

Rostek, Marzena Joanna

**Working Paper**

## How Do Income Distributions Change in Europe?

LIS Working Paper Series, No. 240

**Provided in Cooperation with:**

Luxembourg Income Study (LIS)

*Suggested Citation:* Rostek, Marzena Joanna (2000) : How Do Income Distributions Change in Europe?, LIS Working Paper Series, No. 240, Luxembourg Income Study (LIS), Luxembourg

This Version is available at:

<https://hdl.handle.net/10419/160912>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

**Luxembourg Income Study  
Working Paper No. 240**

**How Do Income Distributions  
Change in Europe?**

**Marzena Joanna Rostek**

**May 2000**



Marzena Joanna Rostek

## HOW DO INCOME DISTRIBUTIONS CHANGE IN EUROPE?

### Abstract

*This study explores income changes in nine European countries. First, in order to rank the distributions of income, we apply dominance criteria. Then, we extend this standard approach and examine the differences between Lorenz and Generalised Lorenz curves by means of nonparametric methods. We trace movements in relative position of particular income groups by investigating the direction and magnitude of the impact of changes in income that took place. Analysis of the inequality distributions allows us to reveal the information concealed in these curves, but not provided by the ranking. Apart from the time- and country-specific phenomena, we identify some regular, consistent across time and space patterns.*

*To assess the consistency of our results, we employ different specifications of equivalence scales. In addition, this sensitivity analysis provides evidence of a U- and inverted J-shape pattern displayed by the value of inequality index, when the parameters of the equivalence scale are changed.*

May 2000

I am grateful to Professor Dr Stefan Dercon for his helpful comments.

## Table of contents

1. Introduction .....	3
2. Theory .....	6
3. Data .....	8
4. Methodology .....	9
4.1 Kernel functions .....	13
4.2 Distribution properties of income share changes .....	15
5. Empirical analysis .....	20
5.1 Ranking income distributions by means of Lorenz and Generalised Lorenz dominance criteria .....	21
5.2 Analysis of changes in income shares .....	23
5.3 Density estimation of changes in income shares .....	27
6. Sensitivity analysis .....	31
7. Conclusions .....	35
References .....	37
Appendix 1	
Appendix 2	
Appendix 3	
Appendix 4	
Appendix 5	

## 1. Introduction

This paper examines trends in income distribution in Finland, France, Germany, Italy, Luxembourg, the Netherlands, Norway, Sweden and the United Kingdom. The question we aim to answer is whether the standard of living improved in these countries during the last decades. We do not only attempt to provide a yes/not answer, but we also look into the nature of changes that occurred.

Throughout the paper, we focus particular attention on one of the aspects of income distribution, namely inequality. Although as such it is not a measure of the standard of living, examination of it sheds more light on income dispersion. On the other hand, inequality as a relative concept does not take real standards of living into account. Thus, inquiring into inequality alone may lead to misleading inference<sup>1</sup>. Therefore, we examine it both by itself and together with the changes in mean income.

In the literature, the analysis of income patterns (the distributions themselves and changes over time) is typically based on the Lorenz curves and inequality measures (with the most commonly used: Gini and Atkinson's indices, Theil's entropy measure, coefficient of variation and interquantile ratio). This kind of studies provides evidence on the direction of overall change, without going into more detailed description, unless different population groups are examined (however, within them the description remains limited). (Generalised) Lorenz dominance allows for drawing meaningful conclusions but only if the distributions can be unambiguously ranked. Inequality indices can be helpful, especially in the inconclusive cases. Nevertheless, they still give only general statements. So much for the analysis of income changes? There still remain interesting questions to be answered. No matter whether the comparison of the (Generalised) Lorenz curves is conclusive or not, we can examine which income groups were affected and compare the strength of the impact. This is the task that we undertake in this study. Obviously, we do not claim that these questions have never been asked and answered, but the comparison is

---

<sup>1</sup> In fact, inequality can be treated in two ways. Firstly, it can be considered as a purely relative concept. The second approach encompasses the previous one as it allows considering inequality in a broader sense and in particular, to take account of absolute differences between income distributions that render into "within"-differences. As a result, such an approach would discriminate between the distributions with the same Lorenz curves with one being a multiple of the other. This distinction is critical for the relation between Lorenz and Generalised Lorenz curves (on which the analysis in this paper is based) and their efficiency.

commonly made by examining population subgroups, determined *ex ante* and it is very limited in exploring changes within them. We will extend the standard analysis of Lorenz and Generalised Lorenz curves and investigate the patterns displayed by the changes in relative income shares, derived from the former curves as well as absolute shares in the average income level, based on the latter ones. In particular, we estimate the densities of relative changes in income shares by means of kernel method. This enables us to identify the groups affected in a different way, without imposing the division of the population, and to explore the depth and frequency with which the changes occurred.

Cross-sectional comparison of income distribution in mid 1990's including Finland, Germany and Sweden, to mention the countries analysed in this study, can be found in De Nardi et al. (2000), for Germany (1984) – in Kazakov (1997). Ruiz-Huerta et al. (1999) examine inequality during two decades since mid 1970's for Germany, France, Italy, Norway, Sweden and the United Kingdom, Osberg (2000) – for Germany, Sweden and the United Kingdom. Gottschalk and Smeeding (1997) provide the review of evidence and the analysis of cross-country trends in income inequality, including all countries examined in this study. For illuminating theoretical and methodological discussion on the relation between polarisation and inequality see Wolfson (1994) and also Esteban et al. (1999), who provide empirical application as well (including Germany, Sweden and the United Kingdom).

The structure of the paper is as follows. First, we provide theoretical and methodological background to the economics of income distribution. We identify general properties of the distributions of the changes in income shares. They are more concentrated and tend to have fatter tails than the normal distribution. The skewness reveals whether increases or decreases in the shares prevailed. These effects are stronger for the absolute series compared to the relative ones. Next, we use Lorenz and Generalised Lorenz dominance criteria to compare income distributions of nine European countries. We find evidence of a change in income trends at the turn of the 1980's and the 1990's. Ranking of distributions suggests that in the second half of the former decade, there was an improvement in the standard of living in Finland, Luxembourg, the Netherlands, Sweden and the United Kingdom. All these countries, except for the United Kingdom, as well as Germany and Italy experienced deterioration in terms of the income level in the first half of the next decade. In France, the tendency was opposite. For Norway the evidence is mixed.

Subsequently, in order to gain more insight into income dynamics, we further analyse the series of income shares and estimate kernel densities of the first differenced changes in the Lorenz and Generalised Lorenz curves. The results point again to the opposite patterns in the 1980's versus the 1990's. In Finland, Luxembourg, the Netherlands, Sweden and the United Kingdom (*nota bene* the same countries where the outcome of the ranking was favourable to the society as a whole), relative redistribution in favour of the poorer income groups took place in the late 1980's and the opposite process could be observed at the beginning of the 1990's (with the exception of Finland). In Italy (in the earlier of the two periods) and in Norway (in the later one) these were middle income groups that became relatively better off. Moreover, we detect interesting regularities across countries, for instance in the way the very bottom and top parts of the income distributions were affected. We also find that in contrast to the 1980's, when detailed analysis of the within-distribution changes suggests interchange of the direction of impact on consecutive income groups, in the 1990's there was systematically a sizeable range of income for which the changes were similar. Furthermore, we observe that although relatively small changes dominate in all the examined cases, large magnitudes of impact (up to a sign) are more frequent than intermediate ones. This effect arises for both, relative and absolute changes. We also analyse the evolution of the group affected in a similar way in terms of their share in the average level of welfare.

We proceed by investigating the sensitivity of the results to changes in the equivalence scales. The rankings based on the Generalised Lorenz criterion are much more robust. In addition to testing the robustness of the results, we provide evidence of the U- and inverted J-shapes exhibited by inequality, measured by Gini coefficient, which arise as a result of the joint impact of the within-household economies of scale and household composition. Finally, we conclude.

## 2. Theory

In order to examine the dynamics of income in the 9 European countries, this study will exploit the concepts of first and second order stochastic dominance<sup>2</sup>. The key analytical device that serves as a base for other tools is the Lorenz curve.

The use of Lorenz curves to measure income inequality was first justified by Atkinson (1970), who adapted the notions from the theory of choice under uncertainty introduced by Rothschild and Stiglitz (1970). He showed that partial ordering of income distributions according to the Lorenz criterion is identical to the ordering implied by social welfare for additively separable social welfare functions regardless of the form of individual utility functions as long as they are identical and concave. Dasgupta, Sen and Starrett (1973) demonstrated that this result is valid with less demanding assumptions and the welfare functions need not be additive<sup>3</sup>.

Since the notion of dominance is crucial to our analysis, it is now useful to present it more formally.

Consider nonzero density function  $f_t(y)$  on the income interval  $[y_t^1; y_t^N]$ , (2.1)

with  $y_t^1 < y_t^N$  and mean  $\mu_t$ ,  $t=1,2$ <sup>4</sup>.

This implies:  $\forall p \in (0;1) \exists!$  income level  $x$  ranked with  $p$ :  $p = F_t(x)$ .

Lorenz curve is defined as follows:

$$L_t(p) = \begin{cases} \frac{\int_0^x y f_t(y) dy}{\int_0^\infty y f_t(y) dy} & \text{for } p \in (0;1) \\ 0 & \text{for } p = 0 \\ 1 & \text{for } p = 1 \end{cases} \quad (2.2)$$

<sup>2</sup> First degree stochastic dominance was developed by Quirk and Saposnik (1962) and second degree stochastic dominance - by Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970).

<sup>3</sup> S-concavity is sufficient. They also showed that Atkinson's theorem can be used in international comparisons.

<sup>4</sup> We use index  $t$ , since in our paper we compare income distributions over time.



$F_2 \geq_L F_1$  (distribution  $F_2$  Lorenz dominates distribution  $F_1$ ) if  $\forall_{p \in [0;1]} L_2(p) \geq L_1(p)$  and  $F_1 \neq F_2$ . The dominating Lorenz curves represent a more egalitarian distribution, as according to the Dalton (1920) principle of transfers the lower curve can be transformed into the upper one by means of equalising transfers<sup>5</sup>.

However, as it is commonly known, Lorenz dominance does not give a complete ordering of distributions. In the case of intersecting Lorenz curves the conclusions are ambiguous. Moreover, because the Lorenz curves are unaffected by the mean of the distribution, they can only be used to rank distributions in terms of inequality, not social welfare.

Generalised Lorenz (henceforth GL) curves - developed independently by Shorrocks (1983) and Kakwani (1984) - enable overcoming these deficiencies. The GL curve is derived from the Lorenz curve by scaling it up by the mean income:

$$GL_i(p) = \mu_i L_i(p). \quad (2.3)$$

$F_2 \geq_{GL} F_1$  (distribution  $F_2$  GL dominates distribution  $F_1$ ) if  $\forall_{p \in [0;1]} GL_2(p) \geq GL_1(p)$ .

Although GL dominance is equivalent to the second-order dominance, which is weaker than the first-order one, it may allow ranking distributions when the stronger concept yields inconclusive results. At the same time, this equivalence implies that GL dominance translates into preference by all increasing S-concave social welfare functions (see e.g. Thistle 1989, Deaton 1997). Taking into account that the Lorenz curve determines the distribution function up to location and scale (Iritani and Kuga 1983), GL curve - obtained by mean-scaling of Lorenz curve - completely determines the distribution function, and as a result is dual to it.

Nevertheless, if the GL curves cross, the ranking will depend on the specification of the given social welfare function and more precisely, on the potential trade-off between more equality and more mean. Thus, (Generalised) Lorenz curves provide only a partial ordering of income distributions. In contrast, any single-number measures allow a complete ordering and consequently can be used to rank the distributions, whose (Generalised) Lorenz curves cross. However, as first indicated by Atkinson (1970) and

---

<sup>5</sup> It is worth mentioning that this principle does not require quasi-concavity. S-concavity suffices, which is compatible with the conditions necessary for the Atkinson theorem to be valid, as shown by Dasgupta, Sen and Starrett (1973).

Sen (1973), they all imply some a priori value judgements about the distribution, which reflect in the discrepancies between inequality differences indicated by these measures. Specifically, Atkinson's index is sensitive to inequality changes in the lowest part of the distribution, Gini – around the median and Theil as well as coefficient of variation - in the top part. As long as the Lorenz curves of the distribution do not cross, the qualitative conclusions as to the direction of the inequality change, derived on the basis of all these measures are the same. Nevertheless, when the Lorenz curves intersect, these indicators can differ in the direction of the change in inequality between two distributions.

### **3. Data**

The data used in this paper come from the Luxembourg Income Study<sup>6</sup>. It comprises cross-section datasets through time collected by means of household surveys - almost 100 surveys for 26 countries spanning the period 1970 – 95. We will use 36 of these datasets to analyse changes in the standard of living for 9 countries (Finland, France, Germany, Italy, Luxembourg, the Netherlands, Norway, Sweden and the United Kingdom). Sample sizes are given in Appendix 1.

The approach we adopt allows us not to face the problem of ensuring simultaneity of samples to guarantee consistency<sup>7</sup>.

Annual disposable income is the variable that serves as a base for analysis in this paper. Its main advantage is that the definition varies little from country to country.

In comparative studies, it is necessary to take international differences in price structures into account, which is even more important when the standard of living is being evaluated. Since transforming nominal values into a common currency by means of official exchange rates does not ensure comparability with regard to purchasing power, the standard procedure is to use PPP's. We do this to convert national currencies into 1991 US dollars.

---

<sup>6</sup> The Luxembourg Income Study is a cooperative research project with 26 country members. It was founded in 1983 under the joint sponsorship of the government of the Grand Duchy of Luxembourg and the Centre for Population, Poverty and Policy Studies (CEPS). The main purpose of the project was to create a comparable micro-level data set.

<sup>7</sup>In fact, the LIS database poses severe limitations in this respect. Even if it did not, given that the economic cycles are not perfectly synchronised across the countries, data do not always refer to directly comparable economic periods.

Applying PPP involves an implicit assumption that such conversion, intended to reflect differences in purchasing power, is the same for households at all points in the distribution and as a result - for all household members. To establish income comparisons in real terms, an ideal method would be to use specific price indices for each household, as proposed by Ruiz-Castillo (1995), in order to analyse the distributive effects of inflation better. Dowrick and Quiggin (1994) suggested using detailed price and consumption data to rank countries according to the revealed preference approach, since as they argued, PPP-adjusted income may provide poor basis for the analysis of economic well-being. Nevertheless, this approach suffers from data limitations.

We use PPP's for consumption, not GDP. To provide more than intuitive (considering that we examine the standard of living) justification, we point to inappropriateness of using GDP index on micro data when there are cross-country differences in the tax-financed provision of public goods. Although included in GDP, they are not covered by the money income received by households. This incompatibility might affect measures of relative inequality.

Negative income values have been recoded to zero. As an alternative, one could use truncated distributions (see e.g. Cowell, Litchfield and Mercader-Prats 1999, Ruiz-Huerta, Martinez and Ayala 1999), where extreme observations are eliminated. However, since in our analysis we stress the inequality aspect, such a method might prove detrimental to the generality of the results<sup>8</sup>.

#### **4. Methodology**

The primary unit on which the data are based is a household. To provide more meaningful comparisons, we apply equivalence scales. In the first part of our analysis we use household income *per capita*. Next, in view of the evidence of studies in the 1990's on the non-monotonic relationship between inequality or poverty estimates and the parameters in equivalence scales, we apply more advanced scales.

Two seminal papers by Buhmann, Rainwater, Schmaus and Smeeding (1988) and Coulter,

---

<sup>8</sup> All observations are weighted by the product of 'hweight' (LIS sample weight) and the household size. For details about the structure of LIS database see <http://www.lis.ceps.lu>.

Cowell and Jenkins (1992), which initiated research on this issue, demonstrated that the value of inequality and poverty indices display a U-shaped pattern with the change of the economies of scale parameter. This finding was corroborated in subsequent studies (see e.g. Jenkins and Cowell 1994, Duclos and Mercader-Prats 1996, Mercader 1993, Figini 1998, Jänti and Danziger 1999, Ruiz-Castillo 1994). While in empirical studies it is common to take account of either household composition or economies of scale that arise within it, we decided to include both adjustments in the equivalence scales and in this way to make the examination of changes in income shares more relevant<sup>9</sup>.

We proceed using the following formula of parametric approximation:

$$Y_A = \frac{Y_D}{(\delta_1 N_1 + \delta_2 N_2 + \dots + \delta_k N_k)^\varepsilon} \quad (4.1)$$

where  $Y_D$  and  $Y_A$  denote disposable household income and adjusted income,  $N_i$  ( $i=1, \dots, k$ ) stands for the number of members of household subgroup,  $\delta_i$  ( $i=1, \dots, k$ ) indicates relative weight given to it and  $\varepsilon$  designates elasticity of equivalent income with respect to household size, i.e. represents the economies of scale within the household. In our study, we use 2-parameter version of this formula, applying OECD equivalence scale. Although this scale is an improvement compared to unweighted household size, it still involves a strong assumption of being constant across all characteristics other than number of subgroup members. Allowing  $\varepsilon$  to take values different from 1 enables us to overcome the main drawback of the OECD scale, when used by itself, i.e. not distinguishing between the relativities related to family size and to the needs of particular household members. The adjustment factor (denominator) in (4.1) becomes:

$$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon \quad (4.2)$$

where  $N_A$  and  $N_C$  stand for the number of adults and children in a household, respectively. We employ the range of the economies of scale parameter values:  $\varepsilon = 0, 0.25, 0.5, 0.75, 1$ . We do this primarily to test the robustness of the results to the specification of the equivalence scale as well as to check what is the pattern exhibited by the values of inequality index. For the analysis of sensitivity of the equivalence scale to changes in relative weights of household members, see Jenkins and Cowell (1994), Figini (1998).

---

<sup>9</sup> It is, however, disputable how the resources are shared among household members. This is an issue of extreme importance, since unequal sharing unequivocally increases inequality.

Our paper utilises the concepts of stochastic dominance to compare income distributions. We start the analysis with applying this standard apparatus of Lorenz curves (together with derived from them Gini coefficients) and their generalised version. They are used to make, where possible, conclusive statements about whether an improvement took place or not. Although inference about an overall change in inequality between periods could already be made from the Gini coefficients, a single measure does not allow for drawing conclusions about the changes in relative positions within the population. Moreover, one must bear in mind that Gini index is sensitive to inequality changes around the distribution median. In order to gain more insight into the changes in inequality, we extend this procedure to inquire into the character of changes in the distribution of income (which groups have gained and which have lost in relative terms, by how much). To this end, we analyse the distributions of inequality changes. More specifically, we investigate the scaled by the time interval for the sake of comparability (on a yearly basis) differences between the Lorenz and Generalised Lorenz curves as well as the first differences of these series. Using this latter method, we will be able to analyse changes that refer to the 1%-shares of population instead of the cumulative ones. First, we analyse the shares' changes in the order in which they appear directly in the (Generalised) Lorenz curves. Subsequently, in order to provide explicit information about the essence of each distribution, namely frequencies, we estimate the densities of the first-differenced changes in the Lorenz and GL curves. We do this by means of nonparametric methods, more specifically - kernel functions. Using the definitions (2.2) and (2.3) we can write the series, whose densities we estimate as<sup>10</sup>:

$$\Delta_k = \Delta_t \left[ L_t(p_j) - L_t(p_{j-1}) \right] = \left( \frac{x_{2,j}}{\int_0^\infty y f_2(y) dy} - \frac{x_{2,j-1}}{\int_0^\infty y f_2(y) dy} \right) - \left( \frac{x_{1,j}}{\int_0^\infty y f_1(y) dy} - \frac{x_{1,j-1}}{\int_0^\infty y f_1(y) dy} \right) \quad (4.3)$$

where  $j=1, \dots, 100$ ;  $k=j$ , for  $j=2, \dots, 100$ ;  $\Delta_t$  stands for differencing with respect to  $t$ ;  $x_{1,j}$  and  $x_{2,j}$  are income levels in the distributions from the earlier and later period, respectively, ranked with  $p_j$ :  $p_j = F_t(x_j)$ ,  $t = 1, 2$ .

---

<sup>10</sup> For the reasons of brevity, in analysis that follows we consider only the interesting cases with  $p \in (0; 1)$ .

Alternatively:

$$\Delta_k = \Delta_j [L_2(p_j) - L_1(p_j)] = \left( \frac{\int_0^{x_{2,j}} y f_2(y) dy}{\int_0^\infty y f_2(y) dy} - \frac{\int_0^{x_{1,j}} y f_1(y) dy}{\int_0^\infty y f_1(y) dy} \right) - \left( \frac{\int_0^{x_{2,j-1}} y f_2(y) dy}{\int_0^\infty y f_2(y) dy} - \frac{\int_0^{x_{1,j-1}} y f_1(y) dy}{\int_0^\infty y f_1(y) dy} \right) \quad (4.4)$$

Obviously, these two formulas are equivalent, but each of them allows for looking at our series from a different point of view. According to (4.3),  $\Delta_k$  is interpreted as a difference between income shares for a given percentage group of population, and to (4.4) – as a difference between income share changes for consecutive cumulative population shares. Both interpretations refer to income groups. Analogously, series obtained from the GL curves are defined as:

$$\Delta_{Gk} = \Delta_t [GL_t(p_j) - GL_t(p_{j-1})] = \left( \mu_2 \frac{\int_0^{x_{2,j}} y f_2(y) dy}{\int_0^\infty y f_2(y) dy} - \mu_2 \frac{\int_0^{x_{2,j-1}} y f_2(y) dy}{\int_0^\infty y f_2(y) dy} \right) - \left( \mu_1 \frac{\int_0^{x_{1,j}} y f_1(y) dy}{\int_0^\infty y f_1(y) dy} - \mu_1 \frac{\int_0^{x_{1,j-1}} y f_1(y) dy}{\int_0^\infty y f_1(y) dy} \right) \quad (4.5)$$

or

$$\Delta_{Gk} = \Delta_j [GL_2(p_j) - GL_1(p_j)] = \left( \mu_2 \frac{\int_0^{x_{2,j}} y f_2(y) dy}{\int_0^\infty y f_2(y) dy} - \mu_1 \frac{\int_0^{x_{1,j}} y f_1(y) dy}{\int_0^\infty y f_1(y) dy} \right) - \left( \mu_2 \frac{\int_0^{x_{2,j-1}} y f_2(y) dy}{\int_0^\infty y f_2(y) dy} - \mu_1 \frac{\int_0^{x_{1,j-1}} y f_1(y) dy}{\int_0^\infty y f_1(y) dy} \right) \quad (4.6)$$

We will refer most of the time to the interpretation of (4.3) and (4.5).

We use the series based on the Lorenz curves to answer questions that refer to the relative position of population subgroups. We compare the impact of changes on the poorer and the richer as well as the impact of inequality changes across the population in more detail. Analysis of first-differenced changes between the Generalised Lorenz curves enables us to give the sense of absolute income distance (and thus to interpret the results in terms of welfare or the standard of living, as it is achieved by the ranking distributions by means of Shorrocks's theorem). These distributions of the differences between the shares in the average social welfare (standard of living) provide important information that could not be inferred from the ranking. Specifically, they show discrepancies in the strength of impact of the change, help to analyse the evolution of the population fraction that was affected in

a similar way, detect relative changes reversed to the direction pointed by the GL criterion and identify groups that experienced them<sup>11</sup>.

Another advantage of the method we use is that it can be applied also to the cases where the ranking based on the (Generalised) Lorenz dominance is inconclusive.

#### 4.1 Kernel functions

The simplest nonparametric density estimate of a distribution is of course histogram. However, histograms are far from being perfect in density approximation. Since they are sensitive to the choices of origin as well as the number of bins and their width (to a large extent arbitrary), the shape of the distribution differs conditionally on this choice, which of course is not a desirable property. More importantly, histograms as devices developed to represent discrete distributions are not continuous and consequently, incompatibility problem arises, when they are used to estimate densities of continuous variables. This has its implications not only for the shape of the distribution, but also for differentiation. Therefore, in order to avoid these deficiencies, we will estimate the densities using the kernel method. For the sample of  $N$  observations  $(x_1, \dots, x_N)$ , kernel density estimate at point  $x$  is:

$$f(x) = \frac{1}{Nh} \sum_{i=1}^N K\left(\frac{x - x_i}{h}\right)$$

where  $h$  is the bandwidth and  $K(\cdot)$  denotes a kernel function. Kernels smooth estimates compared to histograms, by putting less weight on observations that are further from the point being evaluated. From the range of kernel functions, each giving different weights to the points inside the band, we employ Epanechnikov kernel for all series, showing the results of alternative specification for one case. The kernel functions we use are specified as follows:

Epanechnikov: 
$$\frac{3}{4}(1 - u^2)I(|u| \leq 1)$$

Triangular: 
$$(1 - |u|)I(|u| \leq 1)$$

---

<sup>11</sup> It is important not to misinterpret conclusions that follow from the analysis of the differenced changes between the (Generalised) Lorenz curves. Changes in the direction opposite to this pointed by the dominance criterion do not contradict the conclusions about the ranking, since they refer to the relative position of a given income group.

Uniform:	$\frac{1}{2}I( u  \leq 1)$
Normal (Gaussian):	$\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)$
Biweight (Quartic):	$\frac{15}{16}(1-u^2)^2I( u  \leq 1)$
Triweight:	$\frac{35}{32}(1-u^2)^3I( u  \leq 1)$
Cosinus:	$\frac{\pi}{4}\cos(\frac{\pi}{2}u)I( u  \leq 1)$

where  $u$  is the argument of the kernel function.

It is not, however, the choice of the kernel function that is decisive in determining the shape of the estimated density, but the choice of bandwidth as it controls the smoothness of the density estimate. Considering that oversmoothed estimates are biased and undersmoothed – too variable, in order to find a compromise to the trade-off between variation and bias we investigate the sensitivity of our estimates to the bandwidth selection. Specifically, we plot three density estimates using bandwidths  $0.5h$ ,  $h$ , and  $1.5h$ , where  $h$  is a data-based (not artificially imposed) bandwidth computed using method proposed by Silverman (1986)<sup>12,13</sup> :

$$h = 0.9kN^{-1/5} \min\{s, R/1.34\}$$

where factor  $k$  is a canonical bandwidth transformation,  $N$  - the number of observations,  $s$  - the standard deviation, and  $R$  - interquartile range of the series. Then, as suggested by Deaton (1997), we assess visually whether the plots are under- or oversmoothed, to find the threshold which does not seem to generate bias in either way<sup>14</sup>.

## 4.2 Distribution properties of income share changes

We have already justified why we will use more advanced kernels than the histogram. As far as nonparametric methods themselves are concerned, they have the benefit of not

<sup>12</sup>The canonical bandwidth-transformation adjusts the bandwidth in a different way across various kernel functions so that the automatic density estimates have approximately the same amount of smoothness.

<sup>13</sup>There is no general rule for the appropriate choice of the bandwidth. An alternative to the kind of grid search we make is to choose a somewhat small bandwidth and thus undersmooth since it is easier for the eye to smooth (Silverman 1986).

<sup>14</sup> Depending on the purpose of the analysis, it could be desirable to extend this procedure to cross-validation (see e.g. Hamilton 1994, Silverman 1986) by comparing mean-squared errors between the estimates for different bandwidths and choosing the bandwidth that minimises error.



imposing any explicit functional form of the densities. This is important, since according to our knowledge, there are no studies applying this kind of analysis to the series that we examine. This is the reason why we find it instructive to check whether they exhibit any common patterns. As by doing this, we want to determine more general properties of the distributions in question, we provide this analysis in the part of the paper devoted to methodology. Of course, these properties will be useful for the interpretation of the results from the kernel estimation that follows.

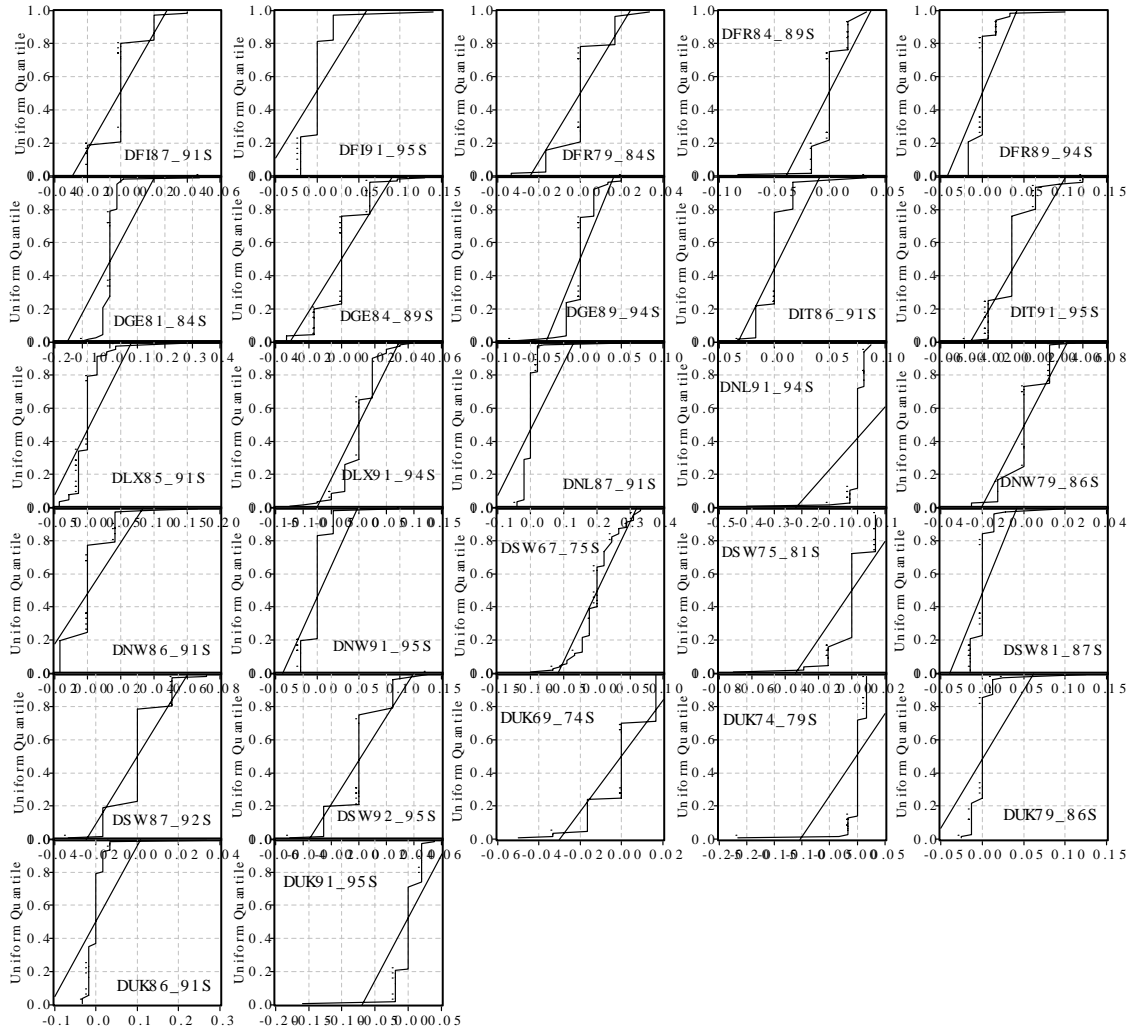
To form some expectations about the distributions, we apply the quantile-quantile (QQ)-plots to the first-differenced changes in the Lorenz curves (Figures 4.1 and 4.2) and subsequently, in GL curves (Figure 4.3). The quantiles of the chosen series are plot against the quantiles of a theoretical distribution. The normal and uniform distributions will serve as a point of reference. The former is always useful as a benchmark. The latter has an interesting interpretation. In general, since the considered distributions show the frequencies of a given change in the income share, the more the distribution of the changes in (Generalised) Lorenz curve values deviates from the uniform density, the more similar the impact of the income changes on the relative position of groups placed in different parts of the distribution.

This stage of our study will enable us to make an assessment of the shape of densities that we will estimate. We have decided to carry the analysis in this order (first, to form some expectations about the properties of the examined densities and after that - estimate them) in order to avoid obviously undesirable hasty conclusions and biased interpretation.

First, we compare our sample quantiles of the series derived from the Lorenz curves with the uniform ones and detect some empirical regularities.

Figure 4.1 Quantile-quantile plots of scaled first-difference changes between Lorenz curves

(in relation to uniform distribution)



1. The quantile-quantile plot is a tool for comparing two distributions. If they are the same, the QQ-plot should lie on the 45°-line. The pattern of deviation from linearity provides an indication of the nature of the discrepancy.
2. “S” stands for “scaled by the number of years”.

In the Figure 4.1 we observe that QQ-plots are not smooth, they bend sharply instead, which indicates that the magnitudes of the changes in relative income shares are centred around some values, rather than spanned on an interval. The regression lines, although approximately parallel to the 45° one, are systematically either much higher or lower than this borderline of corresponding quantiles. This means that our empirical distributions are more concentrated than the rectangular density function (which was to be expected). This also reveals that the distributions are skewed. Intuitively, the sign of the skewness must be related to the sign of the relative change, i.e. whether an improvement or deterioration prevailed.

Figure 4.2 Quantile-quantile plots of scaled first-differenced changes between Lorenz curves  
(in relation to normal distribution)<sup>1,2</sup>

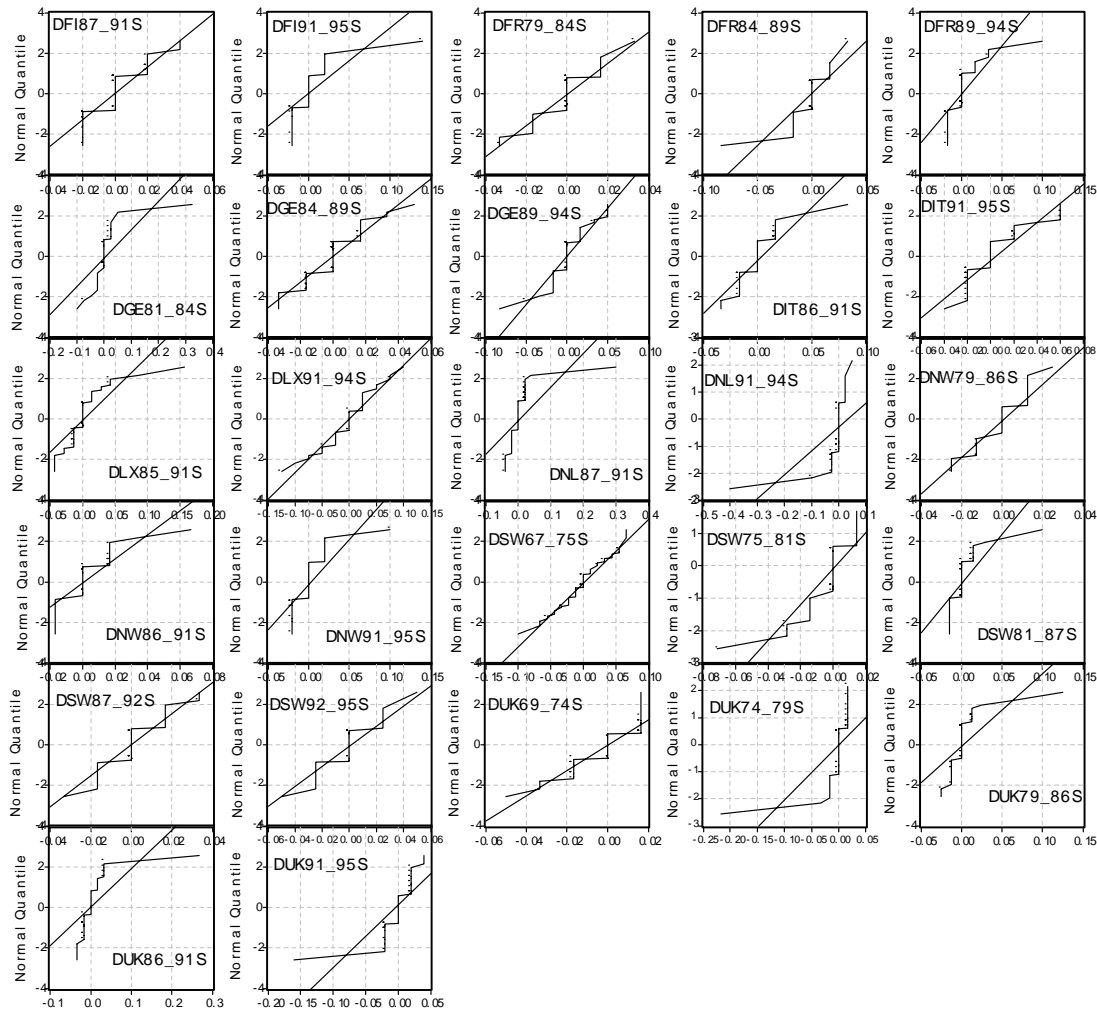


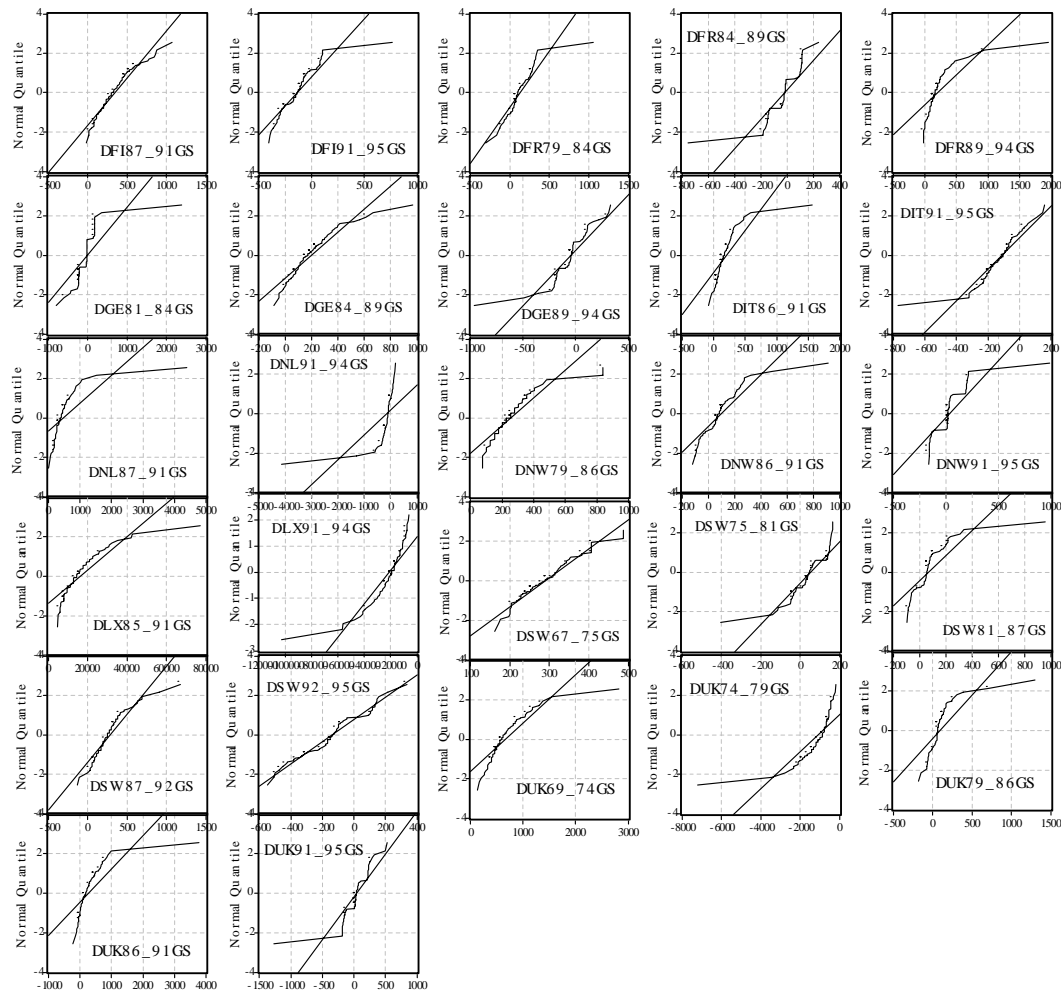
Figure 4.2 presents the results of the same exercise as above, but this time we relate the sample distributions to the normal distribution. Again, a few common characteristics can be noticed. If we compare the new plots with those from the Figure 4.1, we observe that slopes are less steep (to ease the comparison, regression lines are provided). More in detail, in contrast to the previous example, where the slopes tend to be bigger than  $45^\circ$ , magnitudes close to that value seem to prevail. Since the QQ-plots in the Figure 4.2 start above the  $45^\circ$ -line and/or end below it, we conclude that at least one of the tails is thicker than in the normal distribution<sup>15</sup>. Small transitions from concavity into convexity of the QQ-plots with the concave or convex pattern point to the existence of more than one peak.

<sup>15</sup> There are two exceptions: the Netherlands (1991-94) and the United Kingdom (1974-79). The kind of estimation we are doing requires data of particularly high quality. Less precise data may result in smaller range of changes in income shares, whose distributions we examine. While working with our data, we have noticed that the series for the Netherlands are relatively less precise.

Moreover, in all but one (Norway 1979-86) cases the estimated kurtosis statistics (Appendix 2A) exceed 3, which indicates that the distributions are leptokurtic and thus more peaked than the normal.

To assess how important are the described above departures from normal distribution, we perform Jarque-Bera test (that is based on both considered parameters, i.e. kurtosis and skewness). The results (see Appendix 3A) show that the null hypothesis of normality is rejected in 20 cases at 1% level of significance and one more at 5% level.

Figure 4.3 Quantile-quantile plots of scaled first-differenced changes between Generalised Lorenz curves (in relation to normal distribution)<sup>1</sup>



1. “G” refers to series derived from the Generalised Lorenz curves

In the next step, we explore the distributional properties of the differenced series between the GL curves. In view of the large departures from the uniform density of the relative

changes, deviations of these series are even more likely. Therefore, we decide to compare our sample distributions with the normal one only.

Greater smoothness of the QQ-plots in Figure 4.3 indicates more diversified range of the presently examined series. All plots deviate from a straight line and rule of deviation from linearity is invariant to the time period and country choice. They are either concave or convex and this feature is in more distinct than in the series derived from the Lorenz curves. The concave shape of the QQ-plot for indicates that the distribution of the series is positively skewed with a long right tail, the convex one points to negative skewness. Estimated kurtosis of the distribution (Appendix 3B) are greater than 3 (with two exceptions for Sweden, substantially), which points to greater concentration compared to the normal distribution. Another similar to the previously analysed series property is that most of the QQ-plots are above the 45°-line at the left end and below it at the right end, which suggest that the likelihood of relatively large or small changes is greater than would be expected under a normal distribution. However, the conclusions about fat-tails property are not straightforward, since although to capture the higher than normal kurtosis, more fat-tailed distributions could be used to model the considered series<sup>16</sup>, there is no consensus on the precise amount of probability mass in the tails of the distribution. More importantly, although we aim at describing densities of differenced changes in GL curves, it is not our purpose to find the precise specification, but to provide some useful insights into the nature of changes in the income distribution.<sup>17</sup>

Analogously to the changes in relative shares, we test for normality of absolute series (Appendix 3B). The test rejected the null hypothesis at 1% level of significance in all but two cases (Sweden 1967-75, 1992-95). With these 2 exceptions, p-values for  $\chi^2$  statistics are 0.00000. Compared to the previous series, absolute shares deviate from the normal distribution even more, in the sense of greater concentration.

---

<sup>16</sup> Examples include t-Student, the discrete mixtures of normals and the exponential distribution.

<sup>17</sup> Modelling these series would require extreme caution. The series we examine do not have to be (and usually they are not) strictly random, since they can be to a large extent influenced by policy. Moreover, the pattern of the series in question could be due not only to economic forces, which affect income but also to the measurement error. Any potential model would have to account for these issues. Obviously, there is no unique distribution representing the behaviour of the changes in income shares and attempts to obtain it would be pointless (even if by some chance successful, they would be biased). They are determined by policies as well as purely economic market forces. Therefore, particular cases should be considered separately. Choosing nonparametric methods, we implicitly allow for richer representation. Nonetheless, as it is shown in this study, some common features can be determined.

An overall conclusion from the above comparisons is that the densities of both relative and absolute income changes across the income subgroups of population are more concentrated than the normal, have fatter tails and are skewed (as we expect, depending on whether positive or negative changes prevailed). These effects are stronger for the distributions of the absolute share changes.

One issue deserves to be highlighted. The area under the density function for the differences between the relative shares (not differenced) can be related to the change in inequality, since it corresponds to half of the difference in Gini coefficients. The mean of changes in (cumulated) relative income shares is proportional to the difference in Gini coefficients associated with the Lorenz curves being compared. This matches intuition: inequality (measured by Gini coefficient) changes in proportion to the shift in average income share.

## **5. Empirical analysis**

In our analysis, we use 100-point Lorenz curves. We deliberately decide not to increase the number of these points. Abstracting from economic interpretation of our analysis, increasing the number of observations is of course in general beneficial to the precision of estimates. However, we have checked that this would make the curves unnecessarily undersmoothed without adding gains for the interpretation that would counterbalance this disadvantage. Willing to ensure high precision of the estimates on one hand, and to avoid any distortions that could prove detrimental on the other, we face two opposing effects of expanding the size of the set used in estimation. Therefore, we have chosen to base our analysis on 100-element curves and in order to decrease the probability of potential deviations as well as increase the reliability of the conclusions that will follow (both in and from our analysis) we will employ nonparametric methods that use the available observation much more efficiently. Apart from the above-mentioned advantages of using the nonparametric method, this choice has also the benefit of retaining the favourable properties that are no longer mutually exclusive and making our references to density curves estimated from these points more trustworthy. Moreover, these are percentages that are useful to refer to in interpretation, not promilles.

## 5.1 Ranking income distributions by means of Lorenz and Generalised Lorenz dominance criteria

In this section, we apply the Lorenz and subsequently, GL dominance analysis to rank the distributions of income of the 9 countries whose changes in the standard of living we investigate.

Table 5.1 summarises the results of comparing the Lorenz curves. The method used was to examine the differences of values related to a given population percentage, divided by the length of time interval<sup>18</sup>. Corresponding figures are presented in Appendix 4A. Apart from the Lorenz curves themselves, the differences are plot.

Table 5.1 Ranking of Lorenz curves

COUNTRY	SAMPLE PERIODS				
<b>Finland</b>				1987-91	1991-95
				-	_*
<b>France</b>			1979-84	1984-89	1989-94
			0	0	0
<b>Germany</b>			1981-84	1984-89	1989-94
			0	0	0
<b>Italy</b>				1986-91	1991-95
				0	_*
<b>Luxembourg</b>				1985-91	1991-94
				0	0
<b>Netherlands</b>				1987-91	1991-94
				0	0
<b>Norway</b>			1979-86	1986-91	1991-95
			0	-	0
<b>Sweden</b>	1967-75	1975-81	1981-87	1987-92	1992-95
	_*	_*	-	0	0
<b>United Kingdom</b>	1969-74	1974-79	1979-86	1986-91	1991-95
	_*	0	-	0	0

1. '+' denotes dominance of the Lorenz curve corresponding to the later year, '-' – to the earlier year, '0' – inconclusive ranking.
2. '\*' indicates these cases where the directions of the change in inequality and the mean are opposite (unambiguous ranking).

The evidence from the examined samples is conclusive in 9 out of 27 cases. It shows that in the 1970's income inequality decreased in Sweden and in the first part of this decade, also in the United Kingdom. In contrast, in the 1980's, these two countries experienced an

<sup>18</sup> In his paper from 1983 Shorrocks made pairwise comparisons in 10 points of the Lorenz curves. Here, a different approach is adopted. Instead of comparing values in 10 discrete cases, 100 points for each curve is computed, enabling a continuous approximation. This substantially reduces the probability that other intersections may take place within the ranges and thus, makes the conclusions more reliable.

increase in inequality. In the first half of the 1990's, the unambiguous results point to inequality increase in Italy and Finland. Nevertheless, irrespective of whether the ranking is conclusive or not, we can inquire into the direction of the inequality changes by means of the Gini coefficients. The results (see Appendix 6) show inequality increase for one decade since mid 1980's in Finland, the Netherlands, Norway, Sweden and the United Kingdom. In France, Germany and Italy, the second half of the 1980's with decreasing inequality was followed by the reversed tendency in the first half of the 1990's. In Luxembourg, the trend was opposite.

However, among the conclusive (with respect to inequality change) cases, 4 are characterised by increasing inequality and at the same time – increase in the absolute value of income (for mean income see Appendix 2). Therefore, normative conclusions are dubious without additional assumptions. On the whole, using the corollary of the Atkinson's theorem (1970), only 5 out of 27 pairwise comparisons indicate that income distributions become unequivocally more or less preferable<sup>19, 20</sup>.

The next step is to check how (if at all) the conclusions change when the Generalised Lorenz curves are used. The results are reported in Table 5.2. Graphs are presented in Appendix 4B.

Table 5.2 Ranking of Generalised Lorenz curves

COUNTRY	SAMPLE PERIODS				
<b>Finland</b>				1987-91	1991-95
				+	-
<b>France</b>			1979-84	1984-89	1989-94
			0	-	+
<b>Germany</b>			1981-84	1984-89	1989-94
			0	0	-
<b>Italy</b>				1986-91	1991-95
				0	-
<b>Luxembourg</b>				1985-91	1991-94
				+	-
<b>Netherlands</b>				1987-91	1991-94
				+	-
<b>Norway</b>			1979-86	1986-91	1991-95
			+	0	0
<b>Sweden</b>	1967-75	1975-81	1981-87	1987-92	1992-95
	+	+	0	+	-

<sup>19</sup> Although the data are not based on cohort samples, large number of observations should justify the generalisation made.

<sup>20</sup> To be precise, the conclusion from Atkinson's *theorem* for these cases would be that income distributions with higher mean and lower inequality are preferred to the distributions to which they are being compared times the ratio of means.



<b>United Kingdom</b>	1969-74	1974-79	1979-86	1986-91	1991-95
	+	-	0	+	0

1. '+' denotes dominance of the Lorenz curve corresponding to the later year, '-' – to the earlier year, '0' – inconclusive ranking.

When the Lorenz curves are transformed into their generalised form, 12 comparisons became conclusive and 3 inconclusive. Interestingly, the direction of dominance changes for the turn of the 1980's and the 1990's in Finland: although the analysis of absolute changes revealed an inequality increase, according to the ranking of distributions with the adjustment for the mean income, there was an improvement for the whole society. Inspection of the table reveals that the GL curves intersect in only 9 cases. Thus, in the two-thirds of pairwise comparisons Shorrocks' theorem (1983) yields unambiguous ranking (the rate for Lorenz curves was less than 20%)<sup>21</sup>. However, 30% of cases still remain unresolved - the change in average income for inconclusive cases is not sufficient to offset the variation in relative income shares.

Apart from the conclusions drawn from the analysis on the basis of Lorenz curves, which remain valid with one noted exception for Finland, Table 5.2 gives evidence of the improvement (in terms of income) in the late 1980's and the beginning of the following decade in Finland, Luxembourg, the Netherlands, Sweden and the United Kingdom. In the first half of 1990's the trend was opposite in 6 out of 9 countries. These exceptions are Norway and the United Kingdom, where the ranking is inconclusive, and more interestingly France, where the pattern of changes was opposite to the other countries.

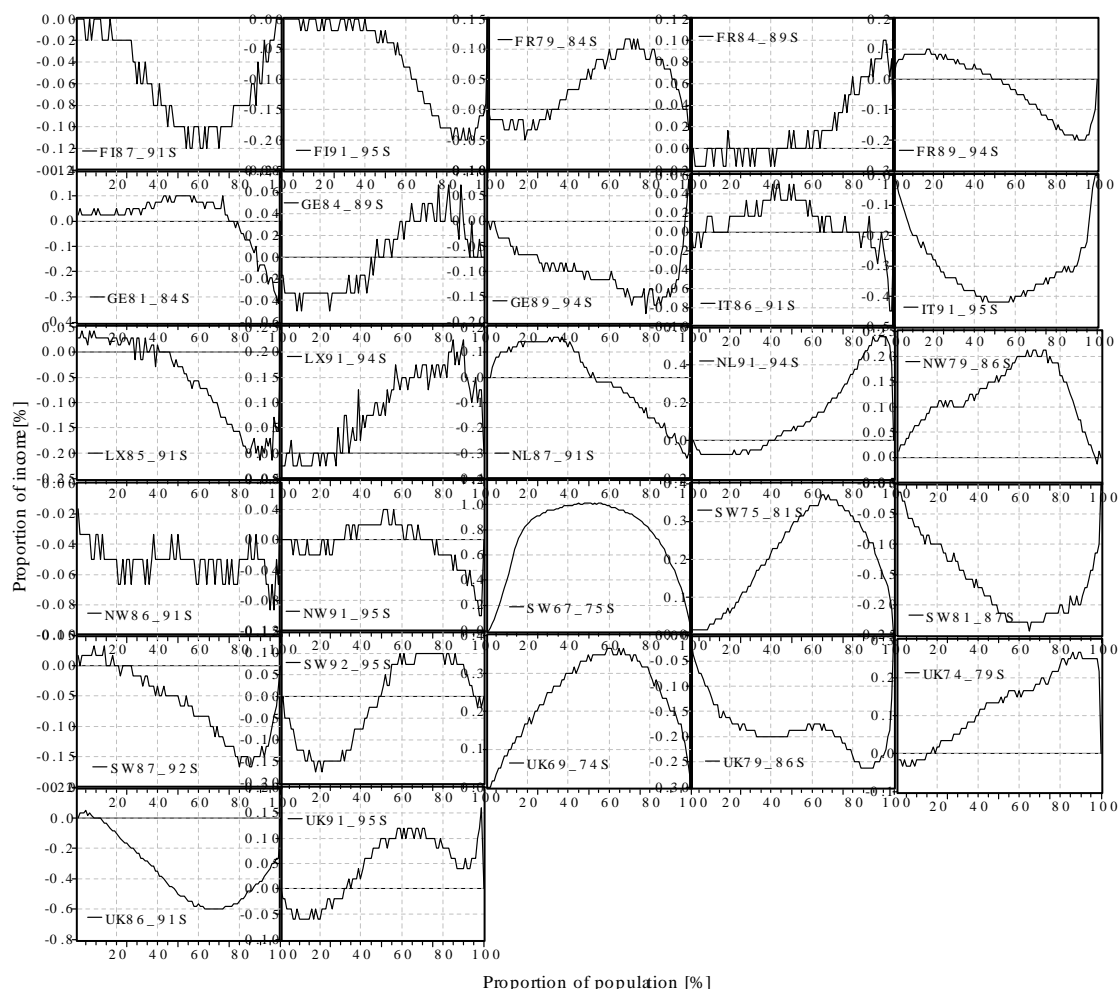
In the following sections, we extend the analysis of income inequality to the examination of changes at various points of the distribution. We start with more detailed - than on the basis of (Generalised) Lorenz dominance – description of the patterns of inequality changes. Next, we estimate the distribution densities of these series.

## 5.2 Analysis of changes in income shares

In order to provide a more thorough description of the changes in income, in the Figure 5.1 we plot the scaled by the time interval differences between the Lorenz curves' values for any pairwise comparison made.

<sup>21</sup> The rates obtained by Kakwani in his study (1984) analysing 72 countries (and thus involving 2556 pairwise comparisons) were as follows: cases resolved by Atkinson's theorem and corollary – 47%, Shorrocks' theorem – 37%, indecisive – 16%.

Figure 5.1 Scaled changes in relative income shares per cumulative shares of population



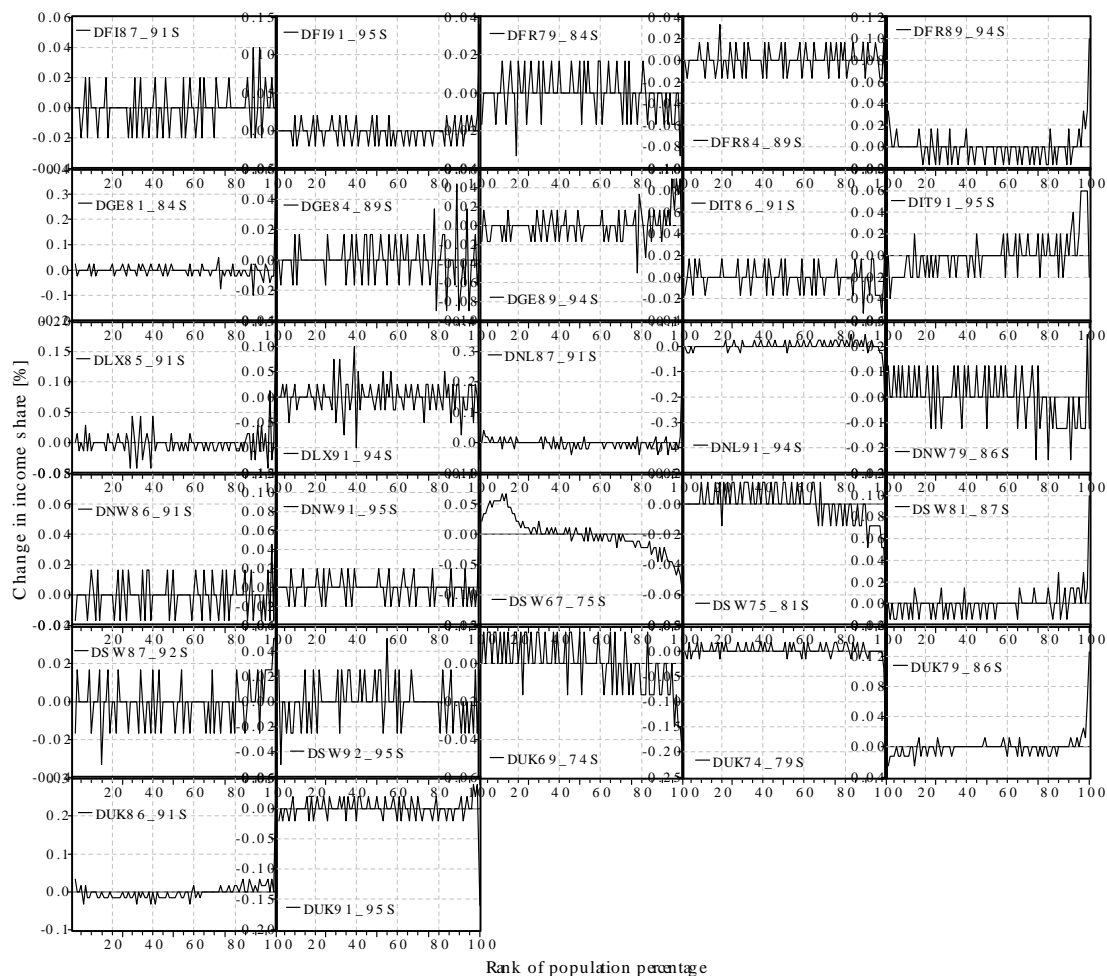
Presence of parts of plots on two sides of the zero-line indicates general pattern of redistribution - in relative terms, however. Therefore it is worthwhile to use the evidence from the analysis based on the (Generalised) Lorenz dominance (Tables 5.1 and 5.2) and check how the presence of redistribution (and its direction) translates into the preference of the distribution of income in absolute terms.

Of course, “relative redistribution” of income took place in all cases, where Lorenz dominance criterion was inconclusive. In the majority of samples, where the Generalised Lorenz dominance has yielded unambiguous ranking, the more preferred distribution is also the one with the relative position more in favour of the poorer and *vice versa*. More in detail, apart from the observed above change in inequality at the turn of the decades, we notice that in the second half of the 1980’s, the income share of the poorer group of society increased in Luxembourg, the Netherlands, Sweden and the United Kingdom. This is also true for Finland in the sense that the decrease in the share of the bottom income group is much smaller compared to the higher income part of the distribution (one can say

that it is relative redistribution of relative change). The evidence from the examined samples points to the opposite process in all these countries (except for Finland) in first half of the 1990's. For France and Germany this trend is reversed. Moreover, we notice an interesting pattern of change in "relative redistribution" for Italy 1986-91 and Norway 1991-95, where the relative position of the group of population in the middle of distribution improved, "at the cost of" the richest and the poorest. As a matter of fact, the bottom and top parts of the distributions were affected in an opposite way in Finland (1987-95). In Germany and Italy the impact reverses at the turn of the 1980's and the 1990's. The very extreme groups systematically experience the change in the same direction in France (1979-94), Luxembourg (1985-94), the Netherlands (1987-94), Norway (1986-95), Sweden (1987-95) and the United Kingdom (1986-95), but only in the two noted above cases the impact on the highest income group is strong enough to remarkably counterbalance the effect on the middle part.

In order to obtain explicit picture of the extent and direction of changes that affected consecutive income subgroups (underlying the cumulative effects observed above), in the Figure 5.2 we plot first-differenced series of differences between the Lorenz curves.

Figure 5.2 Changes in relative income shares per population percentage shares



The pattern of relative movements is much more complex than for the cumulative shares of population. However, the goal of this latter investigation was to determine the changes in relative position of the poorer versus the richer. The present technique not only confirms the findings of the effects on the extremes of the distribution, but it allows us to thoroughly ascribe the increases and decreases in relative income shares to all the income groups. The overall effect the poor – the rich depends on which kind of changes dominates and concentration of changes of identical sign in different parts of the distribution is not necessary for it to arise, which is the case in the second half of the 1980's for France, Germany, Luxembourg, the Netherlands and Sweden. In contrast to this period, when in all these countries and also in Finland and Italy we observe that the changes in the relative position are spread across the population in such a way that improvement and deterioration interchange, with the exception of Luxembourg, at the beginning of the 1990's it is possible to identify one distinct income group, who was affected in the same way (in terms

of the sign of the relative change). Throughout the 1970's, in Sweden and the United Kingdom these were clearly the richest that lost in relative terms.

Interestingly, we can assess the magnitudes of changes in relative income shares, which are systematically around 0-0.03%, rarely larger than 0.05% (up to a sign). However, the range of oscillations is not uniform across all the income groups. In contrast to the other countries, where the variation of relative changes tends to increase at the extremes of the distribution, in Luxembourg (1985-94) the second quarter of the population experienced the largest change. This leads us to the next step, i.e. assessing the frequencies with which particular values of changes occur.

### 5.3 Density estimation of changes in income shares

Now, when we have the intuition about the properties of the distributions of the changes in income, we will estimate densities of the differenced changes between the Lorenz and Generalised Lorenz curves (thus we will use points related to respective population percentages) by means of kernel method. To ensure consistency with the procedure that we have followed so far, we evaluate density functions in 100 equispaced points.

We check seven specifications of kernel function, with triple choice of bandwidth for each one. In Appendix 5 (A1, A2, B1, B2) we present the density estimates for the differences between the Lorenz (A) and Generalised Lorenz curves (B) by means of the Epanechnikov kernel (respectively A1, B1) and for the sake of comparison - all seven densities for one series, the United Kingdom 1991-95 (A2, B2)<sup>22</sup>. Overall impression with respect to the bandwidth effect is that only the reference choice ( $h$ ) appears neither to overvalue nor to undervalue the magnitudes of frequencies and the span of density, balancing the trade-off that arises between them<sup>23</sup>. Moreover, only this band gives the shapes consistent across all the specifications (with respect to the number and height of peaks, continuity etc.).

The forms of kernel functions have stronger effects on the shapes of estimated distributions of series derived from Lorenz curves compared to the GL curves, where the only estimate distinct from the others, which differs only in a very subtle way, is that by means of the uniform kernel. Densities based on the GL differences are much less

---

<sup>22</sup> Observations we provide for this one example are representative - we have checked for other countries and periods.

<sup>23</sup> The sensitivity of the estimated densities to the data precision can be seen in the plots in Appendix 5.

variable, which is intuitive in view of their construction. For the Lorenz curves cases, we notice that Gaussian and triweight choices tend to oversmooth the densities (when we consider all three bandwidth values). In contrast, triangular and uniform kernels lead to artificially peaked curves. We regard as reasonable approximation Epanechnikov and cosine kernels, which yield to very similar (basically indistinguishable) estimates that are neither unnecessarily distorted nor too smooth (which is important, considering that these are changes in income shares that are the essence of our analysis). Properties of the shapes in quartic (biweight) specification are intermediate, although closer to the smoother estimates. Greater efficiency of the Epanechnikov kernel in terms of minimising mean integrated square error may serve as an additional argument supporting our choice (see Silverman 1986, where this kernel function is compared to the biweight, triangular, Gaussian and rectangular kernels).

The first striking feature we notice is systematically 3-peaked shape of the densities of the changes in the relative income share. We have checked that the mode of the distribution is always at 0. The associated with it group, whose relative position did not change, consistently constitutes around 30-40% population<sup>24</sup>. At the same time there were two distinct groups that were affected in an opposite way. As far as the changes themselves are concerned, the observed pattern also implies that extreme changes in the relative income shares were more frequent than the moderate. Together, these properties corroborate the anticipated features of fat-tailness and relatively high concentration, determined on the basis of the quantiles analysis.

The results from the analysis of dominance and examination of the differences between the Lorenz curves are reflected in the skewness of the distribution, as was to be expected. The estimated densities of income changes tend to skew negatively, if the income distribution became more preferred by the whole society (according to the Lorenz dominance criterion) or improvements prevailed, and *vice versa*.

Interestingly, although the probability mass associated with the positive and negative changes is not the same, the magnitudes of these values are symmetric around 0, irrespective of whether the distribution became more or less preferred (if at all). In this

---

<sup>24</sup> The frequency corresponding to the mode ranged in our samples from 29% to 43%, with Sweden (1967-75) – 13% and Italy (1986-91) – 47%.

sense, the pattern of entropy (see Shannon 1949) seems similar across time and space. However, to establish the degrees of uncertainty to which the distributions correspond, it is necessary to look at the domain. We notice that in all the countries the range of the relative income changes was larger in the first half of the 1990's compared to the preceding period. Moreover, the impact of the changes on society at that time was different not only with respect to magnitudes, but also more diversified (more continuous density, larger intervals in the domain)<sup>25</sup>.

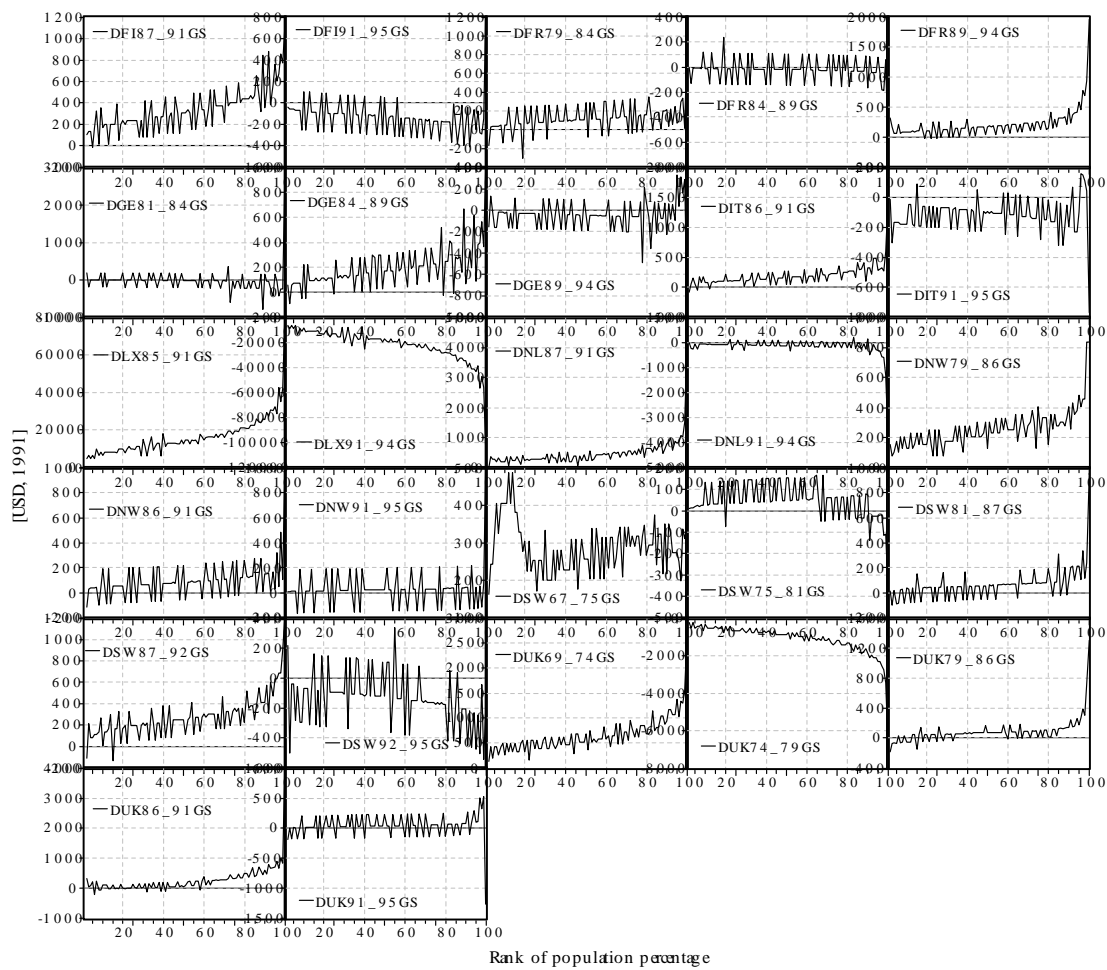
The estimated densities of the series derived from the GL curves are skewed with a large probability mass on the interval of the sign corresponding to the direction of changes. In this way the analysis of these densities can be treated as a kind of alternative of the comparison based on the GL dominance. However, they give more details than a simple ranking.

The densities tend to have one distinct peak that dominates, instead of being concentrated around separated (and narrow) intervals. Nevertheless, although these distributions are more continuous, there is an interval of unusually high or low values (often separated from the remaining part of domain). We conclude that there was systematically a fraction of society who experienced exceptionally (compared to the rest of society) large improvement or deterioration. Thus, also in these series the observed above extreme values effect is present. The sign of these unusually large (in absolute terms) values was not always as one could expect from the GL ranking, however. For instance, although in Finland (1991-95) and Sweden (1975-81, 1987-92) the distribution dominance points to an improvement for the society as a whole, a part of population fractions was affected in an opposite way, in terms of their share in the average level of welfare (standard of living). When we analyse the estimated densities together with the series of the changes in GL curves ordered according to the values of income (Figure 5.3) we can identify these groups. These are, respectively, income groups in the first half, top part and bottom end of the distribution. Similarly, while in Sweden (1992-95), according to the GL criterion the income distribution became less preferred, a considerable group (middle income levels) became better-off in terms of their share in the average level of welfare. These observations enrich the picture of income changes with important information.

---

<sup>25</sup> However, this could be to a large extent due to the data quality.

Figure 5.3 Changes in average income share per population percentage share



Furthermore, we can trace the strength of the impact of income changes over time. An interesting example is Norway, where from the end of the 1970's until mid 1990's it became more and more extreme. Similar pattern can be observed in Sweden. On the other hand, the mass probability at the mode was approximately constant over time in Sweden for the whole period of two decades from mid 1970's and for one decade in Norway from mid 1980's. This means that the proportion of population affected in a similar way remained basically the same. In France (1979-94), Germany (1981-94), Luxembourg (1985-94) and Italy (1986-95) this fraction became smaller and therefore the changes in income – more diversified. The opposite phenomenon can be observed in Finland (1987-95). Interestingly, during the period 1979-95 in the United Kingdom, the value of the considered fraction followed a U-shape, whereas in Norway – a reversed U-shape.



## 6. Sensitivity analysis

Thus far, we have carried out the analysis in terms of household income *per capita*. It is interesting (and advisable) to check whether and if yes - how the conclusions from our analysis change when we apply equivalence scales. As a result, in what follows, the inference refers to equivalent household members.

We allow the intensity of the equivalence scale to vary from 0 to 1, i.e. from household income household income *per capita* (with the adjustment for household composition). These extreme cases (interesting from theoretical point of view, not realistic, however) represent, respectively, perfect economies of scale and lack of scale economies. Table 6.1 presents the results of the Lorenz dominance rankings, repeated five times, using the equivalence scale with elasticity of equivalent income with respect to household size taking values 0, 0.25, 0.5, 0.75 and 1. We include also the results from the first ranking, based on household income *per capita*. Analogously, Table 6.2 presents the results for the GL dominance analysis.

Table 6.1 Sensitivity of Lorenz dominance analysis to changes in equivalence scale

COUNTRY	SAMPLE PERIODS														
Finland										1987-91			1991-95		
			0							-	0	-	_*	0	0
										-	0	0	0	0	0
France							1979-84			1984-89			1989-94		
							0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0
Germany							1981-84			1984-89			1989-94		
							0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0
Italy										1986-91			1991-95		
										0	0	0	_*	_*	_*
										0	+	0	_*	_*	_*
Luxembourg										1985-91			1991-94		
										0	0	0	0	+	+
										0	0	0	0	0	0
Netherlands										1987-91			1991-94		
										0	0	0	0	0	0
										0	0	0	0	0	0
Norway							1979-86			1986-91			1991-95		
							0	0	0	-	0	0	0	_*	-
							0	0	0	0	0	0	-	-	-
Sweden	1967-75			1975-81			1981-87			1987-92			1992-95		
	+	+	+	+	+	+	-	-	-	0	0	0	0	0	0
	0	+	+	+	+	+	-	-	-	0	0	0	0	0	0
United Kingdom	1969-74			1974-79			1979-86			1986-91			1991-95		

	+	*	-	-	0	0	0	-	0	0	0	0	0	0	0	0	0
	0	0	+	*	0	0	0	-	-	-	0	0	0	0	0	0	0

1. Rankings for each sample are based on the following equivalence scales:

$N, \varepsilon=1$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0.25$
$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0.5$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0.75$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=1$

where:  $N$ -household size=  $N_A + N_C$ ,  $N_A$  -number of adults in a household,  $N_C$  -number of children in a household,  $\varepsilon$  - parameter of within-household economies of scale.

2. '+' denotes dominance of the Lorenz curve corresponding to the later year, '-' - to the earlier year, '0' - inconclusive ranking.
3. '\*' denotes these cases where the directions of the change in inequality and the mean are opposite (unambiguous ranking).

Table 6.1 reveals that with the change in equivalence scale, only part of the results remains robust. The most striking change is for the United Kingdom (1969-74), where for large economies of scale (small parameter  $\varepsilon$ ), the ranking points to deterioration, while with no adjustment for scale economies, it suggest that there was improvement. Often, conclusive result appears only for some parameter values. For Finland (1991-95) and Norway (1986-91), however, neither ranking based on the OECD equivalence scale confirms unambiguity achieved with *per capita* adjustment. For Italy (1986-91), Luxembourg (1991-94) and Norway (1991-95) new rankings can be established, in the latter sample – most robust.

Table 6.2 Sensitivity of Generalised Lorenz dominance analysis  
to changes in equivalence scale

COUNTRY	SAMPLE PERIODS											
Finland							1987-91			1991-95		
							+	+	+	-	0	0
							+	0	+	0	0	-
France							1979-84			1984-89		
							0	0	0	-	-	-
							0	0	0	-	-	-
Germany							1981-84			1984-89		
							0	0	0	0	0	0
							0	0	0	0	0	0
Italy							1986-91			1991-95		
							0	+	+	-	-	-
							+	+	+	-	-	-
Luxembourg							1985-91			1991-94		
							+	+	+	-	+	+
							+	+	+	0	0	0
Netherlands							1987-91			1991-94		
							+	+	+	-	-	-
							+	+	+	-	-	-
Norway							1979-86			1986-91		
							+	+	+	0	0	0
							+	+	+	0	0	0

Sweden	1967-75			1975-81			1981-87			1987-92			1992-95		
	+	+	+	+	+	+	0	0	0	+	+	+	-	-	-
	+	+	+	+	+	+	0	0	0	+	+	+	-	-	-
United Kingdom	1969-74			1974-79			1979-86			1986-91			1991-95		
	+	0	+	-	-	-	0	0	0	+	0	0	0	0	0
	+	+	+	-	-	-	0	0	0	0	0	+	0	0	0

1. Rankings for each sample are based on the following equivalence scales:

$N, \varepsilon=1$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0.25$
$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0.5$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=0.75$	$(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon, \varepsilon=1$

where:  $N$ -household size=  $N_A + N_C$ ,  $N_A$  -number of adults in a household,  $N_C$  -number of children in a household,  $\varepsilon$  - parameter of within-household economies of scale

2. '+' denotes dominance of the Lorenz curve corresponding to the later year, '-' – to the earlier year, '0' – inconclusive ranking.

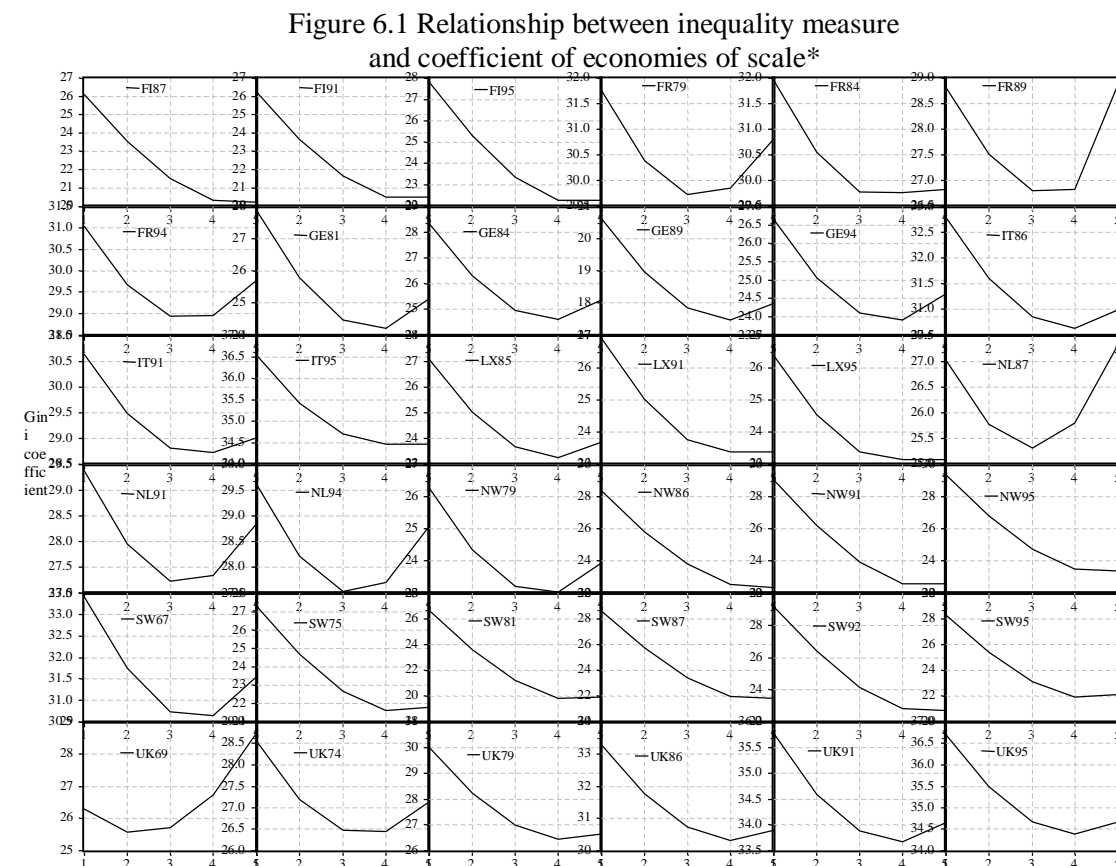
Table 6.2 shows that rankings based on the GL dominance criterion are much less sensitive to changes in equivalence scale specification. Also here new rankings arise: Norway (1991-95) and Italy (1986-91). In the former case only full adjustment for economies of scale points to unanimous preference for the earlier distribution, in the latter case - for all values of parameter  $\varepsilon$  we can conclude that improvement took place. For Finland (1991-95) and the United Kingdom (1986-91) only rankings with no adjustment for the economies of scale are conclusive. Moreover, we can observe that the change of preference for Finland (1987-91) – when the criterion accounts for the value of mean income - is robust.

This part of analysis suggests that one must be cautious in drawing conclusions using only one equivalence scale specification. Interestingly, the previous and present conclusive rankings do not necessarily correspond to each other. Moreover, accounting for household structure and economies of scale can not only enhance classification of distributions or make it inconclusive, but also change the direction of preference. It points to the necessity of checking various scales and parameters to be able to make reliable inference.

Due to the specification of the equivalence scale we use, the relationship between the Gini coefficients and economies of scale parameter we observe (Figure 6.1, Appendix 6) is more complex than that considered in Coulter et al. (1992). Some profiles display a distinct U-shape pattern, some exhibit an inverted J-shape. Considering only the relation between inequality indices and the scale parameter  $\varepsilon$ , the U-shape arises from two opposing effects, when  $\varepsilon$  is changed from 0 to 1. With the increase in the economies of

scale parameter, the ratio income/size decreases more for rich units (concentration effect, Coulter et al. 1992). At the same time changes in ranking become more frequent (re-ranking effect). The former process (equalising the distribution) is stronger for lower values of  $\varepsilon$ , the latter dominates from a certain threshold (for an illuminating explanation, see Figini 1998). Accounting for the composition of household induces an additional effect. In general, the stylised fact (in contrast to the theoretical relationship established by Coulter et al., 1992) is that with the increase of the weight given to adults or children, when computing equivalent income, a J- or an inverted J-shape can emerge (see Figini 1998). The ultimate profile is a result of interplay between the size of within-household economies of scale and household composition. Since we use fixed values of the household member's weights, we do not explicitly examine the impact of altering the importance attached to particular age groups. Nevertheless, we do this implicitly, since by changing the parameter of the scale economies, we modify the extent to which household composition influences the value of inequality index.

In the cases with the U-shape, our estimated Gini coefficients have minimum for  $\varepsilon$  0.5 and 0.75 (except for United Kingdom 1969)<sup>26</sup>.



<sup>26</sup> Figini (1998) did this kind of a grid search for 10 values of  $\varepsilon$  and found the minimum value of Gini coefficient at the value of economies of scale parameter between 0.6 and 0.7 (with one-parameter equivalence scale).

\*Inequality is measured by Gini coefficient. Numbers 1,..., 5 on the horizontal axis refer to the values of parameter of scale economies within a household of 0, 0.25, 0.5, 0.75 and 1, respectively, in:  $(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon$ , where: N-household size=  $N_A + N_C$ ,  $N_A$  -number of adults in a household,  $N_C$  - number of children in a household,  $\varepsilon$  - parameter of within-household economies of scale.

## 7. Conclusions

This paper inquired into changes in income distributions that took place in nine European countries over the last decades. The approach we adopted is a convenient and intuitively appealing way to investigate the direction and depth of income changes in detail. It does not only offer a kind of alternative to the dominance analysis, but also provides answers to different questions. Examining the distributions of differences between relative income shares gives a more exhaustive picture of changes in inequalities. In addition to the information revealed by the Gini coefficient (its starting and/or ending values and change) that refers to inequality in general, this approach gives insight into the constituents of the change in inequality, the mechanisms that contribute to it. It is useful both in the cases where the ranking of distributions is inconclusive, since it provides information about the nature of changes that have occurred without the need of assessment of overall change, as well as in the ambiguous according to the dominance criteria cases, revealing information that cannot be deducted from the outcome of the ranking, viz. identification of the groups affected in a different way, the strength of this impact etc.

Even after applying the Generalised Lorenz dominance criterion, still 30% of pairwise comparisons remained unresolved. Using a range of tools, we pursued our analysis to explore more and more detailed dimensions of these trends. Examination of the shares ordered according to income values was an intermediate step towards estimating densities, which however was not redundant, as it allowed us to explicitly identify groups with different income level. The information about the range of changes and frequencies culminated in the densities.

We found evidence of the opposite tendencies in the 1980's versus the 1990's. These effects are present in the overall direction of income movements (improvement/deterioration) as well as in the impact of changes on the poorer relatively to the richer and changes in the relative position of consecutive income groups. We found the change of patterns of relative redistribution at the turn of the decades in

Finland, Luxembourg, the Netherlands, Sweden and the United Kingdom. Together with the improvement according to the GL dominance criterion, the poorer became relatively better-off in these countries in the 1980's. In contrast, the deterioration at the beginning of the next decade resulted in relative redistribution in favour of the richer (with the exception of the United Kingdom, where the ranking was inconclusive in the period 1991-95 and Finland, where position of the richer income groups systematically improved in relative terms). The opposite changes took place in France. Moreover, we detected some properties systematically present in the distribution of income changes. One of the most consistent phenomena is the extreme values effect, which can be observed both in relative and absolute changes (only in Luxembourg the middle income groups experienced relatively larger changes). It arises not only in terms of income groups, but also in the frequencies of the changes. We also found that in contrast to the 1980's, in the first half of the 1990's there was one distinct group affected in a similar way. Looking at the frequency aspect of the impact in more detail, we found that in France (1979-94), Germany (1981-94), Luxembourg (1985-94) and Italy (1986-95) the changes in income became less and less uniform, in Finland (1987-95) – less diversified, in Sweden (1975-95) and Norway (1986-95) – the fraction of population affected in the same way remained approximately constant.

An interesting extension of this study would be to approach the examined topic more from the standpoint of welfare theory and investigate how the specification of the social welfare function influences the conclusions about the change in the standard of living. In contrast to the ranking of distributions by means of the dominance criteria, where this is not relevant (according to Atkinson's theorem), the procedure we proposed provides the opportunity to do this, since it does not require conclusive rankings. Another issue that deserves to be explored is the identification of the mechanisms underlying the changes that we traced. In a sense, our analysis is a step towards finding the explanation, since by providing much more detailed information than the ranking of distributions, it indicates potential causes of the changes. In this way it serves a double purpose of giving empirical evidence and pointing to areas crucial for specifying the reasons of the observed processes.

## References

- Atkinson, A.B. (1970), "On the measurement of inequality", *Journal of Economic Theory* 2, pp. 244-263.
- Atkinson, A.B. (1987), "On the Measurement of Poverty", *Econometrica* 55, pp.749–764.
- Bowman A. W. (1984), "An Alternative Method of Cross-Validation for the Smoothing of Density Estimates", *Biometrika* 71, pp.353-60.
- Buhmann B., Rainwater L., Schmaus G., Smeeding T., (1988), "Equivalence Scales, Well-being, Inequality and Poverty: Sensitivity Estimates across Ten Countries using the Luxembourg Income Study Database", *Review of Income and Wealth* 34(1), pp.115-42.
- Coulter F.A.E., Cowell F.A., Jenkins S. P., (1992), "Equivalence Scale Relativities and the Extent of Inequality and Poverty", *Economic Journal* 102, pp.1067-82.
- Cowell, F. A., S. P. Jenkins, J. A. Litchfield (1996), "The changing shape of the UK income distribution: kernel density estimates", in: *New inequalities. The changing distribution of income and wealth in the United Kingdom*, ed. J. Hills, Cambridge: Cambridge University Press.
- Dasgupta, P., A. Sen, D. Starrett (1973), "Notes on the Measurement of Inequality", *Journal of Economic Theory* 6, pp.180-7.
- Davidson, R. and J.Y. Duclos (1998), "Statistical Inference for Stochastic Dominance and the for the Measurement of Poverty and Inequality", *Cahier de recherche* 98-05, D'épartement d'économie, Université Laval. <http://www.ecn.ulaval.ca/~jyves>
- Deaton, A. (1997), *The Analysis of Household Surveys. A Microeconomic Approach to Development Policy*, Baltimore, London: John Hopkins University Press.
- De Nardi, M., L. Ren, C. Wei (2000), "On the Distribution of Income in Five Countries", Maxwell School of Citizenship and Public Affairs, Syracuse University, Syracuse, New York 13244-1020, LIS Working Paper No.227.
- Dowrick, S., J. Quiggin (1994), "International comparison of living standards and tastes: A revealed preference analysis", *American Economic Review* 84(1), pp.332-341.
- Duclos, J. Y., M. Mercader (1996), "Household Needs and Poverty: With Application to Spain and the UK", *Cahier de recherche* 96-14, CRÉFA, Département d'économie, Université Laval, juin 1996.
- Duclos, J. Y., M. Mercader (1993), "Household Composition and Classes of Equivalence Scales: With Application to Spain and the UK", University of Cambridge, The Microsimulation Unit, No. MU 9403.

Esteban, J., C. Gradín, D. Ray (1999), "Extensions of a Measure of Polarisation with an Application to the Income Distribution of Five OECD Countries", LIS Working Papers, No. 218. November 1999.

Figini, P. (1998), "Inequality Measures, Equivalence Scales and Adjustment for Household Size and Composition", Maxwell School of Citizenship and Public Affairs, Syracuse University, Syracuse, New York 13244-1090, LIS Working Paper No.185.

Foster, J.E. and A.F. Shorrocks (1988), "Inequality and Poverty Orderings", *European Economic Review*, 32, pp.654–662.

Gottschalk, P. and T.M. Smeeding (1997), "Cross-National Comparisons of Earnings and Income Inequality", *Journal of Economic Literature*, 35, pp.633–687.

Hadar J., W. R. Russell (1969), "Rules for ordering uncertain prospects", *American Economic Review*, March 1969, pp.25-34.

Hamilton, J. D. (1994), *Time Series Analysis*, Princeton, New Jersey: Princeton University Press.

Hanoch, G., H. Levy (1969), "The Efficiency Analysis of Choices Under Risk", *Review of Economic Studies*, July 1969, pp.333-46.

Iritani, J., K. Kuga (1983), "The Duality Between the Lorenz Curve and the Income Distribution Functions", *Economic Studies Quarterly*, April 1983, pp.9-21.

Jäntti, M., S. Danziger (1999), "Income Poverty in Advanced Countries", Maxwell School of Citizenship and Public Affairs, Syracuse University, Syracuse, New York 13244-1020, LIS Working Paper No.193.

Jenkins, S. P., F. Cowell (1994), "Parametric equivalence scales and scale relativities", *Economic Journal* 104 (425), pp.891-900.

Jenkins, S. P., P. J. Lambert (1993), "Ranking income distribution when needs differ", *Review of Income and Wealth* 39(4), pp. 337-356.

Jenkins, S. P, P. J. Lambert (1997). "Three 'I's of Poverty Curves, With an Analysis of UK Poverty Trends", *Oxford Economic Papers*, 49, pp.317–327.

Kakwani, N. C. (1984), "Welfare ranking of income distributions", in *Advances in Econometrics*, edited by R. L. Basmann, G. F. Rhodes, Jr., Greenwich: JAI Press, Vol.3, pp. 191-213.

Kazakov, E. A. (1997), "Changes in the Standard of Living, Connected with Ageing and Retirement: Comparative Perspective (USA, Germany, Russia)", Institute for Complex Social Studies, Moscow, LIS Working Paper No.193.

Lambert, P. J. (1993), "The Distribution and Redistribution of Income. A Mathematical Analysis", Manchester, New York: Manchester University Press.



Mercader, M. (1993), “Low Income Population in Spain and a Comparison With France and the UK”, Discussion Paper, No. 95, Welfare State Programme, STICERD, LSE.

Osberg, L. (2000), “Long Run Trends in Economic Inequality in Five Countries – A Birth Cohort View”, Maxwell School of Citizenship and Public Affairs, Syracuse University, Syracuse, New York 13244-1020, LIS Working Papers No. 222.

Pagan, A., A. Ullah (1999), *Nonparametric Econometrics*, Cambridge University Press.

Quirk, J. P., J. Saposnik (1962), “Admissibility and Measurable Utility Functions”, *Review of Economic Studies*, 1962, pp.140-46.

Rothschild, M., J. E. Stiglitz (1970), “Increasing Risk I: A Definition”, *Journal of Economic Theory*, 1970, pp. 225-43.

Rudemo M. (1982), “Empirical Choice of Histograms and Kernel Density Estimators”, *Scandinavian Journal of Statistics* 9, pp.65-78.

Ruiz-Castillo, J. (1994), “The distribution of expenditure in Spain, 1973-74 to 1980-81”, Universidad Carlos III, Working Paper 94-10.

Ruiz-Huerta, J., R. Martinez, L. Ayala (1999), “Inequality, Growth and Welfare: An International Comparison”, Maxwell School of Citizenship and Public Affairs, Syracuse University, Syracuse, New York 13244-1020, LIS Working Papers No. 215.

Shannon, C. E., W. Weaver (1949), “The Mathematical Theory of Communication”, University of Illinois Press.

Thistle, P. D. (1989), “Ranking Distributions with Generalized Lorenz Curves”, *Southern Economic Journal* 56, pp.1-12.

Wolfson, M.C. (1994), “When Inequalities Diverge”, *American Economic Review* 84, Papers and Proceedings, pp.353-358.

## APPENDIX 1. Data sample sizes

TableA1.1 Sample sizes (LIS database)

COUNTRY	CODE	SAMPLE SIZE					
<b>Finland</b>	FI				<b>1987</b>	<b>1991</b>	<b>1995</b>
					11863	11749	9262
<b>France</b>	FR			<b>1979</b>	<b>1984</b>	<b>1989</b>	<b>1994</b>
				11044	12693	9038	11294
<b>Germany</b>	GE			<b>1981</b>	<b>1984</b>	<b>1989</b>	<b>1994</b>
				2862	5159	4187	6045
<b>Italy</b>	IT				<b>1986</b>	<b>1991</b>	<b>1995</b>
					8022	8188	8135
<b>Luxembourg</b>	LX				<b>1985</b>	<b>1991</b>	<b>1994</b>
					2049	1957	1813
<b>Netherlands</b>	NL				<b>1987</b>	<b>1991</b>	<b>1994</b>
					4190	4378	5187
<b>Norway</b>	NW			<b>1979</b>	<b>1986</b>	<b>1991</b>	<b>1995</b>
				10414	4975	8073	10127
<b>Sweden</b>	SW	<b>1967</b>	<b>1975</b>	<b>1981</b>	<b>1987</b>	<b>1992</b>	<b>1995</b>
		5921	10306	9625	9530	12484	16260
<b>United Kingdom</b>	UK	<b>1969</b>	<b>1974</b>	<b>1979</b>	<b>1986</b>	<b>1991</b>	<b>1995</b>
		7005	6695	6777	7178	7056	6797

## APPENDIX 2. Sample mean income

Table A2.1 Mean income in samples from LIS database [USD, PPP 1991]

COUNTRY	SAMPLE PERIODS					
<b>Finland</b>				<b>1987</b>	<b>1991</b>	<b>1995</b>
				7670,875	9350,937	8651,633
<b>France</b>			<b>1979</b>	<b>1984</b>	<b>1989</b>	<b>1994</b>
			7135,908	7630,285	7496,028	8751,927
<b>Germany</b>			<b>1981</b>	<b>1984</b>	<b>1989</b>	<b>1994</b>
			7700,304	7606,929	8749,0788	8443,297
<b>Italy</b>				<b>1986</b>	<b>1991</b>	<b>1995</b>
				6122,955	7271,043	6693,897
<b>Luxembourg</b>				<b>1985</b>	<b>1991</b>	<b>1994</b>
				8955,101	121847,9	13890,23
<b>Netherlands</b>				<b>1987</b>	<b>1991</b>	<b>1994</b>
				6590,429	8917,602	8288,769
<b>Norway</b>			<b>1979</b>	<b>1986</b>	<b>1991</b>	<b>1995</b>
			6077,605	8108,246	8666,236	8798,703
<b>Sweden</b>	<b>1967</b>	<b>1975</b>	<b>1981</b>	<b>1987</b>	<b>1992</b>	<b>1995</b>
	4733,76	7300,028	7645,956	8077,415	9730,412	9182,219
<b>United Kingdom</b>	<b>1969</b>	<b>1974</b>	<b>1979</b>	<b>1986</b>	<b>1991</b>	<b>1995</b>
	9942,641	13849,09	7226,487	7891,246	9475,643	9704,759

### APPENDIX 3. Distribution properties of changes in income shares

#### A. Shares derived from Lorenz curves

Table A3.1 Jarque-Berra test for relative income shares

COUNTRY	SAMPLE PERIODS				
<b>Finland</b>				<b>1987-91</b>	<b>1991-95</b>
Kurtosis				3.477,	29.846,
Skewness				0.409	3.916
$\chi^2$ -statistic (p-value)				3.700 (0.15716)	3226.038 (0.00000)**
<b>France</b>			<b>1979-84</b>	<b>1984-89</b>	<b>1989-94</b>
Kurtosis			3.406,	13.063,	19.955,
Skewness			-0.169	-1.857	3.021
$\chi^2$ -statistic (p-value)			1.149 (0.56297)	474.549 (0.00000)**	1336.316 (0.00000)**
<b>Germany</b>			<b>1981-84</b>	<b>1984-89</b>	<b>1989-94</b>
Kurtosis			47.179,	4.214,	7.907,
Skewness			5.384	0.180	-0.540
$\chi^2$ -statistic (p-value)			8529.382, (0.00000)**	6.618 (0.03655)*	104.135 (0.00000)**
<b>Italy</b>				<b>1986-91</b>	<b>1991-95</b>
Kurtosis				13.292,	5.388,
Skewness				1.759	1.199
$\chi^2$ -statistic (p-value)				487.960 (0.00000)**	47.250 (0.00000)**
<b>Luxembourg</b>				<b>1985-91</b>	<b>1991-94</b>
Kurtosis				20.310,	5.354,
Skewness				3.117	-0.560
$\chi^2$ -statistic (p-value)				1396.391 (0.00000)**	28.042 (0.00000)**
<b>Netherlands</b>				<b>1987-91</b>	<b>1991-94</b>
Kurtosis				61.419,	66.128,
Skewness				6.875	-7.382
$\chi^2$ -statistic (p-value)				14857.760 (0.00000)**	17337.750 (0.00000)**
<b>Norway</b>			<b>1979-86</b>	<b>1986-91</b>	<b>1991-95</b>
Kurtosis			2.888,	7.793,	1815.000,
Skewness			-0.233	1.295	2.569
$\chi^2$ -statistic (p-value)			0.947167 (0.62277)	122.4579 (0.00000)**	1.055.680 (0.00000)**
<b>Sweden</b>	<b>1967-75</b>	<b>1975-81</b>	<b>1981-87</b>	<b>1987-92</b>	<b>1992-95</b>
Kurtosis	3.846,	10.738,	29.33.178,	3.322,	3.183,
Skewness	-0.147	-1.864	3.969	0.179	-0.013
$\chi^2$ -statistic (p-value)	3.310.833 (0.19101)	304.297 (0.00000)**	3120.076 (0.00000)**	0.955 (0.620479)	0.141 (0.93206)
<b>United Kingdom</b>	<b>1969-74</b>	<b>1974-79</b>	<b>1979-86</b>	<b>1986-91</b>	<b>1991-95</b>
Kurtosis	3.596,	63.833,	36.956,	55.012,	30.695,
Skewness	-0.714	-7.128	-7.128	6.372	-3.918
$\chi^2$ -statistic (p-value)	9.868 (0.00720)**	16103.610 (0.00000)**	5165.749 (0.00000)**	11828.890 (0.00000)**	3417.186 (0.00000)**

“\*\*” denotes significance at 5%-level, “\*\*\*”- at 1%-level

## B. Shares derived from Generalised Lorenz curves

Table A1.2 Jarque-Berra test for shares in average income level

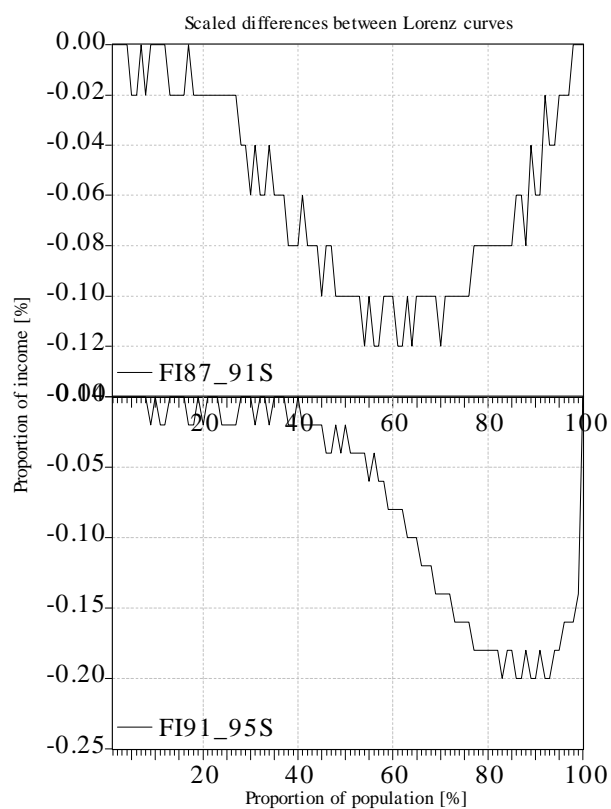
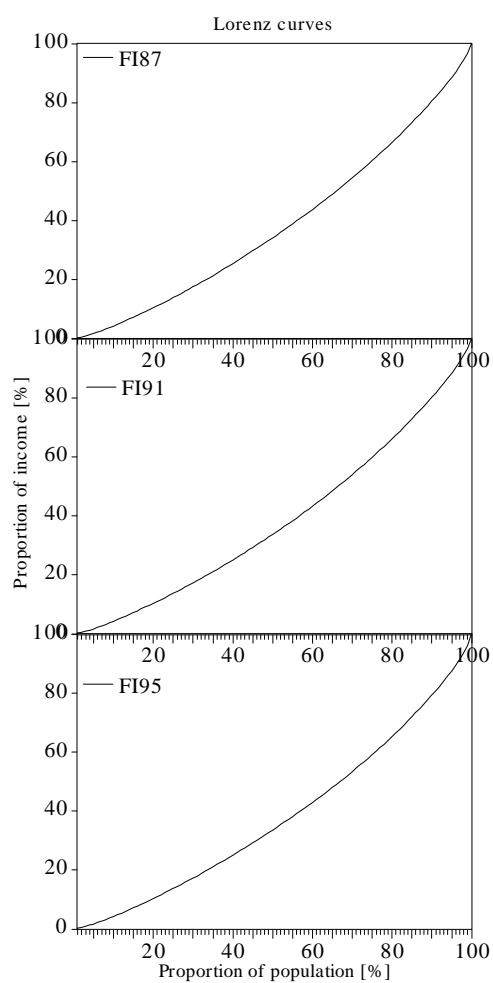
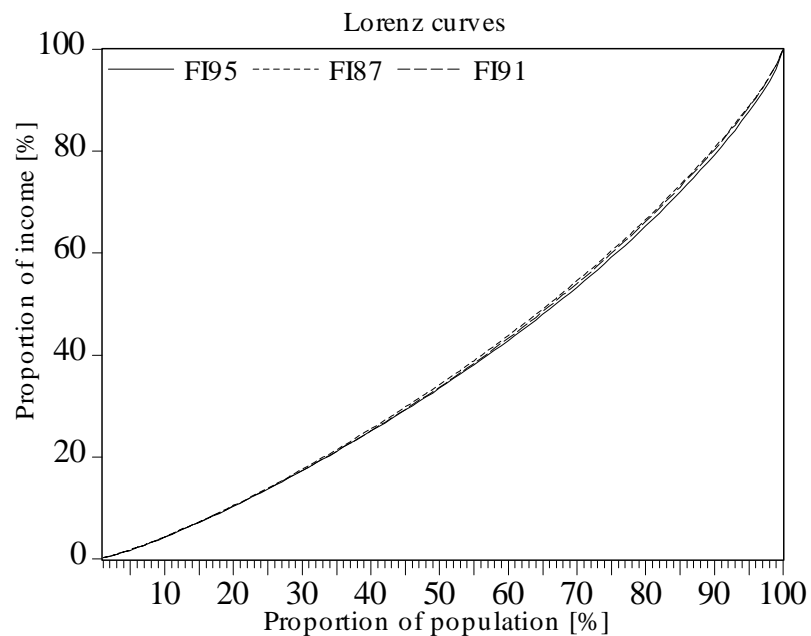
<b>COUNTRY</b>	<b>SAMPLE PERIODS</b>				
<b>Finland</b>				<b>1987-91</b>	<b>1991-95</b>
Kurtosis				4.517	13.379
Skewness				1.019	1.990
$\chi^2$ -statistic (p-value)				26.621 (0.00000)**	509.675 (0.00000)**
<b>France</b>			<b>1979-84</b>	<b>1984-89</b>	<b>1989-94</b>
Kurtosis			14.129	18.011	31.971
Skewness			1.848	-2.510	4.604
$\chi^2$ -statistic (p-value)			567.310 (0.00000)**	1033.168 (0.00000)**	3812.085 (0.00000)**
<b>Germany</b>			<b>1981-84</b>	<b>1984-89</b>	<b>1989-94</b>
Kurtosis			44.257	7.983	13.689
Skewness			5.076	1.631	-1.836
$\chi^2$ -statistic (p-value)			7.447 (0.00000)**	146.340 (0.00000)**	526.992 (0.00000)**
<b>Italy</b>				<b>1986-91</b>	<b>1991-95</b>
Kurtosis				35.220	11.761
Skewness				4.615	-1.514
$\chi^2$ -statistic (p-value)				4633.696 (0.00000)**	354.479 (0.00000)**
<b>Luxembourg</b>				<b>1985-91</b>	<b>1991-94</b>
Kurtosis				16.224	15.894
Skewness				2.820	-2.773
$\chi^2$ -statistic (p-value)				852.524 (0.00000)**	812.631 (0.00000)**
<b>Netherlands</b>				<b>1987-91</b>	<b>1991-94</b>
Kurtosis				51.949	66.740
Skewness				6.230	-7.522
$\chi^2$ -statistic (p-value)				10523.840 (0.00000)**	17692.340 (0.00000)**
<b>Norway</b>			<b>1979-86</b>	<b>1986-91</b>	<b>1991-95</b>
Kurtosis			10.533	15.646	22.531
Skewness			2.054	2.455	3.096
$\chi^2$ -statistic (p-value)			303.726 (0.00000)**	759.057 (0.00000)**	1731.615 (0.00000)**
<b>Sweden</b>	<b>1967-75</b>	<b>1975-81</b>	<b>1981-87</b>	<b>1987-92</b>	<b>1992-95</b>
Kurtosis	3.233	9.710	27.742	7.862	3.051
Skewness	0.516	-1.577	3.837	1.540	-0.026
$\chi^2$ -statistic (p-value)	4.623 (0.09909)	226.759 (0.00000)**	2768.250 (0.00000)**	136.619 (0.00000)**	0.022 (0.98903)
<b>United Kingdom</b>	<b>1969-74</b>	<b>1974-79</b>	<b>1979-86</b>	<b>1986-91</b>	<b>1991-95</b>
Kurtosis	16.164	27.958	33.300	44.481	20.654
Skewness	2.736	27.958	4.678	-2.652	5.524
$\chi^2$ -statistic (p-value)	838.268 (0.00000)**	2846.374 (0.00000)**	4148.175 (0.00000)**	1401.739 (0.00000)**	7.601 (0.00000)**

“\*\*” denotes significance at 1%-level

## APPENDIX 4. Ranking of income distributions by means of dominance criteria

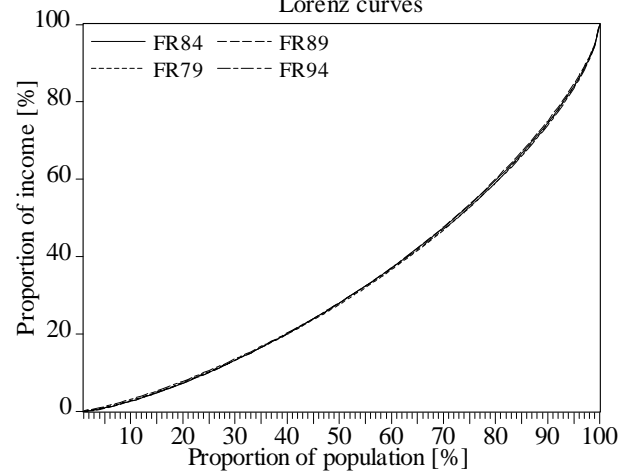
### A. Lorenz dominance analysis

#### FINLAND

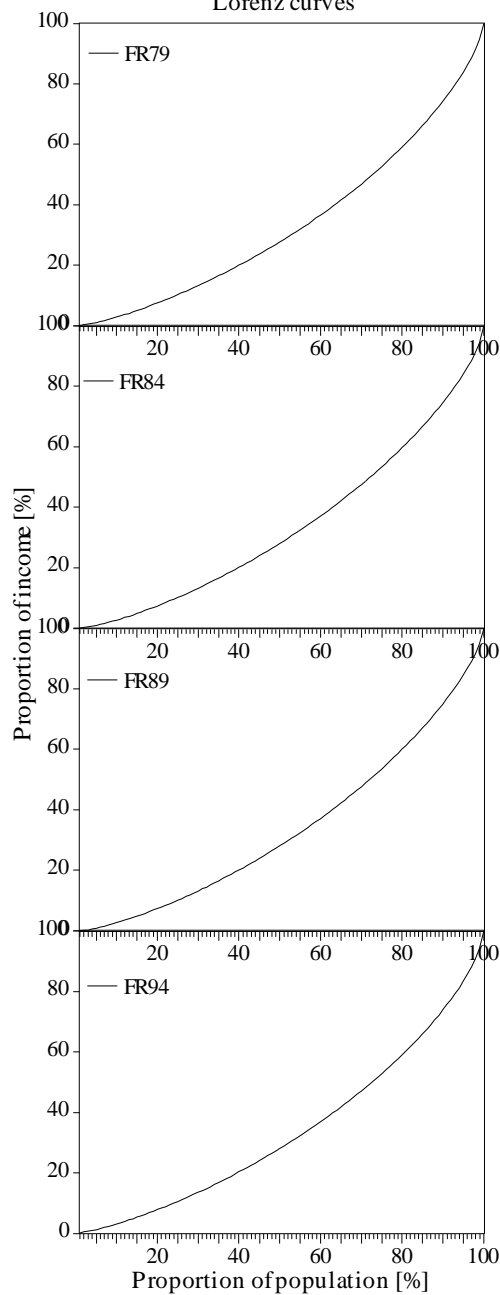


# FRANCE

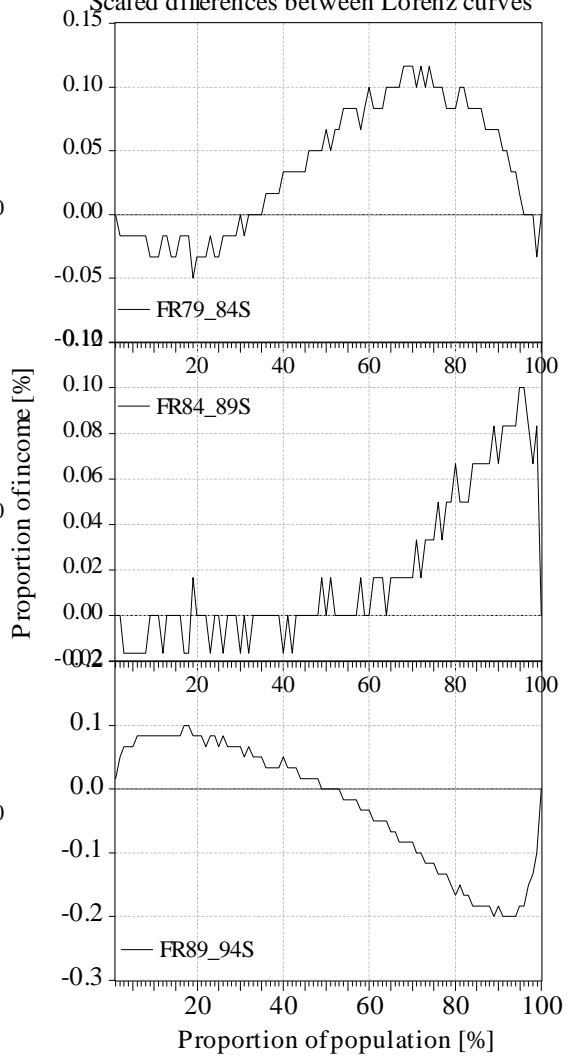
Lorenz curves



Lorenz curves

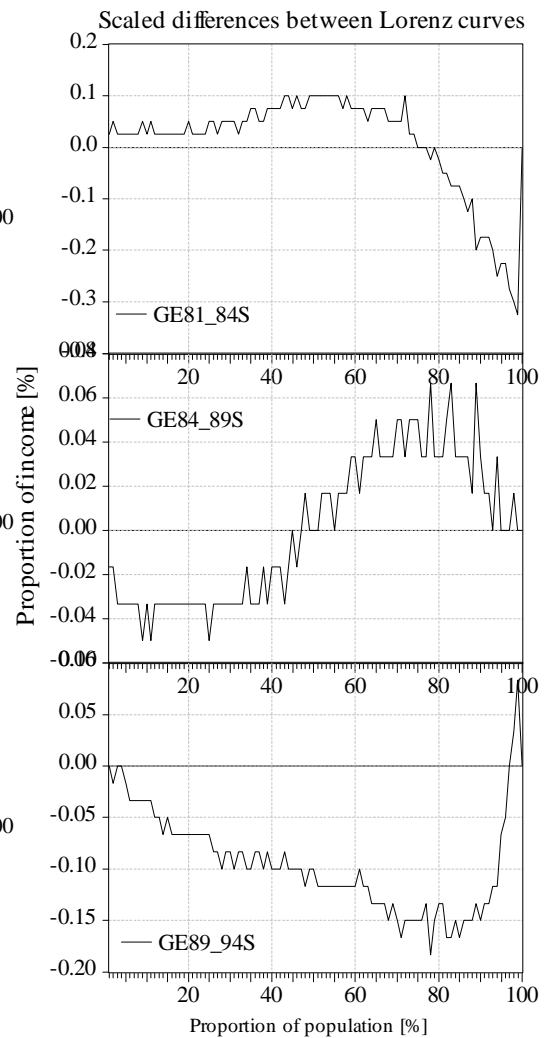
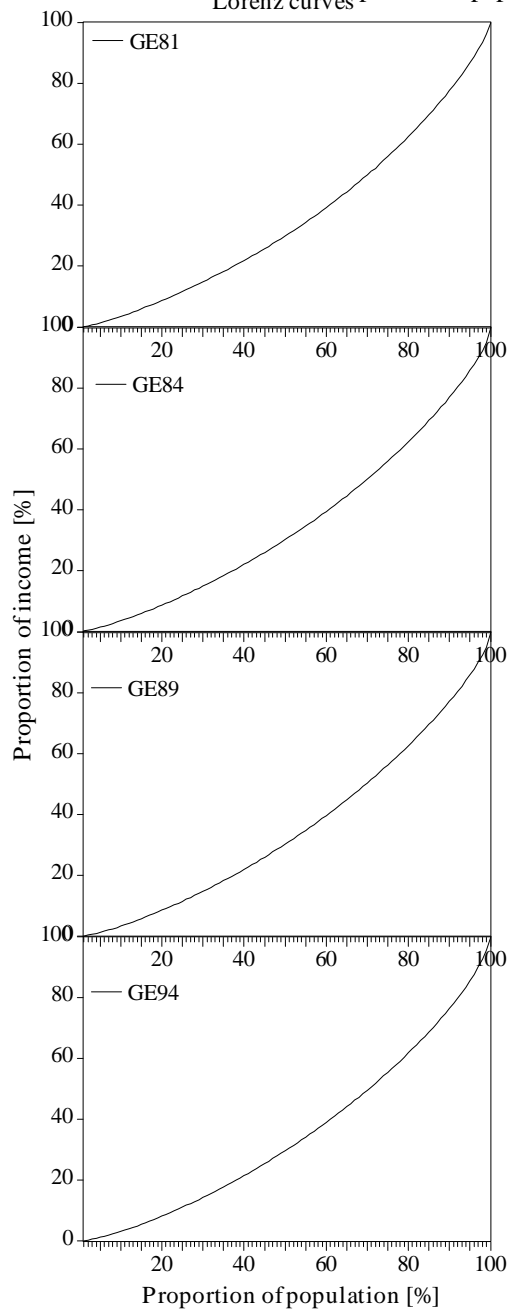
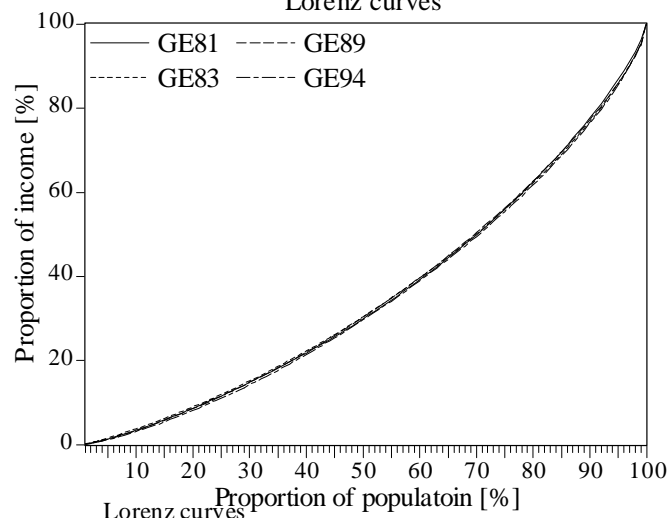


Scaled differences between Lorenz curves



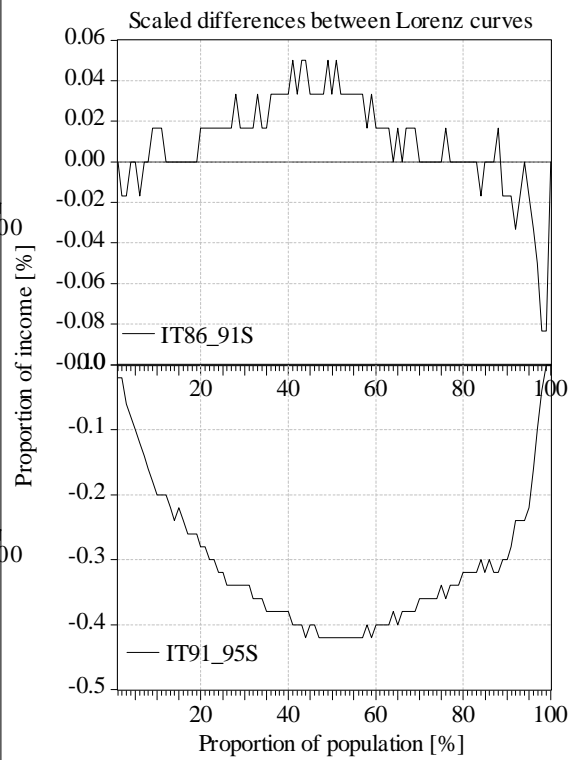
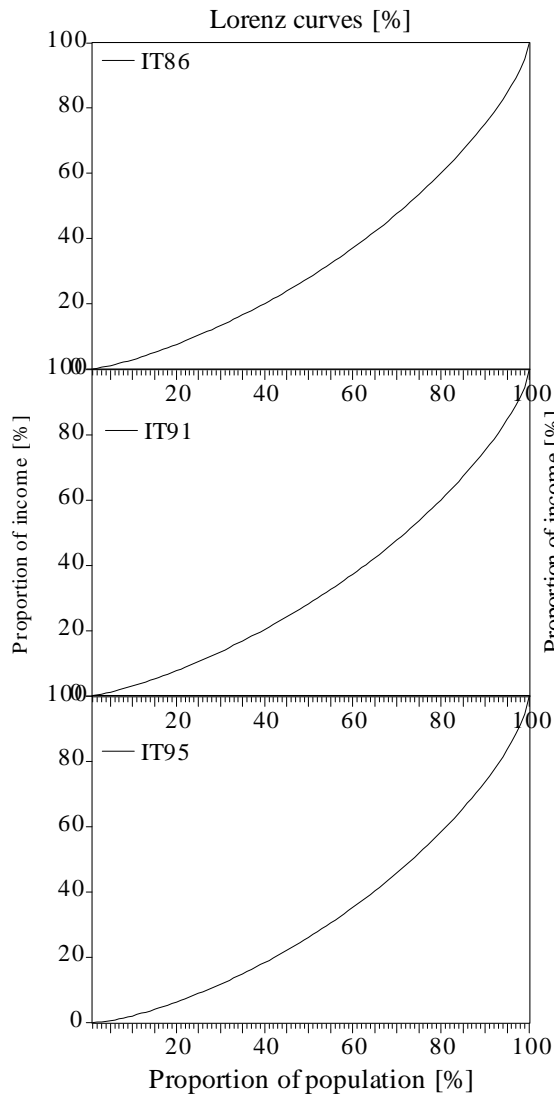
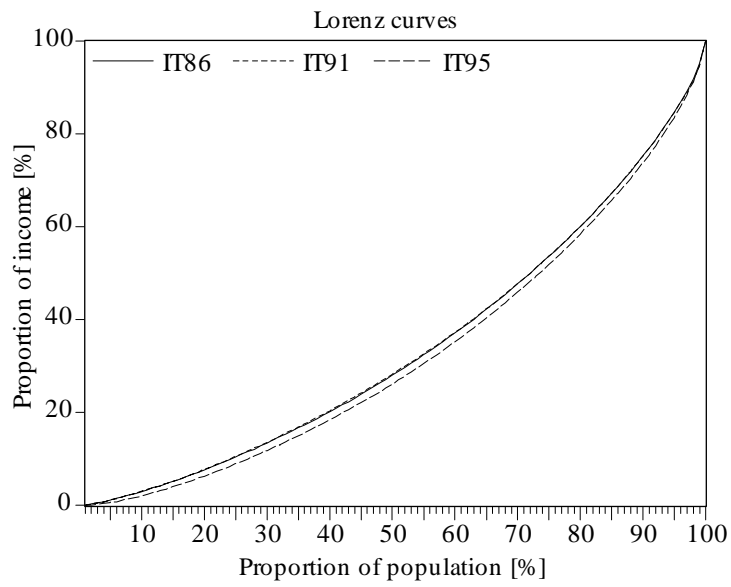
# GERMANY

## Lorenz curves

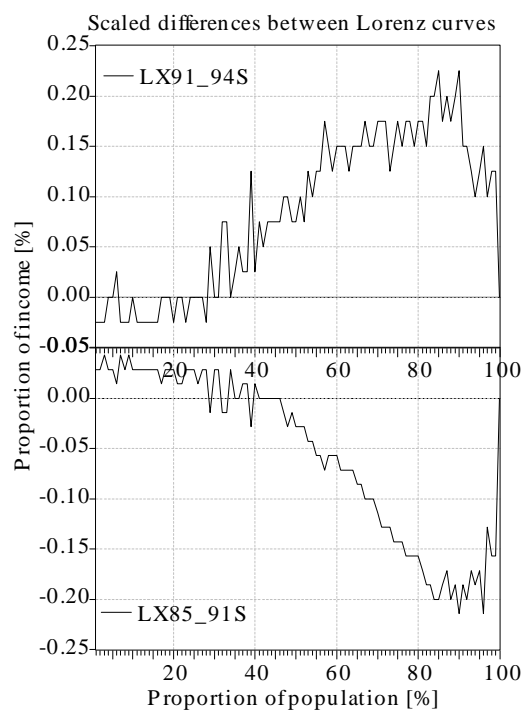
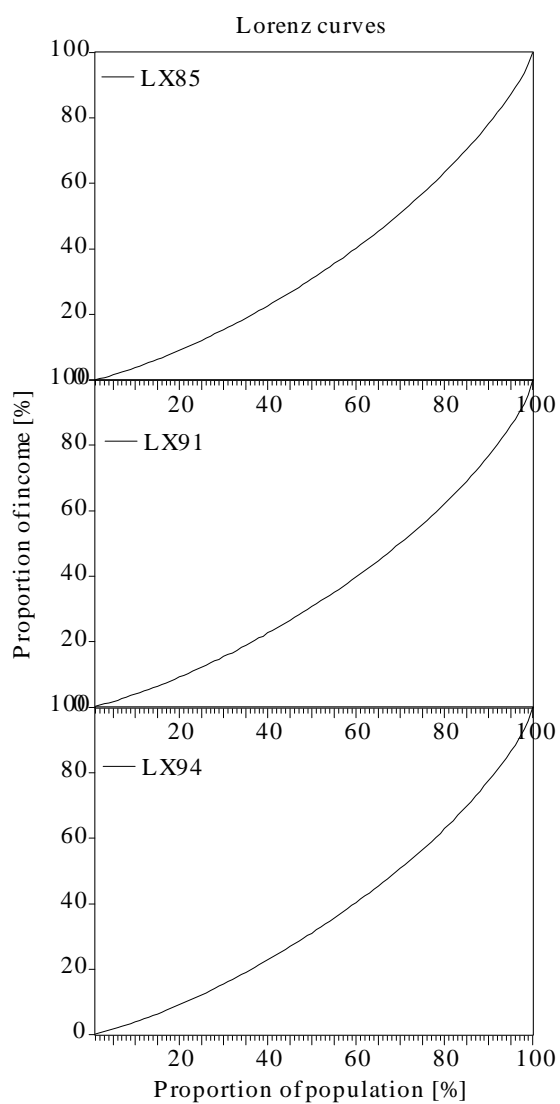
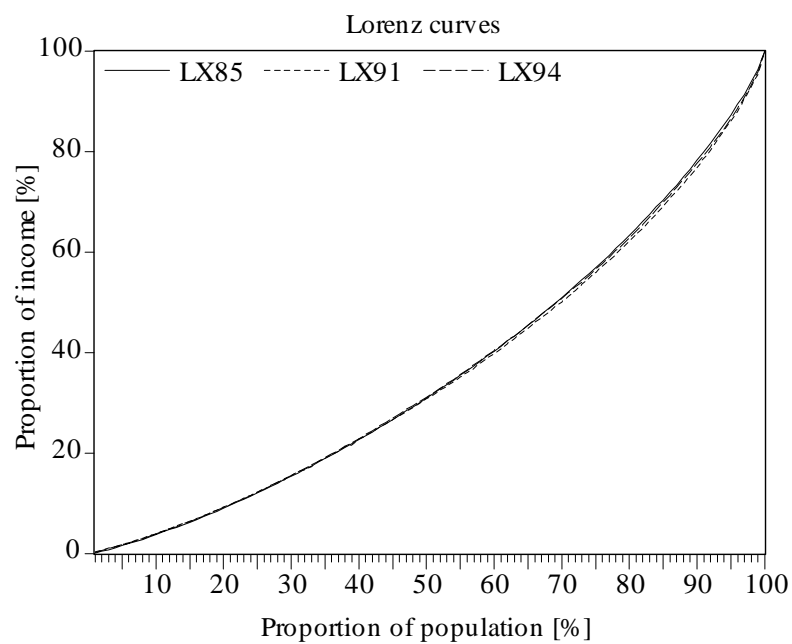




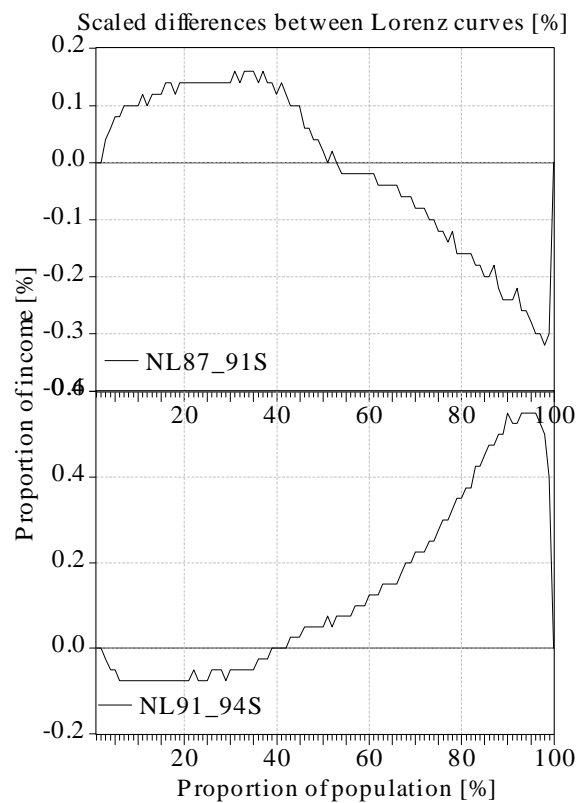
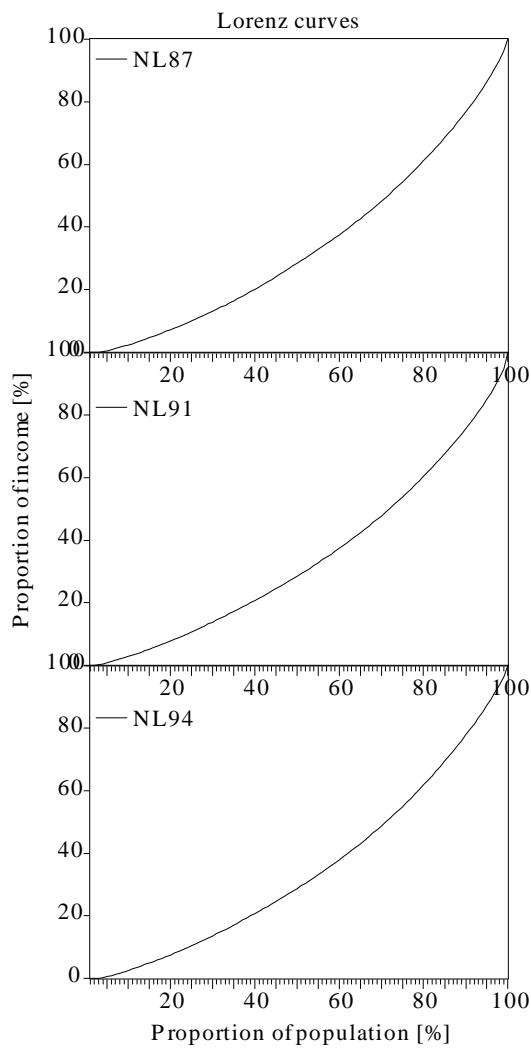
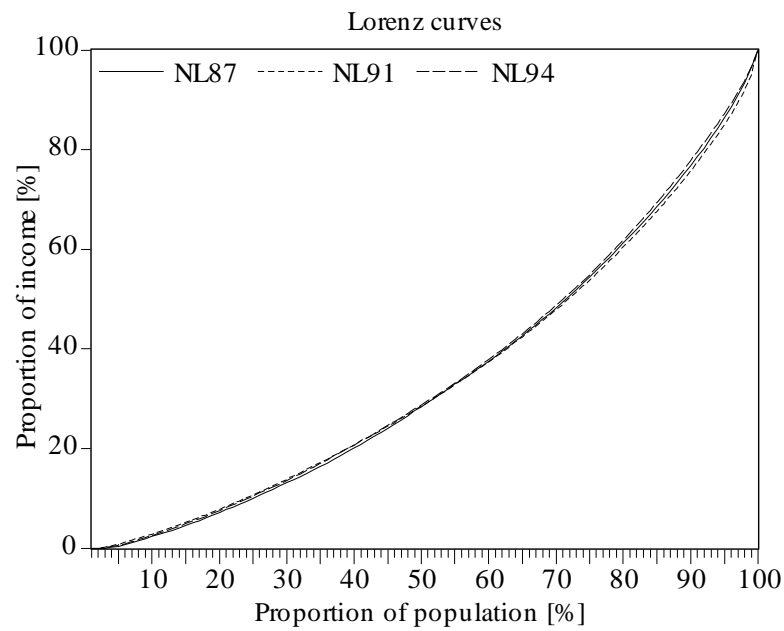
# ITALY



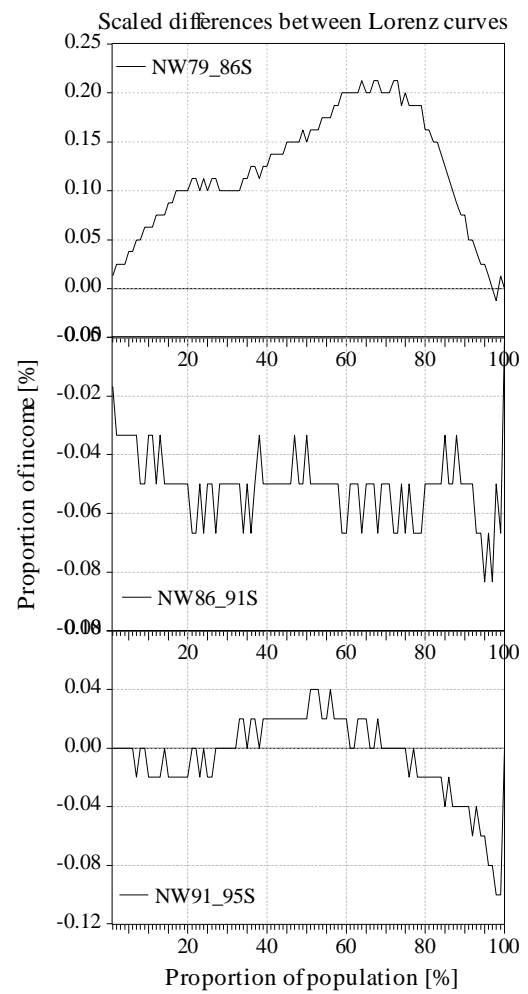
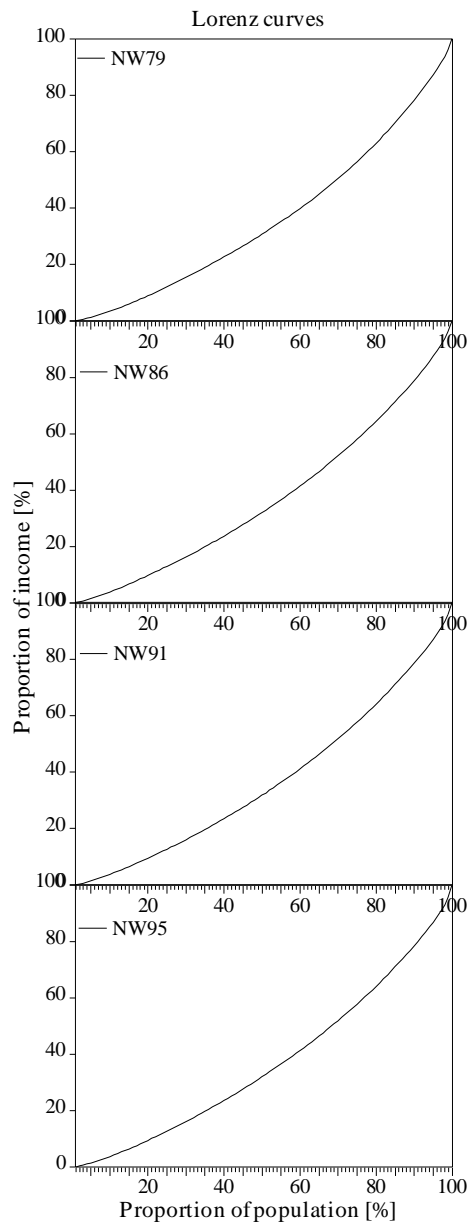
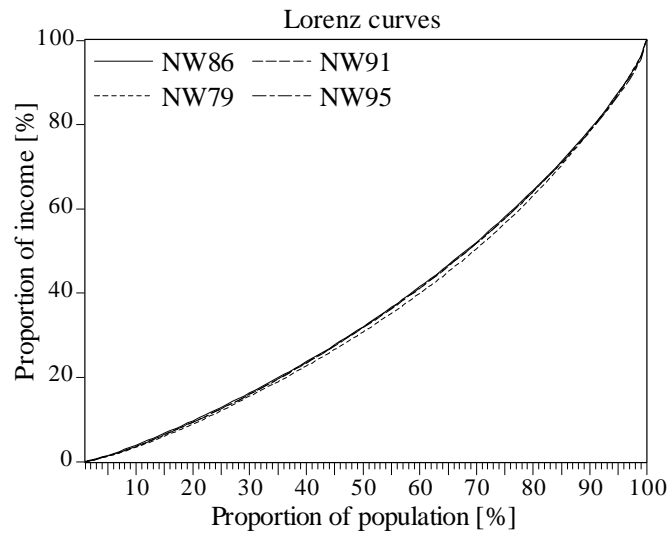
# LUXEMBOURG



# NETHERLANDS

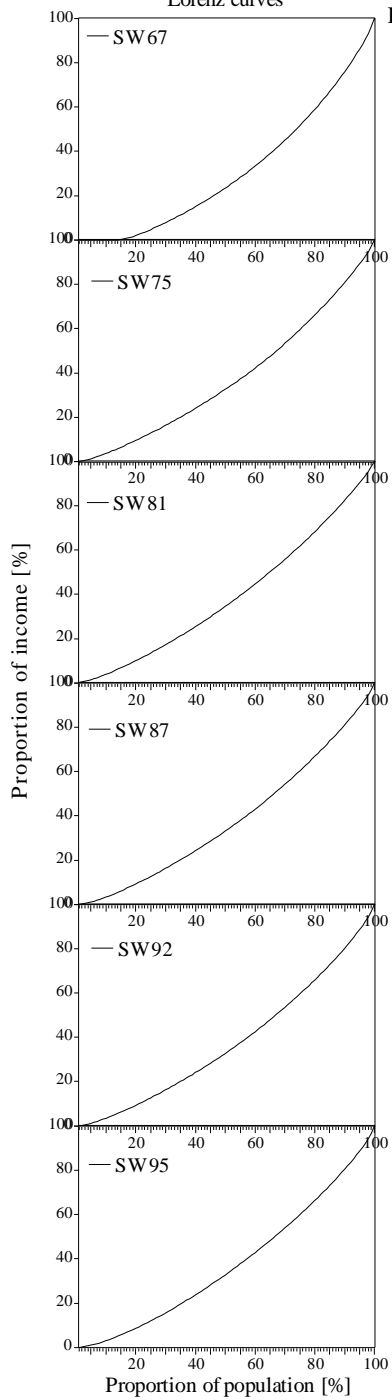
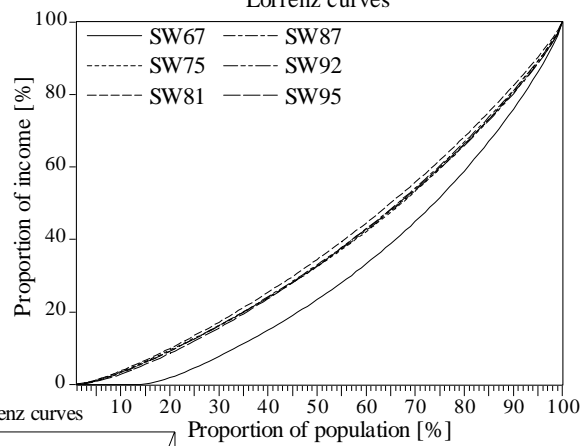


# NORWAY

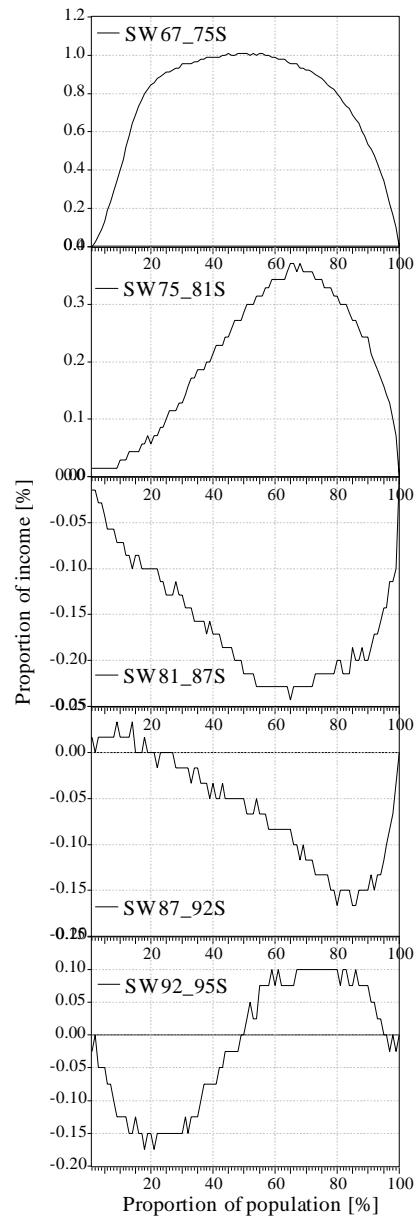


# SWEDEN

Lorenz curves

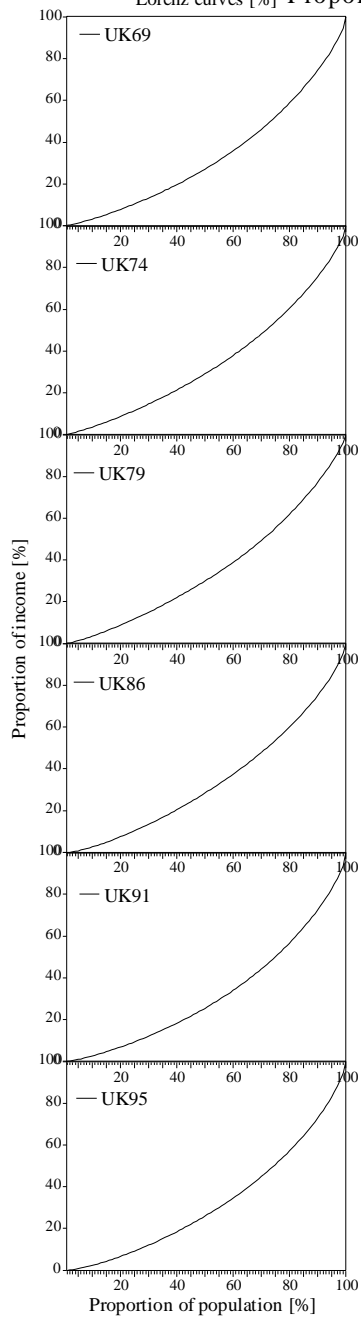
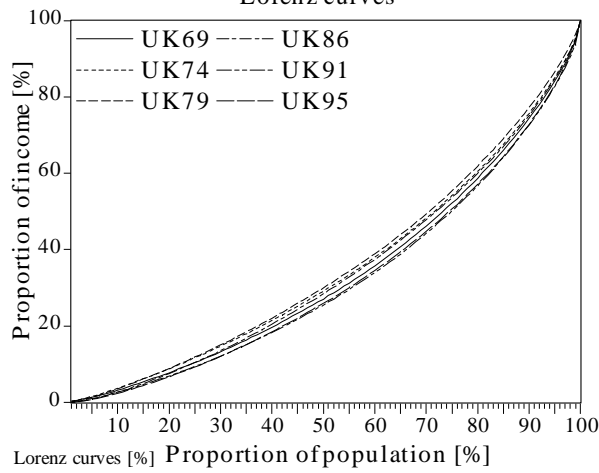


Scaled differences between Lorenz curves

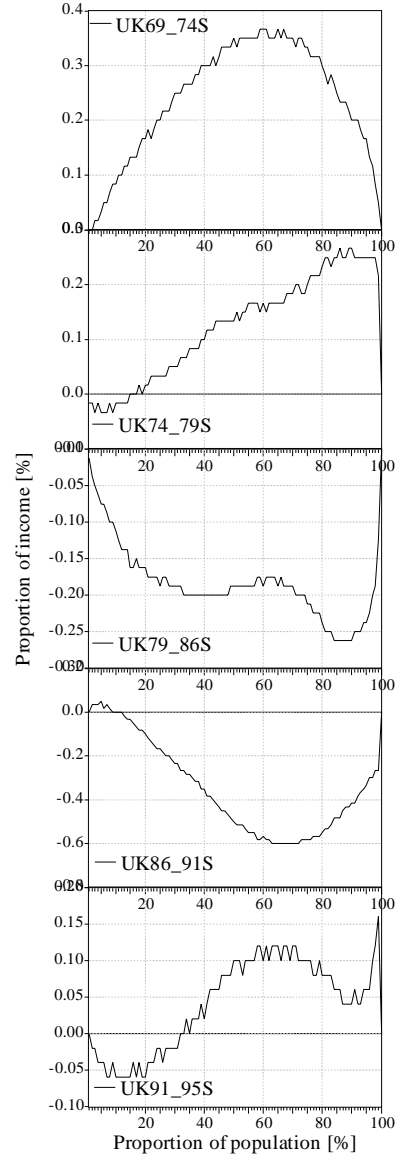


# UNITED KINGDOM

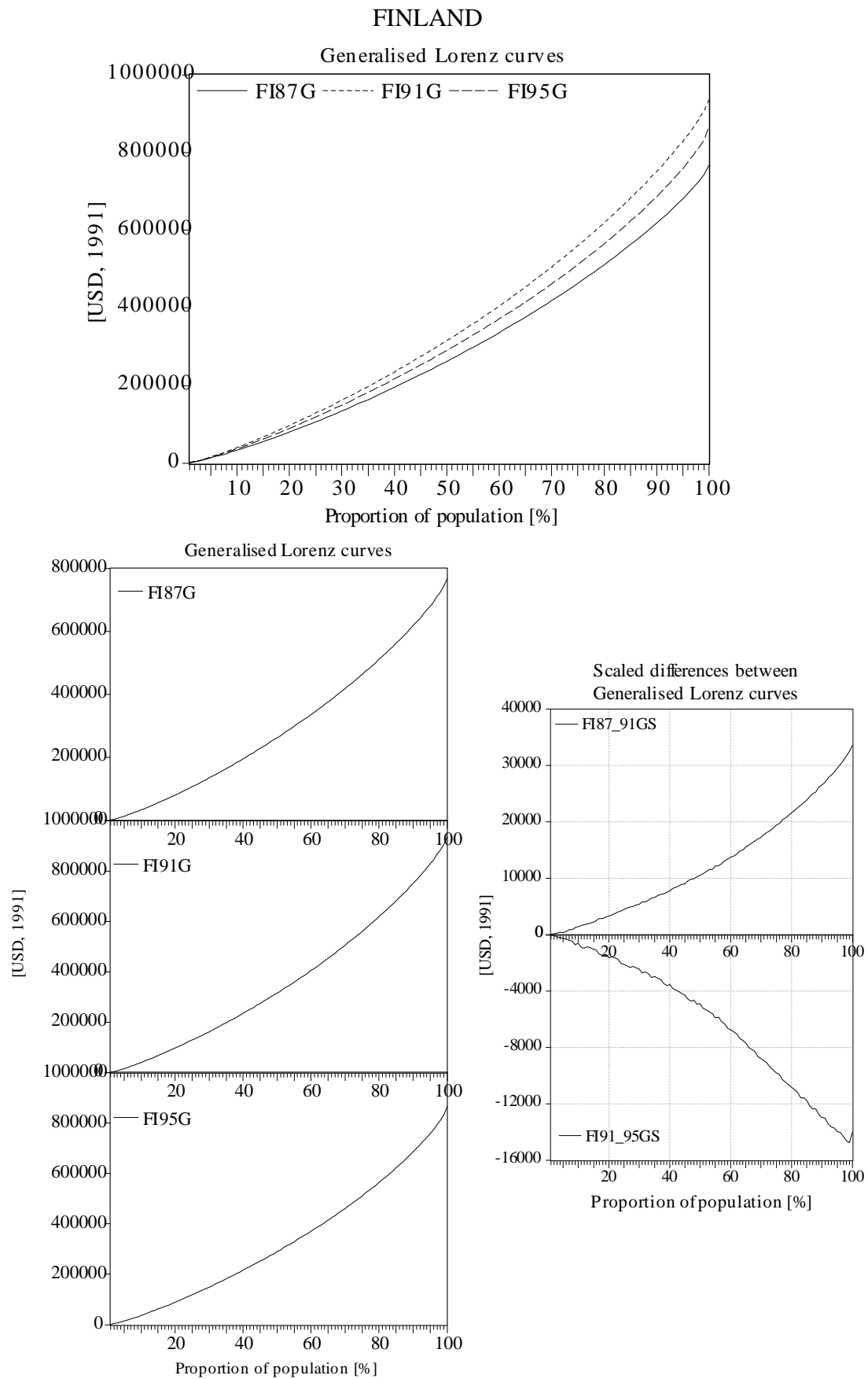
Lorenz curves



Scaled differences between Lorenz curves

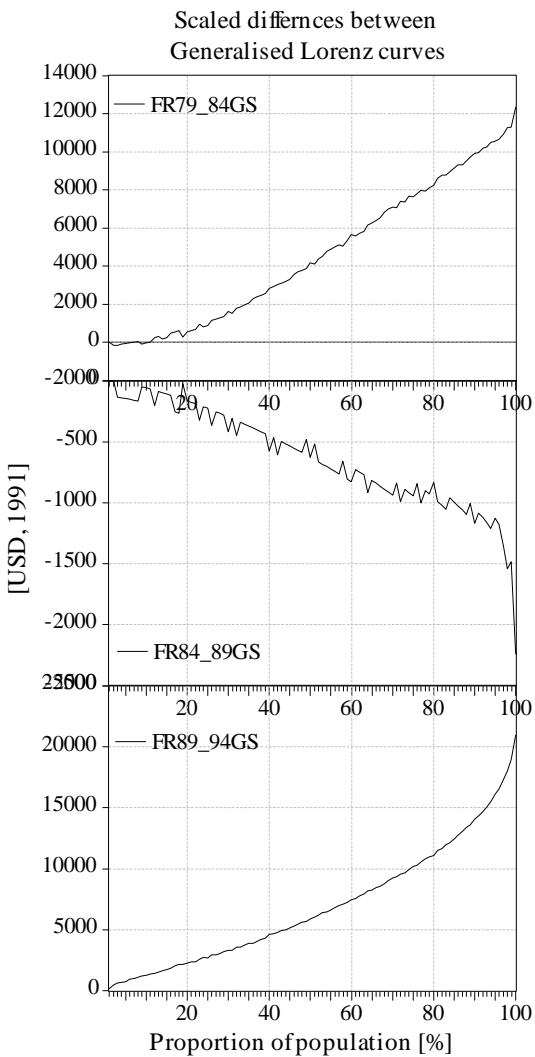
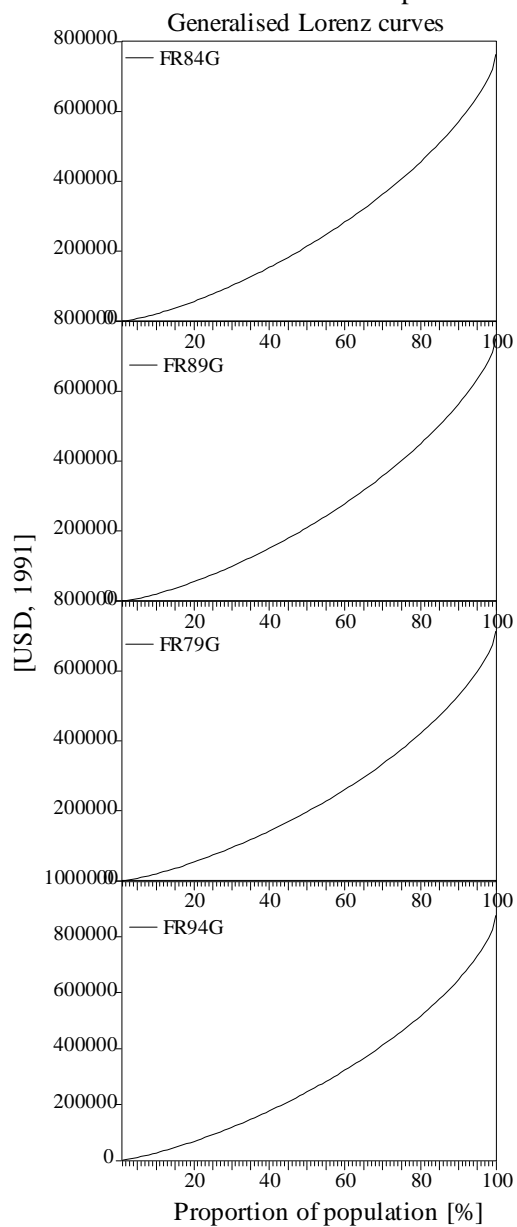
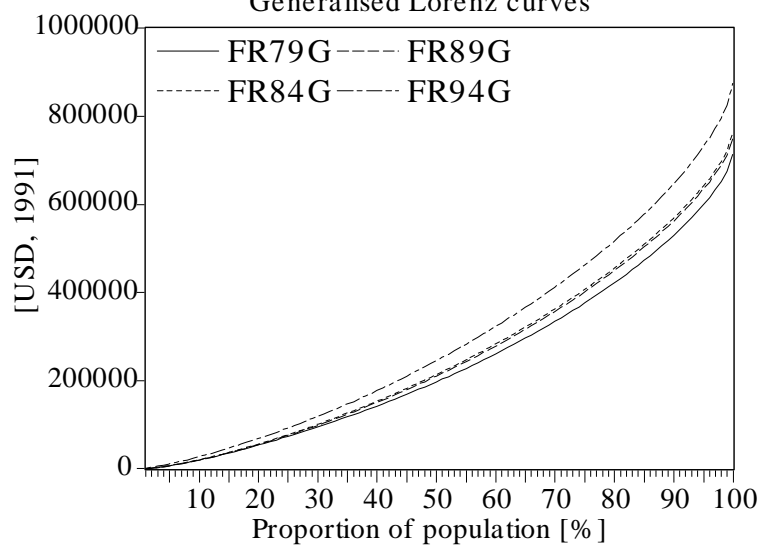


**B. Generalised Lorenz doinance**



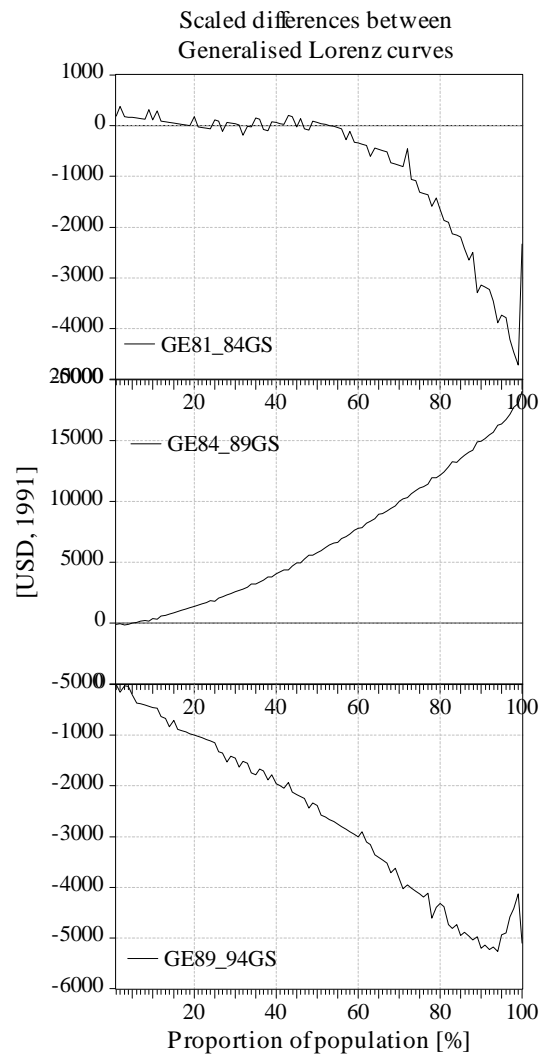
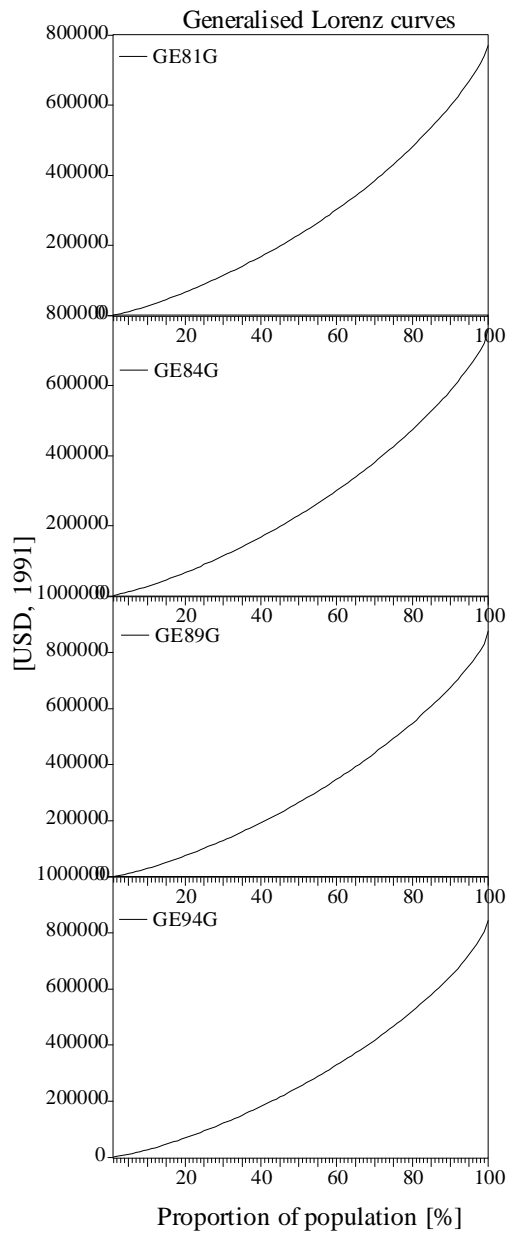
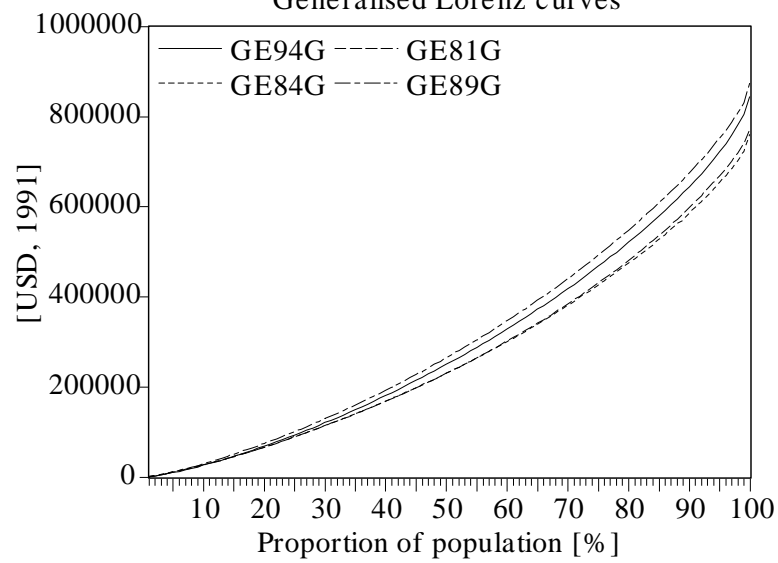
# FRANCE

## Generalised Lorenz curves

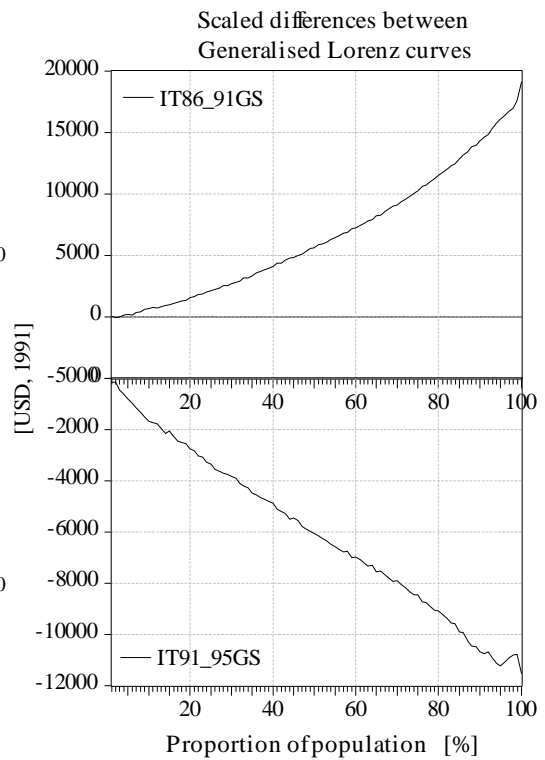
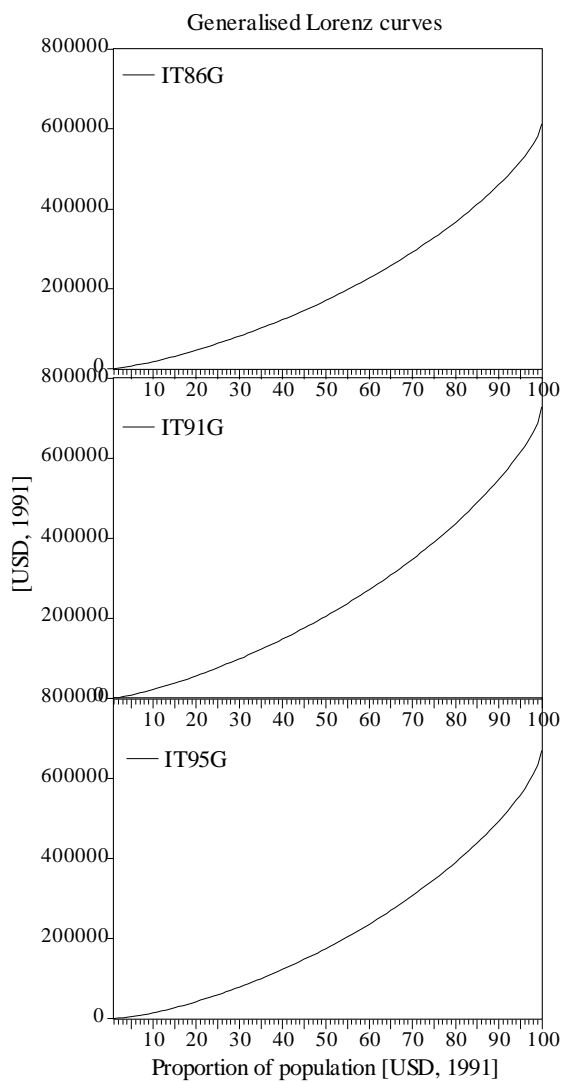
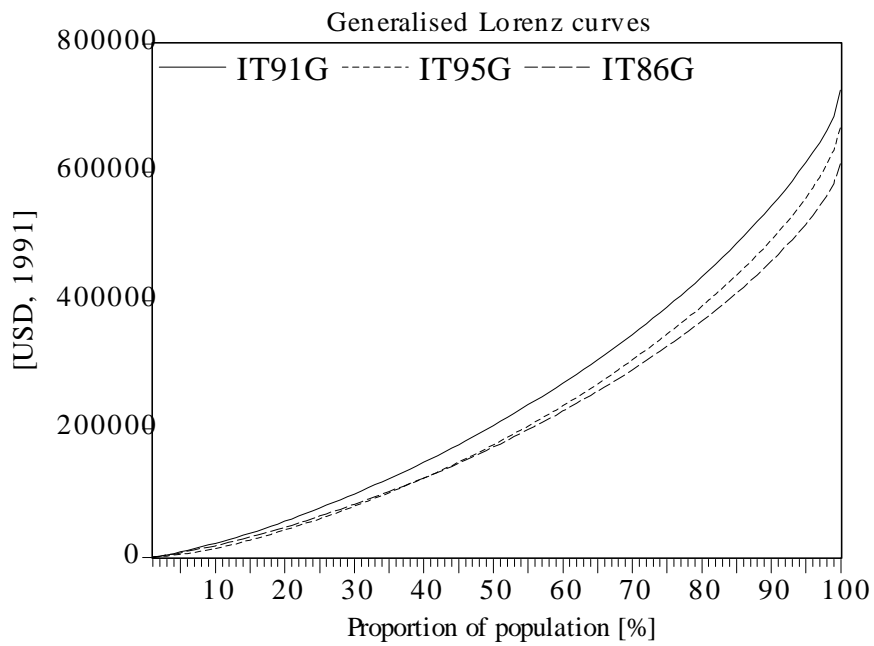




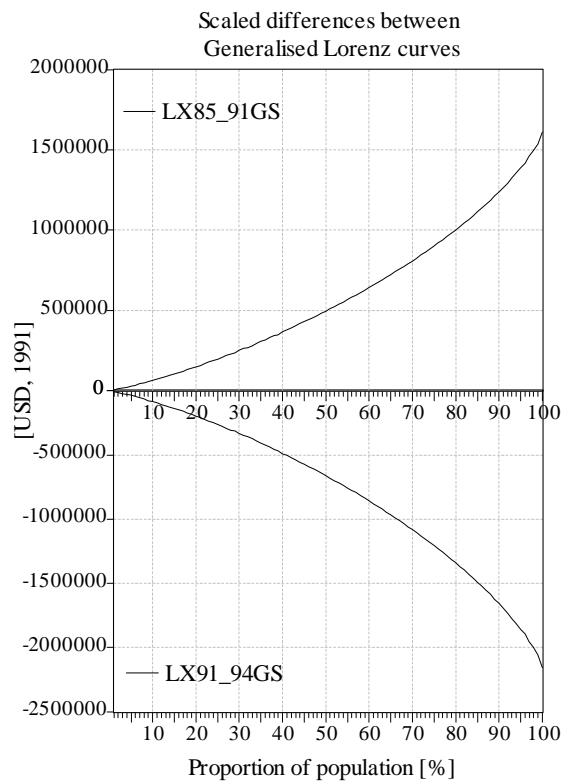
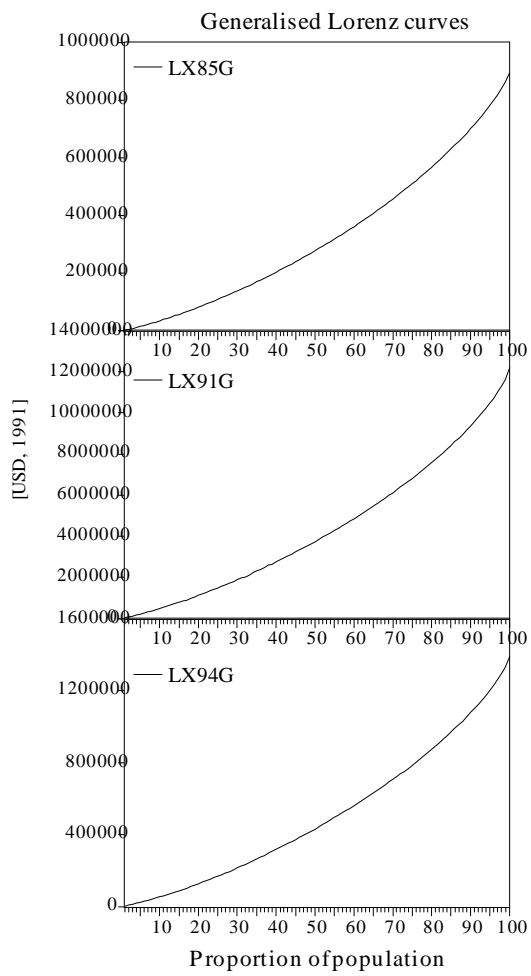
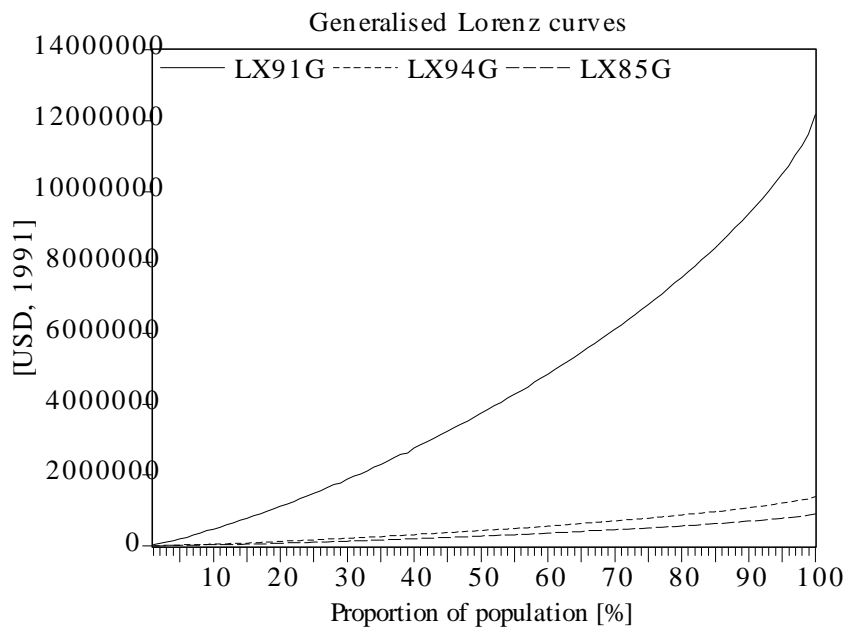
# GERMANY Generalised Lorenz curves



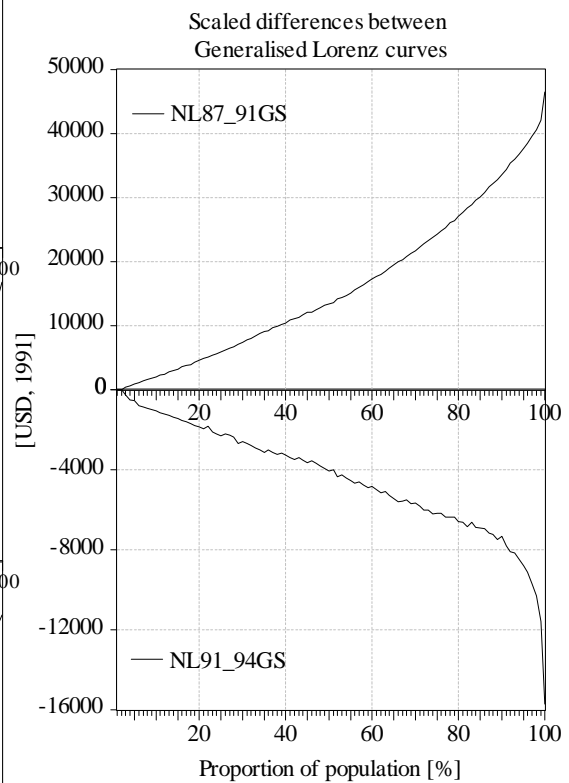
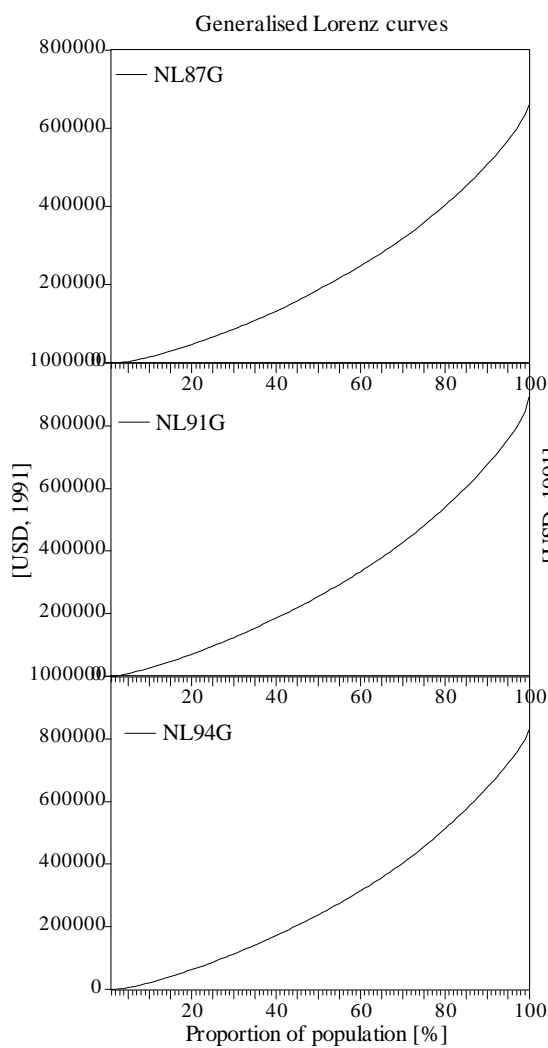
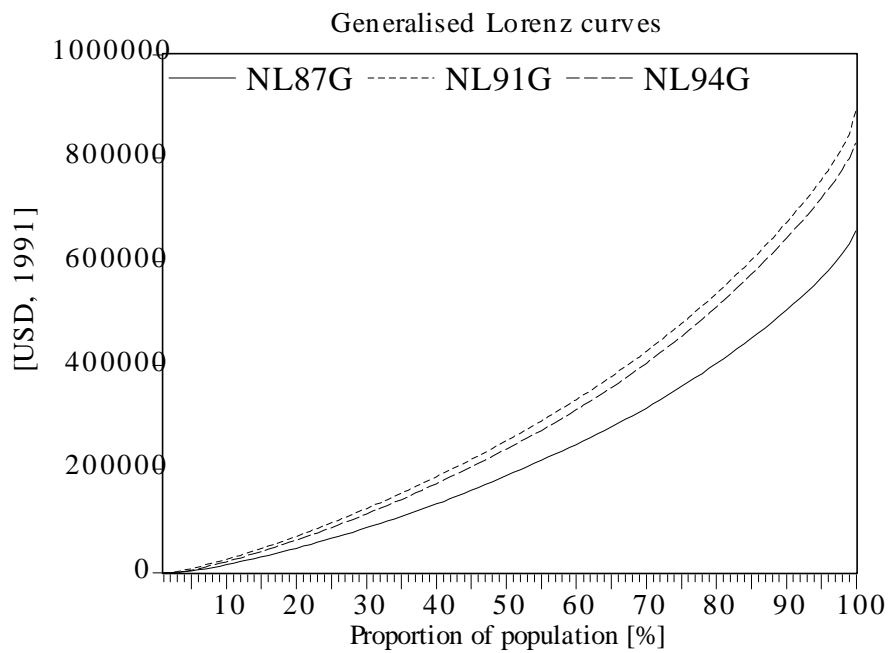
# ITALY



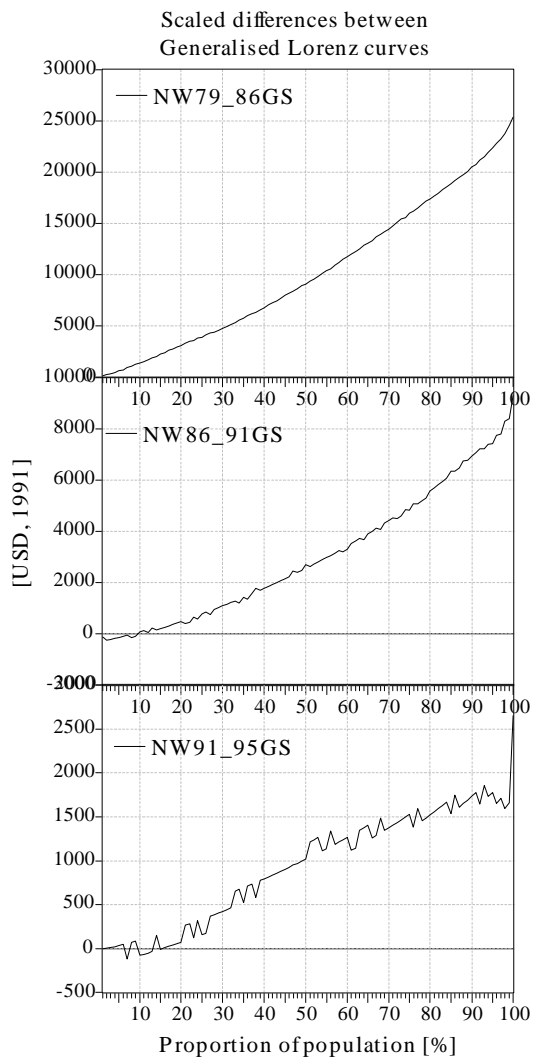
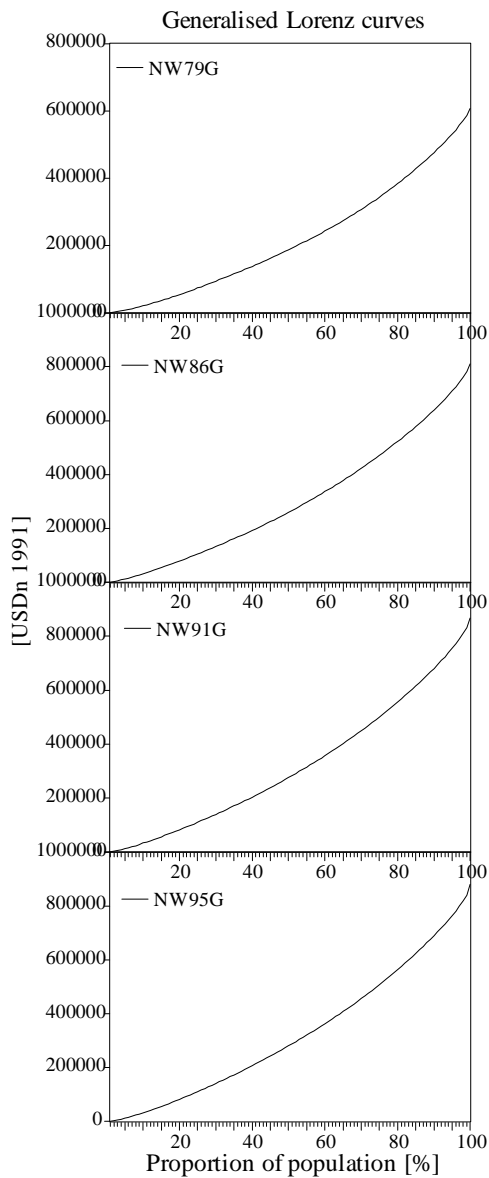
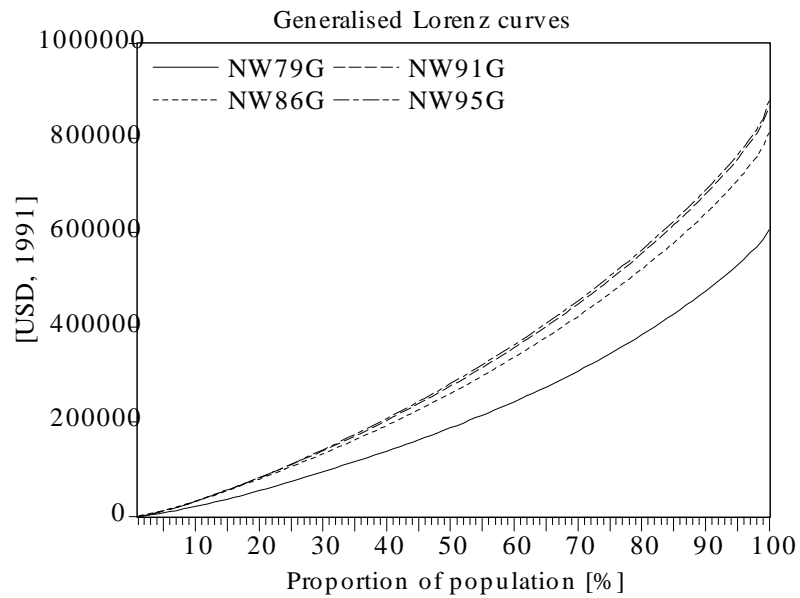
# LUXEMBOURG



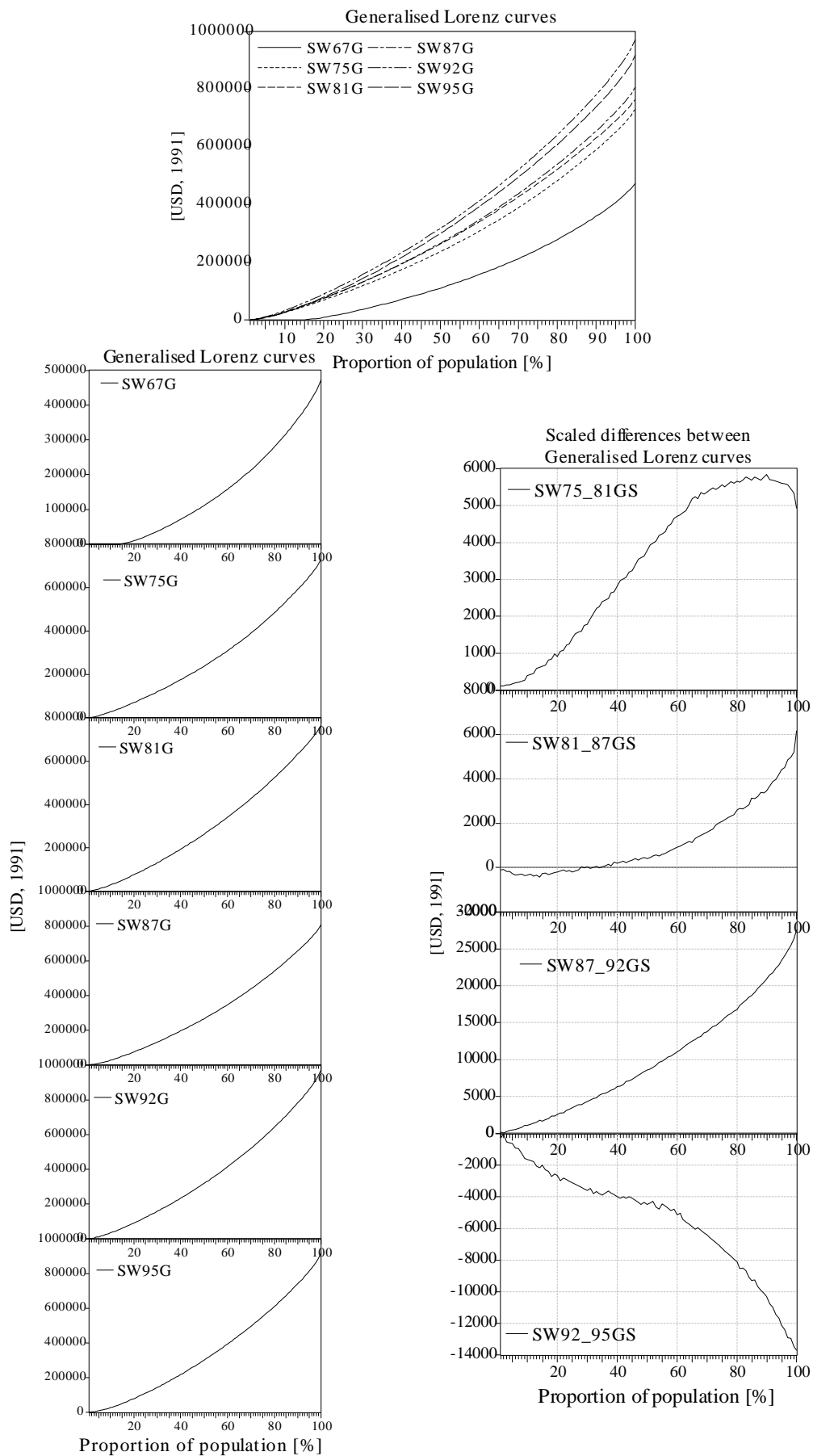
# NETHERLANDS



# NORWAY

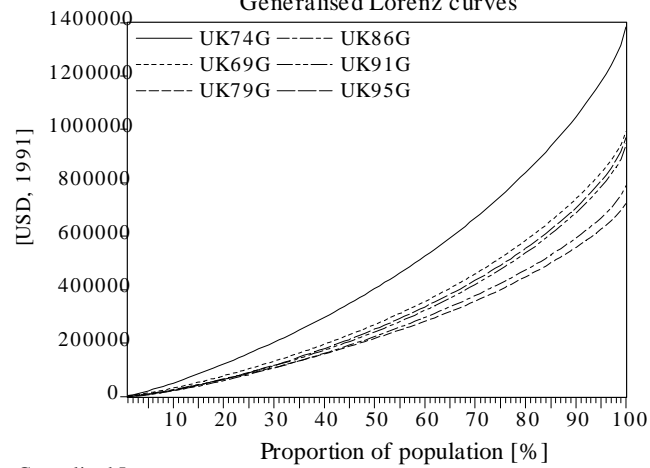


# SWEDEN

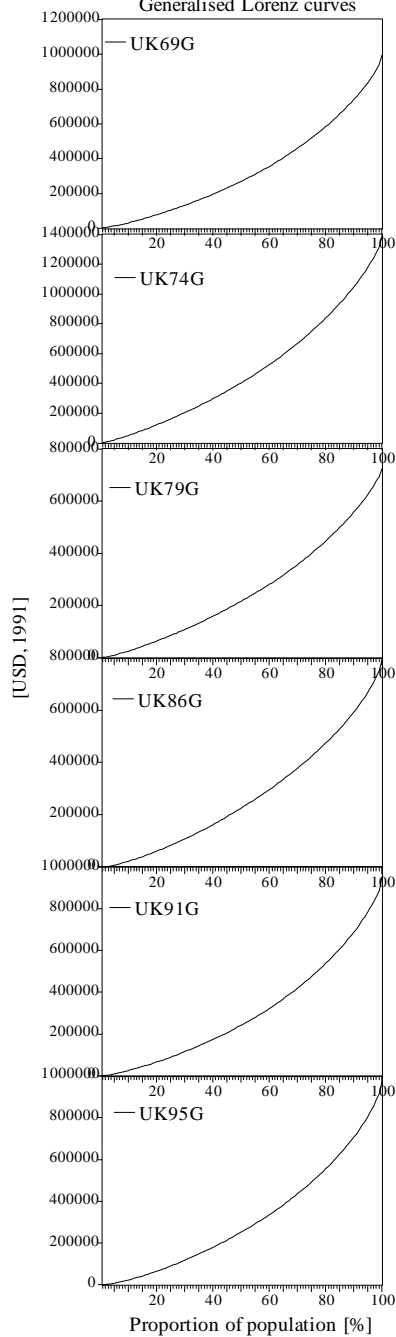


# UNITED KINGDOM

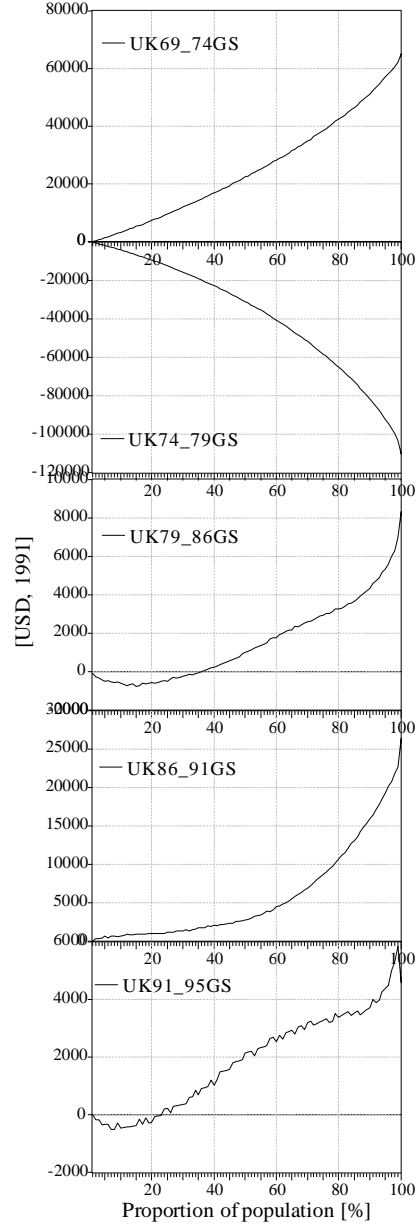
Generalised Lorenz curves



Generalised Lorenz curves

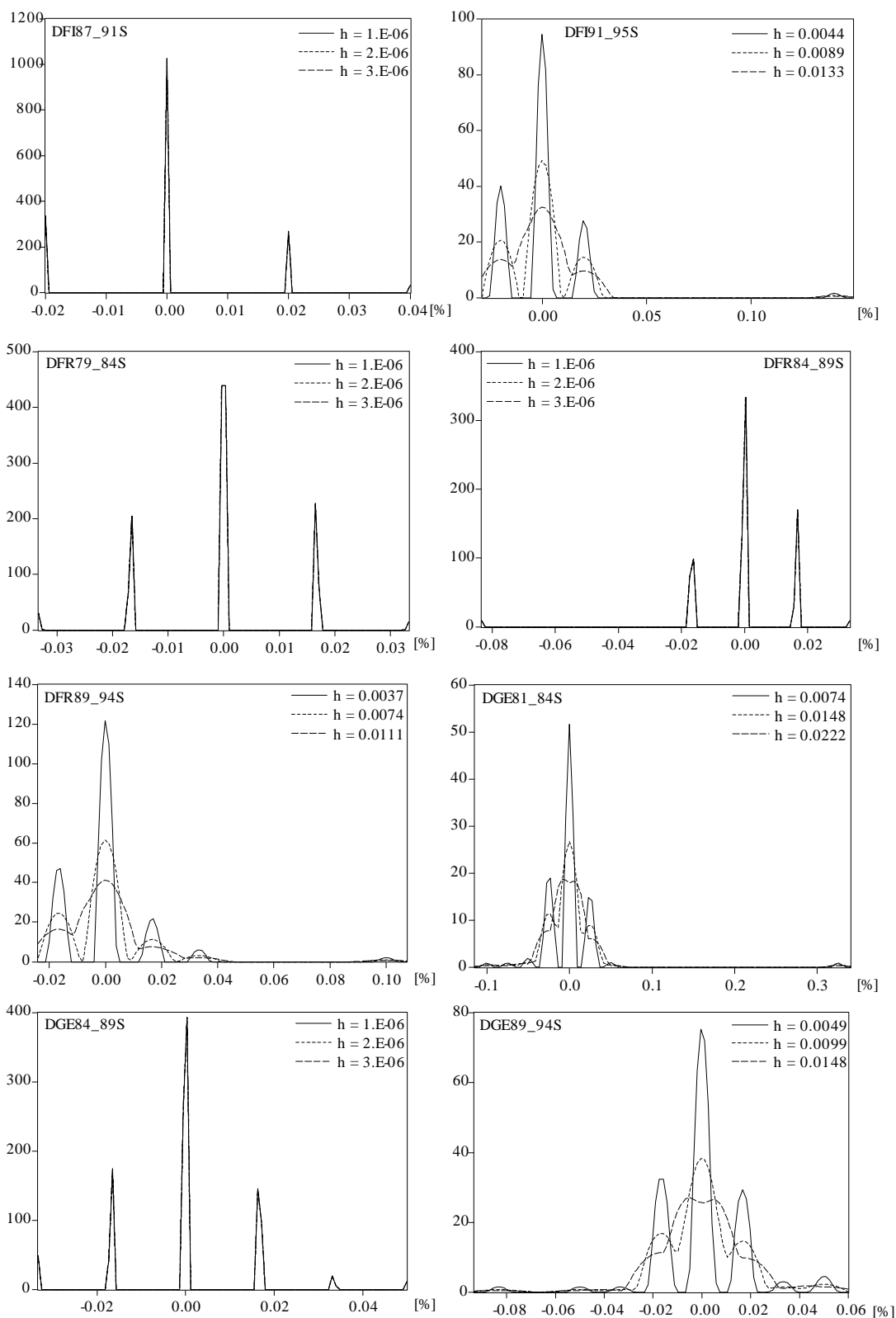


Scaled differences between Generalised Lorenz curves

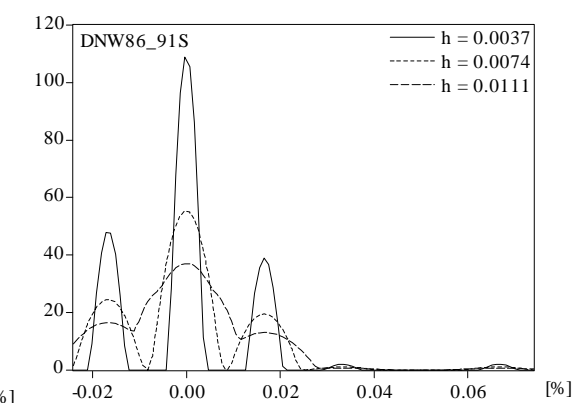
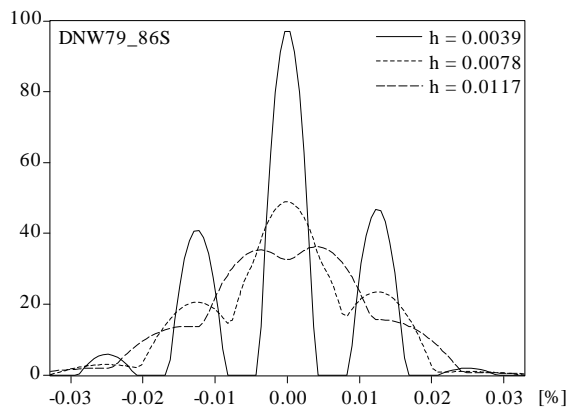
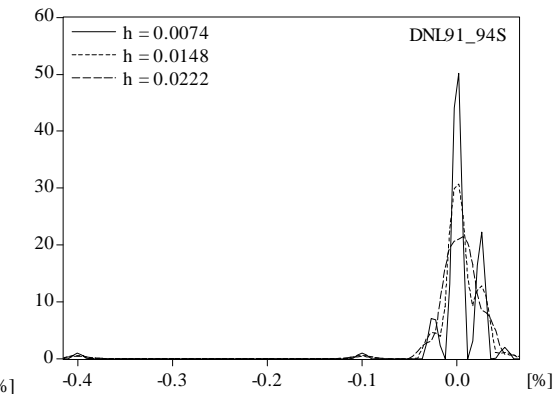
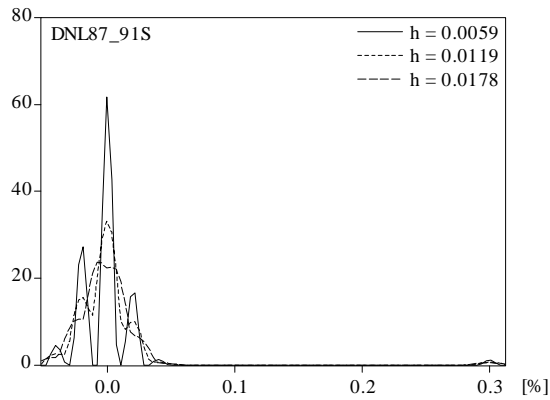
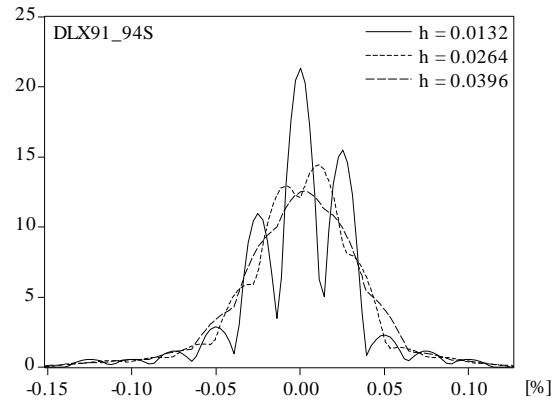
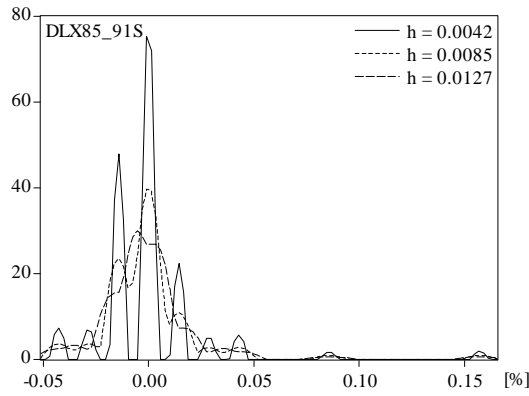
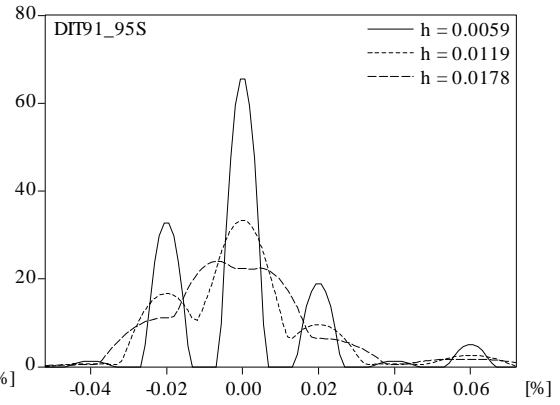
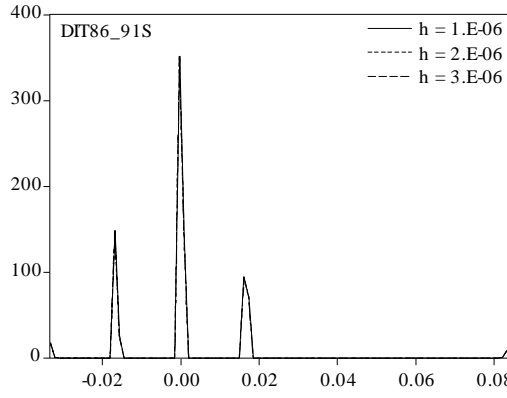


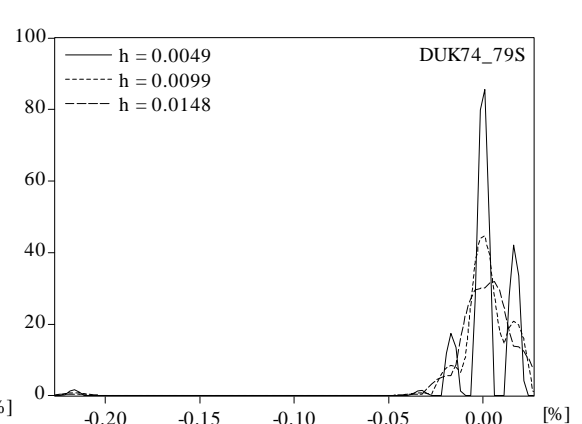
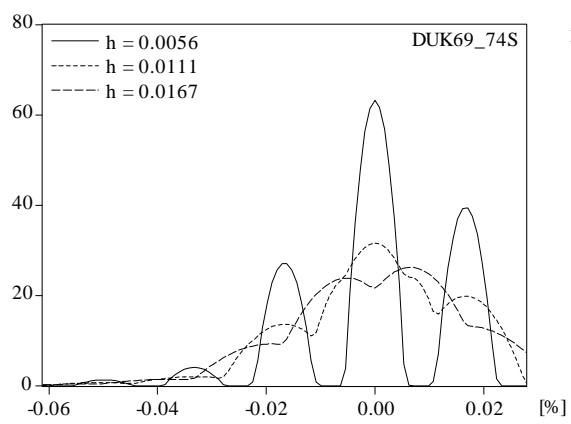
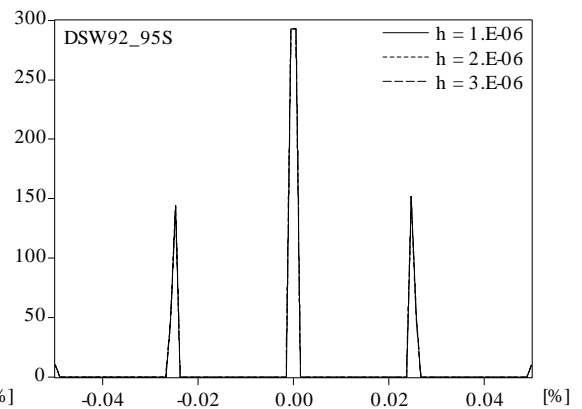
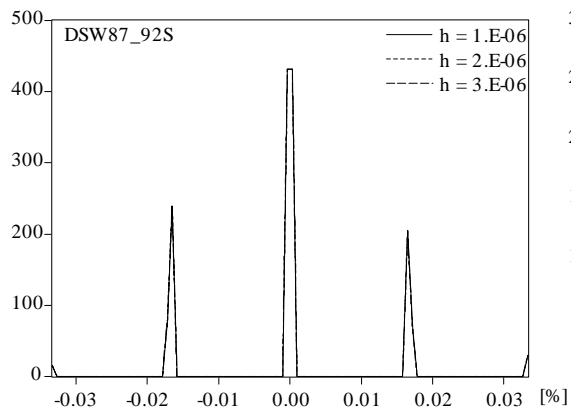
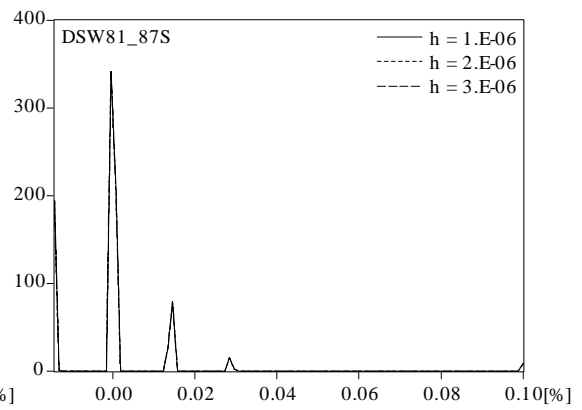
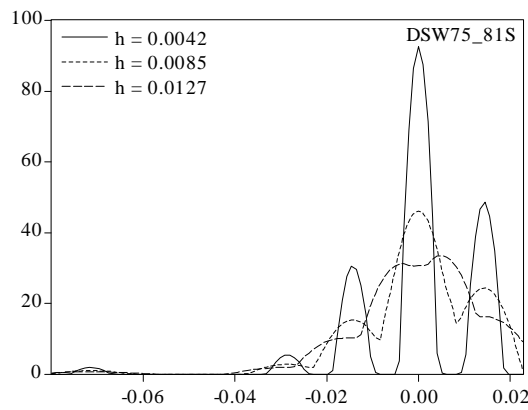
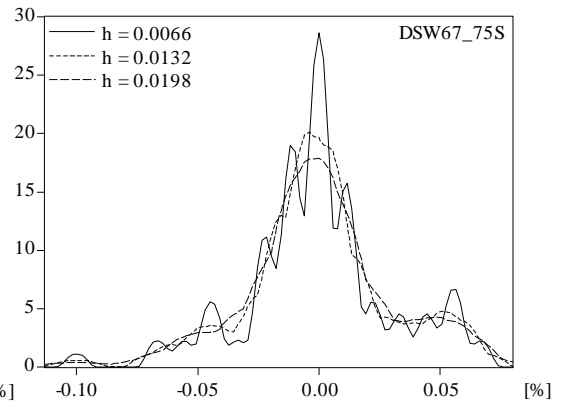
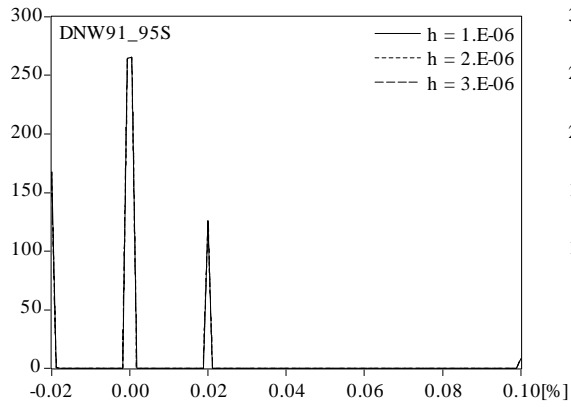
**APPENDIX 5. Densities of relative income shares**  
**estimated by means of kernel method**  
**(series derived from: A - Lorenz curves, B – Generalised Lorenz curves,**  
**kernel function: 1 – Epanechnikov, 2 – other)\***  
 (Probability should be read as a product of the bandwidth ( $h$ ) and value on the vertical axis)

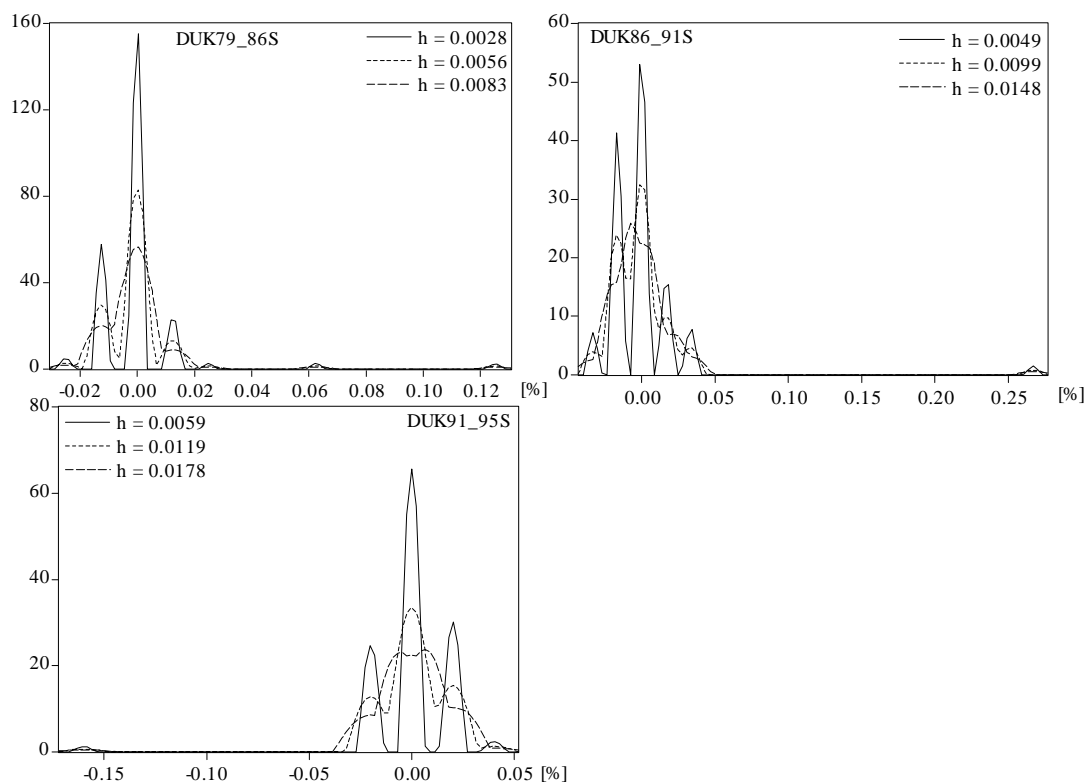
**A1.**



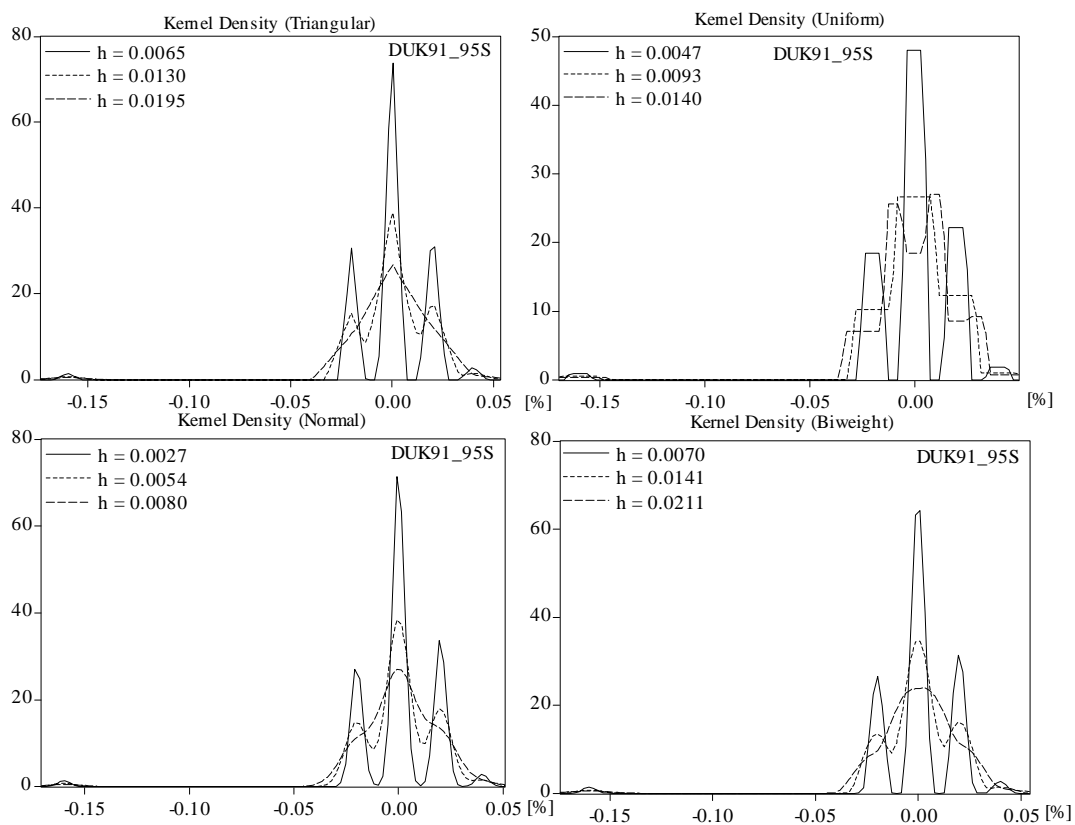


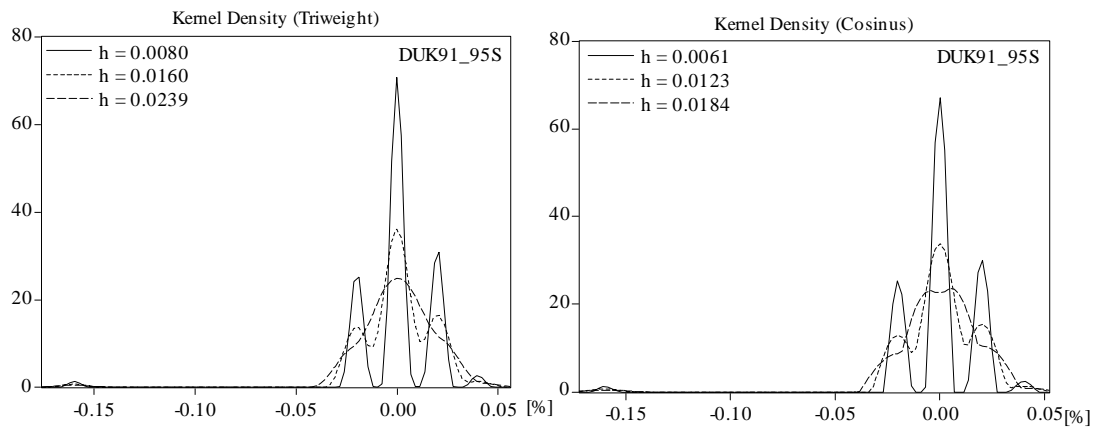




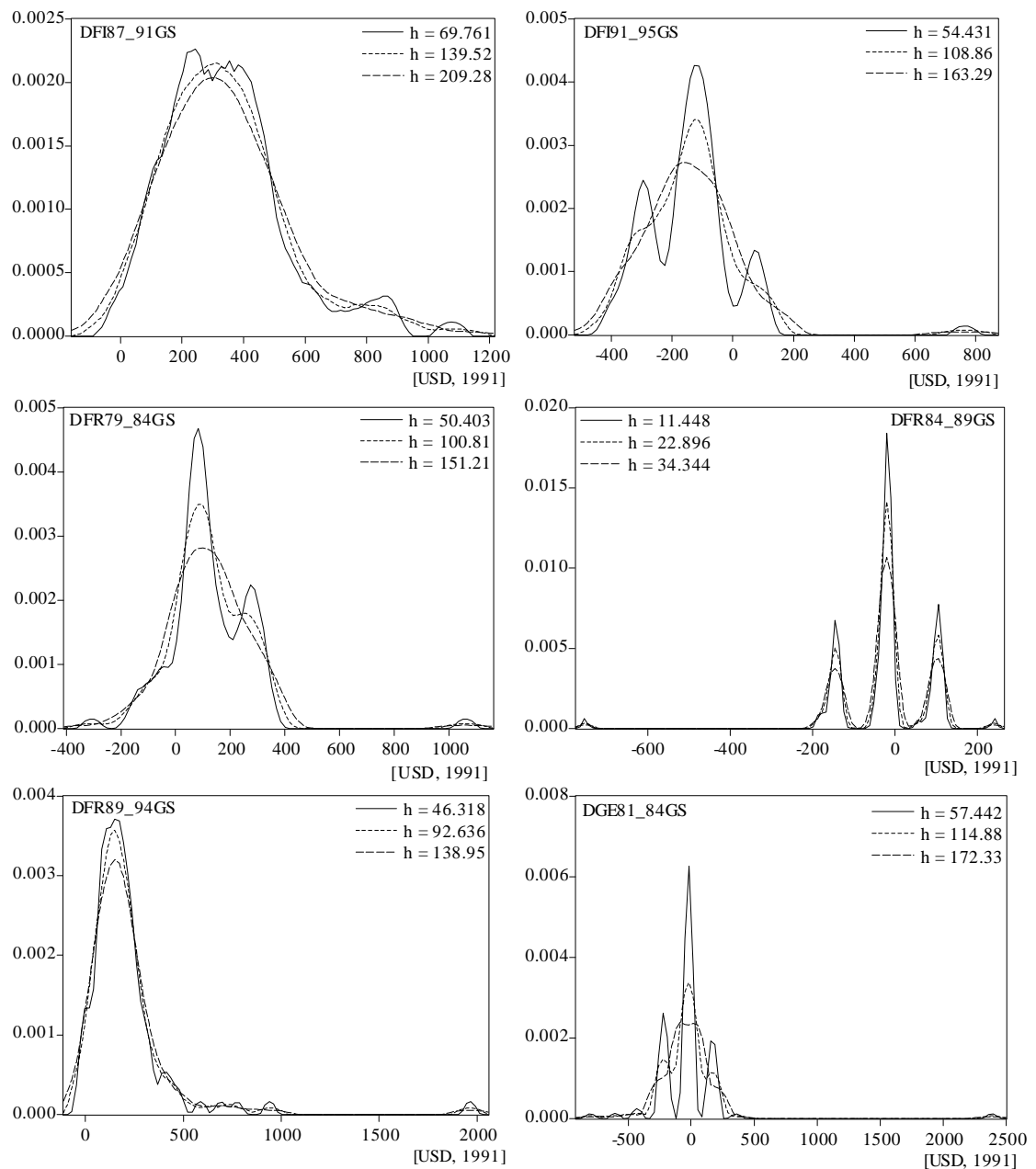


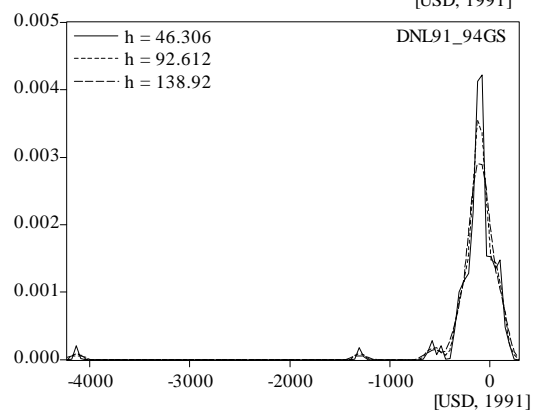
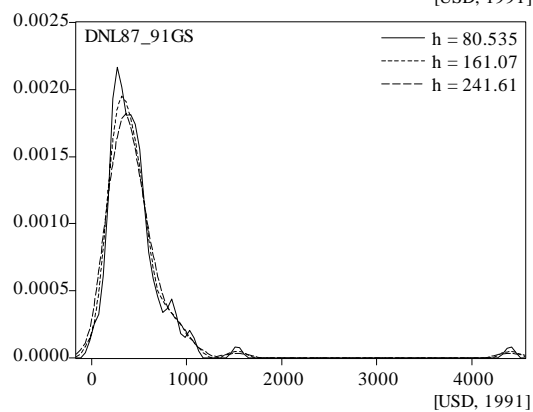
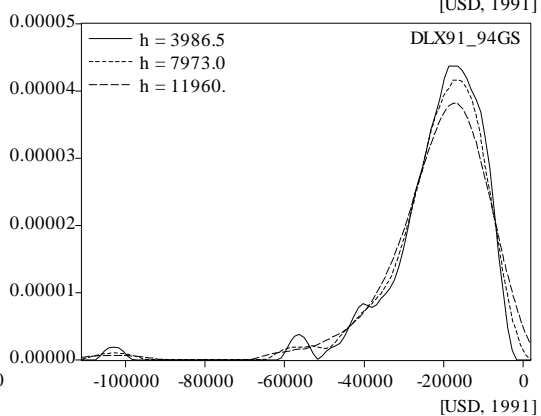
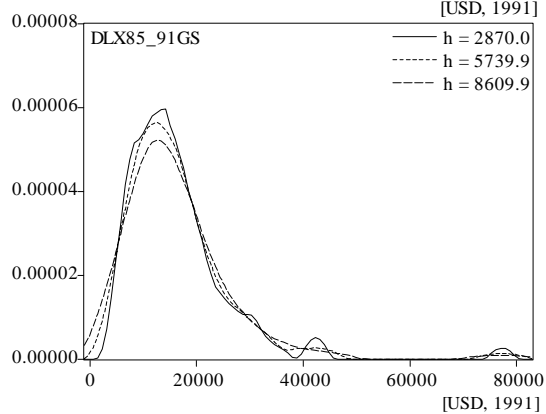
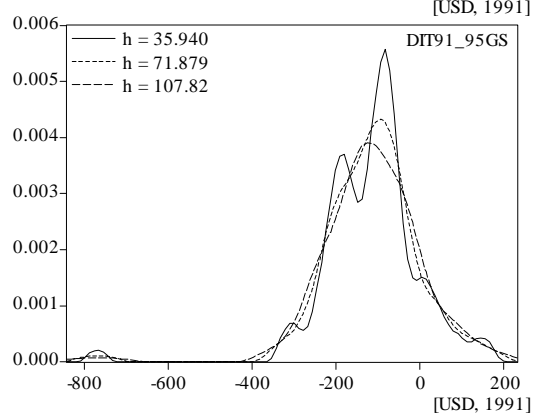
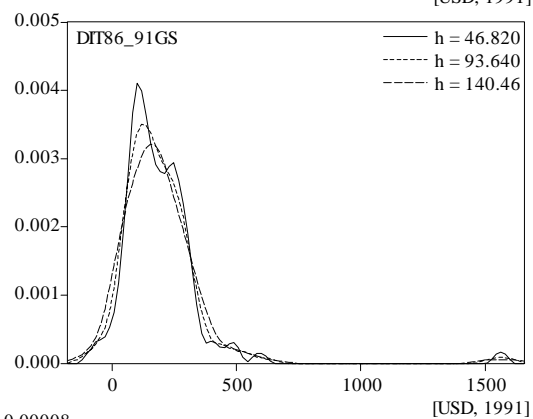
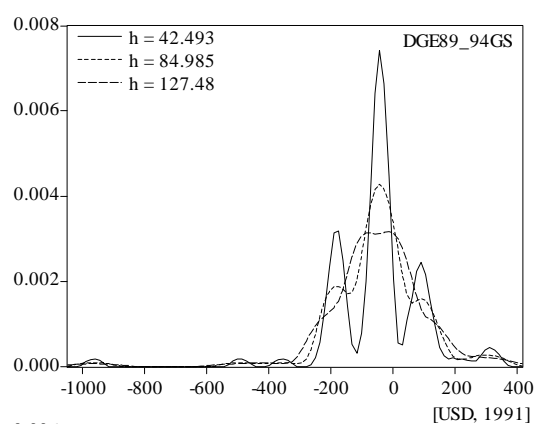
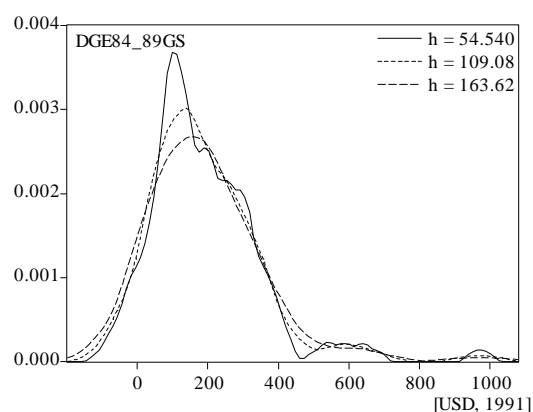
**A2.**

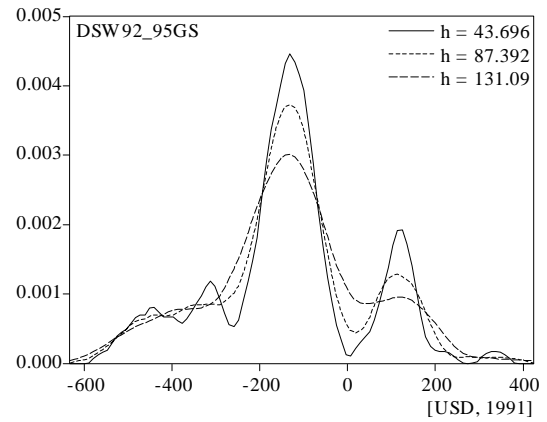
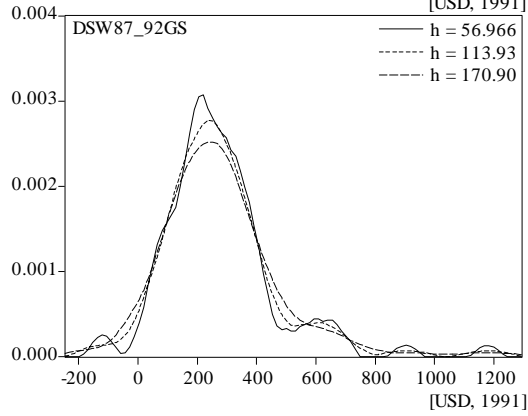
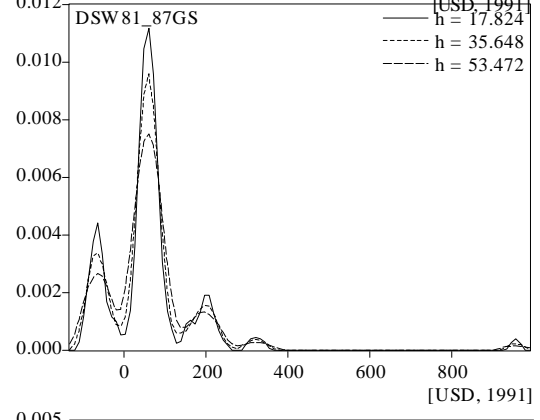
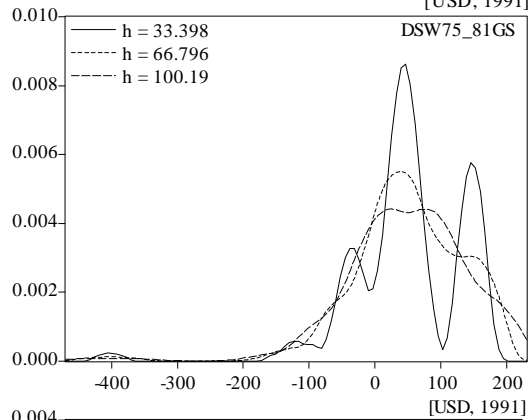
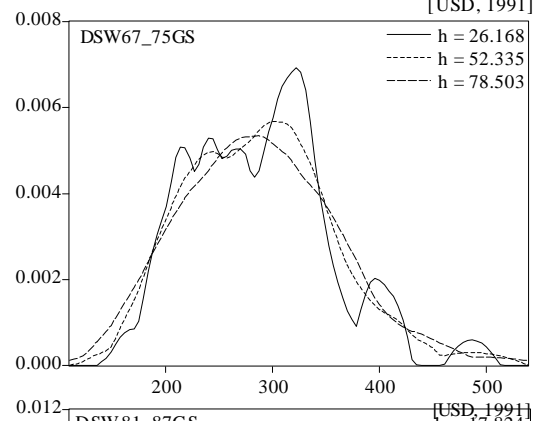
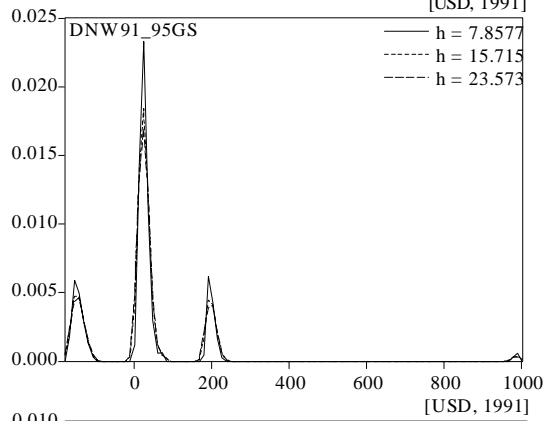
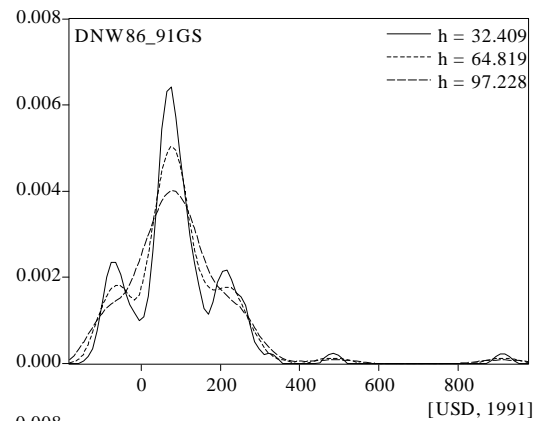
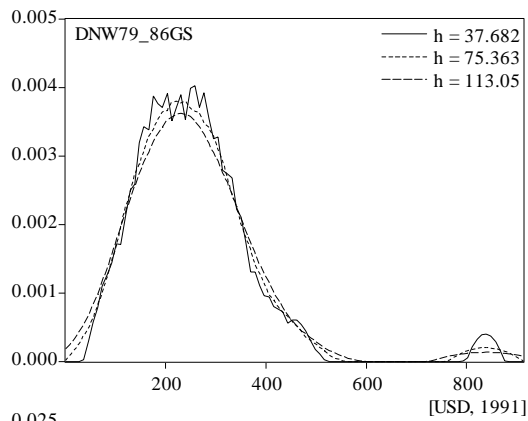


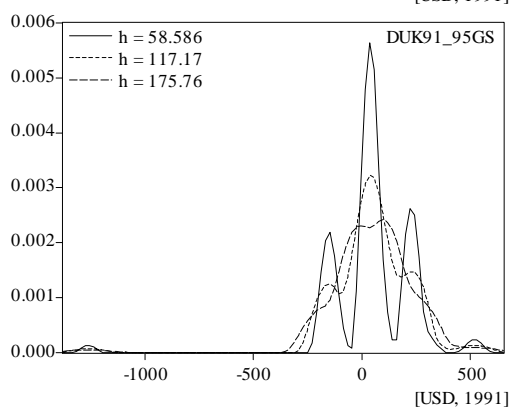
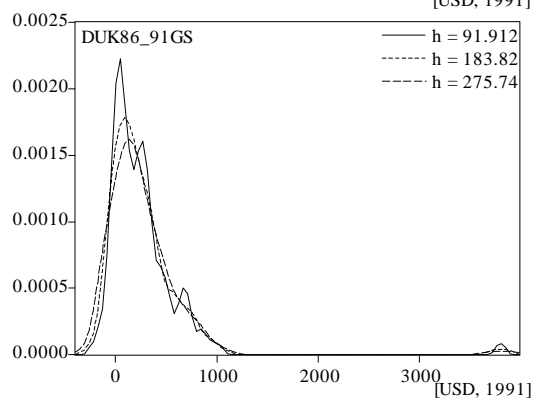
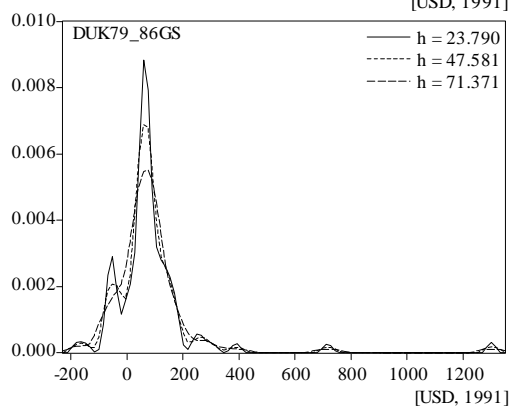
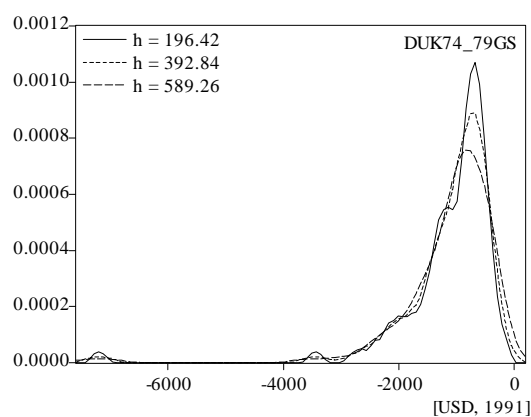
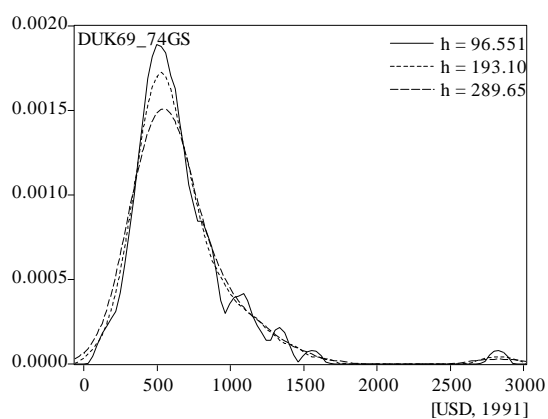


## B1.

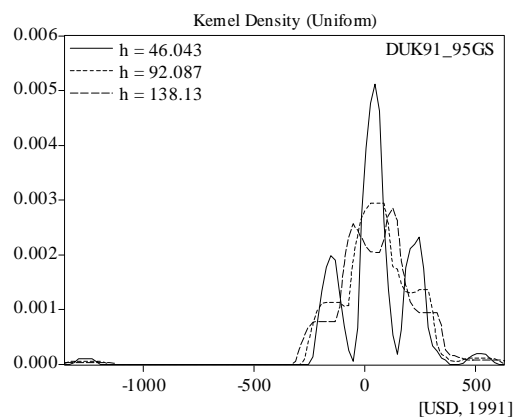
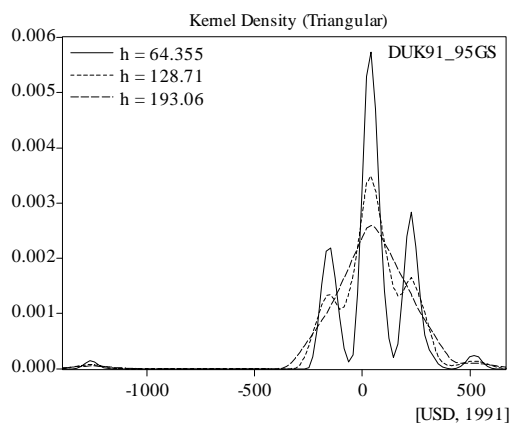


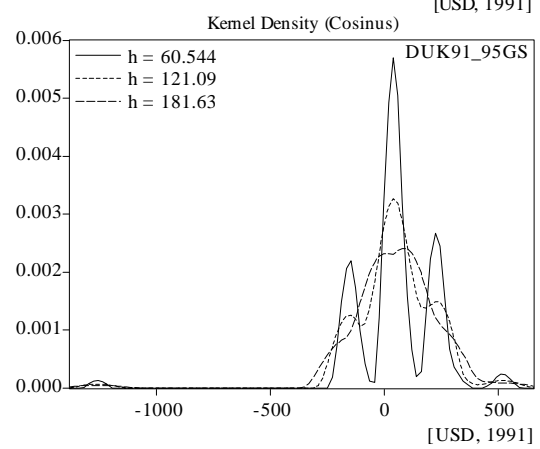
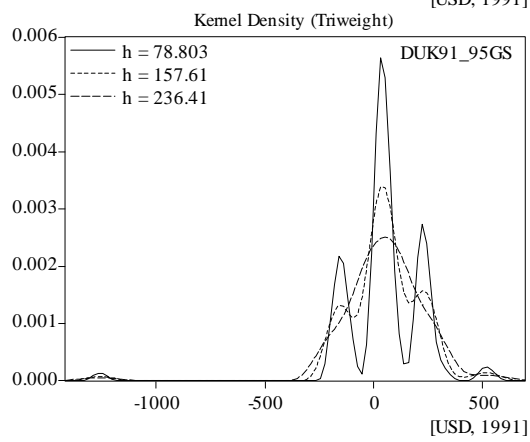
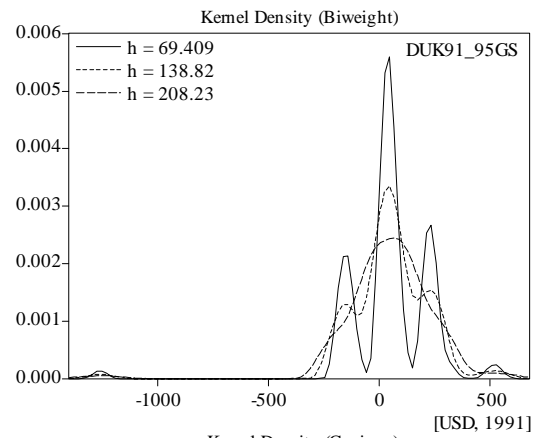
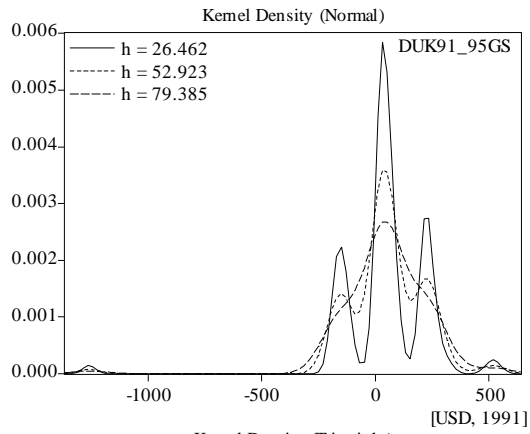






**B2.**







## APPENDIX 6. Gini coefficients – sensitivity analysis

Table A6.1 Gini coefficients \*

COUNTRY	SAMPLE PERIOD	1	2	3	4	5	6
<b>Finland</b>	1987	22,4613	26,0895	23,5226	21,5027	20,3086	20,2097
	1991	23,0023	26,2353	23,6436	21,6398	20,512	20,5117
	1995	24,6447	27,8053	25,2849	23,3466	22,2681	22,296
<b>France</b>	1979	33,4937	31,7462	30,3901	29,72	29,8489	30,8067
	1984	33,2786	31,9481	30,5517	29,7783	29,7683	29,8286
	1989	30,1391	28,8067	27,5124	26,8087	26,8243	28,8953
	1994	32,4022	31,0305	29,6772	28,9406	28,9484	29,7724
<b>Germany</b>	1981	28,3371	27,8724	25,777	24,4681	24,2053	25,12
	1984	28,2647	28,329	26,2854	24,9599	24,5959	25,3414
	1989	20,2191	20,6116	18,9548	17,8365	17,472	17,979
	1994	27,3556	26,6478	25,0616	24,0893	23,9082	24,62
<b>Italy</b>	1986	32,5157	32,7763	31,6001	30,8537	30,628	30,9947
	1991	31,226	30,6303	29,48	28,8121	28,7304	29,0078
	1995	36,3494	36,5376	35,4241	34,7062	34,4704	34,4704
<b>Luxembourg</b>	1985	26,7852	27,0813	25,0303	23,6774	23,2606	23,8402
	1991	26,9303	26,9058	25,0089	23,7578	23,3762	23,3762
	1994	26,9281	26,3744	24,5292	23,379	23,1408	23,1408
<b>Netrerlands</b>	1987	31,5286	27,0171	25,7725	25,3064	25,7982	27,335
	1991	31,8793	29,371	27,9546	27,2259	27,3356	28,358
	1994	32,2317	29,6114	28,2089	27,5259	27,7031	28,7894
<b>Norway</b>	1979	27,4665	26,2477	24,3509	23,2014	23,0278	23,911
	1986	24,6116	28,3366	25,8141	23,7891	22,5412	22,3199
	1991	25,4362	29,0443	26,2081	23,9178	22,5846	22,5527
	1995	25,8828	29,3759	26,7967	24,7261	23,4848	23,3642
<b>Sweden</b>	1967	34,034	33,4094	31,7464	30,7362	30,6348	31,5616
	1975	24,5055	27,2975	24,6751	22,6663	21,617	21,7644
	1981	23,291	26,6559	23,6065	21,1702	19,8111	19,9201
	1987	24,554	28,6318	25,745	23,4011	21,9708	21,8062
	1992	25,4179	29,1543	26,3727	24,157	22,8376	22,7005
	1995	25,5431	28,2794	25,3946	23,1366	21,9356	22,1365
	1995	25,5431	28,2794	25,3946	23,1366	21,9356	22,1365
<b>United Kingdom</b>	1969	33,6886	26,3004	25,5809	25,7076	26,731	28,6541
	1974	30,0137	28,5567	27,198	26,4721	26,4448	27,1322
	1979	29,0036	30,0212	28,2269	26,9899	26,4302	26,6326
	1986	33,1023	33,2813	31,7615	30,7301	30,3161	30,6226
	1991	36,2272	35,7806	34,6021	33,8751	33,6721	34,0452
	1995	36,8245	36,7093	35,4831	34,6788	34,3866	34,6663

\*Gini coefficients were based on the income adjusted by means of the following equivalence scales:

1. $N$ , $\varepsilon=1$	2. $(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon$ , $\varepsilon=0$	3. $(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon$ , $\varepsilon=0.25$
4. $(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon$ , $\varepsilon=0.5$	5. $(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon$ , $\varepsilon=0.75$	6. $(1 + 0.7(N_A - 1) + 0.5N_C)^\varepsilon$ , $\varepsilon=1$

where:  $N$ -household size=  $N_A + N_C$ ,  $N_A$  -number of adults in a household,  $N_C$  -number of children in a household,  $\varepsilon$  - parameter of within-household economies of scale