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**A New Type of Model  
for Multi-Item Capacitated Dynamic  
Lotsizing and Scheduling**

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**Abstract:** Three types of multi-item capacitated dynamic lot sizing and scheduling models are well established until now: The capacitated lot sizing problem, the discrete lot sizing and scheduling problem as well as the continuous setup lot sizing problem. An analysis of the underlying fundamental assumptions provides the basis for introducing a new model type, the proportional lot sizing and scheduling problem. This model type is well suited for incorporating some of the extensions relevant with respect to practice: Setup times, sequence dependent setup costs (times), multiple machines as well as multiple stages. Last but not least we show how make-or-buy decisions may be modelled within this framework. This is relevant regarding three important aspects: First it is more general, second it is of practical relevance, third it provides the basis for overcoming difficulties associated with feasibility problems. The last fact opens up new avenues especially for the development of efficient local search procedures.

**Keywords:** Production planning, lot sizing, scheduling, setup times, multiple machines, multiple stages, make-or-buy decisions

## 1. Introduction

Production decisions in manufacturing have to determine how machines and other resources should be used for the production of items. These decisions are often separated into the aggregated planning level (see, for instance, [29, 44]) and the detailed scheduling level. Regarding the latter the most important problem is to balance the tradeoff between setup and inventory costs via lot sizing and scheduling.

Lot sizing and scheduling addresses the problem of determining the sequence and size of production lots (batches) for one or more items on one or more machines in a single- or multi-stage manufacturing system. The time horizon usually is divided into a finite number of time periods with equal lengths. There are capacity constraints for the machines. Demand is assumed to be known per period. The problem is to find a minimal cost production schedule, where machine setup and holding costs for inventory are considered to be most relevant.

Three types of lot sizing and scheduling models are well-known from literature: The "Capacitated Lot sizing Problem" (CLSP), the "Discrete Lot sizing and Scheduling Problem" (DLSP) as well as the "Continuous Setup Lot sizing Problem" (CSLP).

Each model type is based on a fundamental assumption which is more or less crucial. A discussion of these assumptions based on an instructive example provides the basis for the introduction of a new type of model, the "Proportional Lot sizing and Scheduling Problem" (PLSP).

The paper is organized as follows: In section 2 we formulate the CLSP, the DLSP as well as the CSLP in its generic form, present the underlying assumptions and demonstrate the relative (dis-)advantages based on an example. In section 3 the PLSP is derived, once more in its generic form. Section 4 provides extensions of the PLSP regarding the incorporation of setup times, sequence dependent setup costs (times),

multiple machines as well as multi-stage production systems. In Section 5 make-or-buy decisions are modelled in the PLSP context. The main focus of the paper is on modelling. Despite it should be noticed that the incorporation of make-or-buy decisions has very attractive features with respect to the feasibility problem.

The main difference between the well-known Economic Lotsizing and Scheduling Problem (ELSP; see [10, 17, 19, 31, 32]) not dealt with here and the models considered here is that ELSP assumes time to be continuous, demand for items to be constant and the planning horizon to be infinite.

## 2. Basic Models

The models to be considered in section 2 are based on the following *assumptions*: We consider a single-stage system, where a number of different items  $j = 1, \dots, J$  have to be manufactured on one machine (corresponds to a single capacity constraint). The time horizon  $T$  is segmented into a finite number of equal length time periods  $t = 1, \dots, T$ . Demand  $d_{jt}$  for item  $j$  in period  $t$  is known in advance deterministically. The setup cost  $s_j$  for item  $j$  are incurred whenever production of a batch starts. The holding costs  $h_j$  per unit of item  $j$  are identical for all periods.

The following example with  $J = 3$  items will be used in this section. Table 1 provides the demands  $d_{jt}$  (missing entries are zero), the holding costs  $h_j$  as well as the setup costs  $s_j$ , respectively. The data will be used in two different ways:

- First we use  $T = 5$  with a period length of e.g. two weeks for the CLSP. Here positive demand occurs at the end of periods  $t = 1, 3$  and  $5$ , respectively. The capacity of the machine  $C_t$  is 100 (hours) for  $t = 1, \dots, 5$ . The holding costs have to be multiplied with the inventory occurring at the end of the (rough) periods  $1, 2, \dots, 5$ .
- Second we use  $T = 10$  for the DLSP, the CSLP, and the PLSP, correspondingly with half of the above period length (one week). Positive demand occurs at the end of periods  $t = 2, 6$  and  $10$ , respectively. The capacity of the machine  $C_t$  is 50 (hours) for  $t = 1, \dots, 10$ . The (lower) holding costs have to be multiplied with the inventory occurring at the end of the (fine) periods  $1, 2, \dots, 10$ .

The production speed  $p_j$  is either set to 2 (hours per unit) for item  $j$  in the CLSP and the PLSP or (equivalent) to 50 (units per period) for the DLSP and the CSLP.

Disregarding inner-period inventory changes and counting the end-of-period inventory regardless of the scaling of the time horizon provides the basis for a "fair" comparison of the models.

### 2.1 The Capacitated Lotsizing Problem

The CLSP is based on the following *assumption*:

- A1. There is exactly one setup for each item which is produced in the period.



$s_j$  : the setup costs for item  $j$ ,

$x_{jt}$  : a binary variable indicating whether setup occurs for item  $j$  in period  $t$  ( $x_{jt} = 1$ ) or not ( $x_{jt} = 0$ ).

The  $I_{j0}$  are given data for  $j = 1, \dots, J$ , representing the initial stock ( $I_{j0} := 0 \forall j$  w.l.o.g.).

Table 2 provides the optimal solution of the CLSP applied to the example of Table 1 based on  $T = 5$ ; we get  $Z_{\text{CLSP}}^* = 2070$  as optimal objective function value. (A "splitting" of the CLSP quantities based on  $T=10$  would lead to a slightly increased optimal objective function value of 2115.)

Table 2: Optimal solution of the CLSP for the example of Table 1

$t$	1	2	3	4	5
$q_{1t}$	30		90		30
$q_{2t}$		30			70
$q_{3t}$		40		60	

This solution is rather poor compared with the results obtained below in section 3. This is mainly due to the inability of the CLSP to look back for the machine status in the preceeding period resulting in very high setup costs.

Let us now briefly discuss some literature related to the CLSP. Heuristic procedures are provided by [11, 16, 26, 36]. Decomposition approaches based on Lagrangean relaxation can be found in [13, 39, 52, 53]. In [20] a shortest path-based reformulation technique (with tight LP-bounds and thus few branches) has been presented which allows to solve CLSPs with up to  $J = 200$  items and  $T = 10$  periods to optimality with LINDO. The cost increase due to cyclical scheduling for the (noncyclical) CLSP is analyzed in [9]. A recent survey can be found in [41].

In [13, 24] it is shown that the CLSP is NP-hard. In [40] it is proved that for the multi-stage CLSP with setup times even the *feasibility problem* is NP-complete. This fact shows drastically how difficult it is to design "common sense" heuristics for this type of problem.

Finally we want to stress the following important fact relating to the CLSP: For the single-stage CLSP it is quite easy to determine (period by period) a feasible schedule corresponding to the (however determined) lot sizes. Despite the NP-completeness of the feasibility problem of multi-stage generalizations of the CLSP it is highly difficult to find (in a second step) a feasible schedule corresponding with the lot sizes (determined in a first step). Thus the models for simultaneous lotsizing and scheduling presented in the following seem to be preferable regarding their intrinsic scheduling capability.

## 2.2 The Discrete Lotsizing and Scheduling Problem

The DLSP is based on the following *assumption*:

A2. The production process always runs full periods without changeover.

This "all or nothing production" implies that at most one item can be produced per period, setup costs have to be paid at the beginning of the periods only and lot sizes are multiples of a full period production. In contrast to the CLSP the DLSP considers the sequence of the lots. Assumption A2 restricts the application of the DLSP to short term production scheduling with periods as days or shifts.

Mathematically the DLSP can be stated as follows:

$$\min \quad \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (7)$$

$$\text{s.t.} \quad I_{j,t-1} + p_j y_{jt} - I_{jt} = d_{jt} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (8)$$

$$\sum_{j=1}^J y_{jt} \leq 1 \quad (t = 1, \dots, T) \quad (9)$$

$$x_{jt} - y_{jt} + y_{j,t-1} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (10)$$

$$I_{jt}, x_{jt} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (11)$$

$$y_{jt} \in \{0, 1\} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (12)$$

where

$x_{jt}$  : a variable indicating whether setup for item  $j$  takes place in period  $t$  ( $x_{jt} > 0$ ) or not ( $x_{jt} = 0$ ),

$y_{jt}$  : a binary variable indicating the setup state, i.e. the machine produces item  $j$  in period  $t$  ( $y_{jt} = 1$ ) or not ( $y_{jt} = 0$ ),

$p_j$  : the production speed for item  $j$  (units per period),

and  $d_{jt}$ ,  $h_j$ ,  $I_{jt}$  as well as  $s_j$  have the same meaning as for the CLSP.

Note that in the case of nonnegative setup costs the setup variable  $x_{jt}$  ( $\geq 0$ ) takes only the binary values 0 (no setup costs incurred) or 1 (setup costs incurred) due to inequality (10).

The  $y_{j0}$  are given data for  $j = 1, \dots, J$ , representing the initial status of the machine ( $y_{j0} := 0 \forall j$  w.l.o.g). Furthermore, the machine can be left in any status at the end of the horizon.

Table 3 provides the optimal solution of the DLSP applied to the example of Table 1 based on  $T = 10$ ; we get  $Z_{DLSP}^* = 2140$  as optimal objective function value.

Table 3: Optimal solution of the DLSP for the example of Table 1

t	1	2	3	4	5	6	7	8	9	10
$p_1 \cdot y_{1t}$		50	50	50						
$p_2 \cdot y_{2t}$						50				50
$p_3 \cdot y_{3t}$					50				50	

This solution once more is rather poor compared with the results presented below. The two reasons are: The DLSP does not look back for more than one period (the interchange of the quantities of the periods 9 and 10 as well as the splitting of  $d_{26}$  would decrease the optimal objective function value to 1955); furthermore batches are too large in some periods thus increasing holding costs.

Now we are going to discuss briefly some references related to the DLSP. The first mixed binary programming models based on assumption A2 have been developed some decades ago (see e.g. [15, 27], but there was no method for solving medium- to large-size problem instances. A2 has been assumed in [37], too, where a column generation procedure is developed which is practicable only for problems where the number of periods is substantially smaller than the number of products. A dynamic programming algorithm for a special case of the DLSP is presented in [25].

Recently, an exact branch-and-bound approach based on Lagrangean relaxation of the capacity constraints (9) has been presented in [22]. The resulting subproblems (which are separately studied in [58]) have been solved by dynamic programming. This approach is capable of solving problems with (e.g.)  $J = 12$ ,  $T = 122$  or  $J = 3$ ,  $T = 250$  to optimality on a (slow) personal computer within a reasonable amount of time.

The approach of [22] has been extended to the DLSP with sequence dependent setup costs in [23]. Column generation based heuristics for the DLSP with setup times are presented in [12].

A six field notation for the short classification of a variety of DLSPs is presented in [47, 48]. Additionally, in both sources the complexity of feasibility problems associated with the DLSP are addressed. It is proved that the one machine DLSP with nonzero setup times as well as the parallel machine problem with zero setup times are NP-complete.

### 2.3 The Continuous Setup Lotsizing Problem

The CSLP is based on the following *assumption*:

A3. At most one item can be produced per period.

This implies that either there is one setup at the beginning of a period or no setup occurs at all within the period. Moreover, lot sizes are continuous quantities between



zero and the maximum possible production of item  $j$  in period  $t$ . Like the DLSP the CSLP is restricted to short term production scheduling.

Mathematically the **CSLP** can be stated as follows:

$$\min \quad \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (13)$$

$$\text{s.t.} \quad I_{j,t-1} + q_{jt} - I_{jt} = d_{jt} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (14)$$

$$\sum_{j=1}^J y_{jt} = 1 \quad (t = 1, \dots, T) \quad (15)$$

$$x_{jt} - y_{jt} + y_{j,t-1} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (16)$$

$$p_j y_{jt} - q_{jt} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (17)$$

$$I_{jt}, q_{jt}, x_{jt} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (18)$$

$$y_{jt} \in \{0, 1\} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (19)$$

where

$y_{jt}$  : a binary variable indicating whether the machine is setup for item  $j$  in period  $t$  ( $y_{jt} = 1$ ) or not ( $y_{jt} = 0$ ),

$x_{jt}, p_j$  are defined as in the DLSP and the other symbols are the same as in the CLSP.

Table 4 provides the optimal solution of the CSLP applied to the example of Table 1 based on  $T = 10$ ; we get  $Z_{\text{CSLP}}^* = 1910$  as optimal objective function value.

Table 4: Optimal solution of the CSLP for the example of Table 1

$t$	1	2	3	4	5	6	7	8	9	10
$q_{1t}$		30	30	50						40
$q_{2t}$						30	20	50		
$q_{3t}$					50				50	

Once more this solution is poor compared with the results presented for the PLSP below. The reason is that the CSLP does not make use of capacities not fully used by the production of one item.

The single-item CSLP has been studied in [7, 33]. In [34] a branch-and-bound approach based on Lagrangean relaxation of the capacity constraints (17) has been presented, along with rather discouraging computational experiences even for small-sized

problems. MPSARX, an MP-system with automatic cut generation capabilities [45, 55] allows to solve medium-sized problems to optimality.

Recently, a parallel machine CSLP has been studied in [43]. In [42] a cutting plane algorithm is presented for solving the Changeover Cost Scheduling Problem (CSP), which is closely related to the CSLP, too.

### 3. The Proportional Lotsizing and Scheduling Model

The PLSP is designed to overcome the shortcomings of the models presented in section 2. So we are now going to describe a type of model which produces continuous lot sizes over one or several, adjacent or non-adjacent periods where the setup costs are correctly calculated by looking back several periods. Moreover, we allow one changeover within each period. The PLSP is based on the following assumption:

A4. At most one changeover is allowed within each period.

The three cases that may occur are: No setup in a period, setup at the beginning of a period, or setup during a period. Like the DLSP and the CSLP, the PLSP is restricted to short term production scheduling.

Mathematically the PLSP can be stated as follows:

$$\min \quad \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (20)$$

$$\text{s.t.} \quad I_{j,t-1} + (q_{jt}^1 + q_{jt}^2) - I_{jt} = d_{jt} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (21)$$

$$\sum_{j=1}^J y_{jt} = 1 \quad (t = 1, \dots, T) \quad (22)$$

$$x_{jt} - y_{jt} + y_{j,t-1} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (23)$$

$$B \cdot y_{jt} - q_{j,t+1}^1 - q_{jt}^2 \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (24)$$

$$\sum_{j=1}^J p_j (q_{jt}^1 + q_{jt}^2) \leq C_t \quad (t = 1, \dots, T) \quad (25)$$

$$I_{jt}, q_{jt}^1, q_{jt}^2, x_{jt} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (26)$$

$$y_{jt} \in \{0, 1\} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (27)$$

where

$p_j$  : the production speed (hours per unit) of item  $j$ ,

$q_{jt}^1$  : the quantity of (the first) item  $j$  to be produced in period  $t$ ,

$q_{jt}^2$  : the quantity of (the second) item  $j$  to be produced in period  $t$ ,

and  $x_{jt}$ ,  $y_{jt}$  are defined as in the CSLP and the other symbols are the same as in the CLSP.

In the PLSP we distinguish between two items to be producible in period  $t$ . The first item  $j$  fills up the demand gap left over from the preceeding period  $t-1$ ; it is produced with a quantity of  $q_{jt}^1$  units. For the second item  $k$  the machine is going to be setup in period  $t$  after a number of time units *proportional* to  $q_{jt}^1 / C_t$ ; then it is producing a quantity of  $q_{kt}^2$  units. The splitting of the machine capacity for the production of two products within one period proportional to the quantities needed motivates the name of the model.

Like in the CSLP the  $y_{j0}$  are given data (w.l.o.g.  $y_{j0} := 0 \ \forall j$ ) and the machine can remain in any status at the end of the horizon. Furthermore we assume that  $q_{j1}^1 := q_{j,T+1}^1 := 0 \ \forall j$ .

Table 5 provides the optimal solution of the PLSP applied to the example of Table 1 based on  $T = 10$ ; we get  $Z_{\text{PLSP}}^* = 1710$  as optimal objective function value.

Table 5: Optimal solution of the PLSP for the example of Table 1

$t$	1	2	3	4	5	6	7	8	9	10
$q_{1t}^1/q_{1t}^2$		0/30		50/0	50/0	20/0				
$q_{2t}^1/q_{2t}^2$						0/30		30/0	40/0	
$q_{3t}^1/q_{3t}^2$	0/20	20/0							0/10	50/0

In the terminology of [20] the CLSP is a big bucket problem, the DLSP and the CSLP are small bucket problems, and the PLSP is a small bucket problem, too. In the uncapacitated single-item case the DLSP, the CSLP, and the PLSP reduce to the classical Wagner-Whitin model [57] which now is solvable in linear time  $O(T)$  [21, 56].

Let us now discuss the relations between short-term lotsizing and scheduling models, i.e. the DLSP, the CSLP as well as the PLSP in more detail:

- Regarding the "all or nothing production" assumption of the DLSP and the feasibility of continuous lot sizes for the CSLP it follows that the set of feasible solutions of the DLSP is a subset of the set of feasible solutions of the CSLP.
- It can be seen easily that the CSLP is a special case of the PLSP for  $q_{jt}^1 := 0$  for all  $j$  and  $t$ , where constraints (24) may be replaced by (17) and (25) may be dropped. Thus the set of feasible solutions of the CSLP is a subset of the set of feasible solutions of the PLSP.

These facts relating to sets of feasible solutions lead to the following inequalities:

$$Z_{\text{DLSP}}^* \geq Z_{\text{CSLP}}^* \geq Z_{\text{PLSP}}^*$$

Thus, despite the effort which has to be spent for the solution of the different models, the PLSP is considered to be superior to the DLSP and the CSLP regarding quality of solutions.

#### 4. Some Extensions of the PLSP

In section 3 we dealt with a basic version of the PLSP, i.e. a single-stage single-machine model without setup times. Now we are going to present various extensions of the PLSP. The incorporation of safety stocks and negative inventories could - at least in the single-stage case - be done easily and is therefore omitted.

##### 4.1 The PLSP with Setup Times

Disregarding setup times may be very crucial in practice; in the case of scarce machine times as well as substantial setup times infeasible production schedules could be the result. In order to avoid such cases we are forced to consider the PLSPST, i.e. the PLSP with setup times.

Mathematically the PLSPST can be stated as follows:

$$\min \quad \sum_{j=1}^J \sum_{t=1}^T (SC_{jt} + h_j I_{jt}) \quad (28)$$

$$\text{s.t.} \quad I_{j,t-1} + (q_{jt}^1 + q_{jt}^2) - I_{jt} = d_{jt} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (29)$$

$$B \cdot z_{jt} - st_j (y_{j,t} - y_{j,t-\delta-1}) + \sum_{\tau=t-\delta}^t w_{j\tau} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T; \delta = 0, \dots, sp_j) \quad (30)$$

$$\sum_{j=1}^J y_{jt} = 1 \quad (t = 1, \dots, T) \quad (31)$$

$$B \cdot y_{jt} - w_{jt} - p_j (q_{j,t+1}^1 + q_{jt}^2) \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (32)$$

$$B (1 - z_{jt}) - (q_{j,t}^1 + q_{j,t}^2) \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (33)$$

$$SC_{jt} - sc_j (y_{jt} - y_{j,t-1}) \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (34)$$

$$\sum_{j=1}^J [p_j (q_{jt}^1 + q_{jt}^2) + w_{jt}] \leq C_t \quad (t = 1, \dots, T) \quad (35)$$

$$I_{jt}, q_{jt}^1, q_{jt}^2, SC_{jt}, w_{jt} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (36)$$

$$y_{jt}, z_{jt} \in \{0, 1\} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (37)$$

where

- $sc_j$  : the setup costs for item  $j$ ,
- $SC_{jt}$  : the setup costs for item  $j$  to be paid in period  $t$ ,
- $sp_j$  : the maximum number of setup periods necessary for item  $j$ ,
- $st_j$  : the setup times (hours) for item  $j$ ,
- $w_{jt}$  : the setup times (hours) for item  $j$  required in period  $t$ ,
- $y_{jt}$  : binary variable indicating the setup state of the machine,
- $z_{jt}$  : a binary variable indicating whether setup has been finished for item  $j$  in period  $t$  ( $z_{jt} = 0$ ) or not ( $z_{jt} = 1$ ),

and the other symbols are the same as for the PLSP. Note that  $C_t$ ,  $st_j$  and  $w_{jt}$  must have appropriate dimensions. That means, if e.g.  $C_t$  is measured in hours per period, then  $st_j$  and  $w_{jt}$  must be measured in hours, too.

The objective function (28) counts, in conjunction with (34), for setup and holding costs. (29) are ordinary balance constraints. (30) "distributes" the setup times over some periods. (32) secures that setup and production of the "first" and the "second" item are related to the machine state. According to (33) nothing is produced in "full" setup periods. (35) are the capacity constraints. Note that the setup times are not restricted to be multiples of period lengths.

$sp_j$  may be set equal to  $\lceil st_j / \min \{C_t > 0 \mid t = 1, \dots, T\} \rceil$  with  $\lceil a \rceil$  as the smallest integer greater than or equal to  $a$ .

Until now only very little research has been published on lotsizing (and scheduling) with setup times. The CLSP with setup times has been considered in [54]. In [12] for the DLSP with setup times (which are restricted to be multiples of period lengths) column generation based heuristics are presented.

#### 4.2 The PLSP with Sequence Dependent Setup Costs

In many real life problems setup costs are not sequence independent. We are now going to present the PLSPSDSC, i.e. the PLSP with sequence dependent setup costs.

Mathematically the PLSPSDSC can be stated as follows:

$$\min \sum_{j=1}^J \sum_{t=1}^T (SC_{jt} + h_j I_{jt}) \quad (38)$$

$$\text{s.t.} \quad I_{j,t-1} + (q_{jt}^1 + q_{jt}^2) - I_{jt} = d_{jt} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (39)$$

$$\sum_{j=1}^J y_{jt} = 1 \quad (t = 0, \dots, T) \quad (40)$$

$$SC_{jt} - sc_{jj} y_{jt} + \sum_{i=1}^J sc_{ij} y_{i,t-1} \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (41)$$

$$B \cdot y_{jt} - q_{j,t+1}^1 - q_{jt}^2 \geq 0 \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (42)$$

$$\sum_{j=1}^J p_j (q_{jt}^1 + q_{jt}^2) \leq C_t \quad (t = 1, \dots, T) \quad (43)$$

$$I_{jt}, q_{jt}^1, q_{jt}^2 \geq 0 \quad (t = 1, \dots, T) \quad (44)$$

$$y_{jt} \in \{0, 1\} \quad (j = 1, \dots, J; t = 1, \dots, T) \quad (45)$$

where

$sc_{ij} : \max \{ \overline{sc}_{\alpha\beta} \mid \alpha, \beta = 1, \dots, J \} - \overline{sc}_{ij}$  (setup cost "regret"),

$\overline{sc}_{ij} : \text{the setup costs, when item } i \text{ (in period } t-1) \text{ is "followed" by item } j \text{ (in period } t),$

$SC_{jt}$  is defined as in the PLSPST and the other symbols are the same as for the PLSP.

(40) requires that the machine is setup for one of the items at the beginning ( $t = 0$ ). Thus setup state change is calculated correctly in (41) for period  $t = 1$ .

Until now only little research has been done on problems with sequence dependent setup costs, too. In [49] a DLSP-like model with sequence dependent setup costs is considered and LP-approximations are analyzed. In [23] the approach of [22] is extended to the DLSP with sequence dependent setup costs. A model which requires a lot of additional variables is presented in [14].

#### 4.3 The PLSP with Multiple Machines

In practice lotsizing and scheduling has to take account of scarce multiple machines. Here we will consider the PLSPMM, i.e. the PLSP with multiple machines.

Mathematically the **PLSPMM** can be stated as follows:

$$\min \quad \sum_{j=1}^J \sum_{m \in M_j} s_{jm} \sum_{t=1}^T x_{jmt} + \sum_{j=1}^J \sum_{t=1}^T h_j I_{jt} \quad (46)$$

$$\text{s.t.} \quad I_{j,t-1} + \sum_{m \in M_j} (q_{jmt}^1 + q_{jmt}^2) - I_{jt} = d_{jt} \quad (j=1, \dots, J; t=1, \dots, T) \quad (47)$$

$$\sum_{j \in J_m} y_{jmt} = 1 \quad (m=1, \dots, M; t=1, \dots, T) \quad (48)$$

$$x_{jmt} - y_{jmt} + y_{jm,t-1} \geq 0 \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (49)$$

$$B \cdot y_{jmt} - q_{jm,t+1}^1 + q_{jmt}^2 \geq 0 \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (50)$$

$$\sum_{j \in J_m} p_{jm} (q_{jmt}^1 + q_{jmt}^2) \leq C_{mt} \quad (m=1, \dots, M; t=1, \dots, T) \quad (51)$$

$$I_{jt}, q_{jmt}^1, q_{jmt}^2, x_{jmt} \geq 0 \quad (j=1,\dots,J; m \in M_j; t=1,\dots,T) \quad (52)$$

$$y_{jmt} \in \{0,1\} \quad (j=1,\dots,J; m \in M_j; t=1,\dots,T) \quad (53)$$

where

$M$  : the number of machines ( $m = 1,\dots,M$ ),

$M_j$  : the set of machines capable of producing item  $j$ ,

$J_m$  : the set of items producible on machine  $m$ ,

and the other symbols are the same as in the PLSP with the straightforward machine index extension.

The CLSP with multiple machines has been dealt with in, among others, [40]. The DLSP with multiple machines is analyzed in [48]. Note that the multiple machine extension becomes especially relevant in the multi-stage context.

#### 4.4 The Multi-Stage PLSP

In most real-life manufacturing situations the production process is covering several stages. Here we turn back to the single-machine situation dealt with up to section 4.2; this rigid assumption will be relaxed in section 5. We are going to formulate the PLSPMS, i.e. the multi-stage PLSP.

Mathematically the **PLSPMS** can be stated as follows:

$$\min \quad \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (54)$$

$$\begin{aligned} \text{s.t.} \quad & I_{j,t-1} + (q_{j,t-1}^1 + q_{j,t-1}^2) - \\ & \sum_{i \in \mathcal{N}_j} a_{ji} (q_{it}^1 + q_{it}^2) - I_{jt} = d_{jt} \quad (j = 1,\dots,J; t = 1,\dots,T) \\ & \text{and (22) - (27).} \end{aligned} \quad (55)$$

where

$a_{ji}$  : the number of units of item  $j$  required to produce one unit of item  $i$ ,

$\mathcal{N}_j$  : the set of immediate successor items of item  $j$ ,

and the other symbols are the same as for the PLSP.

Constraints (55) secure inventory balance in the multi-stage system, where the production lead time is assumed to be one period and we additionally have to provide in period  $t$  the number of items  $j$  required for the production of successor items  $i \in \mathcal{N}_j$  in period  $t$ .

One of the first multi-stage models is due to [27]. Multi-level lotsizing problems are modelled as constrained generalized networks in [50]. Uncapacitated multi-stage

lotsizing problems are solved via shortest path relaxations in [2]. The efficiency of single-stage heuristics in a multi-stage setting is analyzed in [8]. Heuristics for serial multi-stage systems are provided in [4]. LP-based heuristics can be found in [6, 40]. Simulated annealing as a heuristic method is evaluated in [35]. Further special heuristics may be found in [1, 30, 46, 51]. More details are provided in the survey [3].

## 5. An Integrated PLSP Model With Make-or-Buy Decisions

In the following we will combine together the different aspects treated above in an integrated model. One additional important task of modelling will be the incorporation of make-or-buy decisions.

### 5.1 Fundamental Implications of Make-or-Buy Decisions

Make-or-buy decisions are rather important regarding at least three aspects, i.e. generality, practical relevance and feasibility.

- Until now we focused on the production system (capacities, capabilities etc.) only. The incorporation of at least one "external" machine with high capacity (and production costs) obviously leads (*ceteris paribus*) to a more general model.
- In practice the items necessary for the finished products are not all manufactured by the company itself. Moreover, there often take place technical, capacity-oriented as well as economic considerations in order to decide about the right choice of make-or-buy on one or several of the manufacturing stages.
- As discussed in section 2 even the single-stage single-machine DLSP with nonzero setup times, the single-stage multiple machine DLSP without setup times as well as (beside others) the multi-stage CLSP with nonzero setup times belong to the class of the NP-complete feasibility problems. This is even more worse than NP-completeness of the corresponding optimization problem regarding the fact that in practice one often would be "lucky" with "good" feasible solutions. It is quite clear that the "feasibility curse" is one of the main drawbacks of the more "realistic" models. In the following we will overcome the feasibility problems associated with nonzero setup times or multiple machines in the context of modelling make-or-buy decisions.

### 5.2 The General PLSP

In the following we will present the PLSPG, i.e. the general PLSP which integrates setup times, sequence dependency of setup times and costs, multiple machines, multiple-stages as well as make-or-buy decisions.

Mathematically the PLSPG can be stated as follows:

$$\min \sum_{j=1}^J \sum_{t=1}^T \left[ h_j I_{jt} + \sum_{m \in M_j \cup M} [ SC_{jmt} + k_{jm} (q_{jmt}^1 + q_{jmt}^2) ] \right] \quad (56)$$



$$\text{s.t. } I_{j,t-1} + \sum_{m \in M_j} (q_{jmt,t-v_{jm}}^1 + q_{jmt,t-v_{jm}}^2) -$$

$$\sum_{i \in J_j} a_{ji} \sum_{m \in M_i \cup M} (q_{imt}^1 + q_{imt}^2) - I_{jt} = d_{jt} \quad (j=1, \dots, J; t=1, \dots, T) \quad (57)$$

$$\sum_{j \in J_m} y_{jmt} = 1 \quad (m=1, \dots, M-1; t=0, \dots, T) \quad (58)$$

$$SC_{jmt} - sc_{jjm} y_{jmt} + \sum_{i \in J_m} sc_{ijm} y_{imt-1} \geq 0 \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (59)$$

$$B \cdot z_{jmt} - st_{jjm} y_{jmt,t-\delta} + \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T;$$

$$\sum_{i \in J_m} st_{ijm} y_{imt,t-\delta-1} + \sum_{\tau=t-\delta}^t w_{jmt} \tau \geq 0 \quad \delta=0, \dots, sp_{jm}) \quad (60)$$

$$B \cdot y_{jmt} - w_{jmt} - p_{jm} (q_{jmt,t-1}^1 + q_{jmt}^2) \geq 0 \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (61)$$

$$B (1 - z_{jmt}) - (q_{jmt}^1 + q_{jmt}^2) \geq 0 \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (62)$$

$$\sum_{j=1}^J [p_{jm} (q_{jmt}^1 + q_{jmt}^2) + w_{jmt}] \leq C_{mt} \quad (m=1, \dots, M-1; t=1, \dots, T) \quad (63)$$

$$I_{jt}, SC_{jmt}, w_{jmt} \geq 0 \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (64)$$

$$q_{jmt}^1, q_{jmt}^2 \geq 0 \quad (j=1, \dots, J; m=1, \dots, M; t=1, \dots, T) \quad (65)$$

$$y_{jmt}, z_{jmt} \in \{0, 1\} \quad (j=1, \dots, J; m \in M_j; t=1, \dots, T) \quad (66)$$

where

$M-1$  : the number of machines within the production system (internal "make" machines),

$M$  : the additional machine providing "unlimited" capacity (external "buy" machine),

$M_j$  : the set of internal machines capable of producing item  $j$ ,

$J_m$  : the set of items producable on machine  $m$  ( $J_M := \{j \mid j = 1, \dots, J\}$ ),

$C_{mt}$  : the available capacity of the internal machine  $m$  in period  $t$ ,

$p_{jm}$  : the speed of the internal machine  $m$  for producing item  $j$  (units per period),

$k_{jm}$  : the costs of producing one unit of item  $j$  on machine  $m$ , i.e. the variable production costs ( $k_{jM} \gg k_{jm}$  for  $m = 1, \dots, M-1$ ),

$st_{ijm}$  :  $\max \{\overline{st}_{\alpha\beta\gamma} \mid \gamma = 1, \dots, M-1; \alpha, \beta \in J_\gamma\} - \overline{st}_{ijm}$  (setup time "regret"),

$\overline{st}_{ijm}$  : the setup time, when item  $i$  (in period  $t-1$ ) is "followed" by item  $j$  (in period  $t$ ),

$v_{jm}$  : the production lead time for item  $j$  on the internal machine  $m \in M_j$  ( $v_{jm} \geq 1$ ),

and the other symbols are the same as the one defined above with the (if necessary) straightforward machine index extension.

Like in the PLSPSDSC in  $t = 0$  every machine must be setup for one of the items, thus allowing correct calculation of sequence dependent setup costs and times.

It should be noticed that it would be equivalent to our formulation to establish a very high capacity of the "buy" machine ( $C_{Mt} \gg C_{mt}$  for  $m = 1, \dots, M-1$ ) in period  $t$ , as well as to define the speed of the external machine in producing item  $j$  to be very fast (i.e.  $p_{jM} \gg p_{jm}$  for  $m = 1, \dots, M-1$ ).

Note that in multi-stage and multi-machine generalizations of the CLSP (where the production speed is not assumed to be finite !) it is quite difficult to specify the minimum necessary production lead time. In this case it would be necessary to count only for the quantities produced in the last period (week) resulting in the MRP-inherent bad consequences of drastic overestimation of leadtimes.

In multi-stage generalizations of the DLSP it is supposed to be very difficult to "tune" the quantities necessary on different stages due to the only allowed fixed quantities.

The way of modelling make-or-buy decisions chosen in the PLSPG is a very simple one and could be made much more sophisticated (a specific "buy" machine for subsets of items with specific costs, lead times etc.). But even in this simple form it overcomes the difficulties due to feasibility problems (associated with nonzero setup times and multiple machines) sketched out in section 2. In the presence of plentiful feasible solutions it is not hopeless to spend effort in the development of greedy- and/or local-search procedures.

At present very little research papers explicitly deal with make-or-buy decisions in the lotsizing context. In [38] it is analyzed for single-stage, single-item problems with resource constraints. Note that the concept of modelling "overtime" (see [5, 54]) of internal machines (or skilled labour) is closely related but does not directly address this aspect.

## 6. Future Work

In this paper we have presented a new type of lotsizing and scheduling model, the proportional lotsizing and scheduling problem (PLSP), which is a generalization of the discrete lotsizing and scheduling model as well as of the continuous setup lotsizing model. Various types of generalizations of the basic PLSP have been presented here. For these models three types of methods should be developed in the future:

- Exact methods for solving small to medium-size problems in order to provide benchmarks for heuristics; two promising MPS-based concepts are here that of variable redefinition [20] or automatic cut generation [45].
- Fast greedy or semi-greedy procedures [18, 28] in order to construct "good" feasible schedules for large real-life problem instances; such procedures could provide the basis for the enhancement of MRP-systems.

- Local search procedures for generating near optimal solutions for medium- to large-size problems, based on concepts such as genetic algorithms, simulated annealing [35] or tabu search.

It is quite clear that especially for the last two classes of methods the concept of make-or-buy decisions is of fundamental importance: They render a lot of feasible solutions and thus make neighbourhood search applicable.

## References

- [1] Afentakis, P., "A parallel heuristic algorithm for lot-sizing in multistage production systems", *IIE Transactions*, Vol. 19 (1987), pp. 34-42.
- [2] Afentakis, P., Gavish, B., "Optimal lot-sizing algorithms for complex product structures", *Operations Research*, Vol. 34 (1986), pp. 237-249.
- [3] Bahl, H.C., Ritzman, L.P., Gupta, J.N.D., "Determining lot sizes and resource requirements: A review", *Operations Research*, Vol. 35 (1987), pp. 329-345.
- [4] Billington, P.J., Blackburn, J.D., Maes, J., Millen, R.A., Van Wassenhove, L.N., "Multi-product scheduling in multi-stage serial systems", in: Chikán, A., Lovell, M.C. (Eds.): *The economics of inventory management*, Amsterdam 1988, pp. 345-355.
- [5] Billington, P.J., McClain, J.O., Thomas, L.J., "Mathematical programming approaches to capacity - constrained MRP systems: Review, formulation and problem reduction", *Management Science*, Vol. 29 (1983), pp. 1126-1141.
- [6] Billington, P.J., McClain, J.O., Thomas, L.J., "Heuristics for multilevel lot-sizing with a bottleneck", *Management Science*, Vol. 32 (1986), pp. 989-1006.
- [7] Bitran, G.R., Matsuo, H., "The multi-item capacitated lot size problem: Error bounds on Manne's formulation", *Management Science*, Vol. 32 (1986), pp. 350-359.
- [8] Blackburn, J.D., Millen, R.A., "An evaluation of heuristic performance in multi-stage lot-sizing systems", *International J. of Production Research*, Vol. 23 (1985), pp. 857-866.
- [9] Campbell, G.M., Mabert, V.A., "Cyclical schedules for capacitated lot sizing with dynamic demands", *Management Science*, Vol. 37 (1991), pp. 409-427.
- [10] Carreno, J.J., "Economic lot scheduling for multiple products on parallel identical processors", *Management Science*, Vol. 36 (1990), pp. 348-358.
- [11] Cattrysse, D., Maes, J., Van Wassenhove, L.N., "Set partitioning and column generation heuristics for capacitated dynamic lotsizing", *European J. of Operational Research*, Vol. 46 (1990), pp. 38-47.
- [12] Cattrysse, D., Salomon, M., Kuik, R., Van Wassenhove, L.N., "Heuristics for the discrete lotsizing and scheduling problem with setup times", Discussion Paper, Erasmus-Universiteit Rotterdam 1990.
- [13] Chen, W.-H., Thizy, J.-M., "Analysis of relaxations for the multi-item capacitated lot-sizing problem", *Annals of Operations Research*, Vol. 26 (1990), pp. 29-72.
- [14] Dilts, D.M., Ramsing, K.D., "Joint lot sizing and scheduling of multiple items with sequence-dependent setup costs", *Decision Sciences*, Vol. 20 (1989), pp. 120-133.
- [15] Dinkelbach, W., *"Zum Problem der Produktionsplanung in Ein- und Mehrproduktunternehmen"*, Würzburg-Wien 1964.
- [16] Dixon, P.S., Silver, E.A., "A heuristic solution procedure for the multi-item, single-level limited capacity, lot-sizing problem", *J. of Operations Management*, Vol. 2 (1981), pp. 23-39.
- [17] Dobson, G., "The economic lot-scheduling problem: Achieving feasibility using time-varying lot sizes", *Operations Research*, Vol. 35 (1987), pp. 764-771.

- [18] Drexel, A., "Scheduling of project networks by job assignment", *Management Science*, Vol. 37 (1991), pp. 1590-1602.
- [19] Elmaghraby, S.E., "The economic lot scheduling problem (ELSP): Review and extensions", *Management Science*, Vol. 24 (1978), pp. 587-598.
- [20] Eppen, G.D., Martin, R.K., "Solving multi-item capacitated lot-sizing problems using variable redefinition", *Operations Research*, Vol. 35 (1987), pp. 832-848.
- [21] Federgruen, A., Tzur, M., "A simple forward algorithm to solve general dynamic lot sizing models with  $n$  periods in  $O(n \log n)$  or  $O(n)$  time", *Management Science*, Vol. 37 (1991), pp. 909-925.
- [22] Fleischmann, B., "The discrete lot-sizing and scheduling problem", *European J. of Operational Research*, Vol. 44 (1990), pp. 337-348.
- [23] Fleischmann, B., Popp, Th., "Das dynamische Losgrößenproblem mit reihenfolgeabhängigen Rüstkosten", in: Pressmar, D. et al. (Eds.): *Operations Research Proceedings 1988*, Berlin 1989, pp. 510-515.
- [24] Florian, M., Lenstra, J.K., Rinnooy Kan, A.H.G., "Deterministic production planning: Algorithms and complexity", *Management Science*, Vol. 26 (1980), pp. 669-679.
- [25] Gascon, A., Leachman, R.C., "A dynamic programming solution to the dynamic multi-item, single-machine scheduling problem", *Operations Research*, Vol. 36 (1988), pp. 50-56.
- [26] Günther, H.-O., "Planning lot sizes and capacity requirements in a single stage production system", *European J. of Operational Research*, Vol. 31 (1987), pp. 223-231.
- [27] Haehling von Lanzanauer, C., "A production scheduling model by bivalent linear programming", *Management Science*, Vol. 17 (1970), pp. 105-111.
- [28] Hart, J.P., Shogan, A.W., "Semi-greedy heuristics: An empirical study", *Operations Research Letters*, Vol. 6 (1987), pp. 107-114.
- [29] Hax, A.C., Candea, D., *Production and inventory management*, Englewood Cliffs (New Jersey) 1984.
- [30] Heinrich, C.E., *Mehrstufige Losgrößenplanung in hierarchisch strukturierten Produktionsplanungssystemen*, Berlin 1987.
- [31] Hsu, W.-L., "On the general feasibility test of scheduling lot sizes for several products on one machine", *Management Science*, Vol. 29 (1983), pp. 93-105.
- [32] Jones, P.C., Inman, R.R., "When is the economic lot scheduling problem easy?", *IIE Transactions*, Vol. 21 (1989), pp. 11-20.
- [33] Karmarkar, U.S., Kekre, S., Kekre, S., "The dynamic lot-sizing problem with startup and reservation costs", *Operations Research*, Vol. 35 (1987), pp. 389-398.
- [34] Karmarkar, U.S., Schrage, L., "The deterministic dynamic product cycling problem", *Operations Research*, Vol. 33 (1985), pp. 326-345.
- [35] Kuik, R., Salomon, M., "Multi-level lot-sizing problem: Evaluation of a simulated-annealing heuristic", *European J. of Operational Research*, Vol. 45 (1990) pp. 25-37.
- [36] Lambrecht, M.R., Vanderveken, H., "Heuristic procedures for the single operation multi-item loading problem", *AIIE Transactions*, Vol. 11 (1979), pp. 319-326.
- [37] Lasdon, L.S., Terjung, R.C., "An efficient algorithm for multi-item scheduling", *Operations Research*, Vol. 19 (1971), pp. 946-969.
- [38] Lee, S.-B., Zipkin, P.H., "A dynamic lot-size model with make-or-buy decisions", *Management Science*, Vol. 35 (1989), pp. 447-458.
- [39] Lozano, S., Larraneta, J., Onieva, L., "Primal-dual approach to the single level capacitated lot-sizing problem", *European J. of Operational Research*, Vol. 51 (1991), pp. 354-366.

- [40] Maes, J., McClain, J.O., Van Wassenhove, L.N., "Multilevel capacitated lotsizing complexity and LP-based heuristics", *European J. of Operational Research*, Vol. 53 (1991), pp. 131-148.
- [41] Maes, J., Van Wassenhove, L.N., "Multi-item single-level capacitated dynamic lot-sizing heuristics: A general review", *J. of the Operational Research Society*, Vol. 39 (1988), pp. 991-1004.
- [42] Magnanti, T.L., Vachani, R., "A strong cutting plane algorithm for production scheduling with changeover costs", *Operations Research*, Vol. 38 (1990), pp. 456-473.
- [43] de Matta, R., Guignard, M., "Production scheduling with sequence-independent changeover cost", Working Paper, The Wharton School, University of Pennsylvania 1989.
- [44] McClain, J.O., Thomas, L.J., Weiss, E.N., "Efficient solutions to a linear programming model for production scheduling with capacity constraints and no initial stock", *IIE Transactions*, Vol. 21 (1989), pp. 144-152.
- [45] Pochet, Y., Wolsey, L.A., "Solving multi-item lot-sizing problems using strong cutting planes", *Management Science*, Vol. 37 (1991), pp. 53-67.
- [46] Roll, Y., Karni, R., "Multi-item, multi-level lot sizing with an aggregate capacity constraint", *European J. of Operational Research*, Vol. 51 (1991), pp. 73-87.
- [47] Salomon, M., "Deterministic lotsizing models for production planning", Berlin 1991.
- [48] Salomon, M., Kroon, L.G., Kuik, R., Van Wassenhove, L.N., "Some extensions of the discrete lotsizing and scheduling problem", *Management Science*, Vol. 37 (1991), pp. 801-812.
- [49] Schrage, L., "The multiproduct lot scheduling problem", in: Dempster, M.A.H., Lenstra, J.K., Rinnooy Kan, A.H.G. (Eds.), "Deterministic and stochastic scheduling", Dordrecht / Holland, 1982, pp. 233-244.
- [50] Steinberg, E., Napier, H.A., "Optimal multi-level lot sizing for requirements planning systems", *Management Science*, Vol. 26 (1980), pp. 1258-1271.
- [51] Tempelmeier, H., Helber, S., "A heuristic for dynamic multi-item multi-level capacitated lotsizing for general product structures", Discussion Paper, TH Braunschweig 1992.
- [52] Thizy, J.-M., "Analysis of Lagrangian decomposition for the multi-item capacitated lot-sizing problem", *INFOR*, Vol. 29 (1991), pp. 271-283.
- [53] Thizy, J.-M., Van Wassenhove, L.N., "Lagrangean relaxation for the multi-item capacitated lot-sizing problem: A heuristic implementation", *IIE Transactions*, Vol. 17 (1985), pp. 308-313.
- [54] Trigeiro, W.W., Thomas, L.J., McClain, J.O., "Capacitated lot sizing with setup times", *Management Science*, Vol. 35 (1989), pp. 353-366.
- [55] Van Roy, T.J., Wolsey, L.A., "Solving mixed integer programming problems using automatic reformulation", *Operations Research*, Vol. 35 (1987), pp. 45-57.
- [56] Wagelmans, A., van Hoesel, S., Kolen, A., "Economic lotsizing: An  $O(n \log n)$  algorithm that runs in linear time in the Wagner-Whitin case", *Operations Research*, Vol. 40 (1992), Suppl. No. 1, pp. S145-S156.
- [57] Wagner, H.M., Whitin, T.H., "Dynamic version of the economic lot size model", *Management Science*, Vol. 5 (1958), pp. 89-96.
- [58] Wolsey, L.A., "Uncapacitated lot-sizing problems with start-up costs", *Operations Research*, Vol. 37 (1989), pp. 741-747.