

Mangelsdorff, Lukas; Weber, Martin

Working Paper — Digitized Version

Testing choquet expected utility

Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 285

Provided in Cooperation with:

Christian-Albrechts-University of Kiel, Institute of Business Administration

Suggested Citation: Mangelsdorff, Lukas; Weber, Martin (1992) : Testing choquet expected utility, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 285, Universität Kiel, Institut für Betriebswirtschaftslehre, Kiel

This Version is available at:

<https://hdl.handle.net/10419/161412>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Nr. 285

TESTING CHOQUET EXPECTED UTILITY

Lukas Mangelsdorff
Martin Weber

July 1992

Dipl.- Volkswirt Lukas Mangelsdorff, Prof. Dr. Martin Weber,
Lehrstuhl für Allg. BWL und Entscheidungsforschung, Universität Kiel,
Olshausenstr. 40-60, 2300 Kiel, Germany

We like to thank Prof. Graham Loomes very much for his helpful comments and pointing out some errors in a former version of the paper. Funds for paying the participants of the experiment were provided by the Graduiertenkolleg für Betriebswirtschaft of the University of Kiel.

Abstract:

Many theories have been developed to model decision behavior under ambiguity. In this paper we empirically investigate theories which are based on non-additive probabilities, i.e. Choquet expected utility (CEU) theories. We first replicated Ellsberg-paradox behavior. Then we elicited the individual non-additive probabilities, the so called capacities. Those capacities did not have all properties theoretically required. Finally we found that CEU is not really superior to EU in predicting the participants' decisions.

Zusammenfassung:

Betrachtet werden Entscheidungen unter Ambiguität, d.h. Entscheidungen unter Unsicherheit über Wahrscheinlichkeiten. Wir untersuchen empirisch die Theorien, die Entscheidungsverhalten bei Ambiguität durch nicht-additive Wahrscheinlichkeiten abbilden (Choquet-Erwartungsnutzentheorie). Zunächst replizieren wir sogenanntes Ellsberg-paradoxes Verhalten. Die sich aus der Theorie für die Ermittlung der nicht-additiven Wahrscheinlichkeiten, der Kapazitäten, ergebenden Implikationen werden nur teilweise bestätigt. Abschließend wird gezeigt, daß die Choquet-Erwartungsnutzentheorie das Entscheidungsverhalten der Experimententeilnehmer nicht besser als die Erwartungsnutzentheorie vorhersagt.

Key words:

Ellsberg paradox, ambiguity, Choquet expected utility, non-additive probabilities, experimental economics

Schlüsselwörter:

Ellsberg-Paradoxon, Ambiguität, Choquet-Erwartungsnutzen, nicht-additive Wahrscheinlichkeiten, experimentelle Wirtschaftsforschung

1 Introduction

Many theorists make a distinction between risk and uncertainty. A situation with sure or known probabilities can be illustrated by the following example. Consider a lottery where you win if a white ball is drawn from an urn which contains exactly 50 white and 50 yellow balls. Certainly this is a situation under risk and there is no reason not to be sure that the probability of winning is .5. By contrast, consider an urn which contains 100 white or yellow balls in an unknown proportion. You are allowed to pick the colour you want to win.

If the amount of winning is the same for both urns most people will choose the first urn over the second - they dislike being unsure about the probability of winning in the second urn. Of course, according to standard subjective expected utility theory you should choose the second urn or be indifferent between both urns. This phenomenon of ambiguity aversion was first highlighted by Ellsberg (1961). He showed that ambiguity aversion leads to a violation of Savage's sure-thing principle (Savage 1954) and thus cannot be described by subjective expected utility.

Quite a few empirical investigations have convincingly demonstrated that decision makers can have a (non-neutral) attitude towards ambiguity. Also many theories have been proposed to model attitudes towards ambiguity. For an overview on recent developments in decision making under ambiguity see Camerer and Weber (1992).

One notable approach to modelling decision making under ambiguity makes use of the concept of non-additive probabilities. This class of theories is called Choquet expected utility theories¹. In this paper we will experimentally test Choquet expected utility theories. We will compare different ways to

1) For Choquet expected utility see Schmeidler (1989). See also Gilboa (1987), Schmeidler (1986), Wakker (1989) and Nakamura (1990).

elicit non-additive probabilities, test implications of the theory and try to predict decision makers' behavior in decision situations under ambiguity. As a representative for Choquet expected utility theories we will mainly focus upon Schmeidler (1989).

In section 2 we give a brief description of Choquet expected utility. We explain in more detail ways to elicit and calculate non-additive probabilities, i.e. capacities, for each decision maker. In section 3 our hypotheses are given and the experimental design is described. The results are presented in section 4. As we got some striking results which could be due to an order effect of our design we replicated parts of the study slightly changing the design. These results are presented in section 5.

2 Choquet Expected Utility (CEU)

2.1 The Theory

Ellsberg (1961) considered an urn which contained 30 black balls and 60 red or yellow balls where the proportion of red and yellow balls is unknown. Thus the urn altogether contains 90 balls.

	BLACK	RED	YELLOW
Lottery A	50 DM	0 DM	0 DM
Lottery B	0 DM	50 DM	0 DM
Lottery C	50 DM	0 DM	50 DM
Lottery D	0 DM	50 DM	50 DM

Table 1: Ellsberg's three-colour problem

We defined four lotteries on the basis of the Ellsberg urn (see table 1). Lottery A pays DM 50 if a black ball is drawn from the urn. Lottery B pays DM 50 if a red ball is drawn from

the urn.

When having to choose between A and B most people prefer lottery A over lottery B. Now both lotteries are varied in the same way. One may also win DM 50 if a yellow ball is drawn from the urn (lotteries C and D). According to Savage's sure-thing principle (Savage 1954) this alteration should not lead to a change in preferences. However, a substantial number of people prefer lottery D over lottery C, a clear violation of subjective expected utility theory. The decision pattern can be explained in terms of aversion to ambiguity. The alteration of lotteries A and B to lotteries C and D implies a shift of ambiguity from lottery B (which is not chosen) to lottery C (which is not chosen). It can be shown that the standard decision pattern is not compatible with an additive probability measure.

Relaxing the independence axiom of Anscombe and Aumann (1963) only to hold for comonotonic acts Schmeidler (1989) derives a utility theory which allows for non-additive probabilities. Two acts (alternatives) are comonotone if the outcomes of the two acts under different states do not move in opposite directions. More formally, acts f and g are comonotonic if for any pair of states s and t , $f(s) \geq f(t)$ implies $g(s) \geq g(t)$. According to Schmeidler preferences can be represented by the integral in (1)²:

$$f \preceq g \Leftrightarrow \int_S u(f(s)) d\pi(s) \leq \int_S u(g(s)) d\pi(s), \quad (1)$$

where u is a real valued utility function of the outcomes, and π is a not necessarily additive (probability) measure with the following properties:

- $\pi(S) = 1$ with S = set of all states
- $\pi(\emptyset) = 0$

2) See also Nakamura (1990) and Sarin, Wakker (1991).

- $\pi(s) \leq \pi(t)$ if $s \subseteq t$.

The integration has to be done in a way first described by Choquet (1955) in order to account for the non-additivity of π . Therefore this way of calculating expected utility is referred to as Choquet expected utility (CEU). Because of its non-additivity π does not fulfill the requirements of a probability measure and is therefore called a capacity.

For the case of discrete states (as in Ellsberg's urn) the Choquet integral reduces to (2):

$$u_1 \pi(s_1) + \sum_{i=2}^n u_i [\pi(\bigcup_{j=1}^i s_j) - \pi(\bigcup_{j=1}^{i-1} s_j)], \quad (2)$$

with $u_i = u[f(s_i)]$, $u_1 \geq u_2 \geq \dots \geq u_n$

When calculating Choquet expected utility of a lottery one first has to rank the different states s_i according to their attractiveness. Note, that the ranking of states could differ from lottery to lottery. In CEU the utility of an outcome of a state u_i is weighted with something one might call difference of transformed cumulative probabilities. In case of additive capacities, i.e. probabilities, CEU reduces to (subjective) expected utility. As CEU contains expected utility as a special case u has to be the traditional utility function.

We now demonstrate how CEU is able to model the standard preference pattern in Ellsberg's three-colour paradox (A preferred to B, D preferred to C). Let u be a standard utility function with $u(50)=1$ and $u(0)=0$. The Choquet expected utility of lottery A can be calculated in the following way:

$$\begin{aligned} CEU(A) &= u(50) \pi(BLACK) \\ &+ u(0) [\pi(BLACK \cup RED) - \pi(BLACK)] \\ &+ u(0) [\pi(BLACK \cup RED \cup YELLOW) - \pi(BLACK \cup RED)] \\ &= \pi(BLACK) \end{aligned} \quad (3)$$

Similarly $CEU(B)$ is equal to $\pi(RED)$. A preference for A over B implies nothing else than that the capacity for drawing a

black ball is greater than the capacity for drawing a red ball out of the urn. Since $CEU(C)$ is equal to $\pi(\text{BLACK} \cup \text{YELLOW})$ and $CEU(D)$ is equal to $\pi(\text{RED} \cup \text{YELLOW})$ a preference of D over C implies that the capacity $\pi(\text{RED} \cup \text{YELLOW})$ is greater than $\pi(\text{BLACK} \cup \text{YELLOW})$. This is compatible with the monotonicity requirement imposed on π , whereas it would not allow for π being an additive probability measure.

2.2 Capacities

In our empirical study we considered two decision problems: the three-colour Ellsberg urn and a problem where subjects had to bet on the share price of the Japanese Dai Ichi Kangyo Bank. As this company is not well known among German students we expected a high ambiguity aversion for this decision problem³. We will show how to elicit the capacities for both decision problems. First, we will consider the Dai Ichi Kangyo Bank case as there are two fundamentally different ways to elicit the capacities. Later we will mainly concentrate on this case. The Ellsberg case is slightly more complex since there are more events for which the capacities have to be determined. We will assume that the decision maker's utility function is known.

2.2.1 Dai Ichi Kangyo Bank

In the Dai Ichi Kangyo Bank case the participants had to choose between four pairs of lotteries. Each pair consisted of one lottery defined on the share price of the Dai Ichi Kangyo Bank and of one lottery with sure probabilities. The following example may serve as an illustration:

Lottery I: You will get DM 50, if on the 25th of November,

3) Heath and Tversky (1991) as well as Keppe and Weber (1991) found that ambiguity aversion is related to the judged knowledge of the event.

1991⁴, at Tokio stock exchange the price of a share of Dai Ichi Kangyo Bank, one of the world's ten largest banks, was higher than (>) Yen 2600. Otherwise you will get nothing.

Lottery J: You will get DM 50, if a black ball is drawn out of an urn containing 50 black and 50 white balls. Otherwise you will get nothing.

Calculating $CEU(I)$ and $CEU(J)$ ⁵ we get:

$$\begin{aligned} CEU(I) \\ = u(50) \pi(>) \end{aligned} \quad (4)$$

$$\begin{aligned} CEU(J) \\ = u(50) \frac{1}{2} \end{aligned} \quad (5)$$

If the decision maker is indifferent between I and J the capacity $\pi(>)$ is equal to $1/2$ since (4) and (5) have to be equal. If the decision maker prefers I or J one lottery has to be changed to derive an indifference statement. Based on the indifference statement the capacity can be calculated. A lottery can be changed by changing the chance of winning or by changing the amount to be won. We will first demonstrate how the capacity can be calculated if the chance of winning is changed.

Changing Chances of Winning

If I is preferred to J the decision maker is asked how many black balls have to be added to make him indifferent between I

4) The experiment was done on November 29th, 1991.

5) For nonambiguous events we set the capacities to be equal to the relative likelihood.

and J^6 . Taking X as the number of black balls which have to be added $\pi(>)$ is given by the following equation:

$$\pi(>) = \frac{50+X}{100+X} \quad (6)$$

Preferring I over J means preferring the more ambiguous lottery. Therefore, the capacity for the ambiguous event has to be greater than $1/2$.

Things are very much similar if a subject prefers J over I . Taking X as the number of black balls which have to be taken out $\pi(>)$ is then given by the following formula:

$$\pi(>) = \frac{50-X}{100-X} \quad (7)$$

Changing Amount to be Won

Instead of changing the chances of winning we can also change the DM amounts subjects can win ("DM statements").

If I is preferred over J , lottery J has to be made more attractive. This can be done by increasing the amount of money to be won if a black ball is drawn. The individual is asked how much money s/he must receive ($= 50 + Y$ DM) if a black ball is drawn in order to be indifferent between this new lottery J' and I . $\pi(>)$ is determined by the following equation:

$$\pi(>) = \frac{u(50+Y)}{u(50)} \quad (8)$$

6) The participants were explicitly told that adding black balls (taking black balls out of the urn) does not mean that they substitute white balls (they are substituted by white balls). Thus a change of the number of black balls also leads to a change of the total number of (black and white) balls. One could think of other devices to elicit the probability equivalents, e.g. a probability wheel.

In the case J is preferred over I the amount to be won if a black ball is drawn has to be reduced, thus Y in (8) is negative.

2.2.2 Ellsberg

For three elementary events in Ellsberg's three-colour problem (drawing a black, a red, and a yellow ball), there are $2^3=8$ events for which the capacities have to be determined. As the capacities do not need to be additive, the determination gets more complicated than in the case of additive probabilities. The capacity $\pi(\text{BLACK} \cup \text{YELLOW})$, e.g., can no longer be determined by adding $\pi(\text{BLACK})$ and $\pi(\text{YELLOW})$.

For some of the eight events the capacities are straight forward (Schmeidler 1989, pp. 571). Especially for nonambiguous events the capacities should be equal to the relative likelihoods:

- $\pi(\emptyset) = 0$
- $\pi(\text{BLACK}) = 1/3$
- $\pi(\text{YELLOW} \cup \text{RED}) = 2/3$
- $\pi(\text{BLACK} \cup \text{YELLOW} \cup \text{RED}) = 1$

In addition we assume the capacities to be identical for symmetric events⁷:

- $\pi(\text{RED}) = \pi(\text{YELLOW})$
- $\pi(\text{BLACK} \cup \text{RED}) = \pi(\text{BLACK} \cup \text{YELLOW})$.

We will demonstrate how to calculate $\pi(\text{BLACK} \cup \text{YELLOW})$ assuming that $\pi(\text{YELLOW})$ had been previously determined. The decision maker is asked to choose between lotteries C and D (see table 1). Calculating $\text{CEU}(C)$ and $\text{CEU}(D)$ we get:

7) In this setting we assume that subjects have no color preference.

$$\begin{aligned}
CEU(C) &= u(50) \pi(BLACK) \\
&+ u(50) [\pi(BLACK \cup YELLOW) - \pi(BLACK)] \\
&= u(50) \pi(BLACK \cup YELLOW)
\end{aligned} \tag{9}$$

$$\begin{aligned}
CEU(D) &= u(50) \pi(RED) \\
&+ u(50) [\pi(RED \cup YELLOW) - \pi(RED)] \\
&= u(50) \frac{2}{3}
\end{aligned} \tag{10}$$

If the decision maker is indifferent between C and D the capacity $\pi(BLACK \cup YELLOW)$ is equal to $2/3$ since (9) and (10) have to be equal. If the decision maker prefers C or D one lottery has to be changed to derive an indifference statement. In the Ellsberg case the capacities can only be determined by changing the amount to be won. A change in the number of balls, i.e. a change in the chances of winning, leads to different ambiguous or unambiguous events. The event is, e.g., not any more "drawing a black ball out of the original Ellsberg urn" but "drawing a black ball out of the changed urn".

If C is preferred over D lottery C has to be made less attractive. This can be done by reducing the amount of money to be won if a black ball is drawn. The individual is asked how much money s/he must receive ($= 50 - Y$ DM) if a black ball is drawn in order to be indifferent between this new lottery C' and D. As $50 - Y$ is smaller than 50, a change in the ranking of desirability of the states occurs and $CEU(C')$ has to be calculated the following way:

$$\begin{aligned}
CEU(C') &= u(50) \pi(YELLOW) \\
&+ u(50 - Y) [\pi(BLACK \cup YELLOW) - \pi(YELLOW)]
\end{aligned} \tag{11}$$

Lottery D has not been changed. In case of indifference between C' and D, (11) is equal to (10). We get:

$$\pi(BLACK \cup YELLOW) = \pi(YELLOW) + \frac{u(50)}{u(50 - Y)} \left[\frac{2}{3} - \pi(YELLOW) \right] \tag{12}$$

In the case D is preferred over C the amount to be won has to be increased until the decision maker is indifferent between the new lottery C'' and the lottery D. At the point of indifference the amount to be won in C'' ($= 50 + Z$) is greater than 50, and $CEU(C'')$ has to be calculated the following way:

$$\begin{aligned} CEU(C'') &= u(50+Z) \pi(BLACK) \\ &+ u(50) [\pi(BLACK \cup YELLOW) - \pi(BLACK)] \end{aligned} \quad (13)$$

In case of indifference between C'' and D, (13) is equal to (10), leading to:

$$\pi(BLACK \cup YELLOW) = 1 - \frac{1}{3} \frac{u(50+Z)}{u(50)} \quad (14)$$

The other capacities of the Ellsberg case can be derived similarly.

3 Hypotheses and Experimental Design

3.1 Hypotheses

The objective of our investigation is to test certain implications of Choquet expected utility experimentally. In addition we wanted to test the predictive power of the theory.

From previous empirical research we know that decision makers show some degree of ambiguity aversion when confronted with Ellsberg's three-colour problem and with a lottery based on something like the Dai Ichi Kangyo Bank's stock price. This ambiguity aversion has to be reflected in the capacities.

Hypothesis 1:

Capacities for ambiguous events differ from probabilities which could have been derived according to the principle of insufficient reason.

Choquet expected utility does not distinguish between the two ways of deriving capacities presented in section 2.2.1. Hypothesis 2 explicitly states this cross-validation argument.

Hypothesis 2:

Capacities elicited on the basis of chances of winning are the same as capacities elicited on the basis of the amount to be won.

CEU does not distinguish between gain and loss domain. As there is evidence (see Camerer and Weber 1992) that subjects have different degree of ambiguity aversion in both domains hypothesis 3 is formulated.

Hypothesis 3:

Capacities are identical when derived from gain or loss statements.

Finally, we will investigate if CEU is able to predict the intuitive preferences in decision situations with ambiguity. The fact that CEU can model behavior towards ambiguity does not imply that in a certain decision situation CEU is really able to predict better than expected utility (see Weber and Camerer 1987 for a similar discussion for non-EU theories).

Hypothesis 4:

CEU is better able to predict subjects' decisions than expected utility.

3.2 Experimental Design

In the first part of the questionnaire (elicitation sample) we asked questions to derive the individual parameters (capacities and utility function). Based on these parameters we tested hypotheses 1-3. Then we presented some more alternatives (validation sample) of which we tried to predict the evaluation. We used two different sets of events (Ellsberg

case and Dai Ichi Kangyo Bank case).

Elicitation Sample

In the Dai Ichi Kangyo Bank case the subjects had to choose between four pairs of lotteries (I-J as shown above, K-L, M-N, and Q-P). In lottery pair K-L lottery K only differs from lottery I in so far as one wins DM 50 if the share price was smaller than or equal to (\leq) Yen 2600. L is identical to J. Lottery pairs M-N and P-Q are identical to pairs I-J and K-L, however, all amounts were negative. After each decision subjects had to answer the question what changes had to be made in order to be indifferent between the lotteries. We asked for the number of balls to be changed and for the amount of money to be changed.

In the Ellsberg case subjects had to choose between lotteries A and B, and C and D (see table 1). The participants had to choose between two more pairs of lotteries E and F, and G and H. Those were identical to the lottery pairs A-B and C-D, however, they involved losses rather than gains. After each comparison the participants had to answer the question what changes in the amount to be won had to be made in order to be indifferent between the lotteries.

The subjects' utility functions were determined using a mid-point splitting procedure. First a subject had to fill in the certainty equivalent, X, for a 50-50 lottery of DM 0 and DM 100. Next they were asked for the certainty equivalents for two 50-50 lotteries of 0 DM and DM X, and DM X and DM 100. We determined the utility function for [0 DM, 100 DM] as well as for [-100 DM, 0 DM]⁸.

8) In a few cases subjects' answers to determine the capacities would have required a utility function with a wider range than DM -100 or DM 100. To still be able to map DM-values into utilities we linearly extrapolated the utility function.

Validation Sample

The validation sample consisted of eight Ellsberg lotteries (numbers 1 to 8) and eight Dai Ichi Kangyo Bank lotteries (numbers 9 to 16). In the Ellsberg case subjects had to evaluate lotteries which paid DM 0 or DM 100 (DM -100) depending on the ball picked. The Dai Ichi lotteries paid DM 0 or DM 100 (DM -100) as well as DM 0 or DM 50 (DM - 50). Assuming equal probabilities for elementary events the expected values range from DM -66 to DM 66 in the Ellsberg case and from DM -50 to DM 50 for the Dai Ichi Kangyo Bank. The lotteries' outcomes were either only positive (including zero) or only negative (including zero).

For each lottery subjects were asked for their certainty equivalent. The number elicited had to be positive or negative depending on whether the lottery had a positive or a negative expected value. We asked for the certainty equivalent in the following way:

"We are interested how you evaluate the following lotteries compared to a sure payment. Please, tell us the DM amount you must get with certainty so that you are indifferent between this amount and the lottery. If the lottery consists of negative outcomes, tell us the DM amount you would have to pay, so that you are indifferent between paying this amount and the lottery."

Procedure

The experiment was run as a questionnaire. The eight pairs of lotteries in the elicitation sample were presented first. The pairs were given in an order so that negative and positive amounts varied as well as the type of event. Then the utility functions were elicited. At the end of the questionnaire the validation sample was presented to our subjects.

Altogether 74 students answered the questionnaire. All students were master students in business or economics. They were guided through most parts of the questionnaire by the experi-

menter. They took on average 40 to 50 minutes to fill in the questionnaire. For their participation subjects were paid a flat fee of DM 15. We excluded five participants from the analysis. They either gave incomplete answers in various parts of the questionnaire or they were prepared to pay more for a lottery than its best possible consequence.

4 Results

4.1 Ellsberg-Paradox Behavior

We first wanted to check if our subjects show some attitude towards ambiguity. Whether people behave in an Ellsberg-paradox way can be analysed by looking at the decision patterns of the lottery pairs A-B vs. C-D (positive amounts) and pairs E-F vs. G-H (negative amounts). A (SEU) rational decision maker should prefer A and C (E and G), or B and D (F and H) or be indifferent. An ambiguity averse subject should prefer A and F, an ambiguity seeker B and E. Those being indifferent in one pair and indicating preference in the second pair are labelled "others" in table 2.

Decision patterns	Number of people choosing the pattern			
	A-B, C-D positive	E-F, G-H negative	I-J, K-L positive	M-N, P-Q negative
SEU rational	24	28	17	27
Ambiguity averse	34	16	41	23
Ambiguity seeking	3	18	7	12
Others	8	7	4	7
Σ	69	69	69	69

Table 2: Ellsberg behavior

Table 2 shows that for positive amounts 54% show some attitude towards ambiguity, mainly being ambiguity averse. Only 35% behave according to SEU. For negative amounts we see a slight

increase in SEU rational subjects and a strong increase in ambiguity seeking subjects.

For the Dai Ichi Kangyo Bank case let us consider the lottery pairs I-J and K-L. A preference for I and L, for J and K and two indifference statements can be explained by SEU⁹. A preference for I and K shows ambiguity seeking, a preference for J and L ambiguity aversion. Again, answers indicating indifference only once are labelled "others". Table 2 shows that for positive amounts only 25% behave SEU rational whereas 70% show an attitude towards ambiguity, mostly indicating ambiguity aversion. For negative amounts around 39% of our subjects show a decision pattern compatible with SEU whereas 51% show some attitude towards ambiguity (2/3 ambiguity averse and 1/3 ambiguity seeking).

The results clearly demonstrate that subjects have an attitude towards ambiguity and they also indicate that the aggregate behavior is different for gains and losses. Taking the Ellsberg and the Dai Ichi Kangyo Bank case together we did a chi-square test of the equality of the distributions of the decision patterns for gains and losses. The empirical value, 53.56, is far higher than the theoretical one of 11.34 ($\alpha = .01$, three degrees of freedom). Thus there is much support for rejecting the null hypothesis that there is no difference in the attitudes towards ambiguity for gains and losses.

4.2 Calculation of Capacities

Hypothesis 1

Hypothesis 1 states that the capacities should be different from the probabilities derived using the principle of insuffi-

9) According to SEU a preference I over J would mean, that the probability of the price of the Dai Ichi Kangyo Bank on the 25th of November, 1991, being greater than 2600 Yen is more than .5. Consequently one should prefer lottery L over K, since the probability of winning in lottery K is less than .5.

cient reason. As we argued before we only need to calculate $\pi(\text{RED})$ and $\pi(\text{BLACK} \cup \text{RED})$.

Calculating the capacities on the basis of DM statements we found that some subjects violated the monotonicity requirements. The following restrictions have to be fulfilled:

- $\pi(\text{BLACK}) = 1/3 \leq \pi(\text{BLACK} \cup \text{RED}) \leq 1$ and
- $0 \leq \pi(\text{RED}) \leq \pi(\text{BLACK} \cup \text{RED})$

In the Ellsberg case for positive outcomes we had eleven subjects who violated the first restriction. We therefore set those values for $\pi(\text{BLACK} \cup \text{RED})$ equal to 1 which were greater than 1, and those smaller than $1/3$ were adjusted to $1/3$. In the Ellsberg case for negative outcomes we had four people with $\pi(\text{RED})$ smaller than 0. Those values were replaced by 0. In the Dai Ichi Kangyo Bank case for positive outcomes we calculated one value for $\pi(>)$ greater than 1, which we replaced by 1. For negative outcomes we got four values smaller than 0, which we replaced by 0.

	positive outcomes		negative outcomes	
	Ellsberg			
	$\pi(\text{RED})$	$\pi(\text{BLACK} \cup \text{RED})$	$\pi(\text{RED})$	$\pi(\text{BLACK} \cup \text{RED})$
prob	.33	.67	.33	.67
DM	.30**	.56**	.34	.64
t	-2.65	-5.20	.54	-1.66
	Dai Ichi Kangyo Bank			
	$\pi(>)$	$\pi(\leq)$	$\pi(>)$	$\pi(\leq)$
prob	.5	.5	.5	.5
DM	.41**	.40**	.46*	.50
t	-4.28	-5.15	-2.06	-.17
balls	.43**	.44**	.49	.51
t	-5.07	-5.39	-1.10	.71

Table 3: Average capacities

In table 3 we present the average capacities for the Ellsberg case as well as for the Dai Ichi Kangyo Bank case¹⁰. To test hypothesis 1 we tested against $H_0: \pi(..)$ equal to probability of insufficient reason, in all 12 different cases. A * (**) indicates a significant difference on the .05 (.01) level. All mean capacities were significantly lower than the probabilities if the capacities were derived from lotteries based on positive amounts. Especially in the Dai Ichi case the capacities reflect a considerable ambiguity aversion. For capacities derived from negative amounts a significant difference was only found in one case.

Hypothesis 2

Hypothesis 2 stated that in the Dai Ichi Kangyo Bank case capacities estimated on DM basis and on ball basis are identical. To test this hypothesis, for each person we calculated the differences for corresponding capacities and tested for their significance ($H_0: \pi(..|DM \text{ statement}) - \pi(..|ball \text{ statement}) = 0$). As can be seen in table 3 for each of the four average capacities the capacity based on changing the number of balls is larger than the capacity based on changing the amount to be won. This difference is significant for:

- $\pi(>)$, positive amounts ($\alpha < .05$, $t = -2.24$)
- $\pi(\leq)$, positive amounts ($\alpha < .01$, $t = -3.10$)
- $\pi(>)$, negative amounts ($\alpha < .05$, $t = -2.40$)

The above cases are three out of the five cases where at least one of the two assessment procedures yielded a significant difference between capacities and probabilities (see table 3). Remember that the largest attitude towards ambiguity was observed for Dai Ichi Kangyo Bank if the lotteries were designed with positive amounts. For these cases we find that changing balls will yield in significantly higher capacities than changing amounts of money, i.e. the degree of ambiguity aver-

10) All numbers were rounded to the second digit. The tests, however, were done using four digits.

sion is larger for changing amounts than for changing balls.

Hypothesis 3

Hypothesis 3 stated that capacities are identical for gains and losses. As with hypothesis 2 we calculated the individual differences for corresponding capacities and tested for their significance ($H_0: \pi(..|positive\ outcomes) - \pi(..|negative\ outcomes) = 0$). Again, as we can see from table 3, the average capacities for positive amounts are always smaller than the one for negative. This is significant for:

- $\pi(RED)$, DM ($\alpha < .05$, $t = -2.19$)
- $\pi(BLACK\ or\ RED)$, DM ($\alpha < .01$, $t = -3.10$)
- $\pi(>)$, DM ($\alpha < .1$, $t = -1.80$)
- $\pi(\leq)$, DM ($\alpha < .01$, $t = -3.90$)
- $\pi(>)$, ball ($\alpha < .01$, $t = -3.27$)
- $\pi(\leq)$, ball ($\alpha < .01$, $t = -4.90$)

As the difference between capacities based on positive and negative amounts is always significantly different from zero we have to reject hypothesis 3. Similar to decision making under risk (Kahneman and Tversky 1979) we find that people get less ambiguity averse when we come into the domain of loss. On average we find ambiguity neutrality for losses with quite a few people being ambiguity seeking.

4.3 Prediction

Our validation sample consisted of 16 lotteries. For each of these lotteries we elicited the certainty equivalent. The goal of our study (hypothesis 4) was to investigate which theory or assessment procedure is best able to predict the certainty equivalents. As there were 69 subjects we altogether had 1104 observations to test the predictive power of the different theories and assessment procedures.

The following list gives the different values we calculated

for each of the 16 lotteries:

CE:	Certainty equivalent of a lottery;
U(CE):	Utility of the certainty equivalent;
CEU_DM:	Choquet expected utility of a lottery based on capacities derived by DM statements;
CEU_ball:	Choquet expected utility of a lottery based on capacities derived by ball statements; holds only for the Dai Ichi Kangyo Bank case;
EU:	Expected utility of a lottery based on probabilities according to the principle of insufficient reason;
EV:	Expected value of a lottery based on probabilities according to the principle of insufficient reason.

When calculating the individual values of CEU_DM and CEU_ball we used the capacities which we determined for positive (negative) outcomes if the lottery of the validation sample consisted of positive (negative) outcomes.

Ellsberg								
Lottery	1	2	3	4	5	6	7	8
EV in DM	33	33	-33	-33	67	-67	67	-67
CEU_DM	.33	.30	-.33	-.36	.67	-.67	.56	-.66
EU	.33	.33	-.33	-.33	.67	-.67	.67	-.67
U(CE)	.40	.36	-.41	-.42	.66	-.64	.65	-.64
CE in DM	37	33	-30	-31	62	-54	62	-53
Dai Ichi Kangyo Bank								
Lottery	9	12	11	15	16	14	13	10
EV in DM	50	50	-50	-50	25	25	-25	-25
CEU_DM	.41	.40	-.50	-.54	.22	.22	-.31	-.32
CEU_ball	.43	.44	-.49	-.51	.23	.24	-.30	-.31
EU	.5	.5	-.5	-.5	.27	.27	-.30	-.30
U(CE)	.48	.49	-.55	-.54	.29	.26	-.33	-.34
CE in DM	45	46	-44	-44	25	23	-21	-22

Table 4: Average values for the validation sample

In table 4 the expected value of a lottery is identical for all subjects. It only serves to describe the lottery. The values CEU_DM, CEU_ball, and EU were calculated using each subjects capacities and utility function. CE was observed, however, we had to transform it to $U(CE)$ in order to measure the observed values on the same scale as the calculated values. When determining the utility function the average certainty equivalent for a 50-50 lottery yielding DM 0 and DM 100 was DM 46.1, thus showing some risk aversion, for negative amounts the average certainty equivalent was DM -39.8, thus showing somewhat stronger risk seeking.

To test hypothesis 4 we calculate for each calculated value (CEU_DM and EU) the difference between the calculated value and the observed value transformed by u :

$$\begin{aligned} \text{CEU_DM} - U(CE), \\ \text{EU} - U(CE). \end{aligned}$$

In case of an absolutely perfect fit of a theory, the corresponding difference should be 0 for all observations. The less significantly it diverges from 0 the better the predictive power of the theory.

For $\text{CEU_DM} - U(CE)$ we found that the mean is significantly smaller than zero ($\alpha < .01$, $t = -3.44$). For $\text{EU} - U(CE)$ we found that the mean is significantly larger than zero ($\alpha < .05$, $t = 2.55$). Overall neither EU nor CEU seem to be good predictors.

Contrary to EU, CEU is able to predict people's behavior in case they have a non-neutral attitude towards ambiguity. To sharpen our analysis we will investigate the predictive power for those 34 subjects who showed ambiguity aversion in the Ellsberg case (see table 2). Contrary to what one would expect now EU predicts best, i.e. the mean of $\text{EU} - U(CE)$ is not significantly different from zero, whereas $\text{CEU_DM} - U(CE)$ is significantly smaller than zero ($\alpha < .01$, $t = -5.02$).

To differentiate our analysis we will consider capacities derived from positive and negative amounts separately. Remember, that capacities derived from negative (positive) amounts show hardly any (strong) ambiguity aversion. As we found the strongest ambiguity aversion for Dai Ichi Kangyo Bank we will concentrate on these events.

	CEU_DM - U(CE)	CEU_ball - U(CE)	EU - U(CE)
Positive outcomes			
Mean, σ^2	-.070, .021	-.047, .014	.002, .014
t	-7.93**	-6.56**	.34
Negative outcomes			
Mean, σ^2	.021, .038	.036, .026	.037, .022
t	1.74	3.72**	4.15**

Table 5: Predictive power for positive and negative Dai Ichi Kangyo Bank lotteries

Table 5 shows the surprising results that CEU_DM predicts best for negative amounts and EU predicts best for positive amounts. This is clearly not satisfying. While determining the capacities we have observed a strong ambiguity effect for ambiguous lotteries with positive amounts. Those effects can not be modelled by EU.

Comparing certainty equivalents for positive and negative lotteries for Dai Ichi Kangyo Bank in table 4 we find that the values for corresponding lotteries are remarkably identical. However, as the degree of risk seeking is on average stronger than the degree of risk aversion and we also observed ambiguity aversion in the latter case we can not make a general conclusion.

To gain some insight what was going on we focus on the values for positive lotteries (lotteries number 9, 12, 14, 16) in table 4. Taking risk aversion and ambiguity aversion into account subjects gave certainty equivalents which were way too

high. For the last lottery subjects were on average even willing to pay more than the expected value.

Let us, e.g., consider lottery 14 of the validation sample, which is identical to lottery K of the elicitation sample. When choosing between the ambiguous lottery K and lottery L, a lottery with a 50-50 chance for winning DM 50 or nothing, people preferred lottery L. To be indifferent between both lotteries, the winning amount of lottery L (DM 50) would have to be reduced by on average DM 9.22 (ambiguity premium). Thus the participants were indifferent between an ambiguous lottery giving them an uncertain chance of winning DM 50 and a lottery where they could win approximately DM 41 with a .5 chance. Taking the participants' risk aversion into account subjects should pay less than DM 20 for lottery K (= lottery 14). As table 4 shows subjects on average gave a certainty equivalent of DM 23.12.

A possible explanation for this result might be an order effect: all 69 participants had to answer the questions in the same order, and thereby they answered the questions of the validation sample in the end. To control for a potential order effect we run a second experiment.

5 Additional Investigation

As the Dai Ichi Kangyo Bank events with positive amounts of winning showed the strongest ambiguity aversion we only considered these events in our second study. We reversed the order of the questionnaire:

- First, we asked for certainty equivalents for the lotteries 9, 12, 14, and 16.
- Next, we elicited the utility function for the interval [DM 0, DM 100].
- Finally, we let them choose between the lotteries I and J and between the lotteries K and L.

The questionnaire was identical to the first one, however, we

changed the date of the stock price to January 13th, 1992¹¹, and the amount to Yen 2000. These slightly changed lotteries are referred to as lotteries 9', 12', 14', and 16'.

The participants were not paid to fill in the questionnaire. 35 students studying mostly business and economics participated in the experiment, five participants had to be taken out of the analysis, mainly because they violated dominance requirements.

The second experiment showed similar results when we tested hypothesis 1 and 2. Out of 30 subjects 11 had decision patterns compatible with SEU, 15 were ambiguity averse, 3 ambiguity seeking and 1 was categorized as "others" (see table 2 for the results of the first experiment). The capacities based on ball statements and on DM statements were significantly smaller than .5 (see table 3 for the first experiment):

For DM: $\pi(>) = .42$ ($\alpha < .05$, $t = -2.09$)

$\pi(\leq) = .39$ ($\alpha < .01$, $t = -3.45$)

For balls: $\pi(>) = .43$ ($\alpha < .05$, $t = -2.55$)

$\pi(\leq) = .41$ ($\alpha < .01$, $t = -3.21$)

For $\pi(>)$ we got no significant difference between ball and DM capacities, whereas for $\pi(\leq)$ the capacities derived from balls were significantly larger than those derived from DM statements ($\alpha < .05$, $t = -2.10$).

Table 6 gives the calculated and observed values for lotteries 9', 12', 14', and 16'.¹² Again we tested for i) CEU_DM - U(CE), for ii) CEU_ball - U(CE) and for iii) EU - U(CE) whether the mean difference was significantly different from zero.

11) This was the second author's 40th birthday. Presents and congratulations are still welcome.

12) 17 participants were given the lotteries of the holdout sample in the sequence 9', 12', 14', and 16', and the other 13 participants were given these lotteries in the sequence 14', 16', 9', and 12'. As we found no significant differences in the certainty equivalents of these two different groups we will abandon this distinction.

We found that i) and ii) were significantly smaller than zero. As in the first experiment the mean of $EU - U(CE)$ was not significantly different from zero (see table 7).

Dai Ichi Kangyo Bank				
Lottery	9'	12'	16'	14'
EV in DM	50	50	25	25
CEU_DM	.42	.39	.24	.22
CEU_ball	.43	.41	.25	.24
EU	.5	.5	.28	.28
U(CE)	.50	.46	.29	.28
CE in DM	45	41	24	23

Table 6: Additional investigation: average values for the validation sample

	CEU_DM - U(CE)	CEU_ball - U(CE)	EU - U(CE)
Mean, σ^2	-.0618**, .047	-.0476*, .043	.009, .036
t	-3.09	-2.50	.54

Table 7: Additional investigation: test of significance

Both experiments show that the certainty equivalent of an ambiguous lottery can not be predicted with Choquet expected utility. Using only attitude towards risk (and neglecting attitude towards ambiguity) or using only attitude towards ambiguity (and neglecting attitude towards risk) we were able to predict the certainty equivalents.¹³

13) As in the first experiment we want to consider lottery 14' in more detail. Our subjects on average gave a certainty equivalent of DM 22.80. In order to be indifferent between lottery K' (= lottery 14') and lottery L the winning amount of lottery L has to be reduced by on average DM 9.27 (ambiguity premium). Thus the participants were indifferent between the ambiguous lottery K' and a lottery with a .5 chance of winning DM 41, a lottery with an expected value of approximately DM 20. As the participants were on average risk averse they should have a certainty equivalent clearly less than DM 20 for lottery K'.

6 Discussion

In our study we empirically tested some behavioral implications of Choquet expected utility theory. One main goal of our study was to show that it is possible to assess capacities for everyday decision situations. We described two assessment procedures to elicit and calculate capacities. To derive indifference statements necessary to calculate capacities one can change the amount to be won (or to be lost) or the chances of winning (or losing), whereby the Ellsberg case allows only for the first method. Both assessment procedures gave results that were significantly different from probabilities which could be derived by the principle of insufficient reason. The degree of subadditivity can serve as a measure for the ambiguity aversion of our subjects.

We found a significant difference in attitude towards ambiguity in the gain and in the loss domain. Subjects showed a considerable aversion to ambiguity for lotteries on positive amounts and (on average) no ambiguity aversion for losses. A descriptive theory of decisions under uncertainty has to take this result into account.

In testing the predictive power of CEU models we did not find a superiority of CEU models over EU. This could be attributed to a general robustness of EU (see Currim and Sarin 1990). This is clearly not satisfying: especially for those types of lotteries where subjects showed a strong ambiguity effect we were not able to predict with CEU better than with EU. For lotteries with positive outcomes the certainty equivalents elicited were way too large compared to the values calculated using capacities and utility function. Capacities were calculated from statements where subjects had only to think about their ambiguity aversion. Giving certainty equivalents they had to do both - think about risk aversion and think about ambiguity aversion. Perhaps this additional complexity was enough to influence the results. Clearly, this question is

worth being studied further.

References

- Anscombe, F.J. and R.J. Aumann (1963), A definition of subjective probability, Annals of Mathematical Statistics, 34, 199-205
- Camerer, C.F. and M. Weber (1992), Recent developments in modelling preferences under uncertainty and ambiguity, Journal of Risk and Uncertainty, in print
- Choquet, G. (1955), Theory of capacities, Ann. Inst. Fourier, 5, 131-295
- Currim, I.S. and R.K. Sarin (1989), Prospect versus utility, Management Science, 35, 22-41
- Currim, I.S. and R.K. Sarin (1990), Robustness of expected utility model in predicting individual choices, Working paper
- Ellsberg, D. (1961), Risk, ambiguity, and the Savage axioms, Quarterly Journal of Economics, 75, 643-669
- Gilboa, I. (1987), Expected utility theory with purely subjective non-additive probabilities, Journal of Mathematical Economics, 16, 65-88
- Gilboa, I. and D. Schmeidler (1989), Maximin expected utility with a non-unique prior, Journal of Mathematical Economics, 18, 141-153
- Heath, C. and A. Tversky (1991), Preference and belief: ambiguity and competence in choice under uncertainty, Journal of Risk and Uncertainty, 4, 5-28
- Hogarth, R.M. and H.J. Einhorn (1990), Venture theory: a model of decision weights, Management Science, 36, 780-803
- Kahneman, D. and A. Tversky (1979), Prospect theory: an analysis of decision under risk, Econometrica, 47, 263-291
- Keppe, H.-J. and M. Weber (1991), Judged knowledge and ambiguity aversion, Working paper No. 277, Institut für Betriebswirtschaftslehre, University of Kiel
- Nakamura, Y. (1990), Subjective expected utility with non-additive probabilities on finite state space, Journal of Economic Theory, 51, 346-365
- Sarin, R.K. and P.P. Wakker (1991), Incorporating attitudes towards ambiguity in Savage's set-up, Working paper

- Savage, L.J. (1954), The foundations of statistics, 1st edition, New York
- Schmeidler, D. (1986), Integral representation without additivity, Proceedings of the American Mathematical Society, 97, 255-261
- Schmeidler, D. (1989), Subjective probability and expected utility without additivity, Econometrica, 57, 571-587
- Wakker, P.P. (1989), Continuous subjective expected utility with non-additive probabilities, Journal of Mathematical Economics, 18, 1-27
- Weber, M. and C.F. Camerer (1987), Recent developments in modelling preferences under risk, OR-Spektrum, 9, 129-151