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De-Biasing Strategic Communication

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De-Biasing Strategic Communication

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Abstract

This paper studies the effect of disclosing conflicts of interests on strategic communication when the sender has lying costs. I present a simple economic mechanism under which such disclosure often leads to more informative, but at the same time also to more biased messages. This benefits rational receivers while it exerts a negative externality from them on naive or delegating receivers; disclosure is thus *not* a Pareto-improvement among receivers. I identify general conditions of the information structure under which this effect manifests and show that whenever it does, full disclosure is socially inefficient. The results hold independently of the degree of receivers' risk-aversion and for arbitrary precession of the disclosure statement.

Keywords: strategic communication, misreporting, conflict of interest, disclosure

JEL Classification: D82, D83, L51

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1 Introduction

A substantial part of the world's economic activity deals with the elicitation of information by experts and its communication to non-experts. Examples include stock analysts, researchers, consultants or managers reporting to shareholders. Too often, experts face a conflict of interests (henceforth COI) such as sale commissions or affiliations which make their reports prone to be biased. Inefficiencies then arise because of two main reasons: First, receivers of such information may ignore the expert's COI and make poor choices by following biased information. Second, receivers lack information about the COI, e.g. its relative magnitude and the direction of the bias it induces. Without such information, they cannot accurately correct the expert's advice. They may then rationally decide to ignore the expert's message, at least partially, such that valuable information is lost. Disclosure of COIs promises to be a simple remedy to this problem. The idea is that information about the expert's COI helps at least those receivers who can use it to correct for a potential bias. It is also tempting to policy makers as it carries the, as I will show incorrect, intuition that flattening information asymmetries is always desirable and should at least not hurt anyone. From a regulatory view, disclosure is also an appealing option as it is less paternalistic and less costly to regulators than direct supervision and regulation.¹ A prominent example for such a policy is contained in the Sarbanes-Oxley-Act which was enacted in 2002 as a response to prior corporate frauds, in particular among financial analysts. Among its adopted regulations is the requirement to "[...]disclose in each research report, as applicable, conflicts of interest that are known or should have been known by the securities analysts[...]" United States Congress (2002, Sec. 501b).

The objective of this paper is to describe an economic mechanism which shows how and why such disclosure often can lead to consequences opposite to those intended. It does so by considering a communication game in which both, the superior information an expert owns and the COI he faces are his private type. The model allows some receivers to be naive with regard to the sender's COI while others are fully rational, in a Bayesian sense. Alternatively, naivety in this setup is equivalent to delegation of decision to an expert, e.g. to a managed fund. The combination of these factors then unveils a simple channel under which disclosure can lead to more biased information transmission which hurts the naive receivers of expert information who do not anticipate the strategic effects of disclosure.

In a work which examines the communication by financial analysts, Malmendier and Shanthikumar (2014) show that they *strategically* inflate their stock recommendations and are not

¹Fung et al. (2007) lists further reasons for why disclosure regulations seem appealing, numerous specific examples, and also examples for their failure.

just overly optimistic, a feature which is maintained in the analysis of this paper. To do so, they use data on analyst recommendations which covers a period before and after the Sarbanes-Oxley-Act. Their analysis shows that the strategic bias did not disappear after the act was put into action.² Similarly, Mullainathan et al. (2012) conducted an audit study and show that in period after the act came into effect, financial advice has remained of poor quality. Clean, causal evidence for negative effects of disclosure comes from Cain et al. (2005): In their experiment, subjects in the role of an experts had ample opportunities to study a jar filled with coins. These subjects then advised others who had to estimate the amount of money inside the jar but could not examine it before-hand. Their paper first confirms the straightforward intuition that when the experts' pay is based on the final estimate's accuracy, the advice and estimates are better than when the experts are paid based on how high estimates are. They however also show that when receivers are made aware of the experts' incentive to induce a high estimate, the experts' bias *increases*. On average, receivers do not account for this effect and end up making worse decisions than without disclosure. This finding on the adverse effects of disclosure have also been replicated in similar setups by Koch and Schmidt (2010), Inderst et al. (2010), and Cain et al. (2011).³ The effect identified here is in line with these observations.

To understand the source of this adverse effect, consider an analyst ("he") who knows a share's fundamental value. When sending a message about it, he faces a COI to overstate it, such as through a sales commission. Suppose also he faces lying costs of doing so, e.g. reputational and/or expected legal costs. The magnitude of the bias in his message is then determined by equalizing the marginal costs of lying to the marginal return of doing so. The latter is given by the commission weighted with the average marginal reaction receivers to the sender's message. Now regard a client ("she") who receives a message from the sender and is aware of the bias. She can try to de-bias it by correcting for the expected bias. However, since she faces uncertainty regarding the commission's actual size or even its direction, she can even worsen the outcome by such de-biasing when the actual bias differs from the expected one. Facing such a risk, rational receivers of biased messages will then act based on a combination between the imperfectly de-biased message and her prior about the state of the world. The weight which a rational receivers then puts on the sender's imperfectly de-biased message is inversely related to the strategic uncertainty regarding the sender's COI. Disclosing the COI decreases this strategic uncertainty and thus leads the receiver to put more weight on the sender's message after de-biasing it.

²See Malmendier and Shanthikumar (2014), p.1298: They state that their measure of strategic bias remains sizable and positive for affiliated analysts when they split the sample by August 2001, the date when the scandals became public and which contributed to the enactment of the Sarbanes-Oxley-Act less than a year later.

³For a further review on the failure of disclosure and psychological approaches to these findings, see Loewenstein et al. (2014).

However, this translates into a larger marginal reaction to the sender's message. As explained above, this marginal reaction of receivers scales the sender's bias; it is therefore increased by disclosure. Delegating or naive receivers who do not account for the strategic effects of biasing and de-biasing communication are then hurt by this increase.

The above reasoning combines two main insights: First, the reaction to the sender's message by rational, risk-averse receivers depends on the quality of information they can extract from it. Second, an expert who faces a COI and has lying costs biases his message in proportion to the reaction it induces. Both of these effects are simple in their economic intuition. Combined however, they deliver the surprising result that increasing transparency can be a bad idea when the disclosed information is not used by everyone. In particular, it proves the idea that disclosing COIs cannot be bad for any receiver to be wrong when not all receivers are fully rational. Disclosure can then also lead to less efficiency, depending on the relative share of receiver types. In this paper, I model this effect in a framework which allows for arbitrary degrees of risk-aversion as well as arbitrary quality of the disclosure process. I identify general conditions under which this effect manifests and thereby allows to evaluate the welfare consequences of disclosure. The key variable to identify the consequences of disclosure is the correlation between the expert's COI and the information on which he has superior knowledge. I find that among the settings in which disclosure backfires are all environments in which this correlation is at least weakly positive. It is also shown that in all these settings, full disclosure is never optimal from an social efficiency point of view. When this correlation is negative, there can be cases in which disclosure is a Pareto-improvement among all receivers and only in these cases, full disclosure is efficient.

Related literature: By analyzing the consequences of disclosing an expert's COI, this paper contributes to the literature on strategic communication. In their seminal work on the topic, Crawford and Sobel (1982) characterize communication equilibria to be partitional: In equilibrium, the sender endogenously partitions the state space and announces a message which maps to the partition which contains the actual state of the world. The lower the number of partitions in equilibrium, the greater is the loss in informativeness of his message. This result applies independent of the specific meaning of language, i.e. how exactly states map to messages by the sender and back from messages into actions by receivers.⁴ It is however assumed that, whatever it exactly is, such a meaning of language is common knowledge. In the example of a financial analysts who announces "I expect share X will pay Y this year" many people would

⁴See Sobel (2013) for an overview of the rich literature which has utilized and extended the partitioning result. Also see the section on pragmatics therein for further discussion on what language and its meaning actually are in the context of strategic communication.

understand its meaning to be literal, thus that X is the share's actual performance or at least the analyst's best estimate. This is also the implicit assumption of studies which investigate the bias of financial analysts and which allows them to conclude that their messages are upward biased (Hayes, 1998; Michaely and Womack, 1999; Malmendier and Shanthikumar, 2014). However, the partitioning result in combination with such a literal meaning (and understanding) of language implies that, on average, the message and the realized state of the world should not differ.

To reconcile a literal meaning of messages and biased messages, one or both of the two crucial assumptions which underlie the partitioning result need to be changed. Addressing them, Kartik et al. (2007) and Kartik (2009) show respectively that these assumptions are the boundedness of the state space and cheap talk, i.e. no costs of lying.⁵ Capturing their insights, this paper allows for an unbounded support, for example when the variable on which the sender has private information is normally distributed. It also allows for lying costs. Similar to those used by Kartik (2009) these costs penalize messages which differ from the state of the world, they thus capture a literal interpretation of language as in the above example. This allows to meaningfully define a sender's bias as the deviation from his message to the true state of the world. Also, it reflects the norm that communication should be as honest and as straightforward as possible, in particular that language should not be inflated. This in turn justifies the behavior of naive receivers who trust the sender by choosing their action based on his message's face value.

Different from the above theoretical works which all assume that the sender's COI is common knowledge, this work's focus is on cases when it is his private information. It therefore also adds to the literature on strategic communication with strategic uncertainty. Morgan and Stocken (2003) consider cheap talk with a compact state space when the sender's privately known COI is represented by a binary variable which is independent of the state of the world. In this setup, they find that the sender's messaging strategy remains partitional.⁶ Close to the focus of this work is Li and Madarasz (2008) as they explicitly consider the consequences of decreasing strategic uncertainty. The sender's COI is again assumed to be binary and independent of a uniformly distributed state of the world. Since talk is also assumed to be cheap, i.e. there are no lying costs, equilibria remain partitional. They show that disclosure can decrease informational efficiency, thus the number of partitions. The reason for this is that the sender's COI is assumed

⁵Kartik et al. (2007) show that under general conditions, unbounded support is sufficient for the sender's messaging strategy to be continuous and (upward) biased. Kartik (2009) considers the case when the state space is compact but the sender has lying costs. He shows that equilibria are partially separating of the "LSHP (low types separate and high types pool)"-form: When the sender is upward-biased, he exaggerates his statement by a fixed bias if the state is below a certain threshold. If it is above that threshold, the sender only announces the partition of the upper subset of the state space in which the true state lies.

⁶In the theoretical part of his work, Blanes i Vidal (2003) also considers a binary COI which is independent of a normally distributed state of the world. He finds that receivers can never react to the sender's message in every state of the world. However, under a variety of parameter conditions issues of equilibrium non-existence arise.

to have an expected value of zero; without disclosure it is thus only strategic uncertainty which complicates communication. They then show that common knowledge of the COI's non-zero value through disclosure turns out to be a greater impediment to efficient communication than expecting it to be zero. Inderst and Ottaviani (2012) is the second paper to study explicitly the consequences of lowering strategic uncertainty in a setting of advice-giving for product choices. They model the origin of the sender's COI as commissions paid by product providers to experts who in turn advise customers on which of two competing products suits them best – the state of the world and the expert's message are therefore binary. Their results show that disclosing COIs reduces the provision of commissions but less so in relative terms for the inferior product. Consequently, the relative bias rises upon disclosure and consumers make worse decisions.

The framework used here allows to model strategic communication when both, the state of the world and the sender's COI are represented by continuous, possibly correlated variables and most importantly, when there are costs of lying. The mechanism underlying the adverse effect of disclosure is also different from the ones described above. It derives from linking the larger reaction to a biased message after disclosure to the sender's endogenous choice of the bias. For this, lying costs are an essential feature. The model's specific form extends a linear-quadratic-normal framework as used by Fischer and Verrecchia (2000). They consider a manager who announces the earnings of the company he works for when there is uncertainty regarding the motives for biasing his report. It extends it to elliptical distributions as recently used by Deimen and Szalay (2014) who study strategic communication when players cannot agree on the relative importance of different information they hold.⁷ Within this wider class of models it is the first to focus on disclosure of COIs. To do so, it extends the analysis of communication games in this class along three main dimensions: First, it examines not only the informational content of the sender's message but also his bias – the deviation from a honest, literally meant message. This is important for studying the consequences on receivers, especially those who are strategically unsophisticated. Modeling such naive receivers and their strategic effect on the sender's messaging strategy allows to clearly identify the negative consequences of disclosure. Second, and in contrast to the preceding literature, this work also studies explicitly the role of negative correlation between the variable of interest, here the sender's COI and the state of the world. Apart from analyzing the resulting equilibria more generally, it is shown that only with such a negative correlation, disclosure of COIs can improve welfare for both, rational and naive receivers. Third, it explicitly handles disclosure of COIs through a signal of arbitrary

⁷Kartik and Frankel (2015) recently adapted this approach to study more broadly the consequences of stake sizes on the informativeness of one-dimensional signaling for two-dimensional types. Technically also related, Bénabou and Tirole (2006) study signaling of pro-social motivation within the linear-quadratic framework and jointly normally distributed variables.

precision. Just performing comparative statics with respect to a single parameter, e.g. the variance which describes uncertainty for the sender's COI, overlooks the, potentially opposing, effect of disclosure on correlated variables. In contrast, the approach presented here allows to analyze the effects of disclosure on the whole distribution of priors, including correlated variables.

The next section outlines the model's structure and evaluates the assumptions made with respect to several settings of expert advice. In Section 3, I derive the equilibrium behavior of senders and receivers when the sender's COI is undisclosed. Section 4 covers the case of disclosure. Section 5 synthesizes the preceding analysis and assesses the consequences of disclosure on receivers and overall efficiency. Section 6 concludes by summarizing the main insights and discussing their policy implications.

2 The model

Consider a mass of non-experts, henceforth called receivers. Every receiver would like to know the state of the world denoted by the random variable $s \in S \subseteq \mathbb{R}$, because she has to take a decision $d \in S$ whose return is dependent on the realization of s . For example, s might represent an asset's return and d the receiver's optimal position into this asset. Consequently, the receiver suffers a loss which is the greater, the more her decision and the actual state of the world are misaligned. To capture this, assume that a receiver's ex-post utility, given her decision d and the state of the world s , can be represented by the loss function L . More precisely, assume that

$$u^R(d; s) = L(d - s) \tag{1}$$

where $L : \mathbb{R} \rightarrow \mathbb{R}_-$ is assumed to be \mathcal{C}^2 , strictly concave, and symmetric around zero. Without loss of generality, it is also assumed that $L(0) = 0$. As the leading example, consider $L(d - s) = -\frac{1}{2}(d - s)^2$, the quadratic loss function.⁸

Receivers do not know s and therefore refer to a risk-neutral sender who knows its value. The sender communicates via a public message $m \in M = S$ about s . In doing so, he faces costs of lying which are captured by a \mathcal{C}^2 , strictly concave loss function $K : M \times S \rightarrow \mathbb{R}_-$ with image $K(m; s) \in \mathbb{R}$. This image is uniquely maximized at $m = s$, reflecting that the meaning of the message is assumed to be literal. This cost of lying may stem from different sources, including the expected legal costs of being captured lying, reputational concerns, or moral considerations.⁹

⁸This is the canonical example put forward by Crawford and Sobel (1982) and used in much of the literature on strategic communication. Ottaviani (2000) shows that this specific function covers the case of a receiver with exponential utility who makes an investment d into a risky asset of which he knows its variance but not its expected value s .

⁹For a rationale of reputational costs, see Sobel (1985) and Morris (2001). Evidence that many people have

If the above were the sender's only strategic incentive he would be honest and always send $m = s$. Receivers would then just follow the message and implement their optimal choice. However, such strong influence of the sender on the receivers' decisions can be exploited. The sender can be paid to induce either a high or low decision among receivers, e.g. via sales commissions. To see how such a COI creates a bias consider the aggregate decision of receivers, e.g. expected aggregate demand, which the sender can possibly influence with his message and denote it by $D(m)$.¹⁰ The COI of the sender is then denoted by a variable $c \in C \subseteq \mathbb{R}$. It scales in proportion to the potential aggregate decision he can influence and relative to the costs of lying K . The sender then has to choose a message $m \in M$ which maximizes the following expected utility function:

$$\mathbb{E}[u^S(m; s, c)] = cD(m) + K(m; s) \quad (2)$$

Note that for a (temporarily) fixed supply, shifting demand also implies a shift in its price. By appropriate scaling of c , this allows to capture situations where the sender's COI lies in eventually influencing prices, as in Fischer and Verrecchia (2000). Also note that the commission is additive and proportional to the demand. This differs from other approaches which assume that the receiver and sender have the same utility functions which differ only in their bliss points. In particular, such approaches assume that the sender's utility decreases the more the receiver's action differs in *either* direction from the sender's bliss point.¹¹ For the example used previously, this would mean that a financial analyst facing sales commissions would try to induce a specific demand up to that bliss point but not more. In contrast, the assumption here is that a COI induces the sender to shift demand as much as possible, subject to the costs of lying and the loss of credulity due to his resulting bias. More exactly, when $D(m)$ is differentiable, the sender's optimal message m^* has to solve the following first-order condition:

$$cD'(m) = -\frac{\partial K(m; s)}{\partial m} \quad (3)$$

As a direct consequence, there is no truth-telling when there is a COI and receivers react to the sender, thus when $cD'(m) \neq 0$ holds. Furthermore, by concavity of $K(m, s)$ in m around s , the larger the magnitude of this marginal reaction, the larger is the bias, defined as the distance between the message sent m and the actual state s . In consequence, every shift in marginal

a preference for being honest per se is provided, among others, in Erat and Gneezy (2012), López-Pérez and Spiegelman (2012), and Abeler et al. (2014).

¹⁰For example, when there is a continuum of receivers with mass one, each receiver indexed by i and having demand $d_i(m)$, then $D(m) = \mathbb{E}[d_i(m)]$.

¹¹In the framework of this paper, this would be $u^S(d, s, b) = L(d - (s + b)) + K(m; s)$ with $b \neq 0$. In such a setting, naive receivers who choose their action equal to the sender's message in itself constitute a limit on the sender's exaggeration (see Ottaviani and Squintani, 2006; Kartik et al., 2007).

demand, e.g. when some receivers react to strategically valueable information such as disclosed COIs, increases the sender's bias.

To examine this general effect in more detail, I will henceforth assume the specific loss function $K(m, s) = -\frac{1}{2}(m-s)^2$. This function is chosen because it captures the above considerations and proxies well for lying costs which originate from social preferences, moral concerns against lying, or reputational concerns in a stage version of a repeated game.¹² Second, the above conditions show that its size, relative to the sender's lying costs, can be adjusted simply by choosing c and its distribution accordingly. It then allows a tractable analysis in closed form and increased clarity. The main reason for this is that the above first-order condition then simplifies to the following, intuitive form:

$$m^* = s + cD'(m^*) \quad (4)$$

Note that the sender's bias $cD'(m)$ and the state of the world s enter the sender's strategy additively and separately. However, they can be correlated in terms of the data-generating process. The following describes the underlying informational setting, including scenarios in which a non-zero correlation between s and c matters in more detail.

Information: Both, the state of the world s and the sender's COI c are his private information. They are assumed to be drawn from an elliptical distribution with support over $K \times S = \mathbb{R}^2$. The standard example is the (multivariate) normal distribution, but others such as the heavier-tailed logistic or student-t also fall into this class of distributions and are often used in financial and risk modeling.¹³ A specific elliptical distribution F such as the normal one can be parametrized by denoting it $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$ where $\boldsymbol{\eta}$ represents the vector of expected values and $\boldsymbol{\Sigma}$ the variance-covariance matrix. Elliptical distributions have three properties which will be important in the present context:

- E1) Elliptically distributed random variables are closed under linear transformations, i.e. a linear transformation of jointly F -distributed variables is itself F -distributed.
- E2) A random variable, conditional on the realization of another random variable, both being jointly elliptically distributed according to $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$, is also distributed according to F . The

¹² When L is also quadratic, this cost function captures concerns for the utility of a receiver who follow the sender's message at face value. Kartik (2009) uses this specific form of costs as a prominent example for capturing costs of lying per se. Reputational concerns can also be proxied by it: If the actual value of s became knowledge ex-post, the squared distance of s and m is part of the nominator of the sender's coefficient of determination (R^2) which one would obtain by regressing the state of the world on his prior messages; the sender credulity is thus decreasing in this squared distance.

¹³See Embrechts et al. (2002), also for the potential caveats of doing so.

vector of expected values and the variance-covariance matrix of the resulting conditional distribution are linear transformations of $\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}$.

E3) Elliptical distributions are symmetric around their vector of expected values.

A more formal statement of these properties, together with a precise definition of elliptical distributions and references can be found in the appendix. I assume that the moments of $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$ from which the sender type (s, c) is drawn are finite and real. They are denoted as follows:

$$\boldsymbol{\eta} = \begin{bmatrix} \bar{s} \\ \bar{c} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_s^2 & \sigma_{sc} \\ \sigma_{sc} & \sigma_c^2 \end{bmatrix}$$

When appropriate, I will refer to correlation of s and c instead of its covariance σ_{sc} . To make things interesting, I assume it not to be perfect, i.e. $|\text{Corr}[s, c]| = \frac{|\sigma_{sc}|}{\sigma_s \sigma_c} < 1$. For the same reason, I assume that the diagonal elements of $\boldsymbol{\Sigma}$ are strictly positive. Otherwise, the receiver's inference problem would effectively become one-dimensional or vanish entirely.

The above information structure is suited to naturally model how players, in particular rational receivers, arrive at their prior. First, assuming unbounded support for (s, c) means that no commonly known bounds on the state space are required. In contrast, assuming compact support implies mutual understanding on such sharp bounds. Sometimes, this is straightforward and reasonable, e.g. when the sender communicates on how much of one's wealth should be invested into a certain asset. However, once leverages become available or when s reflects asset returns, appropriate bounds are not so clear-cut. A solution to this is then to assume that all real values are theoretically possible while "unrealistic" or "unreasonable" extreme realizations receive appropriately low probabilities by choosing the above moment parameters accordingly. Another situation which is captured by this framework is when past information is used, e.g. by using (4) as a structural model to arrive at a prior. Resulting estimates of (s, c) are then, by the central limit theorem, approximately normally distributed. One could also assume compact support, for example the uniform distribution is also elliptical. For the sake of simplicity and without much loss of generality, I however maintain the assumption of unbounded support.¹⁴

As an import feature, this framework allows to handle the case of $\sigma_{sc} \neq 0$, e.g. when the COI is dependent on the state of the world. As an example, consider the example of a financial analyst or a manager who reports on an asset for which he holds call-options. By inducing a high demand he can potentially raise the asset's price such that it exceeds the option's strike

¹⁴Note that by choosing the distribution's parameters appropriately, the probability that its realizations are within some compact set can be made arbitrarily high. For the uniform distributions as an elliptical one see the second example in Gómez et al. (2003), pp. 359/360 and Kartik and Frankel (2015). All crucial results reported here also apply to a bounded support.

price. This would then allow him to earn on the option. However, this only works if the fundamental value of the asset represented by s is high enough such that he can "bridge" the distance necessary to meet the option's strike price via the effect his message has on demand. If this is not feasible, thus when s is sufficient low, then there is essentially no COI. In consequence, s and c are positively correlated. If the sender owns the asset himself, a negative correlation can follow. The reason is that when the asset has low value, he wants to get rid of it. Before selling the asset, he then faces a COI to bias his report upwards as to increase demand and thereby the sale price he earns.¹⁵

Rational and naive receivers: I will now turn to the demand side and analyse reactions to the sender's message in detail. The above shows that COIs induce the sender to not report truthfully. How should receivers then take such a distortion into account and how in turn, should the sender adjust his signal to the receivers' reaction? A receiver who rational, in a Bayesian sense, should do so by basing decision on the information he can extract from the sender's message:

$$d_r(m) \equiv \operatorname{argmax}_{d \in S} E[L(s, d) | m] \quad (5)$$

In the case of a quadratic loss function, this is clearly the conditional expectation of s given m . The following result, due to Deimen and Szalay (2014), shows that this generalizes to the strictly concave, symmetric loss function L when the message m is elliptically distributed:

Lemma 1. *Suppose m is elliptically distributed. It then holds that $d_r(m) = E[s | m]$.*

Proof. the proof adapts the proof of part i) of lemma 2 in Deimen and Szalay (2014) to this paper's setup and can be found in the appendix.

The optimal decision $d_r(m)$ is that of fully rational, Bayesian receiver who is capable of adjusting the effect of the sender's bias on his message and connecting it to the common prior. While some receivers, e.g. institutional ones, can act in such a manner, empirical evidence shows that most individuals often do not anticipate and correct for others' strategic behavior (see, for

¹⁵Another scenario which is noteworthy to be mentioned and which can be easily captured in the framework is scientific fraud: First, the reactions to empirical research are strongly based on test statistics with elliptical distributions, for example normally or student-t-distributed regression coefficients. Second, outright cheating such as making up data or more subtle techniques such as selective sampling and data mining are methods with which these test statistics *are* manipulated (Fanelli, 2009; Steen, 2011; Simonsohn, 2014). However, ethical concerns as well as reputation and legal sanctions create costs of doing so. Third, COIs to influence decisions based on these statistics are common as they map one-to-one into statistical significance. Pressure to publish and publication bias towards statistically significant results then create an incentive to use the above methods in order to inflate test statistics (Simmons et al., 2011). Incentives to decrease such statistics do also exist, e.g. in an antitrust case where the decisive measure is a company's market share and the researcher is affiliated with that company. Finally, disclosure policies are in practice for many journals and other receivers of scientific work. Taken together, the effect of disclosure policies on fraud in science constitute another example for which the above framework can be applied.

example Brown et al., 2012; Brocas et al., 2014). In addition, behaving in a Bayesian manner does not only require skills but also information to form a prior. Just listening to an expert and following his advice does not require any such information. If the money at stake and/or the expected bias are small, relative to the cost of conducting Bayesian inference, receivers may even rationally prefer to just follow an expert or delegate their decisions (Sims, 2003). In line with such reasoning, Malmendier and Shanthikumar (2007) show that small investors such as private households follow analysts' optimistic recommendation more closely than bigger, institutional ones. To capture these observations, I allow for the possibility that there are naive receivers who take the sender's signal at face value. Their action is given by $d_n(m) = m$ as in Ottaviani and Squintani (2006) and Kartik et al. (2007). I denote the share of naive or delegating receivers by $\mu \in [0, 1]$.¹⁶ The mass of rational receivers is therefore given by $1 - \mu$ which yields the following expected demand function for the receiver:

$$D(m) = \mu m + (1 - \mu)d_r(m) \quad (6)$$

Note that this approach also captures a scenario in which a risk-neutral sender faces a single receiver but does not know whether this receiver is naive or rational. Denoting the probability for the former case with μ and for the latter with $1 - \mu$, the expected demand for the sender would then be the same. Also note that acting based on the face value of an expert's message is strategically equivalent to delegating one's decision, e.g. putting one's money into an investment company's actively managed fund. Rational receivers would instead listen to the company's experts but eventually make investment decisions on their own.

3 Undisclosed conflicts of interest

The communication game with undisclosed commissions has the following timing:

- 1) the sender's type (s, c) is draw from F and privately observed by the sender,
- 2) the sender sends a signal m about s ,
- 3) receivers observe m , if rational update their belief about s , then choose d ,
- 4) payoffs are realized.

¹⁶By appropriate scaling of μ , one can always account for situations where naive or delegating receivers do not react one-to-one, e.g. when $d_n(m)$ is a positive affine transformation with $d'_n(m) = r > 0$. As an example, suppose that there is a mass 0.5 of naive receivers who follow the signal, on average, in proportion $r = 0.6$. From the sender's point of view, this is the same as if there were a mass 0.2 of receivers who ignore him, mass 0.3 of naives who follow one-to-one, and a mass 0.5 of rational receivers. Using $\mu = \frac{0.3}{0.8}$ would then be strategically equivalent.

I look for a Perfect Bayesian Equilibrium of this game. It consists of a pair of equilibrium strategies $m^* : S \times C \rightarrow M$ for the sender and $d_r^* : M \rightarrow S$ for a rational receiver such that each player's expected utility is maximized, given the other players' strategy when their beliefs are formed by Bayes' rule. Naive receivers are assumed to have a dominant strategy of following the sender, their beliefs therefore do not matter. The key equilibrium belief in this context is a rational receiver's belief about s , denoted $E[s|m^*] \equiv E[s|m]_{m=m^*(s,c)}$. This is the conditional expectation of s given an image m which is formed by the sender's equilibrium messaging strategy $m^*(s, c)$.¹⁷

I will use this equilibrium concept under different settings of common knowledge, henceforth called information structures \mathcal{I} . The information structure of the game with undisclosed incentives is given by $\mathcal{I}_U = \{F(\boldsymbol{\eta}, \boldsymbol{\Sigma}), \mu\}$, the game's fundamental parameters and their joint distribution. If commissions are disclosed, I will denote this information structure \mathcal{I}_D and will later specify it more exactly. Whenever I use the expectations operator or terms based on it such as variance or covariance, it is with respect to the respective information structure. For example, $E[c] = \bar{c}$ when $\mathcal{I} = \mathcal{I}_U$ but this will not hold with disclosed COIs (see next section).

Rational receivers maximize their expected utility by choosing $d_r(m)$. By assumption, naive receivers choose $d_n(m) = m$. From (6) one then gets the following expected utility for the sender:

$$U^S(s, c, m) = c(\mu m + (1 - \mu)d_r(m)) - \frac{1}{2}(m - s)^2 \quad (7)$$

The sender's message directly affects the naive receivers' demand and his lying cost. In addition, it indirectly influences the rational receiver's demand via its effect on $d_r(m)$. The optimal messaging strategy $m^*(s, c)$ needs to weight these factors. Analogous to the definition of $E[s|m^*]$ and (5), I define a rational receiver's optimal decision, given a m , formed by the sender's equilibrium messaging strategy $m^*(s, c)$, as follows:

$$d_r^*(m) \equiv \operatorname{argmax}_{d \in S} E[L(d - s)|m]_{m=m^*(s,c)}$$

In an equilibrium in which receivers' reaction are differentiable with respect to the sender's message, his underlying equilibrium strategy then has to solve the following first-order condition for every equilibrium message m^* formed by it:

$$m = s + c(\mu + (1 - \mu)d_r^{*'}(m)) \quad (8)$$

¹⁷Note that a complete belief profile over the sender's type also requires to specify an analogously-defined $E[c|m]_{m=m^*(s,c)}$. Since this is however payoff-irrelevant for either player, I omit it for the sake of easier notation and space.

The expression $d_r^{*'}(m)$ in the above denotes the marginal change in the action which maximizes a rational receivers' expected utility due to a change in the message when this is an image of the equilibrium messaging strategy. The sender therefore has to take into account that the rational receiver's marginal reaction to the message does not rely only on the message sent but also on the underlying messaging strategy. If for example, the equilibrium messaging strategy is very uninformative, $d_r^{*'}(m)$ should be relatively low. The sender thus trades off his incentive to bias the signal in order to affect receiver's decision with the loss in informativeness and therefore their reaction to that message. In addition he also faces costs of lying. To derive how the sender behaves optimally given these opposing incentives, I define the *equilibrium inference coefficient* ρ^* as a measure of the equilibrium strategy's informativeness:

$$\rho^* \equiv \frac{\text{Cov}[s, m^*]}{\text{Var}[m^*]} \equiv \frac{\text{Cov}[s, m]|_{m=m^*(s,c)}}{\text{Var}[m]|_{m=m^*(s,c)}} \quad (9)$$

This equilibrium parameter measures how much of the change in the equilibrium message refers to an actual change in the state of world s . Throughout this paper, I will focus on the case that ρ^* is a real, strictly positive number. I therefore rule out completely uninformative messaging strategies in which the message m and s do not covary, e.g. when the sender always sends the same or a completely random message, regardless of his type. It also means that messaging strategies which generate a negative correlation of the message with the state of the world are not considered. In such situation, it would be common knowledge that the sender tells the opposite of what is really going on – a feature which is unlikely to happen in an information market with experts.

Using this concept, one can show that in equilibrium, the inference coefficient has to be equal to the marginal reaction of rational receivers' to the message, thus that $d_r^{*'}(m) = \rho^*$ holds. Proving this relationship constitutes the main building block for the following proposition which characterizes the players' equilibrium actions and the relevant equilibrium beliefs:

Proposition 1. *Every pure-strategy Perfect Bayesian Equilibrium of the communication game with strategies $m^*(s, c)$ for the sender and $d_r^*(m) \in \mathcal{C}^2$ for rational receivers takes the form of*

$$m^*(s, c) = s + c(\mu + (1 - \mu)\rho^*) \quad (10)$$

$$d_r^*(m) = (1 - \rho^*)E[s] + \rho^*(m - E[c](\mu + (1 - \mu)\rho^*)) \quad (11)$$

The rational receivers equilibrium belief w.r.t. to s is given by $E[s|m^] = d_r^*(m)$.*

Proof. see appendix

The above result characterizes equilibria in which a rational receiver's reaction to the sender's

message is smooth in the sense that it is twice continuously differentiable. Note from (6) that assuming a smooth strategy for rational receivers is a necessary condition for the overall demand $D(m)$ to be smooth. I focus on such smooth, pure strategy equilibria because it is sufficient but yet relatively general to demonstrate the main point of this paper, the adverse effects of disclosure. It also captures the idea that arbitrarily small changes in the sender's message should yield no effect on demand.

In such an equilibrium, the sender's action takes an intuitive form. The sender announces the state of the world and adds a bias, given by $m^*(s, c) - s = c(\mu + (1 - \mu)\rho^*)$. This bias equals the marginal change in expected demand due to a change in the message, weighted with the sender's COI. Part of this expected demand is the rational receivers' belief about s . It consists of two parts: The first, weighted with $1 - \rho^*$, is their prior about the true state of the world, given by $E[s]$. The other part of her inference is given by the received message corrected for the expected bias $E[c](\mu + (1 - \mu)\rho^*)$. This is weighted with ρ^* , the informativeness of the message given the sender's equilibrium strategy. In consequence, the marginal reaction of rational receiver due to a change in the message is given by ρ^* . This equilibrium parameter will be particularly important for the following analysis.

Note that the correction for the expected bias is based on the expected commission. It can thus be wrong in both, direction and magnitude. This possible failure in rational receiver's de-biasing of the message and her risk-aversion provides the reason why she often does not react one-to-one to the corrected message. Whenever $\rho^* \in (0, 1)$ applies, a rational receiver strategically ignores the senders message and puts weight on her prior such that information is left unused. For illustration, consider a situation where σ_s^2 is almost zero, thus almost no fundamental uncertainty exists: In this case, $\text{Cov}[s, m^*]$ will also be close to zero and so is ρ^* . This would imply that a rational receiver acts almost entirely according to her prior $E[s]$. The reason is that the actual state of the world s is very close to the prior in such a situation. Any variation in the signal can then only be due to the sender's bias. Just following $E[s]$ is then optimal for the rational receiver since it brings her action very close to the true state of the world. In equilibrium, the sender takes such low values of ρ^* and the receiver's expected reaction into account by scaling down his bias. The ratioanle for this is that if receivers do not react to the message anyhow, there is no reason to bias one's message, given that lying is costly. Generally, lower values of ρ^* , thus a lower informativeness of the message, will lead to decrease in the sender's bias while higher informativeness increases it. Such reasoning is consistent with interpreting the equilibrium inference coefficient as the coefficient from a linear regression of the state of the world on the message: Both, a regression coefficient and ρ^* describe the marginal change in the conditional expectation of a dependent variable due to a marginal change in the

independent variable. The crucial difference is that in a regression, this refers to an exogenous change while here, it is the change in the endogenously determined equilibrium message.

Using the functional form as stated in proposition 1 for the messaging strategy can then be used to solve for the equilibrium inference coefficient:

Proposition 2. *In the above game with undisclosed commissions, the equilibrium inference coefficient $\rho_U^* = \frac{\text{Cov}[s, m^*]}{\text{Var}[m^*]}$ is a fixed point to*

$$g_U(\rho) = \frac{\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \quad (12)$$

Any fixed point $\rho_U^ \in (0, 1]$ is unique while a fixed point $\rho_U^* > 1$ is either unique or one of three such points.*

Proof. see appendix

The above result, together with proposition 1 completely describes the game's equilibrium strategies and beliefs. With the parameters contained in \mathcal{I}_U , precise expressions for the players' actions and relevant beliefs can be computed from these results. However, one can bound the inference coefficient which shapes such equilibria from more general features of the information structure:

Lemma 2. *A fixed point $\rho_U^* > 0$ to (12) exists if and only if $\sigma_{sc} > \tau^*$ for some $\tau^* < 0$.*

Proof. see appendix

As argued before, assuming that the inference coefficients is positively valued, thus that $\sigma_{sc} > \tau^*$ is natural for many applications of the model. I will therefore assume from now on that the above condition holds. From (11) in the preceding proposition one gets that a positive value of the inference coefficient means that a rational receivers' action d_r covaries with the sender's message m which in turn covaries with the state of the world s . The above lemma links this covariance in s and the messages and reactions about it to the game's information structure. It shows that any non-negative correlation between the state of the world and the COI is sufficient for ρ_U^* to be positive, thus for the message to be positively correlated with s . However, even with a negative correlation $\sigma_{sc} \in (\tau^*, 0)$ this can be the case since the sender's cost of lying counteracts equilibria with a negative correlation of the sender's message and s .

Proposition 2 also shows that multiple equilibria can only arise when the equilibrium inference coefficient is larger than one. The following result examines these cases in more detail and provides general conditions under which ρ_U^* is above and below the value of one, a threshold which will be important in the following analysis:

Lemma 3. *Suppose $\rho_U^* > 0$. Then $\sigma_c^2 \geq \sigma_s^2$ is a sufficient and $\sigma_{sc} \geq -\sigma_c^2$ is a necessary and sufficient condition for $\rho_U^* \leq 1$ to hold.*

Proof. see appendix

As with the preceding lemma, the result maps the game's parameters into behavior and back. It implies that more uncertainty about the COI than about the state of the world or any positive covariance between these variable are sufficient for a rational receiver's demand to be a unique, strictly convex combination between her prior and the sender's message, corrected for the expected bias.

Non-convex combinations are however still possible. If σ_{sc} is contained in $(\tau^*, -\sigma_c^2)$, the above lemma implies that rational receivers "over-react" – a change in the sender's message induces a change in rational receiver's demand greater than that original change in the message. To understand how such over-reaction occurs in equilibrium, note that such sufficiently negative values of σ_{sc} imply that one expects the sender to have a relatively strong incentive to push demand into a direction which is inversely related to the actual state of world s . However, since $\sigma_{sc} > \tau^*$ is also assumed to hold, it is not strong enough to induce a non-positive correlation between the message and s . In equilibrium, rational receivers anticipate this positive correlation between the message and s . They counteract the bias, which they expect to be strong and opposing in direction to s , by over-reacting to the corrected message. Since such correction involves correcting for the expected bias and receivers have concave utility, the expected disutility caused by such possible mis-correction is the greater, the more likely extreme values of c are. Thus, when the COI is too unpredictable relative to fundamental uncertainty, i.e. when $\sigma_c^2 \geq \sigma_s^2$, over-reaction cannot occur. The limit to such expectation-based corrections are also reached when $\sigma_{sc} \leq \tau^*$. In this case, the expected bias is so strong and opposed in direction to s that the risk of mis-correction outweighs the benefits of over-reacting.

Figure 1 illustrates these findings. It depicts the equilibrium inference coefficient for possible correlation of s and c . The three lines represent different values of σ_s^2 , higher ones representing larger variance. This ordering reflects that ceteris paribus, higher variation in s means that it explains more variation for a given messaging strategy and therefore increases the inference drawn from a message, as measured by ρ_U^* . Reflecting the previous lemma, it also shows that for any positive correlations and when $\sigma_c^2 > \sigma_s^2$, the equilibrium inference coefficient is always contained in the unit interval. The left region of the upper two lines then portrays parameter constellations where the inference coefficient is larger than one. The figure also portrays the normalized cut-off value $\tau^*/(\sigma_s\sigma_c)$ for positive equilibrium inference coefficients as a vertical line. For values of $\text{Corr}[s, c]$ below it, an equilibrium in which s and m^* are positively correlated

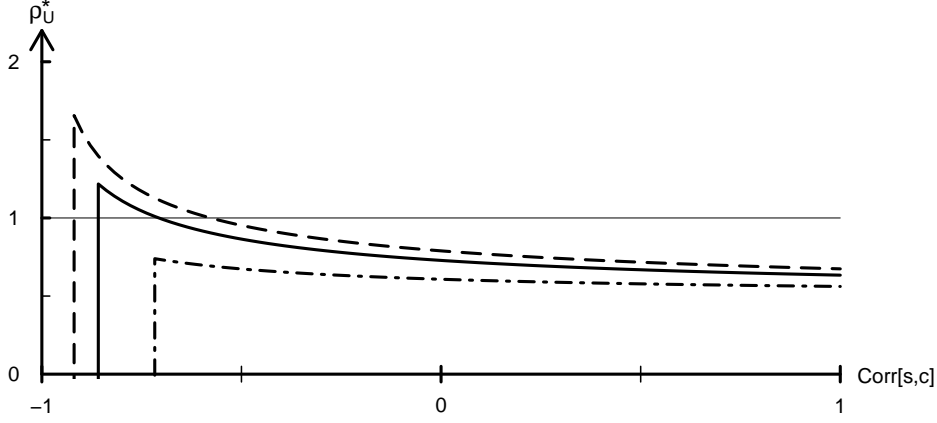


Figure 1: Positive equilibrium inference coefficients ρ_U^* over $\text{Corr}[s, c]$
Parameters: $\mu = 0.5$, $\sigma_c^2 = 1$, and $\sigma_s^2 = 1/2/3$ (bottom/middle/top line).

and in which rational receivers follow the corrected message does not exist.

4 Disclosed conflicts of interests

The above results show that when the sender's COI is his private knowledge, his equilibrium message is biased. Naive receivers who do not account for this bias are deceived by the sender and make wrong decisions. Rational receivers try to correct for the expected bias, but whenever the sender's COI differs from its expected value, thus almost surely, their following action is also sub-optimal. A tentative remedy to this is that the sender has to disclose his COI. Disclosure of COIs will be modeled through a signal \tilde{c} about c which receivers observe before they make their choice. The signal is given by $\tilde{c} = c + \epsilon$ where ϵ is an uncorrelated error term with the same elliptical distribution as (s, c) , an expected value of zero and finite variance $\sigma_\epsilon^2 \geq 0$. This variance therefore measures the quality of this signal. When it is zero the signal is perfectly accurate about the sender's COI, a setting I will henceforth call *full disclosure*. Conversely, *imperfect disclosure* describes situations where the signal error's variance is positive. The timing in the game with a disclosed COI is then as follows:

- 1a) the sender's type and the signal error (s, c, ϵ) are drawn,
- 1b) $\tilde{c} = c + \epsilon$ becomes common knowledge, (s, c) is privately observed by the sender,
- 2) the sender sends a signal m about s ,
- 3) receivers observe m , if rational update their belief about s , then choose d ,
- 4) payoffs are realized.

The only difference to the case of undisclosed COIs is therefore the signal \tilde{c} which allows rational receivers to update their beliefs regarding the value of c . The procedure for this is similar to the

signal extraction from the sender's message. The updated belief regarding c is a combination between the receiver's prior, given by \bar{c} and the signal \tilde{c} with weight for the latter chosen according to its precision. In analogy to the equilibrium inference coefficient one can define the following measure for the signal's informativeness:

$$\rho_c \equiv \frac{\text{Cov}[c, \tilde{c}]}{\text{Var}[\tilde{c}]} = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_\epsilon^2} \quad (13)$$

It reflects how much variation in c can be explained by the signal \tilde{c} about it. Accordingly, ρ_c is key for the distribution of the sender's type, conditional on the received signal about his COI:

Lemma 4. *The posterior distribution of $(s, c | \tilde{c})$ is given by $F(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\Sigma}})$ with*

$$\hat{\boldsymbol{\eta}} = \begin{bmatrix} \bar{s} + (\sigma_{sc}/\sigma_c^2)(\tilde{c} - \bar{c})\rho_c \\ \bar{c}(1 - \rho_c) + \tilde{c}\rho_c \end{bmatrix} \quad \text{and} \quad \hat{\boldsymbol{\Sigma}} = \begin{bmatrix} \sigma_s^2(1 - \rho_c(\text{Corr}[s, c])^2) & \sigma_{sc}(1 - \rho_c) \\ \sigma_{sc}(1 - \rho_c) & \sigma_c^2(1 - \rho_c) \end{bmatrix}$$

Proof. see appendix

Note that if c is correlated with the state of the world, the signal about also affects the posterior moments of s since it then also contains information about this variable. In the special case of full disclosure, $\rho_c = 1$ applies and the updated expectation for the COI equals the observed signal, as it is equal to the actual value of c . Since this eliminates any uncertainty about c , all second moments which contain this variable will then also shrink to zero. The effect of reducing uncertainty however applies to any signal about c and second moments are generally decreasing in its precision $1/\sigma_\epsilon^2$ via the parameter ρ_c .

To see how these *informational* consequences of disclosure translate into *strategic* ones, note that this new posterior distribution becomes common knowledge. In the previously analyzed communication game with undisclosed COIs, all results depended only on the information structure $\mathcal{I}_U = \{F(\boldsymbol{\eta}, \boldsymbol{\Sigma}), \mu\}$. After \tilde{c} has been observed and processed, the new information structure is given by $\mathcal{I}_D = \{F(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\Sigma}}, \tilde{c}), \mu\}$. Recall that expectations are with respect to the information structure, e.g. with disclosed COIs it holds that $E[c] = \bar{c}(1 - \rho_c) + \tilde{c}\rho_c$. Given this, one can proceed analogously as for undisclosed COI to determine equilibrium behavior by using \mathcal{I}_D instead of \mathcal{I}_U . In particular, proposition 1 applies and the equilibrium messaging strategy is of the linear form of the state of the world plus an endogenously determined bias. The equilibrium inference coefficient can then be determined analogously to the case of undisclosed COIs:

Proposition 3. Suppose $\rho_U^* > 0$ exists. Then there exists at least one value $\rho_D^* = \frac{\text{Cov}[s, m^*|\tilde{c}]}{\text{Var}[m^*|\tilde{c}]} > 0$. With full disclosure it is uniquely given by $\rho_D^* = 1$, with imperfect disclosure as a fixed point to

$$g_D(\rho) = \frac{\phi\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\phi\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \quad (14)$$

where $\phi \equiv \frac{1 - \rho_c(\text{Corr}[s, c])^2}{1 - \rho_c} > 1$.

Proof. see appendix

Together with proposition 1, this result completely characterizes equilibrium behavior in games with disclosed COIs. It shows that existence of an equilibrium with a positive inference coefficient under undisclosed COIs is sufficient to establish existence of a similar equilibrium when COIs are disclosed.

In the special case of full disclosure, the inference coefficient, and thereby the marginal reaction of rational receivers to the sender's message, is equal to one. The sender's equilibrium strategy is then given by $m^*(s, c) = s + c$ while rational receivers infer $E[s|m, \tilde{c}] = m - \tilde{c}$. Since with full disclosure $\tilde{c} = c$ applies, rational receivers therefore infer and implement the true state of the world and thus achieve maximum utility, the sender's message is however still biased.

With imperfect disclosure, the equilibrium inference coefficient ρ_D^* solves (14). The only difference to the characterization of inference coefficients with undisclosed COIs as stated in (12) is the additional term $\phi > 1$. Comparing $\hat{\Sigma}$ to Σ , one can see that the ratio which ϕ represent measures how much strategic uncertainty regarding the COI c remains after disclosure, relative to fundamental uncertainty regarding the state of the world s . To illustrate this, consider a signal \tilde{c} which is very uninformative, resulting in ρ_c close to zero. Rational receivers infer equally few about either kind of uncertainty with regard to s or c from such a signal, and the posterior is almost equal the prior. Also, ϕ is close to one and a solution ρ_D^* is then very close to ρ_U^* as (12) and (14) are almost the same term. As the signal quality increases, so do ρ_c and ϕ and the posterior's second moments decrease. However, $\phi > 1$ implies that the direct effect on reducing strategic uncertainty is always stronger than the signal's effect on decreasing fundamental uncertainty.

As in the case of full disclosure, multiple fixed points ρ_D^* to (14) can exist. The following lemma describes them and how they relate to the possible values of ρ_U^* :

Lemma 5. Suppose $\rho_U^* > 0$ exists. For any information structure \mathcal{I}_U and any signal quality of \tilde{c} , the following cases can occur:

- a) When there is a (unique) $\rho_U^* \in (0, 1)$, there is a unique solution $\rho_D^* \in (\rho_U^*, 1)$.
- b) When there is a (unique) $\rho_U^* = 1$, there is a unique solution $\rho_D^* = \rho_U^* = 1$.

c) When there is a unique $\rho_U^* > 1$, there is a unique solution $\rho_D^* \in (1, \rho_U^*)$.

d) When there are three values $\rho_U^* > 1$, there are either a unique or three solutions ρ_D^* :

If ρ_D^* is unique, then $1 < \rho_D^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$.

If there are three fixed points ρ_D^* , then $1 < \rho_{D,1}^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{D,2}^* < \rho_{D,3}^* < \rho_{U,3}^*$.

Proof. see appendix.

To understand the pattern which the above results reflect, first reconsider the case of full disclosure: Rational receivers can extract the true state from the sender's message and react fully to it, as captured by $\rho_D^* = 1$. When $\rho_U^* > 1$ applied before disclosure the inference shrinks towards one. When $\rho_U^* < 1$ applied before it increases towards this threshold. The cases a) through c) of the above proposition generalize this to the case of imperfect disclosure. Upon disclosure, the inference coefficient moves closer towards one or, for the special case that it equaled one with undisclosed COIs, remains at this value. Result d) then covers the case when there are multiple equilibrium inference coefficients, and therefore multiple equilibria, before disclosure. For the first subcase of d), the above pattern goes through and upon disclosure the new, unique equilibrium inference coefficient is closer to one than all those which existed before disclosure. To treat the effect of disclosure in the other subcase, some definitions will be useful. First, one can organize the possible equilibrium coefficients as follows:

Definition 1. Let $\rho_{D,k}^*$ and $\rho_{U,k}^*$ with $k \in \{1, 2, 3\}$ denote the positive-valued fixed points to (12) and (14), respectively, ordered by increasing value. In case that such a fixed point is unique, any such ρ_k^* denotes its value.

Then, one can order the equilibria according to the stability of the solutions they represent:

Definition 2. Let $g(\rho)$ be a continuously differentiable, real-valued function. A fixed point ρ_k^* to $g(\rho)$ is called asymptotically stable if and only if $|g'(\rho)| < 1$ at $\rho = \rho_k^*$.

From the above definition it follows that asymptotically stable equilibrium inference coefficients are locally unique and converge back to their original value after any sufficiently small perturbation. For example, they are robust to iterative search procedures and adaptive dynamics. Although intrinsically a dynamic concept, this criterion has a long history of being used in the analysis of one-shot situations, e.g. via (general) equilibrium tâtonnement processes and recently also in strategic communication.¹⁸ The following then relates this notion of asymptotically stable equilibrium inference coefficients, to the equilibria which they characterize:

¹⁸See Blume and Board (2014) who treat endogenously chosen vagueness in a one-shot communication game and use this concept. On p.869, they provide references on how asymptotic stability relates more generally to one-shot situations, in particular to Samuelson's correspondence principle.

Definition 3. Let the tuple $\mathcal{E}(\rho_k^*)$ collect strategies and beliefs as specified in proposition 1 with $\rho_k^* = \rho_{U(D)}^* > 0$ as described in proposition 2 (proposition 3) when $\mathcal{I} = \mathcal{I}_U$ ($\mathcal{I} = \mathcal{I}_D$). $\mathcal{E}(\rho_k^*)$ is then called an asymptotically stable equilibrium if and only if the corresponding ρ_k^* is asymptotically stable.

This notion allows to rank multiple equilibria as characterized by their equilibrium inference coefficients according to their asymptotic stability:

Lemma 6. An equilibrium $\mathcal{E}(\rho_k^*)$ is asymptotically stable if and only if $k \in \{1, 3\}$.

Proof. see appendix

The above, together with lemma 5 implies that the only unstable equilibria is the medium one when $\rho_U^* > 1$ and there are multiple equilibria (case d) of proposition 5). All other equilibria are asymptotically stable. In consequence, the following can be stated:

Corollary 1. In any asymptotically stable equilibrium $\mathcal{E}(\rho_k^*)$, the equilibrium coefficient strictly increases (decreases) upon disclosure if and only if $\rho_U^* < 1$ ($\rho_U^* > 1$).

As a direct consequence of the preceding analysis, there is a non-monotone effect of disclosure on the sender's bias, via the equilibrium inference coefficient. The following section explores this key effect of disclosure in more detail, in particular how it affects the welfare of different receiver types and overall efficiency.

5 Consequences of disclosure

In the following, I will take an ex-ante view on receivers' utility, thus before a draw of the sender's type takes place. I start with naive receivers. They are agnostic about the sender's bias and do not account for the strategic change since for them, $d_n^*(m) = m$ holds. Recall that receivers' utility decreases in the distance of their decision and the state of the world. Since the sender's equilibrium strategy is to announce the state of the world plus a bias, the argument of naive receivers' utility then equals this bias, given by $m^*(s, c) - s$. Their expected utility in equilibrium $\mathcal{E}(\rho_k^*)$ is therefore equal to

$$\mathbb{E}[u_n^R(\mathcal{E}(\rho_k^*))] = \mathbb{E}[L(c(\mu + (1 - \mu)\rho^*))] = \mathbb{E}[L(|c|(\mu + (1 - \mu)\rho^*))] < 0 \quad (15)$$

where the second equality follows from the fact that only c can be non-positive, while L is negatively valued and symmetric around its maximum of zero. The expected utility of naive receivers therefore strictly decreases in ρ^* and the following immediately emerges:

Corollary 2. *Upon disclosure of COIs, the expected utility of naive receivers decreases (increases) in every asymptotically stable equilibrium $\mathcal{E}(\rho_k^*)$ if and only if $\rho_U < 1$ ($\rho_U > 1$).*

The result shows that in any stable solution to the communication game, naive receivers can only benefit from disclosure when the equilibrium inference coefficient is larger than one, e.g. only if none of the conditions stated in lemma 3 is fulfilled. Otherwise, naive receiver suffer from disclosure.

To evaluate the overall effect of disclosure, one needs to also look on its effects for rational receivers. These receivers correct for the expected bias and therefore implement what they infer to be the expected state of the world. However, the risk of this implementation being wrong yields (expected) disutility whose magnitude depends on the concavity of L and the volatility of the sender's bias. To obtain a tractable measure for rational receivers' expected utility, one can exploit the property E2 of elliptically distributed variables, namely that they inherit their distribution to linear combinations formed from them. In particular, it follows from (10) that the rational receiver's decision error $d_r^* - s$ is elliptically distributed. Using an approach similar to Meyer (1987), one can then show that this is sufficient to represent rational receivers' expected utility as mean-variance preferences. This then admits the following representation of rational receivers' expected utility:

Lemma 7. *The expected utility of rational receivers in equilibrium $\mathcal{E}(\rho_k^*)$ is given by*

$$\mathbb{E}[u_r^R(\mathcal{E}(\rho_k^*))] = \mathcal{L} \left(\sigma_s^2 \left[1 - (\text{Corr}[s, m]_{m=m^*(s,c)})^2 \right] \right) \leq 0$$

where $\mathcal{L} \in \mathcal{C}$ obeys $\mathcal{L}'(x) \leq 0$ for any $x \geq 0$. Furthermore, $\mathbb{E}[u_r^R(\mathcal{E}(\rho_k^*))] = 0$ if and only if there is full disclosure of the sender's COI.

Proof. see appendix

The lemma allows to represent the expected utility of rational receivers by a loss function with a single argument. This argument only depends on the information structures and the equilibrium inference coefficient derived from it. It is given by the fundamental uncertainty regarding σ_s^2 , scaled down by the squared correlation of the state of the world with the sender's equilibrium message. Note that the empirical analog to this measure, the squared sample correlation of s and m^* , equals the coefficient of determination one would obtain if one regressed past values of s on the sender's message. In consequence, the more the sender's message is correlated with the variable it is supposed to report on, the better off are rational receivers.

This formulation of the sender's expected utility helps in analyzing the opposing effects of disclosure. When ρ^* increases upon disclosure, this reflects a greater informativeness of the

sender's message. However, the sender strategically anticipates this and in turn, also increases his bias' magnitude which makes the rational receivers' inference more complicated again. Conversely, when ρ^* decreases, so does the bias. But does such a decrease in the inference coefficient then not imply that also the message's informativeness, and therefore the rational receivers' utility, decreases? Using the above lemma, the following result then shows that in both scenarios, the net effect of disclosure on rational receivers' expected utility is positive:

Proposition 4. *Upon disclosure of COIs, the expected utility of rational receivers increases in every asymptotically stable equilibrium $\mathcal{E}(\rho_k^*)$.*

Proof. see appendix

While this is good news from the perspective of rational receivers, the following results immediately follows from the preceding ones. It shows that disclosure often requires to trade of naive receivers' expected utility against the expected utility of rational ones:

Corollary 3. *In any asymptotically stable equilibrium $\mathcal{E}(\rho_k^*)$, disclosure of COIs is a Pareto-improvement among receivers if and only if $\rho_U^* \geq 1$.*

Thus, only when the inference coefficient is at least one, then all receivers benefit from disclosure. If this is not the case, e.g. when any of the conditions in lemma 3 apply, naive receivers will suffer from disclosure and a Pareto-criterion forbids it. A policy maker who can influence disclosure and the quality of disclosed information may want to resort to other criteria such as efficiency in this case. I capture such an efficiency criterion by assuming that a policy maker wants to maximize a welfare function of the following form where the weights w_n and w_r are assumed to be positive, w_K to be non-negative:

$$W(\mathcal{E}(\rho_k^*)) = w_n \cdot E[u_n^R(\mathcal{E}(\rho_k^*))] + w_r \cdot E[u_r^R(\mathcal{E}(\rho_k^*))] - w_K \cdot E[(c(\mu + \rho_k^*(1 - \mu)))^2] \quad (16)$$

In the above, the first two terms capture the expected utility of naive and rational receivers, respectively. The third term allows to capture the sender's expected cost of lying $E[K(m; s)]$. A straightforward choice is the respective population share, i.e. $w_n = \mu$, $w_r = 1 - \mu$, and $w_K = 0$. Other weights can capture further considerations: If Bayesian inference and the steps involved therein are costly for rational receivers, a fully informative but biased message is not optimal. In face of such de-biasing costs, only a truthful, unbiased message would be optimal. A choice $w_n > \mu$ would then reflect the relative importance of these costs, relative to the pure informational content of the sender's message and in addition to its effect on naive receivers. Setting $w_K > 0$ allows to capture when the sender's expected costs of lying are relevant for a

policy maker.¹⁹

Generally, the exact decision of whether disclosure should occur and how precise it should be can only be answered when specific weights and utility functions are assumed. However, some policy-relevant statements with respect to the effect of disclosure on W can be made even when the exact values of these parameters are unknown:

Proposition 5. *Full disclosure never (always) maximizes social welfare W in an asymptotically stable equilibrium $\mathcal{E}(\rho_k^*)$ when $\rho_U^* < 1$ ($\rho_U^* \geq 1$).*

Proof. see appendix

The second statement follow directly from the previous corollary. The first result, when $\rho_U^* < 1$, follows from the fact that receivers have strictly concave utility. They can achieve maximum utility only with full disclosure (see lemma 7). When, near this optimum, some noise is added to the signal \tilde{c} the resulting loss is then always smaller than the gain in expected utility this induces for naive receivers, via the associated decrease in ρ^* . Note that while full disclosure is often not optimal, the reverse reasoning does not work and no or imperfect disclosure can be optimal, in a second-best sense. Determining precise criteria for this however requires more specific assumptions on the informational environment and preferences. An example when no disclosure is best is contained in the appendix.

The preceding results which examine the consequences of disclosing COIs depend crucially on whether ρ_U^* is above or below the threshold of one. Besides computing this value via (12), lemma 3 allows to determine this directly from the second moments of the sender's type distribution. However, even knowledge of these parameters is not necessary either for an outside observer to yield testable predictions and to make informed decisions. One only needs to observe aggregate players' behavior. The reason for this is that $\rho_D^* > 1$ is easily verified to be a necessary and sufficient condition for the overall demand to react over-proportionally to a message, i.e. $D'(m) > 1$. If in contrast the market is observed to react under-proportionally to new messages, thus when $D'(m) < 1$, it follows from (6) that $\rho_U^* < 1$ has to hold. For example, observing stronger than one-to-one reactions to messages by experts is a sufficient condition to mandate full disclosure. Observing less than one-to-one reactions however indicates that full disclosure will increase average reaction upon disclosure, thereby hurt naive receivers, and is thus not optimal from the point of maximizing welfare.

¹⁹Apart from a welfare function which take into account the lying costs per se, such costs may also matter from a policy point of view when they represent a reduced form of the sender's reputation. This becomes economically relevant when the loss in the sender's credulity impedes his economic function of information elicitation and dissemination. Alternatively, if the sender's cost of lying come from a situation where he has to invest own funds, e.g. own equity, according to his advice, these costs then reflect the cost of such mis-allocated capital.

6 Conclusion

"Discounting advice appropriately for a disclosed conflict of interest requires a mental model of advisor behavior[...]" is what Loewenstein et al. (2011), p.424 conclude in their work on the failure of disclosing COIs. This paper does so in a setting where an expert who faces a private COI communicates the value of a random variable of interest. He does so by reporting to uninformed receivers, some of which are naive towards the resulting bias and facing lying costs when he biases his message. The model provides a parsimonious framework for studying the effect of disclosing the sender's COI either fully or imperfectly within a wide class of distributions and for arbitrary degrees of the receivers' risk aversion.

I find that disclosure fulfills the aim of informing *rational* receivers: Information about the sender's COI helps them to learn more from the sender's biased message and to choose actions which match better the actual state of the world. On the downside however, this paper's core result shows that exactly this desired effect of disclosure backfires on receivers who are not strategically sophisticated. It does so because in equilibrium, the average reaction to the biased signal and the sender's bias are mutually dependent. Upon disclosure, the reaction by rational receivers to the sender's message often increases and so does the bias contained in this message. Naive receivers who do not account for this are then hurt by disclosure. Disclosure thus amplifies a negative externality which rational receivers exert on their naive peer and therefore hurts those which are most vulnerable to strategic biases.

I also determine more precisely when and how these adverse effects of disclosure manifest. In terms of economic fundamentals, this is always the case when the state of the world and the sender's COI are weakly positively correlated. Another sufficient condition for disclosure to backfire is when strategic uncertainty regarding the sender's COI exceeds fundamental uncertainty regarding the variable which describes the state of the world. In terms of observed behavior, this happens when an expert's message does not induce an equivalent, e.g. one-to-one, reaction among receivers. Only when receivers "over-react", that is when they react, on average, stronger than one-to-one to changes in the sender's message, then disclosure is an improvement among all, rational *and* naive, receivers. This is also the only case when full disclosure is optimal from an efficiency point of view. In all other cases, a less than perfect signal about the sender's COI, potentially even an uninformative one, is optimal for maximizing efficiency. Taken together, the results derived in this work support the conclusion that in many settings of strategic communication, the inefficiencies which arise from experts' conflicts of interest are best solved by eliminating rather than by just announcing them.

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Appendix

Features of elliptically distributed random variables

The following definition is obtained from definition 1 in the survey on elliptical distributions by Gómez et al. (2003) and Theorem 4 ii) therein:

Definition: A random vector $\mathbf{x} \in \mathbb{R}^n$ is elliptically distributed with expected value $\boldsymbol{\eta} \in \mathbb{R}^n$, positive definite variance-covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{n \times n}$, and the Lebesgue-measurable function $g : [0, \infty) \rightarrow [0, \infty)$ s.t. $\int_0^\infty t^{\frac{n}{2}-1} g(t) dt < \infty$ as parameters, if it has the density function

$$f(\mathbf{x}; \boldsymbol{\eta}, \tilde{\boldsymbol{\Sigma}}, g) = c_n |\tilde{\boldsymbol{\Sigma}}|^{-\frac{1}{2}} g\left((\mathbf{x} - \boldsymbol{\eta})^T \tilde{\boldsymbol{\Sigma}}^{-1} (\mathbf{x} - \boldsymbol{\eta})\right)$$

where $c_n = \Gamma(\frac{n}{2}) / \left(\phi^{\frac{n}{2}} \int_0^\infty t^{\frac{n}{2}-1} g(t) dt\right)$, and $\tilde{\boldsymbol{\Sigma}} \propto \boldsymbol{\Sigma}$.

The exact form of the distribution therefore depends on the density generator g . In the context of this paper it is assumed to be implicitly defined by the specific elliptical distribution F which is assumed. The generic example is when F denotes the (multivariate) normal distribution which would imply that $g(t) = \exp(-\frac{1}{2}t)$. Other examples include the multivariate logistic, student-t or power exponential families of distributions.

The results in this paper do not depend on the specific distribution F as long as it is elliptical, but just on its first two moments, $\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}$. To illustrate them, consider a random vector $\mathbf{x} \in \mathbb{R}^n$ with $n \geq 2$ which is elliptically distributed according to $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$. Also consider two non-empty partitions $[\mathbf{x}_1, \mathbf{x}_2]$ of this vector. Partition analogously the corresponding vector of expected values as $\boldsymbol{\eta} = (\boldsymbol{\eta}_1, \boldsymbol{\eta}_2)$ and the variance-covariance matrix $\boldsymbol{\Sigma}$ into blocks $(\boldsymbol{\Sigma}_{11}, \boldsymbol{\Sigma}_{12}, \boldsymbol{\Sigma}_{21}, \boldsymbol{\Sigma}_{22})$. Then, the following properties hold for \mathbf{x} :

E1: *linear combinations of elements of \mathbf{x} are distributed according to F*

E2: *$(\mathbf{x}_2 | \mathbf{x}_1)$ is distributed according to*

$$F(\boldsymbol{\eta}_2 + \boldsymbol{\Sigma}_{21} \boldsymbol{\Sigma}_{11}^{-1} (\mathbf{x}_1 - \boldsymbol{\eta}_1), \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21} \boldsymbol{\Sigma}_{11}^{-1} \boldsymbol{\Sigma}_{12})$$

E3: *\mathbf{x} is symmetrically distributed around $\boldsymbol{\eta}$*

E3 follows from the above density function. Properties E1 and E2 are consequences of Theorem 5 and Theorem 8, respectively in Gómez et al. (2003) which also contains further references on the original research establishing these properties for elliptical distributions. The three statements about elliptical distributions in section 3 then reflect the above properties. It will be useful to

note that for the special case that $(\mathbf{x}_2|\mathbf{x}_1) \in \mathbb{R}^2$, E2 implies that $(x_2|x_1)$ is distributed according to

$$F\left(E[x_2] + (x_1 - E[x_1]) \frac{\text{Cov}[x_1, x_2]}{\text{Var}[x_1]}, \text{Var}[x_2] (1 - (\text{Corr}[x_1, x_2])^2)\right)$$

Proof of Lemma 1

When m is distributed according to F , it is jointly elliptically distributed with s . By E2, the resulting distribution of s conditional on m , denoted by its pdf $f(s|m)$, is then also elliptical. Furthermore, E3 implies that $f(s|m)$ is symmetric around $E[s|m]$. By definition, it then has to hold that $d_r = \arg\max_{d \in S} \int_{\mathbb{R}} L(d - s) f(s|m) ds$. The necessary FOC for a candidate solution $d_r = E[s|m]$ is given by

$$0 = \int_{\mathbb{R}} L'(d_r - s) f(s|m) ds = \int_{-\infty}^{+\infty} L'(E[s|m] - s) f(s|m) ds$$

and is also sufficient as L is strictly concave. To verify that this FOC applies for this candidate solution note that by being strictly concave, L is single peaked and symmetric around its bliss point s . Let $\Delta \leq 0$ be the absolute deviation of the candidate solution from the optimal choice, i.e. $\Delta = |d_r - s|$. By symmetry of L around zero it holds that $L'(\Delta) = -L'(-\Delta)$. Since $f(s|m)$ is symmetric around $E[s|m] = d_r$ it then follows that

$$L'(\Delta) f(d_r - \Delta|m) = -L'(-\Delta) f(d_r + \Delta|m) \leq 0$$

for any $\Delta \geq 0$. Integrating over all $\Delta \in \mathbb{R}_+$ then validates that the above FOC actually holds. Since L is single-peaked, it is also the only solution. \square

Proof of Proposition 1

The following proof constructs a pure strategy equilibrium when the implied demand $D(m)$ is twice continuously differentiable. To do so, it proceeds in three steps. Step 1 solves the rational receiver's problem to choose his optimal action given that the sender's message contains information about s . Step 2 determines how such signal extraction by rational receivers manifests in equilibrium when the sender anticipates this process. Step 3 combines these results to obtain equilibrium actions and beliefs.

Step 1: Consider a candidate equilibrium messaging strategy $\tilde{m}(s, c)$ such that $D(m) = \mu m + (1 - \mu)\tilde{d}_r(m)$ with $\tilde{d}_r(m) \equiv \arg\max_{d \in S} E[L(d - s)|m]_{m=\tilde{m}(s, c)}$ exists and is twice differentiable w.r.t. m . By (8), \tilde{m} has to solve $\tilde{m} = s + c(\mu + (1 - \mu)\tilde{d}_r^{*'}(\tilde{m}))$. Note that given the message \tilde{m} and the candidate equilibrium messaging function $\tilde{d}_r, \tilde{d}_r^*(\tilde{m})$ is a non-random image of the

function $\tilde{d}'_r(m)$. The associated messaging function which yielded \tilde{m} is thus given by

$$\tilde{m}(s, c) = s + c(\mu + (1 - \mu)\tilde{d}'_r(\tilde{m}))$$

By E1, it is elliptically distributed according to F .

Lemma 1 then implies that $\tilde{d}_r(m) = E[s|m]_{m=\tilde{m}(s,c)}$. Using E2 then yields that for a given equilibrium message \tilde{m} , it has to hold that

$$\tilde{d}_r(m) = E[s] + \left(m - E[s] + E[c](\mu + (1 - \mu)\tilde{d}'_r(m)) \right) \frac{\text{Cov}[s, m]_{m=\tilde{m}(s,c)}}{\text{Var}[m]_{m=\tilde{m}(s,c)}} \quad (17)$$

with expected value $E[m]_{m=\tilde{m}(s,c)} = E[s] + E[c](\mu + (1 - \mu)\tilde{d}'_r(m))$ and the associated equilibrium parameters $\text{Cov}[s, m]_{m=\tilde{m}(s,c)}$ and $\text{Var}[m]_{m=\tilde{m}(s,c)}$.

Step 2: In equilibrium, $\tilde{d}_r^*(m)$ has to solve the first-order linear differential equation

$$\tilde{d}_r'(m) = \left(1 - E[c](1 - \mu)\tilde{d}_r''(m) \right) \tilde{\rho}$$

with $\tilde{\rho} \equiv \frac{\text{Cov}[s, m]_{m=\tilde{m}(s,c)}}{\text{Var}[m]_{m=\tilde{m}(s,c)}}$ which is obtained by differentiating (17) w.r.t. m .

When $\tilde{d}_r'(m) = 0$, it follows that $\tilde{d}_r(m) = \tilde{\rho} = 0$. Similarly, if $E[c] = 0$, then $\tilde{d}_r'(m) = \tilde{\rho}$. Now suppose that $\tilde{\rho}E[c] \neq 0$. One then gets $\tilde{d}_r'(m)$ as the solution to the above differential equation given by

$$\tilde{d}_r'(m) = \rho + \xi \cdot \exp\left(-\frac{m}{(1 - \mu)E[c]\tilde{\rho}}\right)$$

where ξ is an integration factor. To determine its value, integrate the obtained $\tilde{d}_r'(m)$ over M to get

$$E[s|m]_{m=\tilde{m}(s,c)} = \int_{-\infty}^{+\infty} \tilde{d}_r'(m) dm = m\tilde{\rho} - \xi(1 - \mu)E[c]\tilde{\rho} \cdot \exp\left(-\frac{m}{(1 - \mu)E[c]\tilde{\rho}}\right) + \tilde{K} \quad (18)$$

where \tilde{K} is a constant of integration. This can be plugged into the sender's expected utility (7) to obtain the following expression for $U^S(s, c, m)|_{m=\tilde{m}(s,c)}$:

$$c\mu m + c(1 - \mu) \left[m\tilde{\rho} - \xi(1 - \mu)E[c]\tilde{\rho} \cdot \exp\left(-\frac{m}{(1 - \mu)E[c]\tilde{\rho}}\right) + \tilde{K} \right] - \frac{1}{2}(m - s)^2 \quad (19)$$

To determine ξ , I start with the case that $c > 0$. In this case, $U^S(s, c, m)$ is increasing in $E[s|m^*]$, the term above in square brackets. If $\tilde{\rho}E[c] > 0$ the sender's expected utility decreases exponentially in m while all other terms involving m are either linear or quadratic. If $\xi < 0$, the sender would then maximize his expected utility by choosing $m \rightarrow -\infty$ and there is no

equilibrium. Therefore, $\xi \geq 0$ has to hold in this case for any equilibrium. For $\xi > 0$ however, $U^S(s, c, m)$ would be lower than with $\xi = 0$. Since ξ is part of the endogenous inference of the sender's signal, he will not send a signal which allows such an inference. It follows that with $c > 0$ and $\rho E[c] > 0$, only $\xi = 0$ can be the equilibrium integration factor.

Continue to suppose that $c > 0$ but now $\rho E[c] < 0$ holds. Reverse to the the preceding reasoning, $E^*[s|m]$ now increases exponentially in m which implies a global maximum of the sender's expected utility at $m \rightarrow +\infty$ whenever $\xi > 0$. Thus, for an equilibrium $\xi \leq 0$ has to hold. Again, any strictly negative value of ξ would decrease the sender's expected utility. Messaging strategies allowing such inference are therefore not chosen by the sender and $\xi = 0$ holds in any equilibrium with $c > 0$ and $\rho E[c] < 0$.

For the case that $c < 0$, $U^S(s, c, m)$ is decreasing in $E[s|m^*]$. The same reasoning as for the case of $c > 0$ but with reversed signs can then be repeated which rules out any $\xi \neq 0$ in equilibrium when $c < 0$ and $\tilde{\rho} E[c] \neq 0$.

Eventually, when $c = 0$ the inference $E[s|m]$ does not enter $U^S(s, c, m)$ and therefore does neither affect the sender's action nor the receiver's reaction to it and one can assume w.l.o.g. $\xi = 0$. It therefore has to hold in any equilibrium that $\xi = 0$ and therefore $\tilde{d}_r(m) = \tilde{\rho}$.

Step 3: Given the above, one can determine the integration constant

$$\tilde{K} = E[s] - (E[s] + E[c] (\mu + (1 - \mu)\tilde{\rho}))\tilde{\rho}$$

by combining (17) and (18). Using $\xi = 0$ then allows to write (19) as

$$U^S(s, c, m) = mc(\mu + (1 - \mu)) - \frac{1}{2}(m - s)^2 + c(1 - \mu)K$$

It is then easily verified that the unique message which maximizes the above expression is given by $m = s + c(\mu + (1 - \mu))\tilde{\rho}$. In equilibrium, it thus holds that $m^*(s, c) = s + c(\mu + (1 - \mu))\tilde{\rho}$ with $\rho^* = \tilde{\rho} = d_r^{*'}(m) = \frac{\text{Cov}[s, m]_{m=m^*(s, c)}}{\text{Var}[m]_{m=m^*(s, c)}}$ as stated in (10). Using $\tilde{\rho} = \rho^*$, $\xi = 0$, and the above expression for \tilde{K} on (18) then yields the rational receivers belief and strategy as stated in (11). \square

Proof of Proposition 2, part 1

By using $m^*(s, c) = s + c(\mu + (1 - \mu)\rho)$ from proposition 1 and the definition of $\rho^* = \frac{\text{Cov}[s, m^*]}{\text{Var}[m^*]}$, the latter must be a solution to

$$\begin{aligned} \rho &= \frac{\text{Cov}[s, m^*]}{\text{Var}[m^*]} = \frac{\text{Cov}[s, m]_{m=m^*(s, c)}}{\text{Var}[m]_{m=m^*(s, c)}} = \frac{\text{E}[(s - \text{E}[s])((s - \text{E}[s]) + (\mu + (1 - \mu)\rho)(c - \text{E}[c]))]}{\text{E}[((s - \text{E}[s]) + (\mu + (1 - \mu)\rho)(c - \text{E}[c]))^2]} \\ &= \frac{\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \end{aligned}$$

which yields (12). It will be useful to define this fixed point expression via $g_U(\rho) = N(\rho)/D(\rho)$ with $N(\rho) = \sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}$ and $D(\rho) = \sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2$ reflecting the above nominator and denominator. The following properties then hold for the rational function $g_U(\rho) = N(\rho)/D(\rho)$:

Property a): $N(\rho)/D(\rho)$ is continuous with $D(\rho) > 0$ for all $\rho \in \mathbb{R}$.

Proof: Since both, $D(\rho)$ and $N(\rho)$ are continuous in ρ , it is sufficient to show that $D(\rho) > 0$ always holds. Suppose to the contrary it would not. Rearranging $D(\rho)$, this would require that $\rho^2 + a\rho + b = 0$ with

$$a = \frac{2(\sigma_{sc} + \mu\sigma_c^2)}{(1 - \mu)\sigma_c^2} \quad b = \frac{\sigma_s^2 + 2\mu\sigma_{sc} + \mu^2\sigma_c^2}{(1 - \mu)^2\sigma_c^2}$$

has at least one real solution, thus that $(a/2)^2 - b \geq 0$ holds. Plugging in and rearranging, this yields $(\sigma_{sc}/(\sigma_c\sigma_s))^2 \geq 1$ - a contraction to $|\text{Corr}[s, c]| < 1$.

Property b): $\lim_{\rho \rightarrow +\infty} \left(\frac{N(\rho)}{D(\rho)} \right) = 0^-$ if $\sigma_{sc} < 0$ and $\lim_{\rho \rightarrow +\infty} \left(\frac{N(\rho)}{D(\rho)} \right) = 0^+$ if $\sigma_{sc} \geq 0$

Proof: $N(\rho)$ strictly decreases (weakly increases) linearly in ρ when $\sigma_{sc} < 0$ ($\sigma_{sc} \geq 0$) and therefore attains negative (positive) values for ρ large enough. From a), $D(\rho)$ is strictly positive and it grows quadratically in ρ . Therefore, for large values of ρ , the ratio $N(\rho)/D(\rho)$ is negative (positive) and arbitrarily close to zero.

Property c): $\frac{N(\rho)}{D(\rho)}$ has at most two extreme points.

Proof: Any extreme point has to set the first derivative

$$\begin{aligned} \left(\frac{N(\rho)}{D(\rho)} \right)' &= \frac{(1 - \mu)\sigma_{sc}D(\rho) - 2(1 - \mu)N(\rho)(\sigma_{sc} + (\mu + (1 - \mu)\rho)\sigma_c^2)}{(D(\rho))^2} \\ &= \frac{(1 - \mu)}{D(\rho)} \cdot \left(\sigma_{sc} - \frac{N(\rho)}{D(\rho)} \cdot 2(\sigma_{sc} + (\mu + (1 - \mu)\rho)\sigma_c^2) \right) \end{aligned} \tag{20}$$

equal to zero. By a), the first factor is non-zero. Extreme points therefore have to solve

$$\sigma_{sc}D(\rho) = N(\rho) \cdot 2(\sigma_{sc} + (\mu + (1 - \mu)\rho)\sigma_c^2)$$

Plugging in the functions for $N(\rho)$ and $D(\rho)$ yields an equation which is quadratic in ρ and thus has at most two real solutions.

Before I continue with remaining part of the proof of proposition 2, I first proof lemma 2 and 3. While they are stated in the main text after proposition 2 for reasons of better exposure, they only rely on properties proved so far.

Proof of Lemma 2

Necessitiy: By property a) as derived above it follows that for a fixed point ρ_U^* which solves $g_U(\rho_U^*) = N(\rho_U^*)/D(\rho_U^*) > 0$, $N(\rho_U^*) > 0$ has to hold. This is equivalent to $\sigma_{sc} > \tau(\rho_U^*)$ where $\tau(\rho) = -\sigma_s^2/(\mu + (1 - \mu)\rho) < 0$ is defined for any $\rho > -\mu/(1 - \mu)$. Note that $\tau'(\rho) > 0$ for all $\rho \geq 0$. For $\rho_U^* > 0$ it therefore has to hold that $\sigma_{sc} > \tau^* \equiv \tau(\rho_U^*)$ with $\tau^* < 0$.

Sufficiency: To see that $\sigma_{sc} > \tau^*$ is also sufficient for (12) to have a solution $\rho_U^* > 0$, note that by the above reasoning $\sigma_{sc} > \tau(\rho^*) > \tau(0)$ holds and therefore, $\text{Cov}[s, m^*]|_{\rho=0} = N(0) > 0$ applies. Since $\text{Var}[m^*]_{\rho=0} = D(0) > 0$, it then follows that $g_U(0) = N(0)/D(0) > 0$. Together with continuity and a limit of zero of $g_U(\rho)$ as derived in properties a) and b) above, this means that there has to be at least one fixed point, i.e. at least one intersection of $g_U(\rho)$ with the 45-degree line over \mathbb{R}_{++} .

Proof of Lemma 3

I start with the necessary and sufficient condition: By proposition 2, $\rho_U^* \leq 1$ conditional on $\rho_U^* > 0$, holds if and only if

$$\left. \frac{\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \right|_{\rho=\rho_U^*} \leq 1$$

This condition simplifies, such that $\rho_U^* > 0$ is equivalent to

$$\sigma_{sc} \geq -(\mu + (1 - \mu)\rho_U^*)\sigma_c^2 \tag{21}$$

The above inequality becomes more slack for higher, positive values of ρ_U^* . Replacing it by $\rho_U^* = 1$, the upper bound on the desired value range, yields the stated condition $\sigma_{sc} \geq -\sigma_c^2$.

To see the second sufficient condition, note that this is equivalent to $\text{Corr}[s, c] \geq -\sigma_c/\sigma_s$ and therefore that it always holds when $\sigma_c^2 \geq \sigma_s^2$. \square

Proof of Proposition 2, part 2

A solution ρ_U^* to (12) requires an intersection of the 45-degree line and $N(\rho)/D(\rho)$. Note that every such fixed point has to be a root of the cubic equation $k(\rho) = \rho D(\rho) - N(\rho) = A\rho^3 + B\rho^2 + C\rho + D = 0$ with coefficients

$$A = (1 - \mu)^2 \sigma_c^2, \quad B = 2(1 - \mu)(\sigma_{sc} + \mu \sigma_c^2), \quad C = \sigma_s^2 + \mu^2 \sigma_c^2 + (3\mu - 1)\sigma_{sc}, \quad D = -\sigma_s^2 - \mu \sigma_{sc}.$$

To examine multiplicity of such roots, I use the following result:

Theorem. (Descarte's rule of signs) *Consider a n -degree polynomial $p(x) = \sum_{k=0}^n c_k \cdot x^k$ with real coefficients. Order the non-zero coefficients c_k in an descending order of the exponent of k . The number of positive, real roots of the polynomial is less by an even number or equal to the number of sign changes between successive coefficients in this ordering.*

It always holds that $A > 0$. Furthermore, by lemma 2, a solution $\rho_U > 0$ implies $D < 0$ because $-D = \text{Cov}[s, m^*]|_{\rho=0} = N(0) > 0$. By the sign rule, the only configuration for more than one sign change, given that $A > 0 > D$, is $C > 0 > B$. Thus, there are either one or three positive roots corresponding to fixed points of $g_U(\rho)$.

Multiple fixed points therefore require $B < 0$, thus $\sigma_{sc} < 0$. Suppose they exist. By property a) and b) derived in the first part of this proof, this means that $g_U(\rho) = N(\rho)/D(\rho)$ continuously approaches zero from below when ρ becomes large enough. Also, it has been shown that $N(0)/D(0) > 0$ (see proof of lemma 2). Together, this implies that g_U has to have a negatively valued local minimum on \mathbb{R}_{++} denoted by ρ_- , i.e. $g_U(\rho_-) = N(\rho_-)/D(\rho_-) < 0$. If ρ_- is the only extreme value over \mathbb{R}_{++} this implies only one intersection with the 45-degree line, thus a unique fixed point. If it is not the unique extreme value, by property c) derived in the first part of the proof, there is exactly one other extreme value of g_U over \mathbb{R}_{++} . It has to be a local maximum since ρ_- is a local minimum; I denote its location ρ_+ . By property b) it then has to hold that $0 < \rho_+ < \rho_-$. Furthermore, since ρ_+ is a local maximum and $N(0)/D(0) > 0$ it follows that $g_U(\rho_+) = N(\rho_+)/D(\rho_+) > 0$.

Accordingly, $g_U(\rho) = N(\rho)/D(\rho)$ has to be non-increasing between its local maximum value $g_U(\rho^+) > 0$ and its minimum value $g_U(\rho^-) < 0$. Because $\sigma_{sc} < 0$ was assumed, it approaches its limit of zero from below on $[\rho_-, +\infty)$ (see property b) in the first part of the proof) and therefore cuts the 45-degree line exactly once and this has to be over (ρ_+, ρ_-) . Multiple, positively valued

fixed points of $g_U(\rho)$ number to three such such that their coordinates can be denoted w.l.o.g. by $0 < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$. It therefore has to hold that

$$0 < \rho_{U,1}^* < \rho_{U,2}^* < \rho_+ < \rho_{U,3}^* < \rho_-$$

By property c), there is no further extreme point over $[\rho_{U,1}^*, \rho_{U,2}^*] \subset (0, \rho_+)$ while $0 < g_U(0) < g_U(\rho_+)$. It follows that $g_U(\rho)$ is non-decreasing on $[\rho_{U,1}^*, \rho_{U,2}^*]$. Three fixed points of $g_U(\rho)$ at $\rho_{U,1}^* < \rho_{U,2}^*$ and $\rho_{U,3} \in (\rho_+, \rho_-)$ then imply that $g_U(\rho)$ cuts the 45-degree line (which has slope 1) thrice: First, twice from below and then from above, thus

$$(N(\rho)/D(\rho))'|_{\rho=\rho_{U,3}^*} < 0 < (N(\rho)/D(\rho))'|_{\rho=\rho_{U,1}^*} < 1 < (N(\rho)/D(\rho))'|_{\rho=\rho_{U,2}^*} \quad (22)$$

Using the fact that if this indeed an equilibrium, $\rho_{U,1}^* = N(\rho_{U,1}^*)/D(\rho_{U,1}^*)$ has to hold, the requirement of a positive slope greater at $\rho_{U,1}^* > 0$ translates via (20) into

$$\left(\frac{N(\rho)}{D(\rho)} \right)' \Big|_{\rho=\rho_{U,1}^*} = \frac{(1-\mu)}{D(\rho_{U,1}^*)} \cdot (\sigma_{sc} - 2\rho_{U,1}^*(\sigma_{sc} + (\mu + (1-\mu)\rho_{U,1}^*)\sigma_c^2)) > 0$$

For this to hold, $\sigma_{sc} + (\mu + (1-\mu)\rho_{U,1}^*)\sigma_c^2 < 0$ is a necessary condition as $\rho_{U,1}^* > 0 > \sigma_{sc}$. Multiplying by $(\mu + (1-\mu)\rho_{U,1}^*) > 0$ yields the equivalent necessary condition

$$(\mu + (1-\mu)\rho_{U,1}^*)\sigma_{sc} + (\mu + (1-\mu)\rho_{U,1}^*)^2\sigma_c^2 = D(\rho_{U,1}^*) - N(\rho_{U,1}^*) < 0$$

Rearranging this inequality then yields that $1 < N(\rho_{U,1}^*)/D(\rho_{U,1}^*) = \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$ is a necessary condition for multiple fixed points ρ_U^* . \square

Proof of Lemma 4

Following the assumptions regarding ϵ , it holds that

$$\begin{pmatrix} s \\ c \\ \epsilon \end{pmatrix} \sim F \left(\begin{bmatrix} \bar{s} \\ \bar{c} \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \sigma_{sc} & 0 \\ \sigma_{sc} & \sigma_c^2 & 0 \\ 0 & 0 & \sigma_\epsilon^2 \end{bmatrix} \right)$$

Using E1 establishes that $\tilde{c} = c + \epsilon$ is distributed according to F and so is the random vector (\tilde{c}, s, c) . Note that because ϵ is independent and unbiased, $\text{Cov}[s, \tilde{c}] = \text{E}[(s - \text{E}[s])(c + \epsilon - \text{E}[c])] = \text{E}[(s - \bar{s})(c - \bar{c})] = \sigma_{sc}$, $\text{Var}[\tilde{c}] = \text{E}[(c + \epsilon - \text{E}[c])^2] = \text{E}[(c + \epsilon - \bar{c})^2] = \sigma_c^2 + \sigma_\epsilon^2$, and

$\text{Cov}[c, \tilde{c}] = \mathbb{E}[(c - \mathbb{E}[c])(c + \epsilon - \bar{c})] = \mathbb{E}[(c - \bar{c})(c - \bar{c})] = \sigma_c^2$. The following can then be stated:

$$\begin{pmatrix} \tilde{c} \\ s \\ c \end{pmatrix} \sim F \left(\begin{bmatrix} \bar{c} \\ \bar{s} \\ \bar{c} \end{bmatrix}, \begin{bmatrix} \sigma_c^2 + \sigma_\epsilon^2 & \sigma_{sc} & \sigma_c^2 \\ \sigma_{sc} & \sigma_s^2 & \sigma_{sc} \\ \sigma_c^2 & \sigma_{sc} & \sigma_c^2 \end{bmatrix} \right)$$

Using E2, as stated in the beginning of this appendix, with the parameters from the above distribution then yields after some rearranging the stated conditional moments for $(s, c | \tilde{c})$. \square

Proof of Proposition 3

Using again the equilibrium mapping $m^*(s, c)$ from proposition 1, now with information structure \mathcal{I}_D and the associated conditional distribution of $(s, c | \tilde{c})$, means that with disclosed COIs an inference coefficient ρ_D^* has to solve

$$\begin{aligned} \rho &= \frac{\text{Cov}[s, m^* | \tilde{c}]}{\text{Var}[m^* | \tilde{c}]} \\ &= \frac{(1 - \rho_c(\text{Corr}[s, c])^2) \sigma_s^2 + (\mu + (1 - \mu)\rho)(1 - \rho_c)\sigma_{sc}}{(1 - \rho_c(\text{Corr}[s, c])^2) \sigma_s^2 + 2(\mu + (1 - \mu)\rho)(1 - \rho_c)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2(1 - \rho_c)\sigma_c^2} \\ &= \frac{\left(\frac{1 - \rho_c(\text{Corr}[s, c])^2}{1 - \rho_c} \right) \sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\left(\frac{1 - \rho_c(\text{Corr}[s, c])^2}{1 - \rho_c} \right) \sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \end{aligned}$$

Under full disclosure, $\sigma_\epsilon^2 = 0$ holds and $\rho_c = 1$ applies such that a unique solution $\rho = \rho_D^* = 1$ follows from the second line. The last transformation assumes $\rho_c \neq 1$, i.e. $\sigma_\epsilon^2 > 0$. Substituting ϕ for $(1 - \rho_c(\text{Corr}[s, c])^2)/(1 - \rho_c)$ then yields (14). For the remainder of the proof imperfect disclosure with $\rho_c \in (0, 1)$, thus $\phi > 1$, is assumed.

In analogy to first part of the proof of proposition 2, one can then define $g_D(\rho) = \tilde{N}(\rho)/\tilde{D}(\rho)$ with $\tilde{N}(\rho) = \phi\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc} > N(\rho)$ and $\tilde{D}(\rho) = \phi\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2 > D(\rho)$. It is easily verified that $\tilde{N}(\rho)/\tilde{D}(\rho)$ inherits the properties a) through c) of $N(\rho)/D(\rho)$ stated in that proof. Continuing, one can prove, analogously to the proof of lemma 2, that a sufficient condition for $\rho_D^* > 0$ to exists is given by the condition $\sigma_{sc} > \tilde{\tau}(\rho_D^*)$ with $\tilde{\tau}(\rho) = -\phi\sigma_s^2/(\mu + (1 - \mu)\rho)$. Existence of $\rho_U^* > 0$ is equivalent to $\sigma_{sc} > \tau(\rho_U^*)$ (see proof of lemma 2). It then follows that $\rho_U^* > 0$ is sufficient for $\rho_D^* > 0$ to exists, since this implies that $\sigma_{sc} > \tau(\rho_U^*)$ and $\tau(\rho) = -\sigma_s^2/(\mu + (1 - \mu)\rho) > \tilde{\tau}(\rho) = -\phi\sigma_s^2/(\mu + (1 - \mu)\rho)$ holds for any $\rho > 0$.

Similar to the second part of the the proof of proposition 2, where fixed points to $g_U(\rho)$ were expressed as roots to $k(\rho)$, one can define the cubic function $\tilde{k}(\rho) = \rho\tilde{D}(\rho) - \tilde{N}(\rho) =$

$\tilde{A}\rho^3 + \tilde{B}\rho^2 + \tilde{C}\rho + \tilde{D} = 0$ with coefficients

$$\tilde{A} = A > 0, \quad \tilde{B} = B, \quad \tilde{C} = \phi\sigma_s^2 + \mu^2\sigma_c^2 + (3\mu - 1)\sigma_{sc} > C, \quad \tilde{D} = -\phi\sigma_s^2 - \mu\sigma_{sc} < D < 0$$

where A through D are defined as in the second part of the proof of proposition 2. Roots to $\tilde{k}(\rho)$ then correspond to fixed point of $g_D(\rho)$. Applying the sign rule again implies that there are either one or three such roots. \square

Proof of Lemma 5

The proof uses the cubic function \tilde{k} with coefficients \tilde{A} through \tilde{D} as defined in the proof of proposition 3 above. Its roots denote fixed points to $g_D(\rho)$ as stated in (14). Similarly, it uses the cubic function k as defined with coefficients A through D in the second part of the proof of proposition 2 whose roots denote fixed points to $g_U(\rho)$ as stated in (12). Furthermore, from proposition 2 that g_U has either one or three fixed points, with solutions $\rho_U^* \in (0, 1]$ being unique.

Given these prerequisites, note that $\tilde{k}(0) = \tilde{D} < k(0) = D < 0$ and $\tilde{k}'(\rho) = 3\tilde{A}\rho^2 + \tilde{B}\rho + \tilde{C} > k'(\rho) = 3A\rho^2 + B\rho + C$ for all $\rho \in \mathbb{R}_+$. Furthermore, $\tilde{k}(\rho) = k(\rho)$ if and only if $\rho = 1$. It therefore holds that $k(\rho) > \tilde{k}(\rho)$ for all $\rho \in (0, 1)$ and $k(\rho) < \tilde{k}(\rho)$ for all $\rho > 1$.

Taken together, the above means that if there is a (unique) root $\rho_U^* \in (0, 1)$ of k , there must be a unique root \tilde{k} on $(\rho_U^*, 1)$ and if $\rho_U^* = 1$, $\rho_D^* = 1$ applies. To see that a root $\rho_D^* < 1$ is unique, one can repeat the same reasoning as in the second part of the proof of proposition 2 to show that multiple solutions require all of them to have a value larger than one. This proves cases a) and b).

For case c), thus when there is a unique $\rho_U^* > 1$, by proposition 2 and the above reasoning, $\tilde{k}(1) = k(1) < 0$ has to hold. A unique root of k at $\rho_U^* > 1$ implies that g_U never cuts the real line again on $(\rho_U^*, +\infty)$. Neither does \tilde{k} since $\tilde{k}(\rho) > k(\rho)$ for $\rho > 1$. This, in addition with $\tilde{k}(1) = k(1) < 0$, however means also that \tilde{k} cuts the real line once over $(1, \rho_U^*)$ which proves case c).

Now consider case d), i.e that there are three positively-valued fixed points to $g_U(\rho)$. By proposition 2, their coordinates have to obey $1 < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$. The continuous, cubic function k obeys $k(0) = D < 0$ (see the second part of the proof of proposition 2). This implies that k cuts the real line from below at $\rho_{U,1}^*$, from above at $\rho_{U,2}^*$, and again from below at $\rho_{U,3}^*$. Since it is a continuous polynomial, it has to have a local maximum and minimum in between these points. They are denoted by ρ_-^k and ρ_+^k , respectively. It thus holds that $1 < \rho_{U,1}^* < \rho_+^k < \rho_{U,2}^* < \rho_-^k < \rho_{U,3}^*$. If \tilde{k} also has three roots, denoted by $\rho_{D,1}^* < \rho_{D,2}^* < \rho_{D,3}^*$, it is

a similarly-shaped polynomial by analogous reasoning. Therefore, \tilde{k} cuts the real line from below at $\rho_{D,1}^*$, from above at $\rho_{D,2}^*$, and from below at $\rho_{D,3}^*$. From $\tilde{k}(1) = k(1) < 0$ and $\tilde{k}(\rho) > k(\rho)$ for $\rho > 1$, it follows that when \tilde{k} cuts the real line from below (above), it has to do so at lower (higher) values than k . For three roots of \tilde{k} , this implies that

$$1 < \rho_{D,1}^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{D,2}^* < \rho_{D,3}^* < \rho_{U,3}^*$$

which proves the second part of case d). If \tilde{k} has only one root (two have been ruled out by the sign rule), $\tilde{k}(1) = k(1) < 0$ and $\tilde{k}(\rho) > k(\rho)$ again imply that it cuts the real line from below, i.e. at lower coordinates than k . It follows that $1 < \rho_{D,1}^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$ which proves the first part of case d). \square

Proof of Lemma 6

Another way of finding fixed points to $g_U(\rho)$ is to find roots to the function $f(\rho) = g_U(\rho) - \rho$ which is continuous and for which $f(0) = g_U(0) = N(0)/D(0) > 0$ holds (see the first part of the proof of proposition 2). It follows that for f to have three roots, this function has to cut the real line from above at $\rho_{U,1}^*$, from below at $\rho_{U,2}^*$, and again from above at $\rho_{U,3}^*$. This implies $f'(\rho_{U,1}^*) < 0$, $f'(\rho_{U,2}^*) > 0$, and $f'(\rho_{U,3}^*) < 0$. It therefore holds that $f'(\rho_{U,1}^*) = g'_U(\rho_{U,1}^*) - 1 < 0$, $f'(\rho_{U,3}^*) = g'_U(\rho_{U,3}^*) - 1 < 0$ and $f'(\rho_{U,2}^*) = g'_U(\rho_{U,2}^*) - 1 > 0$. This proves asymptotic stability of $\rho_{U,1}^*$ and $\rho_{U,3}^*$, and asymptotic instability of $\rho_{U,2}^*$.

For the case of undisclosed commissions one can repeat the above procedure analogously by using $g_D(\rho) = \tilde{N}(\rho)/\tilde{D}(\rho)$ with $\tilde{N}(\rho)$ and $\tilde{D}(\rho)$ as defined in the proof of proposition 3. In particular, when $\rho_U^* > 0$ exists, $\tilde{N}(\rho) > N(\rho) > 0$ holds for all $\rho > 0$. By the same reasoning, $\tilde{D}(\rho) > D(\rho) > 0$ applies such that $g_D(0) = \tilde{N}(0)/\tilde{D}(0) > 0$. Preceding as above, one then define $\tilde{f}(\rho) = g_D(\rho) - \rho$ whose roots correspond to fixed points of $g_D(\rho)$. In particular, $\tilde{f}(0) = \tilde{N}(0)/\tilde{D}(0) > 0$ applies. Repeating the above reasoning regarding the shape of \tilde{f} and the implications on its derivatives for the function \tilde{f} , one can then show the stated asymptotic stability and asymptotic instability of for fixed points to $g_D(\rho)$ analogously to the above. \square

Proof of Lemma 7

The argument of the sender's (expected) utility is given by

$$\begin{aligned} z &\equiv d_r^*(m) - s = (1 - \rho^*)E[s] + \rho^*[m^*(s, c) - E[c](\mu + (1 - \mu)\rho^*)] - s \\ &= -(s - E[s]) + \rho^*[m^*(s, c) - E[s] - E[c](\mu + (1 - \mu)\rho^*)] \\ &= -(s - E[s]) + (m^*(s, c) - E[m^*(s, c)])\rho^* \end{aligned} \tag{23}$$

Note that by (10), $m^*(s, c)$ is a linear transformation of the vector (s, c) and by E1, it is thus distributed according to F . Similarly, s is also distributed according to F . In consequence, z is distributed according to $F(E[z], \text{Var}[z])$. One can then normalize z via the linear transformation $\hat{z}(z) = z/\sqrt{\text{Var}[z]} - E[z]$ such that \hat{z} follows $F(0, 1)$ whose density will be denoted $f(\hat{z})$. The expected utility of rational receivers can then be expressed as

$$E[L(z)] = \int_{-\infty}^{+\infty} L\left(E[z] + \hat{z}\sqrt{\text{Var}[z]}\right) f(\hat{z}) d\hat{z} \equiv V\left(E[z], \sqrt{\text{Var}[z]}\right) \leq 0$$

From (23) it follows that $E[z] = 0$. One can thus define the univariate function $\mathcal{L}(\text{Var}[y]) \equiv V(0, \text{Var}[z]) \leq 0$ which denotes a rational receiver's expected utility and for which it holds that

$$\mathcal{L}'(\text{Var}[z]) = \frac{\partial V\left(E[z], \sqrt{\text{Var}[z]}\right)}{\partial \text{Var}[z]} \Big|_{E[z]=0} = \frac{1}{2\sqrt{\text{Var}[z]}} \cdot \int_{-\infty}^{+\infty} \left[\hat{z} \cdot L'\left(\hat{z}\sqrt{\text{Var}[z]}\right)\right] f(\hat{z}) d\hat{z}$$

Since L is strictly concave and symmetric around zero, $\text{sgn}[\hat{z}] = -\text{sgn}\left[L'\left(\hat{z}\sqrt{\text{Var}[z]}\right)\right]$ and therefore, the above expression is non-positive. In addition, \hat{f} is symmetric around zero and $\mathcal{L}'(\text{Var}[z]) = 0$ if and only if $\text{Var}[z] = 0$. It has been shown in the main text that under full disclosure, $u_r^R(\rho^*, \cdot) = 0$ holds since rational receivers can then extract s from the message and implement their optimal choice. Full disclosure therefore implies $\mathcal{L}(0) = 0$.

To see that full disclosure is also necessary for $\mathcal{L}(0) = 0$ to hold, note from the above that this requires $\text{Var}[z] = 0$ and therefore $d_r^*(m) = s$. Suppose that this held under imperfect disclosure. For $d_r^*(m) = s$ to apply in this case, (11) requires both $\rho^* = 1$ and $c = E[c]$ to hold simultaneously for *any* realization (s, c) . The latter statement is a contradiction to the fact that under imperfect disclosure with $\rho_c \in (0, 1)$, $\text{Var}[c|\tilde{c}] > 0$ and $\text{Var}[s|\tilde{c}] > 0$ applies (see lemma 4).

To see the alternative representation of the argument $x \equiv \text{Var}[z]$, note that by using the definition of ρ^* one gets the following:

$$\begin{aligned} x = \text{Var}[z] &= \text{Var}[d_r^*(m) - s] \\ &= E[(-(s - E[s]) + (m^*(s, c) - E[m^*(s, c)]))\rho^*]^2 \\ &= (\sigma_s^2 - 2\rho^* \text{Cov}[s, m^*] + (\rho^*)^2 \text{Var}[m^*]) \\ &= \sigma_s^2 - \rho^* \text{Cov}[s, m^*] \end{aligned}$$

From the law of total variance and using again the definition of ρ^* , it also holds that

$$\begin{aligned}
\mathbb{E}[\text{Var}[s|m^*]] &= \text{Var}[s] - \text{Var}[\mathbb{E}[s|m^*]] \\
&= \sigma_s^2 - \mathbb{E}[(d_r^*(m) - \mathbb{E}[s])^2] \\
&= \sigma_s^2 - \mathbb{E}[(m^* - \mathbb{E}[m^*])\rho^*]^2 \\
&= \sigma_s^2 - (\rho^*)^2 \text{Var}[m^*] \\
&= \sigma_s^2 - \rho^* \text{Cov}[s, m^*] \quad (= x) \\
&= \sigma_s^2 - \frac{\text{Cov}[s, m^*]^2}{\text{Var}[m^*]} \\
&= \sigma_s^2 (1 - \text{Corr}[s, m^*]^2) \geq 0
\end{aligned}$$

where $\text{Corr}[s, m^*] = \text{Corr}[s, m]_{m=m^*(s,c)} = \text{Cov}[s, m^*]/(\sigma_s \sqrt{\text{Var}[m^*]})$. □

Proof of Proposition 4

Lemma 7 shows that the expected utility of rational receivers strictly increases in $\text{Corr}[s, m^*]^2$. For equilibria with $\rho^* > 0$ and therefore $\text{Cov}[s, m^*] > 0$, it is then sufficient to show that $\text{Corr}[s, m^*] > 0$ increases upon disclosure. For this note that

$$\text{Corr}[s, m^*] = \frac{\text{Cov}[s, m^*]}{\text{Var}[m^*]} \cdot \frac{\sqrt{\text{Var}[m^*]}}{\sigma_s} = \rho^* \cdot \frac{\sqrt{\text{Var}[m^*]}}{\sigma_s} \quad (24)$$

I First consider the case that $1 \geq \rho_D^* > \rho_U^* > 0$. According to (10) one gets

$$D(\rho^*) = \text{Var}[m^*] = \text{Var}[s + c(\mu + (1 - \mu)\rho^*)] = \text{Var}[\sigma_s^2 + 2(\mu + (1 - \mu)\rho^*)\sigma_{sc} + (\mu + (1 - \mu)\rho^*)^2\sigma_c^2] \quad (25)$$

Since the first factor on the RHS of (24) increases upon disclosure, it is then sufficient to show that also $D(\rho_D^*) > D(\rho_U^*)$ holds. From the fact that $\sigma_{sc} + (\mu + (1 - \mu)\rho_U^*)\sigma_c^2 > 0$ is a necessary and sufficient condition for $\rho_U^* \in (0, 1)$ (see proof of lemma 3) this then follows from

$$D'(\rho^*)|_{\rho^*=\rho_U^*} = 2(1 - \mu) \cdot (\sigma_{sc} + (\mu + (1 - \mu)\rho_U^*)\sigma_c^2) > 0 \quad (26)$$

Now consider the case of asymptotically stable equilibria with $\rho_U^* > 1$. From lemma 5 and 6 it then follows that for such equilibria $1 < \rho_D^* < \rho_U^*$ holds, thus disclosure *decreases* ρ^* . To show that $\text{Corr}[s, m^*]$ does also increase upon disclosure in this case, I will use again that in equilibrium $\rho^* = N(\rho^*)/D(\rho^*)$ holds with

$$N(\rho^*) = \text{Cov}[s, m^*(s, c)] = \text{Cov}[s, s + c(\mu + (1 - \mu)\rho^*)] = \sigma_s^2 + (\mu + (1 - \mu)\rho^*)\sigma_{sc} \quad (27)$$

and $D(\rho^*) = \text{Var}[m^*(s, c)]$ as defined in (25). From (24) one gets that

$$\left. \frac{\text{Corr}[s, m^*(s, c)]}{\partial \rho^*} \right|_{\rho^* = \rho_U^*} = \left[\left(\frac{N(\rho^*)}{D(\rho^*)} \right)' \cdot \left(\frac{\sqrt{D(\rho^*)}}{\sigma_s} \right) + \left(\frac{N(\rho^*)}{D(\rho^*)} \right) \cdot \left(\frac{D(\rho^*)'}{2\sigma_s \sqrt{D(\rho^*)}} \right) \right] \Big|_{\rho^* = \rho_U^*} \quad (28)$$

To determine the sign of the above, note that by multiplying it with $\sqrt{D(\rho^*)} > 0$ and using $\rho^* = N(\rho^*)/D(\rho^*)$ again, its sign is given by

$$\text{sgn} \left[N(\rho^*)'|_{\rho^* = \rho_U^*} - \rho^* \cdot \frac{D(\rho^*)'|_{\rho^* = \rho_U^*}}{2} \right] \Big|_{\rho = \rho_U^*} = \text{sgn} [\sigma_{sc} - \rho_U^* (\sigma_{sc} + (\mu + (1 - \mu)\rho_U^*)\sigma_c^2)]$$

Substituting the above RHS with $\rho_U^* = N(\rho_U^*)/D(\rho_U^*)$ and this again with (27) and (25) then yields after multiplying it by $D(\rho_U^*) > 0$ (and some transformations) that the sign of the above equals

$$\text{sgn}[(\sigma_{sc}^2 - \sigma_c^2 \sigma_s^2)] = \text{sgn}[(\text{Corr}[s, c]^2 - 1)] < 0$$

In consequence, a decrease in $\rho^* > 1$ upon disclosure increases $\text{Corr}[s, m^*(s, c)]$.

Finally, consider the case of $\rho_U^* = 1$. By case c) in lemma 5, the inference coefficient then remains constant upon disclosure. Furthermore, by lemma 3, it has to apply that $\sigma_c^2 = -\sigma_{sc}$. Proposition 1 then implies that $E[s|m^*] = m^*(s, c) - E[c] = s + c - E[c]$. From lemma 7 and its proof, $x = \sigma_s^2 (1 - \text{Corr}[s, m^*]^2) = E[\text{Var}[s|m^*]]$ is the argument of the loss function \mathcal{L} which describes that rational receivers expected utility. Applying again the law of total variance yields

$$\begin{aligned} E[\text{Var}[s|m^*]] &= \text{Var}[s] - \text{Var}[E[s|m^*]] \\ &= \text{Var}[s] - E[(s + c - E[c] - E[s + c - E[c]])^2] \\ &= -2\text{Cov}[s, c] - \text{Var}[c] = \sigma_c^2 \end{aligned}$$

with undisclosed COIs. By analogous reasoning and using the posteriors from lemma 4 one obtains for disclosed COIs, after the signal \tilde{c} has been obtained, $E[\text{Var}[s|m^*, \tilde{c}]] = (1 - \rho_c)\sigma_c^2$ follows. With full or imperfect disclosure $\rho_c \in (0, 1]$ applies and the rational receiver's expected utility \mathcal{L} strictly increases when its argument strictly decreases. \square

Proof of Proposition 5

I start with the case of $w_K = 0$ and denote, with slight abuse of notation, $W(\sigma_\epsilon^2) \equiv W(\mathcal{E}(\rho_D^*(\sigma_\epsilon^2, \cdot)))$ via the analogously defined $E[u_r^R(\sigma_\epsilon^2)] \equiv E[u_r^R(\mathcal{E}(\rho_D^*(\sigma_\epsilon^2, \cdot)))]$ and $E[u_n^R(\sigma_\epsilon^2)] \equiv E[u_n^R(\mathcal{E}(\rho_D^*(\sigma_\epsilon^2, \cdot)))]$. This reflects that in the case of disclosed COIs, the coefficient ρ_D^* is the only term which contains σ_ϵ^2 via the function ϕ (see proposition 3). Using $\tilde{N}(\rho)$ and $\tilde{D}(\rho)$ as defined in the proof of

proposition 3, together with $\rho_D^* = \tilde{N}(\rho)/\tilde{D}(\rho)$ then yields

$$\frac{\partial \rho_D^*}{\partial \phi} = \frac{\partial \left(\tilde{N}(\rho_D^*)/\tilde{D}(\rho_D^*) \right)}{\partial \phi} = \frac{\sigma_s^2 \tilde{D}(\rho_D^*) - \tilde{N}(\rho_D^*) \sigma_s^2}{(\tilde{D}(\rho_D^*))^2} = \frac{(1 - \rho_D^*) \sigma_s^2}{\tilde{D}(\rho_D^*)}$$

Since $\partial \phi / \partial \sigma_\epsilon^2 = (\partial \phi / \partial \rho_c) \cdot (\partial \rho_c / \partial \sigma_\epsilon^2) < 0$ it follows from lemma 5 that

$$\text{sgn} \left[\frac{\partial \rho_D^*}{\partial \sigma_\epsilon^2} \right] = -\text{sgn} \left[\frac{\partial \rho_D^*}{\partial \phi} \right] = \text{sgn} [\rho_D^* - 1] = \text{sgn} [\rho_U^* - 1]$$

Since $\frac{E[u_n^R(\rho^*)]}{\partial \rho^*}$ is positive (negative) if and only if $\rho_U^* > 1$ ($\rho_U^* < 1$) one then gets from (15) and the above for any $\sigma_\epsilon^2 \geq 0$ the following:

$$\text{sgn} \left[\frac{E[u_n^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \right] = \text{sgn} \left[\frac{E[u_n^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \right] = \text{sgn} \left[\frac{E[u_n^R(\rho^*)]}{\partial \rho^*} \Big|_{\rho^*=\rho_D^*} \cdot \frac{\partial \rho_D^*}{\partial \sigma_\epsilon^2} \right] = -\text{sgn} [\rho_U^* - 1] \quad (29)$$

When $\rho_U^* \in (0, 1)$, every decrease in σ_ϵ^2 therefore hurts naive receivers. In contrast, it has been shown in the main text that when there is full disclosure, i.e. $\sigma_\epsilon^2 = 0$, rational receivers achieve their maximum utility, thus $E[u_r^R(0)]' = 0$ holds. The first part of the statement then follows from showing that when $\rho_U^* \in (0, 1)$, there exists a $\Delta > 0$ such that starting from full disclosure with $\sigma_\epsilon^2 = 0$, the gradual increase of this variance to $\sigma_\epsilon^2 = \Delta$ increases $W(\sigma_\epsilon^2) = w_r \cdot E[u_r^R(\sigma_\epsilon^2)] + w_n \cdot E[u_n^R(\sigma_\epsilon^2)]$. This is equivalent to showing that $\lim_{\Delta \rightarrow 0^+} (W(\Delta) - W(0))$ is positive which, for any such small $\Delta > 0$, follows from

$$\begin{aligned} \text{sgn} \left[\lim_{\Delta \rightarrow 0^+} \left(\frac{W(\Delta) - W(0)}{\Delta} \right) \right] &= \text{sgn} \left[\sum_{j=r,n} w_j \cdot \lim_{\Delta \rightarrow 0^+} \left(\frac{E[u_j^R(\Delta)] - E[u_j^R(0)]}{\Delta} \right) \right] \\ &= \text{sgn} [w_r \cdot E[u_r^R(0)]' + w_n \cdot E[u_n^R(0)]'] \\ &= \text{sgn} [w_n \cdot E[u_n^R(0)]'] = -\text{sgn} [\rho_U^* - 1] > 0 \end{aligned}$$

For the case that $w_K > 0$, note that the above proof applies for any loss function $u_n^R(\sigma_\epsilon^2) = L(\sigma_\epsilon^2)$ which is strictly concave and symmetric around zero. It therefore also holds when in addition to $E[u_n^R(\sigma_\epsilon^2)]$, positive weight is assigned to $E[K(m, s)|_{m=m^*(s,c)}] = -E[c(\mu + \rho^*(\sigma_\epsilon^2, \cdot)(1 - \mu))^2]$. This then yield the first part of the proposition.

The second statement is then an immediate consequence of the fact that according to (29), given any imperfect disclosure, further increasing the signal precision (decreasing σ_ϵ^2) helps naive receivers when $\rho_U^* > 1$ and that full disclosure maximizes the utility of rational receivers (see lemma 7). \square

Example for non-disclosure to be optimal

As a concrete example for a scenario where non-disclosure is optimal, consider the parameters $\sigma_s^2 = \sigma_c^2 = 1$, $\bar{s} = \bar{c} = \sigma_{sc} = 0$, together with $\mu = w_n = w_r = 0.5$, $w_k = 0$, and the loss function $L(d - s) = -(d - s)^2$. Plugging this into (12) and solving yields $\rho_U^* \approx 0.6$. Following lemma 5, disclosure then increases the inference coefficient. Using proposition 1 and (16) yields

$$\begin{aligned}
W &= -0.5 \left(\mathbb{E}[(\rho[m^*(s, c) - \bar{c}(\mu + (1 - \mu)\rho)] + (1 - \rho)\bar{s} - s)^2] + \mathbb{E}[(m^*(s, c) - s)^2] \right) \Big|_{\rho=\rho^*} \\
&= -0.5 \left(\mathbb{E}[(\rho m^*(s, c) - s)^2] + \mathbb{E}[(m^*(s, c) - s)^2] \right) \Big|_{\rho=\rho^*} \\
&= -0.5 \left(\mathbb{E}[(s(\rho - 1) + c\rho(0.5 + 0.5\rho))^2] + \mathbb{E}[(c(0.5 + 0.5\rho))^2] \right) \Big|_{\rho=\rho^*} \\
&= -0.5 \left((\rho - 1)^2 \mathbb{E}[s^2] + 2(\rho - 1)\rho(0.5 + 0.5\rho)\mathbb{E}[sc] + (\rho^2 + 1)(0.5 + 0.5\rho)^2 \mathbb{E}[c^2] \right) \Big|_{\rho=\rho^*} \\
&= -0.5 \left((\rho - 1)^2 + (\rho^2 + 1)(0.5 + 0.5\rho)^2 \right) \Big|_{\rho=\rho^*}
\end{aligned}$$

which is easily verified to be strictly decreasing in ρ when $\rho > 0.4$. Therefore, non-disclosure maximizes W .