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**Asymmetric Information in
Frictional Markets for Liquidity:
On the Non-Equivalence of Credit and Sale**

Florian Madison

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ASYMMETRIC INFORMATION IN FRICTIONAL MARKETS FOR LIQUIDITY

ON THE NON-EQUIVALENCE OF CREDIT AND SALE[†]

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February 24, 2016

WORK IN PROGRESS

ABSTRACT: In the aftermath of the global financial crisis, the market for unsecured credit literally dried out and collateral secured debt became the most widely used concept to coinsure against liquidity shocks. However, since financial assets are usually unproductive, the question comes up why institutions in the need of cash do not just simply sell these assets rather than using them as collateral. The aim of this paper is to develop a non-equivalence between secured credit and outright sale in the presence of asymmetric information and to show through a signaling game, why the willingness to deposit assets as collateral is a best response.

JEL CLASSIFICATION: D82, E44, G12, G21

KEYWORDS: Liquidity, Asymmetric Information, Collateral, Undeclared Equilibrium

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1. INTRODUCTION

In the aftermath of the global financial crisis, the market for unsecured credit literally dried out and collateralized debt became the most common source of financing. Thereby, different financial securities including sovereign bonds, corporate bonds, as well as asset backed securities, are used to raise liquidity and serve as security in case of default. The main shelter for these belongings is the over-the-counter interbank market, where banks with excess liquidity provide credit to other banks in the need of cash (see Gorton and Metrick (2012*b*) and Gorton and Metrick (2012*a*)), which allows them to coinsure against liquidity shocks and prevents solvent banks from illiquidity (as shown in Allen and Gale (2000) and Leitner (2005)). However, since financial assets are usually unproductive, the question comes up why institutions in the need of cash do not just simply sell these assets rather than entering a bilateral secured credit contract. The presumption in this work is that this behavior is triggered by asymmetric information about the future payoff of the respective asset.

The aim of this paper is to develop a new monetarist model including an asset market with asymmetric information, where outright sale and secured credit coexist. Thereby, the objective is to achieve a self-regulation in the market for liquidity through signaling. More specifically, the goal is to show that in the presence of hidden information concerning the future payoff of the deposited collateral, the decision to favor a bilateral secured credit contract to an outright sale of assets represents an optimal choice on the part of the decision maker, i.e. the party in need of liquidity. Hence, there exists a non-equivalence between secured credit and outright sale under adverse selection.

The basic model used for this analysis is the search-theoretic framework established by Lagos and Wright (2005) and the extension introduced by Rocheteau and Wright (2005). Additionally, in order to account for the information sensitivity of the collateral, asymmetric information is introduced, which extends the model as done in Rocheteau (2011). Thereby, at the beginning of each period agents are privately informed about the future output of the assets they're currently holding, similar to the fruits of a Lucas (1978) tree. Given that, the informed party makes the offer, consisting of secured credit and/or sale, which determines the signaling structure of the bargaining game using Perfect Bayesian Equilibria (PBE) and the uninformed party has to decide whether or not to accept the proposed terms of trade. However, different to Rocheteau (2011), the equilibrium outcome is not only restricted to the Riley (1979) least-inefficient separating equilibrium induced through the Cho and Kreps (1987) Intuitive Criterion. Instead, we allow as well for Hellwig (1987) efficient pooling equilibria by introducing the refinement of the 'undefeated equilibrium' established by Mailath et al. (1993).¹ The decision, which of the two equilibria (separating or pooling) to chose, is solely taken by the high-value asset holder and partially depends on the distribution of types in the economy. While at an outright sale of assets, depending on the fraction of low-types, the cost of separation

¹At the same time, but completely independent, Ayushi Bajaj, a phd student of Guillaume Rocheteau at University of California, Irvine has developed a framework using the same equilibrium refinement (Bajaj (2015)). However, the content and focus of her paper differ strongly.

vary and high types may become better off offering a pooling contract, at a collateral secured credit contract, this distribution is completely irrelevant.

The results show that by considering a signaling game with Perfect Bayesian Equilibria in a pure asset sale economy and a high share of low-value asset holders in the market, all but the least-inefficient separating equilibrium can be dismissed. Thus, given the chosen refinement, all pooling equilibria are ruled out and high-value asset holders effectively separate themselves from the low-value consumers through asset-retention. However, this separation comes at the cost of sacrificing a first-best equilibrium due to the resulting inefficiently low consumption. On the contrary, if the share of low-value asset holders is relatively small, allocations change to an efficient pooling equilibrium, since the high-type is not willing to take the cost of separation anymore. Considering a secured credit economy, high as well as low types, prefer entering a pooling credit contract rather than selling their asset today at a pooling price, no matter the proportion of low-types in the economy. Given the counterparty's rational expectations indirectly determining the credit contract's terms of trade, both types will honor their credit obligations and agree to buy back the deposited collateral. While a low-type is indifferent between a pooling credit contract and his complete-information offer (Akerlof (1970) market for lemons), the chosen terms of trade allow the high-type to inherit the high future dividend of his pledged asset. Hence, through offering a bilateral secured credit contract, we obtain a self-selecting first-best equilibrium outcome in an economy with asymmetric information, justifying the observed behavior of banks on the over-the-counter market for liquidity.

There is a broad literature analyzing collateral secured credit contracts. Bester (1985) in his work shows that collateral allows agents on the interbank market to overcome asymmetric information about the solvency of the respective counterparty willing to borrow. Flannery (1996), however, argues that while other market participants may know about the overall solvency of the respective counterparty, they're not aware of the individual assets in the portfolio. Given that, a lender might end up getting the low quality assets which may lead to a lemons problem, since a bank in need of liquidity only liquidates the bad assets. As a result, loans may not generate their full value on the interbank market (haircuts). Kocherlakota (2001) in his work uses collateral to enforce credit contracts and discourage lenders from default. Ferraris and Watanabe (2008) introduce collateral secured loans in a monetary economy, while Ferraris and Watanabe (2011) extend the given framework by introducing collateral fluctuations. Berentsen and Waller (2011) analyze the trade-off of liquidity constrained agents between selling assets and borrowing by comparing outside (selling) to inside (credit) bonds. Thereby, they conclude that any allocation in an economy with inside bonds can be replicated by an economy with outside bonds, but not vice versa. Monnet and Narajabad (2012) however show that if agents face uncertainty about the value of holding the security in the future, they prefer to conduct a repurchase agreement instead of just selling the asset, while Parlatore (2015) analyzes this trade-off in the presence of uncertainty about the outcome of the financed risky projects. Gottardi et al. (2015) in their latest work model repurchase agreements and show that haircuts on repos can increase without any counterparty risk.

Concerning the organization of the paper, section 2 will give an introduction into the environment of the discussed theoretical framework. Section 3 then studies the bargaining game in an economy where liquidity is acquired through the outright sale of assets, bilateral secured credit, or a combination of both. Within this section, the concept of the undefeated equilibrium established by Mailath et al. (1993) is explained in detail and applied. Using these insights, section 4 then analyzes the stationary equilibrium. Section 5 discusses the resulting properties including a numerical analysis and the conclusion can be found in section 6.

2. ENVIRONMENT

The model is based on the framework established by Lagos and Wright (2005) and the extensions introduced by Rocheteau and Wright (2005) and Rocheteau (2011). Time is discrete, starts at $t = 0$ and continues forever. There is a continuum $[0, 1]$ of infinitely lived agents and each period is divided into two subperiods (markets). The first subperiod is called the settlement market (CM), where agents produce and consume a general good, redeem the claims of the previous period and adjust their portfolios in a centralized market. The second subperiod is the decentralized goods market (DM), in which agents trade assets against (search-) goods through bilateral sale agreements or lending and borrowing against collateral. Thereby, the buyer proposes a take-it-or-leave-it offer to the seller. If the offer is accepted, the trade takes place. The discount factor across periods is defined as $\beta \in (0, 1)$, where $\beta = (1 + r)^{-1}$ and r is the time rate of discount. In the following, the timing of events and the two markets are discussed in detail, starting with the centralized settlement market (see Figure 1).

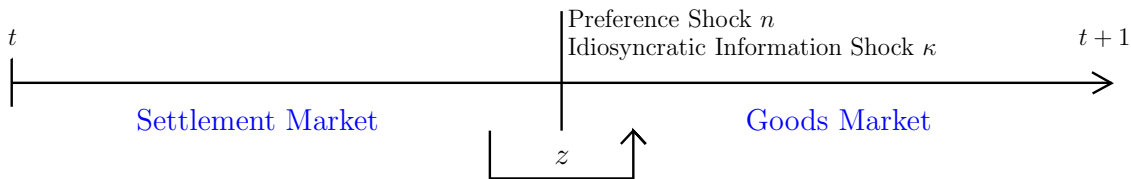


Figure 1: Environment

In the first subperiod (CM), all agents produce and consume a general good z , settle claims and adjust their portfolios. The general goods are produced solely through inputs of labor according to a constant return-to-scale technology, whereas the production of one general consumption good z demands one unit of labor h leading to one unit of disutility $-h$, i.e. there is no initial endowment $A > 0$ as done in Rocheteau (2011). Contrary to the basic framework established by Lagos and Wright (2005), general goods (henceforth assets) are storable and serve as medium of exchange/collateral in the subsequent goods market.

At the beginning of the second subperiod (DM), agents are subject to an idiosyncratic shock determining their role in the goods market. With probability $n \in (0, 1)$ agents want to produce but cannot consume the search good q , while with complement probability $1 - n$ agents want to consume, but cannot produce. We refer to consumers as buyers (consuming in market 2 and producing in market 1) and to producers as sellers (producing in market 2 and consuming in market 1). A buyer derives utility $u(q_b)$ from goods market consumption, where $u(q_b)$ is continuously differentiable and satisfies the Inada (1963) conditions and q_b denotes consumption in market 2.² A producer incurs a linear utility cost $c(q_s)$ of $-q_s$ for producing q_s units of output. All market participants and trades in the goods market are anonymous, lack commitment, and trading histories are private information, which precludes unsecured bilateral credit between consumers and producers. Given there is a double coincidence of wants problem, a medium of exchange for direct settlement or collateral secured credit is inevitable for trade to occur, as shown by Kocherlakota (1998), Wallace (2001) and Shi (2006) among others.³

Additional to the preference shock, each agent is privately informed about the terminal value κ of his asset in the subsequent settlement market. The realization of this idiosyncratic information shock is common to all assets held by an agent, but independent across agents. Thereby, we impose a stochastic flow of output (real goods) $\kappa \geq 0$, which can take on two values $0 \leq \kappa^L \leq \kappa^H \leq \infty$. With probability $\pi \in (0, 1)$, the terminal output of an asset is $\kappa = \kappa^H$ ($\pi \equiv \text{Prob}[\kappa = \kappa^H]$), while with complement probability $(1 - \pi)$, $\kappa = \kappa^L$. Given that, the expected output of an asset is $R = \bar{\kappa} = \pi\kappa^H + (1 - \pi)\kappa^L$. Based on the preference shock and the idiosyncratic information shock, agents trade assets against goods.

SOCIAL OPTIMUM: Before moving on to the bargaining game and deriving the steady-state equilibria of the model, we consider the first-best solution. Without loss of generality, it is assumed that the social planner treats all agents symmetrically and maximizes their average lifetime utility, subject to the market clearing condition:

$$(1 - \beta)W = [(1 - n)u(q_b) - nq_s] + (\beta R - 1)z \quad (1)$$

$$\text{s.t.} \quad (1 - n)q_b = nq_s. \quad (2)$$

While the first term on the right hand side of (1) denotes the expected utility from consuming and producing in the goods market, the second term represents the utility of producing assets and receiving the expected payoff R in the settlement market of the subsequent period $t + 1$. It needs to be imposed that $\beta R \leq 1$, since otherwise ($\beta R > 1$) agents would store infinite amounts of goods, which is inconsistent with equilibrium. Since all agents enter the period with the same amount

²Inada (1963) conditions are: the function is continuously differentiable, strictly increasing in q_b and concave [$u'(q_b) > 0 > u''(q_b)$]. The value of the function at $q_b = 0$ is zero, the limit of the derivative towards zero is positive infinity and the limit of the derivative towards positive infinity is zero.

³As will be shown, in the presence of asymmetric information, these two means of payment are not equivalent, which contradicts the results of Lagos (2010).

of assets, q_b and q_s is the same for all of them. Therefore, it is straightforward that $q_s = \frac{1-n}{n}q_b$. Solving the planner's problem, subject to the feasibility constraint, the first-best allocation, denoted by (q^*, z^*) satisfies the following first order conditions:

$$u'(q^*) = 1 \quad (3)$$

$$\beta R = 1. \quad (4)$$

This implies that at the optimum, the marginal utility of consumption in the goods market ($= u'(q^*)$) equals the marginal cost of production ($= 1$), while the marginal utility of consumption in the settlement market ($= 1$) equals the discounted value of the marginal returns on assets ($= \beta R$), which corresponds to the Friedman rule.

3. BARGAINING GAME

Consumers and producers, both holding a portfolio of information sensitive assets z , meet and trade, using their assets as medium of exchange. Thereby, payment takes place either through direct settlement (outright sale), through a bilateral secured credit contract, where buyers in the goods market (DM) pledge their real asset (or a fraction of it) as collateral and agree to fulfill their credit obligation in the subsequent settlement market (CM) at a price defined today, or a mixture of both.⁴ Since all credit contracts need to be backed by collateral in case of a default, agents face a borrowing constraint, as defined by Kiyotaki and Moore (1997) in a non-monetary economy.

SETTLEMENT MARKET: $W(z, l; \kappa)$ denotes the expected value of entering the settlement market with z units of assets at time t , their corresponding value κ , and l units of credit obligations from the previous period, where the subscripts b and s stand for the buyer and the seller respectively. Thereby, it is assumed that the consumer's portfolio is common knowledge in the match, but not the terminal value of the asset:

$$W(z, l; \kappa) = -h + l_s - l_b + \max_{z'} \mathbb{E}V(z'; \kappa') \quad (5)$$

$$\text{s.t. } z' = h + z[i\kappa^H + (1-i)\kappa^L] \quad (6)$$

where $V(z; \kappa)$ is the value of entering the goods market with z units of the general good and i is an indicator function with $i = \{0, 1\}$. The expected value corresponds to the terminal output of the assets $\kappa \in \{\kappa^L, \kappa^H\}$ at the beginning of the subsequent period $t + 1$. Solving the budget constraint

⁴The secured credit contract mechanism is thereby equivalent to a repurchase agreement (repo), where agents sell an asset against liquidity, but at the same time agree to buy it back at a later point in time and at a price determined today.

after h and plugging in to the value function $W(z, l; \kappa)$ yields:

$$W(z, l; \kappa) = z[i\kappa^H + (1-i)\kappa^L] - l_b + l_s + \max_{z'} \{-z' + \mathbb{E}V(z'; \kappa')\}. \quad (7)$$

whereas $W(z, l; \kappa)$ is linear in wealth with intersect $W(0, 0; \kappa)$ and slope $\bar{\kappa}$. Moreover, the agent's portfolio choice in the subsequent goods market z' is independent of his current asset holdings when entering the market z . The first-order and the envelope conditions are:

$$\mathbb{E}V_{z'} = 1; \quad W_z = [i\kappa^H + (1-i)\kappa^L]; \quad W_{l_b} = -1; \quad W_{l_s} = 1 \quad (8)$$

where W_z , W_{l_s} , W_{l_b} and $V_{z'}$ are the partial derivatives of $W(z, l; \kappa)$ and $V(z'; \kappa')$ with respect to z , l_s , l_b and z' .

GOODS MARKET: The bargaining game between the buyer and the seller in the goods market has the structure of a signaling game (see Figure 2), i.e. the informed agent (buyer) moves first (makes the offer). A strategy for the buyer is to specify an offer $(q_b, l_b, y_b) \in \mathcal{F} = \mathbb{R}_+ \times [0, z]$, where q_b is the amount of search goods received, l_b is the credit obligation of real assets to the seller in period $t+1$ and y_b is the proportion of assets sold/ deposited as collateral (perfectly divisible), all as a function of the buyer's type κ^χ , i.e. its private information about the future output of the assets, where $\chi \in \{L, H\}$. Thereby, the transfer of assets is constrained by the agent's current portfolio z , which is in line with the lotteries introduced by Berentsen et al. (2002), where agents in a bilateral trade are able to offer their asset probabilistically. A strategy for the seller is to define an acceptance rule that specifies a set $\mathcal{A}_s \subseteq \mathcal{F}$ of acceptable offers.

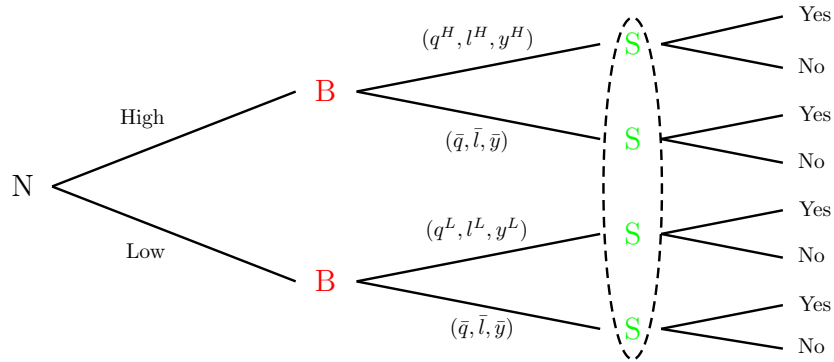


Figure 2: Signaling game (uninformed producers)

The buyer's payoff in the dividend state κ is:

$$\begin{aligned} V_b(z; \kappa_b) &= \left\{ u(q_b) + \beta \left[(1-d)W(z - y_{1,b}, l_b; \kappa_b) + dW(z - y_{1,b} - y_{2,b}, 0; \kappa_b) \right] \right\} \mathbb{I}_{\mathcal{A}}(q_b, l_b, y_b) \\ &+ \beta W(z, 0; \kappa_b) (1 - \mathbb{I}_{\mathcal{A}}(q_b, l_b, y_b)) \end{aligned} \quad (9)$$

where $y_{1,b} \in \{0, z - y_{2,b}\}$ is the proportion of assets used as a direct means of payment and $y_{2,b} \in \{0, z - y_{1,b}\}$ the proportion used as collateral, allowing sale and secured credit to coexist in the economy. $\mathbb{I}_{\mathcal{A}_s}(q_b, l_b, y_b)$ is an indicator function equal to 1 iff $(q_b, l_b, y_b) \in \mathcal{A}_s$, i.e. if the buyer's offer is in the seller's set of acceptable offers, and zero otherwise. We allow agents to default on their credit obligation, whereas default is a binary discrete choice variable $d \in \{0; 1\}$, where $d = 0$ denotes repayment and $d = 1$ default (which implies $l_b = 0$).⁵ Using the linearity of the agent's value function in the settlement market, the buyer's payoff in case of trade can be reduced to its surplus: $S_b^X = \left[u(q_b) - \beta y_{1,b} \kappa_b - \beta \left[(1-d)l_b + dy_{2,b} \kappa_b \right] \right] \mathbb{I}_{\mathcal{A}_s}(q_b, l_b, y_b)$. Similarly, the producer's payoff is:

$$\begin{aligned} V_s(z; \kappa_s) &= \left\{ -q_s + \beta \left[(1-d)W(z + y_{1,s}, l_s; \kappa_s) + dW(z + y_{1,s} + y_{2,s}, 0; \kappa_s) \right] \right\} \mathbb{I}_{\mathcal{A}_s}(q_b, l_b, y_b) \\ &+ \beta W(z, 0; \kappa_s) (1 - \mathbb{I}_{\mathcal{A}_s}(q_b, l_b, y_b)). \end{aligned} \quad (10)$$

which can be reduced to $S_s^X = \left[-q_s + \beta y_{1,s} \kappa_b + \left[(1-d)l_s + dy_{2,s} \kappa_b \right] \right] \mathbb{I}_{\mathcal{A}_s}(q_b, l_b, y_b)$. In order for a producer to accept the offer made by the consumer, it has to form expectations (posterior beliefs) about the terminal output of the consumer's asset. Let $\lambda(q, l, y) \in [0, 1]$ represent the updated belief of a producer that the consumer holds a high-value asset, i.e. $\kappa_b = \kappa_b^H$, conditional on the offer (q_b, l_b, y_b) made. Therefore, the posterior expected value equals to:

$$\mathbb{E}_\lambda[\kappa_b] = \lambda(q, l, y) \kappa_b^H + [1 - \lambda(q, l, y)] \kappa_b^L. \quad (11)$$

Given the seller's updated belief, he optimally chooses whether to accept or reject the buyer's offer. For a given belief system λ , the set of acceptable offers $\mathcal{A}_s(\lambda)$ for the producer is:

$$\mathcal{A}_s(\lambda) = \left\{ (q, l, y) \in \mathcal{F} : -q_s + \beta \left[y_{1,s} + (1-d)l_s + dy_{2,s} \right] \left[\lambda \kappa_b^H + [1 - \lambda] \kappa_b^L \right] \geq 0 \right\} \quad (12)$$

i.e. the producer has to have a positive expected surplus from trading in the goods market. Assuming a tie-breaking rule according to which a seller agrees to any offer that makes him indifferent between accepting and rejecting, the buyer holding an asset of quality κ_b chooses an offer that maximizes his surplus S_b^X , taking as given the acceptance rule of the seller \mathcal{A}_s . Therefore, the consumer's problem

⁵Thinking about repurchase agreements, $d = 0$ corresponds to the situation where consumers make use of their repurchase option, while at $d = 1$ they don't.

reduces to:

$$\begin{aligned}
S_b^x(z; \kappa_b) = & \max_{d \in \{0;1\}} \left[(1-d) \max_{q_b, l_b, y_{i,b}} \left[u(q_b) - \beta y_{1,b} \kappa_b - \beta l_b \right] \right. \\
& + \quad d \max_{q_b, l_b, y_{i,b}} \left[u(q_b) - \beta y_{1,b} \kappa_b - \beta y_{2,b} \kappa_b \right] \left. \right] \mathbb{I}_{\mathcal{A}_s}(q_b, l_b, y_b) \\
\text{s.t.} \quad & (q_b, l_b, y_b) \in \mathbb{R}_+ \times [0, z] \\
\text{s.t.} \quad & \mathcal{A}_s(\lambda)
\end{aligned} \tag{13}$$

subject to the consumer's budget constraint and the seller's acceptance rule.

We consider Perfect Bayesian Equilibria (PBE), whereas an equilibrium of the bargaining game is a profile of strategies for the buyer and the seller and a system of beliefs $\lambda(q, l, y)$. If (q_b, l_b, y_b) is an offer made in equilibrium, then $\lambda(q, l, y)$ is derived from the producer's prior belief according to Bayes's rule. Without restriction, every allocation in this bargaining game can be a Perfect Bayesian Equilibrium. Therefore, in order to narrow the set, the equilibrium has to be refined. We apply the concept of the *undefeated equilibrium* established by Mailath et al. (1993). Thereby, depending on the distribution of high and low types in the population, the concept selects the least-inefficient separating equilibrium (see Riley (1979)) using the Intuitive criterion established by Cho and Kreps (1987) if it is also a Perfect Sequential Equilibrium outcome, and otherwise the most efficient pooling equilibrium (see Hellwig (1987)). The decision which of the two to select is triggered solely by the surplus of the consumer holding a high-valued asset S_b^H , since low-value asset holders in general prefer pooling against separation, allowing them to increase their surplus on the cost of the high-types. Therefore, depending on the high type's surplus, one defeats (Pareto dominates) the other, or vice versa, which eliminates pareto inefficiencies.⁶

Figure 3 demonstrates this line of reasoning, whereas the lines S_b^H and S_b^L reflect indifference curves of the respective types. Thereby, the graph on the left hand side shows an undominated separating equilibrium and the graph on the right hand side a dominated one (i.e. a pooling equilibrium). The decisive factor which of the two to choose is the high-type's surplus and this again depends on the distribution of types in the economy (influencing the expected payoff $\mathbb{E}[\kappa] = \pi \kappa_b^H + (1 - \pi) \kappa_b^L$). In the following, the two possible equilibria are derived, starting with the separating equilibrium.

⁶In order to get a better understanding for the concept of the undefeated equilibrium, a simple Spence (1973) job market signaling example is provided in [Appendix A](#).

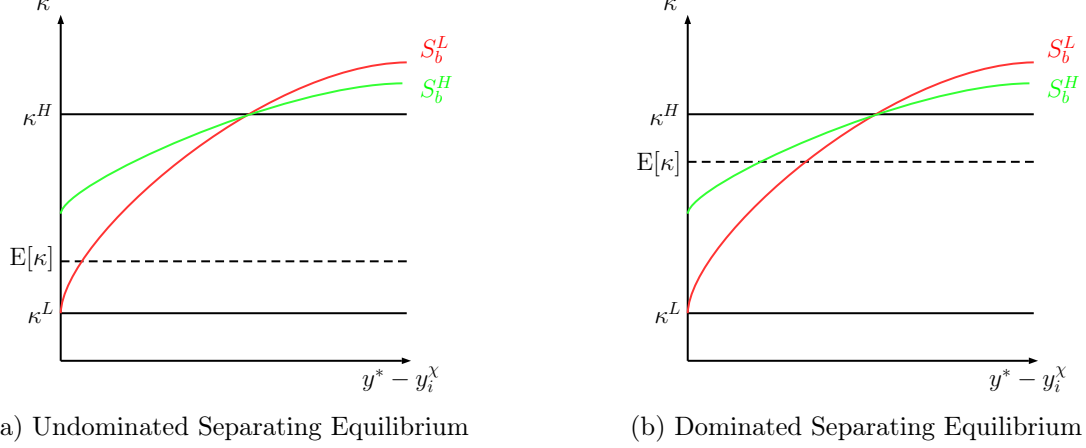


Figure 3: Undominated and dominated separating equilibria

3.1 SEPARATING OFFER

The proposed equilibrium (q, l, y) fails the Cho and Kreps (1987) Intuitive Criterion if there exists an out-of-equilibrium offer $(\tilde{q}, \tilde{l}, \tilde{y})$ such that the following is true:

$$\begin{aligned}
 u(\tilde{q}_b) - \beta [\tilde{y}_{1,b}\kappa_b^H + (1-d)\tilde{l}_b + d\tilde{y}_{2,b}\kappa_b^H] &> u(q_b^H) - \beta [y_{1,b}^H\kappa_b^H + (1-d)l_b^H + dy_{2,b}^H\kappa_b^H] \\
 u(\tilde{q}_b) - \beta [\tilde{y}_{1,b}\kappa_b^L + (1-d)\tilde{l}_b + d\tilde{y}_{2,b}\kappa_b^L] &< u(q_b^L) - \beta [y_{1,b}^L\kappa_b^L + (1-d)l_b^L + dy_{2,b}^L\kappa_b^L] \\
 -\tilde{q}_s + \beta [\tilde{y}_{1,s}\kappa_b^H + (1-d)\tilde{l}_s + \tilde{y}_{2,s}\kappa_b^H] &\geq 0
 \end{aligned} \tag{14}$$

whereas the first inequality corresponds to the buyer holding a high-value asset, the second inequality to the low type and the third to the producer forming beliefs. If this would not hold, the out-of-equilibrium offer $(\tilde{q}, \tilde{l}, \tilde{y})$ would make the H-type strictly better off, the L-type strictly worse off and the offer would still be accepted by the producer, since he believes that it comes from a high-type. This is due to the fact that the low-type has no incentive to make such an offer.

DEFINITION 1: An equilibrium of this bargaining game (separating) is a pair of strategies and a belief system, $\{[q(\kappa), l(\kappa), y(\kappa)], \mathcal{A}_s, \lambda\}$, such that:

- (i) The terms of trade $[q(\kappa), l(\kappa), y(\kappa)]$ are a solution to the consumer's maximization problem (13) with $\kappa \in \{\kappa^L, \kappa^H\}$.
- (ii) The seller's acceptance rule $\mathcal{A}_s(\lambda)$ is given by (12).
- (iii) The belief system $\lambda(q, l, y)$ satisfies Bayes' rule and the Intuitive Criterion (14).

CONSUMER'S PROBLEM - LOW VALUE ASSET: In equilibrium, the low-type consumer can do no worse than to make the same offer he would under complete information, since this offer is always

acceptable to the producer, independent of his beliefs. However, at the same time, the low-type consumer cannot do any better than the complete-information offer, since otherwise the offer would have to be pooled with a high-type offer ($\tilde{y} < y^L$). Such offers are ruled out by the Intuitive criterion. Therefore, the problem of a buyer holding a low-value asset takes the following form:

$$\begin{aligned}
S_b^L(z; \kappa_b^L) &= \max_{q_{i,b}^L, l_b^L, y_{i,b}^L, d \in \{0,1\}} u(q_b^L) - \beta y_{1,b}^L \kappa_b^L - \beta [(1-d)l_b^L + dy_{2,b}^L \kappa_b^L] \\
\text{s.t.} \quad &-q_{1,s}^L + \beta y_{1,s}^L \kappa_b^L \geq 0 \\
\text{s.t.} \quad &-q_{2,s}^L + \beta [(1-d)l_s^L + dy_{2,s}^L \kappa_b^L] \geq 0 \\
\text{s.t.} \quad &0 \leq y_1^L + y_2^L \leq z
\end{aligned} \tag{15}$$

where $q_b^X = q_{1,b}^X + q_{2,b}^X$, $y_b^X = y_{1,b}^X + y_{2,b}^X$ and $S_b^L(z; \kappa_b^L)$ is the surplus of a consumer holding a low-dividend asset entering the goods market, subject to the seller's participation constraints and a no-debt constraint. From the second constraint one can see that $l_b^X \leq y_{2,b}^X \kappa^X$ has to hold in equilibrium, since otherwise strategic default would be incentivized, no matter the chosen offer.⁷ (q_1^L, y_1^L) is the allocation obtained through an exclusive outright sale of assets and (q_2^L, l^L, y_2^L) results through bilateral secured credit. Assuming market clearing $q_b^L = q_s^L$, $l_b^L = l_s^L$ and $y_b^L = y_s^L$, the solution to (15) is:

$$q_i^L = \min[q^*, \beta z \kappa_b^L] \tag{16}$$

$$y_i^L = \min\left[\frac{q_i^L}{\beta \kappa_b^L}, z - y_{-i}^L\right] \tag{17}$$

where q^* solves $u'(q) = 1$ and $i \in \{1, 2\}$ is the indicator for sale or credit. Since the buyer makes a take-it-or-leave-it offer to the seller, he appropriates the entire surplus of the match. If the asset return of the low-type buyer is sufficiently large ($\beta z \kappa_b^L \geq q^*$), trade is efficient, i.e. $q^L = q^*$ (benchmark). However, if the asset return is lower than the seller's cost of production ($\beta z \kappa_b^L < q^*$), $q^L < q^*$.⁸ Since we look at a complete information offer, consumers and producers are indifferent between outright sale and secured credit, as well as between repayment ($d = 0$) and default ($d = 1$) in the latter case.

CONSUMER'S PROBLEM - HIGH VALUE ASSET: A high-type consumer's offer (q_b^H, l_b^H, y_b^H) will be an equilibrium offer, if the low-type has no incentive to offer it instead of (q_b^L, l_b^L, y_b^L) . Hence, the

⁷This corresponds to the Kiyotaki and Moore (1997) borrowing constraint in non-monetary economies.

⁸If we had complete information about the asset quality in the goods market, the high-value asset holder would incorporate a weakly higher quantity than the low-type $q^H \geq q^L$, and at the same time spend a lower fraction of his assets $y^H \leq y^L$.

high-type's maximization problem takes the following form:

$$\begin{aligned}
S_b^H(z; \kappa_b^H) &= \max_{q_{i,b}^H, l_b^H, y_{i,b}^H, d \in \{0;1\}} u(q_b^H) - \beta y_{1,b}^H \kappa_b^H - \beta [(1-d)l_b^H + dy_{2,b}^H \kappa_b^H] \\
\text{s.t.} \quad &-q_{1,s}^H + \beta y_{1,s}^H \kappa_b^H \geq 0 \\
\text{s.t.} \quad &-q_{2,s}^H + \beta [(1-d)l_s^H + dy_{2,s}^H \kappa_b^H] \geq 0 \\
\text{s.t.} \quad &u(q_b^H) - \beta y_{1,b}^H \kappa_b^L - \beta [(1-d)l_b^H(\kappa_b^L) + dy_{2,b}^H \kappa_b^L] \leq S_b^L(z; \kappa_b^L) \\
\text{s.t.} \quad &0 \leq y_1^H + y_2^H \leq z
\end{aligned} \tag{18}$$

where the first two constraints are the seller's participation constraints, the third is an incentive compatibility constraint ensuring that the low-type consumer cannot make himself better off by offering (q_b^H, l_b^H, y_b^H) instead of (q_b^L, l_b^L, y_b^L) and the fourth is a no-debt constraint. Since without the incentive compatibility constraint the consumer's problem would be identical to the complete information case, this constraint has to bind in equilibrium. Otherwise, the low-type buyer would have an incentive to mimic the offer of the high-type and conversely, the producer would not have the correct belief that he faces a high-value asset holder anymore. From the maximization problem of the low-type buyer, we know that the participation constraints hold as equality (zero profit cond.). Solving the maximization problem, implying the market clearing conditions $y_s^H = y_b^H$, $l_s^H = l_b^H$ and $q_s^H = q_b^H$, the following equilibrium allocations emerge:

$$q_i^H = \min \left[\frac{\kappa_b^H}{\kappa_b^L} \left[u(q_i^H) - S_b^L(z; \kappa_b^L) \right], \beta z \kappa_b^H \right] \tag{19}$$

$$y_i^H = \min \left[\frac{q_i^H}{\beta \kappa_b^H}, z - y_i^L \right]. \tag{20}$$

Rewriting the incentive compatibility constraint:

$$u(q_i^H) - \frac{\kappa_b^L}{\kappa_b^H} q_i^H = u(q_i^L) - q_i^L \tag{21}$$

one can see that q_i^H is uniquely determined and has to fulfill: $q_i^H < q_i^L$, i.e. $q_i^H \in (0, q_i^L)$, where the solution (q_i^H, l_i^H, y_i^H) corresponds to the lowest possible q_i^H that solves equation (21).⁹ Using the fact that $q_i^H < q_i^L$, which implies that $q_i^H = \beta y_i^H \kappa_b^H < q_i^L = \beta y_i^L \kappa_b^L$, it immediately follows that $y_i^H < y_i^L \leq z$. Hence, high-type consumers perform asset retention in order to separate themselves from the low-type consumers in the market, i.e. they consume less than the low-type consumers.¹⁰ By doing this, buyers signal the high future payoff of their assets and hence secure better terms of

⁹This can be verified as follows: If $q_i^H = 0$, the left hand side of equation (21) is lower than the right hand side, and if $q_i^H = q_i^L$, the opposite holds. Additional to that, for all $q_i^H \leq q^*$, the left hand side is increasing in q_i^H .

¹⁰This is in line with the result in Demarzo and Duffie (1999).

trade (better prices $\frac{q_i^H}{\beta y_i^H} = \kappa_b^H > \kappa_b^L = \frac{q_i^L}{\beta y_i^L}$) in the goods market.¹¹

LEMMA 1: *All but the least-inefficient separating equilibrium can be dismissed. [Proof in Appendix B](#)*

Only the least-inefficient separating offer survives the Cho and Kreps (1987) Intuitive Criterion. This is due to the fact that it rules out pooling equilibria and therefore the buyer holding a low-value asset cannot do any better than revealing his type by making a complete-information offer to the seller. Being aware of that, the high-type can use this offer as a reference (benchmark) to separate himself from the low-type (see Figure 3a).

PROPOSITION 1: *In a separating equilibrium, the expected lifetime utility of a consumer V_b^χ with $\chi \in (L, H)$ is equivalent for outright sale \mathcal{S} , secured credit \mathcal{C} and strategic default \mathcal{D} , i.e. $V_{b,\mathcal{S}}^\chi = V_{b,\mathcal{C}}^\chi = V_{b,\mathcal{D}}^\chi$, but $V_{b,\mathcal{S}}^\chi \neq V_{b,\mathcal{S}}^{-\chi}$, $V_{b,\mathcal{C}}^\chi \neq V_{b,\mathcal{C}}^{-\chi}$ and $V_{b,\mathcal{D}}^\chi \neq V_{b,\mathcal{D}}^{-\chi}$.¹²*

The results of the bargaining game have shown, that in a separating equilibrium, a consumer holding an asset $\kappa_b \in \{\kappa_b^L, \kappa_b^H\}$ is indifferent between using the asset as a direct medium of exchange $V_{b,\mathcal{S}}^\chi$ or engaging in a bilateral secured credit contract $V_{b,\mathcal{C}}^\chi = V_{b,\mathcal{D}}^\chi$.¹³ The reason for that is the fact that through the performed separation (asset retention), the low- as well as the high-type is indifferent between outright sale and secured credit, i.e. the terms of trade for both settlement methods are identical. This supports the equivalence result published by Lagos (2010).

Comparing the two allocations, the following normative properties of the equilibrium emerge. Under complete information, the economy would achieve its first best solution if $z\kappa_b^L \geq q_s^*$, since the value of the low-dividend asset is large enough to allow for the efficient consumption q^* (hence implying that $z\kappa_b^H > q^*$). However, under incomplete information, first best is not achievable anymore, i.e. the equilibrium allocation is inefficient. This is due to the inefficiently low consumption of the buyers holding a high-dividend asset $q^H < q^* = q^L$. In the case where the asset holdings are lower than the seller's cost of production (i.e. $z\kappa_b^L < q^*$), consumption is inefficiently low in all DM matches, $q^H < q^L < q^*$.¹⁴ These results hold for the case where $\kappa^H > \kappa^L > 0$. However, if $\kappa^L = 0$, there will be no trade in the economy anymore, i.e. $q^H = q^L = y^H = y^L = 0$, as shown in Nosal and Wallace (2007).

¹¹This result corresponds to Gresham's Law by showing that the existence of lemons in the market crowds out good assets.

¹²The subscripts \mathcal{D} , \mathcal{S} and \mathcal{C} stand for 'strategic default', 'sale' and 'credit' respectively.

¹³As well, the agent is indifferent between repayment and default for the same reason as in the low-type's maximization problem.

¹⁴An incentive-feasible first-best allocation ($q^L = q^H = q^*$) under private information using the Cho and Kreps (1987) intuitive criterion is not achievable, since this would require pooling offers.

3.2 EFFICIENT POOLING OFFER

Having determined the least-inefficient separating equilibrium in the goods market, we can now move on to the most efficient pooling equilibrium $(\bar{q}, \bar{l}, \bar{y})$, which is defined as the one equilibrium where the surplus of a high-value asset holder S_b^H is maximized.¹⁵

DEFINITION 2: *An equilibrium of this bargaining game (pooling) is a pair of strategies and a belief system, $\{[q(\kappa), l(\kappa), y(\kappa)], \mathcal{A}_s, \lambda\}$, such that:*

- (i) *The terms of trade $[\bar{q}(\kappa), \bar{l}(\kappa), \bar{y}(\kappa)]$ are a solution to the consumer's maximization problem (13) with $\kappa = \kappa^H$.*
- (ii) *The seller's acceptance rule $\mathcal{A}_s(\lambda)$ is given by (12).*

The problem of the consumer holding a high-valued asset is:

$$\begin{aligned}
 \bar{S}_b^H(z; \kappa_b^H) &= \max_{\bar{q}_{i,b}, \bar{l}_b, \bar{y}_{i,b}, d \in \{0;1\}} u(\bar{q}_b) - \beta \bar{y}_{1,b} \kappa_b^H - \beta \left[(1-d) \bar{l}_b + d \bar{y}_{2,b} \kappa_b^H \right] \\
 \text{s.t.} \quad & -\bar{q}_{1,s} + \beta \bar{y}_{1,s} [\pi \kappa_b^H + (1-\pi) \kappa_b^L] \geq 0 \\
 \text{s.t.} \quad & -\bar{q}_{2,s} + \beta \left[(1-d) \bar{l}_s + d \bar{y}_{2,s} \kappa_b^L \right] \geq 0 \\
 \text{s.t.} \quad & 0 \leq \bar{y}_1 + \bar{y}_2 \leq z,
 \end{aligned} \tag{22}$$

where the first two constraints are the seller's participation constraints and the third a no-debt constraint. As one can see, the seller only accepts the buyer's offer, if the costs of production are covered in expectation, which is due to the fact that the pooling offer does not reveal the type of the consumer's asset and in line with the seller's acceptance rule (12) defined earlier. However, looking at the seller's participation constraint for a credit contract $(\bar{q}_{2,b}, \bar{l}_b, \bar{y}_{2,b})$, one can see that in case of a default, the constraint has to explicitly hold for a low-value asset holder κ_b^L (instead of the expected value $\mathbb{E}_\lambda[\kappa_b] = \lambda \kappa_b^H + (1-\lambda) \kappa_b^L$). This is due to the seller's rational expectations forecasting a strategic default on the part of the low-type otherwise.¹⁶ Given that, updating his beliefs, the above participation constraint results. Solving the buyer's maximization problem, implying $\bar{q}_b = \bar{q}_s$, $\bar{l}_b = \bar{l}_s$

¹⁵We restrict our attention to this kind of pooling equilibrium and abstain from all other possible equilibria, as defined in Mailath et al. (1993).

¹⁶If the seller's participation constraint was $-\bar{q}_{2,s} + \beta \left[(1-d) \bar{l}_s + d \bar{y}_{2,s} [\pi \kappa_b^H + (1-\pi) \kappa_b^L] \right] \geq 0$, the low value asset holder engaging in a bilateral secured credit contract would always have an incentive to default, since the future claim $\beta \bar{y}_2 [\pi \kappa_b^H + (1-\pi) \kappa_b^L]$ is higher than the actual value of the deposited collateral $\beta \bar{y}_2 \kappa_b^L$, incentivizing default.

and $\bar{y}_b = \bar{y}_s$, the following equilibrium allocations emerge:

$$\bar{q}_1 = \min \left[\bar{q}_1, \beta z [\pi \kappa_b^H + (1 - \pi) \kappa_b^L] \right] \quad (23)$$

$$\bar{q}_2 = \min \left[q^*, \beta z \kappa_b^L \right] \quad (24)$$

$$\bar{y}_1 = \min \left[\frac{\bar{q}_1}{\beta [\pi \kappa_b^H + (1 - \pi) \kappa_b^L]}, z - \bar{y}_2 \right] \quad (25)$$

$$\bar{y}_2 = \min \left[\frac{\bar{q}_2}{\beta \kappa_b^L}, z - \bar{y}_1 \right]. \quad (26)$$

where $\bar{q}_1 < q^*$ if $\pi < 1$ and $\bar{q}_2 \leq q^* \forall \pi$. If the constraint is binding, $\bar{q}_1 = \beta z [\pi \kappa_b^H + (1 - \pi) \kappa_b^L]$ and $\bar{y}_1 = z$. However, if the constraint is non-binding, $u'(\bar{q}_1; \kappa_b^H) = \frac{\kappa_b^H}{\pi \kappa_b^H + (1 - \pi) \kappa_b^L} > 1$ if $\pi < 1$. Thus, in a pooling equilibrium, a consumer is not able to reach its first-best consumption by offering an outright sale contract, i.e. $\bar{q}_1 < q^*$ and $\bar{y}_1 = \frac{\bar{q}_1}{\beta [\pi \kappa_b^H + (1 - \pi) \kappa_b^L]}$, if there is a positive number of low-types in the economy. Anyhow, engaging in a secured credit contract instead, the high as well as the low types are able to obtain their first-best quantity $\bar{q}_2 = q^*$ (given $q^* \leq \beta z \kappa_b^L$), independent of the distribution of types in the economy.

PROPOSITION 2: *In a pooling equilibrium, the high-value asset holder κ_b^H strictly prefers a collateral secured credit contract, since $\bar{V}_{b,C}^H > \bar{V}_{b,S}^H > \bar{V}_{b,D}^H \forall \pi \neq 1$. If $\pi = 1$, then $\bar{V}_{b,C}^H = \bar{V}_{b,S}^H = \bar{V}_{b,D}^H = V_b^{H*}$.*

The results of the bargaining game show that the high type prefers engaging in a bilateral secured credit contract rather than selling the asset at pooling price below value, i.e. $\bar{\kappa}_b < \kappa_b^H$. The reason for that is the following. Given the fact that the seller is not able to distinguish between the two consumers, he demands an amount \bar{y} of assets to be pledged, covering his production costs $-\bar{q}_s$ in expectation. Hence, there occurs overcollateralization on the part of the high-value asset holder by depositing too many assets ($\bar{y}_{2,b} = y^{L*} > y^{H*}$), while the low-type mimicking the offer pledges the same amount as in the complete-information case elaborated before (due to the seller's rational expectations). However, since the high-type is aware of his assets' high future dividend, he has an incentive to fulfill his credit obligation $\beta \bar{y} \kappa_b^L$ in order to inherit the assets' output $\beta \bar{y} \kappa_b^H$ in period $t + 1$. The low-type, however, is indifferent between repayment and default, since the pooling offer corresponds to its complete-information offer $(\bar{q}_2, \bar{l}, \bar{y}_2) = (q^L, l^L, y^L)$. In other words, high-types can use the pooling secured credit agreement as a bypass for future income streams without sacrificing the high dividend of the pledged asset. At the same time, the producer does not have to fear any losses through default on the part of the low-type.

LEMMA 2: *In a pooling secured credit equilibrium $(\bar{q}_2, \bar{l}, \bar{y}_2)$, we observe 'overcollateralization' on the part of the high type κ_b^H .*

Applying the above result to the low-value asset holders, the following condition has to be fulfilled:

$u(q^L) - \beta y_2^L \kappa_b^L \leq u(\bar{q}) - \beta \bar{y}_2 \kappa_b^L$, i.e. the low-type's payoff is at least as high as under the complete information offer. Otherwise, the low-value asset holder would have an incentive to deviate. Given that, for all offers that are strictly preferred to $(\bar{q}, \bar{l}, \bar{y})$ by the low type, but would make the high-types worse off, $\lambda = 0$. For all offers that make the low-type strictly worse off than $(q^L, l^L, y^L) = (q^*, l^*, y^*)$, $\lambda = 1$. For all other, however, $\lambda = \pi$.

PROPOSITION 3: *In a pooling equilibrium, a low-value asset holder κ_b^L strictly prefers a secured credit contract $\bar{V}_{b,C}^L = \bar{V}_{b,D}^L > \bar{V}_{b,S}^L \forall \pi < \tilde{\pi}^L$. However, if $\pi > \tilde{\pi}^L$, then $\bar{V}_{b,S}^L > \bar{V}_{b,C}^L = \bar{V}_{b,D}^L$.*

Given the fact that we only consider efficient pooling equilibria maximizing the high-type's surplus, there exists a threshold $\tilde{\pi}^L$ below which low-value asset holders are better off 'revealing' their type by offering a collateral secured credit contract, leading to the first-best outcome $q^L = q^*$. As soon as the amount of high-types exceeds this threshold, i.e. $\pi > \tilde{\pi}^L$, pooling outright sale is the preferred settlement method, i.e. $\bar{V}_{b,S}^L > \bar{V}_{b,C}^L$ (see Figure 4). This is due to the prevailing information asymmetry allowing the low-types to benefit on the cost of the high types. However, given earlier results, a producer is aware of the fact that a high-value asset holder will never offer a pooling sale contract and therefore associates the consumer's offer to a low-type revealing his true asset return κ_b^L . Thus, by offering a pooling sale contract, the low-type enters an Akerlof (1970) market for lemons. Given this line of reasoning, the low-value asset holder's only options for all $\pi > \tilde{\pi}^L$ are making a complete information offer or engaging in a pooling secured credit contract (since $\bar{V}_{b,C}^L = \bar{V}_{b,D}^L = V_b^{L*}$), which supports the properties of the *efficient pooling* equilibrium.

4. STATIONARY EQUILIBRIUM

Having determined the terms of trade in the goods market (DM) as a function of the agent's private information and his asset balance accumulated in the CM, the buyer's portfolio choice in the settlement market (CM) can now be determined. Thereby, the terms of trade $\{q(z; \kappa_b), l(z; \kappa_b), y(z; \kappa_b)\}$ are functions of the consumer's portfolio and his private signal only and solve the buyer's respective maximization problem dependent on $\kappa_b \in \{\kappa_b^L, \kappa_b^H\}$.¹⁷ The expected lifetime utility of a buyer entering the goods market with z units of the real asset and the rate of return κ_b is:

$$V_b(z; \kappa_b) = u[q(z; \kappa_b)] + \beta W_b[z, l(z; \kappa_b); \kappa_b] \quad (27)$$

¹⁷From the bargaining solution we know that the seller's portfolio composition is irrelevant in equilibrium.

where $q \in \{q^L, q^H, \bar{q}\}$, $y \in \{y^L, y^H, \bar{y}\}$ and $l \in \{l^L, l^H, \bar{l}\}$. Due to the linearity of the CM value functions, the expected lifetime utilities can be rewritten as follows:

$$V_b(z; \kappa_b) = \left[u[q(z; \kappa_b)] - \beta \kappa_b y_1(z; \kappa_b) - \beta [(1-d)l(z; \kappa_b) + d\kappa_b y_2(z; \kappa_b)] \right] + \beta \kappa_b z + \beta W_b(0, 0; \kappa_b) \quad (28)$$

Substituting the value function of a buyer entering the goods market into the settlement value function (7), the buyer chooses its portfolio to maximize his expected surplus in the DM, net of the cost of holding real balances, according to:

$$\begin{aligned} \max_z \Bigg\{ & -(1 - \beta \bar{\kappa}_b)z + \pi \left[u(q(z; \kappa_b^H)) - \beta \kappa_b^H y_1(z; \kappa_b^H) - \beta [(1-d)l(z; \kappa_b^H) + d\kappa_b^H y_2(z; \kappa_b^H)] \right] \\ & + (1 - \pi) \left[u(q(z; \kappa_b^L)) - \beta \kappa_b^L y_1(z; \kappa_b^L) - \beta [(1-d)l(z; \kappa_b^L) + d\kappa_b^L y_2(z; \kappa_b^L)] \right] \Bigg\} \end{aligned} \quad (29)$$

where $\bar{\kappa}_b = \pi \kappa_b^H + (1 - \pi) \kappa_b^L$ and $(1 - \beta \bar{\kappa}_b)$ is the cost of holding real assets.

DEFINITION 3: *An equilibrium in this economy is defined as a list of portfolios and the terms of trade in the goods market $\{z, [q(\cdot), l(\cdot), y(\cdot)]\}$ such that:*

- (i) z is a solution to (29).
- (ii) For all $z \in \mathbb{R}_+$, $\{q(z; \kappa_b), l(z; \kappa_b), y(z; \kappa_b)\}$ is a solution to (15) and (18) in the separating equilibrium and (22) in the pooling equilibrium.

SEPARATING EQUILIBRIA: Plugging in the solution to the bargaining games (15) and (18), the buyer's maximization problem changes to:

$$\max_z \left\{ -(1 - \beta \bar{\kappa}_b)z + \pi \left[u(q(z; \kappa_b^H)) - q(z; \kappa_b^H) \right] + (1 - \pi) \left[u(q(z; \kappa_b^L)) - \frac{\kappa_b^L}{\kappa_b^H} q(z; \kappa_b^H) \right] \right\}. \quad (30)$$

LEMMA 3: *The portfolio choice of a buyer in a separating equilibrium is the solution to:*

$$\frac{\partial}{\partial z} : \quad -(1 - \beta \bar{\kappa}_b) + \kappa_b^H u'(q(z; \kappa_b^H)) - \bar{\kappa}_b = 0 \quad \text{if } z > 0. \quad (31)$$

If $1 > \beta \bar{\kappa}_b$, i.e. the price of the real asset is greater than its fundamental value, the solutions to the buyer's maximization problem takes the above form (31). Hence, in order for an agent to hold a real asset, the cost of holding the asset must be equal to the expected marginal benefit obtained in the DM. If $1 = \beta \bar{\kappa}_b$, $z \in [0, \infty)$ and if $1 < \beta \bar{\kappa}_b$, there is no solution.

POOLING EQUILIBRIA: Rearranging terms in (29) and substituting the solutions of the bargaining

game (22) leads to the buyers' maximization problem:

$$\max_z \left\{ -(1 - \beta \bar{\kappa}_b) + u(\bar{q}(z; \kappa_b)) - \bar{q}(z; \kappa_b) \right\}. \quad (32)$$

LEMMA 4: *The portfolio choice of a buyer in a pooling equilibrium is the solution to:*

$$\frac{\partial}{\partial z} : \quad -(1 - \beta \bar{\kappa}_b) + u'(\bar{q}(z; \kappa_b)) - 1 = 0 \quad \text{if } z > 0. \quad (33)$$

Given Lemma 3 and 4, the following normative properties emerge. If $1 < \beta \bar{\kappa}_b$, i.e. the fundamental value exceeds the price of the asset, then there is no solution to problem (29), as the consumer would demand an infinite amount of assets. However, if $1 > \beta \bar{\kappa}_b$, i.e. assets are costly to hold, a solution to the buyer's maximization problem exists. Buyers will not carry more real balances into the DM than they expect to spend and hence $y = z$. If $1 = \beta \bar{\kappa}_b$, however, assets are costless to carry and buyers will bring enough real balances z to purchase their first-best consumption q^* ($z \geq y$).

5. DISCUSSION

After having determined the bargaining solution of the goods market' matches and the respective outcomes in a separating and in a pooling equilibrium independently, we now want to focus on the consumer's decision, which of the two to choose and under what conditions. To support the theoretical results, a numerical analysis of the derived framework is conducted. In order to proceed, a functional form of the utility function $u(q)$ has to be determined. The specification applied is denoted below, while the used parameter values are pooled in [Table 1](#).

$$\begin{aligned} u(q) &= 2\sqrt{q} \\ c(q) &= q \end{aligned}$$

Parameter	Value	Comment
β	0.97	Riskless return on safe bonds is 3%
κ^L	0.8	Assumption
κ^H	1	Assumption

Table 1: Parameter values

Using the specified utility function and the parameter values, the following graphical results emerge ([Figure 4](#)), where the horizontal axis denotes the fraction of high-types π in the economy and the vertical axis the resulting expected lifetime utility of the consumer V_b^X .¹⁸

¹⁸An extended analysis including the obtained day goods quantities q , the amount of assets transferred y , and the

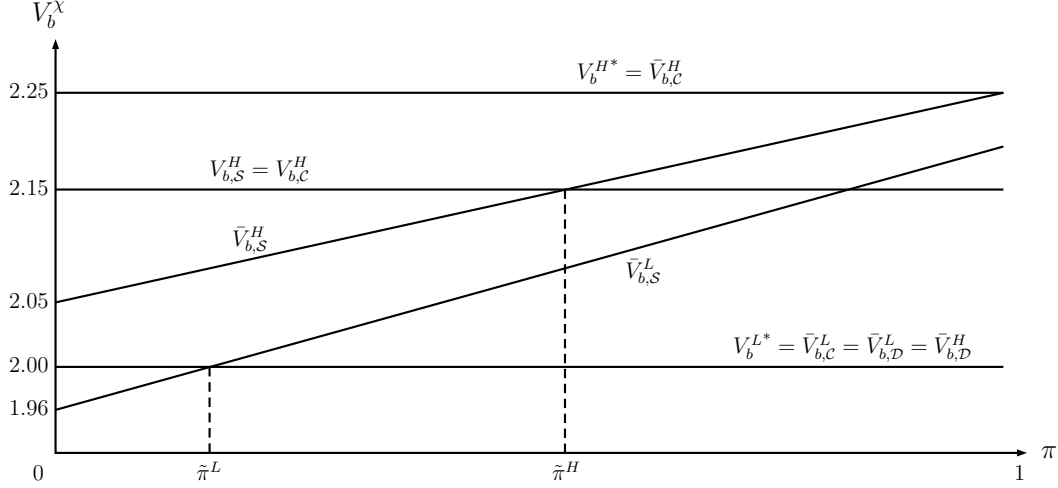


Figure 4: Consumers' expected lifetime value V_b^χ

The outcomes of the bargaining game have shown that agents in a separating equilibrium are indifferent between outright sale and secured credit $V_{b,S}^\chi = V_{b,C}^\chi = V_{b,D}^\chi$, since the respective terms of trade are equivalent for both settlement methods. In a pooling equilibrium, however, high types prefer entering a bilateral secured credit contract, while low types favor an outright sale, as soon as the fraction of high types in the economy exceeds a certain threshold $\pi > \tilde{\pi}^L$ (see Proposition 3). However, since this deviation would immediately reveal the low-value asset holder's type, he's indifferent between engaging in a pooling secured credit contract and his complete information offer $V_{b,C}^L = V_{b,D}^L = V^{L*}$. Being aware of these results, we now focus on the choice between offering a pooling or a separating contract, using Definition 4.

DEFINITION 4: The buyer's offer $(q_b^\chi, l_b^\chi, y_b^\chi)$, where $\chi \in (L, H)$, and the resulting equilibrium in the bargaining game with private information is (q_b^H, l_b^H, y_b^H) in (18) if $V_b^H > \bar{V}_b^H$ (and hence (q_b^L, l_b^L, y_b^L) in (15)) and $(\bar{q}_b, \bar{l}_b, \bar{y}_b)$ in (22) if $\bar{V}_b^H > V_b^H$.

The choice between a separating and a pooling equilibrium only depends on the surplus of the buyer holding a high-valued asset S_b^H . Thereby, provided that the low-type buyer's incentives and the seller's participation constraint are fulfilled, the high-value asset holder chooses his preferred offer (q_b, l_b, y_b) , which follows the refinement defined by Mailath et al. (1993). Considering first a pure asset sale economy, the following lemma needs to hold:

PROPOSITION 4: In a pure asset sale economy, for given asset values $0 < \kappa^L < \kappa^H$, there exists a threshold value $\tilde{\pi}^H \in (0, 1)$ such that the equilibrium is pooling if $\tilde{\pi}^H \leq \pi$ and separating if $\tilde{\pi}^H > \pi$.

Proof in Appendix D

Given that, if the fraction of high value-asset holders π is smaller than the threshold value $\tilde{\pi}^H$,

resulting asset prices $\theta = \frac{q}{y}$ can be found in Appendix C.

high-types are better off separating themselves from the low-types $V_b^H > \bar{V}_{b,S}^H$. However, if the fraction of low-value asset holders $(1 - \pi)$ is particularly small, offering a pooling contract $(\bar{q}, \bar{l}, \bar{y})$ is beneficial, since it allows the high-type avoiding the cost of separation, yielding $\bar{V}_{b,S}^H > V_b^H$. In sum, one can conclude that only if the economy is perceived to be relatively sound by its respective agents, high types are willing to offer a pooling outright sale contract. From proposition 3 we know that low-types would prefer a pooling sale contract for all $\tilde{\pi}^L < \pi < \tilde{\pi}^H$. However, due to rational expectations, this would immediately reveal their true type κ_b^L and hence, they would deviate to a complete information offer, which is in line with the high type's strategy in the first place and the properties of the *efficient pooling* equilibrium applied in Mailath et al. (1993). Having analyzed the properties of a pure asset sale economy, we now focus on the credit economy.

PROPOSITION 5: *In a pure secured credit economy, for given asset values $0 < \kappa^L < \kappa^H$, the bargaining choice is independent of the distribution of types in the economy. Agents always prefer a pooling $(\bar{q}, \bar{l}, \bar{y})$ over a separating equilibrium (q^H, y^H, l^H) , since $\bar{V}_{b,C}^H > V_b^H \forall \pi$.*

The logic behind Proposition 5 is straight forward and originates from the chosen 'undefeated equilibrium' refinement. Since the choice whether to pool or not is solely made by the high-types in the economy and since offering a bilateral secured credit contract allows this very same to reach first-best consumption q^* for all π (see Proposition 2), agents offer a pooling contract, no matter the distribution of types in the economy. Being aware of Proposition 3, one already knows that pooling credit as well maximizes the low-type's surplus for all π , justifying Theorem 1.

THEOREM 1: *In an asset market with asymmetric information about the assets' future dividend $\kappa_b \in \{\kappa_b^H, \kappa_b^L\}$, where $0 < \kappa^L < \kappa^H$, agents prefer engaging in a pooling collateral secured credit contract \bar{V}_b^X with $(\bar{q}, \bar{l}, \bar{y})$ over any other sale or secured credit agreement. Thereby, high-types honor their debt ($d = 0$) yielding $\bar{V}_{b,C}^H > \bar{V}_{b,D}^H$, while low-types are indifferent between repayment and strategic default ($d = 1$), since $\bar{V}_{b,C}^L = \bar{V}_{b,D}^L = V^{L*}$ and $(\bar{q}, \bar{l}, \bar{y}) = (q^L, l^L, y^L)$.*

6. CONCLUSION

The aim of this paper is to explain the prevalent behavior of banks engaging in bilateral secured credit contracts on the interbank market for short-term liquidity. The results show that in the presence of asymmetric information about the assets' future payoff, the introduction of the undefeated equilibrium refinement established by Mailath et al. (1993) allows for a self-selecting mechanism of liquidity bypassing. Thereby, high as well as low value asset holders strictly prefer a pooling collateral secured credit contract against any other settlement method, as soon as there exists a positive amount of low-value asset holders (lemons) in the market.

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APPENDIX

A: SPENCE (1973) JOB MARKET EXAMPLE: UNDEFEATED EQUILIBRIUM

In order to get a better understanding for the concept of the *undefeated equilibrium*, we take a look at a simple Spence job market signaling example where workers get utility $u(w, e, t) = w - \frac{e}{t}$ with w being the wage, e being the education level and t the ability of the respective worker (see Spence (1973)). Assuming $T = \{1, 2\}$, the prior probability of being a worker with ability t is π_t . Given this environment, there can be two kinds of equilibria: separating with $e_1 = 0$ and $e_2 \in [1, 2]$ and pooling with $\bar{e} \in [0, \pi_2]$. From Riley (1979) we know that any separating equilibrium with $e_2 \neq 1$ is defeated by the separating equilibrium with $e_2 = 1$. Additional to that, we know that any pooling equilibrium with $\bar{e} \neq 0$ is defeated by the pooling equilibrium where no worker chooses to get educated, i.e. $\bar{e} = 0$. This is due to the fact that in a pooling equilibrium, the wage is the same for all, no matter the education level.

Intuitively, it should be clear that low-ability workers are strictly worse off when signaling is possible, compared to an economy where it's not. This is due to the fact that in both cases, they do not incur costs of education (since $e_1 = 0$), but when signaling is possible they receive a wage $w_1 = t_1$ instead of $\mathbb{E}\{t\}$. For the high-ability worker, however, it depends on the distribution in the population. If $\pi_2 < \frac{1}{2}$, i.e. there are more low-types than high-types, the high-ability worker prefers the separating equilibrium to any pooling equilibrium and therefore the least-inefficient separating equilibrium (Riley equilibrium) is the only *undefeated equilibrium*. However, if $\pi_2 > \frac{1}{2}$, the high ability workers receive a higher utility in a pooling equilibrium with $\bar{e} = 0$ than in the least inefficient separating equilibrium. The reason for this outcome is that if the fraction of low-ability workers in the economy is low, a high-ability worker might be better off offering a pooling contract (\bar{w}, \bar{e}) instead of taking the cost of separating himself from an arbitrarily small amount of low-types in the economy. This pooling equilibrium is thus the only *undefeated equilibrium* in this environment since any separating equilibrium would yield a lower payoff to the high-ability worker. Finally, assuming $\pi_2 = \frac{1}{2}$, both the least inefficient separating equilibrium (Riley) and the pooling equilibrium with $\bar{e} = 0$ are *undefeated* since the high ability worker prefers the Riley equilibrium to any pooling equilibrium with strictly positive education.

B: PROOF OF LEMMA 1

LEMMA 1: *All but the least-inefficient separating equilibrium can be dismissed.*

As shown in section 3.1, if the high-value asset holder chooses to offer a separating contract, then by imposing the Cho and Kreps (1987) intuitive criterion, there is no equilibrium of the bargaining game with a pooling offer $(\bar{q}, \bar{l}, \bar{y})$. In order to proof this, consider the left panel of Figure 5. Assume that both agents, high as well as low-types, make the same pooling offer $(\bar{q}, \bar{l}, \bar{y}) \neq (0, 0, 0)$ leading

to surpluses $S_b^H = u(\bar{q}) - \beta\kappa^H\bar{y}$ and $S_b^L = u(\bar{q}) - \beta\kappa^L\bar{y}$ and the offer is accepted by the seller. The corresponding indifference curves are displayed in Figure 5 and denote the set of offers (q, l, y) that generate the equilibrium surplus. Thereby, one can see that the S_b^L is steeper than S_b^H which is due to the Spence-Mirrlees single crossing property and key for obtaining a separating equilibrium. The participation constraint of a seller who beliefs that he is facing a high-type buyer is given by $S_s^H \equiv \{(q, l, y) : -c(q) + \beta\kappa^H y \geq 0\}$. The proposed equilibrium offer $(\bar{q}, \bar{l}, \bar{y})$ is located above S_s^H , since, by Bayes' rule, it is only accepted if $\lambda < 1$, the seller assumes that there are some low-types in the economy. The shaded-area indicates the set of offers that increase the utility of the high-type, decrease the utility of the low-type and are acceptable to the seller, i.e. fulfill $\mathcal{A}_s(\lambda)$ assuming that $\lambda = 1$. These offers violate the Cho and Kreps (1987) intuitive criterion and therefore fulfill (14). In other words, a high-type would be able to make himself better off by reducing his transfer of assets and his goods market consumption (by deviating from $(\bar{q}, \bar{l}, \bar{y})$), while making a low-type strictly worse off, i.e. a low-type would not choose such an offer because his asset is worth less than the one of the high-type. Additional to that, a seller would still accept the proposed offer, since he beliefs that it comes from a high type.

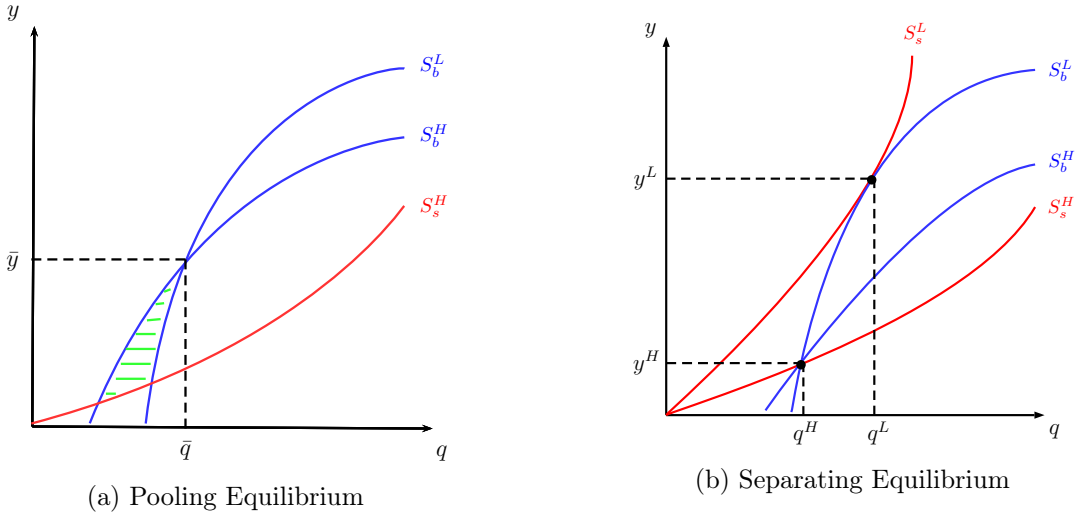


Figure 5: Pooling versus Separating Equilibria

Given that, pooling offers are not compatible with equilibrium and therefore, if an equilibrium exists, it has to be characterized by separating offers, i.e. different offers for the high and the low types. The respective offers are described in (15) and (18) and illustrated in the right panel of Figure 5. The offer of a low-type is at the tangency point of the seller's participation constraint facing a low-type, $S_s^L \equiv -c(q) + \beta\kappa^L y \geq 0$ and the surplus of the buyer holding a low-value asset S_b^L , i.e. the low-type makes a complete information offer. The high-type buyer however has to make an offer to the left (and including) of S_b^L and above (and including) S_s^H in order to satisfy the seller's participation constraint and the incentive compatibility condition. The corresponding utility maximizing offer is at the intersection of these two curves, implying $q^H < q^L \leq q^*$. Given that, buyers holding a

high-value asset always consume less than the low-types and retain a fraction of their high-valued assets $y^H < y^L \leq z$. ■

C: NUMERICAL ANALYSIS

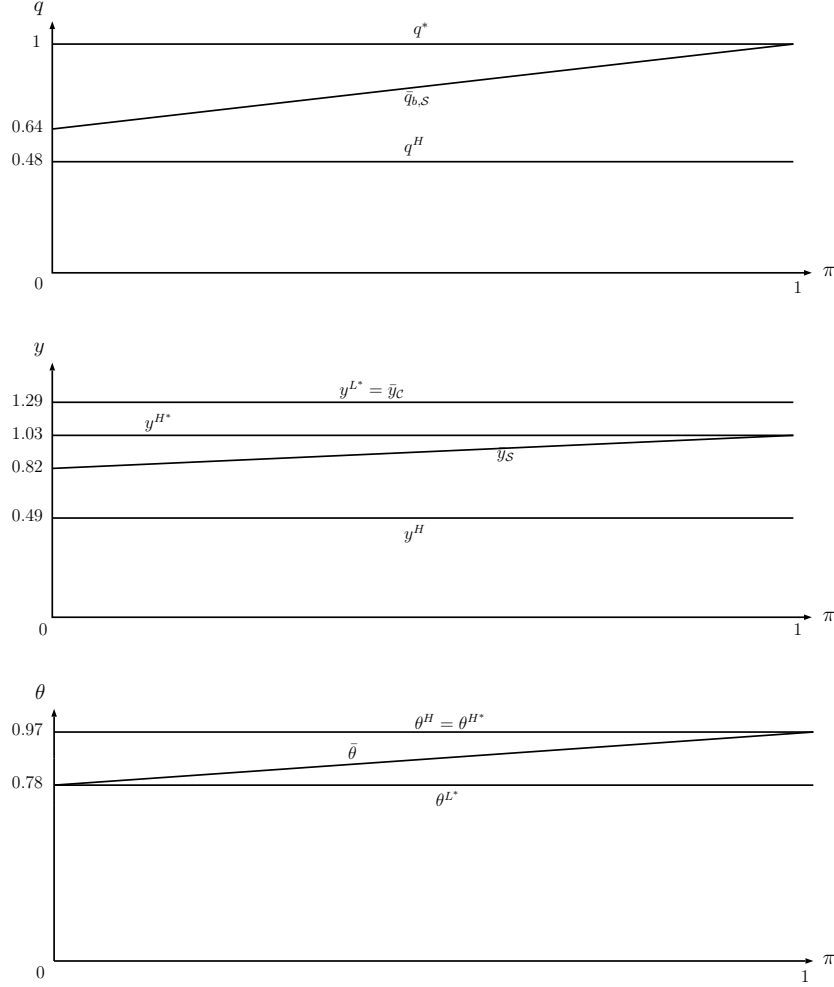


Figure 6: Numerical Analysis

D: PROOF OF PROPOSITION 4

PROPOSITION 4: *In a pure asset sale economy, for given asset values $0 < \kappa^L < \kappa^H$, there exists a threshold value $\tilde{\pi}^H \in (0, 1)$ such that the equilibrium is pooling if $\tilde{\pi}^H \leq \pi$ and separating if $\tilde{\pi}^H > \pi$.*

The maximization problem of the high-value asset holder offering a separating contract (18) has shown that its payoff $V_b^H(z; \kappa)$ is independent of π . In contrast, from (22) with $\chi = H$, the payoff of the high-value asset holder offering pooling sale contract (\bar{q}, \bar{y}) contract, yielding $\bar{V}_{b,S}^H(z; \kappa)$, is strictly decreasing in the amount of low-types $(1 - \pi)$ in the economy.

Assuming the population only consists of high-types, i.e. $\pi = 1$, then (18) corresponds to (22) with the difference that in the separating case the incentive compatibility constraint is binding. Given that, if $\pi = 1$ it holds that $\bar{V}_{b,\mathcal{S}}^H(z; \kappa) > V_b^H(z; \kappa)$. Considering now the case where there are only low types in the economy, i.e. $\pi = 0$, the following has to hold: $\bar{V}_{b,\mathcal{S}}^H(z; \kappa) < V_b^H(z; \kappa)$.

Given that, there exists a threshold value $\tilde{\pi}^H \in (0, 1)$ such that the payoff of the high-types in the separating and in the pooling equilibrium are equal $\bar{V}_{b,\mathcal{S}}^H(z; \kappa) = V_b^H(z; \kappa)$. Then, for all $\tilde{\pi}^H \leq \pi$, $\bar{V}_{b,\mathcal{S}}^H(z; \kappa) > V_b^H(z; \kappa)$ and the equilibrium is pooling, and if $\tilde{\pi}^H > \pi$, $\bar{V}_{b,\mathcal{S}}^H(z; \kappa) < V_b^H(z; \kappa)$ and the equilibrium is separating. ■