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Improving Value-at-Risk Estimation from the Normal EGARCH Model

Mahsa Gorji¹, Rasoul Sajjad²

ABSTRACT

Returns in financial assets display consistent excess kurtosis and skewness, implying the presence of large fluctuations not forecasted by Gaussian models. This paper applies a resampling method based on the bootstrap and a bias-correction step to improve Value-at-Risk (VaR) forecasting ability of the n-EGARCH (normal EGARCH) model and correct the VaR for both long and short positions. Our aim is to utilize the advantages of this model, but still use the bootstrap resampling method to accurate for the tendency of the model to miscalculate the VaR. Empirical results indicate that the bias-correction method can improve the n-GARCH and n-EGARCH VaR forecasts so much that the acquired VaR predictions are different from the proposed probability. Additionally, allowing asymmetry in the conditional variance using the EGARCH model with normal distribution instead of GARCH improves the performance of the bias-correction method in forecasting the VaR for almost all considered indices. Moreover, the bias-corrected n-EGARCH model performs better than the simple t-EGARCH model. Thus, it seems that this model can take account of both the asymmetry in the conditional variance and leptokurtosis in returns distribution. However, we find that the superiority of the bias-corrected n-EGARCH model over the t-EGARCH model is not completely confirmed for short positions based on the censored likelihood scoring rule.

KEY WORDS: Bootstrap; EGARCH; GARCH; Value-at-Risk

JEL Classification: C58, G32, C15

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1. Introduction

Since the Basle Committee (1995; 1996) began allowing banks to implement internal VaR models for calculating capital requirements, various methods have been proposed to achieve this purpose. However, the theoretical and computational complexity of these methods has also been raised. Furthermore, various methods have been suggested for modeling conditional variance, and a large number of candidate distributions have been considered for modeling empirical

features of the returns (Alexander, 2001; Bao, Lee, & Saltoğlu, 2007). Although more complex shapes of the tails have the potential preference of increased abilities to describe the VaR, they may lead to more uncertainty in the parameters and hence in the VaR estimate itself (Bams, Lehnert, & Wolff, 2005).

The simple method that can consider two characteristics of financial asset returns, namely time-varying volatility and excess kurtosis, is the GARCH model by Engle (1982) and Bollerslev (1986). Researchers beginning with Black (1976) have demonstrated that stock returns are negatively correlated with changes in return volatility. Nevertheless, symmetric models such

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as the GARCH model have difficulties in correctly modeling the tails of the returns distribution (Giot & Laurent, 2003) due to leverage effects. To overcome this shortcoming, various studies have proposed the inclusion different asymmetric terms in the conditional variance equation (Ding, Granger, & Engle, 1993; Engle & Ng, 1993; Glosten, Jagannathan, & Runkle, 1993). Nelson (1991) also proposed the exponential GARCH model, which was re-expressed by Bollerslev and Mikkelsen (1996).

Hartz, Mitnik and Paoletta (2006) have developed a resampling method based on the bootstrap and bias-correction for improving the VaR forecasting ability of the n-GARCH (normal GARCH) model. Their proposed method has improved the VaR forecasts of the n-GARCH model. Our main objective is to extend their study by allowing asymmetry in conditional variance. To this end, we apply the n-EGARCH in addition to the n-GARCH model to consider certain theoretical advantages of this model over the n-GARCH. However, Fama (1965) has demonstrated that return distributions of financial instruments are more leptokurtic than normal distributions and tend to exhibit "fat tails". In addition, empirical studies of high-frequency financial time series demonstrate that the tail behavior of GARCH models remains too short even with standardized Student's *t* innovations (Tsay, 2005). Therefore, we implement the bias-correction procedure based on the bootstrap method to remove the deficiencies of the n-GARCH and n-EGARCH models with respect to appropriate VaR forecasts. We try to conserve the simplicity of these methods, but still use the bootstrap resampling method to accurate for the tendency of these models to miscalculate the VaR. While Hartz et al. (2006), model the long VaR only, we try to extend their analysis by correcting the VaR for both long and short positions based on the aforementioned GARCH models (n-GARCH and n-EGARCH). Additionally, we evaluate models based on the censored likelihood (csl) scoring rule proposed by Diks, Panchenko, and Van Dijk (2011) in addition to the well-known Christoffersen's LR test. Empirical validation shows that considering asymmetry in conditional variance generally leads to improvements in accurately forecasting one-day-ahead VaR based on the bias-correction method for long and short positions, which is somewhat confirmed by the csl scoring rule.

The rest of the paper is organized as follows. Section 2 explains the methodology for estimating the distribution of the VaR point forecast and how it can be used to improve its accuracy. The empirical analysis and evaluation the performance of competing models in forecasting VaR are presented in Section 3. Finally, Section 4 concludes the paper.

2. Forecasting VaR

In general, for the set of equally spaced asset returns, r_t , $t = 1, \dots, T$, the class of ARMA(p, q)-EGARCH(r, s) models is given by

$$r_t = a_0 + \sum_{i=1}^p a_i r_{t-i} + \varepsilon_t + \sum_{j=1}^q b_j \varepsilon_{t-j}, \quad \varepsilon_t = z_t \sigma_t, \quad (1)$$

$$\ln \sigma_t^2 = c_0 + \sum_{i=1}^r c_i \ln \sigma_{t-i}^2 + \sum_{j=1}^s d_j \left[|z_{t-j}| - E|z_{t-j}| \right] + \sum_{j=1}^s \theta_j z_{t-j} \quad (2)$$

where d_j represents the magnitude effect that indicates how much volatility increases autonomously of the direction of the shock. The θ_j is the sign effect. Both z_t and $|z_t| - E|z_t|$ are zero-mean iid sequences with continuous distributions. $E|z_t|$ depends on the assumption made on the unconditional density of z_t . For the standard Gaussian random variable t , $E|z_t| = \sqrt{2/\pi}$. The log of the conditional variance guarantees that forecasts of the conditional variance are non-negative (Nelson, 1991). Some additional properties of the EGARCH model can be found in Nelson (1991).

For a given return series and a chosen model from the n-ARMA-EGARCH class in (1) and (2), the usual conditional VaR forecast is acquired by estimating the unknown parameter vector $\theta = (a_0, \dots, a_p, b_1, \dots, b_q, c_0, \dots, c_r, d_1, \dots, d_s, \theta_1, \dots, \theta_s)$ via conditional maximum likelihood.

We also define the set of estimated standardized residuals $\{\hat{z}_t\}$, as $\hat{z}_t = \hat{\varepsilon}_t / \hat{\sigma}_t$, with

$$\hat{\varepsilon}_t = r_t - \hat{a}_0 - \left(\sum_{i=1}^p \hat{a}_i r_{t-i} \right) - \left(\sum_{j=1}^q \hat{b}_j \hat{\varepsilon}_{t-j} \right), \quad (3)$$

$$\ln \hat{\sigma}_t^2 = \hat{c}_0 + \sum_{i=1}^r \hat{c}_i \ln \hat{\sigma}_{t-i}^2 + \sum_{j=1}^s \hat{d}_j \left[|\hat{z}_{t-j}| - E|\hat{z}_{t-j}| \right] + \sum_{j=1}^s \hat{\theta}_j \hat{z}_{t-j} \quad (4)$$

then the h -step-ahead (time horizon, $h \in \mathbb{Z}$) VaR forecast is obtained by

$$\hat{v}_\lambda = \hat{v}_\lambda(h, T) = \Phi^{-1}(\lambda; \hat{\mu}_{T+h}, \hat{\sigma}_{T+h}^2), \quad (5)$$

where $\Phi^{-1}(\lambda; \mu, \sigma^2)$, indicates the inverse cdf of the standard normal distribution with mean μ variance σ^2 and λ , a given probability level, and

$$\hat{\mu}_{T+h} = \hat{a}_0 + \sum_{i=1}^p \hat{a}_i r_{T+h-i} + \sum_{j=1}^q \hat{b}_j \hat{\varepsilon}_{T+h-j}, \quad (6)$$

$$\ln \hat{\sigma}_{T+h}^2 = \hat{c}_0 + \sum_{i=1}^r \hat{c}_i \ln \hat{\sigma}_{T+h-i}^2 + \sum_{j=1}^s \hat{d}_j \left[\left| \hat{z}_{T+h-j} \right| - E \left| z_{t-j} \right| \right] + \sum_{j=1}^s \hat{\theta}_j \hat{z}_{T+h-j} \quad (7)$$

The VaR for a short position is similarly computed where the same definition is used for the right tail of the distribution function, i.e., $1-\lambda$ substitutes for λ .

2.1. VaR forecast distribution

For implementing the bias-correction method we need to estimate VaR forecast distribution in addition to a VaR point estimator (\hat{v}_λ). Therefore, as Hartz et al. (2006) proposed, we apply the bootstrap method to this end. This method coincides with that described in Pascual, Romo and Ruiz (2006), Reeves (2005) and Trucíos and Hotta (2016). It is connected to the filtered historical simulation method proposed by Barone-Adesi, Giannopoulos and Vosper (1999; 2002) and the bootstrap methodology described by Dowd (2005).

Despite that the sampling distribution of the VaR point forecast is unknown and intangible, the bootstrap method provides the possibility for its approximation. Because, assuming the true data generating process is constant over time, the bias caused by the use of the inappropriate but simple n-GARCH model will display certain regularities and thus can be corrected based on a set of past VaR bootstrap distributions (Hartz et al., 2006). This is the assumption used in the bias-correction method described in Section 2.2.

For implementing B bootstrap iterations, the b th replication, $b = 1, \dots, B$, we should consider the following steps as Hartz et al. (2006) suggested:

Step 0: For a chosen set of values p, q, r, s (for which $p = r = s = 1$ and $q = 0$ is most common), obtain QML parameter vector estimate $\hat{\theta}$, estimated standardized residuals $\{\hat{z}_t\}$, then forecast VaR for a certain h (we consider $h=1$) by (5).

Step 1: Simulate the (b) th of B , n-ARMA(p, q)-EGARCH(r, s) time series, $\{r_t^{(b)}\}$, of length T , using the estimated parameter vector $\hat{\theta}$ and bootstrapped standardized residuals sequence $\{\hat{z}_t^{(b)}\}$. To eliminate the effect of initial values, we simulate $T+\ell$ series and then discard first ℓ observations ($\ell=T$).

Step 2: Obtain the QML parameter vector estimate $\hat{\theta}^{(b)}$ based on n-ARMA(p, q)-EGARCH(r, s) using the simulated time series $\{r_t^{(b)}\}$.

Step 3: Estimate a resampled VaR estimate, $\hat{v}_\lambda^{(b)}(h, T)$, using the *original series* $\{r_t\}$, and the bootstrap parameter vector estimate $\hat{\theta}^{(b)}$, via

$$\hat{\varepsilon}_t^{(b)} = r_t - \hat{a}_0^{(b)} - \left(\sum_{i=1}^p \hat{a}_i^{(b)} r_{t-i} \right) - \left(\sum_{j=1}^q \hat{b}_j^{(b)} \hat{\varepsilon}_{t-j}^{(b)} \right), \quad t = 1, \dots, T,$$

$$\ln \hat{\sigma}_t^{2(b)} = \hat{c}_0^{(b)} + \sum_{i=1}^r \hat{c}_i^{(b)} \ln \hat{\sigma}_{t-i}^{2(b)} + \sum_{j=1}^s \hat{d}_j^{(b)} \left[\left| \hat{z}_{t-j}^{(b)} \right| - E \left| z_{t-j} \right| \right] + \sum_{j=1}^s \hat{\theta}_j^{(b)} \hat{z}_{t-j}^{(b)}$$

and to calculate bootstrapped VaR forecast

$$\hat{v}_\lambda^{(b)}(h, T) = \Phi^{-1} \left(\lambda, \hat{\mu}_{T+h}^{(b)}, \hat{\sigma}_{T+h}^{2(b)} \right),$$

where

$$\hat{\mu}_{T+h}^{(b)} = \hat{a}_0^{(b)} + \sum_{i=1}^p \hat{a}_i^{(b)} r_{T+h-i} + \sum_{j=1}^q \hat{b}_j^{(b)} \hat{\varepsilon}_{T+h-j}^{(b)},$$

$$\ln \hat{\sigma}_{T+h}^{2(b)} = \hat{c}_0^{(b)} + \sum_{i=1}^r \hat{c}_i^{(b)} \ln \hat{\sigma}_{T+h-i}^{2(b)} + \sum_{j=1}^s \hat{d}_j^{(b)} \left[\left| \hat{z}_{T+h-j}^{(b)} \right| - E \left| z_{t-j} \right| \right] + \sum_{j=1}^s \hat{\theta}_j^{(b)} \hat{z}_{T+h-j}^{(b)},$$

are calculated using conditional expectations for unobserved values.

The primary VaR prediction, $\hat{v}_\lambda^{(0)}$, and the B bootstrapped VaR predictions, $\hat{v}_\lambda^{(b)}$, $b=1, \dots, B$, can be used to construct an empirical distribution function of VaR estimation. This function is given by

$$\hat{F}_k(a; \lambda, h, T) = \frac{1}{B+1} \sum_{b=0}^B \psi_{(-\infty, a)}(\hat{v}_\lambda^{(b)}(h, T)), \quad (8)$$

where $\psi(\cdot)$ is the indicator function. This function could be used to construct a bootstrap confidence interval for the VaR; for instance Christoffersen and Gonçalves (2005) propose to use the bootstrap for constructing confidence intervals of a conditional VaR estimator. Nieto and Ruiz (2010) suggest a new bootstrap procedure to obtain prediction intervals of future VaR and Expected Shortfall (ES), as well. We use the bootstrap method as described in the previous section to obtain a more accurate VaR based on n-GARCH models (n-GARCH and n-EGARCH) as much as possible.

2.2. Bias-correcting VaR forecasts

The corrective method uses the VaR distributions (approximated based on the illustrated bootstrapped method) and an objective function, which is described by definition of the VaR. The VaR is defined to be the worst possible loss from an investment over a target horizon and for a given probability level (Crouhy, Galai, & Mark, 2001). Therefore, the evident criterion to construct this function is the observed frequency of exceptions, or past realized returns that are less (higher) than or equal to the predicted VaR for long (short) positions.

For a given probability level λ , the observed frequency of exceptions, denoted \hat{f} , for a set of successive VaR predictions for long positions obtained from the usual n-GARCH models between times, say, τ_1 and τ_2 , and the equivalent realized returns is given by

$$\hat{f} = \frac{1}{\tau_2 - \tau_1 + 1} \sum_{t=\tau_1}^{\tau_2} \psi_{(-\infty, \hat{v}(t)]}(r_{t+h}), \quad \hat{v}(t) = \hat{v}_\lambda(h, t) \quad (9)$$

The observed frequency of exceptions for a short position is similarly computed where the same definition is used for the right tail of the distribution function, i.e., $\psi_{(\hat{v}(t), +\infty)}$ substitutes for $\psi_{(-\infty, \hat{v}(t)]}$.

The observed frequency of exceptions is less (higher) than the real risk level λ , if the VaR forecasts calculated by the n-GARCH models tend to overestimate (underestimate) the risk. Therefore, the logic behind the bias-correction method is to find the quantile of past VaR distributions which causes observed frequency of exceptions conforms to (as close as possible) a given risk level.

Let $\{\hat{v}_\lambda^{[b]}(h, t)\}_{b=0}^B$ be the sorted VaR predictions, with length $(B+1)$ produced by the resampling algorithm with the original n-GARCH models forecast, $\hat{v}_\lambda^{(0)}(h, t)$, added, i.e., $\hat{v}_\lambda^{[b]}(h, t) \leq \hat{v}_\lambda^{[b+1]}(h, t)$ for $b = 0, \dots, B-1$. Calculating the correct quantile of the VaR distribution \hat{F}_k is equivalent to find the largest index b for the long VaR (the smallest for the short VaR), denoted b^* , such that for the conforming series $\{\hat{v}_\lambda^{[b^*]}(h, t)\}_{t=\tau_1}^{\tau_2}$, the observed frequency of exceptions is less than or equal to the given risk level. Therefore, we need a certain number of past VaR distribution predictions that precede the actual VaR prediction of interest. As Hartz et al. (2006) have described, we consider a moving window procedure for finding the proper quantile of the VaR distribution produced by the resampling al-

gorithm. Common to all models is the construction of a moving window in which the model is recalculated for each window period, removing the first observations and adding new ones as the window advances. L is the fixed number of preceding VaR forecast distributions that we consider for finding the proper quantile for the h -step-ahead prediction for the downfall risk made at time T . Accordingly, the optimal quantile is described as

$$b^* = \max\{b : b \in \{0, 1, \dots, B\}\} \text{ s.t. } \frac{1}{L} \sum_{t=T-L+1}^{T-h} \psi_{(-\infty, \hat{v}_\lambda^{[b]}(h, t))}(r_{t+h}) \leq \lambda. \quad (10)$$

where b^* denotes the greatest quantile of the last L feasible VaR distributions for which the conforming series of VaR predictions, $\{\hat{v}_\lambda^{[b^*]}(h, t)\}$, leads to an observed frequency of VaR exceptions that is equal to (or just smaller) than the given risk level. The b^* for a short position is similarly computed where it determines the smallest quantile of the VaR distributions and the $\psi_{(\hat{v}_\lambda^{[b]}(h, t), +\infty)}$ substitutes for $\psi_{(-\infty, \hat{v}_\lambda^{[b]}(h, t))}$.

As the data generating process is not constant over time, calculating an optimal L would be reliable only for particular segment of a particular data set (Hartz et al., 2006). Thus, following the suggestion of Hartz et al. (2006) concerning the choice of L , we assume two sizes of the moving window (L) to examine whether this criterion could affect the results of our study.

We consider two values, $L=250$ and 500 , (one and two years of trading data, respectively) to perform the bias-correction method.

3. Empirical analysis

We consider the daily percentage log-return series, defined by $r_t = (\ln p_t - \ln p_{t-1}) \times 100$, where p_t is the closing price index on day t . The data set analyzed in this paper comprises daily observations on three major stock market indices' returns; namely, the FTSE Composite, NIKKEI 225 and CAC (hereafter the FTSE, NIKKEI and CAC). The starting date for the series are August 27, 1999 for the FTSE; April 09, 1999 for the NIKKEI; and November 05, 1999 for the CAC.

The usual descriptive statistics of the data are given in Table 1. The moments of the stock index returns are shown along with the results of an aggregate autocorrelation (Ljung-Box) test for returns and their squares. As seen, for the FTSE and NIKKEI the skewness is significant and negative, indicating a possible leverage

Table 1. Moments of the FTSE, NIKKEI and CAC returns along with aggregate autocorrelation test results.

	FTSE	NIKKEI	CAC
Mean	0.0008	-0.004	-0.007
Minimum	-9.26	-12.11	-9.47
Maximum	9.38	13.23	10.59
Std. Dev.	1.276	1.565	1.550
Skewness	-0.126	-0.419	0.028
Kurtosis	8.715	9.361	7.540
Ljung-Box test for returns			
Q-stat(12)	78.81 {0}	13.85 {0.31}	56.95 {0}
Q-stat(24)	102.2 {0}	44.34 {0.007}	71.28 {0}
Q-stat(36)	153.52 {0}	61.00 {0.006}	120.97 {0}
Ljung-Box test for squared returns			
Q-stat(12)	2797 {0}	3051 {0}	1988 {0}
Q-stat(24)	4340 {0}	3860 {0}	3138 {0}
Q-stat(36)	5351 {0}	4114 {0}	3907 {0}

Note:

p -values in curly braces.

Q-stat (q) denotes a modified Ljung-Box type statistic.

effect in data, but for the CAC the skewness is positive, and the kurtosis is significantly higher than that of a Gaussian distribution (excess kurtosis), indicating fat-tailed returns for all series. This suggests the need for a fat-tailed distribution, for example Student's t , to describe the returns' conditional distribution. Furthermore, imposing an asymmetric parameter in the conditional variance equation enables us to capture possible leverage effects in the data. In addition, the large Q-statistics up to 12, 24 and 36 orders strongly reject

the null hypothesis of no serial correlation in both returns and squared returns for the FTSE and CAC but only in returns for the NIKKEI index.

The estimation period is set to $T = 1000$, which corresponds to about four years of daily returns data for estimating the parameters of models. $B = 1000$ bootstrap replications are used for calculating the VaR forecast distributions.

For each series, we use $p = 2000$ out-of-sample values, and the last forecast is made for July 16, 2013 for

each series. The models are used to estimate one-day-ahead VaR ($h=1$) of both long and short trading positions (left and right tails of returns distribution) with different probabilities (at different tail quantiles): 0.5%, 1%, 5%, 95%, 99% and 99.5%.

For implementing the bias-correction method, we use the two values $L = 250$ and 500 . So, an additional number of $L=500$ VaR forecast distributions resulting in a total of $P+L=2500$ VaR forecast distributions are considered.

In addition to the n-AR(1)-GARCH(1,1) and n-AR(1)-EGARCH(1,1) models, we consider the AR(1)-EGARCH(1,1) with the Generalized Error Distribution (GED) and t-AR(1)-EGARCH(1,1) (with Student's t innovations) to assess the performance of the bias-correction method.¹ The original form of this model is used for the comparison due to its ability to consider the asymmetry in the conditional variance and the excess kurtosis relative to the normal distribution. In this model, the conditional variance obtained is similar to the n-EGARCH model (i.e., Equation (2)), while the $E|z_t|$ for a Student's t distribution with $\nu > 2$ degrees of freedom is given by

$$E(|z_t|) = \frac{2\sqrt{\nu-2}\Gamma((\nu+1)/2)}{(\nu-1)\Gamma(\nu/2)\sqrt{\pi}}, \quad (11)$$

where $\Gamma(x)$ is the usual gamma function.

The corresponding VaR prediction for the t-AR(1)-EGARCH(1,1) model is given by

$$\hat{v}_\lambda = \hat{v}_\lambda(h, T) = \hat{\mu}_{T+h} + \hat{\sigma}_{T+h} F_t^{-1}(\lambda; \hat{\nu}), \quad (12)$$

where $F_t^{-1}(\lambda; \nu)$ is the inverse of the cumulative distribution function of the t -distribution with ν degrees of freedom and a standardized variance of one.

In the remainder of the paper, we first examine the VaR forecast distributions generated by the bootstrap algorithm. Then, we compare the VaR estimations of the competing models.

3.1. VaR forecast distribution

Figures 1–3 present the first VaR forecast distributions included in our analysis for the FTSE, NIKKEI and CAC for both long and short trading positions. The upper part of the figures refers to the VaR forecast distribution for the n-GARCH model, while the lower part of the figures refers to the VaR forecast

distribution for the n-EGARCH model. Additionally, the vertical lines exhibit the usual n-GARCH and n-EGARCH VaR predictions, \hat{v}^0 .

As seen from figures 1, 2 and 3, the risk in VaR forecasts for both long and short trading positions increases for smaller probability levels. The usual VaR predictions do not exactly coincide with the mode of the VaR forecast distributions. For instance, for the first VaR forecasts made for all series, the usual n-EGARCH VaR forecasts are smaller than the mode of the VaR forecast distributions for both long and short trading positions. On the other hand, the first VaR forecasts for all series from the usual n-GARCH model for short trading position are greater than the mode of the corresponding VaR forecast distributions.

3.2. Performance of Models in Forecasting VaR

For assessing the accuracy of competing methods, we define the Boolean sequence as

$$I_{t+1} = \psi_{(-\infty, \hat{v}_\lambda(1,t))}(r_{t+1}), \quad (13)$$

where $\hat{v}_\lambda(1,t)$ is one-step-ahead VaR predictions for λ , a given probability level, and r_{t+1} is the observed return. Christoffersen (1998) showed that evaluating interval forecasts can be reduced to examining whether the Boolean sequence, $\{I_t\}_{t=1}^T$, satisfies the unconditional coverage (UC) and independence (IND) properties.

Let $T_1 = \sum_{t=1}^T I_{t+1}$, the number of violations, and the failure rate be expressed as $\hat{f} = T^{-1} \sum_{t=1}^T I_{t+1} = T_1 / T$. For

testing the UC, the proper likelihood ratio statistic, also called the Kupiec (1995) LR unconditional test, (under the null hypothesis that, $\hat{f} = \lambda$) is

$$LR_{uc} = 2 \ln \left[(1 - \hat{f})^{T-T_1} (\hat{f})^{T_1} \right] - 2 \ln \left[(1 - \lambda)^{T-T_1} (\lambda)^{T_1} \right] \sim \chi_1^2,$$

$$P_{un} = 1 - F_{\chi_1^2}(LR_{un}), \quad (14)$$

For P_{un} below the desired significance level, the null hypothesis is rejected.

The LR_{uc} test adapted from Christoffersen (1998) is used to test the conditional coverage.

We compare models based on the p -values, and models with higher p -values are preferred.

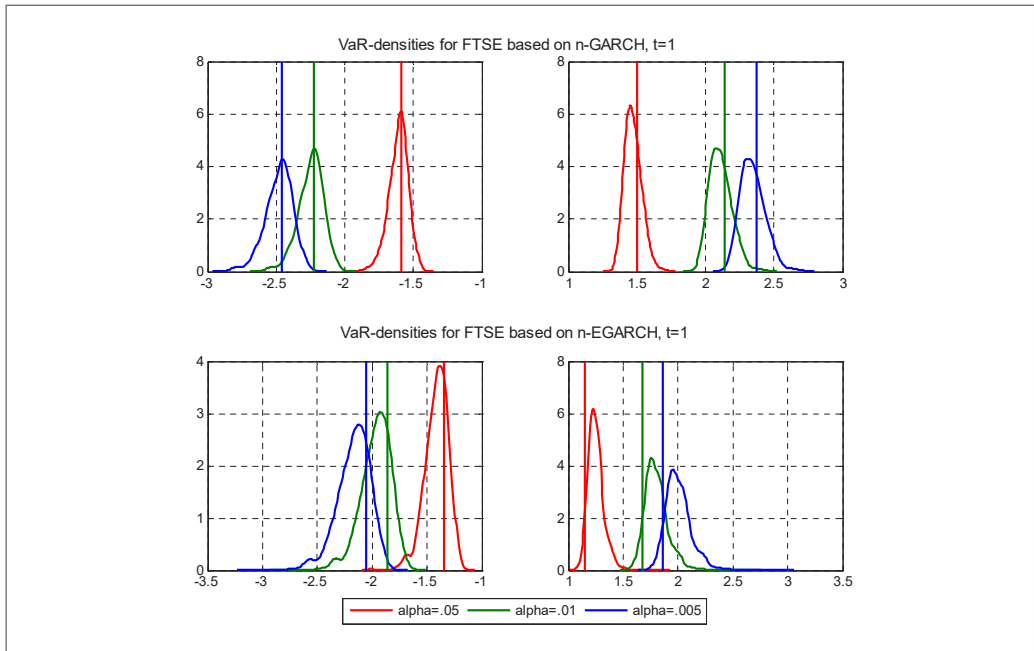


Figure 1. Vertical lines refer to the usual n-GARCH and n-EGARCH VaR predictions, \hat{v}^0 . VaR forecast distributions for the first forecast - FTSE.

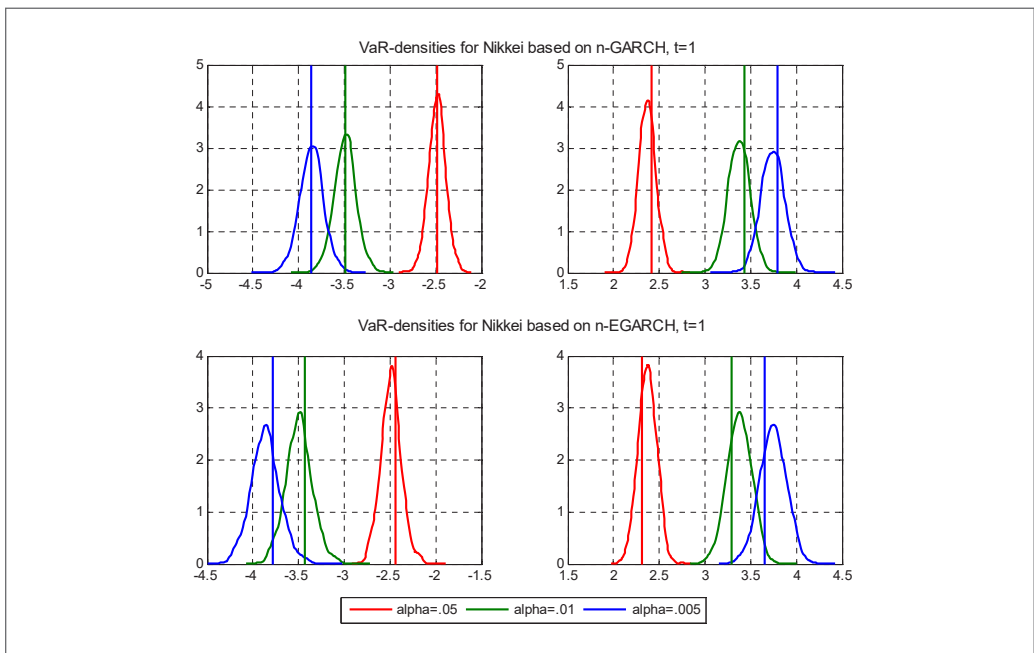


Figure 2. Vertical lines refer to the usual n-GARCH and n-EGARCH VaR predictions, \hat{v}^0 . VaR forecast distributions for the first forecast - NIKKEI.

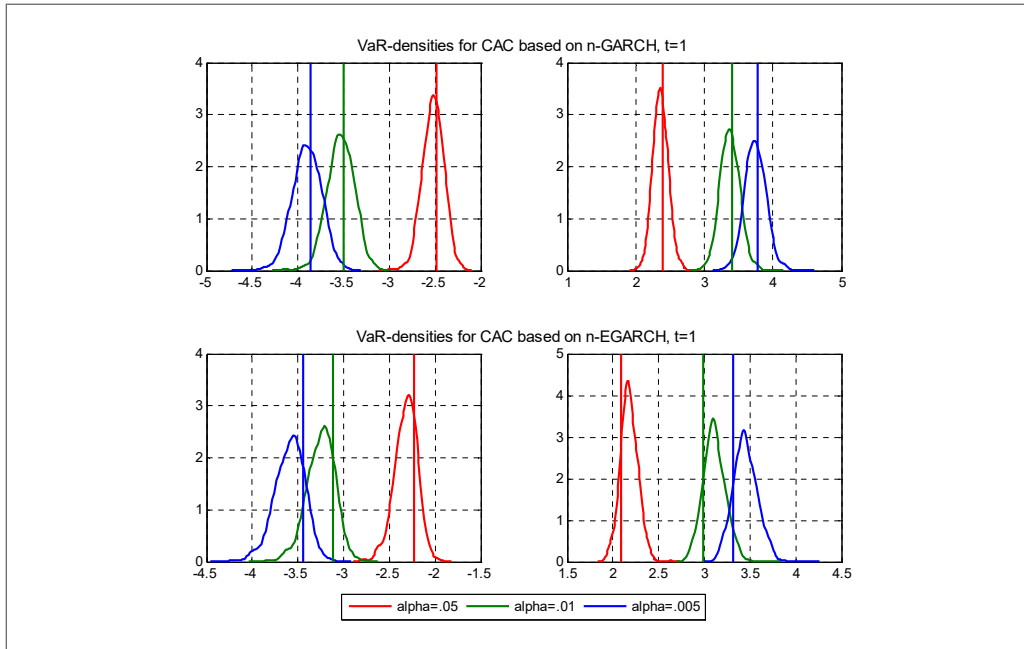


Figure 3. Vertical lines refer to the usual n-GARCH and n-EGARCH VaR predictions, \hat{r}^0 . VaR forecast distributions for the first forecast - CAC.

In addition to unconditional and conditional tests we consider a csl scoring rule suggested by Diks et al. (2011) to assess the performance of the bias-correction method. They have shown that this scoring rule is handy as the main interest lies in comparing the accuracy of density forecasts for a specific region, such as the left tail in financial risk management. While they have considered long VaR only, we also evaluate our models for both long and short positions. The censored likelihood (csl) score function is given by

$$S^{\text{csl}}(\hat{a}_t; r_{t+1}) = w_t(r_{t+1}) \log \hat{a}_t(r_{t+1}) + (1 - w_t(r_{t+1})) \log(1 - \int w_t(s) \hat{a}_t(s) ds), \quad (15)$$

$$d_{t+1}^{\text{csl}} = S^{\text{csl}}(\hat{a}_t; r_{t+1}) - S^{\text{csl}}(\hat{b}_t; r_{t+1}),$$

where \hat{a}_t and \hat{b}_t are two competing density forecasts and $w_t(r)$ is a threshold weight function $w_t(r) = I(r \leq \hat{r}_t^a)$. Additionally, corresponding realizations of the variable r_{t+1} are accessible for $t = n, n+1, \dots, T-1$. Then, \hat{a}_t and \hat{b}_t are comparable based on their average scores

difference d , by testing if their difference is statistically significant².

In the following, we first concentrate on the measures LR_{cc} , LR_{ind} and LR_{uc} . The results for the seven competing models are given in Tables 2–4 for the FTSE, NIKKEI and CAC.

We first examine the capability of the bias-correction procedure to improve the VaR forecasts of the usual n-GARCH and n-EGARCH models. Comparing the forecasts from the usual models with the bias-corrected forecasts, we find that the P_{uc} -values for at least five out of the 6 specified probabilities for $L = 500$ are superior (larger) to those for the usual n-GARCH model, while for the n-EGARCH model the p -values for at least four out of the 6 cases for both window lengths are superior to those for the usual n-EGARCH model.

We found no P_{uc} -values below the 5% significance level for the bias-corrected VaR predictions of the n-EGARCH model with $L = 500$ for all series except the NIKKEI at 99%, while for the usual VaR forecasts, we

Table 2. Unconditional coverage, independence, conditional coverage—FTSE.

VaR models	\hat{f}	p -values			\hat{f}	p -values			
		P_{cc}	P_{ind}	P_{un}		P_{cc}	P_{ind}	P_{un}	$*/**/*$
VaR 5%					VaR 95%				
\hat{f}_N	0.0645	0.0133	0.4802	0.0043	0.0420	0.0600	0.0952	0.0918	5/3/3
\hat{f}_{N-500}^*	0.0545	0.4343	0.3598	0.3624	0.0495	0.0794	0.0245	0.9182	2/1/0
\hat{f}_{EGARCH}	0.0605	0.1052	0.7070	0.0367	0.0435	0.0803	0.0742	0.1730	4/4/2
$\hat{f}_{EGARCH-500}^*$	0.0505	0.6627	0.3674	0.9184	0.0520	0.1870	0.0742	0.6834	0/0/0
$\hat{f}_{t-EGARCH}$	0.0650	0.0120	0.6866	0.0032	0.0450	0.0951	0.0572	0.2970	5/5/4
VaR 1%					VaR 99%				
\hat{f}_N	0.0210	<0.0001	0.2852	<0.0001	0.0070	0.3258	0.6456	0.1541	
\hat{f}_{N-500}^*	0.0145	0.0321	0.0699	0.0581	0.0130	0.3054	0.3990	0.1974	
\hat{f}_{EGARCH}	0.0205	0.0001	0.7898	<0.0001	0.0075	0.4440	0.6229	0.2397	
$\hat{f}_{EGARCH-500}^*$	0.0135	0.2235	0.3813	0.1353	0.0120	0.5049	0.4357	0.3835	
$\hat{f}_{t-EGARCH}$	0.0210	<0.0001	0.8094	<0.0001	0.0050	0.0429	0.7395	0.0129	
VaR 0.5%					VaR 99.5%				
\hat{f}_N	0.0155	<0.0001	0.4905	<0.0001	0.0025	0.2111	0.8623	0.0792	
\hat{f}_{N-500}^*	0.0095	0.0331	0.5356	0.0112	0.0070	0.4403	0.6456	0.2319	
\hat{f}_{EGARCH}	0.0115	0.0015	0.4548	0.0004	0.0020	0.0950	0.8874	0.0304	
$\hat{f}_{EGARCH-500}^*$	0.0060	0.7654	0.6920	0.5388	0.0045	0.9074	0.7636	0.7471	
$\hat{f}_{t-EGARCH}$	0.0120	0.0006	0.4357	0.0001	0.0010	0.0082	0.9382	0.0020	

Note: \hat{f}_N : observed downfall probability for n-AR(1)-GARCH(1,1) model; \hat{f}_{EGARCH} : observed downfall probability for n-AR(1)-EGARCH(1,1) model; $\hat{f}_{t-EGARCH}$: observed downfall probability for t-AR(1)-EGARCH(1,1) model; \hat{f}_{N-500}^* ($\hat{f}_{EGARCH-500}^*$): observed downfall probability for calibrated n-AR(1)-GARCH(1,1) (n-AR(1)-EGARCH(1,1)) model with L=500; P_{un} (P_{ind} , P_{cc}): probability of observing a sample with higher unconditional coverage test statistic (independence test statistic, conditional coverage test statistic). The results for both long and short positions are reported in the left and right panel of the table, respectively. */**/***/***: Number of *p*-values for unconditional coverage are smaller than 0.10/0.05/0.01.

have at least 2 *p*-values below the 1% level and one *p*-value below the 0.01%.

The results show that the usual VaR models based on normal innovations have difficulties with modeling large returns. In particular, the usual normal GARCH models consistently underestimate the return (risk)

for all series for long positions. In other words, failure numbers are much greater than expected at a given quantile in the case of normal innovations.

Regarding independence of the VaR exceptions over time, for both the usual n-GARCH and n-EGARCH VaR forecast violations or those from the bias-correc-

Table 3. Unconditional coverage, independence, conditional coverage—NIKKEI.

VaR models	\hat{f}	<i>p</i> -values			\hat{f}	<i>p</i> -values			
		P_{cc}	P_{ind}	P_{un}		P_{cc}	P_{ind}	P_{un}	*/**/**
VaR 5%					VaR 95%				
\hat{f}_N	0.0605	0.1024	0.6599	0.0367	0.0430	0.2589	0.4619	0.1415	5/5/3
\hat{f}_{N-500}^*	0.0540	0.4759	0.3629	0.4175	0.0570	0.2923	0.4874	0.1596	0/0/0
\hat{f}_{EGARCH}	0.0595	0.1415	0.5720	0.0581	0.0550	0.2789	0.2157	0.3123	4/2/2
$\hat{f}_{EGARCH-500}^*$	0.0555	0.3158	0.3001	0.2671	0.0595	0.0537	0.1329	0.0581	2/1/0
$\hat{f}_{t-EGARCH}$	0.0620	0.0552	0.7064	0.0174	0.0555	0.1617	0.1204	0.2671	4/3/1
VaR 1%					VaR 99%				
\hat{f}_N	0.0160	0.0123	0.1036	0.0131	0.0045	0.0205	0.7636	0.0056	
\hat{f}_{N-500}^*	0.0125	0.3355	0.3138	0.2794	0.0130	0.3054	0.3990	0.1974	
\hat{f}_{EGARCH}	0.0220	<0.0001	0.3342	<0.0001	0.0070	0.3258	0.6456	0.1541	
$\hat{f}_{EGARCH-500}^*$	0.0130	0.2771	0.3413	0.1974	0.0150	0.0242	0.0801	0.0364	
$\hat{f}_{t-EGARCH}$	0.0180	0.0049	0.6493	0.0012	0.0055	0.0812	0.7156	0.0270	
VaR 0.5%					VaR 99.5%				
\hat{f}_N	0.0100	0.0087	0.1918	0.0053	0.0010	0.0082	0.9382	0.0020	
\hat{f}_{N-500}^*	0.0060	0.1392	0.0590	0.5388	0.0065	0.6037	0.6686	0.3634	
\hat{f}_{EGARCH}	0.0110	0.0023	0.2373	0.0010	0.0025	0.2111	0.8623	0.0792	
$\hat{f}_{EGARCH-500}^*$	0.0050	0.1172	0.0384	1.0000	0.0070	0.4403	0.6456	0.2319	
$\hat{f}_{t-EGARCH}$	0.0075	0.0866	0.0993	0.1401	0.0025	0.2111	0.8623	0.0792	

Note: See the legend of Table 2 for explanations.

tion method, independence cannot be rejected; in other words, the occurrence of violations for VaR forecasts have no systematic pattern.

Considering the conditional coverage test statistic, we see that, for all series, the *p*-values for at least five out of the six specified probabilities for *L* = 500 are superior to those for the usual n-GARCH model, while for the n-EGARCH model the *p*-values for at least four out of the six cases for both window lengths are supe-

rior to those for the usual n-EGARCH model. Summarizing the results for the conditional coverage, the bias-correction method can improve the n-GARCH and n-EGARCH VaR forecasts so much that the VaR predictions acquired are different from the proposed probability.

Comparing the unconditional coverage results for the bias-correction method based on the n-EGARCH model with those based on the n-GARCH model, we

Table 4. Unconditional coverage, independence, conditional coverage—CAC.

VaR models	\hat{f}	p -values			\hat{f}	p -values			
		P_{cc}	P_{ind}	P_{un}		P_{cc}	P_{ind}	P_{un}	*/**/***
VaR 5%					VaR 95%				
\hat{f}_N	0.0760	<0.0001	0.5474	<0.0001	0.0400	0.0335	0.1306	0.0338	4/4/3
\hat{f}_{N-500}^*	0.0575	0.2389	0.4389	0.1324	0.0520	0.0469	0.0147	0.6834	1/0/0
\hat{f}_{EGARCH}	0.0715	0.0001	0.4849	<0.0001	0.0450	0.0951	0.0572	0.2970	3/3/3
$\hat{f}_{EGARCH-500}^*$	0.0520	0.4741	0.2495	0.6834	0.0485	0.3093	0.1335	0.7571	0/0/0
$\hat{f}_{t-EGARCH}$	0.0750	<0.0001	0.3454	<0.0001	0.0455	0.0981	0.0523	0.3487	3/3/3
VaR 1%					VaR 99%				
\hat{f}_N	0.0200	0.0001	0.1957	<0.0001	0.0095	0.8045	0.5356	0.8207	
\hat{f}_{N-500}^*	0.0095	0.8045	0.5356	0.8207	0.0145	0.1067	0.3474	0.0581	
\hat{f}_{EGARCH}	0.0215	<0.0001	0.1643	<0.0001	0.0115	0.4289	0.2617	0.5102	
$\hat{f}_{EGARCH-500}^*$	0.0135	0.2235	0.3813	0.1353	0.0120	0.3883	0.2873	0.3835	
$\hat{f}_{t-EGARCH}$	0.0185	0.0014	0.2313	0.0006	0.0080	0.1875	0.1153	0.3517	
VaR 0.5%					VaR 99.5%				
\hat{f}_N	0.0100	0.0166	0.5148	0.0053	0.0060	0.7654	0.6920	0.5388	
\hat{f}_{N-500}^*	0.0055	0.8913	0.7156	0.7551	0.0060	0.7654	0.6920	0.5388	
\hat{f}_{EGARCH}	0.0145	<0.0001	0.3474	<0.0001	0.0055	0.8913	0.7156	0.7551	
$\hat{f}_{EGARCH-500}^*$	0.0065	0.6037	0.6686	0.3634	0.0050	0.9462	0.7395	1.0000	
$\hat{f}_{t-EGARCH}$	0.0110	0.0036	0.4744	0.0010	0.0050	0.9462	0.7395	1.0000	

Note: See the legend of Table 2 for explanations.

conclude that the n-EGARCH model outperforms the n-GARCH model as for all but the NIKKEI index: the *p*-values for at least four out of the six specified probabilities are superior to those for the n-GARCH model with $L = 500$. Similar results hold for the conditional coverage test statistic for all series. Additionally, the bias-correction method based on the n-EGARCH model is the only model never rejected by any of the three performance tests of Christoffersen (1998) for all the

specified probabilities (even at 99.5% and 0.5% tails) and considered indices.

Consequently, the bias-correction method based on the n-EGARCH model leads to an acceptable performance for out-of-sample VaR prediction. In other words, it seems that allowing for an asymmetric response of the conditional variance to positive and negative shocks generally yields improvement in the VaR performance of the bias-correction method. Therefore,

Table 5. Average score differences and tests of equal predictive accuracy for the censored likelihood (csl).

	FTSE	NIKKEI	CAC	FTSE	NIKKEI	CAC
	VaR 5%			VaR 95%		
$d_{N, 500}$	-0.0507	-0.0401	-0.0720	-0.1252	-0.2692	-0.1791
Test stat.	-5.8568	-4.4134	-0.0001	-0.0023	-0.0004	-0.0018
$d_{EGARCH, 500}$	-0.0569	-0.0181	-0.0793	-0.0179	-0.1373	-0.0395
Test stat.	-6.4187	-4.6009	-0.0001	-4.2077	-0.0002	-0.0002
$d_{N,EGARCH-500}$	-0.0007	0.0204	-0.0059	-0.0539	-0.0976	-0.1273
Test stat.	-8.2003	2.6194	-8.8945	-6.0130	-5.2386	-8.6942
$d_{EGARCH-1,500}$	-0.0259	0.0283	-0.0540	0.0519	-0.1897	-0.0572
Test stat.	-2.3995	1.8529	-6.6502	8.7729	-0.0001	-2.6532
	VaR 1%			VaR 99%		
$d_{N, 500}$	-0.0478	-0.0268	-0.0521	-0.4432	-0.6463	-0.3534
Test stat.	-5.7177	-5.3513	-0.0001	-0.0010	-0.0005	-0.0004
$d_{EGARCH, 500}$	-0.0572	-0.0057	-0.0392	-0.2748	-0.5455	-0.1229
Test stat.	-3.9568	-3.9915	-0.0001	-0.0007	-0.0002	-0.0002
$d_{N,EGARCH-500}$	-0.0100	0.0031	0.0270	-0.0688	-0.2125	-0.2187
Test stat.	-1.3473	1.5694	7.7423	-2.0162	-2.6433	-4.2149
$d_{EGARCH-1,500}$	-0.0272	-0.0002	-0.0046	0.4595	-0.1076	0.3828
Test stat.	-5.9952	-3.5245	-1.2083	2.7004	-2.0393	5.2561
	VaR 0.5%			VaR 99.5%		
$d_{N, 500}$	-0.0358	-0.0386	-0.0269	-0.5041	-0.7674	-0.1196
Test stat.	-0.0002	-7.3189	-0.0002	-0.0005	-0.0004	-4.7315
$d_{EGARCH, 500}$	-0.0446	-0.0066	-0.0457	-0.3150	-0.5415	0.4784
Test stat.	-6.0635	-4.0795	-0.0001	-0.0004	-0.0003	0.0001
$d_{N,EGARCH-500}$	-0.0244	0.0019	0.0079	-0.0574	-0.1053	0.0808
Test stat.	-3.4634	1.6062	5.2479	-1.3984	-2.5569	8.9946
$d_{EGARCH-1,500}$	-0.0249	0.0062	-0.0116	0.7539	0.1648	1.2717
Test stat.	-8.1588	1.0638	-2.7084	2.9685	1.3557	5.5332

Note: $d_{N, 500}$ ($d_{EGARCH, 500}$): average score difference based on n-AR(1)-GARCH(1,1) (n-AR(1)-EGARCH(1,1)) model relative to calibrated n-AR(1)-GARCH(1,1) (n-AR(1)-EGARCH(1,1)) model with L=500; $d_{N,EGARCH-500}$: average score difference for calibrated n-AR(1)-GARCH(1,1) relative to calibrated n-AR(1)-EGARCH(1,1) model with L=500; $d_{EGARCH-1,500}$: average score difference for t-AR(1)-EGARCH(1,1) model relative to calibrated n-AR(1)-EGARCH(1,1) model with L=500. Additionally, the corresponding test statistics are shown. The results for both long and short positions are reported in the left and right panel of the table, respectively.

this probably tells us that considering asymmetry in conditional variance is crucial in determining a proper value of VaR.

The results slightly prefer the longer window of length $L = 500$, with a higher number of greater p -values and a smaller number of significant test statistics. Because the results hold for all series, we can say that the method is not openly sensitive to the choice of L . However, Hartz et al. (2006) found that the results slightly prefer the shorter window of length $L = 250$.

With respect to the results of the t-EGARCH model for the LR_{uc} test, we see that the t-EGARCH VaR forecasts for only one specified probability for the NIKKEI index are superior to the bias-corrected n-EGARCH VaR forecasts, while they are never better for the FTSE and CAC. Similar results hold for the conditional coverage test statistic for all series while the independence test statistics indicate no systematic problems. According to empirical results, the bias-correction method based on the n-EGARCH model performs better than the t-EGARCH model. Therefore, it seems that there is no need to consider a fat-tailed distribution to describe the returns' conditional distribution.

Table 5 presents the average score differences d with the accompanying tests of equal predictive accuracy for models relative to the bias-corrected n-GARCH and n-EGARCH models. The score difference d is computed by subtracting the score of the bias-corrected n-EGARCH from the score of n-EGARCH, bias-corrected n-GARCH, and t-EGARCH models, such that negative values of d indicate better predictive ability of the bias-corrected n-EGARCH model. Additionally, the score difference d for the n-GARCH and bias-corrected n-GARCH models is calculated and interpreted in a similar way. For all series, the csl scoring rule suggests superior or equal predictive ability of the bias-correction method in comparison with usual n-GARCH models. Comparing the results for the bias-correction method based on the n-EGARCH model with those based on the n-GARCH model, we conclude that the n-EGARCH model outperforms the n-GARCH model for both the FTSE and NIKKEI indexes (except the NIKKEI at 99%), but evidence is weaker when we consider the CAC index. On the other hand, the bias-correction method based on the n-EGARCH model performs better than the t-EGARCH model for the NIKKEI, while this is not true for short FTSE and CAC positions.

4. Conclusions

In this paper, we extend the study by Hartz et al. (2006) to take account of asymmetry in conditional variance and correct the VaR for both long and short positions. We focus on three extreme percentiles $\alpha = 0.5\%$, 1% and 5% in the empirical study. Our results are robust to the chosen bias-correction window length, with a slight preference for the longer window length of $L=500$ for the three real return series investigated.

Our findings support those found in the study by Hartz et al. (2006). They obtain similar results with the other data, finding that the bias-correction method based on the n-GARCH model performs better than the usual n-GARCH model. This is also confirmed by the csl scoring rule for both GARCH models which has not been considered by Hartz et al. (2006). Our empirical study shows that the bias-correction method based on the n-EGARCH model instead of n-GARCH leads to improvements in correctly forecasting one-day-ahead VaR for long and short positions of almost all real return series investigated based on the three performance tests of Christoffersen (1998), while the independence of the VaR violations is unaffected by this method. We found that the bias-corrected n-EGARCH model is the only model never rejected by any of the three performance tests for all the specified probabilities and real return series investigated. Overall, it seems that allowing for an asymmetric response of the conditional variance to positive and negative shocks yields an improvement in the VaR performance based on the bias-correction method in terms of the three performance tests, but the improvement is not highly confirmed based on the csl scoring rule.

Moreover, the bias-correction method based on the n-EGARCH model performs better than the t-EGARCH model based on three performance tests. This is also true even for the NIKKEI with the most skewness and kurtosis in our data investigated. Thus, it seems that the bias-corrected n-EGARCH model can take account of both the asymmetry in the conditional variance and leptokurtosis in return distribution. However, we observe that the superiority of this model is not maintained in terms of predictive power according to the csl scoring rule for short positions. Thus, further research is advisable to adjust the standard VaR predictions of the n-GARCH models based on csl scoring rule rather than the observed frequency

of violations. Additionally, future work could extend the methodology to other types of GARCH models such as the power (G)ARCH models of Brooks, Faff, McKenzie and Mitchell (2000) or several GARCH models considered by Loudon, Watt and Yadav (2000).

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Endnotes

- 1 For the sake of brevity, only the results for Student's *t* distribution are presented in this paper. The results for the GED, which are very similar to those for the Student's *t* distribution, are available from the authors upon request.
- 2 For a more technical definition of the method, please refer to Diks et al. (2011).

