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## Lorenz and Stochastic Dominance Comparisons of European Income Distribution

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# Luxembourg Income Study Working Paper Series

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European Income Distribution

John Bishop, Peter Formby and Ryoichi Sakano

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**LORENZ AND STOCHASTIC DOMINANCE COMPARISONS OF EUROPEAN  
INCOME DISTRIBUTIONS**

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## I. INTRODUCTION

The last three decades of the 20th century represent a period of intense world change with economic integration proceeding at an unprecedented rate. The process of integration has been under way in the U.S. and Europe for some time and there are compelling reasons to expect freely flowing labor and capital within a united and effectively integrated economic system to lead to eventual convergence of income distributions and welfare among the separate regions of the U.S. and distinct nations of Europe. For example, the U.S. South historically was much poorer and had greater income inequality than the rest of the U.S., but Bishop, Formby and Thistle (1992) provide evidence that the emergence of a national labor market in the following WW II and the interregional flow of resources led to the convergence of regional income distributions in the U.S. during the decade of the 1970s. Similar processes will no doubt be at work in European development. In this paper we investigate the degree to which income distributions and welfare in five European countries differ from one another in 1980 and 1985.

Recent developments in applied welfare economics and statistical inference procedures for dominance relations among income distributions are used to gauge the differences in and degree of convergence of income distributions in France, Germany, The Netherlands Sweden and the U.K. We apply the same methodology as Bishop, Formby and Thistle (1992) and make Lorenz, Generalized Lorenz and rank dominance comparisons among countries and across time. A distinguishing characteristic of this approach is that it makes use of Luxembourg Income Survey (LIS) microdata applies inference based stochastic dominance method to rank European income distributions. We extend earlier work by Bishop, Formby and Smith (1991) who also use LIS data and apply Beach et al.'s inference tests. The extensions are of three sorts. First, we make stochastic dominance comparisons as well as Lorenz orderings.<sup>1</sup> Second, we analyze the changes in distributions across time. Third, we explore whether there is any evidence of convergence or divergence in European income distributions of the sort that Bishop, Formby and Thistle (1992) have detected for the U.S. By applying inference tests for rank and GL dominance we also extend the work of Shorrocks's (1983) and Bishop, Formby and Thistle

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1. Bishop, Formby and Smith (1990) compare Lorenz curves of nine countries including the five analyzed here. But they consider only the circa 1980 data and do not make stochastic dominance comparisons.

(1991), who use simple numerical comparisons to construct ranking of distributions across countries.

The paper is organized as follows. Section II briefly reviews the Lorenz and stochastic dominance approach to rankings of income distributions and welfare. Section III discusses the data and statistical inference procedures. Section IV presents the empirical results. The final section provides brief concluding remarks.

## **II. DOMINANCE COMPARISONS OF INCOME DISTRIBUTIONS AND WELFARE**

The last two decades have witnessed important advances in understanding the welfare implications of the distribution of income and inequality. Dominance techniques for ranking income distributions stand at the forefront of these developments. Important theoretical contributions have been made by Kolm (1969), Atkinson (1970), Shorrocks (1983) and Saposnik (1981, 1983). The generality and elegant simplicity of the dominance approach has given rise to a large number of papers that extend the stochastic dominance methodology for ranking income distributions and welfare.

Beginning with the work of Atkinson (1970) and Kolm (1970) income distributions have been ranked in a fashion analogous to the stochastic dominance rankings of probability distributions under uncertainty, which were developed in financial economics in the 1960s. Atkinson and Kolm share the original contribution that establishes Lorenz dominance as an important welfare ranking criterion. However in its original form the dominance principle connecting income distributions and welfare was subject to two severe restrictions, which were relaxed by contributions of Dasgupta Sen and Starrett (1973), Shorrocks (1983) and Saposnik (1981, 1983). Dasgupta, Sen and Starrett (1973) show that the Lorenz dominance theorem applies to all increasing and S-concave social welfare functions, which comprise a far more general class than those considered in Atkinson's original dominance theorem. Shorrocks (1983) and Saposnik (1981, 1983) extend the dominance principle to address the empirically important cases where there are differences in mean incomes of the distributions being compared. Shorrocks proposes generalized Lorenz dominance, which requires that one rescaled Lorenz curve dominate another, where the scalar is mean income. Generalized Lorenz dominance is equivalent to second degree stochastic dominance. Saposnik (1981, 1983) proposes rank dominance, a more

basic and less restrictive dominance principle, which is based upon the strong Pareto principle and is equivalent to first degree stochastic dominance. Bishop, Formby and Thistle (1991) show that much of the power contained in generalized Lorenz dominance in ordering empirical income distributions is actually contained in rank dominance.

It is useful to formally establish the welfare content of income distributions that can be ordered using the dominance approach. Following Atkinson (1970) we assume the relationship between the distribution of income and standard of living is summarized in a social welfare or social evaluation function, which represents the ethical judgments regarding income distributions. Both first and second degree stochastic dominance impose restrictions on the welfare function, which are detailed below.

First degree stochastic dominance (FSD) is underpinned by the strong Pareto principle and anonymity, two assumptions which should have a wide degree of acceptance. In addition, for purposes of the empirical analysis in this section we adopt the population principle (Dalton, (1920), Dasgupta, Sen and Starrett, (1973) and Sen (1967)). Together these assumptions imply that the statistical cumulative distribution function (cdf) for income contains sufficient information for ranking social states. Formally, let  $F$  denote the income cdf. The inverse distribution function or quantile function,  $X(p) := \inf\{x:F(x) \geq p\}$ ,  $p \in [0, 1]$ , yields individuals' incomes in increasing order.

We denote the class of anonymous, increasing welfare functions as  $W_p$ . Saposnik (1981, 1983) provides the following theorem on rank dominance.

**THEOREM 1:**  $X >_R Y$  iff  $w(X) > w(Y)$  for all  $w \in W_p$ .

Thus distribution  $X$  dominates distribution  $Y$  iff  $X(p) \geq Y(p)$  for all  $p \in [0, 1]$ . If for all  $p \in [0, 1]$ ,  $X(p) = Y(p)$ , then  $X$  and  $Y$  have the same income distribution and standard of living. If  $X(p) > Y(p)$  for some  $p$ , and  $X(p) < Y(p)$  for some  $p$  (i.e., the quantile functions cross), the distributions are noncomparable and cannot be ordered using the rank dominance criterion.<sup>2</sup>

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2. Foster and Shorrocks (1988) provide an important corollary to Theorem 1 linking rank dominance (FSD) to the headcount poverty concept.

If quantile functions cross, the analyst can proceed by placing further restrictions on the class of admissible welfare functions by assuming a social preference for equity. This leads to the application of the GL (SSD) criterion. As with rank dominance, the income distribution (cdf) contains all the information necessary to apply the GL criterion. Also like rank dominance, it is more convenient to define the GL function in terms of the inverse function,  $F^{-1}$ . Adapting Gastwith's (1971) definition of the Lorenz curve we can write the GL curve as,

$$G_X(p) = \int_0^p F^{-1}(x)dx = \mu_X L_X(p),$$

where  $L_X(p)$  is the ordinary Lorenz ordinate and  $G_X(1) = \mu_X$ . The GL criterion requires that the class of admissible welfare functions be restricted to only those that are equality-preferring. Dasgupta, Sen and Starrett (1973) demonstrate that this amounts to assuming that the welfare function is S-concave. We denote the class of anonymous, increasing, and S-concave welfare functions, as  $W_E$ . Shorrocks (1983) demonstrates the relationship between GL dominance,  $W_E$ , and second degree stochastic dominance with the following theorem on GL dominance.

**THEOREM 2:**  $X >_{GL} Y$  iff  $w(X) > w(Y)$  for all  $w \in W_E$ .

Income vector  $X$  generalized Lorenz dominates  $Y$ , denoted  $X >_{GL} Y$ , if, and only, if  $G_X(p) \geq G_Y(p)$  for all  $p \in I$ , with at least one strict inequality at some  $p$ . Like ordinary Lorenz curves the GL criterion provides only a partial ordering because crossing generalized Lorenz curves cannot be ranked. Thus, GL curves can be compared in essentially the same manner as ordinary Lorenz curves.<sup>3</sup>

### III. DATA AND STATISTICAL INFERENCE PROCEDURES

The data for this study are taken from two Luxembourg Income Study (LIS) datasets for each of the following countries, France, Germany, The Netherlands Sweden

3. As in the case of rank dominance and headcount poverty, Foster and Shorrocks (1988) provide a corollary which connects the GL criterion to the income gap poverty concept.

and the U.K. The LIS data provides micro observations of national survey data with the distinguishing feature that LIS data is adjusted for definitional differences to make it more comparable than the original national survey data. We use all European countries for which there are at least two data sets on file as of September 1991. Table 1 identifies the original national surveys, the dates of the surveys and shows sample sizes. Detailed descriptions of the data are provided by Buhmann, Rainwater, Schmaus and Smeeding (1988), O'Higgins, Schmaus and Stephenson (1989) and Bishop, Formby and Smith (1990). The measure of income is the LIS concept of family cash income, which is a comprehensive measure of market income minus direct income taxes and payroll taxes plus cash and some non-cash transfers. The income recipient unit is percapita family income in which, for example, a family of five with net cash income of \$40,000 is reported as five incomes of \$8,000 each. In all cases except the UK, the data is weighted by the LIS person weights to more precisely represent the underlying populations.

The LIS incomes are reported in their own country currencies. While this presents no difficulty when the analysis is restricted to Lorenz dominance, the stochastic dominance comparisons are not scale free and require conversion to a single currency. Additionally, as indicated in Table 1 the country specific data sets for both the early time period and the later time period are generally not the same for each of the countries. In the early period the years of the surveys for the various countries range from 1979 to 1983 and we refer to these early datasets as "circa 1980". The second set of surveys are for years ranging from 1984 to 1987 and we refer to these LIS datasets as "circa 1986". Like differences in currencies, the stochastic dominance comparisons in different years require adjustments for the level of incomes. In a manner similar to Shorrocks (1983) and Bishop, Formby and Thistle (1991) we standardize the LIS data sets using the per capita gross domestic product (GDP) estimates of Summers and Heston (1990).<sup>4</sup> For circa 1980 we use the average of 1979, 1980 and 1981 per capita GDP expressed in constant 1985 dollars and measured in terms of international prices. For the circa 1986 datasets we use the average of 1985, 1986

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4. By using GDP to measure the level and the LIS data to measure dispersion of incomes, we are implicitly assuming that a dollars worth of non-transfer expenditures provides a dollars worth of benefits, divided proportionally among income quantiles.



and 1987 per capita GDP expressed in constant 1985 dollars and measured in international prices.

Table 2 provides the circa 1980 and circa 1986 GDP estimates in 1985 US dollars, as well as the percentage changes over this time period. For 1980, percapita GDP is highest in West Germany and lowest in the UK. However, percapita GDP grew fastest in Sweden (14.5 percent) and by 1986 Sweden had surpassed West Germany in percapita GDP. While the UK still had the lowest GDP in 1986 its growth rate (13.8 percent) was second only to that of Sweden.

Before summarizing the estimation procedures and presenting the results, a further caution concerning the data is in order. The LIS data are unquestionably the best available for making international comparisons at the present time; but there are several limitations which warrant emphasis. First, differing customs lead to somewhat different definitions of the family across the sample of LIS countries considered. For a discussion of this point see O'Higgins, Schmaus, and Stephenson (1989) and Bishop, Formby, and Smith (1990). The use of percapita incomes in the comparisons below reduces but does not eliminate all of the difficulties involved in comparing countries with differing family definitions. Another data problem relates to the West German surveys. For the circa 1980 data the German coverage (91.5 percent) is significantly below that of other countries (which range from 95.5 to 99.2 percent) because it excludes households headed by foreign born nationals. This too introduces an element of noncomparability. For the circa 1986 German a different national survey than the one in the earlier period was used (see Table 1), with the result that there is greater comparability to other countries, but perhaps less comparability in the German data across time.

Given the caveats with respect to the data we estimate the Lorenz curve using a vector of sample Lorenz ordinates,  $L_i$ ; we estimate the quantile functions as a step function of the sample decile conditional means,  $\hat{\mu}$ ; and we estimate the GL curves from a vector of decile sample GL ordinates,  $\hat{G}$ , where  $G_0 = 0$ ,  $G_i = \mu L_i$ , and  $G_{10} = \mu$ . To test for differences in Lorenz curves, quantile functions and GL curves we construct 95 percent confidence bands around the sample estimates from the LIS data. The formulae for the asymptotically distribution-free variance expressions used to construct confidence bands for the GL curves and quantile functions are given by Beach and Davidson (1983).

To construct a confidence band around a sample Lorenz curve (quantile function, GL curve), we use the information from the Lorenz ordinates (decile conditional means, GL ordinates) and their standard errors. Since this requires drawing inferences from the union of 10 disjoint subhypotheses, simultaneous inference procedures are appropriate. Following Beach and Richmond's (1985) procedure for ordinary Lorenz curves and Beach and Kaliski's (1986) results for weighted sample data, a joint confidence interval (confidence band) around a Lorenz (quantile function, GL curve) is constructed using the Studentized Maximum Modulus (SMM) variate.<sup>5</sup> The five percent SMM critical value for deciles is 2.80

The comparison of Lorenz curve (quantile function, GL curve) confidence bands allows three possible outcomes. First, if the confidence bands overlap over the entire range the Lorenz curves (quantile functions, GL curves) are not significantly different and are ranked as equal. Second, if the Lorenz curves (quantile functions, GL curves) are not equal but intersect, the Lorenz curves (quantile function, GL curves) "cross" and are noncomparable. Finally, if two Lorenz curves (quantile functions, GL curves) neither cross nor are equal, then a dominance relation exists.<sup>6</sup>

#### IV. EMPIRICAL RESULTS

Tables 3, 4 and 5 provide the necessary information to apply Lorenz dominance (LD), rank dominance (FSD) and GL dominance (SSD) to the five countries in each time period and across time. Table 3 contains the Lorenz ordinates and standard errors used to construct the Lorenz curves and confidence bands. Table 4 contains the decile means and the standard errors used to construct the quantile function confidence bands. Table 5 provides the decile GL ordinates and standard errors and can be used to construct generalized Lorenz curves and confidence bands.

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5. Miller (1981) discusses simultaneous inference and the SMM distribution. Tables for the percentiles of the SMM distribution are provided by Stoline and Ury (1979).

6. Alternatively, a partial order of the Lorenz curves (quantile functions, GL curves) can be constructed using pairwise tests of the conditional means or GL ordinates. For an example of this type of test procedure see Bishop, Formby and Thistle (1989).

The basic tools for our analysis of changes in European income distributions are the Lorenz ordinates, the conditional means, and the generalized Lorenz ordinates. Table 3 presents the circa 1980 and circa 1986 decile Lorenz ordinates for each country. For example, for France circa 1980, the first entry is 0.0270 and the standard error is 0.0019. This means that the bottom ten percent of French families (on a percapita basis) received 2.7 percent of the total income. The last column provides the share of the top ten percent of families. For France circa 1980 the top ten percent of families received 25.99 percent of total income ( $1 - 0.7401$ ), which is the largest of any country for either time period.

In a manner similar to Table 3, Tables 4 and 5 present the decile conditional means and generalized Lorenz ordinates. Each of the decile means and generalized Lorenz ordinates is presented in 1985 US dollars to allow across time and across country comparisons. As discussed above, the decile means are used to make first degree stochastic dominance (FSD) comparisons and generalized Lorenz ordinates are used to make second degree stochastic dominance (SSD) comparisons.

In Table 4, the first entry (France, circa 1980) provides the percapita income of the bottom ten percent of French families (\$2998). The last entry of the first row provides the percapita income top ten percent of French families (\$28911). The first entry in Table 5 is the first decile French generalized Lorenz ordinate, which is simply the overall mean from Table 2 times the cumulative share of income from Table 3 ( $\$300 = \$11122 * 0.0270$ ). The last column of generalized Lorenz ordinates is simply the overall mean.

Table 6 presents the results of the Lorenz and stochastic dominance comparisons for each country individually across time. Table 6a provides an example of the test procedure for the UK while Table 6b provides the Lorenz and stochastic dominance comparisons for all of the countries over time. Columns 1-3 of Table 6a present the British Lorenz ordinates and their test statistics. The Lorenz test statistics are large and negative, implying that the UK circa 1980 incomes are more equally distributed than the UK circa 1986 incomes. Columns 4-6 show the British decile means which are used to approximate the first degree stochastic dominance test. The comparison of the UK decile means from circa 1980 to circa 1986 shows that there is a statistically significant crossing. This crossing implies that no unambiguous conclusion can be drawn concerning Paretian social welfare.

Furthermore, as the crossing occurs at the bottom of the distribution, there is no higher order of stochastic dominance which can rank these two distributions.

The UK results and those for the other countries considered are summarized in Table 6b. We employ the following notation to summarize the changes in the size and dispersion of incomes between circa 1980 and circa 1986. "0" implies no significant difference over time, "X" implies an statistically significant crossing, LD implies circa 1986 Lorenz dominates circa 1980, [LD] implies circa 1980 Lorenz dominates circa 1986, and FSD implies the first degree dominance of circa 1986 over circa 1980. Thus, the "[LD]" for the UK in column 1 of Table 6b represents the finding of increased income inequality between circa 1980 and circa 1986, while the "X" in column 2 implies that the two time periods can not be ranked with stochastic dominance. For France, a "0" in columns 1 and 2 imply that neither the inequality or the size of the decile incomes changed over this time period. Like France, Germany shows no change in income inequality over time, however, rising German incomes result in circa 1986 first degree dominating circa 1980. For the Netherlands, declining inequality of incomes is accompanied by an increase in each of the decile percapita incomes. Finally, Sweden experienced increasing income inequality, together with increases in the decile percapita incomes.

An alternative method for evaluating the changes in European income distributions over time is to make pairwise comparisons at the two points in time and observe if there are any movements in the rankings. Table 7 provides the pairwise Lorenz comparisons for circa 1980 and circa 1986. Here "LD" signifies that the row Lorenz dominates the column at the first degree while "[LD]" signifies that the country in the column dominates the country in the row. It is important to note that in none of the twenty pairwise comparisons do the Lorenz curves cross, eliminating the need to construct summary measures of inequality.

Figure 1 presents a Hesse diagram of Lorenz ordering for both circa 1980 and 1986. For circa 1980 (Figure 1a), Sweden Lorenz dominates all other countries and France and the Netherlands are equivalent to each other and are Lorenz dominated by all other countries. For circa 1986, the most pronounced change is for the UK, which is now Lorenz dominated by all countries except France, to which it is equivalent in terms of inequality. In addition, France is now Lorenz dominated by the Netherlands.

Overall, Figure 1 provides some evidence of converging Lorenz curves. First, consider France, which Table 6 shows had no significant change in income inequality over this time period. For circa 1980, France is dominated by the UK and is not significantly different from the Netherlands. However, by circa 1986 France and the UK are not significantly different and France is dominated by the Netherlands. Finally, while Sweden and Germany continue to dominate all other countries in the late period, Table 6 shows that Sweden experienced an increase in income inequality and Germany no change in income inequality between the two time periods.

Table 8 presents the pairwise stochastic dominance comparisons. Here FSD signifies that the row dominates the column at the first degree while [FSD] signifies that the country in the column dominates the country in the row. SSD signifies a first degree "crossing" together with the second degree dominance of the country in the row over the country in the column. Finally, a "X" indicates both a first and second degree "crossing."

Figure 2 provides a Hesse diagram of the stochastic dominance ordering. A comparison of the rankings in the two time periods provides little evidence of convergence. The most easily identifiable change is the relative position of the UK which in the later period is dominated by all countries at the first degree.

## V. CONCLUSIONS

Application of theoretical and statistical dominance methods to the internationally comparable Luxembourg Income Study (LIS) database to five European income distributions yields comparable measures of inequality and levels of welfare in the late 1970s through the mid 1980s. The evidence on inequality indicates that Lorenz curves became significantly more unequal in Sweden and the U.K., did not change significantly in Germany and France and became more equal in the Netherlands. Even with the rise in inequality Sweden continued to dominate each of the other countries in the circa 1986 comparison. But the changes in Britain and The Netherlands resulted in a reversal of their positions in the Lorenz ordering in 1986 compared to 1980. Overall, the evidence suggests there has been some convergence of Lorenz curves. However, in terms of stochastic dominance we find no evidence of convergence at either the first or second degree. Thus, as of the mid 1980s European economic integration <sup>has not</sup> resulted in a convergence of standards of living as measured by rank and generalized Lorenz dominance.

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**Table 1**  
**LIS Data Sets**

<b>Country</b>	<b>Year</b>	<b>Data Source</b>	<b>Sample Size</b>
France	1979	Survey of Individual Income Tax Returns	11044
France	1984	Survey of Individual Income Tax Returns	12693
Netherlands	1983	Survey of Income and Program Users	4833
Netherlands	1987	Survey of Income and Program Users	6771
Sweden	1981	Income Distribution Survey	9625
Sweden	1987	Income Distribution Survey	9421
Germany	1981	Transfer Survey	2727
Germany	1984	Panel Survey: Wave II	5174
United Kingdom	1979	Family Expenditure Survey	6888
United Kingdom	1986	Family Expenditure Survey	7178



**Table 2**  
**Per capita GDP\***  
**(\$ 1985)**

	Circa 1980	Circa 1986	Percentage Change
France	11122	11623	4.5
Netherlands	10595	11110	4.9
Sweden	10908	12494	14.5
Germany	10959	11909	8.7
United Kingdom	9713	11058	13.8

\*Data Source: Summers and Heston (1990). The circa 1980 values are the average of the 1979, 1980, and 1981 GDP's. Circa 1986 is the average of the 1985, 1986, and 1987 GDP's.

**Table 3**  
**Per capita Income Lorenz Ordinates**

	Deciles								
	1	2	3	4	5	6	7	8	9
<b>A. Circa 1980</b>									
France	0.0270 (0.0019)	0.0734 (0.0030)	0.1310 (0.0043)	0.1980 (0.0056)	0.2753 (0.0070)	0.3645 (0.0084)	0.4674 (0.0099)	0.5887 (0.0112)	0.7401 (0.0122)
Netherlands	0.0213 (0.0013)	0.0722 (0.0016)	0.1319 (0.0018)	0.2004 (0.0021)	0.2782 (0.0024)	0.3675 (0.0027)	0.4723 (0.0030)	0.5975 (0.0031)	0.7545 (0.0030)
Sweden	0.0365 (0.0028)	0.0971 (0.0027)	0.1682 (0.0025)	0.2481 (0.0024)	0.3365 (0.0022)	0.4354 (0.0021)	0.5461 (0.0018)	0.6712 (0.0015)	0.8164 (0.0012)
Germany	0.0376 (0.0008)	0.0911 (0.0013)	0.1540 (0.0018)	0.2255 (0.0023)	0.3062 (0.0028)	0.3970 (0.0032)	0.5022 (0.0036)	0.6239 (0.0038)	0.7727 (0.0037)
United Kingdom	0.0367 (0.0005)	0.0885 (0.0008)	0.1495 (0.0011)	0.2201 (0.0014)	0.3007 (0.0017)	0.3920 (0.0020)	0.4973 (0.0022)	0.6222 (0.0023)	0.7740 (0.0022)
<b>B. Circa 1986</b>									
France	0.0253 (0.0010)	0.0715 (0.0011)	0.1291 (0.0014)	0.1970 (0.0017)	0.2757 (0.0020)	0.3662 (0.0025)	0.4710 (0.0029)	0.5940 (0.0034)	0.7460 (0.0040)
Netherlands	0.0282 (0.0011)	0.0786 (0.0014)	0.1385 (0.0017)	0.2080 (0.0020)	0.2867 (0.0024)	0.3786 (0.0027)	0.4836 (0.0030)	0.6091 (0.0031)	0.7653 (0.0029)
Sweden	0.0332 (0.0015)	0.0936 (0.0016)	0.1643 (0.0017)	0.2432 (0.0018)	0.3306 (0.0019)	0.4272 (0.0021)	0.5366 (0.0023)	0.6619 (0.0025)	0.8065 (0.0028)
Germany	0.0374 (0.0006)	0.0897 (0.0011)	0.1525 (0.0016)	0.2249 (0.0022)	0.3068 (0.0028)	0.3993 (0.0034)	0.5048 (0.0041)	0.6271 (0.0048)	0.7742 (0.0056)
United Kingdom	0.0209 (0.0013)	0.0680 (0.0015)	0.1270 (0.0017)	0.1966 (0.0020)	0.2772 (0.0023)	0.3703 (0.0025)	0.4770 (0.0028)	0.6022 (0.0031)	0.7540 (0.0031)

Table 4

**Per capita Decile Mean Incomes\***  
(in 1985 US Dollars)

	Deciles Means									
	1	2	3	4	5	6	7	8	9	10
<b>A. Circa 1980</b>										
France	2998 (198)	5170 (230)	6398 (286)	7453 (333)	8601 (385)	9919 (445)	11445 (514)	13498 (609)	16830 (764)	28911 (1720)
Netherlands	2254 (142)	5399 (53)	6325 (53)	7254 (65)	8248 (66)	9459 (104)	11107 (120)	13260 (156)	16634 (203)	26013 (455)
Sweden	3982 (315)	6592 (49)	7754 (40)	8708 (44)	9631 (51)	10772 (58)	12061 (68)	13634 (75)	15814 (86)	20008 (141)
Germany	4114 (93)	5861 (75)	6889 (71)	7838 (90)	8835 (86)	9950 (120)	11515 (149)	13334 (174)	16296 (252)	24894 (569)
United Kingdom	3567 (51)	5028 (38)	5928 (44)	6852 (54)	7823 (55)	8869 (69)	10229 (87)	12128 (108)	14747 (136)	21943 (296)
<b>B. Circa 1986</b>										
France	2935 (115)	5378 (40)	6689 (47)	7891 (48)	9147 (56)	10518 (63)	12180 (76)	14297 (91)	17669 (133)	29520 (635)
Netherlands	3135 (125)	5605 (62)	6654 (66)	7724 (71)	8744 (91)	10217 (123)	11671 (104)	13948 (191)	17364 (205)	26086 (459)
Sweden	4142 (200)	7558 (54)	8829 (45)	9858 (49)	10914 (54)	12065 (64)	13673 (87)	15657 (93)	18065 (96)	24168 (436)
Germany	4452 (63)	6223 (62)	7480 (66)	8616 (72)	9748 (73)	11004 (105)	12566 (105)	14552 (139)	17518 (179)	26871 (879)
United Kingdom	2313 (152)	5203 (58)	6531 (58)	7688 (63)	8916 (75)	10294 (81)	11799 (92)	13839 (113)	16792 (164)	27199 (488)

\*The decile means are scaled to the average GDP's of Table 3.

Table 5

**Per capita Generalized Lorenz Ordinates\***  
(in 1985 US Dollars)

A. Circa 1980	Generalized Lorenz Ordinates									
	1	2	3	4	5	6	7	8	9	10
France	300 (20)	817 (29)	1457 (37)	2202 (45)	3062 (51)	4054 (57)	5198 (62)	6548 (67)	8231 (75)	11122 (160)
Netherlands	225 (14)	765 (18)	1398 (22)	2123 (26)	2948 (31)	3894 (38)	5005 (47)	6331 (58)	7994 (72)	10595 (99)
Sweden	398 (32)	1057 (33)	1833 (34)	2704 (35)	3667 (37)	4744 (40)	5950 (43)	7313 (47)	8895 (51)	10908 (56)
Germany	411 (9)	998 (15)	1687 (20)	2470 (27)	3354 (34)	4349 (43)	5500 (54)	6834 (67)	8463 (84)	10959 (119)
United Kingdom	357 (5)	859 (8)	1452 (11)	2137 (16)	2920 (20)	3807 (25)	4830 (32)	6042 (40)	7517 (49)	9713 (67)
B. Circa 1986										
France	293 (11)	831 (13)	1500 (16)	2289 (19)	3204 (23)	4256 (27)	5474 (32)	6903 (39)	8670 (48)	11623 (88)
Netherlands	314 (13)	874 (17)	1539 (21)	2312 (27)	3186 (33)	4208 (42)	5375 (50)	6770 (64)	8506 (78)	11110 (105)
Sweden	414 (20)	1170 (22)	2053 (25)	3039 (27)	4130 (30)	5337 (34)	6704 (39)	8270 (45)	10076 (51)	12494 (72)
Germany	445 (6)	1068 (11)	1816 (17)	2677 (22)	3652 (28)	4752 (36)	6009 (44)	7464 (54)	9216 (66)	11909 (120)
United Kingdom	231 (15)	752 (18)	1405 (22)	2173 (26)	3065 (31)	4094 (37)	5274 (43)	6658 (50)	8337 (61)	11058 (90)

\*The means are scaled to the average GDP's of Table 3.

Table 6

Lorenz and Stochastic Dominance Over Time

A. United Kingdom: Circa 1980 and 1986						
Decile				Decile Means		
	UK 1980	UK 1986	Test Statistic <sup>a</sup>	UK 1980	UK 1986	Test Statistic
1	0.0367	0.0209	-11.12	3567	2313	-7.84
2	0.0885	0.0680	-11.95	5028	5203	2.51
3	0.1495	0.1270	-10.97	5928	6531	8.27
4	0.2201	0.1966	-9.69	6852	7688	10.02
5	0.3007	0.2772	-8.29	7823	8916	11.75
6	0.3920	0.3703	-6.74	8869	10294	13.39
7	0.4973	0.4770	-5.70	10229	11799	12.42
8	0.6222	0.6022	-5.23	12128	13839	10.95
9	0.7740	0.7540	-5.23	14747	16792	9.63
10	1.0000	1.0000	--	21943	27199	9.20

<sup>a</sup>The SMM critical value is 2.77 for k=9 and 2.80 for k=10.

B. Comparisons Over Time <sup>b</sup>		
	Lorenz Dominance	Stochastic Dominance
France	0	0
Netherlands	LD	FSD
Sweden	[LD]	FSD
Germany	0	FSD
UK	[LD]	X

<sup>b</sup>LD implies Circa 1986 Lorenz dominates Circa 1980, [LD] implies the converse, and "0" implies no significant difference. FSD implies first degree dominance over time. "X" implies that the two distributions cannot be ranked at the first or second degree.

Table 7

Pairwise Lorenz Dominance Comparisons<sup>a</sup>  
(Per capita Incomes)

A. Circa 1980

	France	Netherlands	Sweden	Germany
France	*			
Netherlands	0	*		
Sweden	LD	LD	*	
Germany	LD	LD	[LD]	*
United Kingdom	LD	LD	[LD]	[LD]

B. Circa 1986

	France	Netherlands	Sweden	Germany
France	*			
Netherlands	LD	*		
Sweden	LD	LD	*	
Germany	LD	LD	[LD]	*
United Kingdom	0	[LD]	[LD]	[LD]

<sup>a</sup>LD implies that the row Lorenz dominates the column. [LD] implies that the column dominates a row. A "0" implies no significant difference.

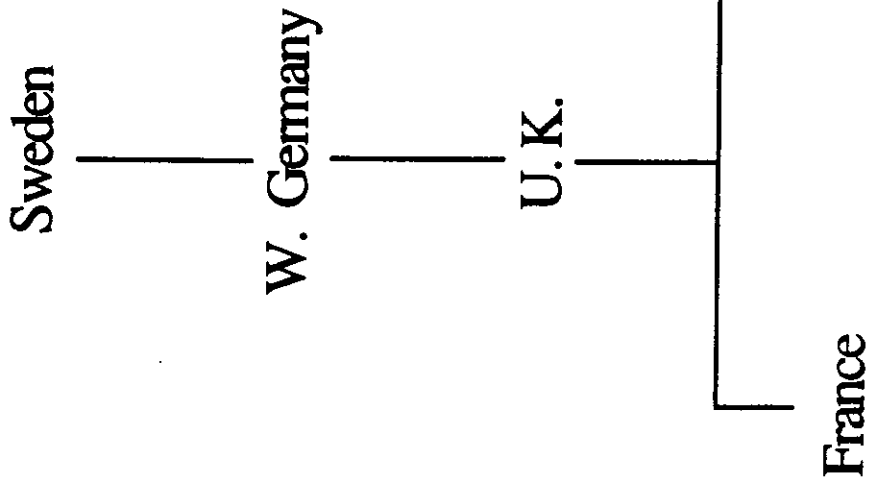
**Table 8**

**Pairwise Stochastic Dominance Comparisons  
(Per capita Incomes)<sup>a</sup>**

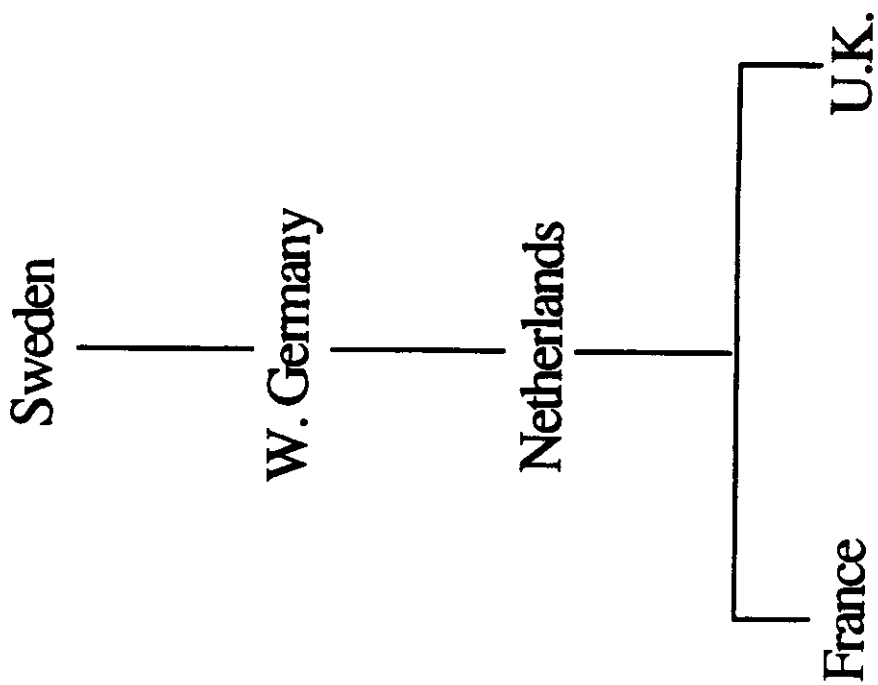
<b>A. Circa 1980</b>				
	<b>France</b>	<b>Netherlands</b>	<b>Sweden</b>	<b>Germany</b>
<b>France</b>	*			
<b>Netherlands</b>	[FSD]	*		
<b>Sweden</b>	SSD	SSD	*	
<b>Germany</b>	FSD	FSD	[SSD]	*
<b>United Kingdom</b>	X	X	[SSD]	[FSD]
<b>B. Circa 1986</b>				
	<b>France</b>	<b>Netherlands</b>	<b>Sweden</b>	<b>Germany</b>
<b>France</b>	*			
<b>Netherlands</b>	[SSD]	*		
<b>Sweden</b>	SSD	SSD	*	
<b>Germany</b>	FSD	FSD	[FSD]	*
<b>United Kingdom</b>	[FSD]	[FSD]	[SSD]	[FSD]
<p><sup>a</sup>FSD implies that the row dominates the column at the first degree.            [FSD] implies that the column dominates the row. SSD denotes second degree stochastic dominance.</p>				

# Figure 1 Lorenz Dominance

a. Circa 1980



b. Circa 1986

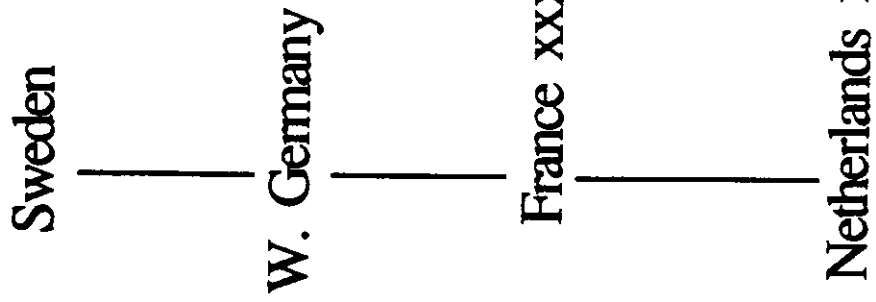


Note: Countries on the same level indicate that the null hypothesis of no difference cannot be rejected.

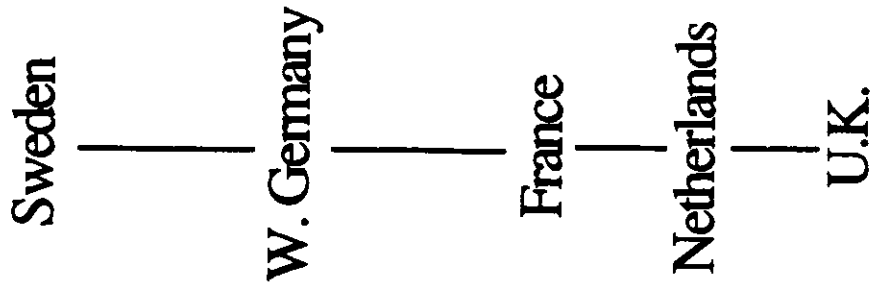


# Figure 2 Stochastic Dominance

a. Circa 1980



b. Circa 1986



Note: x's indicate statistically significant crossings.