectively, the return of stock  $i$  and the value-weighted return of the market index (item vwretd) on day  $d$ , where  $d = 0$  represents the earnings announcement day (quarterly item rdq). For stock  $i$ , at every investment date  $h$ , we use the most recent earnings announcement day as long as the day is at least two days earlier than the investment day (to make sure that  $r_{i(d=1)}$  is available).

## C.2 Value-versus-growth

### C.2.1 Dvol

Dvol is the dollar trading volume in the latest-but-one month (that is, from day  $h - 42$  through day  $h - 22$ ). As in [Chordia et al. \(2001\)](#), we measure it as the natural log of the sum of daily dollar trading volume during that period. Daily dollar trading volume is share price (item prc) times the number of shares traded (item vol). Reversal of Dvol is used as the actual factor.

### C.2.2 ME

[Banz \(1981\)](#) proposes firm size as a factor. We use the market capitalization (ME) of one day before the investment day (that is, on day  $h - 1$ ) as firm size. ME is calculated as price (item prc) times shares outstanding (item csho). Reversal of ME is used as the actual factor

### C.2.3 B/M

[Rosenberg et al. \(1985\)](#) propose book-to-market as a factor. We measure it as the ratio of book equity to market capitalization on the day before the investment day (that is, on day  $h - 1$ ); here, book equity is computed from the most recently announced quarterly data. Our measure of the book equity is the quarterly version of the annual book equity measure in [Davis et al. \(2000\)](#). In particular, it is the book value of common equity (item ceqq) plus the par value of preferred stock (item pstkq), plus balance-sheet deferred taxes and investment tax credit (item txditcq), and then minus the book value of preferred stock. We use redemption value (item pstkrq, zero if missing) for the book value of preferred stock.

### C.2.4 Agr

To construct the [Cooper et al. \(2008\)](#) asset growth (Agr) factor, we divide the total assets (item atq) by 1-quarter-lagged total assets; item atq uses the most recently announced quarterly data. Reversal of Agr is used as the actual factor.

### C.2.5 E/P

Following [Basu \(1983\)](#), earnings-to-price (E/P) is calculated as income before extraordinary items (item `ibq`) divided by the market capitalization (ME) on day  $h - 1$ ; item `ibq` uses the most recently announced quarterly data.

### C.2.6 $\Delta lgr$

Following [Scott et al. \(2005\)](#), we measure change in long-term debt ( $\Delta lgr$ ) as long-term debt (item `lt`) divided by 1-year-lagged long-term debt minus one; item `lt` uses the most recently announced quarterly data. Reversal of  $\Delta lgr$  is used as the actual factor.

### C.2.7 $\Delta ceq$

Following [Scott et al. \(2005\)](#), we measure change in common shareholder equity ( $\Delta ceq$ ) as common shareholder equity (item `ceqq`) divided by 1-quarter-lagged common shareholder equity minus one; item `ceqq` uses the most recently announced quarterly data.

### C.2.8 Cflow

Following [Houge and Loughran \(2000\)](#), we define cash flow from operation (Cflow) as net cash flow from operations in the most recently announced quarter scaled by the average of total assets (item `atq`) for the two previous quarters. Instead of using the item `oancf` (net cash flow from operations) directly, we use net income (item `niq`) minus operating accruals (OA) because these items have a broader coverage than `oancf`, and they have quarterly data. To measure OA, we use the balance-sheet approach of [Sloan \(1996\)](#), that is,

$$OA := (\Delta actq - \Delta cheq) - (\Delta lctq - \Delta dlcq - \Delta txpq) - dpq, \quad (C.2)$$

where  $\Delta$  represents the change in the corresponding item, and items `actq`, `cheq`, `lctq`, `dlcq`, `txpq`, `dpq` are corresponding to the quarterly data of current assets, cash and cash equivalents, current liabilities, debt included in current liabilities (zero if missing), income taxes payable (zero if missing), depreciation and amortization (zero if missing), respectively. Note that the number of stocks for which this factor is available during the first eight investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 9$ .

### C.2.9 Cash

Following [Chandrashekar and Rao \(2009\)](#), cash to price (Cash) is computed as

$$\text{Cash} := (ME + dlттq - atq)/cheq, \quad (C.3)$$

where ME is the market capitalization on day  $h - 1$ , and items `dlттq`, `atq`, and `cheq` are all quarterly data corresponding to long-term debt, total asset, and cash or cash equivalents, respectively; all these items use the most recently announced quarterly data. Reversal of Cash is used as the actual factor.

### C.2.10 D/P

As in [Litzenberger and Ramaswamy \(1979\)](#), dividend yield (D/P) is measured as the total dividends paid out from the previous year (that is, from day  $h - 252$  through day  $h - 1$ ) divided by ME on day  $h - 1$ . The total dividends are calculated by accumulating daily dividends, and the daily dividends is measured as the difference between cum- and ex-dividend returns, which are respectively corresponding to holding period returns (item ret) and return without dividends (item retx), times the 1-day-lagged ME.

### C.2.11 O/P

Following [Boudoukh et al. \(2007\)](#), total payouts (O/P) are dividends on common stock (dvc) plus repurchases of the previous year (that is, from day  $h - 252$  through day  $h - 1$ ) divided by ME on day  $h - 1$ . Repurchases are the total expenditure on the purchase of common and preferred stocks (item prstk) minus the change over the previous year in the value of the net number of preferred stocks outstanding (item pstkrv).

### C.2.12 NO/P

Following [Boudoukh et al. \(2007\)](#), net payouts (NO/P) are the same as total payouts except that the equity issuances have to be subtracted from the total payouts. Equity issuances are the sale of common and preferred stock (item sstk) minus the change over the previous year in the value of the net number of preferred stocks outstanding (item pstkrv).

### C.2.13 SG

[Lakonishok et al. \(1994\)](#) propose sales growth (SG) as a factor. We measure it as the growth rate in sales (item saleq) from quarter  $t - 2$  through quarter  $t - 1$ , where  $t$  denotes the current quarter.

### C.2.14 A/ME

Following [Bhandari \(1988\)](#), A/ME is measured as the ratio of total assets in quarter  $t - 1$  to ME on day  $h - 1$ , where  $t$  denotes the current quarter.

### C.2.15 Aevol

As in [Lerman et al. \(2008\)](#), the abnormal earnings announcement period volume (Aevol) is defined as average daily share trading volume over the three days from  $d = -1$  through  $d = 1$  divided by the average daily share volume over days  $d = -8$  through  $d = -63$ , and then subtracting one, where  $d = 0$  denotes day of the most recent earnings announcement (item rdq):

$$\text{Aevol}_i := \frac{\text{Avg}_{d \in [-1, 1]}(\text{vol}_{id})}{\text{Avg}_{d \in [-63, -8]}(\text{vol}_{id})} - 1 . \quad (\text{C.4})$$

Note that the day of the most recent earnings announcement must be at least two days before the investment day  $h$  (to make sure that  $\text{vol}_{i(d=1)}$  is available).

### C.2.16 Sue

Following [Foster et al. \(1984\)](#), we measure earnings surprise (Sue) as the change in the most recently announced quarterly earnings per share (item epspxq) from its value four quarters ago, divided by the standard deviation of this change in quarterly earnings over the previous eight quarters.

### C.2.17 OB

Following [Gu et al. \(2009\)](#), we measure OB as annual order backlog (item ob) in year  $t - 1$  scaled by the average of total assets (item at) for calendar years  $t - 2$  and  $t - 1$ , where  $t$  denotes the current calendar year. Note that the number of stocks for which this factor is available during the first 65 investment periods is less than 500, and the number is less than 1000 for the entire investment period. As a result, for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 66$  whereas for dimension  $N = 1000$ , we do not consider this factor.

## C.3 Investment

Considering the general negative relation between investment and expected return, all factors in this section are used in reversal.

### C.3.1 Acc

Following [Sloan \(1996\)](#), we measure working capital accruals (Acc) as operating accruals (OA) in quarter  $t - 1$  scaled by the average of total assets (item atq) for quarters  $t - 2$  and  $t - 1$ , where  $t$  denotes the current quarter and OA is the same as in equation (C.2). Note that the number of stocks for which this factor is available during the first eight investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 9$ .

### C.3.2 $\Delta\text{capx}$

Following [Lyandres et al. \(2008\)](#), we measure capital expenditures and inventory ( $\Delta\text{capx}$ ) as changes in gross property, plant, and equipment (item ppeg) plus changes in inventory (item invt) scaled by 1-year-lagged total assets (item at). Note that the number of stocks for which this factor is available during the first two investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 3$ .

### C.3.3 Cii

Following [Thomas and Zhang \(2002\)](#), we measure change in inventory (Cii) as the change in the most recently announced annual inventory from its value one year previous to that, scaled

by the average of total assets (item at).

#### C.3.4 Aci

Following [Titman et al. \(2004\)](#), we measure abnormal corporate investment (Aci) as

$$\text{Aci}_t := \frac{3 * \text{CE}_{t-1}}{\text{CE}_{t-2} + \text{CE}_{t-3} + \text{CE}_{t-4}} - 1, \quad (\text{C.5})$$

where  $t$  denotes the current calendar year and  $\text{CE}_{t-j}$  is capital expenditure (item capx) scaled by sales (item sale) in calendar year  $t - j$ . Note that the number of stocks for which this factor is available during the first three investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 4$ .

#### C.3.5 Nsi

[Pontiff and Woodgate \(2008\)](#) propose net stock issues (Nsi) as a factor. We measure it as the natural log of the ratio of the average split-adjusted shares outstanding over the previous year (that is, from day  $h - 252$  through day  $h - 1$ ) to the average split-adjusted shares outstanding over the year previous to that (that is, from day  $h - 504$  through day  $h - 253$ ). We measure the daily split-adjusted shares outstanding as shares outstanding (item csho) times the adjustment factor (item ajex).

#### C.3.6 Noa

As in [Hirshleifer et al. \(2004\)](#), we measure net operating assets (Noa) as operating assets minus operating liabilities. Operating assets are total assets (item atq) minus cash and short-term investment (item cheq). Operating liabilities are total assets minus debt included in current liabilities (item dlq, zero if missing), minus long-term debt (item dltdq, zero if missing), minus minority interests (item mibq, zero if missing), minus preferred stocks (item pstkq, zero if missing), and minus common equity (item ceq). We use quarterly data instead of annual data.

#### C.3.7 IG

Following [Xing \(2008\)](#), we measure investment growth (IG) as the growth rate in capital expenditure (item capx) from calendar year  $t - 2$  to calendar year  $t - 1$ , where  $t$  denotes the current calendar year.

#### C.3.8 Nxf

Following [Bradshaw et al. \(2006\)](#), we measure net external financing (Nxf) as the sum of net equity financing and net debt financing in year calendar  $t - 1$  scaled by the average of total assets, where  $t$  denotes the current calendar year. Net equity financing is the proceeds from the sale of common and preferred stocks (item sstk) less cash payments for the repurchases of common and preferred stocks (item prstk) less cash payments for dividends (item dv).

Net debt financing is the cash proceeds from the issuance of long-term debt (item dltis) less cash payments for long-term debt reductions (item dltr) plus the net changes in current debt (item dlch, zero if missing). Note that the number of stocks for which this factor is available during the first 13 investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 14$ .

### C.3.9 Cei

Following [Daniel and Titman \(2006\)](#), we define composite issuance (Cei) as the growth rate in market capitalization (ME) during the previous five years (that is, from day  $h - 1260$  through day  $h - 1$ ) not attributable to the stock return. It is calculated as

$$\text{Cei}_t := \log(\text{ME}_t - \text{ME}_{t-5}) - \log r(t - 5, t) , \quad (\text{C.6})$$

where  $r(t - 5, t)$  is the cumulative return on the stock from day  $h - 1260$  through day  $h - 1$ ,  $\text{ME}_t$  is the ME on day  $h - 1$ , and  $\text{ME}_{t-5}$  is the ME on day  $h - 1260$ . Note that the number of stocks for which this factor is available during some middle investment periods (for example, from 08/29/2011 through 12/31/2012) is less than 1000. As a result, for dimension  $N = 1000$ , we do not consider this factor.

### C.3.10 TA/A

Following [Richardson et al. \(2005\)](#), we measure TA/A as total accruals scaled by 1-year-lagged total assets (item at). Total accruals (TA) are calculated as

$$\text{TA} := \Delta \text{WC} + \Delta \text{NCO} + \Delta \text{FIN} , \quad (\text{C.7})$$

where  $\Delta$  represents the change in the corresponding item, and items WC, NCP, FIN are net non-cash working capital, net non-current operating assets, and net financial assets, respectively:

$$\text{WC} := \text{act} - \text{che} - (\text{lct} - \text{dlc}) \quad (\text{C.8})$$

$$\text{NCO} := \text{at} - \text{act} - \text{ivao} - (\text{lt} - \text{lct} - \text{dltt}) \quad (\text{C.9})$$

$$\text{FIN} := \text{ivst} + \text{ivao} - (\text{dltt} + \text{dlc} + \text{pstk}) . \quad (\text{C.10})$$

Here, act, che, lct, dlc, at, ivao, lt, lct, dltt, ivst, pstk are all annual items corresponding to current assets, cash and short-term investment, current liabilities, debt in current liabilities, total assets, long-term investments (zero if missing), total liabilities, current liabilities, long-term debt (zero if missing), short-term investment (zero if missing), and preferred stock (zero if missing), respectively. Note that the number of stocks for which this factor is available during the first 5 investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 6$ .

### C.3.11 Ivg

Following [Belo and Lin \(2012\)](#), we define inventory growth (Ivg) as the growth rate of inventory (item invt) from calendar year  $t - 2$  to year calendar year  $t - 1$ , where  $t$  denotes the current calendar year.

### C.3.12 Poa

Following [Hafzalla et al. \(2011\)](#), percent operating accruals (Poa) is measured as operating accruals (OA) in quarter  $t - 1$ , scaled by net income (item niq) in the same quarter, where  $t$  denotes the current quarter; see equation (C.2) for the definition of OA. Note that the number of stocks for which this factor is available during the first eight investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 9$ .

### C.3.13 Pta

Following [Hafzalla et al. \(2011\)](#), percent total accruals (Pta) is measured as total accruals (TA) scaled by net income (item ni); see equation (C.7) for the definition of TA. Considering the broader coverage, we use annual data instead of quarterly data to calculate this factor. Note that the number of stocks for which this factor is available during the first 6 investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 7$ .

## C.4 Profitability

### C.4.1 $\Delta drev$

Following [Prakash and Sinha \(2013\)](#), we measure change in deferred revenues ( $\Delta drev$ ) as the growth rate of deferred revenues (item drcq) from quarter  $t - 2$  to quarter  $t - 1$ , where  $t$  denotes the current quarter. Note that the number of stocks for which this factor is available is less than 1000 during the entire investment period; therefore, we do not consider this factor for dimension  $N = 1000$ . According to the available number of stocks, for dimension  $N = 100$ , we start the portfolio formation on investment date  $h = 221$  whereas for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 229$ .

### C.4.2 F-score

Following [Piotroski \(2000\)](#), we define F-score as the sum of nine individual binary signals:

$$F := F_{Roa} + F_{\Delta Roa} + F_{Cfo} + F_{Acc} + F_{\Delta Margin} + F_{\Delta Turn} + F_{\Delta Lever} + F_{\Delta Liquid} + F_{EQ} \quad (C.11)$$

where  $Roa$  is income before extraordinary (item ib) scaled by 1-year-lagged total assets (item at);  $\Delta Roa$  is the increase in  $Roa$  compared to the previous year;  $Cfo$  is cash flow from operation (we use funds from operation (item fopt) minus the annual change in working

capital (item wcap) scaled by 1-year-lagged total assets;  $Acc$  is defined as  $Cfo$  minus  $Roa$ ;  $\Delta Margin$  is gross margin (item sale minus cogs, and then divided by sale) in calendar year  $t-1$  less gross margin in calendar year  $t-2$ ;  $\Delta Turn$  is the change in the current calendar year's asset turnover ratio, which is measured as total sales (item sale) scaled by 1-year-lagged total assets (item at), compared to the previous calendar year;  $\Delta Lever$  is the decrease in the current calendar year's lever, which is measured as total long-term debt (item dltd) scaled by average total assets over the previous two calendar years;  $\Delta Liquid$  is the change in the current calendar year's current ratio compared to the previous calendar year, which is measured as the ratio of current assets (item act) to current liabilities (item lct);  $EQ$ , which measures whether the firm issue common equity in the current calendar year, equals the increase in preferred stock (item pstk) minus the sales of common and preferred stocks (item sstk). For our definition, the indicator variable always is equal to 1 if the corresponding variable is positive and is equal to zero otherwise.

### C.4.3 $\Delta PM$

Following Soliman (2008), we measure change in profit margin ( $\Delta PM$ ) as profit margin in quarter  $t-1$  less profit margin in quarter  $t-2$ , where  $t$  denotes the current quarter. Profit margin is operating income after depreciation (item oiadp), scaled by sales (item saleq).

### C.4.4 $Ato$

Following Soliman (2008), we measure asset turnover ( $Ato$ ) as sales (quarterly item saleq), divided by 1-quarter-lagged Noa (net operating assets); see Section C.3.6 for a description of Noa.

### C.4.5 $\Delta tax$

Following Thomas and Zhang (2011), we measure changes in tax expense ( $\Delta tax$ ) as tax expense (item txtq) in quarter  $t$  minus tax expense in quarter  $t-4$ , scaled by total assets (item atq) in quarter  $t-4$ , where  $t$  denotes the current quarter.

### C.4.6 $Roa$

Following Balakrishnan et al. (2010), we measure return on assets ( $Roa$ ) as income before extraordinary items (item ibq) divided by 1-quarter-lagged total assets (item atq).

### C.4.7 $Gma$

Following Novy-Marx (2010), we measure Gross profitability ( $Gma$ ) as sales (item saleq) minus cost of goods sold (item cogsq), then divided by 1-quarter-lagged total assets (item atq).



#### C.4.8 Roic

Following [Brown and Rowe \(2007\)](#), we measure return on invested capital (Roic) as operating income after depreciation (quarterly item oiadpq) divided by 1-quarter-lagged operating assets, which are total assets (item atq) minus cash and short-term investment (item cheq).

#### C.4.9 Roe

Following [Haugen and Baker \(1996\)](#), we measure return on equity (Roe) as income before extraordinary items (quarterly item ibq) divided by 1-quarter-lagged book equity; book equity is computed as in Section [C.2.3](#).

#### C.4.10 Rna

Following [Soliman \(2008\)](#), we measure return on operating assets (Rna) as operating income after depreciation (quarterly item oiadpq) divided by 1-quarter-lagged net operating assets (Noa); see Section [C.3.6](#) for a description of Noa.

#### C.4.11 TI/BI

Following [Green et al. \(2014\)](#), we measure taxable income-to-book income (TI/BI) as pretax income (quarterly item piq) divided by net income (item niq).

#### C.4.12 Cto

Following [Haugen and Baker \(1996\)](#), we measure capital turnover (Cto) as sales (quarterly item saleq) divided by 1-quarter lagged total assets (item atq).

#### C.4.13 O-score

Following [Ohlson \(1980\)](#), we define the O-score as

$$\begin{aligned} O := & -1.32 - 0.407\log(\text{at}) + 6.03\text{tlta} - 1.43\text{wcta} + 0.076\text{clca} \\ & - 1.72\text{oeneg} - 2.37\text{nita} - 1.83\text{futul} + 0.285\text{intwo} - 0.521\text{chin} \end{aligned} \quad (\text{C.12})$$

where  $\text{tlta} := (\text{dlc} + \text{dltt})/\text{at}$ ,  $\text{wcta} := (\text{act} - \text{lct})/\text{at}$ ,  $\text{clca} := \text{lct}/\text{act}$ ,  $\text{nita} := \text{ni}/\text{at}$ , and  $\text{futul} := \text{pi}/\text{lt}$ . that  $\text{oeneg}$  is equal to 1 if  $\text{lt}$  exceeds  $\text{at}$  and is equal to zero otherwise.  $\text{intwo}$  is equal to 1 if  $\text{ni}$  for the last two calendar years is negative and is equal to zero otherwise.  $\text{chine} = (\text{ni}_t - \text{ni}_{t-1})/(|\text{ni}_t| + |\text{ni}_{t-1}|)$ .  $\text{at}$ ,  $\text{dlc}$ ,  $\text{dltt}$ ,  $\text{act}$ ,  $\text{lct}$ ,  $\text{ni}$ ,  $\text{pi}$ ,  $\text{lt}$  are all annual items corresponding to total assets, debt in current liabilities, long-term debt, current assets, current liabilities, net income, pretax income, and total liabilities, respectively. Note that the number of stocks for which this factor is available during the first 5 investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 6$ .

## C.5 Intangible

### C.5.1 Egr

Following [Bazdrech et al. \(2008\)](#), we measure employee growth rate (Egr) as the growth rate in the number of employees (item emp) from calendar year  $t - 2$  to calendar year  $t - 1$ , where  $t$  denotes the current calendar year. Reversal of Egr is used as the actual factor.

### C.5.2 $\Delta\text{ade}$

Following [Chemmanur and Yan \(2009\)](#), we measure change in advertising expense ( $\Delta\text{ade}$ ) as the the natural log of the ratio of advertising expenses in calendar year  $t - 1$  to advertising expenses in calendar year  $t - 2$ , where  $t$  denotes the current calendar year. Note that the number of stocks for which this factor is available during some of the first 181 investment periods is less than 500, and the number available from the 182th investment date to the end is always less than 1000. As a result, for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 182$  whereas for dimension  $N = 1000$ , we do not consider this factor.

### C.5.3 Rdi

Following [Eberhart et al. \(2004\)](#), we measure R&D increase (Rdi) as the growth rate in R&D expenses (item xrd) from calendar year  $t - 2$  to calendar year  $t - 1$ , where  $t$  denotes the current calendar year. Note that the number of stocks for which this factor is available during some of the first 26 investment periods is less than 500, and the number available from the 27th investment date to the end is always less than 1000. As a result, for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 27$  whereas for dimension  $N = 1000$ , we do not consider this factor.

### C.5.4 Ad/M

As in [Chan et al. \(2001\)](#), we measure advertisement expense-to-market (Ad/M) as advertising expenses (item xad) for calendar year  $t - 1$  divided by the market capitalization (ME) on day  $h - 1$ , where  $t$  denotes the current calendar year. Note that the number of stocks for which this factor is available during some of the first 169 investment periods is less than 500, and the number available from the 170th investment date to the end is always less than 1000. As a result, for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 170$  whereas for dimension  $N = 1000$ , we do not consider this factor.

### C.5.5 RD/S

Following [Chan et al. \(2001\)](#), we measure R&D-to-sales (RD/S) as R&D expenses (annual item xrd) divided by sales (item sale). Note that the number of stocks for which this factor is available during some of the first 22 investment periods is less than 500, and the number available from the 23th investment date to the end is always less than 1000. As a result, for

dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 23$  whereas for dimension  $N = 1000$ , we do not consider this factor.

### C.5.6 RD/M

As in [Chan et al. \(2001\)](#), we measure R&D-to-market (RD/M) as R&D expenses (annual item `xrd`) for calendar year  $t - 1$  divided by the market capitalization (ME) on day  $h - 1$ , where  $t$  denotes the current calendar year. Note that the number of stocks for which this factor is available during some of the first 22 investment periods is less than 500, and the number available from the 23th investment date to the end is always less than 1000. As a result, for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 23$  whereas for dimension  $N = 1000$ , we do not consider this factor.

### C.5.7 Rc/A

Following [Li \(2011\)](#), we measure R&D capital-to-assets (Rc/A) as the ratio of R&D capital (Rc) to total assets (item `at`). Rc is a weighted average of R&D expenses (annual item `xrd`) over the last five calendar years with a depreciation rate of 20%:

$$\text{Rc} := \text{xrd}_{t-1} + 0.8\text{xrd}_{t-2} + 0.6\text{xrd}_{t-3} + 0.4\text{xrd}_{t-4} + 0.2\text{xrd}_{t-5} , \quad (\text{C.13})$$

where  $t$  denotes the current calendar year. Note that the number of stocks for which this factor is available during some of the first 30 investment periods is less than 500, and the number available from the 31st investment date to the end is always less than 1000. As a result, for dimension  $N = 500$ , we start the portfolio formation on investment date  $h = 31$  whereas for dimension  $N = 1000$ , we do not consider this factor.

### C.5.8 OL

Following [Novy-Marx \(2011\)](#), we measure operating leverage (OL) as cost of goods sold (quarterly item `cogsq`) plus selling, general, and administrative expenses (item `xsgaq`), then divided by total assets (item `atq`). Note that the number of stocks for which this factor is available during the first 32 investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 33$ .

## C.6 Trading frictions

### C.6.1 Turn

Following [Datar et al. \(1998\)](#), we measure the share turnover (Turn) as its average daily share turnover over the previous six months from  $t - 6$  to  $t - 1$  (that is, from day  $h - 126$  through day  $h - 1$ ). Daily turnover is the number of shares traded (item `vol`) divided by the number of shares outstanding (item `csho`). To account for the institutional features of the way NASDAQ and NYSE volume are reported, we adjust the trading volume for NASDAQ stocks as in [Gao and Ritter \(2010\)](#): Previous to 02/01/2001, we divide NASDAQ volume by 2.0; from

02/01/2001 through 12/31/2001, we divide NASDAQ volume by 1.8; for 2002 and 2003, we divide NASDAQ volume by 1.6; and from 2004 on, we use the original NASDAQ volume. Reversal of Turn is used as the actual factor.

### C.6.2 Tvol

Following [Ang et al. \(2006\)](#), we measure total volatility (Tvol) as the standard deviation of a stock's daily returns over the previous month  $t - 1$  (that is, from day  $h - 21$  through day  $h - 1$ ). Reversal of Tvol is used as the actual factor.

### C.6.3 Avol

Following [Bandyopadhyay et al. \(2010\)](#), we measure accrual volatility (Avol) as the standard deviation of the ratio of total accruals (TA) to total sales (item saleq) over the previous 16 quarters from quarter  $t - 16$  to quarter  $t - 1$ , where  $t$  denotes the current quarter. TA is defined in their equation (7); the only difference is that we use quarterly data here. Reversal of Avol is used as the actual factor. Note that the number of stocks for which this factor is available during the first 27 investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 28$ .

### C.6.4 Cvol

Following [Huang \(2009\)](#), we measure cash flow volatility (Cvol) as the standard deviation of cash flow (CF) over the previous 16 quarters from quarter  $t - 16$  to quarter  $t - 1$ , where  $t$  denotes the current quarter. CF is defined as the sum of income before extraordinary items (item ibq), depreciation and amortization expense (item dpq, zero if missing), and the increase in net non-cash working capital ( $\Delta WC$  in Section [C.3.10](#) with quarterly data). Reversal of Cvol is used as the actual factor. Note that the number of stocks for which this factor is available during the first six investment periods is less than 1000. As a result, for dimension  $N = 1000$ , we start the portfolio formation on investment date  $h = 7$ .

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