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Finders, Keepers?

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Finders, Keepers?
Niko Jaakkola, Daniel Spiro, Arthur A. van Benthem
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Abstract

Natural-resource taxation and investment exhibit cycles in a vast number of countries, driving political turmoil and power shifts. Using a rational-expectations model, we show cycles result from governments’ inability to commit to future taxes and firms’ inability to credibly exit a country indefinitely. A government sets a low initial tax inducing high investments, which in turn prompts it to increase taxes next period. This induces low investment thus low future taxes, and so on. We investigate which factors reinforce cycles and present ways of avoiding them, and document cycles across many countries including detailed case studies of two Latin-American countries.

JEL-Codes: H250, Q350, Q380.

Keywords: resource taxation, tax cycles, limited commitment, expropriation.

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1 Introduction

Taxation of natural resources is a dominant source of government revenue in many countries. More than twenty resource-rich countries obtain three-quarters of their export revenues or half of their government revenues from oil and gas related activities (Venables 2010). The quest for obtaining the associated profits is consequently often the single most important public policy issue in these countries (Boadway and Keen 2010; Hogan and Sturzenegger 2010). These matters are often sufficiently salient to shift political sentiments in the population, to drive political platforms, to determine election outcomes, and to even cause coups or civil wars (Manzano and Monaldi 2008; Venables 2016).

This paper aims to explain a prominent feature of resource markets: cyclicality in resource taxation. Why do finders get to keep more of their discoveries in some periods than others? A stylized description of such cycles is that taxes start out low to induce high investment. Then there is typically a shift in political power, either democratically or not, to a party with a high-tax agenda. The government thus changes the current fiscal terms, either through tax increases or through direct expropriation, which deters investment. Subsequently there is another shift in political power or the incumbent party’s platform, now to politicians setting low taxes, which increases investment, and so on. Thus, resource taxation is a key driver of political turmoil, instability and shifts in power in resource-rich countries. To explain this cyclical tax and investment behavior, we develop the first model of time-consistent resource taxation under limited ability of governments to commit to resource tax rates and firms to commit to exiting. Thus, apart from our main contribution of explaining the empirically prevalent taxation cycles, we also fill an important gap in the resource-economics literature.

A clear illustration of recurring resource-taxation cycles is given by the history of oil and other hydrocarbons in Bolivia. Figure 1 (either panel) plots Bolivia’s effective net resource-income tax rate over the past century. Periods of low tax rates that stimulate investment and production are followed by extremely high taxes and expropriations, which subsequently required the government to offer a favorable fiscal regime to lure back investors. Bolivia has experienced three such cycles, with expropriations in 1937 (Standard Oil), 1969 (Gulf Oil) and 2004-2009 (several foreign companies) and low tax rates in between. Section 2 describes in more detail the history of taxation cycles in Bolivia as well as in Venezuela which expropriated its foreign investors in 1975 and 2007. In both cases, these expropriations were interspersed with long periods of relatively favorable fiscal terms. Bolivia and Venezuela are no exceptions. According to Chua (1995), many Latin American countries have gone through five or more privatization-nationalization cycles.

Other examples abound. Israel offered a low tax rate of 28% to gas exploration firms before the discovery of the large Leviathan gas field in 2010. In anticipation of surging production levels, the Israeli government increased taxes to 42% in 2014 (The Jerusalem Post 2014). After investment dropped, the government recently promised more favorable conditions (Sachs and Boersma 2015; Times of Israel 2015). Argentina’s oil sector has
seen a number of oscillations between low-tax, high-investment periods and nationalizations in the last century. In 2012, Argentina increased taxes and expropriated Repsol’s controlling stake in state-owned company YPF following the discovery of the large Vaca Muerta shale oil and gas field. Consequently, Repsol stopped investing in Argentinian oil in 2014 (Reuters, 2012, 2014). Very recently, a new government promised lower taxes leading international companies to plan on increasing FDI (Forbes, 2015; OilPrice, 2016). There have been two full cycles in Zambia’s copper industry. Low initial taxes and high investment were followed by expropriations under socialist president Kenneth Kaunda in 1969. FDI plummeted and stayed low until the late 1980s, when Zambia abandoned its socialist policies. President Frederick Chiluba, elected in 1991, privatized the copper sector which led to a new wave of FDI and sharply increased production in the 1990s and 2000s. In 2007, the government increased taxes (Chang, Hevia and Loayza, 2016). Peru has seen two cycles in oil and mining during the 20th century (Chua, 1995). Algeria has had two cycles in oil and gas since the 1980s (Oil and Gas Journal, 2000; The Economist, 2007; Reuters, 2013; Layachi, 2013). Chile has experienced two full cycles in copper since the 1960s (Kobrin, 1984; UNCTAD, 2007). Ecuador has seen a full cycle in oil, and Mongolia in copper, since the 1990s (Manzano and Monaldi, 2008; The Mongolist, 2013; Ernst & Young, 2015). Cycles have been documented for many other countries and sectors (e.g., tin, coffee, agriculture, and land development, see Chua (1995); Manzano and Monaldi (2008); Hajzler (2012)).

To explain such cycles we develop a rich yet tractable model with four key assumptions. The first is that governments cannot commit to tax rates for more than a few years. This realistic assumption can be motivated by political-economy considerations (Persson and Tabellini, 1994) or by basic principles of law. As William Blackstone commented on English law: “Statutes against the power of subsequent Parliaments are not binding” (Blackstone, 1765). The second assumption is that firms cannot commit to leaving a country for good following a change in the agreed fiscal terms. This describes large oil majors such as ConocoPhillips and Exxon who, for instance, reinvested in Venezuela after their investments were expropriated in the 1970s. It also describes situations where, if one firm leaves, another firm takes its place as observed, for instance, in Bolivia in the 1950s. The third assumption is that mines are long-lived, relative to the government’s period of commitment. It usually takes several decades from the time a firm starts exploring for resources until it makes a successful discovery, starts a full-scale mining operation, and finally exhausts the resource. This has the implication that old mines, discovered earlier, exist in parallel with newer mines. The fourth assumption is that governments cannot, or do not, differentiate tax rates between different mine vintages that exist in parallel (or, at least, not perfectly). We relax

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1 If governments could fully commit to future tax rates, the problem has an elegant theoretical solution: the first-best outcome would be achieved by auctioning off the exploration rights. This would induce firms to pay the total expected profits and explore efficiently thereafter. Limited commitment is a likely reason for the rare occurrence of pure auction systems for exploration and extraction rights.

2 In the context of natural resources, this principle has been applied in a recent ruling by the Supreme Court of Israel, denying the government the right to tie its own hands with respect to future tax changes for gas companies. The motivation was that a commitment “that binds the government to [...] no changes in legislation and opposing legislative initiatives for 10 years – cannot stand” (Reuters Africa, 2016).
Figure 1: Tax Cycles in Bolivia and (Panel a) Oil and Gas Production or (Panel b) Oil Price

Notes: Tax rate refers to the effective net resource-income tax rate per barrel of oil equivalent. Sources: Jemio (2008) and WoodMackenzie (2012) for tax rates; Jemio (2008), Klein and Peres-Cajías (2014) and International Energy Agency (2016) for production; BP (2015) for prices. Years in which assets are expropriated are coded as a 100% tax rate (though effectively tax rates could be higher than 100% in those years). Gaps in tax rates represent periods in which no foreign firms were producing. In some years, tax rates are plotted as a range instead of a single value, as different rates applied to different projects. Low end and high end ranges indicate the minimum and maximum tax rates (when available), which can differ depending on project characteristics.
this assumption in a model extension.

These assumptions imply that each government faces a trade-off: high taxes maximize profits from old mines but harm new investments and hence profits from new mines. Since mines are long-lived, firms naturally choose investment based both on current and expected future taxes. As a result, a rational government that is unable to commit, when choosing its current tax rate, has to consider the impact of today’s tax rate on all future taxes and on all future investment decisions by the firms.

The model predicts that, following an earlier large discovery, the government will set a high tax to ensure getting a large share of the bonanza. This in turn will inhibit new investments which lowers the future tax base. Hence, in the next period the government refocuses to encouraging new investment and therefore lowers the tax. These high new investments imply a large inelastic tax base in the period after and hence an increase in the tax, and so on.

The model thus predicts cycles in resource taxation and investment in line with the observations described earlier. While not modeled explicitly, the shifts in tax policy can be expected to take the form of either incumbent politicians changing their policy or by new politicians taking over. That is, as observed, resource taxation is a driver of political sentiments in society and our model can thus explain why resource-rich countries often are politically unstable.

The model yields a number of additional predictions. A more backloaded mining profile – i.e., a substantial share of the mining profits coming with a lag – means firms care a lot about the tax tomorrow. Hence, mining investment is insensitive to today’s taxes, implying they are set high. This of course happens in all periods, which implies a high tax level and limited investment throughout. This scenario would apply to projects with large lead times, such as drilling for oil and gas at deep offshore fields or in the Arctic. Conversely, the model predicts that quickly depleted discoveries, such as shale oil, should incur a low tax. The model also predicts that the cycles will be more pronounced in countries that are new exporters of resources. This is because, initially, when current mining activity is low, the government focuses on incentivizing investments by setting an extraordinarily low tax, implying a large increase in new mines and hence a strong revision in taxation later on.

We also offer predictions about the interaction of prices and investment in creating cycles. It is well established, and of course very intuitive, that exogenous price shocks can lead governments to change a tax (see, e.g., Guriev, Kolotilin and Sonin 2011; Stroebel and van Benthem 2013; Chang et al. 2016). Our model accounts for such exogenous price shocks too. But it is noteworthy that many of the observed cycles are not driven by prices. Take the Bolivian example (Figure 1 panel b). The expropriations in 1937 and 1969 did not coincide with high prices. Conversely, the price spikes in the 1970s and 1980s did not coincide with high prices. Conversely, the price spikes in the 1970s and 1980s did not coincide with high prices.

3 An inelastic tax base is consistent with recent evidence that oil production from existing wells is almost completely unresponsive to oil price shocks and, thus, tax rates (Anderson, Kellogg and Salant 2017).

4 As we are agnostic as to the uses of government revenue, the model is consistent with tax collection for corrupt purposes. Besides stealing “official” tax revenues, corrupt politicians may request bribes and other illegal payments. Such side payments are unobserved, but likely to be very small compared to transfers through official taxes or expropriations, and unlikely to interact with the official tax rates and the main channel for cycling in our model (high production).
lead to expropriations. The resource nationalism in the early 2000s started during a period of sharply increasing production before the oil price spike. Likewise the cycles in natural gas taxation in Israel and copper-taxation in Chile were not at all driven by prices and many parts of the mentioned cycles in Algeria, Peru and Argentina were not driven by prices either. Instead, the level of production, installed capital and discoveries seem to be the main drivers of these tax changes. This begs our paper’s question of whether the inner workings of political and economic processes make cycles appear endogenously as an economic and political equilibrium. We show that limited commitment and investment lags provide such a channel. Exogenous price changes, however, have an interesting interaction with the endogenous cycles in our model: a price shock is not what explains the tax cycles per se, but by shocking the economy out of a steady state it initiates new endogenous and persistent cycles in the same way that an unexpected discovery does.

Finally, we show that the result that equilibria will exhibit tax oscillations is not inevitable. Equilibria without cyclicality can be sustained under certain conditions. This could account for why, in practice, tax cycles are very prevalent but not ubiquitous across countries. In one such ‘no-cycling’ equilibrium, the government endogenously commits to setting a low and constant tax. This allows it to capture higher resource revenues than in the cycling equilibrium. This is sustained by firm expectations that, if a high tax is set once, a new cycling equilibrium is being played and by the fact that, once firms hold such expectations, the government’s best response is to cycle taxes. Our results imply that countries with more patient governments (for instance, due to low political turnover) are better able to avoid cycles. Another equilibrium without cycles is characterized by full tax differentiation between old and new mines. In that case, governments would always tax existing production at 100% while taxing new mines at a low rate. Perhaps surprisingly, this increases tax revenues. In reality, while certain governments have resorted to some form of differentiated taxes, this seems to be an exception rather than the rule. Perfect tax differentiation with persistent expropriation of old mines is highly unusual, potentially due to reputational spillovers, a lack of continued capital investments in older facilities, the costs of shifting ownership and operations of old mines to the state, and moral hazard. Thus tax and investment cycles persist.

Following a brief description of the paper’s relation to previous research, Section 2 presents rich descriptions of repeated cycles of taxes and investment in Bolivia and Venezuela. In Section 3 we present the basic model along with comparative statics with respect to the mining profile and the resource price. Here, we assume the government does not care about future revenues. Section 4 extends the model to a case in which the government does care about tax revenues in the more distant future, showing tax cycles appear here too. Section 5 studies how a government can endogenously commit to setting low constant taxes and avoid cycling and in Section 6 we show how tax differentiation can avoid cycling and increase the total tax revenues. Section 7 concludes.
1.1 Relation to previous research

The prime purpose of this paper is to present an endogenous mechanism to explain resource taxation cycles and their interaction with investment cycles. We are by no means the first to analyze resource taxation (Lund, 2009; Boadway and Keen, 2010) or the “natural resources trap” (as Hogan and Sturzenegger (2010) dub the difficult dynamic hold-up problem associated with resource investment). However, we do develop the first model of time-consistent resource taxation, under limited ability of governments to commit to resource tax rates and firms to commit to exiting, which endogenously explains cycles and repeated exit and entry in equilibrium.

The papers most related to ours include a number of important contributions analyzing dynamic commitment problems such as Thomas and Worrall (1994) and Bohn and Deacon (2000). These and other papers yield important insights about resource contracts and taxation when governments cannot commit. We build on this literature by relaxing two unsatisfactory features of existing models: exogenous expropriations (such as in Bohn and Deacon (2000)) and the assumption that individual firms can effectively punish an expropriating host government (e.g., by the threat of autarky such as in Thomas and Worrall (1994)). The first feature is clearly important to relax when trying to explain why and when tax changes occur. The second feature runs counter to the large number of observations (a few of which we have already highlighted) in which either the same firm or a new firm enters the country a few years after an expropriation has taken place. In relaxing these assumptions, we take the standard approach in dynamic public finance and macroeconomics.

Many resource-taxation models study the effect of expropriation risk on private investment. For example, Bohn and Deacon (2000) and Wernerfelt and Zeckhauser (2010) analyze how the risk of expropriation affects the speed of extraction and optimal contracts. However, being mainly interested in the reaction of firms, the risk of expropriation in their models is exogenous. Aghion and Quesada (2010), Engel and Fischer (2010) and Rigobon (2010) also assume exogenous expropriations. None of these papers can explain tax cycles. In contrast, our paper explicitly models the interaction between successive governments that, like firms, hold rational expectations. We thus endogenize the tax (in the extreme case, expropriation) and the reactions of future governments. We also extend earlier analyses to include different

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5 Our setup is similar to Klein, Krusell and Rios-Rull (2008) who describe the lack of commitment as a “game between successive governments.” See also, for instance, Benhabib and Rustichini (1997) and Ortigueira (2000) for similar setups. Since the seminal paper of Kydland and Prescott (1977), a large part of the dynamic public finance literature has been analyzing various forms of commitment problems (e.g., Persson and Tabellini 1994; Reis 2013). The main approach in this literature is to focus on capital as a generator of output. Since resource extraction creates few jobs and is, in many countries, performed by non-domestically owned firms, we treat the resource sector primarily as a source of government income implying a dynamic Laffer trade-off under limited commitment. Related to our focus on taxation cycles, Hassler, Krusell, Storesletten and Zilibotti (2008) analyze circumstances under which oscillatory human-capital taxes are optimal from a normative perspective. Our analysis is positive and the tax cycles in our setting are not optimal. Aguiar, Amador and Gopinath (2009) extend the model of Thomas and Worrall (1994) to include sovereign debt, and find that the threat of expropriation (of both debt and FDI) can induce investment cycles when the government is less patient than consumers.

6 Other papers focus on the question of how to make resource taxes neutral (e.g., Campbell and Lindner 1985; Fan 1987), which is less related to our work. Yet other papers study optimal contracts (e.g., Baldursson and Von der Fehr 2015) and optimal taxation (e.g., Daubanes and Lasserre 2015).
mining profiles and price changes, factors obviously important for resource markets.

Endogenous expropriation has been considered in the seminal paper on foreign direct investment by Thomas and Worrall (1994), in which a government is unable to commit even in the short run. They show how the fear of being condemned to autarky can help a government suppress its urge to expropriate. However, as positive shocks make expropriation ever more lucrative, the firm is forced to ratchet up investment; taxation only kicks in (at a constant rate) once the maximal level of investment is reached. In other words, their model neither exhibits cycles, nor repeated exit and entry, in equilibrium. Instead, investment and taxes increase over time. A similar issue arises in Stroebel and van Benthem (2013), in which expropriations occur with positive probability, after which the country remains in autarky forever, so that expropriation happens only once. We want to explain the empirically relevant features of repeated cycles of resource investment and taxation, and repeated exit and entry by firms. To do this, we differ from the previous literature by assuming the government can commit in the short term, and that resource firms are unable to credibly exit the market for good. This is a better description of the investment conditions and firm behavior in many politically unstable, resource-holding countries.

2 Episodes of repeated tax and investment cycles

We now describe in more detail Bolivia’s and Venezuela’s history of long-run cycles in taxation and investment. These countries’ dealings with foreign oil and gas companies provide instructive case studies and motivation for our model. As emphasized in the introduction, many other countries and natural resources have gone through similar cycles.

2.1 Bolivia

Low taxes spur initial investments: Bolivia opened up its hydrocarbons sector to foreign investors in 1916 and the first foreign oil company (Standard Oil; a predecessor of ExxonMobil) entered in 1921. In that year, the Organic Law on Petroleum set the fiscal terms for the decades to come, mainly consisting of an 11% royalty plus an obligation to return 20 percent of the licensed area back to the state once production began (Jemio, 2008).

Expropriation and low subsequent investment: The relationship between the government and Standard Oil turned sour around the 1932-1935 Chaco War, in which the oil-rich Gran Chaco region was claimed by both Bolivia and Paraguay. Standard Oil started shutting down equipment and moving it out of the country and was generally seen by the public as betraying the Bolivian government. After the war ended, the David Toro government created state-owned company Yacimientos Petrolíferos Fiscales Bolivianos (YPFB) in late 1936 and expropriated Standard Oil’s assets in 1937 (Klein and Peres-Cajias, 2014). This marked the beginning of almost two decades without foreign oil investment; the legislation provided that YPFB could be associated with foreign business, but either for fear of expropriation or lack of interest, no foreign company invested.
**Low taxes, high investment:** Production by YPFB grew in the 1940s and early 1950s, but Bolivia concluded that it could not provide the capital investment needed for a significant expansion of the oil industry. The government therefore offered a favorable fiscal regime to foreign investors (the “New Petroleum Code” of 1955). Standard Oil did not return, but Gulf Oil seized the opportunity. As a result, production grew fast, especially in the mid and late 1960s. Revenues also grew fast despite falling oil prices.

**Resource nationalism, expropriation and low subsequent investment:** This sparked another episode of resource nationalism. Supported by popular resentment against Gulf Oil’s increasing profits, the military government of general Alfredo Ovando Candía expropriated the company in 1969 and transferred its properties to YPFB [Peres-Cajías, 2015]. Oil production fell drastically right after the expropriation of Gulf Oil. Following these dramatic events, Bolivia realized the need for stable investment conditions to boost production, especially since natural gas production and exports to Argentina were about to take off. However, tax rates under Decree Law No. 10170 remained fairly high throughout the 1970s and 1980s. The new fiscal regime had only limited success in bringing back foreign investment in the 1970s and 1980s (Figure 2, Panel a). Bolivia did resist the temptation to expropriate when oil prices soared in the mid and late 1970s.

**Low taxes, high investment:** In 1990, the government reduced the tax rates for certain fields in a further effort to attract private investment. When oil and gas production stagnated and even started to decline in the mid 1990s, the Hydrocarbon Tax Law of 1996 substantially reduced the fiscal take for new fields. In 1996-1997, president Gonzalo Sánchez de Losada even privatized YPFB. All this led to a wave of foreign investment in the natural-gas sector, as clearly illustrated in Figure 2. International oil and gas companies such as BG Group, BP, Petrobras, Repsol and Total entered the country [Valera, 2007].

**Resource nationalism, tax increases and low subsequent investment:** Resource nationalism started yet again by the early 2000s. Even before oil and gas prices were on the rise in the mid 2000s, there was growing public discontent about the profits made by foreign resource firms as their production had increased rapidly. As [Manzano and Monaldi, 2008] put it, private investors in Bolivia and other Latin American oil producing countries were “partially the victims of their own success”. Fifteen years of large private investments had resulted in new resource discoveries which, from the year 2000 onwards, translated to strong growth in production. This created strong incentives to increase government take. President Sánchez de Losada had to resign during the Bolivian gas conflict in 2003, in which the protesters demanded full nationalization of the hydrocarbons sector.

His successor, president Carlos Mesa, held a national referendum – which passed in 2004 – to repeal the existing hydrocarbon law and to increase tax rates on oil and gas companies. In 2005, the referendum was signed into law as the new Hydrocarbon Law No. 7.

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7 In this case, the changes in the tax rates were forced on the government by political unrest and public pressure. In other cases, a government or president can independently initiate fiscal changes, either upon (re-)election or in the middle of an existing term. These various channels are consistent with our model, which does not need to specify explicitly the exact conditions under which a government can change the rules of the game.
3058 which revoked the tax breaks from 1996. The 2005 law also established state ownership of oil and gas at the wellhead and made it mandatory for operators with existing contracts to transfer to the new terms (WoodMackenzie 2012). This ended the significant wave of foreign investment.

The more punitive taxation system still did not satisfy many people who believed that full nationalization was preferable. Following protests in La Paz in May 2005, president Mesa was forced to resign. The sharply increasing gas production had created strong incentives for newly elected president Evo Morales to increase taxes further even at constant oil and gas prices, but the resource price increases of the mid 2000s gave him the perfect opportunity to increase taxes to very high levels in response to popular demand. In 2006, tax rates for some fields increased to as much as 82%. Foreign investment in oil and gas plummeted between 2002 and 2006 (Figure 2). Morales then nationalized certain foreign assets in 2007 as per his election pledge.

**Lowering of taxes to spur investment:** As Morales realized that Bolivia needed three billion dollars in investment to meet its gas-export obligations to Argentina and Brazil, the government quickly softened investment conditions after the nationalizations. In 2010, fiscal terms mostly reverted to those in the 2005 Hydrocarbon Law. The government started offering tax breaks as it feared that hydrocarbon production would stall. By June 2011, fifteen foreign companies had signed contracts for oil and gas exploration, adding up to substantial foreign investment (Figure 2); not a single company pulled out of Bolivia this time (Chávez 2012).

The Bolivian history illustrates a sequence of long-run cycles in taxation and investment, with expropriations in 1937, 1969 and 2004-2009, and low tax rates in between, in line with our model predictions. It also illustrates how production and investment levels are an important driver of cycles, in addition to resource prices.

### 2.2 Venezuela

**Low taxes and a stable tax regime spur initial investment:** Oil was first discovered in Venezuela in 1878, but the first well was not drilled until 1912. Royal Dutch Shell and Standard Oil soon became major oil producers as Venezuela became the second-largest oil producing country in the world. Oil made up more than 90 percent of exports by 1935 (Venezuela Analysis 2003). The First Petroleum Law went into effect in 1922. When production grew between 1922 and 1943, the government realized the need for a stable long-term investment climate. In 1943, Venezuela passed the Hydrocarbons Law, which aimed to ensure that foreign companies could not make greater profits from oil than they paid to the Venezuelan state, yet also allowed the world’s largest oil companies access to Venezuela’s vast reserves at reasonable tax rates for the decades to come. This created stable investment conditions that firmly established the industry and allowed the oil sector to expand rapidly. Between 1944 and 1958, production more than tripled and the annual average growth rate of the net capital stock of the oil industry was 14.3% (Monaldi 2001) (Figure 3 panel a).
Figure 2: Oil and Gas Investment in Bolivia (Panel a) and Venezuela (Panel b)

Notes: Sources: UNCTAD for oil and gas FDI in Bolivia and Venezuela (1994-2012). Monaldi (2001) for oil and gas investment in Venezuela (1950-1997). Monaldi (2001)'s data includes PDVSA investment from 1976 onwards; not just FDI. To compare the two data sources for Venezuela, UNCTAD data in current USD is converted to 1998 Bolivars using exchange rate and GDP deflator data from the World Bank. For consistency reasons, UNCTAD data for Bolivia are also converted to real local currency. See Appendix A for details.
Figure 3: Tax Cycles in Venezuela and (Panel a) Oil and Gas Production or (Panel b) Oil Price

Notes: Tax rate refers to the effective net resource-income tax rate per barrel of oil equivalent. Sources: Monaldi (2001), Manzano and Monaldi (2008) and WoodMackenzie (2012) for tax rates, Monaldi (2001) and International Energy Agency (2016) for production; BP (2015) for prices. Years in which assets are expropriated are coded as a 100% tax rate (though effectively tax rates could be higher than 100% in those years). Gaps in tax rates represent periods in which no foreign firms were producing. Low end and high end ranges indicate the minimum and maximum tax rates (when available), which can differ depending on project characteristics.
Resource nationalism, tax increases and subsequent low investment: This spectacular growth in oil production tempted the government to capture a larger share of the profits. Taxes increased dramatically in the period 1959-1972. Figure 2 (Panel b) shows that, as a result of the increased taxes, oil investment started to decline in the late 1950s and stayed low until 1976, but oil production continued to rise until the early 1970s. It then fell abruptly, though with a significant lag to the reduced investment levels.

In 1973, the oil embargo in the Middle East led to a dramatic increase in oil prices. In 1974, the newly elected president, Carlos Andrés Pérez, used this to promise the population that Venezuela would become a first-world country in just a couple of years. He started nationalizing the oil industry, a process that finished with the creation of Petroleos de Venezuela (PDVSA) in 1976 (Venezuela Analysis, 2003). In the process, Venezuela paid Conoco, Exxon, Gulf Oil, Mobil and Shell only 20 percent of the market value of their assets (Wirth, 2001), that is, a form of expropriation. Conoco left the country, but other firms stayed and signed contracts for training local staff and technological support.

Tax breaks to spur foreign investment: After the expropriation, PDVSA controlled all oil production. PDVSA increased investments dramatically (Figure 2), taking advantage of the prevailing high oil prices. Despite the fast growth, the government realized in the 1980s that foreign investment and expertise was needed to develop the massive heavy oil resources in the Orinoco Belt. Against that background, Venezuela opened up the oil sector to foreign investment again in 1990. Foreign investors were offered tax rates well below the rates of around 80% that PDVSA had been paying during the late 1970s and early 1980s. By 1996, four joint ventures had entered Venezuela and Conoco came back after leaving the country following the expropriations of 1976. By the mid 1990s, private investment had increased substantially (Figure 2) and Venezuela was top on the list for foreign investment in petroleum exploration and production (Manzano and Monaldi, 2008; Hajzler, 2012).

Resource nationalism, nationalization and production decreases: In the early 2000s, resource nationalism was on the rise again. When Hugo Chávez first came to power in 1998, he did not announce any plans for PDVSA. But in 2001, Chávez introduced a new Hydrocarbons Law that increased royalties and forced private investors to sign agreements in which they could only operate in joint ventures with at least 51% PDVSA ownership. Also, when his initial popular support had faded by 2002, Chávez responded to public protests by announcing a re-nationalization of the oil industry. He took control over PDVSA, which was to be managed “by the people and for the people” (Energy Tribune, 2007). In the resulting

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8In 1959, the government share rose from 51% to 65%; a radical break with the 50-50 rule from the 1943 Hydrocarbons Law. In the late 1960s, oil taxes increased further to levels around 71% in 1969. Yet another law increased tax rates to over 78% in 1970. By 1972, tax rates had creeped up to levels around as high as 90%. Up to that point, oil prices had been low and decreasing from $16 per barrel in 1943 to $14 per barrel in 1972. In an attempt to raise global oil prices, Venezuela was instrumental in the formation of the Organization of Petroleum Exporting Countries (OPEC) in 1960.

9This is again consistent with Anderson et al. (2017) who find that investment, not production, is the main margin of adjustment to changing oil prices or tax rates.

10Miguel Espinosa from Conoco’s treasury department explained the decision to come back as follows: “In spite of our previous experience, we were eager to participate in the Venezuelan oil sector once again. We had long-standing commercial relationships with PDVSA – buying their crude to supply our refineries – and strong personal relationships. When the door opened, we took the opportunity” (Esty, 2002).
chaos, oil production fell and Venezuela had to renege on oil deliveries.

While this resource nationalism started in a period of high production and sunk investment (and low oil prices), the sharp increase in oil prices in the mid 2000s added to the government’s desire to impose higher taxes and more restrictions on foreign investors. By 2007 the government had nationalized the oil industry, taking a majority control of all privately operated projects without providing market compensation. ConocoPhillips and ExxonMobil subsequently decided to abandon their assets in the Orinoco Belt and exit the country. BP, Chevron, Statoil and Total accepted that PDVSA increased its share from 40% to 78% (Guriev et al. 2011; Hajzler 2012). Tax rates have remained high since then. In 2008, Venezuela imposed a first windfall tax on incremental revenues for oil prices exceeding $70 per barrel. In 2011, the government introduced a second windfall tax for oil prices between $40 and $70 per barrel (WoodMackenzie 2012). Investment has been low ever since the resource nationalism started in 2002 (Figure 2).

Altogether, the history of Venezuela presents another clear example of long-run cycles of taxation and investment, with expropriations in 1975 and 2007 following long periods of more favorable fiscal regimes. Foreign investment dissipates and foreign investors leave the country when taxes are high but come back later when tax rates are low again.

3 Model of resource taxation with limited commitment

In this section we outline a tractable model of resource taxation and exploration and solve for the equilibrium policies. The overall purpose of our setup is to capture important features of resource markets through four main assumptions. First, governments can only commit to tax rates for a limited period of time. Second, firms cannot commit to permanently exiting a country following changes in the agreed fiscal terms. Third, mines have long lifetimes that extend beyond the government’s commitment period such that old mines, discovered earlier, and new mines exist in parallel. Further, mine development takes time, so that there is a delay between the investment decision and the first revenues. Fourth, we initially assume that taxes do not differentiate between old and new mines, but we relax this assumption later.

3.1 Model setup and Markov-perfect equilibrium

There are two types of agents in the economy: a government wanting to maximize tax revenue, and a unit mass of infinitesimal resource-prospecting firms. The government sets a uniform tax rate for a given period. The firms choose how much to explore.

The time intervals, which we call periods, are indexed by $t \in \mathbb{N}^+$. The sequence of decisions is depicted in Figure 4. The government observes the existing stock of mines and then announces a tax rate, to which it commits for only period $t$, with the objective of maximizing its resource revenues. After the interval has elapsed, the government can freely change the tax. This lack of long-term commitment implies there is in essence a sequence
of governments, each facing a different optimization problem. After the announcement of the tax rate, firms conduct exploration, and any mines open with a lag. This means that within each period there are two subperiods \( s \in \{1, 2\} \). In the first subperiod only the old mines are being extracted from and in the second subperiod extraction is taking place both from the old and the new mines. Note that the government commits to the same tax for the whole period, that is, for both subperiods. Finally, when the current period ends, the old mines close down while the new stay open for the entire next period (i.e., the new mines in period become the old mines in \( t+1 \)).

There is an infinite quantity of land available. A small plot of land can be explored for natural resources by using appropriate factors (e.g., petroleum geologists and drilling rigs, or dynamite and diggers). For tractability we assume firms face quadratic (i.e., convex) internal costs of exploration.\(^{11}\)

Aggregate exploration effort (i.e., investment) in period is denoted by \( e_t \). Since the model is deterministic, \( e_t \) is equivalent to discoveries. Exploration takes place in the first subperiod and every unit of exploration yields a known quantity of resources, normalize to unity. The initial stock is denoted \( e_0 \).

Any discoveries made in period can be exploited in periods \( t \) and \( t+1 \), after which the mine in question closes down. Denote the exogenous (world market) resource price in period \( t \), subperiod \( s \) by \( p_{t,s} \). For now, we assume the price is constant: \( p_{t,s} = p > 0 \) (we consider price shocks in Section 3.3). Assume that the exploration costs are inclusive of the costs of

\(^{11}\)Alternatively, the model could be written in terms of firm entry and external diseconomies of scale, without changing any of the results.
developing the deposit, and that extraction itself is costless. Firms extract during the two periods. In the first period a share \( \delta \in [0, 1] \) of the mine’s content is extracted and in the second period \( 1 - \delta \) is extracted. This way, a small \( \delta \) captures a backloaded mining profile, so that a large share of the extraction from a new mine takes place beyond the commitment period of the current government.

We now construct a Markov-perfect equilibrium: a tax policy rule and an exploration rule which both depend only on the current ‘state’ of the economy. For the government’s problem at time \( t \), the state variable is the pre-existing stock of mines \( e_{t-1} \). For the firm’s problem, due to the sequential move order, the state variable includes both this stock and the tax for the current period \( \tau_t \); however, it will become clear that the former is redundant. Thus, the Markov strategies are described by the rules \( \tau_t^*(e_{t-1}), e_t^*(\tau_t) \).

The representative firm maximizes profits:

\[
\max_{e_t} \pi_t(e_t; \tau_t, \tau_{t+1}^e) \equiv ((1 - \tau_t)\delta + \beta(1 - \tau_{t+1}^e)(1 - \delta)) pe_t - \frac{e_t^2}{2}
\]

in which the firm takes the current and expected taxes \((\tau_t, \tau_{t+1}^e)\) as given. The discount factor used for future revenues is \( \beta \in (0, 1) \).

Equilibrium exploration equals

\[
e_t^*(\tau_t, \tau_{t+1}^e) = ((1 - \tau_t)\delta + \beta(1 - \tau_{t+1}^e)(1 - \delta)) p. \tag{2}
\]

Exploration effort is zero when \( 1 - \tau_t = \beta(1 - \tau_{t+1}^e) = 0 \), i.e., only if there would be full expropriation (100% taxes) both this period and the next. This means that, since firms are forward-looking, they will choose to explore today despite a very high current tax if they foresee a low tax tomorrow.

We assume the government is unconcerned with the revenues obtained in future periods and only wants to maximize the revenues obtained today (we relax this in Section 4). The government recognizes the firms’ reaction function and solves

\[
\max_{\tau_t} \tau_t p \left[ (1 - \delta)e_{t-1} + \delta e_t^*(\tau_t, \tau_{t+1}^e) \right]. \tag{3}
\]

For now, we restrict ourselves to Markovian strategies for the government, that is, tax policies which are a function of past exploration only. With rational expectations, this implies \( \tau_{t+1}^e = \tau_{t+1}^e(e_t) \), so that given any \( \tau_t \), \( \tau_t^* \) defines a fixed-point problem. As long as this has a unique solution, we can write it as \( e_t^*(\tau_t) \). We will limit ourselves to stationary Markov strategies in this section and drop the time subscripts on the tax policy rule.

Assuming (and then verifying in the proof) that the optimal \( \tau_t^* \) is interior and that \( e^*(\tau_t^*) \) is differentiable, the first-order condition is

\[
(1 - \delta)e_{t-1} + \delta e_t^*(\tau_t^*) = -\tau_t^* \delta \frac{de_t^*(\tau_t)}{d\tau_t} \bigg|_{\tau = \tau_t^*}. \tag{4}
\]
In words, the government trades off the extra revenue, given the existing tax base, against the new tax base that shrinks with higher taxes. The existing tax base – the pre-existing mines – are fully inelastic. Since each firm reacts to taxes over multiple periods, the government needs to consider how its tax affects investment and hence taxes tomorrow, which again feeds back on today’s investment. Thus, a succession of short-termist governments are linked by long-lived firms. With no pre-existing mines \((e_{t-1} = 0)\), the government would simply choose to sit at the top of its one-period Laffer curve (where it still needs to take into account how the current tax affects firms’ expectations of future taxes). With a positive pre-existing stock of developed mines, the government prefers to set a higher tax rate.

We will look for equilibria in piecewise linear strategies, using the guess and verify method. To avoid technical issues, in the main text we impose a parameter restriction:

\[
\delta \geq 3(\beta) = \frac{1 - 2\beta + \sqrt{1 + 4\beta}}{4 - 2\beta}.
\] (5)

We can relax this restriction to construct piecewise linear equilibria for smaller \(\delta\) also. These yield no additional intuition, compared to the equilibrium given in the following proposition; to avoid unnecessary technical complications, we relegate them to Appendix C (Proposition C.1).

**Lemma 1.** For \(\delta \geq 3(\beta)\), a linear Markov-perfect equilibrium exists with policy and exploration rules given by

\[
\tau^*_t(e_{t-1}) = \min\{Ae_{t-1} + B, 1\}, \quad e^*_t(\tau_t) = C\tau_t + D
\] (6)

with

\[
A = \frac{1}{p\delta}\left(2\frac{\delta}{1 - \delta} - \beta\frac{1 - \delta}{2} \right), \quad B = \frac{1 + \beta\frac{1 - \delta}{2}}{2 + \beta\frac{1 - \delta}{2}},
\]

\[
C = \frac{p\delta}{2}\left(2 - \beta\left(\frac{1 - \delta}{\delta}\right)^2\right), \quad D = p\delta\left(2 - \beta\left(\frac{1 - \delta}{\delta}\right)^2\right)\left(1 + \frac{\beta\frac{1 - \delta}{2}}{2 + \beta\frac{1 - \delta}{2}}\right).
\] (7)

**Proof.** See Appendix B.

We define the lowest possible level of the tax as \(\tau \equiv B\). Using (6), the tax-transition rule is

\[
\tau^*_t = AD + B + AC\tau_t = \frac{1 + \beta\frac{1 - \delta}{2}}{\delta} - \frac{1 - \delta}{2} \tau_t.
\] (8)

By letting \(\tau_t = \tau^*_t = \tau_{ss}\), it follows immediately that there is a unique steady state at

\[
\tau_{ss} = \frac{B + AD}{1 - AC} = \frac{1 + 2\beta\frac{1 - \delta}{\delta}}{1 + \delta} 2 + \beta\frac{1 - \delta}{\delta}.
\] (9)
Note that the tax transition rule has a slope of $-\frac{1}{2} \frac{1-\delta}{\delta} < 0$. In other words, low taxes today, by inducing higher exploration, lead to high taxes in the next period. Further, the transition is also stable: taxes will converge to the steady state.\(^{13}\)

We summarize the dynamic properties of the benchmark case here:

**Proposition 1.** With a constant resource price $p$, the taxes will cycle around the steady state $\tau_{SS}$.

*Proof.* In the preceding text. \qed

We will now discuss some implications of the proposition.

**Corollary 1.** i) Within a time period there is a positive relationship between the tax rate and the value of the current mines. ii) Within a time period there is a negative relationship between the tax rate and mining investments (i.e., exploration and development of mines).

To illustrate the implications of this corollary consider a country which has just recently discovered that it has some resources but where these have not been explored yet. That is, the initial stock of mines is zero ($e_0 = 0$). For this country $\tau^*_0 = \tau$ which is the lowest possible tax in any period. This means that countries with a newly discovered resource potential will offer a low tax to initiate exploration.

The proposition further implies:

**Corollary 2.** i) The tax in period $t$ is negatively related to the tax in period $t+1$. ii) The number of existing mines in period $t$ is negatively related to the number of existing mines in period $t+1$.

Prediction (i) in Corollary 2 implies that the country with zero initial mines will also be the one that raises taxes the most once discoveries have been made. So the cycles will be particularly strong in less mature resource-producing countries. Prediction (ii) of course implies that investments cycle along with the taxes.

To illustrate the main mechanism of the model consider the case of a flat extraction profile $\delta = \frac{1}{2}$. Then the government will set a tax

$$
\tau_t^* = \begin{cases} 
\frac{2}{p(2-\beta)} e_{t-1} + \frac{1+\beta}{2+\beta} & \text{if } e_{t-1} < \frac{1}{2}, \\
1 & \text{otherwise}.
\end{cases}
$$

This represents the government’s Laffer-type trade-off between getting a large share of the revenues and incentivizing the development of a large tax base. The tax differs from a static Laffer tax in two ways. First, the term $\frac{2}{p(2-\beta)} e_{t-1}$ implies that the government will\(^{12}\) Clearly tax rates are never at 100% for two subsequent periods, so that exploration will always take place. Indeed, 100% taxes can only occur if the initial stock is very high, after which the taxes will oscillate, gradually converging to the steady state. Thus full expropriation can occur in the initial period, if the economy starts with a very large existing stock of active mines; following this, taxes will always be strictly interior.

\(^{13}\)Stability results from the tax-transition slope being lower than 1 in absolute value, which holds as long as $\delta > \frac{1}{3}$. This is ensured by restriction \(\delta \) of The case of an unstable tax transition would lead to an eventual cycle of repeated full expropriations, but this would make the equilibrium non-linear.
set a higher tax since part of the tax base consists of pre-existing, inelastic capital that is unaffected by the current tax. If no old mines exist, this term disappears. To highlight the second difference, suppose that indeed $e_{t-1} = 0$. In a static model, but otherwise similar linear-quadratic specification, the Laffer tax would be $\tau^*_t = \frac{1}{2}$. In our dynamic model, $\tau^*_t = \frac{1+\beta}{2+\beta} \geq \frac{1}{2}$. That is, the tax is higher than the static Laffer tax even if there is no inelastic capital. The reason for this is that patient firms care about future revenues, which mitigates the negative effect of today’s tax rate on investment. First, the firm still expects to obtain some future revenues; second, a higher tax today leads to lower exploration, which means taxes in the next period will be lower. Note that with perfectly impatient firms ($\beta \to 0$) and no pre-existing revenues ($e_{t-1} = 0$), the tax goes to $\tau^*_t \to \frac{1}{2}$. For the same reasons, the steady-state tax $\tau_{ss} = \frac{2}{3} + \frac{2\beta}{3+\beta} \in \left(\frac{2}{3}, \frac{8}{9}\right)$ also exceeds the static Laffer optimum.

### 3.2 Mining profile

We now consider how the mining profile, represented by $\delta$, affects the above results. Recall that a low $\delta$ implies resource revenues are more backloaded, with fraction $1 - \delta$ of the revenues arriving beyond the government’s commitment period.

Note that the steady-state tax $\tau_{ss}$ given in (9) is below unity and decreases in $\delta$. The tax transition rule (8) becomes steeper with low $\delta$, which implies that oscillations decay more slowly for backloaded mining profiles. Furthermore, as $A$ and $B$ are both decreasing in $\delta$ we get the following results:

**Corollary 3.** i) For a given stock of existing mines, the more backloaded the mining profile is, the higher is the tax. ii) Given the stock of existing mines, the more backloaded the mining profile is, the lower is the exploration effort. iii) The more backloaded the mining profile is, the more slowly deviations from a steady-state tax decay.

\[
\frac{\partial}{\partial \delta} \left| \frac{\tau_{ss} - \tau_t}{\tau_t - \tau_{ss}} \right| < 0.
\]

These predictions are intuitive. Consider part (i). When the mining profile is relatively backloaded, then the firm, when deciding on its exploration investment, cares more about future taxes as the mine will now produce much of its value in the next period. The government today knows this and therefore has an incentive to set a high tax to ensure getting a large share of the profits from the old mines. This of course happens in all periods implying that, in general, the tax rate will be high. When the mining profile is sufficiently backloaded, the tax regime becomes so directed at getting at the current mines’ profits that this completely strangles the industry. As an illustration, consider the polar case of a completely backloaded mining profile ($\delta \to 0$). Then the government, knowing that whatever it does will not have an effect on the current-period production from the new mines, only cares about taxing the old mines and sets the tax at an appropriation level $\tau^*_t = 1$. This

\[14\] A bit of algebra shows that the decay rate in the inequality is given by $|AC| = \frac{1}{2} \frac{1-\delta}{\delta}$. 

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of course means there will not be any exploration at all since the firms foresee this in all periods.

The opposite case is one where the mining profile is sufficiently frontloaded so that all the mining occurs in the current period ($\delta \to 1$). In this case the firm is fully responsive to any tax change and there is no linkage between the taxation of subsequent governments. In this case the model converges to the results of a static model, i.e., the Laffer tax of $\tau^* = \frac{1}{2}$. Thus, the high taxes we obtain in the model hinge on 1) mines existing beyond the commitment period of the government and on 2) firms that care about later profits.

The intuition for part (ii) of the corollary is similar. Given the existing stock of mines, more backloaded revenues will reduce the total value of any new discoveries (because of discounting) while increasing the government’s taxes today. As a result, exploration falls with backloadedness.

For part (iii) of the corollary, note that the rate at which oscillations decay is simply the absolute value of the slope of the tax transition $\delta$, which itself depends on how sensitively governments respond to pre-existing mines and firms respond to taxes. An impatient government overseeing a very backloaded resource cares much more about taxing the existing tax base than about encouraging new exploration, and thus responds more sensitively to the stock of pre-existing mines. Equilibrium exploration becomes less responsive to current taxes, as future revenues weigh more and as future taxes are expected to respond more. The government’s increased sensitivity dominates, so that equilibrium taxes become more variable.

There are two ways to interpret the results on the mining profile $\delta$. The first is that they pertain to geological constraints. Capital-intensive and technologically challenging projects, such as offshore drilling, Arctic drilling and ultra-deep drilling, have long lead times between exploration and the start of commercial production. Corollary 3 implies that countries in which these projects represent a large share of hydrocarbon extraction set higher tax rates (and see lower exploration) than countries with frontloaded extraction profiles (e.g., conventional oil).

The second interpretation is that $\delta$ represents the government’s commitment period. If a government is able to commit for many years, then the “current period” applies to a large share of the profits: $\delta$ is large. Corollary 3 then says that countries with stable governments, that is, ones which can be trusted not to change the tax very often, will have lower taxes and more exploration activity. A long commitment period may of course be the result of a stable autocratic regime or characterize a democratic country with low turnover or cohesive political views.

\[15\] Strictly speaking, a longer commitment period also decreases $\beta$ since it postpones the firms’ second-period profits. The derivative $\frac{d\tau_{ss}}{d\delta} > 0$. Hence a long commitment period has two effects, both of which are lowering $\tau_{ss}$: one through decreasing $\beta$ and one through increasing $\delta$. 

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3.3 Price changes

We now turn to the effect of price changes on the tax policy. To highlight the mechanism, Lemma 2 considers an unexpected price change. Suppose the price in all periods is $p$, but in the first subperiod of period $t$ there is a price shock, so that $p_{t,1} \neq p$.

**Lemma 2.** Let $\tilde{p} \equiv \frac{p_{t,1} + p_{t,2}}{2}$ denote the average price in period $t$. Then, the policy rules are given by (6), but with the coefficients indexed by time; for $i \in 0, 1, 2, \ldots$, we have $A_t = \frac{\tilde{p}_t}{p} A$, $A_{t+1+i} = A$, $B_{t+i} = B$, $C_{t+i} = C$, $D_{t+i} = D$.

*Proof.* By applying Lemma 1, rolled forward by one period, it is clear that from period $t+1$ onwards the coefficients are given by (7). The firm’s and government’s problems in period $t$ are then solved by backward induction, exactly as in the proof of Lemma 1.

The above lemma and Proposition 1 yield the following prediction:

**Corollary 4.** i) An unexpected increase in the spot price $p_{t,1}$ raises the current tax. ii) Suppose the economy is in steady state, then one price shock will initiate persistent oscillations.

Part (i) is seen from the fact that an increase in $p_{t,1}$ raises the average price in period $t$, $\tilde{p}_t$, thus raising $A_t$. The predictions are intuitive. An unexpected, temporary, positive price shock in the first subperiod increases the value of existing (old) mines vis-à-vis new mines (which appear only in subperiod 2). Hence, the government becomes more concerned about extracting tax revenues from the existing stock of mines.\footnote{This has been tested by Guriev et al. (2011) and Stroebel and van Benthem (2013), who use a panel data set on expropriation events to provide empirical evidence that a higher oil price is associated with an increased probability of expropriation.}

Part (ii) of the corollary says that, starting from a steady state, a price shock will kick-start cycling. This is illustrated in Figure 5. In periods 1 through 4 the economy is in steady state with a constant price. An exogenous and unexpected price shock happens in period 5, subperiod 1 ($p_{5,1}$ increases). Then the price goes back to the pre-shock level in period 5, subperiod 2 ($p_{5,2}$) and stays there in period 6 and onwards. As can be seen, this raises $\tau_5$ since the government is more directed at appropriating the old mines that have become more valuable, in line with part (i) of the corollary. This dampens exploration investment (there is an immediate dip in exploration in period 5) leading to below steady state old mines in period 6. This, in turn, leads to low taxes and high investment in period 6, and so on. In the particular example in the figure the price shock is such that the government actually expropriates the old mines in period 5.

The key message is that exogenous price shocks interact with the endogenous cycles in an interesting way: price shocks do not explain the tax cycles per se, but by moving the economy away from the steady state, they initiate new endogenous cycles which persist even if no further shocks occur. The same dynamics of course hold for any shock, be it an
Figure 5: Illustration of the effect of a price shock. A one-subperiod positive price shock happens in period five, subperiod one.

unexpectedly large discovery or a new technology that changes the extraction profile It also holds if the price shock is permanent.

A final note on price shocks is that we have illustrated them here, for clarity, with a shock that is unexpected. We have also analyzed a case where a price change is expected to come with a certain probability. The qualitative messages of the previous corollary remain under such an extension – price shocks kick-start cycles.

4 Patient government

In the previous sections, we have assumed that the government is perfectly impatient: it does not care for future tax revenues at all. We will now show that the main results do not depend on this assumption.

\[\text{\textsuperscript{17}}\text{For example, consider the case that a surprisingly large discovery is made or that exploration does not bear the expected fruits. Since our model is deterministic, discoveries are directly determined by the exploration effort of the firms, and the tax and effort oscillate over time until a steady state is reached where the tax and effort are constant. Starting from this steady state, an exogenous and unexpected shock to discoveries will lead to high taxes and therefore low exploration effort next period, thereby starting another cycle (in which both discoveries and taxes are determined endogenously in all periods following the shock).}\]

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Suppose the government discounts future tax revenues with per-period discount factor \( \beta \). Then, the government’s value function, given an existing stock of reserves \( e_{t-1} \), is

\[
V(e_{t-1}) = \max_{\tau_t} ((1 - \delta)\hat{p}_t e_{t-1} + \delta p_{t+2} e^*_{t}(\tau_t)) + \beta V(e^*_{t}(\tau_t)).
\] (10)

Note that, in contrast with (3), the government now also cares about all future tax revenues. We can solve the model as before; details are in Appendix D. To reduce the burden of notation, we normalize \( p = 1 \) and restrict \( \delta > \frac{\sqrt{1+\beta}}{1+\sqrt{1+\beta}} \).

We guess and verify that, in the Markov-perfect equilibrium, the tax policy function and the equilibrium exploration are still linear, i.e., given by (6), and stationary (so that the coefficients \( A, B, C \) and \( D \) are constants). Hence, the structure of these functions is exactly as before.

**Proposition 2.** i) Tax cycles exist for all \( \beta \in (0,1) \). ii) Relative deviations from steady-state taxes decay more slowly, the higher is the discount factor:

\[
\frac{\partial |\tau^*_{t+1} - \tau_{ss}|}{\partial \beta} > 0.
\]

**Proof.** In Appendix D.

Most importantly, this proposition says that tax cycles will exist also if the government is patient. In fact, as part (ii) says, more patient countries should see more persistent tax cycles. Cycles are persistent when the government responds strongly to changes in existing production, and firms respond strongly to tax rates. Increasing patience has two, countervailing effects: it tends to make governments more responsive to changes in existing mines, and firms less responsive to current tax rates. The former prevails, with the net effect being stronger cycling when firms and the government discount the future less.

### 5 Endogenous commitment

Is the result that equilibria will exhibit tax oscillations inevitable? In practice, tax cycles are very prevalent but not ubiquitous across all countries and sectors. We now demonstrate, using our model, mechanisms that would enable countries to escape the cyclicity and capture higher resource profits. First, in this section, we show that there exist equilibria in which the government endogenously commits to setting a low and constant tax. Second, we show in the next section that full tax differentiation between old and new mines can also lead to an equilibrium without cycling.

To show the constant-tax equilibrium with endogenous commitment, consider the problem of the patient government as outlined in the previous section. In the equilibria described

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18 We discuss the intuition further in Appendix D. We have also analyzed a case where the government’s and firms’ discount factors differ and the results remain the same. Claim (ii) of Proposition 2 holds when the derivative is taken with respect to the government’s discount rate (keeping the firm’s discount rate fixed).
there (see Proposition 2 and its proof in Appendix D) the government’s tax both oscillates and is set suboptimally high – it is above the Laffer rate of \( \tau = \frac{1}{2} \) which maximizes the static revenue from new mines. These problems emanate from the government not being able to formally commit to a tax.

To show how the cycles can be avoided, we look at a different class of equilibria. A government may be able to sustain a constant low tax rate by a reputation mechanism. As we are dealing with a continuum of firms all behaving competitively (i.e., not taking into account how their individual actions affect future actions by others), the notion of subgame perfection is technically problematic. We thus use the analogous notion of ‘sustainable equilibrium’ of Chari and Kehoe (1993), developed for contexts such as ours.

**Definition 1.** Denote the time-\( t \) government history by
\[ h_t = (e_0, \tau_1, e_1, \tau_2, e_2, \ldots, \tau_{t-1}, e_{t-1}), \]
and time-\( t \) firm history by \((h_t, \tau_t)\). A ‘sustainable equilibrium’ is a tax policy rule \( \tau^S(h_t) \) and an exploration rule \( e^S(h_t, \tau_t) \) such that: i) given the tax policy rule \( \tau^S \), for all firm histories \((h_t, \tau_t)\) the exploration rule \( e^S \) is optimal for the representative firm; ii) given the exploration rule \( e^S \), for all government histories \( h_t \) the tax policy rule \( \tau^S \) is optimal for the period-\( t \) government.

This definition is essentially analogous to subgame perfection in extended-form games, but it is adapted so as to avoid technical issues related to the competitive and infinitesimal nature of the exploration firms. Note that the Markov-perfect equilibrium we constructed in Section 2 is also a sustainable equilibrium. We again normalize \( p = 1 \). We also restrict \( \delta > \frac{\sqrt{2}}{1 + \sqrt{2}} \), purely to avoid technicalities in formulating the results.

**Proposition 3.** Consider the following strategy for the government:

- Set \( \tau_1 = \frac{1}{2} \). In period \( t > 1 \), set \( \tau_t = \frac{1}{2} \) if \( \tau_i = \frac{1}{2} \) and \( e_i = \frac{1 + \beta}{4} \) for all \( i \in \{1, \ldots, t-1\} \); otherwise, set \( \tau_t \) according to the Markov strategies underlying Proposition 2.

and the following exploration rule for the firms:

- Choose \( e_t = \frac{1 + \beta}{4} \) if \( \tau_t = \frac{1}{2} \) for all \( i \in \{1, \ldots, t\} \) and \( e_j = \frac{1 + \beta}{4} \) for all \( j \in \{1, \ldots, t-1\} \); otherwise, choose \( e_t \) according to the Markov strategies underlying Proposition 2.

Then:

(i) For any \( e_0 \), there exists a \( \beta_C \in (0, 1) \) such that this strategy and exploration rule form a sustainable equilibrium if and only if \( \beta \geq \beta_C \).

(ii) For any \( \beta \in (0, 1) \), there exists an \( e_C \) such that these strategies are not a sustainable equilibrium if \( e_0 > e_C \).

**Proof.** In Appendix E.  \[ \square \]

\[ ^1 \text{Chari and Kehoe (1990) discuss the problems related to subgame perfection, and show how the sustainable equilibrium is also a symmetric perfect Bayesian equilibrium.} \]
Statement (i) of the proposition expresses that a sufficiently patient government can endogenously commit to setting a Laffer-type tax in all periods. This equilibrium is sustained by the expectations of the firms. As long as the government keeps setting a low tax \( \tau = \frac{1}{2} \), firms expect this will continue into the future and hence explore at a relative high rate. The government constantly has the ‘temptation’ to increase taxes to obtain a large share of the old mines. However, should it do so, firms will change expectations about the government’s equilibrium play. More precisely, if the firms observe a high tax, then they will expect the oscillating equilibrium under the Markov strategy is being played and hence explore as if future taxes will oscillate. Given such expectations on the part of firms, it will indeed be optimal also for the government to set taxes that oscillate. Hence the economy switches to the Markov equilibrium with tax and investment cycles. Since oscillations imply a short run gain for the government when appropriating a large part of the old mines, but a lower long-run tax revenue than under the constant \( \tau = \frac{1}{2} \), only a sufficiently patient government will be able to sustain the constant tax as expressed by the proposition. As an illustration, consider a government with \( \beta \) approaching 1. For this government the revenues of the infinite future naturally overshadow any short-run gain, so the government seeks to maximize the long-run tax revenues. Conversely, a government with a low \( \beta \) puts low value on future tax revenues and hence immediately falls for the temptation of attaining a large share of the existing mines.

A closely related result is expressed in statement (ii) which says that a large existing tax base undermines the ability to endogenously commit. To see this, consider a government with a given \( \beta \). If this \( \beta \) is sufficiently high then the constant-tax equilibrium may be attainable. Of course, this relies on the government preferring the constant-tax equilibrium over the oscillating equilibrium. This will not be the case if the existing tax base is very large as the temptation to set a high tax in the first period would be too high.

The results presented here show that the model may account for why, in practice, different countries exhibit oscillating taxes while others do not. In particular, countries with more patient governments (for instance, due to low political turnover or due to shared basic objectives between competing parties) are less likely to exhibit cycles. Related to this, the result that a sufficiently low \( \epsilon_0 \) is required to enable the constant-tax equilibrium can be applied on an intuitive level to any period: a country that has managed to sustain a constant tax may renege by increasing its tax if a large discovery is made.

6 Tax differentiation

In this section we analyze the model in a situation where the government can differentiate taxes based on age – it can set separate taxes on old and new mines – and show that such complete tax differentiation would lead to an equilibrium without cycling. We compare the taxes a government sets in such a setting with the taxes in the baseline (impatient-government) setting. For conciseness, we assume here that \( p = 1 \) and \( \delta = \frac{1}{2} \).
Consider a government in a period $t$. It has the objective function

$$\max_{\tau_{t,\text{old}}, \tau_{t,\text{new}}} \tau_{t,\text{old}} \frac{1}{2} e_{t-1} + \tau_{t,\text{new}} \frac{1}{2} e_t \left( \tau_{t,\text{new}}, \tau_{t+1,\text{old}} \right)$$

(11)

where it chooses a tax, $\tau_{t,\text{old}}$, on the existing stock of mines and a tax, $\tau_{t,\text{new}}$, on new mines explored within the period. It is immediate that, the old mines being a completely inelastic tax base, the government will expropriate them by setting $\tau_{t,\text{old}} = 1$. Hence, $\tau_{t,\text{old}}$ is set independently of $\tau_{t,\text{new}}$. Likewise will be the case for the government in the next period, which will set $\tau_{t+1,\text{old}} = 1$. This, naturally, is foreseen by the representative firm that consequently faces the problem

$$\max_{e_t} (1 - \tau_{t,\text{new}}) \frac{1}{2} e_t - \frac{1}{2} e_t^2$$

which simply says that the firm chooses exploration effort by maximizing current-period revenues only. Taking the first-order condition we get

$$e_t^* = \frac{1 - \tau_{t,\text{new}}}{2}.$$ 

Since $\tau_{t,\text{old}}$ and $\tau_{t,\text{new}}$ are independent of each other, in choosing $\tau_{t,\text{new}}$, the government considers the reaction function of the firm and solves

$$\max_{\tau_{t,\text{new}}} \tau_{t,\text{new}} \frac{1}{2} \left( 1 - \tau_{t,\text{new}} \right)$$

which has the solution

$$\tau_{t,\text{new}}^* = \frac{1}{2},$$

i.e., a Laffer tax on new mines. This in turn implies

$$e_{\text{diff}} \equiv e_t^* = \frac{1}{4}$$

in all periods. Thus, in each period the government expropriates all old mines but sets a rather favorable tax on new mines. Note that there is no cycling in this case.

It is instructive to compare this outcome with the steady-state tax in the baseline case. From (9) we get $\tau_{ss} = \frac{2 + 3\delta}{3 + 2\delta} \in \left( \frac{2}{3}, \frac{8}{9} \right)$ when $\delta = \frac{1}{2}$. This tax is in between the old-mine and new-mine tax of the differentiating government. This way, if the government can tax-differentiate, it is able to better incentivize the firm based on revenues obtained in the current period. But this comes at the cost of being unable to incentivize the firm based on revenues arriving later since such revenues will be expropriated.

Will the two effects jointly imply more or less exploration compared to the non-differentiating case? When evaluating the two effects jointly, the low tax on new mines always overshadows the importance of the high tax on old mines as compared to the single tax in the baseline model. To see this, consider the steady-state exploration effort in the baseline
model. Using (9) in (2) yields
\[ e_{ss} = \left(1 - \frac{2}{3} \frac{2 + 2\beta}{2 + \beta}\right) \frac{1}{2} (1 + \beta). \] (12)

It is straightforward to show that \( e_{ss} \) is decreasing in \( \beta \) and that \( e_{ss}(0) = \frac{1}{6} \) and \( e_{ss}(1) = \frac{1}{9} \), which are both smaller than \( e_{diff} = \frac{1}{4} \). Hence, tax differentiation yields more exploration than non-differentiation. The intuition for this is somewhat subtle and we illustrate it for the polar cases of \( \beta \to 0 \) and \( \beta \to 1 \). Starting with the case of very impatient firms \( (\beta \to 0) \) it should be noted that such firms do not care at all about the revenues in the next period. This means that the tax tomorrow, whether high or low, does not play a role in the firm’s decision. But even so, since the firm obtains revenues from exploration in the current period there will exist old mines in the next period. These mines form an inelastic tax base for the government. To capture these revenues, the non-differentiating government has to raise taxes also on new mines while the tax-differentiating government can get hold of them while still incentivizing new investment (by setting a Laffer tax on new mines). Hence the steady-state tax of the non-differentiating government will be higher than the tax on new mines set by the differentiating government. Since this is the only tax the firm cares about, exploration will be higher when the government differentiates.

Consider now the case of very patient firms \( (\beta \to 1) \). These firms do care about the later revenues and are therefore disincentivized by the expropriatory tax levied on old mines by the tax-differentiating government. However, since these firms care relatively less about today, the non-differentiating government has less reason to set a low tax as the current-period tax works poorly in incentivizing new investments. The non-differentiating government consequently focuses on taxing the inelastic old mines and sets a high \( \tau_{ss} = \frac{2}{9} \). Now, while the tax-differentiating government also faces the problem of taxes on new mines working poorly in incentivizing exploration, it at least retains the possibility of setting this tax separately and hence optimally. This has a positive effect on investments. Thus, the flexibility of differentiated taxes outweighs the value of committing to not fully expropriate next period.

As a final step, we consider the total tax revenues obtained in the case of a differentiating and non-differentiating government respectively. Let \( R \) denote government revenues in a given period. In the differentiating-government case, plugging equilibrium taxes and exploration into (11) yields
\[ R_{diff} = \frac{3}{16}. \]

Similarly, in the non-differentiating government case, plugging the steady-state effort (12) and steady-state tax (9) into (3) yields
\[ R_{ss} = \frac{1}{9} \frac{(2 + 2\beta)(2 - \beta)(1 + \beta)}{(2 + \beta)^2}. \]

\[ ^{20} \text{This is closely connected to our earlier discussion following Corollary 2. Even without any pre-existing mines, the undifferentiated tax exceeds the Laffer tax as firms are patient (} \beta > 0 \text{). With pre-existing mines, the tax is even higher.} \]
which can be shown to be smaller than $R_{diff}$ for all $\beta \in (0, 1)$. The intuition for this follows from the previous results on exploration investment. Since exploration is always higher under the differentiating government which expropriates old mines while optimally taxing new mines, the total tax revenues have to be higher.

This analysis, perhaps surprisingly, suggests that differentiating taxes increases government revenues. Yet in reality full tax differentiation is seldomly observed, and tax and investment cycles persist. We have several hypotheses in mind for why full tax differentiation is rare (e.g., guaranteed expropriation of old mines in all periods could lead to reputational spillovers, a lack of continued capital investments in older production facilities, costs of shifting ownership and operations of old mines to the state, and moral hazard) and view this as an interesting question to explore in future research.

7 Conclusions

This paper has presented the first resource-taxation model with rational expectations by forward-looking firms who cannot commit to exiting a country and governments who cannot commit to tax rates. This is a highly policy-relevant problem, not only in developing countries lacking strong institutions, but also in developed countries.

We have shown how this model predicts repeated cycles of tax rates and investment, which is in line with the empirical reality in a large number of resource-producing countries. We provide two detailed case studies and multiple shorter examples of cyclical taxation. In these examples, the government sets a low tax rate to encourage exploration and investment. This leads to subsequent large discoveries whereby either a new high-tax party seizes power or the current government changes its policy to high taxation. This consequently deters new investment, implying few discoveries and the arrival of a new government with a low-tax platform, and so on. Thus tax cycles are a key driver of political turmoil and changes in political power and platforms and may be an important explanation of the political instability of resource-rich countries. Our model predicts that these cycles will be more pronounced for resources which take longer to develop. We have also analyzed how price changes and increasing government patience affect these tax oscillations. As such, the model offers many testable implications that can be taken to the data.

We also show how our model can sustain equilibria without tax oscillations. Hence, some countries may reach equilibria with cycles while others do not. This is consistent with the
observation that tax cycles are frequently observed across many countries and sectors, while there certainly exist exceptions.

Our model is cast in terms of exhaustible natural resources like oil, gas, metals, and gems, but applies to any setting in which capital is immobile and has a productive lifetime that exceeds the government’s commitment period. Examples from the broader resource sector would include renewable resources such as investments in land development, fisheries, timber or coffee plantations. Examples from the energy sector include power plants, wind farms, pipelines and refineries. Many infrastructure investments, such as privately operated ports, toll roads, railroads, buildings and telecommunications networks, are also long-lived and immobile. Finally, the model applies to capital-intensive manufacturing facilities, such as for automobiles, electronics or steel. Any of these sectors could in principle be subject to cycles following our model and it is well documented that such sectors also exhibit cyclical taxation and investment (e.g., Chua (1995) documents that railroads, electricity grids and other types of infrastructure and sectors have been subject to cycles in a large number of Asian and Latin-American countries).

Our model is rich enough to reflect tax and investment cycles with agents that hold rational expectations, but without the need to exogenously assume expropriations. It is also simple enough to serve as a starting point for further analysis of optimal natural resource taxation under imperfect commitment. For example, the model can be extended to consider changing land prospectivity, imperfect competition among resource extraction firms, different price expectations, endogenous extraction profiles and side payments to corrupt politicians. The model could also include any costs of expropriation – e.g., from reputation loss, international arbitration, and the loss of technological expertise if private investors leave following a full-scale nationalization and production and exploration are left to state-owned companies. One could also introduce more sophisticated taxation schemes that aim at getting around the commitment problem (e.g., exploration subsidies). Hence our model can also be used to study normative issues related to the structure of resource taxation.
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Appendices – online publication only

A Investment data

Figure 2 shows investment flows in oil and gas in Bolivia and Venezuela. Sectoral investment data are hard to collect in a consistent manner over long periods of time, especially for non-OECD countries. UNCTAD is the most comprehensive data source available and reports foreign direct investment in the oil and gas sector, starting in 1974 (Bolivia) and 1994 (Venezuela). For Venezuela, we obtained earlier data on total oil and gas investment flows from Monaldi (2001). Note that, from 1976 onwards, these data represent the sum of investments by national oil company PDVSA and foreign investments. The two series are therefore not perfectly comparable, but the pattern is still highly informative.

As described in Section 2, these data clearly illustrate the mechanism of our model. In both countries, investments are low when taxes are high and the investment regime is unfavorable, and vice versa. Specifically, investment increased with tax reforms in 1990 and especially the new fiscal terms in 1996. Resource nationalism in the early 2000s significantly depressed FDI, after which it stayed low in Venezuela (which has stuck to high taxes) but increased substantially in Bolivia when tax rates went down again in the late 2000s. Investment flows in earlier years are also consistent with our model and the fiscal regime.

B Proof of Lemma 1

Note first that $\delta \geq \frac{1}{2}$, so that the restriction on $\delta$ ensures $2 - \beta \left( \frac{1-\delta}{\delta} \right)^2 > 0$. Thus $A, D > 0$, $C < 0$, and the tax function is weakly increasing. It is also clearly continuous, so given any current tax $\tau_t \in [0, 1]$, the first-order condition (2) defines a unique $\tilde{e}_t(\tau_t)$. Suppose that this $\tilde{e}_t$ yields a next period tax strictly below unity. We can then substitute $\tau_{t+1} = Ae_t + B$ into the FOC, giving the equilibrium exploration as a linear function of $\tau_t$, with coefficients dependent on $A$ and $B$. Substituting this into the government’s FOC (4), we obtain an expression linear in $e_t$, with coefficients functions of $A$ and $B$. For the optimal tax to conform to the policy rule, the coefficient on $e_t$ must equal $A$ and the constant term must equal $B$. We can solve for $A$ and $B$, and substitute these to the expression for $\tilde{e}_t(\tau_t)$ to obtain $C$ and $D$.

To complete the proof, we must confirm that $\tilde{e}_t(\tau)$ indeed yields a tax below unity next period. To check this, we verify that $\tau_{t+1}(\tilde{e}(0)) < 1$, which holds given $\delta > \bar{\delta}$. Given $A > 0$, $C < 0$, then any $\tau > 0$ also yields an interior tax next period. Clearly for very high $e_{t-1}$, the tax policy does not yield an interior solution; given that the government’s problem is strictly concave, this implies for any such $e_{t-1}$ it is optimal to set $\tau_{t}^{*}(e_{t-1}) = 1$.

23 In fact, the spike in FDI from UNCTAD in 1997 is inconsistent with the reported total investment from Monaldi (2001). This is possibly explained by differences in reporting periods or practices, or the conversion between U.S. dollars and local currencies in a period of fluctuating exchange rates and high inflation.
C Equilibria for $\delta < \bar{\delta}(\beta)$

In this section, we demonstrate that piecewise-linear equilibria can also be constructed for some values of $\delta$ less than $\bar{\delta}(\beta)$. The firms’ equilibrium exploration rule is decreasing, but it may have a kink at $\tau = \tilde{\tau}$: for low values of the current tax $\tau \leq \tilde{\tau}$, the firms explore so much that the next period’s taxes hit 100% (see the black curves in Figure C.1). The government’s policy rule is increasing but may be discontinuous; for low values of $e_{t-1} < \hat{e}_1$, the government may set taxes $\tau^*(e_{t-1}) < \hat{\tau}$. These equilibria are illustrated in Figure C.1 and given formally by the following proposition.

**Proposition C.1.** Define $\hat{\delta}(\beta)$ implicitly by

$$
\sqrt{2 - \beta \left(1 - \frac{\delta}{\hat{\delta}}\right)^2} \left(\frac{1}{2} \beta \left(1 - \frac{\delta}{\hat{\delta}}\right)^2 + \left(1 - \frac{\delta}{\hat{\delta}}\right)^{-1}\right) = \sqrt{2} \left(\beta \frac{1 - \frac{\delta}{\hat{\delta}}}{\hat{\delta}} + 1 - \left(1 - \frac{\delta}{\hat{\delta}}\right)^{-1}\right).
$$

For $\delta \in (\delta, \bar{\delta})$, the following strategies constitute an equilibrium:

$$
e^*_t(\tau_t) = \begin{cases} 
(1 - \tau_t)\delta_p & \text{for } \tau_t \in [0, \tilde{\tau}] \\
C\tau_t + D & \text{for } \tau_t \in (\tilde{\tau}, 1)
\end{cases} \quad (C.1)
$$

$$
\tau^*_t(e_{t-1}) = \begin{cases} 
Ee_{t-1} + F & \text{for } e_{t-1} \leq \hat{e}_1 \\
Ae_{t-1} + B & \text{for } e_{t-1} \in (\hat{e}_1, \tilde{e}_2) \\
1 & \text{for } e_{t-1} \geq \tilde{e}_2
\end{cases} \quad (C.2)
$$

with the coefficients $A, B, C$ and $D$ given by (7),

$$E = \frac{1}{2\delta_p} \frac{1 - \delta}{\delta}, \quad F = \frac{1}{2}.$$
The thresholds \( \tilde{\tau} \) and \( \tilde{e}_1 \) are given by

\[
\tilde{\tau} = 1 - \frac{\delta}{1 - \delta} \frac{2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2}{2 + \beta \frac{1 - \delta}{\delta}} < 1
\]

and

\[
\tilde{e}_1 = \frac{\delta p}{2 + \beta \frac{1 - \delta}{\delta}} \left( \frac{1 - \delta}{\delta} \right)^{-2} \left( - \left( 2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2 \right) \right) + \left( \beta \left( \frac{1 - \delta}{\delta} \right)^2 + \frac{1 - \delta}{\delta} - 1 \right) \sqrt{4 - 2 \beta \left( \frac{1 - \delta}{\delta} \right)^2}. \tag{C.3}
\]

Proof. We first show that, given the government’s policy rule, equilibrium exploration will satisfy (C.1). Recall the representative firm’s FOC (2). We plot the average benefit and average cost of exploration in the left panel of Figure C.2, for a given \( \tau_t \), and given the government’s policy rule for \( \tau_{t+1}^* (e_t) \). Equilibrium exploration is given by the intersection, and changes in \( \tau_t \) shift the average benefit vertically. For \( \tau_t < \tilde{\tau} \), the AB curve lies very high and intersects AC in the horizontal section; that is, \( \tau_{t+1}^* = 1 \) and the FOC yields the claimed equilibrium extraction. For \( \tau_t \in (\tilde{\tau}, 1] \), the curves intersect in the region \( e_t \in [\tilde{e}_1, \tilde{e}_2) \), so that \( e_t^* = C \tau_t + D \) as claimed. The threshold \( \tilde{\tau} \) is derived from setting \( \lim_{\tau_t \downarrow \tilde{\tau}} \tau_{t+1}^* (e^* (\tau_t)) = 1 \), or \( \tilde{\tau} = \frac{1 - \beta - AD}{AC} \). The restriction \( \delta > \delta (\beta) \) is derived from the requirement that even a 100% tax puts the economy on the converging path next period, or \( e^* (1) = C + D > \tilde{e}_1 \). The steady state is always stable as \( \delta' (\beta) > 0 \) and \( \delta (0) = \frac{1}{3} \) (ensuring the tax transition curve will always have a slope less than unity in absolute value).

Now let us demonstrate that, given the equilibrium exploration rate, the government will optimally set taxes as according to (C.2). Note first that

\[
\frac{de_t^*}{d\tau_t} = - \frac{p\delta}{1 + p\beta(1 - \delta) \frac{d\tau_{t+1}^*}{de_t^*}} < 0
\]

so that the firms’ equilibrium extraction function is steeper for low \( \tau_t \): for low taxes, next-period taxes are capped at unity, so that a small change in current period taxes is not offset at all by a response in the next-period tax (which are unresponsive, on the margin, to changes in \( e^* (\tau_t) \)). Note also that, as equilibrium exploration is continuous, so is the government’s profit. Thus we can obtain the government’s profits from integrating the difference between marginal benefit and marginal cost of increasing \( \tau_t \), as given by by the LHS and RHS of (4), respectively. We plot these in Figure C.2.

A change in \( e_{t-1} \) shifts the MB curve vertically, leaving MC unchanged. There is a level of \( e_{t-1} = \hat{e}_1 \) such that the profits are equal for either intersection (consider the sizes of the triangles formed by the MB and MC lines and the line \( \tau = \hat{\tau} \)). For \( e_{t-1} < \hat{e}_1 \), it is optimal to set \( \tau^* < \hat{\tau} \). For \( e_{t-1} > \hat{e}_1 \), it is optimal to set \( \tau > \hat{\tau} \). Of course, for very high \( e_{t-1} > \hat{e}_2 \), the taxes are optimally set at the upper limit \( \tau^* = 1 \), with the threshold given
by \( \lim_{\epsilon \to \epsilon_2} \tau^*(\epsilon) = 1 \), or \( \epsilon_2 = \frac{1-B}{A} \).

It is straightforward to confirm that, for \( \epsilon_{t-1} \leq \epsilon_1 \), i.e. setting \( \tau^* < \tilde{\tau} \), the resulting optimal tax rate is given as per (C.2). The case \( \epsilon_{t-1} > \epsilon_1 \) is essentially the case we dealt with in Section 3. What then remains to do is to calculate \( \tilde{\epsilon}_1 \), i.e. the point at which the government obtains equal tax revenues from the optimal taxes below and above \( \tilde{\tau} \). For this, we calculate the values given taxes are set optimally. For \( \tau < \tilde{\tau} \),

\[
V(e_{t-1}, \tau) = \tau^*_{e_{t-1}} \epsilon_{t-1} \left( 1 - \delta \right) e_{t-1} + \delta \left( 1 - \tau^*_{e_{t-1}} \epsilon_{t-1} \right) \delta \rho \left( \left( 1 - \delta \right) e_{t-1} + \rho \delta^2 \left( \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} e_{t-1} \right) \right) \rho
\]

\[
= \left( \frac{1}{2} + \frac{1}{2} \frac{1 - \delta}{\delta} e_{t-1} \right) \left( 1 - \delta \right) e_{t-1} + \rho \delta^2 \left( \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} e_{t-1} \right) \rho
\]

\[
= \left( \delta \rho \left( \frac{1}{2} + \frac{1}{2} \frac{1 - \delta}{\delta} e_{t-1} \right) \right)^2.
\]

For \( \tau > \tilde{\tau} \),

\[
V(e_{t-1}, \tau) = \tau^*_{e_{t-1}} \epsilon_{t-1} \left( 1 - \delta \right) e_{t-1} + \delta \left( C \tau^*_{e_{t-1}} \epsilon_{t-1} + D \right) \rho
\]

\[
= \left( \frac{-D}{2C} - \frac{1}{2C} \frac{1 - \delta}{\delta} e_{t-1} \right) \left( 1 - \delta \right) e_{t-1} + \delta \rho \frac{1 - \delta}{\delta} e_{t-1} - \frac{1}{2} \frac{1 - \delta}{\delta} e_{t-1} \rho
\]

\[
= -\rho \delta C \left( \frac{-D}{2C} - \frac{1}{2C} \frac{1 - \delta}{\delta} e_{t-1} \right)^2
\]

\[
= \left( \rho \delta^2 \right)^2 \left( \frac{2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2}{2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2} - 1 \right) \left( \frac{1 + \beta \frac{1 - \delta}{\delta}}{2 + \beta \frac{1 - \delta}{\delta}} \right)^2 \left( \frac{1 + \beta \frac{1 - \delta}{\delta}}{2 + \beta \frac{1 - \delta}{\delta}} \right)^2.
\]

At \( \tilde{\epsilon}_1 \), these are equal to each other. This implies a quadratic in \( \tilde{\epsilon}_1 \):

\[
\beta \left( \frac{1 - \delta}{\delta} \right)^4 \tilde{\epsilon}_1^2 + \rho \delta \left( \frac{1 - \delta}{\delta} \right)^2 \left( 2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2 \right) \left( \frac{1 + \beta \frac{1 - \delta}{\delta}}{2 + \beta \frac{1 - \delta}{\delta}} \right)^2 \tilde{\epsilon}_1
\]

\[
+ \left( \rho \delta \right)^2 \left( 2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2 \right) \left( 2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2 \right) \left( \frac{1 + \beta \frac{1 - \delta}{\delta}}{2 + \beta \frac{1 - \delta}{\delta}} \right)^2 - 1 \right) = 0
\]

(C.4)

which has the discriminant

\[
D = \left( \frac{2 \beta^2}{2 + \beta \frac{1 - \delta}{\delta}} \right)^2 \left( \frac{2 - \beta \left( \frac{1 - \delta}{\delta} \right)^2}{2 + \beta \frac{1 - \delta}{\delta}} \right)^2 \left( \frac{1 - \delta}{\delta} \right)^4
\]

\[
\cdot \left( 1 - 2 \frac{1 - \delta}{\delta} + (1 - 2 \beta \left( \frac{1 - \delta}{\delta} \right)^2 + 2 \beta \left( \frac{1 - \delta}{\delta} \right)^3 + \beta^2 \left( \frac{1 - \delta}{\delta} \right)^4 \right).
\]

This has the same sign as the expression in the last brackets. We can introduce a new variable \( Q \equiv \frac{1 - \delta}{\delta} + \frac{1 - \delta}{\delta} \), and show that the bracketed expression equals \( \beta^2 \left( Q^2 - \frac{1 + 2 \beta}{2 \beta} \right)^2 \geq 0 \).

Thus the discriminant of (C.4) is weakly positive, so that \( \tilde{\epsilon}_1 \) has either two real roots (one negative, one undetermined) or a single (negative) root. Only a positive \( \tilde{\epsilon}_1 \) makes sense. Furthermore, from (C.4), for \( e > \tilde{\epsilon}_1 \), the profits for the \( \tau > \tilde{\tau} \) regime always exceed the
profits for the $\tau < \tilde{\tau}$ case. The positive root is straightforward to calculate, and equals the expression for $\tilde{e}_1$ given in the proposition. If there is no positive root, then the government’s policy is not discontinuous; even for $e_{t-1} = 0$, it will be optimal to set taxes high enough so that the next period government’s response to equilibrium exploration is to set taxes still below unity (see the middle panel in Figure C.1).

\[ \Box \]

D Details of the patient government case

We here construct the equilibrium for the case with a patient government. We simplify by setting the government’s discount factor equal to the private discount factor: $\beta^G = \beta$. We also normalize $p = 1$, to reduce clutter. To the same end, we also define

\[ x = \frac{1 - \delta}{\delta}. \]

The structure of the proof is exactly analogous to that of Lemma 1: we guess and verify piecewise-linear equilibrium policy rules, imposing a parameter restriction to avoid further complications. More specifically, we conjecture that a strategy and exploration rule of the type (6) solve problems (1) and (10), and proceed to determine the resulting coefficients $A, B, C$ and $D$.

We can apply the envelope theorem to the government’s value function (10) to get

\[ V'(e_{t-1}) = (1 - \delta)\tau^*_t \geq 0. \tag{D.1} \]

Supposing the next period’s government will follow a linear policy function, the first-order condition now takes into account the effect of present exploration on future tax revenues:

\[ (1 - \delta)e_{t-1} + \delta e^*_t (\tau^*_t) + \delta \tau^*_te^*_{t+1}(\tau^*_t) + \beta(1 - \delta)\tau^*_t e^*_{t+1}(\tau^*_t) = 0, \tag{D.2} \]

in which the last term reflects the change in continuation value (obtained using (D.1)).
Substituting in the (supposed linear) policy function and equilibrium exploration effort into (D.2), and solving for \( \tau_t \), we can obtain the coefficients \( A \) and \( B \); we obtain \( C \) and \( D \) by substituting the policy rules into (2) and solving for \( e_t^* \). This yields the system of equations

\[
\begin{align*}
A &= -\frac{1}{C} \frac{x}{2 + \beta x AC} \\
B &= -\frac{1}{C} \frac{D + \beta x (B + AD)C}{2 + \beta x AC} \\
C &= -\frac{1}{1 + \beta (1 - \delta) A} \\
D &= \frac{\delta + \beta (1 - \delta)(1 - B)}{1 + \beta (1 - \delta) A}.
\end{align*}
\]

We can solve for the coefficients. This involves a quadratic in \( AC \), and we rule out one of the roots as it satisfies \( AC < -1 \), which would yield an unstable tax transition; such a case does not conform our assumption that the next period tax rate is always given by the rule \( \tau^*(e) = Ae + B \). The explicit expressions for the coefficients are

\[
\begin{align*}
A &= \frac{1}{1 - \delta} \frac{1 - \sqrt{1 - \beta x^2}}{\beta \sqrt{1 - \beta x^2}} \\
B &= \frac{\sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2}} \\
C &= -\delta \sqrt{1 - \beta x^2} \\
D &= \delta \sqrt{1 - \beta x^2} \frac{1 + \beta x + \sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2}}.
\end{align*}
\]

The tax transition is still \( \tau^*_{t+1}(e_t^*(\tau_t)) = AD + B + AC \tau_t \), and the steady state tax is given by \( \frac{AD + B}{1 - AC} \). We have to impose three restrictions. For the government’s problem to have a solution, we require \( 1 - \beta x^2 \geq 0 \). For the tax transition to be stable, we require \( AC > -1 \). Finally, we must ensure that a zero tax leads to an exploration rate which is then followed by a tax rate strictly below unity, we require \( AD + B < 1 \). This yields the restriction \( x < \frac{1}{\sqrt{1 + \beta}} \), or

\[
\delta > \frac{1}{1 + \frac{1}{\sqrt{1 + \beta}}}.
\]

In fact, (D.3) is sufficient to guarantee the other two restrictions also hold.

We also need the expression for the value function for the proof of Proposition 3. From (D.1), we know that the first derivative of the value function is linear in the state \( e_{t-1} \). We thus expect the value function to be quadratic in the region where \( \tau^*_t(e_{t-1}) < 1 \). So conjecture

\[
V(e_{t-1}) = Z_2 e_{t-1}^2 + Z_1 e_{t-1} + Z_0.
\]

\[\text{24}\] The value function is linear for higher \( e_{t-1} \) and easily calculated, but we do not need it.
We can find the coefficients $Z_2, Z_1$ and $Z_0$ by first calculating the value from the definition by substituting in the optimal policies:

\[
V(e_{t-1}) = \tau^*(e_{t-1})((1 - \delta)e_{t-1} + \delta e^*(\tau^*(e_{t-1}))) + \beta V(e^*(\tau^*(e_{t-1})))
\]

\[
= (Ae_{t-1} + B)((1 - \delta)e_{t-1} + \delta (ACe_{t-1} + BC + D))
\]

\[
+ \beta (Z_2(ACE_{t-1} + BC + D)^2 + Z_1(ACE_{t-1} + BC + D) + Z_0)
\]

Once we put the previous two equations together, we can then equate the coefficients on the quadratic, linear and constant terms of $e_{t-1}$. These are easily solved:

\[
Z_2 = \frac{\delta A(x + AC)}{1 - \beta AC}^2,
\]

\[
Z_1 = \frac{\delta (2ABC + AD + xB) + 2\beta AC(CB + D)Z_2}{1 - \beta AC}.
\]

\[
Z_0 = \frac{\delta B(CB + D) + \beta (CB + D)^2Z_2 + \beta (CB + D)Z_1}{1 - \beta}.
\]

Inserting the expression for $A, B, C$ and $D$, we get

\[
Z_2 = \frac{1}{2} \frac{1 - \delta x^2}{\beta \sqrt{1 - \beta x^2}},
\]

\[
Z_1 = \frac{\delta}{\beta x} (1 - \sqrt{1 - \beta x^2}) \sqrt{1 - \beta x^2},
\]

\[
Z_0 = \frac{1}{2} \frac{\delta^2 (1 + \beta x)^2 \sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2}}.
\]

Proof of Proposition \(\Box\) We see from the tax transition and $AC < 0$ that the tax oscillations occur even with a patient government. Deviations from the steady state die out at the rate

\[
\left| \frac{\tau_{t+1}^* - \tau_{ss}}{\tau_{t} - \tau_{ss}} \right| = |AC|.
\]

From (D.6), it is clear that deviations from the steady state decay faster, the lower is $AC$ in absolute value. We can differentiate $|AC|$ with respect to $\beta$:

\[
\frac{d|AC|}{d\beta} = - \frac{1 - \beta x^2 - \sqrt{1 - \beta x^2}}{\beta^2 x \sqrt{1 - \beta x^2}} < 0
\]

as it is easy to confirm that the numerator is positive. Thus, an increase in the discount factor makes the decay rate slower. \(\Box\)

The reason for the higher persistence of cycles under higher patience is that the rate at which tax cycles disappear is decreasing in the reactivity of the government and the market: highly responsive taxes or investment will cause more persistent cycles. The government indeed becomes more responsive when $\beta$ is high. To see why, take future governments’
decision rules, and the firms’ exploration function, as given. A government today optimally
equates the marginal benefit of increasing the tax rate – the total size of a period’s tax base
– with the marginal cost of lowering exploration, thus shrinking the tax base. For a patient
government, shrinking the tax base is undesirable because it lowers the tax take today and
in the next period. However, as the next period’s government will lower the tax rate in
response to a smaller tax base, this marginal cost is partially offset: higher taxes today
mean lower taxes tomorrow, so that lower investment today hurts less. A more patient
government perceives this offsetting effect on future revenues as more important, so that
high government patience implies a relatively flat marginal cost curve, and thus a greater
tax response to a shift of the marginal benefit curve. Such a shift would result from a change
in the stock of pre-existing mines.

The firms, on the other hand, become less responsive when the discount factor rises.
This is because they weigh future revenues relatively more, so that changes in current tax
rates are less important in determining investment.

In equilibrium, of course, all policy rules adjust. A more responsive tax policy in the
future will constrain the equilibrium exploration rate more tightly. This is because a reduc-
tion in taxes today has a weaker effect on exploration if the next government is expected to
respond to more investment by raising taxes a lot next period. Thus exploration becomes
even less responsive to current taxes \( \frac{\partial C}{\partial \beta} > 0 \), recalling \( C < 0 \). This change in the market
response further increases the government’s incentives to set a highly responsive tax policy:
if raising taxes today has less of an effect on investment, there is a greater incentive to set
them high (if the pre-existing stock is high), reinforcing the direct effect of higher patience.
Overall, then, tax policy is certainly more responsive: \( \frac{\partial A}{\partial \beta} > 0 \). More responsive tax policy
and less responsive investment have countervailing effects on the overall persistence of tax
cycles, but the former dominates the latter, so that increasing government patience makes
cycles more persistent.

E Proof of Proposition 3

The proposed strategies form an equilibrium as longs as the government does not wish to
deviate from the equilibrium strategy, and the exploration firms find it optimal to follow
the exploration rule. The purported equilibrium is constructed from two ‘regimes’: the
‘commitment’ regime under which the government always sets taxes equal to \( \frac{1}{2} \); and a
‘punishment’ regime, which is the Markov-perfect Nash equilibrium (MPNE). A Markov
equilibrium is a special case of sustainable equilibrium—one in which the history dependence
reduces to the dependence only on the current state variable \( e_{t-1} \) (for the government) and
the current tax \( \tau_t \) (for the firm). Thus Proposition 2 establishes that the punishment regime
is, indeed, a sustainable equilibrium, for all histories. Hence, it remains to obtain conditions
under which

(a) no firm would want to deviate in the ‘commitment’ regime;
(b) the government would not want to deviate from the steady state of the commitment regime (which is reached in the period after the initial period); and

(c) the government would not want to deviate in the initial period 0.

Following Proposition 2, we will denote the Markov strategies by \( \tau^*(e) = Ae + B \) and \( e^*(\tau) = C\tau + D \), and the associated MPNE value function by \( V(e) = Z_2e^2 + Z_1e + Z_0 \), with

\[
A = \frac{1}{1 - \delta} \frac{1 - \sqrt{1 - \beta x^2}}{\beta \sqrt{1 - \beta x^2}},
\]
\[
B = \frac{\sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2}},
\]
\[
C = -\delta \sqrt{1 - \beta x^2},
\]
\[
D = \delta \sqrt{1 - \beta x^2} \frac{1 + \beta x + \sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2}},
\]
\[
Z_2 = \frac{1}{2} \frac{1 - \sqrt{1 - \beta x^2}}{\beta \sqrt{1 - \beta x^2}},
\]
\[
Z_1 = \frac{\delta}{\beta x} (1 - \sqrt{1 - \beta x^2}) \sqrt{1 - \beta x^2},
\]
\[
Z_0 = \frac{1}{2} \frac{\delta^2 (1 + \beta x)^2 \sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2}}.
\]

The initial period is taken to be 1. The period-1 value for following the ‘commitment’ regime is denoted by \( V_1^C \), and the value obtained in the steady state of the commitment regime (reached by period 2) is denoted by \( V_t^C, t > 1 \).

In the steady state, the tax rate is constant at \( \tau^{**} = \frac{1}{2} \), and from the firms’ FOC, equilibrium exploration equals

\[
e^{**} = \delta \tau^{**} (1 + \beta x).
\]

This is the unique optimum for the firms, given that firms expect the next period government to follow the purported tax rule, ensuring [a] holds.

We next derive conditions for [b], i.e. that the government does not have a profitable deviation from the steady state of the commitment regime if and only if \( \beta \) is sufficiently high. The government’s flow payoff in the ‘commitment’ regime is \( \tau^{**} e^{**} = \frac{1}{2} e^{**} \), and total discounted payoff is

\[
V_t^C = \frac{1}{1 - \beta} \frac{1}{2} e^{**}, t > 1.
\]

Clearly, as any deviation leads to a Markov continuation, the Markov strategy is the optimal deviation given any initial \( e_0 \). Thus, the difference in the payoffs between the commitment
regime and the optimal deviation is

\[ \Delta V_t \equiv V_t^C(e^{**}) - V(e^{**}) \]

\[ = \frac{1}{2(1 - \beta)} e^{**} - Z_2 e^{**} - Z_1 e^{**} - Z_0 \]

\[ = \frac{\delta(1 + \beta x)}{8(1 - \beta)} \left( 2 - \frac{x^2}{1 + x} (1 - \beta) (1 + \beta x) + 4(1 - \beta x^2) \right) \]

(E.7)

which we obtain by substituting in the above expressions for the coefficients, for \( e^{**} \) and for \( V_t^C \); and using \( \delta = \frac{1}{1 + x} \). It is straightforward to verify that this expression is negative in the limit \( \beta \to 0 \), and goes to infinity as \( \beta \to 1 \). The expression is also continuous in \( \beta \) for admissible parameter values. The value is greater than zero when the term in the brackets is greater than zero, equivalent to

\[ 2(1 + x) \sqrt{1 - \beta x^2} - x^2(1 - \beta)(1 + \beta x) - 2(1 + x)(1 - \beta x^2) \geq 0 \]  

(E.8)

We will show the LHS of (E.8) increases monotonically with \( \beta \). Differentiating with respect to \( \beta \):

\[ \frac{\partial \text{LHS}}{\partial \beta} = x^2 \left( \frac{1 + x}{\sqrt{1 - \beta x^2}} + 3 + x + 2\beta x \right) \]

Note that as \( x^2 < \frac{1}{1 + x} \), \( 1 - \beta x^2 > \frac{1}{1 + x} \), so that \( \sqrt{1 - \beta x^2} > \frac{1}{\sqrt{1 + x}} > \frac{1}{2} \). Thus \( \frac{1 + x}{\sqrt{1 - \beta x^2}} < 2(1 + x) < 3 + x + 2\beta x \), so that the LHS increases monotonically. Together with continuity, and the fact that \( \Delta V_t \) is negative for \( \beta \to 0 \) and is positive as \( \beta \to 1 \), implies that the difference (E.7) changes sign at most once. For values of \( x \) close to 1, \( \Delta V_t < 0 \) for any admissible \( \beta \). However, for larger \( x \), \( \Delta V_t > 0 \) iff \( \beta \) is sufficiently high.

Thus we have shown that, for any high enough \( \delta \), there is a unique \( \beta_1 \) such that, for all \( \beta < \beta_1 \), the Markov payoff from the steady state of the purported commitment equilibrium would yield a higher payoff than following the strategies suggested in the proposition, so that a profitable deviation would exist; and vice versa for \( \beta > \beta_1 \).

We finally turn to (c) and show that the strategies yield a higher payoff in the initial period if and only if \( \beta \) is sufficiently high. We first tackle the case in which the initial stock is not so high that the Markov tax would hit unity. Then, the difference in the initial period
payoffs is

\[ \Delta V_1(e_0) \equiv V_1^C(e_0) - V(e_0) \]

\[ = \frac{1}{2} ((1 - \delta)e_0 + \delta e^*) - \tau^*(e_0) ((1 - \delta)e_0 + \delta e^*(\tau^*(e_0))) \]

\[ + \beta \left( V_{i+1}^C - V(e^*(\tau^*(e_0))) \right) \]

\[ = \delta \left( \left( \frac{1}{2} - Ae_0 - B \right) xe_0 + \frac{1}{2} e^* \right) \]

\[ - (Ae_0 + B) (C(Ae_0 + B) + D) \right) + \beta \frac{1}{2(1 - \beta)} e^* \]

\[ - \beta(Z_2(C(Ae_0 + B) + D)^2 + Z_1(C(Ae_0 + B) + D) + Z_0) \]

This expression can be solved to obtain25

\[ \Delta V_1(e_0) = -\frac{1 - \sqrt{1 - \beta x^2}}{2\beta \sqrt{1 - \beta x^2}} e_0^2 + \frac{\delta x}{2} 1 - \frac{\sqrt{1 - \beta x^2}}{1 + \sqrt{1 - \beta x^2} e_0} \]

\[ + \left( \frac{\delta (1 + \beta x)}{2(1 + \sqrt{1 - \beta x^2})} \right)^2 \frac{\beta x^2}{1 - \beta}. \]

(E.9)

Fixing \( e_0 \) and \( \delta \), the second and third terms are positive and, by inspection, can be seen to increase in \( \beta \). To investigate the behaviour of the first coefficient we differentiate it:

\[ \frac{\partial}{\partial \beta} \left( -\frac{1 - \sqrt{1 - \beta x^2}}{2\beta \sqrt{1 - \beta x^2}} \right) = \frac{2(1 - \beta x^2)(1 - \sqrt{1 - \beta x^2}) - \beta x^2}{4\beta^2(1 - \beta x^2)\sqrt{1 - \beta x^2}} \]

which has the sign of the numerator. Thus, the coefficient increases in \( \beta \) if

\[ -2(1 - \beta x^2)\sqrt{1 - \beta x^2} > \beta x^2 - 2(1 - \beta x^2). \]

This will never hold if the RHS is positive, since the LHS is always negative (since \( \beta x^2 < \frac{1}{2} < 1 \)). First note that the RHS cannot be positive, since that would require that \( x^2 > \frac{2}{\delta^2} \), which will not hold with our restriction \( x^2 < \frac{1}{1+\beta} \). We can then solve the inequality, to find it will hold if \( x^2 < \frac{3}{4\beta} \), which will always hold as we have assumed \( x^2 \leq \frac{1}{1+\beta} < \frac{3}{4\beta} \).

Thus, \( \Delta V_1(e_0) \) is always increasing in \( \beta \). It is easy to show that \( \Delta V_1(e_0) \) is strictly negative as \( \beta \to 0 \) and goes to \(+\infty\) for \( \beta \to 1 \) (see (E.9)). Thus, for any \( \delta \) and \( e_0 \) such that the Markov taxes are strictly interior, there is a unique \( \beta_2 \) such that for \( \beta < \beta_2 \) the Markov strategy provides a higher initial payoff, and for \( \beta > \beta_2 \), the commitment strategy provides

\[ Z_2(BC + D)^2 + Z_1(BC + D) + Z_0 = \frac{\delta^2(1 + \beta x)\sqrt{1 - \beta x^2}}{2\beta(1 - \beta) \left( 1 + \sqrt{1 - \beta x^2} \right)^2} \]

\[ \times \left( 1 + \beta x + \sqrt{1 - \beta x^2}(-1 + 2\beta + \beta x) \right). \]

25Simply substituting and collecting terms. It is useful to note that \( \delta = \frac{1}{1+\beta} \), and that
a higher payoff.

What remains is to show the same for very high initial stocks, so that \( \tau^*(e_0) = 1 \). The difference in the initial period values is then

\[
\Delta V_1(e_0) \equiv V_1^C(e_0) - V(e_0)
\]

\[
= \delta \left( \frac{1}{2} (xe_0 + e^{**}) - 1 (xe_0 + e^*(1)) \right) + \beta \left( V_2^C - V(e^*(1)) \right)
\]

\[
= \delta \left( \frac{1}{2} xe_0 + \frac{1}{2} e^{**} - 1 (C + D) \right) + \beta \frac{1}{2(1 - \beta)} e^{**}
\]

\[
- \beta (Z_2(C + D)^2 + Z_1(C + D) + Z_0)
\]

(E.10)

where \( e^*(1) = C + D \). We can tidy up the expression to obtain

\[
\Delta V_1 = -\frac{\delta}{2} xe_0 + \delta^2 \left( \frac{(1 + \beta x)^2}{4(1 - \beta)} - \frac{\beta}{1 - \beta} \frac{(1 + 2x + \beta x^2)\sqrt{1 - \beta x^2}}{2(1 + \sqrt{1 - \beta x^2})} \right)
\]

\[
= -\frac{\delta}{2} xe_0 + \frac{\delta^2}{4} \frac{(1 + \beta x)^2 + \sqrt{1 - \beta x^2}(1 - 2\beta - 2\beta x - \beta^2 x^2)}{(1 - \beta)(1 + \sqrt{1 - \beta x^2})}
\]

(E.11)

Observe first that

\[
\Delta V_0|_{\beta \to 0} = -\frac{\delta}{2} xe_0 + \delta^2 \leq -\frac{\delta^2}{2} + \frac{\delta^2}{4} < 0
\]

where we have used the fact that we are focusing on cases with \( \tau^*(e_0) = 1 \), i.e. \( e_0 > \frac{1-B}{A} \), implying

\[ xe_0 > \delta \sqrt{1 - \beta x^2}. \]  

(E.12)

Also, \( \lim_{\beta \to 1} \Delta V_1 = \infty \) as the second term in (E.11) is divided by \( (1 - \beta) \). Hence, for very low discount factors, the Markov policy always yields a higher payoff, and for high enough \( \beta \), the commitment regime does. We now proceed to show that \( \Delta V_1 \) can change signs at most once.

We can differentiate (E.11) with respect to \( \beta \) to obtain

\[
\frac{\partial \Delta V_1}{\partial \beta} = \frac{\delta^2}{4} \frac{2 - 2x - \frac{1}{2} x^2 + \beta x^2 + 3 \beta x^3 + \frac{5}{2} \beta^2 x^4 + \sqrt{1 - \beta x^2}(2x + 2\beta x^2)}{\sqrt{1 - \beta x^2}(1 - \beta)(1 + \sqrt{1 - \beta x^2})}
\]

\[
- \frac{\delta^2}{4} \frac{(1 + \beta x)^2 + \sqrt{1 - \beta x^2}(1 - 2\beta - 2\beta x - \beta^2 x^2)}{(1 - \beta)^2(1 + \sqrt{1 - \beta x^2})^2}
\]

\[
\times \frac{-1 - \frac{1}{2} x^2 + \frac{3}{2} \beta x^2 - \sqrt{1 - \beta x^2}}{\sqrt{1 - \beta x^2}}
\]

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We consider the sign of this expression at any point with $\Delta V_1 = 0$. First note that the second line equals the second term in (E.11) divided by $(1 - \beta)(1 + \sqrt{\cdot})$. Hence

$$
\left. \frac{\partial \Delta V_1}{\partial \beta} \right|_{\Delta V_1 = 0} = \frac{\delta^2}{4\sqrt{1 - \beta x^2}(1 - \beta)(1 + \sqrt{1 - \beta x^2})} \left( -2 - 2x - \frac{1}{2} x^2 + \beta x^2 + 3\beta x^3 + \frac{5}{2} \beta^2 x^4 + \sqrt{1 - \beta x^2}(2x + 2\beta x^2) \right.

+ \left. \frac{2x e_0}{\delta} \left( 1 + \frac{1}{2} x^2 - \frac{3}{2} \beta x^2 + \sqrt{1 - \beta x^2} \right) \right).
$$

We now again use (E.12) to show that

$$
\left. \frac{\partial \Delta V_1}{\partial \beta} \right|_{\Delta V_1 = 0} > \frac{\delta^2}{4\sqrt{1 - \beta x^2}(1 - \beta)(1 + \sqrt{1 - \beta x^2})} \left( -2x - \frac{1}{2} x^2 - \beta x^2 + 3\beta x^3 + \frac{5}{2} \beta^2 x^4 + \sqrt{1 - \beta x^2}(2 + 2x + x^2 - \beta x^2) \right).
$$

The RHS is positive if

$$
\sqrt{1 - \beta x^2}(2 + 2x + x^2 - \beta x^2) > 2x + \frac{1}{2} x^2 + \beta x^2 - 3\beta x^3 - \frac{5}{2} \beta^2 x^4
$$

(E.13)

We will show that

$$
\sqrt{1 - \beta x^2}(2 + 2x + x^2 - \beta x^2) > \frac{2x + \frac{1}{2} x^2 + \beta x^2 - 3\beta x^3 - \frac{5}{2} \beta^2 x^4}{\sqrt{1 - \beta x^2}}
$$

which ensures (E.13) holds, as the RHS is always weakly greater than the RHS of (E.13). Multiplying through by $\sqrt{1 - \beta x^2}$, this condition is equivalent to

$$
4 + x^2 - 8\beta x^2 + 2\beta x^3 - 2\beta x^4 + 7\beta^2 x^4 > 0.
$$

It is straightforward to demonstrate that, in the closure of the square $x \in [0, 1], \beta \in (0, 1)$, the only critical points of the LHS are at $x = 0$. As the LHS in continuous in this region, any extreme values have to lie on one of the four edges of the square. Thus we only have to check the minimum values at the edges; the lowest of these is at $(x, \beta) = (0.7, 1)$, at which point the LHS takes the value of $2.457 > 0$. Thus, the inequality holds everywhere, ensuring (E.13) holds also, so that $\Delta V_1$ is increasing in $\beta$ at any point at which $\Delta V_1 = 0$.

Thus, for any $\delta, e_0$ there exists a unique $\beta_3$ such that for $\beta < \beta_3$ the initial payoff from the Markov strategy exceeds the payoff from the commitment regime, and vice versa. This shows that there exists a $\beta_3 \in (0, 1)$ such that for $e_0$ for which $\tau^*(e_0) = 1$, the commitment tax is preferred over the Markov tax.

Define

$$
\beta_{23}(e_0) = \begin{cases} 
\beta_2 & \text{if } \tau^*(e_0) < 1, \\
\beta_3 & \text{otherwise.}
\end{cases}
$$
Endogenous commitment is a SPNE iff $\beta > \beta_C \equiv \max\{\beta_1, \beta_2, \beta_3(e_0)\} \in (0, 1)$ which has a unique solution since $\beta_1, \beta_2, \beta_3$ are all unique for any given $\delta, e_0$. We also need to ensure that the resulting pair $(\beta_C, \delta)$ lies in the feasible region for the patient government case,

$$x \leq \frac{1}{1 + \beta},$$

(E.14)

For low $e_0$, the critical value is $\beta_1$; the intersection of (E.14) and (E.8) is given by $\beta^3 + 2\beta - 1 = 0$, implying that $\delta$ has to be higher than $.547$. For higher $e_0$, first $\beta_2$ and eventually $\beta_3$ become binding. The resulting $\beta_C$ shifts right for any $\delta$; in the limit as $e_0$ becomes arbitrarily high, feasibility is guaranteed by (E.14) for $\beta \to 1$, implying $\delta > \frac{\sqrt{2}}{1 + \sqrt{2}} \approx .586$. This proves Statement (i).

Statement (ii) can easily be seen to be true by inspection of (E.10). For any $\beta, \delta$, the expression is negative for sufficiently large $e_0$. \qed