Bishop, John A.; Formby, M. John P.; Smith, W. James

Working Paper

International Comparisons of Welfare and Poverty: Generalized Lorenz Orderings for Ten Countries

LIS Working Paper Series, No. 45

Provided in Cooperation with:
Luxembourg Income Study (LIS)

Suggested Citation: Bishop, John A.; Formby, M. John P.; Smith, W. James (1990) : International Comparisons of Welfare and Poverty: Generalized Lorenz Orderings for Ten Countries, LIS Working Paper Series, No. 45, Luxembourg Income Study (LIS), Luxembourg

This Version is available at:
https://hdl.handle.net/10419/160717

Luxembourg Income Study Working Paper Series

Working Paper No. 45

International Comparisons of Welfare and Poverty: Generalized Lorenz Orderings for Ten Countries

John Bishop, John Formby and James Smith

February 1990

(scanned copy)

Luxembourg Income Study (LIS), asbl

International Comparisons of Welfare and Poverty: Generalized Lorenz Orderings For Ten Countries

John A. Bishop
Assistant Professor of Economics, East Carolina University,

John P. Formby
Hayes Professor of Economics, University of Alabama

W. James Smith
Professor of Economics, University of Colorado at Denver

Preliminary: Not for citation or quotation

International Comparisons of Welfare and Poverty: Generalized Lorenz Orderings For Ten Countries

John A. Bishop, John P. Formby, and W. James Smith

I. Introduction

In the last two decades, applied welfare economists have made significant advances in understanding the welfare implications of the distribution of income and inequality. The work of Atkinson (1970) stands as a seminal piece in this literature. Atkinson demonstrates that if Lorenz curves with identical mean incomes do not intersect then strong inferences can be made about comparative states of economic welfare. This theorem and its elegant simplicity have given rise to a number of studies, both theoretical and empirical, which extend the concept of dominance beyond Lorenz curves to compare distributions with unequal levels of income as well as differing derees of inequality.

The question of differing income levels has recently been the subject of important contributions by Anthony Shorrocks (1983) and Nanak Kakwani (1984). Shorrocks demonstrates that the original Atkinson theorem can be easily extended to distributions with different levels of income. Specifically, for two distributions with differing mean incomes and Lorenz curves, if the Lorenz curve of distribution one scaled by its mean dominates the second Lorenz curve similarly scaled by its mean, then the first is preferred by all Shur-concave welfare functions which are both efficiency and equality preferring. The new analytical constructs are appropriately referred to as generalized Lorenz curves. In practical application, then, one need only compare Lorenz curves

of inequality correspond to some specific concept of social welfare. As is now widely accepted, the underlying social welfare function should be made explicit and therefore subject to positive analysis and critique. The modern, seminal example of this approach is Anthony Atkinson's famous theorem on Lorenz dominance. The theorem, as extended by Dasgupta, Sen and Starrett (1973), states that, for distributions with equal means, distribution X will be preferred to distribution Y by all increasing, S-concave social welfare functions if, and only if, the Lorenz curve of X lies everywhere above the Lorenz curve for distribution Y, i.e., distribution X Lorenz dominates distribution Y.

In empirical analysis, the assumption of equal means is rarely satisfied and routinely violated. The consequences are serious, not only because such applications represent invalid use of the Atkinson theorem, but because, they ignore the important welfare effects of standard of living. Fortunately, the incorporation of mean income in Lorenz dominance is straightforward as Shorrocks (1983) and Kakwani (1984) recently show. Shorrocks and Kakwani formulate what is now referred to as generalized Lorenz dominance which, in a manner similar to Atkinson, can be used to establish welfare orderings but with standards of living incorporated.

Formally, generalized Lorenz dominance can be summarized by first defining X as an income vector ordered in terms of increasing incomes and p as a set of fractions between 0 and 1. Adapting Gastwirth's (1971) definition of the Lorenz curve we can write the GL curve as:

$$G_X(p) = \int_0^p x(u)du = \mu_X L_X(p), \qquad (1)$$

where $L_X(p)$ is the ordinary Lorenz ordinate and $G_X(1) = \mu_X$. Income vector $\mathbf{X}$ generalized Lorenz dominates $\mathbf{Y}$, denoted $\mathbf{X} >_{GL} \mathbf{Y}$, if, and only, if $G_X(p) \geq GY(p)$ for all $p \in I$, with at least one strict inequality at some p.

We denote the class of anonymous, increasing, S-concave welfare functions as W_E. Shorrocks (1983) proves the following theorem on GL dominance:

Theorem 1: $\mathbf{X} >_{GL} \mathbf{Y}$ iff $w(\mathbf{X}) > w(\mathbf{Y})$ for all $w \in W_E$.

Thus, GL curves can be compared in essentially the same manner as ordinary Lorenz curves. The only difference is that the ordinary Lorenz curve in Atkinson's analysis is replaced by a GL curve, which is simply the ordinary Lorenz curve scaled by mean income. Like ordinary Lorenz dominance, the GL criterion provides only a partial ordering because generalized Lorenz curves which cross cannot be ranked.

Generalized Lorenz dominance also carries important implications for the analysis of poverty as Foster and Shorrocks (1988) demonstrate. The interesting result, pertinent to our purposes, is contained in the following corollary to Theorem 1 which links the GL criterion to the income gap poverty, defined as the weighted sum of the income shortfalls of the poor, or,

$$P(\mathbf{X};\alpha) = (1/(n(x)\alpha)) \sum_{i=1}^{r} (x_i),$$

where αÁ is the poverty line income and r is its corresponding order statistic. For any given α, the income gap criterion is $X >_{P(\alpha)} Y$, iff, $(1/n)\Sigma (x_i) > (1/n)\Sigma (y_i)$, for all i up through r.

Corollary 1: $X >_{GL} Y$ iff $X >_{P(\alpha)} Y$ for all α.[1]

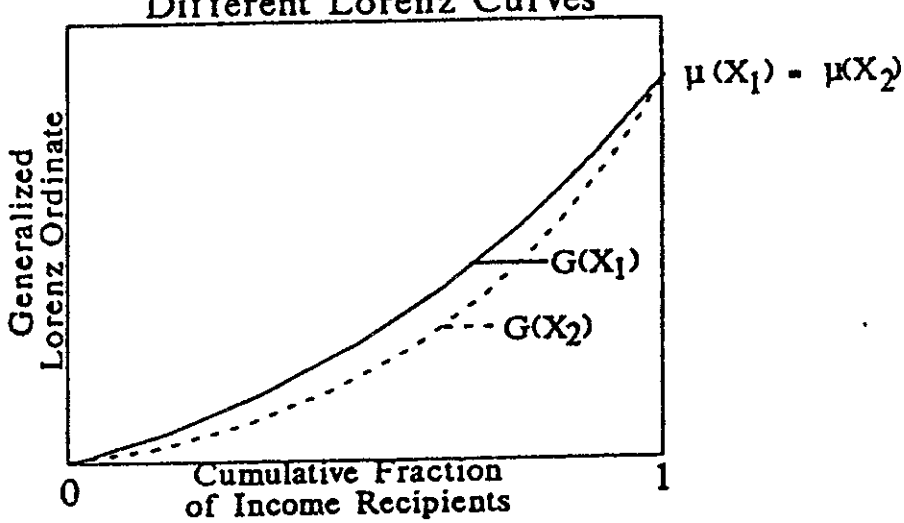
The corollary implies an unambiguous decline in income gap poverty whenever generalized Lorenz dominance is present. Conversely, if distribution X generalized Lorenz dominates distribution Y, then income gap poverty in X cannot exceed that of Y, regardless of the income cutoff, α, used. Thus, truncating the distribution above any arbitrary poverty line, α, and testing for GL dominance on the truncated portion of the distribution provides an ordering in terms of income gap poverty. A strong relationship thus exists between poverty as measured by a cutoff level of income and generalized Lorenz dominance.

These concepts are illustrated in Figures 1 and 2. Figure 1 depicts GL dominance and graphically illustrates the GL criterion's preference for both efficiency and equality. Consider Figure 1a which shows two GL curves, $G(X_1)$ and $G(X_2)$ corresponding to hypothetical income distributions X_1 and X_2 with identical means but differing distributions of income. In the figure, income is measured on the ordinate with cumulative proportion of population on the abscissa. The endpoints of the generalized Lorenz curves are mean incomes, $\mu(X_1)$ and $\mu(X_2)$. In this example, X_1's greater equality in distribution, as reflected in the curvature of the generalized curve, results in X_1 GL dominating X_2.[2] A contrasting situation is shown in the next panel which depicts GL curves for

Figure 1

Generalized Lorenz Dominance

1a. Equal Means and Different Lorenz Curves



1b. Equivalent Lorenz Curves and Different Means

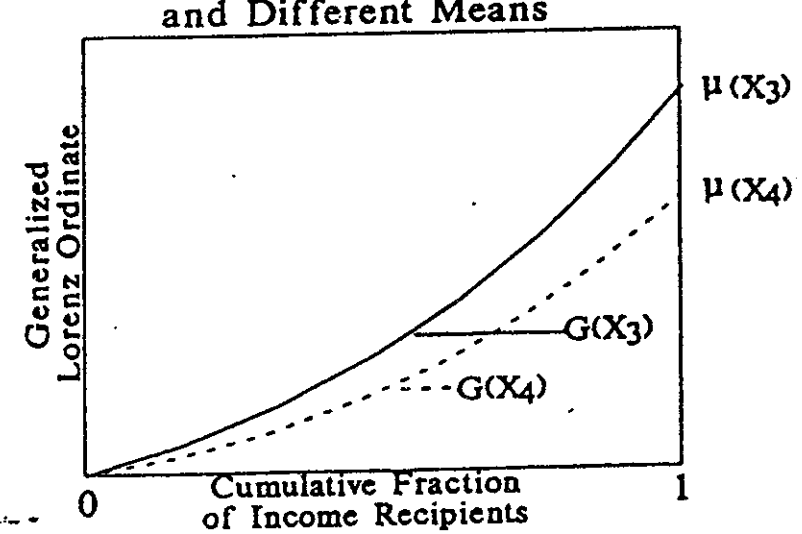
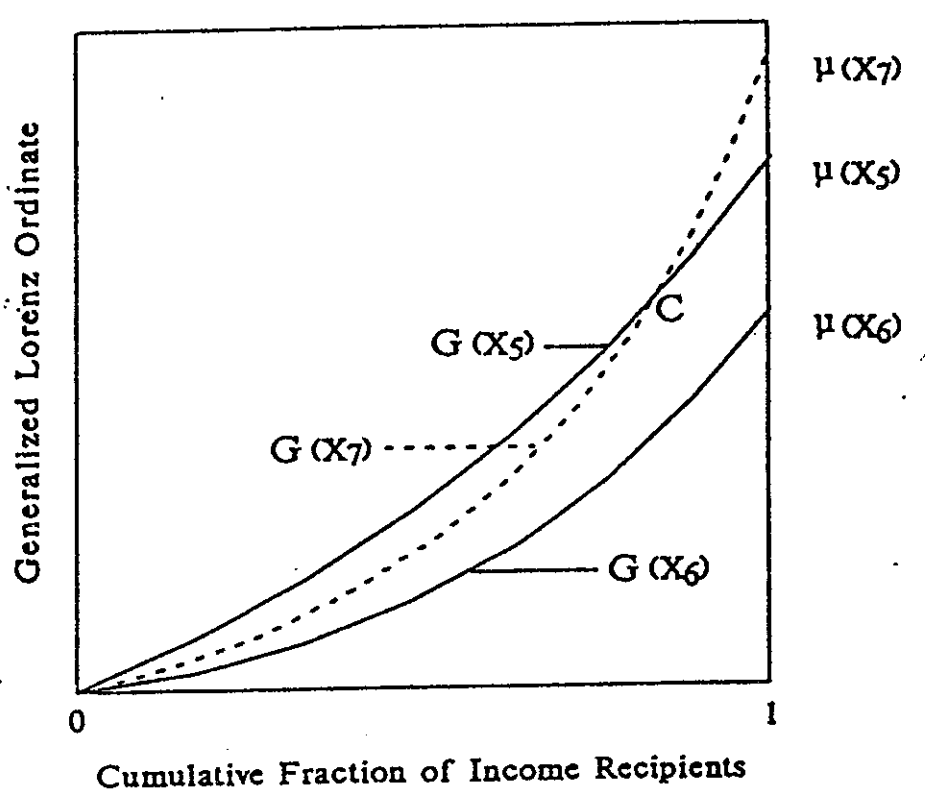


Figure 2

Generalized Lorenz Curves for Three Hypothetical Income Income Distributions -- x_5, x_6 and x_7



two other distributions X_3 and X_4 possessing the same income distributions, but different mean incomes. The X_3 distribution has the greater mean income and dominates X_4.

In most applications both the Lorenz curves and means will differ. This case is illustrated in Figure 2, which depicts three generalized Lorenz curves, $G(X_5)$, $G(X_6)$ and $G(X_7)$, corresponding to hypothetical income distributions X_5, X_6, and X_7. In this example, the GL criterion ranks X_5 above X_6. In contrast, $G(X_5)$ and $G(X_7)$ cross and the conditions of Theorem 1 are not satisfied. These latter two income distributions cannot be ranked using GL dominance. Even in the case of a crossing however, it may be possible to reach important conclusions about poverty. For example, in the case above, if the poverty line is less than or equal to C (the point of intersection), we can conclude that income gap poverty is unambiguously less in X_5 than X_7. This is the case because $G(X_5)$ dominates $G(X_7)$ at all cumulative income levels below C.

III. Statistical Inference

The basis for statistical analysis is the sample GL curve. A sample GL curve is constructed from a vector of k sample GL ordinates, G, where $G_0=0$, $G_i=\mu L_i$, and $G_K=\mu$. To test for differences in GL curves we compare the confidence bands around these sample curves. In constructing these bands we employ results from Beach and Davidson (1983), Beach and Richmond (1985), and Bishop, Chakraborti, and Thistle (1989).

Beach and Davidson (1983) demonstrate the asymptotic normality of $n^{1/2}$ $(G - \hat{G})$ and formulate the dispersion matrix, W, for the

vector of ordinates. Using these results, a 95 percent confidence interval around the ith sample ordinate can be written as:

$$G_i = \hat{G}_i \pm z_{.025}(W_i/n) \qquad (2)$$

for any i = 1,2,...,k. Because we are constructing a confidence band for the entire curve using k ordinates, we draw inferences from the union of k disjoint sub-hypotheses, necessitating the use of simultaneous inference procedures. Following Beach and Richmond's procedure for ordinary Lorenz curves, we construct a joint confidence band around a GL curve by replacing the standard normal distribution in equation (2) with the Studentized Maximum Modulus (SMM) variate.[3]

The comparison of confidence bands has three possible outcomes. First, if the confidence bands overlap over the entire range, the GL curves are not significantly different and are ranked as equal. Second, if the GL curves are not equal but intersect, they are said to "cross" and are noncomparable. Finally, if two curves neither cross nor are equal, then a dominance relation exists. This procedure can be implemented graphically by plotting confidence bands and noting dominance, equality, and crossings.[4]

IV. Empirical Analysis

The LIS data set contains national survey data for 13 countries, corrected for definitional differences in income and income recipient units. This paper analyzes the data from the ten national surveys available for the period circa 1980.[5] Table 1 lists the countries included, the survey coverage, the year

TABLE 1

Overview of LIS Data Sets

Country	Data Set	Sample Size*
Australia	Income and Housing Survey, 1981-82	15,985
Canada	Survey of Consumer Finances, 1981	15,136
France	Family and Professional Life Survey, 1981	3,639
Netherlands	Survey of Income of Program, 1983	4,833
Norway	Norwegian Tax Files, 1979	10,414
Sweden	Swedish Income Distribution Survey, 1981	9,625
Switzerland	Income and Wealth Survey, 1982	7,036
United Kingdom	Family Expenditure Survey, 1979	6,888
United States	Current Population Survey, 1979	15,225
West Germany	Transfer Survey, 1981	2,727

*Number of families in data set.

TABLE 2

Percapita Mean Incomes

Country	GDP $1979	LIS	Adjustment Factor	US $ 1979	Gini
Australia	8152	5542	0.70	3879*	0.3156
Canada	11358	8099	0.67	5428	0.3190
France	9625	23016	0.20	4598	0.3092
Netherlands	8964	11342	0.38	4309	0.3311
Norway	10708	27143	0.19	5156	0.2689
Sweden	8750	34268	0.12	4112	0.2289
Switzerland	9628	19405	0.24	4656	0.3346
West Germany	9619	14549	0.32	4554	0.3355
United Kingdom	8094	2033	1.94	3946	0.2838
United States	11602	5543	1.00	5543	0.3461

*The calculation procedure was as follows:

3879 = [8152/11602]+5543 = [AUSGDP/USGDP]+USLISPC

surveyed, the LIS sample sizes, and data sources. A detailed description of the data is provided in Buhmann et al. (1988).

The income concept utilized in this paper is net cash income, defined as market income plus private and public transfers minus direct (income and payroll) taxes. The recipient unit is the per capita family. In this case, for example, a family of four with net cash income of $40,000 is reported as four incomes of $10,000 each. The data are weighted by the LIS person weights. Differences in the years surveyed (1979 to 1982) also require adjustment of the level of incomes in the LIS data set. In addition, it is necessary to convert various mean incomes of different countries into a single currency. We standardize the LIS data set using the per capita-gross-domestic-product estimates of Summers and Heston (1988).[6] Table 2 provides the per capita-GDP figures, the raw LIS per capita family mean incomes, the adjustment factor, and the LIS mean incomes in 1979 U.S. dollars. Adjusted per capita mean incomes range from $5543 in the United States to $3860 in the United Kingdom. Table 2 also provides Gini coefficients to allow convenient comparison of the degree of income inequality across countries.[7]

A. Generalized Lorenz Dominance

The information necessary to rank GL curves is provided in Table 3 which reports decile GL ordinates and their corresponding standard errors for the ten LIS countries considered.[8] Sample sizes used to construct the confidence bands are the number of primary observations reported in Table 1. The critical value of

TABLE 3

GENERALIZED LORENZ ORDINATES AND STANDARD ERRORS FOR PERCAPITA INCOMES

DECILE	AUSTRALIA	CANADA	FRANCE	NETHER-LANDS	NORWAY	SWEDEN	SWITZER-LAND	WEST GERMANY	UNITED KINGDOM	UNITED STATES
1	101 (1.7)	137 (2.7)	134 (6.3)	98 (4.4)	177 (9.5)	150 (11.9)	108 (15.6)	162 (3.4)	145 (2.1)	97 (5.4)
2	286 (2.9)	388 (4.2)	353 (10.7)	300 (6.2)	469 (10.1)	399 (12.3)	326 (16.3)	390 (5.6)	349 (2.9)	319 (4.4)
3	515 (3.8)	706 (5.7)	617 (14.0)	545 (7.9)	809 (10.8)	692 (12.7)	587 (16.9)	656 (7.8)	590 (4.1)	620 (5.7)
4	781 (4.8)	1076 (7.2)	929 (16.9)	831 (10.1)	1193 (11.6)	1020 (13.3)	893 (17.7)	958 (10.5)	868 (5.6)	991 (7.3)
5	1083 (5.9)	1505 (9.0)	1295 (19.3)	1162 (12.2)	1619 (12.5)	1384 (14.0)	1247 (18.9)	1299 (13.0)	1186 (7.0)	1429 (8.9)
6	1431 (7.4)	1998 (10.9)	1723 (21.3)	1544 (15.5)	2093 (13.9)	1790 (14.9)	1661 (20.6)	1682 (16.5)	1547 (8.8)	1941 (10.6)
7	1843 (9.4)	2572 (13.5)	2221 (22.7)	1999 (19.4)	2630 (15.8)	2245 (16.2)	2147 (23.1)	2126 (21.0)	1962 (11.0)	2543 (12.7)
8	2338 (11.7)	3256 (16.8)	2806 (23.4)	2545 (24.2)	3256 (18.4)	2760 (17.6)	2733 (26.3)	2641 (25.9)	2455 (13.6)	3258 (15.1)
9	2958 (14.6)	4106 (20.6)	3514 (24.0)	3232 (30.1)	4023 (22.2)	3357 (19.2)	3460 (30.2)	3268 (32.6)	3054 (16.4)	4150 (18.5)
10	3879 (19.1)	5428 (28.7)	4602 (43.0)	4309 (41.5)	5156 (36.2)	4112 (21.2)	4656 (56.9)	4554 (451.8)	3946 (22.3)	5543 (24.9)

the SMM statistic at the five percent level of significance as calculated for deciles is 2.77.

The test procedure using 95 percent GL confidence bands is shown in Figure 3 for Canada and Australia. Canada's GL confidence band lies uniformly above that of Australia. We therefore reject the hypotheses of equivalence or crossing and conclude that Canada GL dominates Australia. In Figure 4, however, Norway's band lies above that of the U.S. from the origin to the seventh decile, but overlaps the U.S. band at the eighth decile, and lies below the U.S. band at the ninth and tenth deciles. Thus, the GL curves of the U.S. and Norway cross and the GL criterion is unable to rank the two countries in terms of relative economic well-being.

A summary of the results of the tests for GL dominance are shown in Table 4. A "+" means a country in a row dominates a country in a column. A "-" denotes that a country in a row is dominated by a country in a column. A "X" means that a sgnificant crossing exists. A "O" indicates the null hypothesis of no significant difference between GL curves cannot be rejected. For example, row 2, column 1 shows that Canada dominates Australia, while row 10, column 4 shows that the U.S. and Norway cannot be ranked. The "O" in row 7, column 4 indicates that there is no difference between the GL curves of Switzerland and France. Of the 45 pairwise GL comparisons, 32 (71.1 percent) can be ranked, while 13 result in crossings.[9]

A partial overall ranking can be described by considering consecutive pairwise dominance relations. It is important to stress that this is a more stringent undertaking especially as the

Figure 3

Generalized Lorenz Confidence Bands

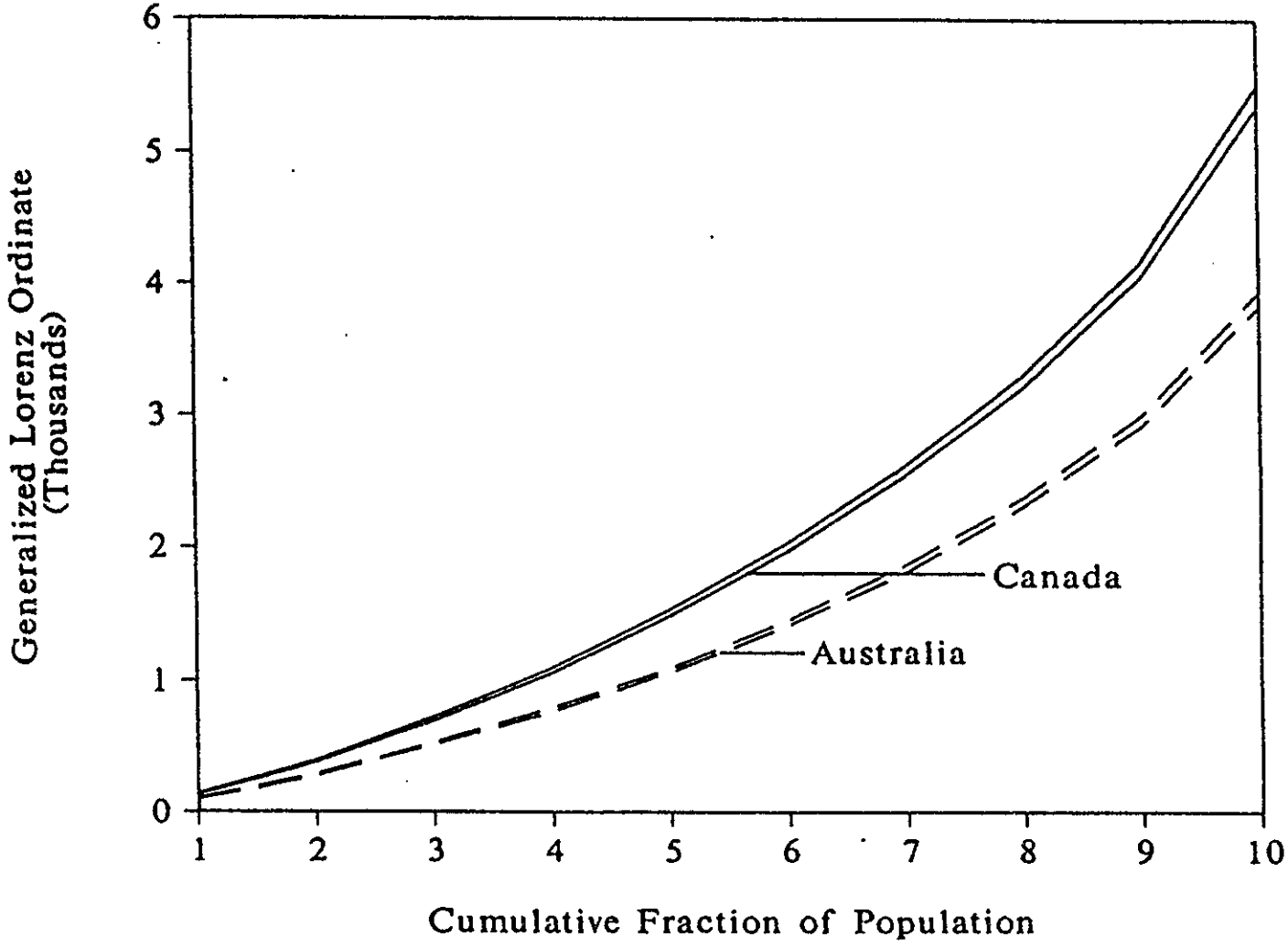


Figure 4

Generalized Lorenz Confidence Bands

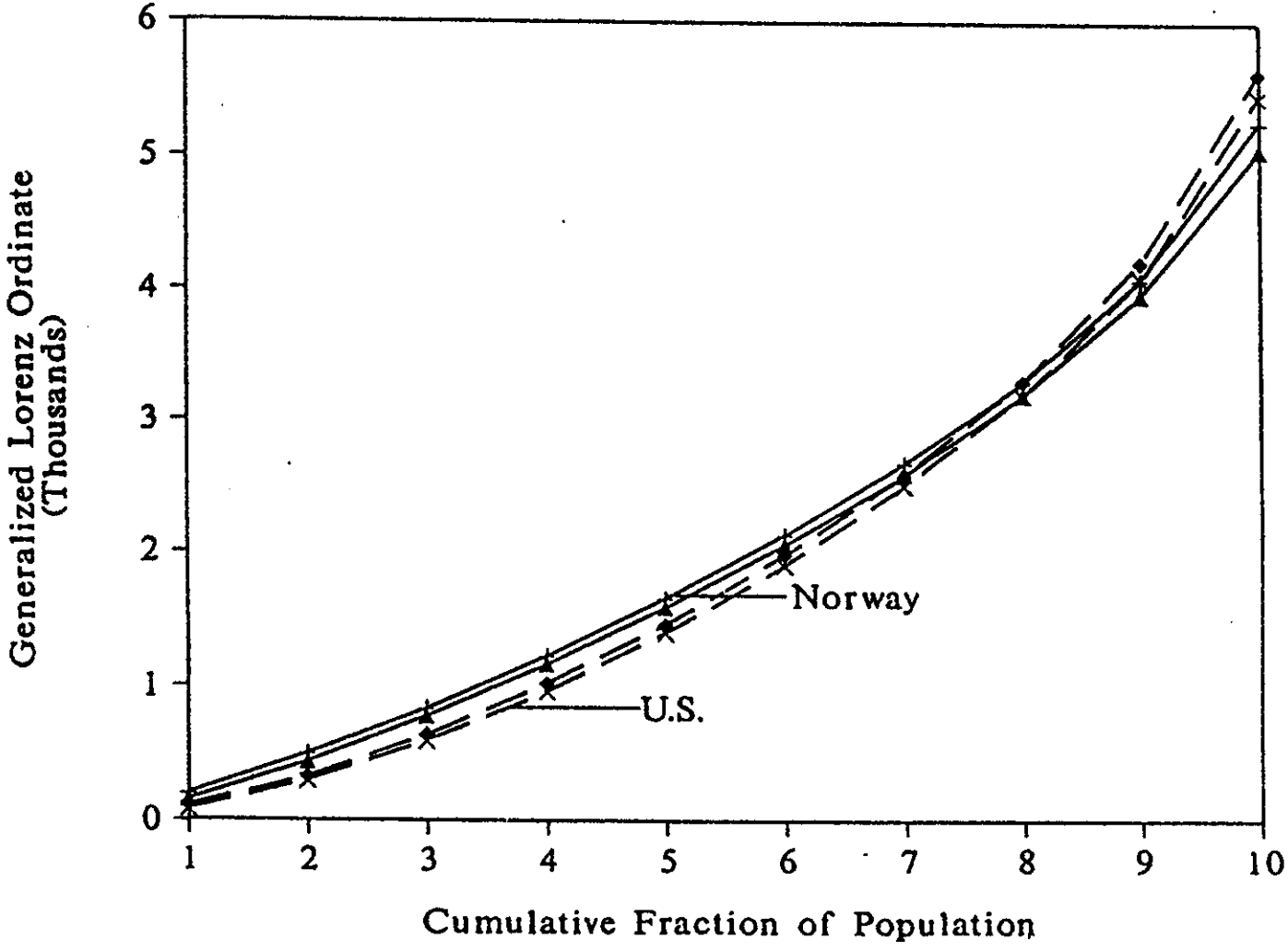


TABLE 4

Percapita GL Dominance

	Australia	Canada	France	Netherlands	Norway	Sweden	Switzerland	West Germany	United Kingdom	United States
Australia	*									
Canada	+	*								
France	+	-	*							
Netherlands	+	-	-	*						
Norway	+	X	+	+	*					
Sweden	+	-	X	X	-	*				
Switzerland	+	-	O	+	-	X	*			
West Germany	+	X	X	+	-	-	X	*		
United Kingdom	+	-	+	X	-	-	-	-	*	
United States	+	X	X	+	X	X	+	X	X	*

NOTE:

A + means the country in the row dominates the country in the column.

A - means the country in the row is dominated by the country in the column.

An X means the distributions cross.

A O means the distributions are equivalent.

number of countries compared increases. The reason, of course, is that crossings interrupt strings of dominance relations.[10] The information in Table 4 can be summarized as follows. Australia is ranked below the other countries primarily because of its low mean income and relatively moderate degree of income inequality. The Netherlands also possesses relatively low incomes and moderate income inequality and is ranked just above Australia. At the other extreme, both Canada and Norway enjoy relatively high incomes as well as relatively moderate to low degrees of income inequality and dominate almost all other countries. However, Canada and Norway cannot be ranked relative to one another due to intersecting GL curves. Canada and Norway both cross the U.S. In addition, Canada also crosses West Germany.

Compared to other LIS countries the U.S. is an interesting case because it possesses very high incomes but also substantial income inequality. These extremes result in a large number of crossings, six of the nine possible. Of the remaining three, the U.S. dominates Australia, The Netherlands and Switzerland. Sweden is also interesting. Its very equitable distribution of income is combined with only a moderate mean incomes with the result that it is dominated by two contries (Canada and Norway), dominates three countries (Australia, West Germany and U.K.) and crosses the other four countries. West Germany crosses the U.S., France and Switzerland, dominates Australia, the Netherlands, and the U.K., and is dominated by Canada, Norway and Sweden. The U.K. with relatively low incomes and low income inequality dominates Australia and France, crosses the U.S. and the Netherlands, and is

dominated by all other countries. Switzerland and France have GL curves which are not significantly different from one another and rank just above The Netherlands and Australia, near the bottom of LIS countries.

B. Income Gap Poverty Rankings

As previously mentioned, Foster and Shorrocks (1988) demonstrate that a income gap poverty ranking is equivalent to the generalized Lorenz ranking over a truncated distribution. This allows a general poverty ordering without specifying a fixed income cutoff. In this section, however, we place a reasonable restriction on the truncation to explore the possibilities of additional rankings. We assume that the poverty group is contained in the bottom three deciles of the income distribution (for interested readers, Table 3 reports sufficient information to specify alternative poverty cutoffs). The poverty rankings are given in Table 5.[11] The notation is same as that used in Table 4.

As expected, a more complete ordering is obtained with the truncated GL distributions. For example, while the U.S., Norway and Canada are noncomparable when comparing the entire GL distributions, Norway dominates both the Canada and the U.S. in terms of income gap poverty. In only two cases (4.4 percent), Canada compared to West Germany and the U.S. compared to the U.K., do the truncated GL curves cross. In four cases (8.9 percent) Switzerland versus France, The Netherlands and the U.S. and the U.K. versus France there are no significant differences in the truncated GL curves. Thus, the GL poverty ordering ranks 43 out 45 (96 percent)of all pairwise comparisons, with 90 percent (39 out

TABLE 5

Percapita Poverty
Deciles 1-3

	Australia	Canada	France	Netherlands	Norway	Sweden	Switzerland	West Germany	United Kingdom	United States
Australia	*									
Canada	+	*								
France	+	-	*							
Netherlands	+	-	-	*						
Norway	+	+	+	+	*					
Sweden	+	-	+	+	-	*				
Switzerland	+	-	O	O	-	-	*			
West Germany	+	X	+	+	-	-	+	*		
United Kingdom	+	-	O	+	-	-	+	-	*	
United States	+	-	-	+	-	-	O	-	X	*

NOTE:

A + means the country in the row dominates the country in the column.

A - means the country in the row is dominated by the country in the column.

An X means the distributions cross.

A O means the distributions are equivalent.

43) of those ranked being classified as "dominance" and the remainder being "equivalent".

The general pattern of poverty gap orderings within the ten LIS countries can be summarized as follows. Norway dominates and Australia is dominated by all other countries. Canada ranks just below Norway, while The Netherlands ranks just above Australia. Compared to its position in terms of rankings based on entire GL curves the high degree of income equality moves Sweden up in the povety gap comparisons of LIS countries. In contrast, the relatively large degree of American inequality moves the U.S. down, with the result that, despite having the highest level of income, the U.S. dominates only Australia and the Netherlands, is equivalent to Switzerland and is dominated by all other countries.

V. Conclusion

This paper applies newly developed theoretical and statistical techniques to rank ten Western countries in terms of standard of living and income inequality. Using the internationally comparable LIS database, we are able to provide ordering of economic well-being in more than seventy percent of the pairwise GL comparisons. In addition, we follow Foster and Shorrocks and use a truncated GL analysis to draw inferences about income gap poverty rankings. In this case we are able to rank ninetysix percent of the truncated distributions.

The results of the statistical tests suggest that Norway and Canada have the highest levels of economic well-being and lowest levels of income gap poverty, while Australia has the lowest level of economic well-being and highest level of income gap poverty.

The high mean income and large degree of inequality in the U.S. results in noncomparability in six of the nine pairwise international comparisons. However, in five of these cases the U.S. is dominated in terms of the truncated GL povety gap criterion.

REFERENCES

Atkinson, A.B. (1970) "On the Measurement of Inequality." *Journal of Economic Theory* 2, 244-263

Beach, C.M. and R. Davidson (1983) "Distribution-free Statistical Inference with Lorenz Curves and Income Shares." *Review of Economic Studies* 50, 723-735

Beach, C. M. and Richmond (1985) "Joint Confidence Intervals for Income Shares and Lorenz Curves." *International Economic Review* 26, 439-450

Bishop, J. A., S. Chakraborti, and P. D. Thistle (1989) "Asymptotically Distribution-Free Statistical Inference For Generalized Lorenz Curves." *Review of Economics and Statistics*, forthcoming

Bishop, J. A., J. P. Formby, and W. J. Smith (1989) "International Comparisons of Income Inequality: Tests For Lorenz Dominance Across Nine Countries." LIS-CEPS Working Paper #26

Dalton, H. (1920) "The Measurement of the Inequality of Incomes." *Economic Journal* 30, 348-361

Dasgupta, P., A. K. Sen, and D. Starrett (1973) "Notes on the Measurement of Inequality." *Journal of Economic Theory* 6, 180-187

Foster, J. E. and A. F. Shorrocks (1988) "Poverty Orderings." *Econometrica* 56, 173-177

Kakwani, N. (1984) "Welfare Ranking of Income Distributions." in R. L. Basmann and G. F. Rhodes, Jr. (eds.), *Advances in Econometrics: Vol. 3* (Greenwich: JAI Press)

Miller, R.G. (1981) *Simultaneous Statistical Inference* 2ed. (New York: Wiley)

Shorrocks, A.F. (1983) "Ranking Income Distributions." *Economica* 50, 3-17

Stoline, M.R. and H.K. Ury (1979) "Tables of the Studentized Maximum Modulus Distribution and an Application to Multiple Comparisons Among Means." *Technometrics* 21, 87-93

Summers, R. and A. Heston (1988) "A New Set of International Comparisons of Real Product and Price Level Estimates For 130 Countries." *Review of Income and Wealth* 34, 1-25

[7] Bishop, Formby, and Smith (1988) test for per capita Lorenz dominance and find that none of these Lorenz curves cross. Thus, the Gini coefficients provide an unambiguous ranking of inequality.

[8] The choice of deciles is, of course, arbitrary. However, given a fixed sample size, increasing the number of quantiles does not necessarily improve the overall test for GL dominance.

[9] The number of numerical crossings (20) is nearly twice that of the number of statistical crossings suggesting that the statistical procedures greatly improve the efficacy of the generalized Lorenz criterion.

[10] Crossings are inconvenient for overall rankings as well as individual rankings. Recall that the statistical tests are conducted separately on two countries at a time. For three countries, e.g. A,B and C, to be ranked in order, it is necessary for A to dominate both B and C and for B to dominate C as well and for no crossings to occur in the pairwise comparisons. It is entirely possible using Beach and Davidson techniques for A to dominate B and B dominate C while at the same time finding that C crosses A. All that is implied here is that A is preferred to B and B to C but no conclusion about A and C can every be definitely reached without specific information on the weights in the welfare function, information which of course is not available. As emphasized previously, crossings translate into noncomparability. Thus it is not surprising that crossings interfere with overall ranking as well.

[11] The five percent SMM critical value for k=3 is 2.38.