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Summary

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Keywords: Utilitarianism, Inequality, Inequity Aversion, Risk Aversion, Intertemporal Welfare, Discounting

JEL Classification: D60, D63, D30

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Welfare as Simple(x) Equity Equivalents*

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March 9, 2017

Abstract

Inequity plays a fundamental role in the evaluation of social welfare in many dimensions. We revisit the concept of inequity, whether across states of world (uncertainty), across individuals (inequality) and across generations (intergenerational equity), using a common framework generalizing the discounted expected utilitarianism approach. We propose a general measure of welfare as equity equivalents and develop the corresponding inequity index. We then allow for different degrees of inequity aversion across the three dimensions to span a simplex of possible inequity preferences and relate it to the recent literature on this topic. We show that the ordering of aggregation across the different dimensions matters for welfare evaluations and that many welfare-theoretical approaches developed in the literature may be seen as special cases of this general framework.

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1 Introduction

The study of inequity has implicitly been the focus of a large part of the economic literature. Problems related to allocation efficiencies, the distribution of consumption, wealth, and other socio-economic variables has occupied economists since the emergence of the discipline. Different dimensions have typically been considered in the study of inequity. The distribution of resources across individuals, be it within national boundaries (distribution of income) or across countries (economic development) continues to be an important subject of research. The distribution of resources over time and the related questions of savings, intergenerational equity stemming from capital dynamics, or the intertemporal use of natural resources has added another layer of potential inequity. Finally, inequity in different “worlds” or “states of nature” due to the presence of risk/uncertainty affecting the realizations of random variables surrounding virtually all economic problems can be regarded as another example of inequity. While these fundamentally different dimensions have historically been considered in isolation in the literature, it is now clear that these dimensions of “inequity” are potentially closely intertwined: intergenerational equity might be correlated with inequality between countries or individuals, uncertainty might affect individuals or countries differently, etc. Focusing on one dimension at a time therefore potentially implies important aspects to be neglected. The objective of this paper is to propose a unified approach to summarize the large existing literature on inequity by considering inequity across the three dimensions of risk, time and individuals. Using this universal framework, we analyze the implications of considering jointly multiple dimensions of the inequity simplex.

While many every-day life issues –not only in economics– can be studied abstracting from multi-dimensional considerations and focusing on preferences in one particular dimension only, other questions intrinsically encompass multi-dimensional components. In particular, issues that concern different individuals or countries, the long-term future, and involve a substantial degree of uncertainty are of great importance. A notable example is the problem of climate change. The use of nuclear technology –military or civilian–, the management of long-run debt policies or social security systems, and the use of genetically modified organisms, antibiotics etc. provide other examples of problems which involve multi-dimensional aspects of distributions.

The discussions about the implications of climate change has spawned a huge literature in normative philosophy and welfare economics. Issues that have been raised in this context include inter-generational equity, and the notion of a social discount rate; the role of inequality and distributional justice; and the role of (deep) uncertainty including the idea of a precautionary principle. The common feature across these seemingly unrelated concepts is that losses and benefits of given policies have to be compared between individuals along different dimensions.

When it comes to applications, economic models used to evaluate the costs and benefits of climate policies implicitly or explicitly assume some welfare criteria to aggregate, measure, or optimize social welfare. These models generally take into account inequality across countries or individuals, intertemporal distributions, and, in some cases, the presence of uncertainty. It is therefore necessary to include preferences over time, risk and regional dimensions in the analysis.¹

In this paper, we review many of the welfare concepts that have been proposed in the theoretical literature and that have been applied notably in the context of climate change. In particular, we propose a way to summarize the different dimensions of inequity by providing a generalized welfare concept based on equity equivalents in different dimensions. We discuss how standard analyses considering a single dimension can be naturally extended when considering the other dimensions simultaneously. The paper is organized as follows. In Section 2, we structure and summarize the existing economic literature that emerged in the last decades according to their focus and the dimension considered. In Section 3, we introduce a common framework that may be used for the study of inequity and show that many of the existing welfare concepts can be considered as special cases of this general framework. We also present a generalized inequity index and provide some numerical estimates of inequity in the three dimensions separately. In Section 4, we consider the case when two dimensions are simultaneously taken into account with potentially different degrees of inequity aversions across dimensions, and we derive some useful results regarding welfare comparisons. We then proceed, in Section 5, to the general three dimensional problem and discuss the meaning of different possible orderings of aggregation. Section 6 concludes the paper.

2 A simple(x) representation of the literature

In this section, we present a graphical representation of the existing literature that has considered inequity in one or more of the three dimensions. Since the climate change problem is a particularly well adapted example that combines the three dimensions of risk, time and space, we especially focus on (but do not limit to) approaches that have been followed in this specific context.

The workhorse model that has typically been used is the one proposed by the expected discounted utilitarian approach. Following this approach, a social welfare function (SWF) may

¹Given that the future is uncertain, projections in regional differences may be varying substantially when looking at possible scenarios for future economic growth for example. A recent example of the significant differences across possible futures is the set of socioeconomic scenarios called Shared Socioeconomic Pathways (SSPs) that have been developed for uses in the assessment of climate change. In that sense, uncertainty about the future state of the world also plays an important role which will affect welfare considerations.

be characterized by considering a single utility function (U), which is aggregated over time (i.e., discounted), states of the world, and across countries or individuals. Specifically, social welfare is defined as the (weighted) sum of individual utilities derived from consumption as

$$SWF = \sum_{t,s,n} q_{tsn} U(c_{tsn}). \quad (1)$$

In this expression, t represents the time dimension, s refers to states of the world and represents the uncertainty dimension, while n refers to individuals or countries.² These dimensions can be considered in isolation or can be combined in different ways. The weights q_{tsn} have different interpretations according to the dimension considered. In the time dimension, they usually refer to discount factors, in the case of uncertainty they represent probabilities, while for aggregating over countries, they typically represent population weights.

The expected discounted utilitarian (hereafter, EDU) approach has several advantages. It is grounded on a number of desirable axioms, defined across the three dimensions separately (see Koopmans (1960), Von Neumann and Morgenstern (1944), and Atkinson (1970)). Combining the respective one-dimensional welfare aggregations across them therefore seems, at first glance, a natural extension and generalization of the one-dimensional concepts. Following the approach proposed by Harsanyi (1955), the EDU SWF formulation presented in (1) has indeed been the most widely used concept to measure welfare in applied modeling.

In recent years however, a large literature has emerged discussing and proposing alternative welfare concepts in the different dimensions. Most of this work has focused on only one or two dimensions at a time, therefore abstracting from the remaining one(s). We briefly summarize a sample of the literature that considered jointly two of the dimensions in Figure 1. We represent the extent to which any of the three dimensions are considered and separated from each other through a three-dimensional simplex where the “origin”, at the back, represents the simple EDU. In this case, a simple threefold (weighted) sum of utilities is considered, as defined in (1). The three remaining points represent welfare on a particular dimension considered in isolation. In general, the location on the simplex reflects to which extent and to which dimensions lies the core of each study. This representation then allows a broad mapping of the existing literature in the field across the three axes as described below. Note that we restrict the analysis to approaches of the Utilitarian type, where welfare can be expressed, in each dimension, as a sum of utilities. More complex welfare functions or approaches in each of the dimension are indicated but go beyond the scope of this paper.

²Throughout this paper we will refer to this spatial dimension as “inequality”, which typically will consider either inequality between individuals or countries.

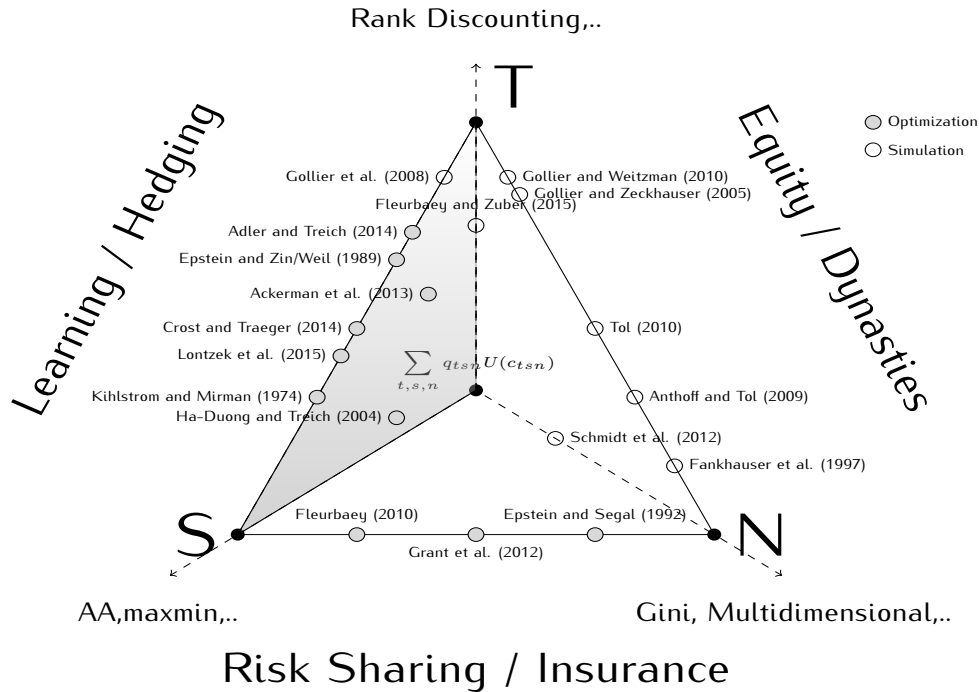


Figure 1: A simplex representation of the existing literature on welfare across the three dimensions

Time In recent years, the need of aggregating welfare over long time horizons has been at the core of the discounting debate. Initially, only the time dimension of the problem has been considered. However, recent extensions and generalizations of the discount rate have included other dimensions of the simplex. In particular, the discussion has been extended to take into account the uncertainty related to economic growth (Gollier et al., 2008) or has focused on individual uncertain discount rates (Gollier and Weitzman, 2010), as well as on the heterogeneity among the discount rates (Gollier and Zeckhauser, 2005). That is, this literature has focused on the study of time preferences, while introducing a limited degree of uncertainty and heterogeneity, respectively (see top segment of the simplex).

Risk/Uncertainty The consideration of risk and uncertainty has been predominantly realized under the expected utility (EU) framework, even though alternative approaches going beyond the linear sum of probability weighted utilities have been proposed (ambiguity averse preferences, maxmin, prospect theory,...). However, even when staying within the linear probability paradigm, as soon as an additional dimension is considered, the aggregation becomes less trivial. Notably, the approaches proposed by Selden (1978); Kreps and Porteus (1978) and later extended by Epstein and Zin (1989); Weil (1989, 1990) disentangling inter-temporal and

risk preferences has been widely applied in many different fields of economics. Their approach may be considered as more oriented towards the time dimension as it is aimed at preserving dynamic consistency and independence of unrealized and past alternatives, while departing from the independence axiom of the EU framework. Alternatively, a less widely-used formulation, which dates back to Kihlstrom and Mirman (1974), also disentangles preferences for risk and time but focuses on maintaining the linearity in probabilities.³

In the context of climate change economics, several authors have used the Epstein-Zin/Weil framework to evaluate welfare across risk and time. Examples of studies using this welfare framework include Ha-Duong and Treich (2004), Crost and Traeger (2014), Ackerman et al. (2013) and Lontzek et al. (2015).⁴ These applications exclusively rely on a global analytical or numerical model considering the world as a whole –often based on Nordhaus’ DICE model⁵– and thus abstract from heterogeneity across countries or individuals. In a more welfare economics oriented literature, extensions towards more general welfare functions, for example considering ex-ante and ex-post concepts of risk aversion in an inter-temporal framework, include Fleurbaey and Zuber (2013) and Adler and Treich (2014).⁶

Inequality The third dimension of the inequity simplex is the inequality across space, that is, between countries or individuals.⁷ It is represented in the right lower corner of the simplex. In many optimization models with regional or individual disaggregation, the use of Negishi weights (Nordhaus and Yang, 1996) –that aims at isolating the issue of a particular policy question at stake from the issue of global inequality and distributional effects– has been the main methodology used to derive general policy recommendations. This approach has however several conceptual flaws (Stanton, 2011), which makes alternative welfare approaches more desirable. In particular, based on the measurement of inequality pioneered by Atkinson (1970) several studies have recently applied similar concepts to the modeling of global policy issues. In the context of climate change, the discussion on “equity weights” has, since Fankhauser et al. (1997), received great attention. The general idea relies on extending standard welfare measures over time to include inequality (in the spirit of Atkinson’s approach) at any point in

³From a practical point of view, this approach has received little attention due to the importance attached to time consistency issues, and to the very convenient recursive formulation proposed by the Epstein-Zin/Weil. A notable exception is for example Bommier (2007).

⁴See also Berger et al. (2016) for an application using an extension of the Epstein-Zin formulation that goes beyond the analysis of risk by considering deep uncertainty.

⁵DICE stands for Dynamic Integrated Climate and Economy, see Nordhaus (1993); Nordhaus and Sztorc (2013) for more details.

⁶Other welfare concepts based on rank discounting (Zuber and Asheim, 2012) or inter-generational egalitarianism (Piacquadio, 2014) or the Rawlsian maxmin welfare function or the Prioritarian SWF approach (Adler, 2011) have been proposed but go beyond the Utilitarian welfare approaches considered here.

⁷This type of inequity is sometimes referred to as ‘intra-generational equity’.

time. Since then, a separation between the inequality concerns from time preferences has been proposed for example by Anthoff and Tol (2009); Tol (2010). These papers explicitly separate inequality aversion between countries from inter-temporal fluctuation aversion, and introduce the concept of “certainty, equity- and balanced growth equivalent” level of consumption as a welfare measure.

From a more theoretical point of view, the separation of risk and inequality (bottom line segment) has been analyzed in Grant et al. (2012) encompassing both the approaches of Fleurbaey (2010) (“expected equally-distributed equivalent-utility”) and the one of Epstein and Segal (1992a) or Diamond et al. (1967) (“generalized utilitarianism”). As we will show, these two approaches basically consider the two possible orderings of aggregation over the two dimensions, leading in consequence to distinct welfare concepts.

While most of the literature has so far focused on one or two of the dimensions, very few recent contributions recently proposed a combination of all the three dimensions at the same time which departs from the standard EDU approach. This is for example the case of Schmidt et al. (2012), who extend the DICE model to take into account both the uncertainty and heterogeneity across populations by means of a simple parametric distribution of income at any point in time. They however do not make explicit the separation between the different attitudes one may have over these dimensions. The recent works by Fleurbaey and Zuber (2015) is another example which considers all the three dimensions in the context of the optimal discount rate, thus focusing on the time dimension.

This literature is therefore only in its early stages. In general, a common feature of the works referenced here is the focus on one or two dimensions and/or the disentanglement of preferences across two dimensions at best –as indicated by the fact that most studies are graphically presented on one of the segments of the simplex in Figure 1. However, if one aims to deal with problems involving the three dimensions simultaneously, the simple (weighted) sum over all three dimensions as defined in (1), which implicitly assumes the same degree of inequity aversion across the three dimensions, might be too restrictive. As Atkinson et al. (2009) empirically showed, preferences for equity are in fact different across the distinct dimensions. The general approach we propose allows for exhibiting distinct equity preferences across the three dimensions. Such disentangled approach appears particularly important for normative statements. In what follows, we show that many of the welfare concepts developed in the literature may be seen as special cases of a more general function, which we define in terms of generalized “equity equivalents”. We deliberately restrict our analysis to the boundaries of the Utilitarian simplex outlined in Figure 1.⁸ We then compare our generalized welfare

⁸It is noteworthy that the generalization of expression (1) we propose does not anymore satisfy a very general

measure with the workhorse simple threefold summation of the EDU, and show how the order of aggregation matters for welfare considerations.

3 A generalized framework for the study of inequity

In this section, we first focus on evaluating distributions of consumption (or income) across one of the three dimensions of the simplex. We start by defining a general framework for the analysis of inequity.

3.1 The setting

Let C be a random variable representing the level of consumption. We denote by $\{C_{tsn}, t \in T, s \in S, n \in N\}$ the collection of consumption levels defined in a three-dimensional state space \mathcal{C} . In this expression, T , S and N are index sets representing respectively the time, state and country/individual dimensions. These sets may be countable or uncountable. For example, the time set T may either represent the (possibly infinite) discrete periods of time or continuous time. A typical element of this collection is denoted c_{tsn} (as the realization of C_{tsn})⁹ and represents the consumption of country (or individual) n in state of the word s at time t . If one or two dimensions are not considered, their respective index sets are empty and we simply use the notations c_i to denote the consumption level with $i \in \{t, s, n\}$, and c_{ij} the consumption level for $i, j, j \neq i \in \{s, t, n\}$. For the sake of simplicity, we will assume that the consumption levels are bounded. Finally, in an slight abuse of notations, we also let t , s and n refer to the dimensional space they index using it as a superscript.

3.2 Welfare as equity equivalents and inequity measures

We now introduce a general framework that will be used for the study of welfare and inequity across the different dimensions of the simplex. To do so, we first start by defining the concept of equity equivalent in one dimension.

Definition 1. The *equity equivalent* in the dimension $i \in \{t, s, n\}$, which in turn represents the time (t), state (s) or national/individual (n) dimension, is the constant level of consumption across dimension i that is deemed worth as much as the consumption level (C) spread across

“independence axiom” e.g., of Harsanyi (1955) across all individuals. Within this paper, we however do not discuss the axiomatic analysis of the different approaches but focus on their implications.

⁹Remark that the order at which indices are taking place does not matter here, so that $c_{tsn} = c_{stn} = c_{nts} = \dots$

dimension i . It is formally defined as:

$$\mathcal{E}^i(C) \equiv f_i^{-1} \mathbb{E}^i [f_i(C)], \quad (2)$$

where $f_i : \mathbb{R} \rightarrow \mathbb{R}$ is a strictly increasing, twice continuously differentiable utility function representing the attitude towards inequity in dimension i ,¹⁰ and \mathbb{E}^i represents the expectation operator taken over dimension i .

This expectation (also called weighted sum) operator takes the form $\mathbb{E}^i [f_i(C)] = \sum_i q_i f_i(c_i)$, where q_i represents the weight attached to element c_i such that $\sum_i q_i = 1$.¹¹ We then define the one-dimensional welfare evaluation function to rank distributions according to this equity equivalent.

Definition 2. The *social welfare function* over dimension i is the function $W^i : \mathcal{C} \rightarrow \mathbb{R}$ of consumption levels distributed across dimension i , defined as

$$W^i(C) \equiv \mathcal{E}^i(C) \quad \forall i \in \{s, t, n\}. \quad (3)$$

It represents a measure of welfare in the sense that, for any $C, C' \in \mathcal{C}$, $W^i(C) \geq W^i(C')$ means that the distribution C of consumption levels across dimension i is deemed socially at least as good as the distribution C' . To simplify the exposition in what follows, when there is no ambiguity, we will use the notation $W^i = W^i(C)$. Note that this notion of welfare is slightly different from the formulation $V^i = \sum_i q_i f_i(c_i)$, which is typically used in the literature to represent welfare, expected utility, or intertemporal (discounted) utility. However, since the equity equivalent is nothing but a strictly monotonic transformation of V^i , it does not alter the ordinal properties of the welfare ranking, and allows both an easier interpretation of welfare and simpler generalizations.¹²

The function (3) is defined over all the elements of the set to which the superscript i refers. Yet, in some instances, it might be useful, to characterize welfare recursively on these elements. This could be done by letting $W_i^i(C)$ be the social welfare function over dimension i (as denoted by the superscript) evaluated recursively at each element of the index set denoted by the running index i as subscript. It is defined recursively as

¹⁰For this utility function we denote the dimension i as a subscript to avoid notation cluttering.

¹¹In the limiting case in which the distribution of C is continuous over dimension i , the \mathbb{E}^i operator is simply defined as the (Lebesgue) integral $\mathbb{E}^i [f_i(C)] = \int f_i(C) dG_i$ with respect to a distribution (or probability measure) G_i .

¹²Another advantage of the equity equivalent formulation is that it leads to numerical values of welfare that are closer to consumption levels in terms of orders of magnitude. This might reveal useful in numerical applications since it can help to improve numerical optimization algorithms.

$$W_i^i = \mathcal{F}^i(C_i, W_{i+1}^i). \quad (4)$$

In this expression, $\mathcal{F}^i : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a so-called *dimension i aggregator* in the sense of Koopmans (1960). It here takes the form

$$\mathcal{F}^i(x, y) = f_i^{-1}(f_i(x) + \hat{q}_{i+1}f_i(y)), \quad (5)$$

where \hat{q}_{i+1} is the weight attached to element $i + 1$ relative to element i . Unsurprisingly, when the function is evaluated over all elements of the index set in dimension i , it is easy to see that expressions (4) and (3) coincide.¹³

One useful characteristic of this general welfare concept is that it can be related to a general measure of inequity. In particular, we can define a general measure of inequity as one minus the ratio of the equity equivalent level of consumption to the arithmetic mean of the distribution.

Definition 3. Formally, the *inequity index* in the dimension $i \in \{t, s, n\}$ is defined as:

$$I^i = 1 - \frac{\mathcal{E}^i(C)}{\mathbb{E}^i[C]} \quad (6)$$

where $\mathbb{E}^i[C] \equiv \sum_i q_i c_i$.

In this expression, $\mathbb{E}^i[C]$ is referred to as the *mean* of the actual distribution of consumption across the dimension considered.¹⁴ Importantly, this inequity index depends on the underlying utility function f_i used to compute the equity equivalent. Unsurprisingly, if the distribution of consumption becomes more equally distributed in dimension i , the equity equivalent of someone who is averse to inequity rises, and the inequity index falls. The inequity index I^i has also the convenient property of being bounded between 0 (complete equity) and 1 (complete inequity).

We are, of course, not the first to have defined such one-dimensional (in)equity measures. In what follows, we show how various existing concepts can be captured under our general framework, and we propose a new interpretation along the time dimension.

Risk/uncertainty If the state of nature dimension is considered ($i = s$), the equity equivalent $\mathcal{E}^s(C)$ corresponds to the standard *certainty equivalent* (CE) defined over risky consumption in the risk theory literature. In this case, s represents the different states of the world, $f_i = v$ is the Von Neumann-Morgenstern utility function capturing risk attitude, \mathbb{E}^s is the expectation operator with q_s being the probability associated with state s , in which the level of consumption

¹³Note that the distinction becomes important in the time dimension as we show in Section 4.

¹⁴Remark that $\mathbb{E}^i[C]$ can equivalently be obtained from (2) in the special case of a neutral attitude towards inequity in dimension i (i.e., if f_i is an affine function).

is c_s . Finally, in this case, $\mathbb{E}^s [C]$ represents the expected value of consumption and I^s is the relative risk premium.¹⁵

Countries/Individuals If the dimension across individuals/countries is considered ($i = n$), the equity equivalent $\mathcal{E}^n(C)$ corresponds to the so-called *equally distributed equivalent* (hereafter EDE, see Atkinson, 1970; Fleurbaey, 2010). The EDE has been defined as the level of consumption which, if it were perfectly equally distributed, would give the same level of welfare as the actual distribution of consumption. In this context, $f_n = g$ is a function representing inequality aversion across countries and q_n corresponds to the weight associated with country n .¹⁶ In this context, I^n corresponds to Atkinson’s (1970) inequality index, while $\mathbb{E}^n [C]$ is just the mean of the actual distribution or per capita consumption.

Time Finally, if the dimension considered is time ($i = t$), the equity equivalent $\mathcal{E}^t(C)$ represents the constant consumption level over time, that would give the same intertemporal utility as the actual stream of consumption $\{C_t, t \in T\}$. In this case, the function $f_t = u \forall t$ represents the attitude towards intertemporal substitution, i.e., it is associated with the willingness to smooth consumption over time. The relative weights, q_t , in this case represent utility discounting and potential changes of population over time.¹⁷ Interestingly, the concept of equity equivalent over the time dimension is closely related to the notion of *balanced growth equivalent* (BGE) introduced by Mirrlees and Stern (1972).¹⁸ In particular, for a zero growth rate of the BGE (i.e. $\lambda = 0$), the two concepts exactly coincide: $BGE_{\lambda=0}(C) \equiv \mathcal{E}^t(C)$. To our knowledge, an equivalent of the inequality index has not yet been defined in the time dimension. However, we can compute it easily from equation (6). This measure I^t thus provides an index of intertemporal inequity of consumption from the social planner’s perspective. This “intertemporal inequity index” can be interpreted as the share of average discounted consumption that is lost in welfare terms due to the unequal distribution over time.

¹⁵The relative risk premium, which has the advantage to be a unit-free measure contrary to the absolute risk premium, corresponds to the share of the mean consumption that one is ready to pay to get rid of a risk on the consumption level. It is implicitly defined $\sum_s q_s u(c_s) = u(\mathbb{E}^s [C] (1 - I^s))$, see for example Eeckhoudt et al. (2005).

¹⁶This weight may for example be proportional to the country’s population size P_n , in which case $q_n = P_n / \sum_{n'} P_{n'}$. Also remark that if different individuals are considered instead of countries, q_n is nothing but the relative weight attached to a given individual.

¹⁷They can for example be expressed as $q_t = \frac{P_t \beta^t}{\sum_{t'} P_{t'} \beta^{t'}}$, where β is the utility discount factor and P_t the size of the population (or generation) living at time t .

¹⁸The BGE measures the level of today’s consumption that, assuming it grows at a constant growth rate λ , would yield the same level of consumption as the given consumption path.

The *equity equivalent* (2) and inequity index (6) therefore represent different concepts depending on which dimension is considered. For each dimension, a particular utility function f_i is used to represent inequity preferences in this particular dimension. As is standard in the economic literature, the concavity of f_i respectively represents risk aversion, inequality aversion and intergenerational inequity aversion when $i = s, n,$ or t . In the spirit of Arrow (1964); Pratt (1964), we can define $\varphi_i = \frac{-cf_i''}{f_i'}$ as a measure of the “relative inequity aversion” in the dimension i . If the utility function is of the isoelastic type, the constant parameter φ_i reflects constant relative inequity aversion:

$$f_i(x) = \begin{cases} \frac{x^{1-\varphi_i}}{1-\varphi_i} & \text{if } \varphi_i \neq 1 \\ \ln(x) & \text{if } \varphi_i = 1 \end{cases}$$

The isoelastic utility function has been widely applied, notably in the context of the Ramsey rule for discounting, the family of Atkinson’s indices of inequality measurement, and as function exhibiting constant relative risk aversion (CRRA). In order to make the dependence of the inequity index on the degree of inequity aversion evident, we will denote it by $I^i(\varphi_i)$ in the following when we refer to the special case of isoelastic functions for f_i . We will also consider three (potentially) different parameters for representing the three dimensions, namely $\varphi_i = \rho$ for risk, $\varphi_i = \gamma$ for inequality across individuals, and $\varphi_i = \eta$ for the (inverse of the) elasticity of intertemporal substitution. In Table 1, we summarize the different measures we discussed in the three dimensions linking them to the existing concepts.

i	$\mathcal{E}^i(C)$	$\mathbb{E}^i[C]$	I^i	q_i	φ_i
s	CE	expected consumption	relative risk premium	$Prob(s)$	ρ
n	EDE	per-capita consumption	Atkinson index	$\frac{P_n}{\sum_{n'} P_{n'}}$	γ
t	$BGE_{\lambda=0}$	average discounted consumption	“Intertemporal inequity index”	$\frac{P_t \beta^t}{\sum_{t'} P_{t'} \beta^{t'}}$	η

Table 1: Equity equivalent and inequity index across the three dimensions

Extension to other dimensions While the three dimensions we discussed are probably the ones which have been the most widely studied, the general approach may be easily extended to any other dimension of interest. As an example, the set i may be extended to incorporate an element m representing an extra dimension of uncertainty known as *model uncertainty* (Marinacci, 2015). In this case, f_m would represent model uncertainty aversion, and q_m would represent the prior belief associated with each potential model. In a similar fashion, in the inequality dimension we could add a second dimension capturing inequality not only between countries, but also within countries potentially with a different utility function. Lastly, in

the time dimension, one could consider considering different equity preferences over one's own generation and other generations.

3.3 Empirical findings

Inequity aversion When it comes to applications and modeling implementations, the function f_i has to be defined and parameterized. While most applications have generally considered utility functions of the isoelastic type to represent preferences over the three dimensions, it is interesting to note that the values of the relative inequity aversion parameters generally chosen have traditionally belonged to rather distinct domains, depending on the dimension under consideration. For instance, Atkinson et al. (2009) showed a substantial variation in estimated degrees of inequity aversion across the three dimensions. Based on a survey among individuals, they typically find that $\gamma < \eta < \rho$. Recently, Groom and Maddison (2013) used different approaches including surveying the literature on these different parameters for the UK finding a best guess value of $\varphi_i = 1.5$ (without differentiating the definitions).

Figure 2 presents the main specifications that have been recently proposed or used in applications to represent social preferences. The different axes represents respectively the values of the relative risk aversion (ρ), the relative inequality aversion (γ), and the relative intergenerational inequity aversion (η).

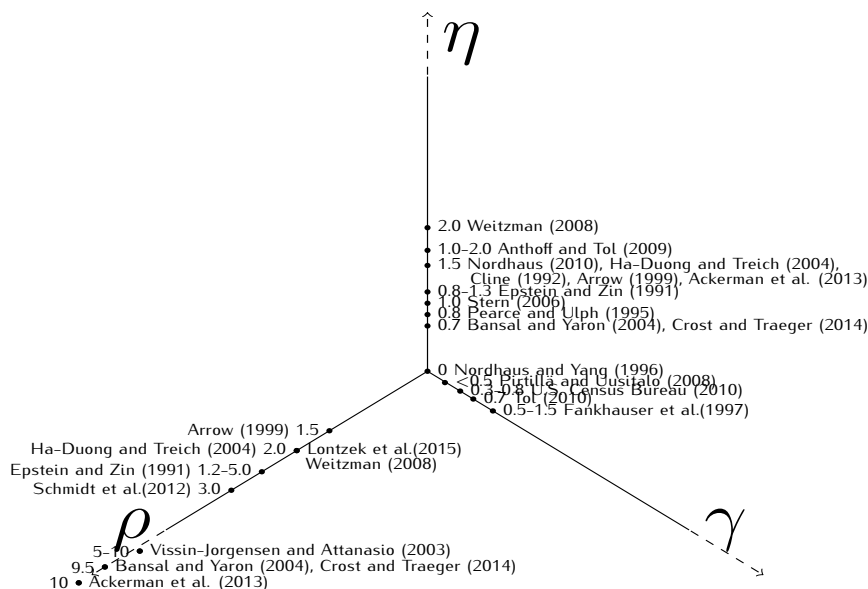


Figure 2: Values of relative risk aversion (ρ), relative inequality aversion (γ), and relative intergenerational inequity aversion (or the inverse of the elasticity of intertemporal substitution) (η) used in recent publications.

As it can be observed, the values of the relative inequity aversion coefficients are very heterogeneous over the different dimensions. They are typically less than 1 when measuring inequality aversion, around 1 for intergenerational inequity aversion, and considerably higher than 1 (and up to 20) for the degree of relative risk aversion.

Level of inequity To illustrate the comparability of the general inequity index, we compute it in a simple exercise in the three dimensions applied to the level of global consumption. In order to obtain comparable results, we focus on the case of $\varphi_i = 1 \forall i$ and thus compute the value of $I^i(1)$ in each dimension.¹⁹

Starting with the inequality between countries, and based on data for 147 countries from World Development Indicator (where population weights are used for the weights q_n), we obtain, for the year 2014, a population-weighted cross-country level of inequity of $I^n(1) = 0.3309$. Considering inequality across individuals within countries, exemplifying for the United States, the U.S. Census Bureau reports household income inequality in 2015 to be equivalent to $I^n(1) = 0.449$ (Proctor et al., 2016). These indices are thus in a similar order of magnitude.

For what concerns intertemporal inequity, we consider the widely cited estimate of Kocherlakota (1996) about fluctuation of consumption, who estimates for the U.S. an average annual growth rate of consumption of $\hat{g} = 1.8\%$. Based on this value, we can compute $\mathcal{E}^t(c)$, once we specify the time horizon to be covered, population weights, and the utility discount rate δ (so that $\beta^t = e^{-\delta t}$). For a time horizon of 100 years, assuming a constant population $P_t = P \forall t$,²⁰ and a utility discount rate of $\delta = 1\%$, we compute $I^t(1) = 0.125$. Note that this index is however increasing in the time horizon $|T|$ unless $\hat{g} = 0$.

For the level of inequity across states of the world, based again Kocherlakota's (1996) estimate and considering a standard deviation of the growth rate of consumption of $\sigma = 3.6\%$ together with the assumption of a geometric Brownian motion (GBM)²¹ for the consumption process, implies that at any point in time t , consumption is log-normally distributed with a variance $\sigma^2 t$. In that case, the inequity index can be analytically computed as $I(\varphi_i) =$

¹⁹Note that for $\varphi_i = 1$, the inequity index is ordinally equivalent to the *generalized entropy index* with parameter 0 (also known as the mean log deviation), which is computed as $GE(0) = -\sum_i q_i \ln \frac{c_i}{\bar{c}}$, in the sense that $I^i(\varphi_i) = 1 - e^{-GE(0)}$. That is, this measure is directly linked to common statistical measures of inequality, see for example Shorrocks (1980).

²⁰For constant population, we have $I^t(\eta) = 1 - \frac{\left(\frac{\delta}{1-e^{-\delta T}}\right)^{\frac{1}{1-\eta}} \left(\frac{1}{g^{(1-\eta)-\delta}} \left[e^{\hat{g}(1-\eta)-\delta} T - 1 \right] \right)^{\frac{1}{1-\eta}}}{\left(\frac{\delta}{1-e^{-\delta T}}\right) \left(\frac{1}{g^{-\delta}} \left[e^{\hat{g}-\delta} T - 1 \right] \right)}$, and it is easy to show that for $\hat{g} = 0$ we obtain $I^t(\eta) = 0$. This shows that intertemporal inequity is fundamentally due to the growth effect. For positive growth rates on the other hand, the index converges to 1 as $T \rightarrow \infty$, if $\hat{g} > \delta$. Otherwise, if $\hat{g} < \delta$, the limiting value for $T \rightarrow \infty$ converges towards a value strictly less than 1, which for $\eta \rightarrow 0$ equals 0. For $\eta \rightarrow \infty$, we get that $I(\infty) \rightarrow \frac{\hat{g}}{\delta} < 1$.

²¹That is, we assume a GBM with $\mu = 1.8\% - \frac{0.036^2}{2}$ and $\sigma = 0.036$.

$1 - e^{-0.5\varphi_i \text{Var}(C)}$, so that we obtain for the 100 year time horizon a value of $I^t(1) = 0.063$, which is a relative risk premium equal to 6.3% of total consumption until the year 2100.

Based on these results, the level of inequity across states of the world at the macro level seems to be lower than the intertemporal inequity of developed economies, which itself seems to be significantly lower than the income distribution across countries or individuals. Note that we compared the three dimensions using the same inequity aversion parameter across the three dimensions precisely to make these indices comparable. With different parameters of inequity aversion across the dimensions, the result might change. In fact, when comparing the available parameter estimates generally considered with the degrees of inequity, it is interesting to remark that higher inequity aversion parameters have typically been found for dimensions where inequity is relatively low, and vice versa. In what follows, we will discuss different ways of disentangling the three dimensions to allow inequity preferences to differ across dimensions.

4 Disentangling two dimensions

As discussed in the introduction, there are many instances in which a social planner might want to consider welfare jointly across different dimensions. It is therefore useful to extend the scope of the general framework allowing for different attitudes to be expressed towards different dimensions of inequity. In this section, we start by considering the two-dimensional problem in which two distinct components of the simplex are represented.

4.1 Welfare and inequity comparisons

As shown in Figure 2, assuming the same inequity aversion over different dimensions considered simultaneously typically leads to either underestimate or overestimate at least one of the coefficients representing inequity aversion. Disentangling two dimensions can therefore be considered as desirable from both a normative and positive perspective.

Once we move from the one-dimensional problem and allow for different preference calibrations, different orders of aggregation become possible. It seems therefore interesting to compare these different orderings in terms of our (in)equity measures. As in the previous section, we start by defining the notions of welfare and inequity in terms of equity equivalents.

Definition 4. The *social welfare function* over the ordered dimensions i and j is defined as the two-dimensional equity equivalent:

$$W^{ij} \equiv \mathcal{E}^i \left(\mathcal{E}^j (C) \right) \quad \forall i, j \neq i \in \{s, t, n\}. \quad (7)$$

This social welfare function represents the preferences of a social planner who first aggregates consumption through dimension j by considering the equity equivalent \mathcal{E}^j , before aggregating it through dimension i using \mathcal{E}^i . In the special case in which the two functions f_i and f_j capturing the social planner's attitudes towards dimensions i and j are identical, this welfare measure collapses to the standard expected discounted utilitarian (EDU) equivalent defined as:

$$W^{\text{EDU}} \equiv f^{-1} \left(\mathbb{E}^i \mathbb{E}^j [f(C)] \right) \quad \forall i, j \neq i \in \{s, t, n\}. \quad (8)$$

In this case, the order of aggregation does not matter and function f represents the attitudes towards inequity in both dimensions i and j . The recursive formulation of (7) evaluated at the element i , now becomes

$$W_i^{ij} = \mathcal{F}^i \left(\mathcal{E}_i^j(C), W_{i+1}^i \right), \quad (9)$$

where we now specify by \mathcal{E}_i^j that the aggregation through dimension j is indexed by the element i at which it is evaluated.

Definition 5. In analogy to expression (6), we define the *two-dimensional inequity index* as:

$$I^{ij} = 1 - \frac{\mathcal{E}^i(\mathcal{E}^j(C))}{\mathbb{E}^i \mathbb{E}^j [C]}, \quad (10)$$

where $\mathbb{E}^i \mathbb{E}^j [C]$ represents the mean consumption taken over both dimensions i and j .

As for the welfare measure, it should be clear that this index is unaffected by the order of aggregation in the case of equal inequity preferences towards dimensions i and j . These results are summarized in the following proposition and its corollary.

Proposition 1. *Let f_i and f_j be functions representing inequity attitude towards dimensions i and j respectively. If the social planner exhibits the same attitude towards dimensions i and j , then the order of aggregation does not matter and the social welfare measure is the standard expected discounted utilitarian equivalent (8).*

Proof. All proofs are relegated to the Appendix. □

Corollary 1. *If the social planner exhibits the same attitude towards dimensions i and j , the two-dimensional inequity index does not depend on the order of aggregation.*

In this case indeed, the equivalence $W^{ij} = W^{ji} = W^{\text{EDU}}$ trivially leads to $I^{ij} = I^{ji}$.

On the contrary, when the attitudes towards different dimensions are distinct, it becomes useful to know how the ordering affects the level of welfare and of inequity. We now derive

some conditions on how the levels of welfare can be compared for the different orderings of aggregation. We first introduce a lemma that is useful for trivial comparisons, before stating a general result on the welfare levels, which holds for any pair of combinations of the dimensions time, states and individuals.

Lemma 1. *Let f_i be a four times continuously differentiable, strictly increasing and strictly concave (convex) function representing inequity aversion over dimension i . Then, the equity equivalent \mathcal{E}^i is concave (convex) as a function of C if and only if the absolute inequity tolerance $-f'_i / -f''_i$ is concave (convex).*

This lemma enables us to compare the two welfare functions W^{ij} and W^{ji} in the trivial case in which the attitude towards one of the two dimensions is inequity neutral. For example if f_j is linear, Lemma 1 tells us that $W^{ij} \geq W^{ji}$ provided that absolute inequity tolerance of f_i (defined as the inverse of its absolute inequity aversion, see for example Gollier (2001)) is concave.²² When both inequity attitudes are non-neutral, more demanding conditions emerge as summarized in the following proposition.

Proposition 2. *Let f_i and f_j be strictly increasing, strictly concave, four times continuously differentiable functions. Then it holds that:*

- (i) $W^{ij} \geq (\leq) W^{ji} \forall C$, if and only if the absolute inequity tolerance of $\phi \equiv f_i \circ f_j^{-1}$ is concave, and f_i is strictly more (less) concave than f_j .
- (ii) $W^{ij} > (<) W^{ji} \forall c_{ij}$ such that $c_{ij} \neq c_{ij'} \forall i, j'$ and $c_{ij} \neq c_{i'j} \forall i', j$, if and only if the absolute inequity tolerance of $\phi \equiv f_i \circ f_j^{-1}$ is strictly concave, and f_i is strictly more (less) concave than f_j .

This result tells us that it is possible to compare the order of aggregation in terms of welfare in the general case under a set of condition on both the relative concavity of the two functions f_i and f_j , and on the concavity of a composition of them. The interpretation of this latter condition is however not trivial. In order to gain further intuition on this result, we focus on the special case in which functions f_i and f_j are of the isoelastic type. Further results concerning the more general class of harmonic absolute inequity aversion functions (also called HARA) are considered in Appendix B.

Proposition 3. *Let f_i and f_j be of the isoelastic type, with coefficients of relative inequity aversion φ_i and φ_j respectively. Then*

²²Note that this will be the case if f_i belongs to the class of functions exhibiting harmonic absolute inequity aversion (called HARA in the risk literature, see Appendix B for more details). This class of function includes the isoelastic (called CRRA in the risk literature), exponential (called CARA in the risk literature) and quadratic functions as special cases.

- (i) $W^{ij} > (<) W^{ji}$ if and only if $\varphi_i > \varphi_j \neq 1$, unless $\exists a \in \mathbb{R}$ s.t. $c_{i'j} = ac_{ij} \forall i, i', j$,
- (ii) $W^{ij} > (<) W^{ji}$ if and only if $\varphi_i > \varphi_j = 1$, unless $\exists b \in \mathbb{R}$ s.t. $c_{i'j} = c_{ij} + b \forall i, i', j$,
- (iii) $W^{ij} = W^{ji}$ if and only if either $\varphi_i = \varphi_j \neq 1$, or $\exists a \in \mathbb{R}$ s.t. $c_{i'j} = ac_{ij} \forall i, i', j$,
- (iv) $W^{ij} = W^{ji}$ if and only if either $\varphi_i = \varphi_j = 1$, or $\exists b \in \mathbb{R}$ s.t. $c_{i'j} = c_{ij} + b \forall i, i', j$.

The intuition behind this result is simple. First remark that two different cases must be separated depending on the degree of inequity aversion across dimension j . The first one ($\varphi_j \neq 1$) implies that the combination $\phi = f_i \circ f_j^{-1}$ is itself of the isoelastic type, while the second ($\varphi_j = 1$) implies ϕ to be of the exponential type (i.e. where the degree of absolute inequity aversion is constant). Then, what Proposition 3 tells us is that welfare will be higher when aggregating first over the dimension which is associated with the lower level of inequity aversion, unless the consumption stream is either multiplicative or additive (depending on the functional form of ϕ) in dimension i , in which case the order of aggregation does not matter.²³ Note also that first parts of (iii) and (iv) are special cases of Proposition 1.

Another important difference between the possible welfare orderings is the correlation structure of C , see the example below and the discussion in Bommier (2007). While clear-cut results are difficult to obtain in general, in the case of independence between the dimensions, it is possible to show the following.

Proposition 4. *If C is independently distributed across dimensions, i.e., $G_{ij}(C) = G_i(C)G_j(C) \forall C$, then, the ordering of aggregation does not matter for measuring welfare.*

In this case indeed, $W^{ij} = W^{ji}$ and the next result trivially follows.

Corollary 2. *Under the assumption of Proposition 4, the two-dimensional inequity index can be computed as $I^{ij} = I^i + I^j - I^i I^j$.²⁴*

In order to give a clearer intuition and to illustrate these results, we now consider the three possible two-dimensional combinations of i and j for aggregating over time and states, and individuals.

²³Remark that a consumption stream that is multiplicative in dimension i in the sense that $\exists a \in \mathbb{R}$ s.t. $c_{i'j} = ac_{ij} \forall i, i', j$, is automatically also multiplicative in dimension j : $\exists a \in \mathbb{R}$ s.t. $c_{i'j} = ac_{ij} \forall i, i', j \iff \exists a' \in \mathbb{R}$ s.t. $c_{ij'} = a'c_{ij} \forall i, j, j'$. The same property holds for the additive case.

²⁴Moreover, we claim, and from numerical experiments it seems to hold, that for any positive correlation, comparing W^{ij} and W^{ji} , assuming f_i more concave than f_j , $W^{ij} > W^{ji}$ if the distribution of C exhibits “more concordance” or has undergone marginal preserving increases in concordance (MPICs) as defined in Tchen (1980).

4.2 The three combinations of two dimensions

We now consider the three possible particular cases of our general two-dimensional framework and relate them to the different formulations, which have been proposed in the literature.²⁵

Time and states If the two dimensions considered simultaneously are risk and time, we recover from (7) various well-known welfare criteria that have been used and applied in many different contexts. For example, consider a two-period model in which first period consumption (C_1) is deterministic, and second period consumption (C_2) is risky. In this case, criterion (7) is equivalent to either the framework proposed by Selden (1978); Kreps and Porteus (1978), or to the preferences used by Kihlstrom and Mirman (1974). To see this, remark that once we first aggregate over states of the world before aggregating through the time dimension, we have $W^{ts} = u^{-1} \left(\frac{1}{1+\beta} u(C_1) + \frac{\beta}{1+\beta} u \circ v^{-1} (\mathbb{E}^s [v(C_2)]) \right)$. This expression, which describes welfare as an equity equivalent, is nothing but a monotonic transformation of the intertemporal welfare criterion proposed in Selden (1978). Similarly, once we change the order aggregation, $W^{st} = v^{-1} \left(\mathbb{E}^s \left[v \circ u^{-1} \left(\frac{1}{1+\beta} u(C_1) + \frac{\beta}{1+\beta} u(C_2) \right) \right] \right)$ corresponds to the equity equivalent version of Kihlstrom and Mirman's (1974) model applied in the intertemporal context, as in Bommier (2007). Finally, when preferences are of the isoelastic type, the formulation W^{ts} is also equivalent to the criterion of the widely used Epstein and Zin (1989); Weil (1989, 1990) preferences. In this case, we directly know, from Proposition 3, that if relative risk aversion is stronger than the inverse of the elasticity of intertemporal substitution, the welfare under Selden's preferences is generally higher than under Kihlstrom and Mirman preferences.

For general problems involving risk and time, formulation (7) may however sometimes be inappropriate. This is the case if a risk is present in different periods of time (as for example if there is learning and there are more than two periods). In such a situation, welfare at time t should be evaluated based on the expectation of future consumption at time t rather than on the expectation taken in each period in time separately, as implicit in the formulation (7). In other words, in order to have a welfare function that has both the desirable properties of being dynamically consistent and independent of both unrealized alternatives and past consumption levels,²⁶ the recursive version of the social welfare function W^{ts} at time t (denoted by the subscript t) should be reformulated as:

$$\hat{W}_t^{ts} = \mathcal{F}^t \left(C_t, \mathcal{E}_t^s \left(\hat{W}_{t+1}^{ts} \right) \right), \quad (11)$$

²⁵Note that as above we will use u , v , and g representing the utility functions over the time, risk, and inequality dimensions, respectively.

²⁶See Epstein and Zin (1989) pp 950-952 for a critical analysis of Kihlstrom and Mirman's (1974) expected utility approach in an intertemporal setting.

where $\mathcal{F}^t : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a temporal aggregator in the sense of Koopmans (1960) as defined in (5), and \mathcal{E}_t^s is the certainty equivalent operator at time t

$$\mathcal{E}_t^s (\hat{W}_{t+1}^{ts}) = v^{-1} \left(\mathbb{E}_t^s \left[v (\hat{W}_{t+1}^{ts}) \right] \right), \quad (12)$$

where \mathbb{E}_t^s is the expectation operator conditional on information available at time t .

In the case of constant population over time,²⁷ exponential discounting is obtained by setting $\hat{q}_{t+1} = \beta$ in (5), where β is the discount factor. A standard specification of the temporal aggregator is, for example, the *constant elasticity of substitution* (CES) aggregator obtained when $f_t(z) = u(z) = z^{1-\eta}/1 - \eta$. In this case, the recursive welfare function (11) simplifies to the additive EDU welfare formulation

$$W_t^{\text{EDU}} = u^{-1} \left(\mathbb{E}_t^s \left[\sum_{\tau} \beta^{\tau} u(c_{t+\tau}) \right] \right), \quad (13)$$

obtained in the special case $u = v$.

Formulation (11) allows for a separation between risk attitude and the degree of intertemporal substitution. Marinacci and Montrucchio (2010) have shown that for a solution of (11) to be unique and globally attractive (i.e., which allows for a solution to be found iteratively starting from any initial point) using CES aggregator is that v exhibits increasing relative risk aversion (IRRA). This includes the widely-used CRRA specification, giving rise to the popular version of (11) of Epstein and Zin (1989); Weil (1989, 1990). Under this specification, it is easy to check that Selden's formulation is recovered when there are only two periods of time.

If instead the problem is seen as one of intergenerational equity and future generations' utility is not discounted (i.e., $\beta=1$) as in Adler and Treich (2014), our formulations W^{ts} corresponds exactly to the notion of "ex-ante prioritarianism", while the Utilitarian case is simply recovered in the special case $u = v$, in accordance with Proposition 1. Interestingly, remark that Bommier (2007) shows that the difference between the two orders of aggregation can be found in the aversion towards intertemporal correlation of risks.

States and individuals/countries Now, instead consider the atemporal problem of a risk being distributed among individuals. that is, the welfare functions W^{sn} and W^{ns} . Following the notation we introduced, these expressions can be developed as

$$W^{sn} = v^{-1} \left(\mathbb{E}^s \left[v \circ g^{-1} \left(\sum_n \frac{P_n}{\sum_{n'} P_{n'}} g(C_n) \right) \right] \right) \text{ and } W^{ns} = g^{-1} \left(\sum_n \frac{P_{nt}}{\sum_{n'} P_{n'}} g \circ v^{-1} \left(\mathbb{E}^s [v(C_n)] \right) \right).$$

These welfare measures can be associated respectively with the notions of "expected equally distributed equivalent" (Fleurbaey, 2010) and "ex-ante egalitarianism" (Diamond et al., 1967;

²⁷Which can be easily generalized by adding population weights to the discount factor.

Epstein and Segal, 1992b). Both collapse to the standard utilitarian criterion when inequality and risk aversion coincide.

These two orders of aggregation can be interpreted as characterizing situations of “global” and “individual” (or country) risk aversion respectively. Loosely speaking, *global risk aversion* considers the risk at the global level and therefore takes into account risk sharing possibilities permitted by the level of inequality between countries, while *individual or country risk aversion* on the other hand only considers the risk at the country/individual level, and considers inequalities in terms of certainty equivalents. In other words, this means that the choice of the welfare criterion implicitly determines how risk is being shared by the social planner. To illustrate the difference between the two concepts, consider the simple scenarios illustrated in Table 2.

	state 1	state 2	Expected consumption
country 1	10	20	15
country 2	20	10	15
Mean	15	15	15

(a) situation A

	state 1	state 2	Expected consumption
country 1	15	5	10
country 2	30	10	20
Mean	22.5	7.5	15

(b) situation B

Table 2: Distribution of consumption in two different cases: global risk aversion (situation A) and individual risk aversion (situation B)

Consider a world made of two countries of equal population size, and only two states of the world, each one realized with equal probability. The consumption levels associated with the two different distributions are presented in Table 2. The average consumption for each state, the expected consumption for each country, and the mean expected consumption $\mathbb{E}^n \mathbb{E}^s (C)$ (which is equal to 15 units in both situations) are also provided. In situation A (left), the mean consumption is certain due to the perfectly negative correlation between the distributions of consumption. In this case, risk aversion should not play any role if the social planner considers the risk affecting only global consumption. In other words, the welfare W^{sn} should not depend on risk attitude. On the contrary, if risk is considered for each country separately, the resulting certainty equivalent will be lower than the expected level of consumption in each country, and the aggregation over both countries will result in a lower level of equivalent consumption. Intuitively, the two concepts therefore yield different results when there is room for welfare enhancing risk sharing transfers. In situation B, the risk affects proportionally one poor (country 1) and one rich country (country 2). In relative terms, the degree of inequality is thus constant across states given that, ex-post, country 2 always consumes twice as much as

country 1.

$\gamma \backslash \rho$	0.7		2	
	W^{sn}	W^{ns}	W^{sn}	W^{ns}
0.7	14.4	14.4	14.4	13.3
2	13.3	14.4	13.3	13.3

(a) situation A

$\gamma \backslash \rho$	0.7		2	
	W^{sn}	W^{ns}	W^{sn}	W^{ns}
0.7	13.0	13.0	12.1	12.1
2	10.8	10.8	10.0	10.0

(b) situation B

Table 3: Welfare and inequity measures with different orders of aggregation and relative inequity aversion coefficients

In Table 3, we compute the welfare in terms of equity equivalents in the case where utility functions are of the isoelastic type, with relative risk (ρ) and inequality (γ) aversion coefficients that can either take value 0.7 or 2. In line with Proposition 1 and its corollary, we can directly check that the order of aggregation does not matter when both parameters ρ and γ are equal. In situation A, as we previously noticed, risk aversion does not affect the welfare measures if the aggregation is first realized according to the individual dimension: W^{sn} (and I^{sn}) do not depend on ρ . In line with Proposition 3, we also see that the order of aggregation is non-neutral and we for example have $W^{sn} = 14.4 > W^{ns} = 13.33$ when $\rho > \gamma$. In terms of the levels of inequity, the values can be easily computed as relative differences from the expected value (which equals 15 in both situations). In situation B, whether risk is considered at the global or country/individual level yields exactly the same level of welfare $W^{sn} = W^{ns}$, in accordance with Proposition 3 (iii).

Time and individuals/countries The third case concerns the joint consideration of inter-temporal equity and inequality. In this case, the general welfare measures W^{tn} and W^{nt} are respectively defined as: $W^{tn} = u^{-1}\left(\sum_t \frac{\bar{P}_t \beta^t}{\sum_{t'} \bar{P}_{t'} \beta^{t'}} u \circ g^{-1}\left(\sum_n \frac{P_{tn}}{\sum_{n'} P_{n'}} g(c_{tn})\right)\right)$ and $W^{nt} = g^{-1}\left(\sum_n \frac{\bar{P}_n}{\sum_{n'} \bar{P}_{n'}} g \circ u^{-1}\left(\sum_t \frac{P_{tn} \beta^t}{\sum_{t'} P_{t'n} \beta^{t'}} u(c_{tn})\right)\right)$, where P_{tn} refers to the population of country n at time t , $\bar{P}_t \equiv \sum_n P_{tn}$ is the total population at time t , and $\bar{P}_n \equiv \sum_t P_{tn} \beta^t$ is the (discounted) intertemporal population size of country n .

If the attitudes towards the two dimensions are identical, the welfare measure (usually referred to as *discounted utilitarian welfare*) is essentially equivalent to the one used in many deterministic numerical models used to study regionally disaggregated welfare maximization problems. This is for example the case in most of the so-called integrated assessment models (IAM) based on welfare maximization used to study the interaction between climate change and the economy. The typical welfare function that has been considered in the literature

under different attitudes towards the two dimensions is W^{tn} , where the aggregation is first realized across countries and then over time. In this case, it is easy to see that under inequality neutrality (i.e. linear function g), the inner part of the welfare criterion simply becomes the average global per-country consumption level at time t . Under this special case, the welfare criterion is equivalent to the idea proposed by Stanton (2011) of simply “maximizing money”. The other possible order of aggregation, W^{nt} , can be seen as as a weighted sum, taken today, over utility of “dynasties” in each country.

This approach is closely related to the recent work of Adler and Treich (2014) who distinguish ex-post and ex-ante prioritarian social welfare functions, which comes down to the same difference of summations across different dimensions, including the concept of spatial heterogeneity.

5 Disentangling the three dimensions

Once one wants to study problems involving different countries (or individuals) at different periods of time and that the outcome is subject to some degree of uncertainty (risk), it is of particular importance to have a proper representation of the welfare and inequity measures. The extension to the three dimensional problem naturally follows from the analysis that precedes.

Definition 6. The *social welfare function* over dimensions i , j and k is defined as the three-dimensional equity equivalent:

$$W^{ijk} \equiv \mathcal{E}^i \left(\mathcal{E}^j \left(\mathcal{E}^k (C) \right) \right) \quad \forall i, j, k \in \{s, t, n\} \text{ such that } i \neq j \neq k. \quad (14)$$

As indicated by the order of the indices in its superscript, this social welfare function represents the preferences of a social planner who first aggregates consumption through dimension k , then through dimension j , before finally aggregating it through dimension i . As before, the standard EDU equivalent consumption represents the preferences of a social planner whose attitudes towards the three dimensions are identical. It is obtained as a special case of (14) where f_i , f_j and f_k are the same and denoted f :

$$W^{\text{EDU}} \equiv f^{-1} \left(\mathbb{E}^i \mathbb{E}^j \mathbb{E}^k [f(C)] \right) \quad \forall i, j, k \in \{s, t, n\} \text{ such that } i \neq j \neq k. \quad (15)$$

This relatively tractable welfare function is similar to the one which is typically used in applied economic models. In this case, the order of aggregation does not alter the social welfare function.

While the social welfare function (14) seems fairly general, it has to be acknowledged that it may, in some instances, need to be refined for use in dynamic settings. In particular, a welfare

function admitting an order of aggregation through the state dimension taking place before time could reveal time inconsistent. In what follows we pay a particular attention to these special cases by adapting the recursive formulation of (14) which is usually written as

$$W_i^{ijk} = \mathcal{F}^i \left(\mathcal{E}_i^j \left(\mathcal{E}_i^k(C) \right), W_{i+1}^i \right), \quad (16)$$

where the aggregations through dimension j and k are now indexed by i (that is, welfare is recursively defined in dimension i as indicated by the subscript).

Remark that we can link this welfare measure as threefold equity equivalent to an extension BGE referred to as the “certainty, equity, and balanced growth equivalent” (CEBGE). This concept has been used by Anthoff and Tol (2009) and Schmidt et al. (2012) in the context of climate change. It measures the level of today’s consumption that, provided it were certain, equally distributed and grew at a constant growth rate λ , would yield the same level of expected discounted utilitarian welfare as the actual consumption path (subject to inequality and uncertainty). Since the CEBGE is by construction is equal across states and countries, attitudes towards risk and inequality do not play any role. Welfare therefore only depends on inequity preferences across time. In particular, welfare in this case can be computed as $W^{ijk} = \mathcal{E}^t (CEBGE(1 + \lambda)^t)$. If f_t is isoelastic, solving this equation for the *CEBGE* given any welfare level W^{ijk} of the actual distribution, leads to $CEBGE = W^{ijk} \left(\sum_t q_t (1 + \lambda)^{(1-\eta)t} \right)^{-\frac{1}{1-\eta}}$, where the weights may be defined as $q_t = \frac{P_t \beta^t}{\sum_{t'} P_{t'} \beta^{t'}}$. Unsurprisingly, when a constant level of consumption over time is considered (i.e. if $\lambda = 0$), we obtain $CEBGE = W^{ijk}$. In that sense, the three-dimensional welfare function we propose can be considered as a special case of the CEBGE with a zero growth rate.

As should be clear from (14), the order of aggregation over the different dimensions is relevant from the social planner’s perspective. In particular, if different attitudes towards the different dimensions are considered, $3! = 6$ potential different welfare orderings of aggregation are admissible. In what follows, we briefly discuss some of these possible orderings.

Global risk aversion We first consider the case of a decision maker considering welfare from a global perspective, when evaluating different states of the world. This decision maker is referred to as “globally risk averse” in the sense that a she considers risky payoffs at the global aggregated level. Within this welfare specification, risk is thus considered only at the aggregated level. The only difference with the Kreps-Porteus/Epstein-Zin/Weil formulation

\hat{W}_t^{ts} defined above is that we now have different individuals, so that the equally distributed equivalent $\mathcal{E}^n(C)$ is itself risky. In this case, the time consistent version of welfare function \hat{W}_t^{tsn} is written recursively as:

$$\hat{W}_t^{tsn} = \mathcal{F}^t \left(\mathcal{E}^n(C_t), \mathcal{E}_t^s \left(\hat{W}_{t+1}^{tsn} \right) \right). \quad (17)$$

As before, \mathcal{F}^t is the temporal aggregator defined as in (5) and \mathcal{E}_t^s is the time t certainty equivalent operator. In the special case in which the social planner is inequality neutral, it is nothing but the adjusted average consumption taken over the different countries/individuals. In this case, the social welfare function \hat{W}_t^{tsn} is nothing but a version of (11) computed for the average per-capita consumption. Intuitively, global risk aversion assumes that the social planner has access to a risk sharing technology (within the bounds of her inequality attitude), see also the application in Emmerling (2014). In particular, this means that negatively correlated risks can be mediated.

Individual risk aversion When orders of aggregation across countries and states of the world are inverted, the decision maker instead considers individual risky consumption, and aggregates the individual certainty equivalents. A social planner is said to exhibit *individual risk aversion*, if she considers risky payoffs at the level of individuals, and then aggregates the certainty equivalents across individuals or countries. In this situation, the recursive form of the welfare function can be written²⁸ as

$$\hat{W}_t^{tns} = \mathcal{F}^t \left(C_t, \mathcal{E}^n \left(\mathcal{E}_t^s \left(\hat{W}_{t+1}^{tns} \right) \right) \right). \quad (18)$$

In this case, if we increase the degree of risk aversion that characterizes the social planner, the implicit welfare weight increases for the country that has the lowest level of consumption in any state of the world.

Dynastic recursive preferences Finally, if the aggregation over countries (or individuals) is realized after the intertemporal aggregation over time, global inequality is evaluated considering welfare defined over the whole time horizon in each region. In this case, the time consistent version of the recursive welfare function is written as

$$\hat{W}_t^{nts} = \mathcal{E}^n \left(\left(\hat{W}_t^{ts} \right)_n \right), \quad (19)$$

²⁸Note that for the first period instead it is due to inequality also for $t = 1$ defined as $\hat{W}_1^{tns} = \mathcal{E}^n(C_1)$

where $(\hat{W}_t^{ts})_n$ is simply the Kreps-Porteus welfare (11) starting at time t , but which is defined for each country separately (as indicated by the index n). That is, the social planner aggregates over welfare of each country/individual that considers expected discounted utility over its (infinite) lifetime similar to a dynasty.

Having defined welfare across the three dimensions, we can now define the index of three-dimensional inequity as follows.

Definition 7. The *three-dimensional inequity index* is defined as:

$$I^{ijk} = 1 - \frac{\mathcal{E}^i \left(\mathcal{E}^j \left(\mathcal{E}^k(C) \right) \right)}{\mathbb{E}^i \mathbb{E}^j \mathbb{E}^k [C]}, \quad (20)$$

where $\mathbb{E}^i \mathbb{E}^j \mathbb{E}^k [C]$ is the mean consumption over the three dimensions.

As before, it has a similar interpretation as the Atkinson index. If inequity preferences differ across the dimensions, we moreover have six different values for this global inequity index depending on the order of aggregation. In a slight abuse of notation, let \tilde{I}^i denote the inequity index in dimension i , which itself is conditional on the distribution in the other dimensions. Then we can rewrite the three-dimensional inequity index as

$$I^{ijk} = 1 - \frac{\mathbb{E}^i \left[\mathbb{E}^j \left[\mathbb{E}^k [C] \left(1 - \tilde{I}^k \right) \right] \left(1 - \tilde{I}^j \right) \right] \left(1 - \tilde{I}^i \right)}{\mathbb{E}^i \mathbb{E}^j \mathbb{E}^k [C]}. \quad (21)$$

While in general, the inequity indices can be computed if the full (up to) three dimensional data is available, we can obtain a simplified version in the special case where the expected inequity indices in one dimension are constant across the other dimensions: $\mathbb{E}^j [\tilde{I}^i] = I^i \forall i \neq j$ and moreover uncorrelated with the average consumption level in any other dimension. For instance, independence of the three-dimensional distributions would be a sufficient condition for this to hold. In this case, following the same argument as in Proposition 4, the three-dimensional inequity index can be decomposed as

$$I^{ijk} = I^i + (1 - I^i) \left(I^j + (1 - I^j) I^k \right), \quad (22)$$

which is symmetric in the dimensions and is therefore independent of the order of aggregation.

We now proceed to the computation of the inequity indices across the three individual dimensions and at the “total” level of inequity across all three dimensions. We use the global data set for GDP and population over 35 years from 1980 to 2014.²⁹ For the global economy

²⁹The data sources are described in detail in Appendix B, where we also report the values also estimated for each country in the data set.

and based on this data, we estimate an average growth rate of per-capita GDP of 1.80% with a standard deviation of 1.30% over the last 35 years. Moreover, averaged over the total time period, across countries we get a value for the population-weighted variance of the logarithm of GDP of $\sigma^2 = 1.294$ for the inequality between countries, which we use as before to fit a log-normal distribution of consumption to compute the inequity index.

Based on these three statistics, we can compute the inequity indices as in Section 3, but now for the global level. We consider the same specification as above, notably a time horizon of 100 years and a utility discount rate of 1%. Table 4 shows the resulting inequity indices for the three dimensions and the total level of inequity if the three dimensions were independent, both for a uniform degree of inequity aversion equal to 1, and for differentiated values. As can be observed, three-dimensional inequity appears to be largely determined by inequality between countries, followed by the inequity across the time dimension and then across the risk domain. Allowing for the parameters to vary (broadly in line with the empirical estimates discussed in Section 3, these differences remain even though the gap decreases.

Dimension	index	$I^i(1)$	specification	$I^i(\varphi_i)$
countries	n	0.480	$\gamma = 0.7$	0.364
risk	s	0.009	$\rho = 8$	0.066
time	t	0.126	$\eta = 1.5$	0.178
TOTAL independent	ijk	0.549	$\{\gamma, \rho, \eta\} = \{0.7, 8, 1.5\}$	0.512

Table 4: Inequity indices for consumption at the global level

6 Conclusion

The concepts of economic welfare and inequity are closely intertwined. In this paper we showed that inequity encompasses different notions depending on whether consumption is spread across states of the world, time periods or individuals. We study welfare and inequity in these dimensions using a generalized framework extending the Utilitarian Expected discounted utility paradigm. In particular, we propose an “equity equivalent” in each dimension as a useful measure of welfare, and an inequity index based on this value. This generalized framework therefore gathers different concepts that have been studied separately in the different dimensions.

We then extend the analysis to the cases where two or three dimensions are jointly considered. In these cases, the order of aggregation matters for welfare evaluation, unless the inequity preferences are identical across all dimensions. In particular, when inequality, risky and time dependent distributions are jointly considered, six different orderings of aggregation are possible, leading to different welfare concepts. In this case, the role of correlations across

the different dimension and the characteristics of the utility functions become important to rank welfare levels.

Empirically, we find that that inequity preferences might well differ. In particular, risk aversion has been found to be far bigger than inequality or intertemporal fluctuation aversion. Moreover, when it comes to the levels of inequity, results based on consumption at the global level suggest that inequality across individuals or countries exhibits the highest degree of inequity, followed by intergenerational inequity and uncertainty of consumption over the 21st century.

Appendix

A Proofs

Proof of Proposition 1. Proposition 1 is trivial and follows directly from the dimension-additive separability of the aggregation (equity equivalent) operator. \square

Proof of Lemma 1. The proof for the case where f_i is strictly convex may be found in Hardy et al. (1952) (Theorem 106, p. 88). The case where f_i is strictly concave is then trivially obtained by introducing the transformation $f_i(x) = -F_i(-x)$, where F_i is increasing and convex (Hennessy and Lapan, 2006). \square

Proof of Proposition 2. First note that function $\phi \equiv f_i \circ f_j^{-1}$ is strictly increasing, and strictly concave (convex) if and only if $-\frac{f_i''}{f_i'} > (<) -\frac{f_j''}{f_j'}$. Then remark that we can write the condition $W^{ij} \geq W^{ji}$ as

$$f_j \circ f_i^{-1} \left(\sum_i q_i f_i \circ f_j^{-1} \left(\sum_j q_j f_j(c_{ij}) \right) \right) \geq \sum_j q_j f_j \circ f_i^{-1} \left(\sum_i q_i f_i(c_{ij}) \right), \quad (\text{A.1})$$

which can be rewritten as

$$\phi^{-1} \left(\sum_i q_i \phi \left(\sum_j q_j f_j(c_{ij}) \right) \right) \geq \sum_j q_j \phi^{-1} \left(\sum_i q_i \phi(f_j(c_{ij})) \right). \quad (\text{A.2})$$

From Lemma 1, we know that the equity equivalent is a concave operator so that $\mathcal{E}^\phi \left(\sum_j q_j f_j(c_{ij}) \right) \geq \sum_j q_j \mathcal{E}^\phi(f_j(c_{ij}))$, which is exactly what is represented in expression (A.2) if we let $\mathcal{E}^\phi(X) \equiv \phi^{-1} \mathbb{E}^i \phi(x)$. This proves case (i). Case (ii) then follows directly from the strict version of Lemma 1. \square

Proof of Proposition 3. First remark that when both f_i and f_j are of the isoelastic type with coefficients of relative aversion respectively φ_i and φ_j , ϕ can be written as

$$\phi(x) = \begin{cases} \frac{1}{1-\varphi_i} \left[(1-\varphi_j)x \right]^{\frac{1-\varphi_i}{1-\varphi_j}}, & \text{if } \varphi_j \neq 1 \\ \frac{e^{(1-\varphi_i)x}}{1-\varphi_i}, & \text{if } \varphi_j = 1. \end{cases} \quad (\text{A.3})$$

In this case, ϕ is itself of the harmonic absolute inequity aversion (HARA) class. In particular, it is an isoelastic function when $\varphi_j \neq 1$, while it is an exponential function when $\varphi_j = 1$. In

accordance with the discussion on general HARA class provided in Appendix B, this function is only defined on the domain \mathbb{R}^+ when $\varphi_j < 1$, and in the domain \mathbb{R}^- when $\varphi_j > 1$. Moreover, remark that this function ϕ is increasing, strictly concave when $\varphi_i > \varphi_j$, strictly convex when $\varphi_i < \varphi_j$, and linear when $\varphi_i = \varphi_j$. The results therefore directly follows from (5) and Corollary (3). In particular, cases (i) and (iii) follow as special cases when $\tau = 0$ which leads to $b = 0$ (since ϕ is of the isoelastic type), and cases (ii) and (iv) follow as special cases when $\tau \rightarrow \infty$ which leads to $a = 1$ (since ϕ is of the exponential type).³⁰ \square

Proof of Proposition 4. We first write W^{ji} , using a continuous distribution if C to simplify notation, as $W^{ji} = f_j^{-1} \int f_j \circ f_i^{-1} \left(\int f_i(C) dG_{i|j} \right) dG_j$ where $dG_{i|j}$ denotes the conditional cumulative distribution of C across dimension i . Given the independence assumption, we have that $dG_{i|j} = dG_i$, so that we obtain $W^{ji} = f_j^{-1} \left(\int f_j dG_j \right) f_i^{-1} \left(\int f_i(c) dG_i \right) = W^{ij}$. That is, in the case of independence it holds that $W^{ij} = \mathcal{E}^i(c) \mathcal{E}^j(c) = \mathcal{E}^j(c) \mathcal{E}^i(c) = W^{ji}$. \square

Proof of Corollary 2. Applying the result of 4 to the definition in the two-dimensional inequity index presented in equation (10) gives $I^{ij} = 1 - \frac{\mathcal{E}^i(C) \mathcal{E}^j(C)}{\mathbb{E}^i \mathbb{E}^j [C]}$. Then, applying the definition of (6) simply leads to $I^{ij} = 1 - (1 - I^i)(1 - I^j)$. \square

B Welfare comparisons in the HARA case

Most of the parametric utility functions in economics, finance and decision theory belong to the class of harmonic absolute inequity aversion utility functions. It is best-known as HARA (harmonic absolute risk aversion) since the dimension considered is generally the risk one. A utility function f_i is said to be of the HARA class if its absolute inequity tolerance (i.e. the inverse of its absolute inequity aversion) is linear in consumption. In particular, f_i is of the HARA class if it can be written as:

$$f_i(x) = \frac{1 - \psi}{\psi} \left[\frac{\alpha x}{1 - \psi} + \eta \right]^\psi, \quad (\text{B.1})$$

which is defined for values of x such that $\frac{\alpha x}{1 - \psi} + \eta > 0$, and strictly increasing and concave when $\alpha > 0$. It is of the isoelastic type when $\eta = 0$, and of the exponential type when $\psi \rightarrow -\infty$. In the special case of HARA functions, Proposition 2 can be expressed as follows.

³⁰Remark that the absolute inequity tolerance of $\phi \equiv f_j \circ f_i^{-1}$ with isoelastic specifications can be computed as $\phi'(x) / -\phi''(x) = (\varphi_j - \varphi_i)(1 - \varphi_i)x$, which is linear in x .

Proposition 5. *Let f_i and f_j be strictly increasing, strictly concave, four times continuously differentiable functions such that $f_i \neq af_j + b$ where $a \neq 0$, $b \in \mathbb{R}$, and let $\phi = f_i \circ f_j^{-1}$ be of the HARA form. Then $W^{ij} = W^{ji}$ if and only if :*

$$y_{ij'} = ay_{ij} + b \quad \forall i, j, j',$$

where $y_{ij} \equiv f_j(c_{ij})$, $a = [\alpha\mathcal{E}^\phi(y_{ij'}) + \tau]/[\alpha\mathcal{E}^\phi(y_{ij}) + \tau]$, $b = [\mathcal{E}^\phi(y_{ij'}) - \mathcal{E}^\phi(y_{ij})]\tau/[\alpha\mathcal{E}^\phi(y_{ij}) + \tau]$, $\tau = (1 - \psi)\eta$ and $\mathcal{E}^\phi(X) \equiv \phi^{-1}(\mathbb{E}^i\phi(X))$.

Proof. First remark that $W^{ij} = W^{ji} \iff \mathcal{E}^\phi\left(\sum_j q_j y_{ij}\right) = \sum_j q_j \mathcal{E}^\phi(y_{ij})$. The proof for the case where there are only two elements in the set indexed by j may be found in Hennessy and Lapan (2006) (Proposition 1 (iii), p. 3). It is then easy to extend the proof to the general cases by proceeding as follows. Subdivide y_{ij} into $y_{ij} = vx_{ij} + (1 - v)x_{ij'}$, with $v \in (0, 1)$, and note that with x_{ij} and $x_{ij'}$, $W^{ij} = W^{ji}$ if and only if $x_{ij'} = a^x x_{ij} + b^x$, where $a^x = [\alpha\mathcal{E}^\phi(x_{ij'}) + \tau]/[\alpha\mathcal{E}^\phi(x_{ij}) + \tau]$, $b^x = [\mathcal{E}^\phi(x_{ij'}) - \mathcal{E}^\phi(x_{ij})]\tau/[\alpha\mathcal{E}^\phi(x_{ij}) + \tau]$. When considering y_{ij} and $y_{ij'}$, we therefore have: $y_{ij'} = a(vx_{ij} + (1 - v)x_{ij'}) + b = a(vx_{ij} + (1 - v)(a^x x_{ij} + b^x)) + b$. Some algebra enables to write it as $y_{ij'} = a^y x_{ij} + b^y$, where $a^y = [\alpha\mathcal{E}^\phi(y_{ij'}) + \tau]/[\alpha\mathcal{E}^\phi(x_{ij}) + \tau]$, $b^y = [\mathcal{E}^\phi(y_{ij'}) - \mathcal{E}^\phi(x_{ij})]\tau/[\alpha\mathcal{E}^\phi(x_{ij}) + \tau]$. Hence if we let $x_{ij} \equiv y_{ij''}$ and $x_{ij'} \equiv y_{ij'''}$ the Proposition's statement holds when there are three elements in the set index by j . Repeating this procedure gives the result. \square

The combination of Propositions 2 and 5 leads us to the following corollary:

Corollary 3. *Let f_i and f_j be strictly increasing, strictly concave, four times continuously differentiable functions such that $f_i \neq af_j + b$ where $a \neq 0$, $b \in \mathbb{R}$, and let $\phi = f_i \circ f_j^{-1}$ be of the HARA form and the conditions of Proposition 5 do not hold, then $W^{ij} > (<)W^{ji}$ if and only if f_i is strictly more (less) concave than f_j .*

Proof. If ϕ is of the HARA form, its absolute risk tolerance is linear in consumption and we therefore know from Proposition 2 case (i) that $W^{ij} \geq (\leq)W^{ji}$ if and only if f_i is strictly more (less) concave than f_j . Moreover, from Proposition 5 and the properties of HARA functions, we also know that $W^{ij} = W^{ji}$ only holds under very specific conditions when $f_i \neq af_j + b$, where $a \neq 0$, $b \in \mathbb{R}$. Combining these two results therefore proves Corollary 3. \square

C Inequity indices across countries

Similar to the results in Section 5 for the global levels of inequity, we now compute the inequity indices across the different dimensions based on individual countries. Figure B.1 shows the

inequity indices based on an inequity aversion parameter of 1 in all the three dimensions ($\varphi_i = 1 \forall i$) using global GDP data. We use the IMF's World Economic Outlook database from October 2016, which covers 186 countries from 1980 until 2015. Similar to the estimation for the United States in section 3 above, we assume a geometric Brownian motion (GBM) for the growth process, and we estimate the mean and variance of this growth process based on the years 1980-2015. Using these estimates, we compute the inequity index over one century in the time dimension (based on the expected growth rates) and risk dimension (based on the volatility of growth). The only additional assumptions we make are a pure rate of time preference of 1% and for the sake of simplicity that of constant population. In Figure B.1, we present the obtained values for intertemporal inequity and uncertainty across all 186 countries using a kernel density estimation across countries in brown (I^t) and green (I^s). The results show a high concentration of inequity in both dimensions around a value of about 0.10, while a significant number of countries exhibit also higher values of inequity of around 0.5. That is, while for most countries, a relatively small degree of intergenerational inequity and uncertainty of consumption at the macro level is found, a significant number of countries exhibit high degrees of inequity in both dimensions due to high (or negative) growth rates and high volatility of growth, respectively.

As for the third dimension of inequity between individuals or countries (I^n), we can consider the global (between country) or between individuals (within countries) inequality. The global level of inequity between countries is estimated at 0.477, as shown by the red bar in the figure . For the within countries computations, we use the latest available (typically between 2009 and 2014) data on income inequality as measured by the Gini coefficient (Source: World Development Indicators), which is available for 153 countries. In order to obtain estimates for our inequity index, we assume a log-normal distribution within each country with parameters (μ, σ) , for which the Gini coefficient G and the inequity index can be computed as $G = 2\Phi(\frac{\sigma}{\sqrt{2}}) - 1$ and $I^i(\varphi_i) = 1 - e^{-0.5\varphi_i\sigma^2}$ respectively. We can then find the inequity index within each country by solving the two equations for each country. We add the resulting density of the estimated within-country inequity indices in blue in Figure B.1. The inequality within countries is estimated between 0.1 and 0.6 according to the inequity measure with a concentration around 0.25. Using the sub-group decomposability of this inequity index (see Bourguignon (1979); Das and Parikh (1981)), we can combine these values with the between country level of inequality and finally obtain a value of 0.605 for the level of global inequality across individuals (pink bar).

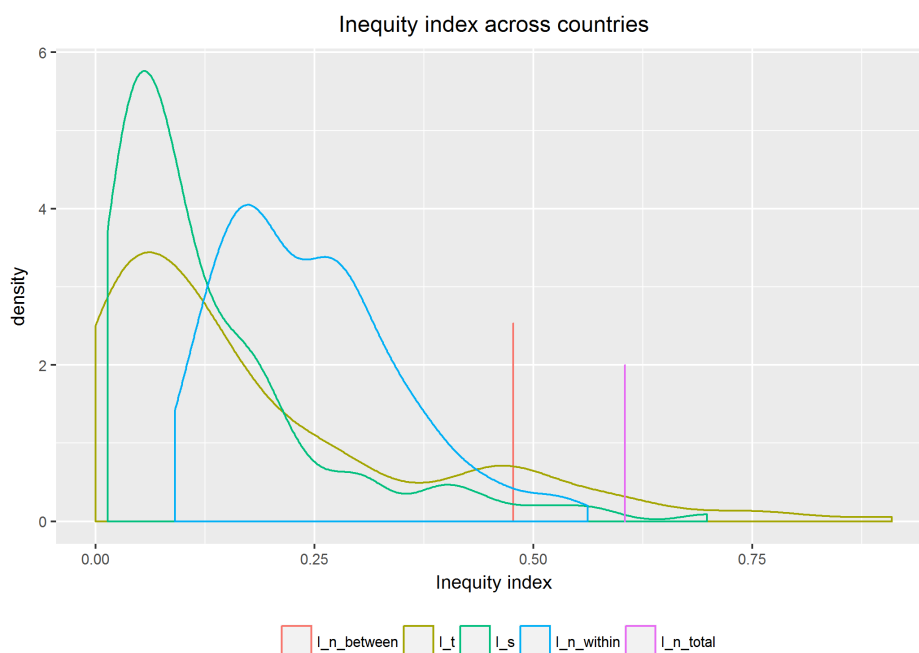


Figure B.1: Kernel density of inequity index across countries in the three dimensions, $\varphi_i = 1.0$, $\delta = 0.01$

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