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Luxembourg Income Study Working Paper Series

Working Paper No. 27

Measuring Inequality and Differing Social Judgments

Anthony Atkinson

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**MEASURING INEQUALITY AND
DIFFERING SOCIAL JUDGMENTS.**

by

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February, 1989

Measuring Inequality and Differing Social Judgments*

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This paper is a revised version of a lecture given at the first Summer Workshop of the Luxembourg Income Study (LIS) in July 1988. This drew in turn on material from graduate lectures given at the London School of Economics since 1980. I am most grateful to John Coder, LIS Technical Director, for supplying tabulations from the LIS data set.

1. Introduction

The topic of this survey - the measurement of inequality - is a highly controversial one. It is a field in which there are large differences in social judgments, which translate themselves into differences in the tools which are applied, such as the measure of inequality or the choice of equivalence scale. Faced with such differences, how should the analyst proceed? One can perhaps distinguish two extremes of approach:

either

the analyst can give a definite answer, by choosing a particular measure of inequality, and a particular equivalence scale, and seek to convince the audience of the reasonableness of this choice.

or

the analyst can draw attention to the difficulties in the choice of inequality measure and equivalence scale, and conclude that nothing definite can be said.

Neither of these approaches is very satisfactory. The first is attractive to the politician, who likes definite answers. Politicians want to be able to say that inequality has gone up (or down). But the mere fact that there are differences in social judgments means that there is likely to be controversy, which is obscured by this approach, and which will feed back into the political debate, where the politician will find opponents quoting different figures. The second makes the differences in judgment transparent, but is useless for the practical policy-maker. The statistician in the government census office cannot simply leave pages in his report blank.

In this survey, I shall be emphasising intermediate routes - ones that recognise that differences in social judgments exist but seek nonetheless to delimit the range of answers. The intermediate approach makes explicit

that fact that the answers depend on social judgments and seeks to describe the empirical information in a way that illuminates the relation between the judgments and the conclusions.

It is of course the case that the answers reached by the intermediate approach are likely to be partial or conditional rather than definite. In particular, we may only be able to make rankings rather than give precise numerical values; and these rankings may only be partial rather than complete. Suppose that we take the distribution of family net income in ten countries covered by the Luxembourg Income Study (LIS), which is the source of the empirical examples in this survey. If one chooses the Gini coefficient, then one obtains a clear answer to the question: how much income inequality is there in the different countries? The results are, ranked in descending order of inequality:

	%
West Germany	37.2
USA	36.8
Australia	34.7
UK	34.7
Canada	34.4
Switzerland	34.1
Israel	33.8
Netherlands	31.5
Norway	31.1
Sweden	29.2

Source: LIS tabulations.

We have, with the figures just quoted, a complete numerical answer: for example, West Germany is 8 percentage points more unequal than Sweden. We may however only be able to make a ranking. That is we may not accept that the Gini is the right quantitative measure of inequality, but we may agree with the ranking shown above. This corresponds to erasing the numbers but leaving the left hand column as an ordering. For a number of purposes, agreement on the ranking may be enough. This may be the case, for example, if we are concerned with different redistributions of income within a

country.

Differences in social judgment may of course lead to differences in the ranking. Suppose for example that, following the Harvard philosopher John Rawls, we are solely concerned with the welfare of the least advantaged group - say the bottom 20%. In this case, the evidence from the LIS data-set shows that the lowest such share is found in the United States, as may be seen from Table 1. So that the US now heads the inequality league, and West Germany now occupies a position inferior only to that of Sweden and Netherlands. Less dramatic are the reversals of ranking of the UK and Canada, and of Norway and the Netherlands. At the same time, the majority of the pair-wise rankings remain unchanged.

Table 1 Cumulative Shares in Total Income for 10 Countries

Country	Share in total income of bottom				
	20%	40%	60%	80%	100%
West Germany	6.5	18.4	34.5	56.4	100.0
USA	4.5	15.8	33.6	59.1	100.0
Australia	5.4	17.1	35.1	60.1	100.0
UK	5.7	17.0	35.2	60.2	100.0
Canada	5.4	17.4	35.6	60.6	100.0
Switzerland	5.8	18.7	36.6	60.1	100.0
Israel	6.0	18.1	36.0	60.5	100.0
Netherlands	7.3	20.4	37.8	61.3	100.0
Norway	6.3	19.1	38.0	63.3	100.0
Sweden	8.1	21.4	38.8	63.3	100.0

Note: countries are ordered according to the value of the Gini coefficient, with the highest value at the top.

If we have to allow for the fact that some people choose the Gini coefficient, and that others judge according to the Rawlsian criterion, then we cannot arrive at a complete ranking of the ten countries. What we have is a - quite extensive - *partial ordering*: ie a ranking which is incomplete. The idea that we may have to settle for less than a complete answer is one

that has been stressed by Sen, particularly in his book On Economic Inequality:

"Most statistical measures of the inequality level assume a high degree of measurement.... It is however possible that the implicit notion of inequality that we carry in our mind is, in fact, much less precise and may correspond to an incomplete quasi-ordering... The notion of inequality has many aspects, and a coincidence of them may permit a clear ranking, but when these different aspects conflict an incomplete ranking may emerge" (1973, p 5).

He goes on to say that "the predisposition [of economists] towards a complete ordering has proved to be a major liability" (1973, p 6).

The approach that I am advocating here is one which seeks to accommodate different social judgments explicitly and to identify the extent of common ground. This common ground may not allow us to provide a complete numerical answer, but it may allow us to say something.

2 Pareto Efficiency and First-Degree Dominance

One key element in the intermediate approach is that of *dominance*. The basic idea is simple. We have a set of social judgments and a set of distributions; and we say that one distribution dominates another if it is preferred (or is no worse) *for all possible social judgments* in the specified set.

The best known example of dominance is that of Pareto efficiency. We say that distribution B is preferred to distribution A in a Pareto sense if

$$y^B_i \geq y^A_i \text{ for all } i \quad (1)$$

where the right (left) hand side denotes the income of a specified person i in the distribution A (B). (Throughout I use "preferred to" as including the case of indifference, and conditions such as (1) are not stated as requiring strict inequality for some i .) We require that everyone be at least as well off. If this is satisfied, and welfare is measured simply by

income, then we can reach a definite conclusion.

What in this case is the specified set of social judgments? The idea of Pareto efficiency is often presented as though it is unproblematic - as though it does not involve any value judgments. This is not the case, even if - as is our starting point here - the evaluation of a social state is assumed to be solely a function of individual welfare levels. Adopting such a *welfarist* approach (see Sen (1977), Section 8), and assuming that the welfare of individual i is represented by his or her income Y_i , we can write the *social welfare function* as

$$W(Y_1, \dots, Y_i, \dots, Y_n).$$

Application of the Pareto principle involves the judgment that the social welfare function is a non-decreasing function of individual incomes: ie W belongs to the set

$$W_0: W \text{ non-decreasing}$$

(It may be noted that we are not requiring that W be *strictly* increasing.) This is an assumption and not necessarily one with which everyone would agree. We may for example be concerned about the *distance* between the top and the bottom. The role of distance is discussed by Wiles (1974), who says that "the visiting economist does not see the Gini coefficient as he walks about the down-town and the slums of a foreign capital" (1974, p 12), and notes some of the advantages of the simple range:

$$(\text{Top} - \text{Bottom}) / \text{Average}$$

Sen describes this as more or less "a non-starter" (1973, p 31), but this seems too dismissive. The development of measures of inequality incorporating notions of distance is an interesting research topic.

If we accept, with this qualification in mind, that W is non-decreasing, we may consider the implications of the further - strong - assumption that W is in addition a *symmetric* function: ie W belongs to the

set

W_1 : W non-decreasing and symmetric

This further assumption is a strong one because it means that a distribution in which Smith gets 100 and Jones gets 200 is equivalent in terms of social welfare to one in which Jones gets 100 and Smith 200. Of course, this is only likely to be a reasonable assumption where Jones and Smith have the same "needs". For example, if the Jones family is twice the size of the Smith family, then the first distribution would give equal per capita incomes but the second would give the Smith family 4 times as much. For the present, I assume that there are no such differences in needs, returning to this subject in Section 5.

The force of this assumption may be seen from a simple example:

	A	B	B'
Person 1	1	3	2
Person 2	3	2	3
Person 3	10	10	10

The distribution B is not preferred to the distribution A according to the Pareto criterion, but is superior if we consider the re-ranked distribution B' and according to the symmetry assumption this re-ranking leaves social welfare unchanged. The reason that we say it is superior is that, if we take the i -th person from the bottom in each distribution and ask how much income they have, then the answer is always larger, or no smaller, in the case of distribution B' than for A.

Represented in terms of a continuous, rather than a discrete, distribution, we have the situation shown in Figure 1, where the range of income is assumed to lie between 0 and a maximum value Y^{\max} , and where we are assuming that the population is the same in all cases being considered. The cumulative distribution function is denoted by

$$F(Y) \quad \text{with } F(0) = 0 \text{ and } F(Y^{\max}) = 1 \quad (2)$$

so that $F(Y)$ is the *proportion* of the population with incomes of Y or less.

(The density function is denoted by $f(Y)$.) The test that we have just applied may be seen by reading *across* from a particular value of the cumulative distribution function F , which corresponds to the rank in the distribution. At any value of $F(Y)$, we require that the income be higher (or equal). Equally, we could have taken a particular value of income, Y , and read *upwards* to check that the cumulative distribution $F(Y)$ was lower (or equal). In other words, whatever income level we choose, there should be a smaller proportion of the population with less than this income: ie for the distribution B to be preferred

$$H_1(Y) = F^B(Y) - F^A(Y) \leq 0 \quad \text{for all } Y \in [0, Y^{\max}] \quad (I)$$

This is known as the first-degree stochastic dominance condition (hence the numbering (I)). (As noted earlier, the case of indifference between A and B is not precluded.)

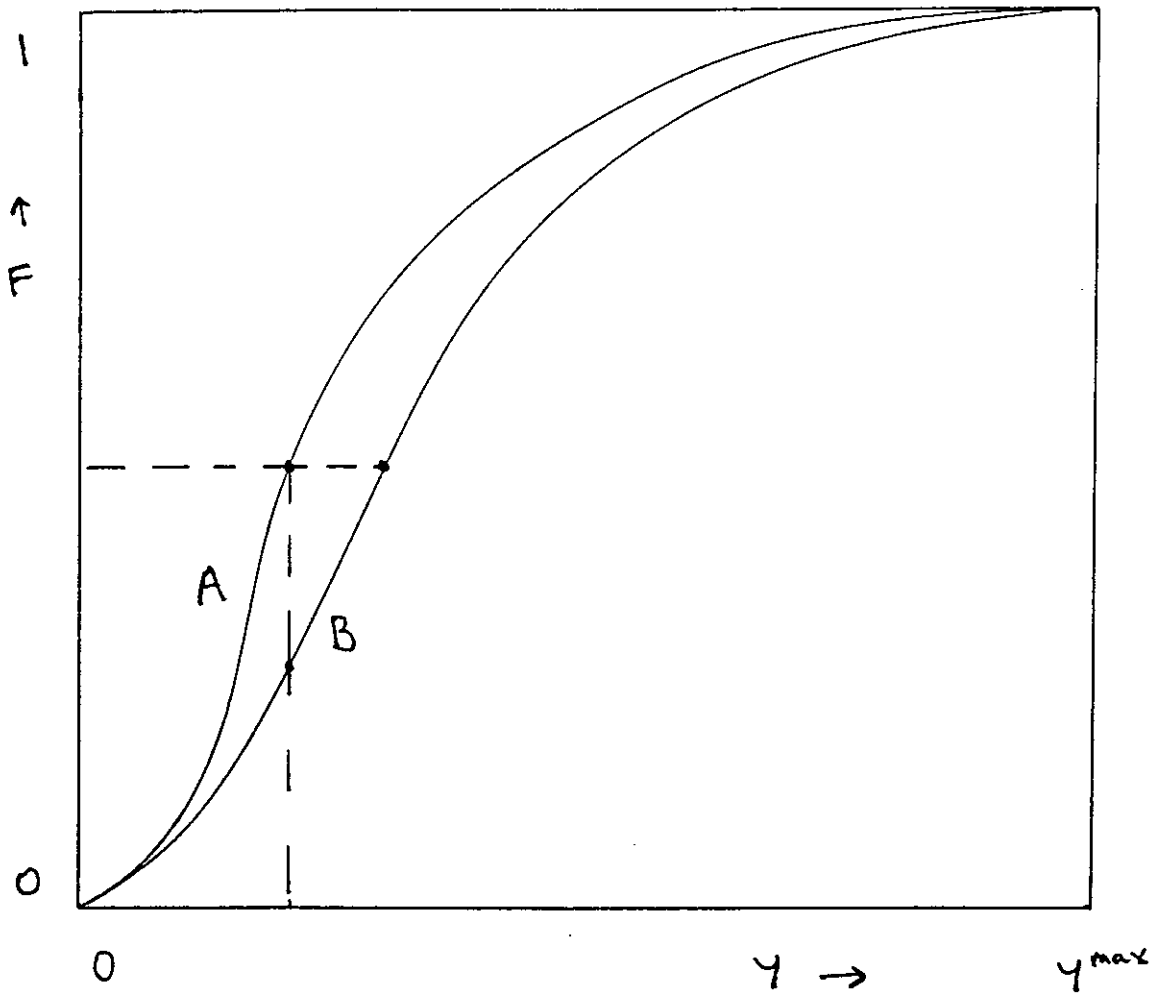
The first-degree condition on the distributions is a strong one, requiring that for a distribution to be strictly better it must have a higher mean income (as in the numerical example where the mean for B and B' exceeds that for A). This may be seen from the fact that the mean, \bar{Y} , may be written:

$$\bar{Y} = Y^{\max} - \int_0^{Y^{\max}} F(y) dy \quad (3)$$

If F is strictly lower at some point, and no higher elsewhere, then the mean must be higher. For subsequent reference, we may note that the difference between the mean of the distribution B and that of A is equal to

$$- \int_0^{Y^{\max}} H_1(y) dy \quad (4)$$

Figure 1 First-Degree Dominance Condition



3 Equity Preference, the Principle of Transfers and Lorenz Curves

We have not yet however introduced any notion of equity preference. Consider two 3-person distributions that have the same mean, modifying our earlier example such that it becomes

	A	B
Person 1	1	2
Person 2	3	3
Person 3	10	9

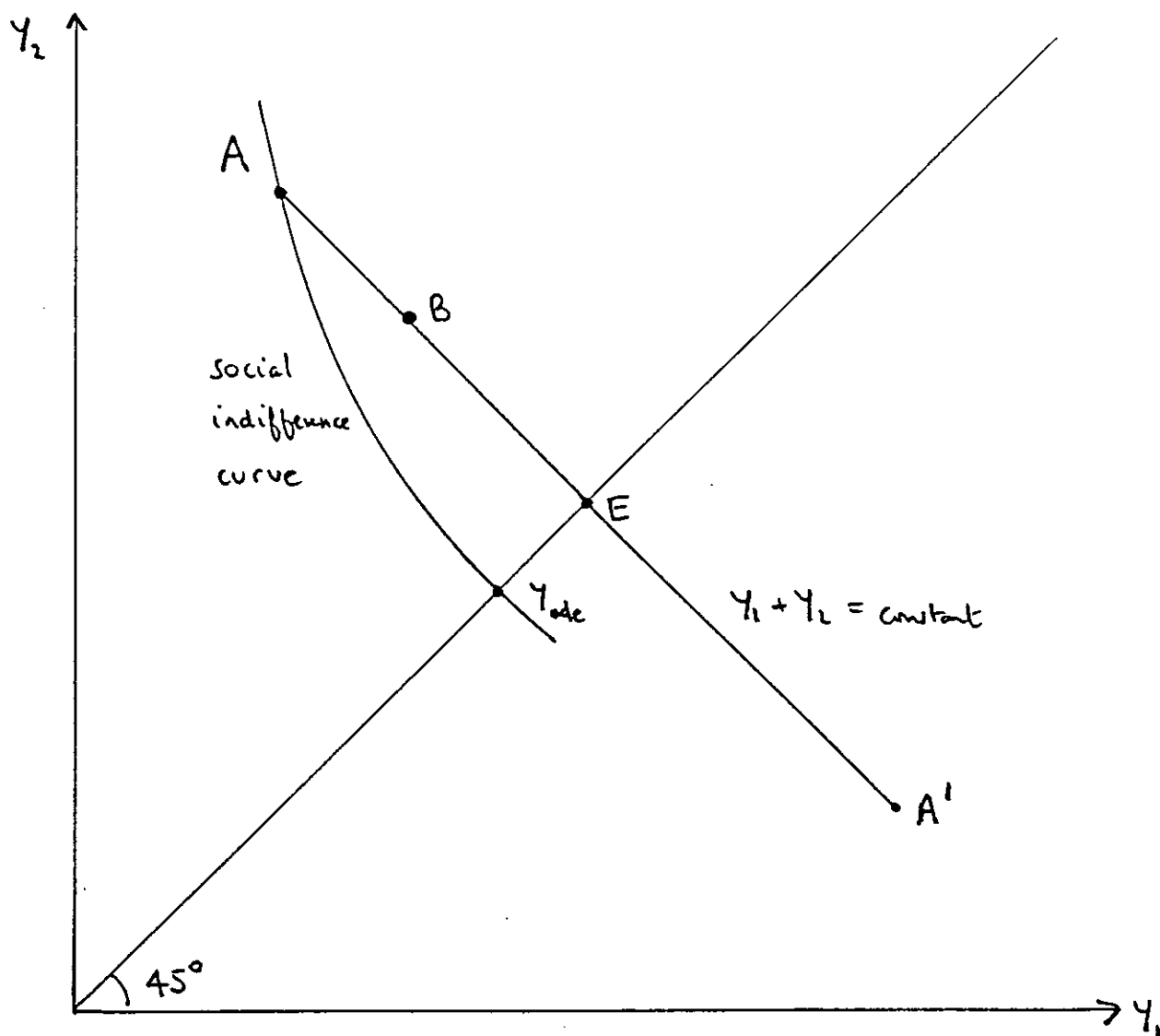
The total income is now the same, and the gain to person 1 comes at the expense of person 3: there has been a *mean-preserving equalising transfer*. In the case of the Rawlsian objective, this is clearly an improvement, since the lowest level of income goes from 1 to 2, and more generally we may expect it to hold for all social welfare functions that are *equity-preferring*, in the sense that they respond positively (or non-negatively) to a mean-preserving equalising transfer.

This is shown geometrically for the two person case in Figure 2. Suppose that the initial distribution is at A where person 2 is better off than person 1. An equalising transfer, $T (\geq 0)$, which has no effect on the pre-tax incomes involves moving along a straight line

$$(Y_2 - T) + (Y_1 + T) = \text{constant} \quad (5)$$

towards the point of equal incomes, E. So that the point B is after such a transfer. The social welfare function is equity preferring if such a move would be preferred: ie the social indifference curve through A never lies above the line AE. It may be noted that the line of equalising transfers extends beyond E, in the sense that, compared with A, all points up to the permutation A' are preferred.

Figure 2 Equalising Transfer in the 2-Person Case



The assumption of equity-preference corresponds to assuming that the social welfare function belongs to the set:

W_2 : W non-decreasing and s -concave

It should be noted (i) that s -concavity is weaker than quasi-concavity, which is the condition that the social indifference curves be convex to the origin, (ii) that symmetry is implied by s -concavity (A being at least as good as A' and vice versa), and (iii) that we are not requiring *strict* s -concavity. With this assumption, we may make use of the following result (see, for example, Dasgupta, Sen and Starrett, 1973 and Rothschild and Stiglitz, 1973) when we are comparing two distributions with the same population and the same mean:

If the distributions A and B have the same total population and the same mean income, the distribution B is preferred for all social welfare functions in W_2

if and only if

B can be obtained from A by a sequence of mean-preserving equalising transfers.

When this condition is satisfied, then the distributions are said to satisfy the *Dalton transfer principle* after Dalton (1920). Alternatively, the condition is referred to as *second-degree dominance*, being equivalent to the condition:

$$H_2(Y) = \int_0^Y H_1(y) dy \leq 0 \text{ for all } Y \in [0, Y^{\max}] \quad (\text{II})$$

Comparing this with condition (I), we can see that it requires the cumulative distribution to be initially below, but it can rise above, providing that the integral remains negative. It may be noted that the integral being strictly negative at interior values of Y is consistent with the two distributions having the same mean (which from equation (4) may be seen to require only that the integral be zero for $Y = Y^{\max}$.)

The problem with the transfer condition is that it is not readily checked with a large micro-data set such as the LIS data-base. The usual solution is to use the well-known *Lorenz curve* (Figure 3), familiar from first year economics textbooks, with the share of total population along the horizontal axis and the share of total income along the vertical. How is the Lorenz curve relevant? The result which provides the Lorenz curve with a rationale is the following:

If the distributions A and B have the same total population and the same mean income, and the Lorenz curve for B lies everywhere above, or co-incides with, that for A, then the distribution B is preferred for all social welfare functions in W_2 .

This result is proved in a less general form in Atkinson (1970) and extended by Dasgupta, Sen and Starrett (1973), Rothschild and Stiglitz (1973), and others. When this condition is satisfied we have the situation of *Lorenz dominance*.

The test of Lorenz dominance is often used in cross-country comparisons. However, the result just described cannot be applied directly, since countries typically differ in size and in mean incomes. These differences may not be apparent in that the Lorenz curve is concerned only with *relative* population shares and with *relative* income shares. So that although the population of Sweden is (approximately) twice that of Norway, we simply allow for this by taking the bottom 20% as a group twice as large, etc. In other words, we are assuming that our judgment about the distribution of income in Norway would be unchanged if each Norwegian immediately acquired a twin with the same income. (The effect on distributional judgments of *replicating* the economy in this way is discussed by Dasgupta et al, 1973, and Sen, 1973, pp 58-60.) Similarly, differences in mean income are excluded from consideration by the assumption that judgments about the distribution are unaffected by scaling

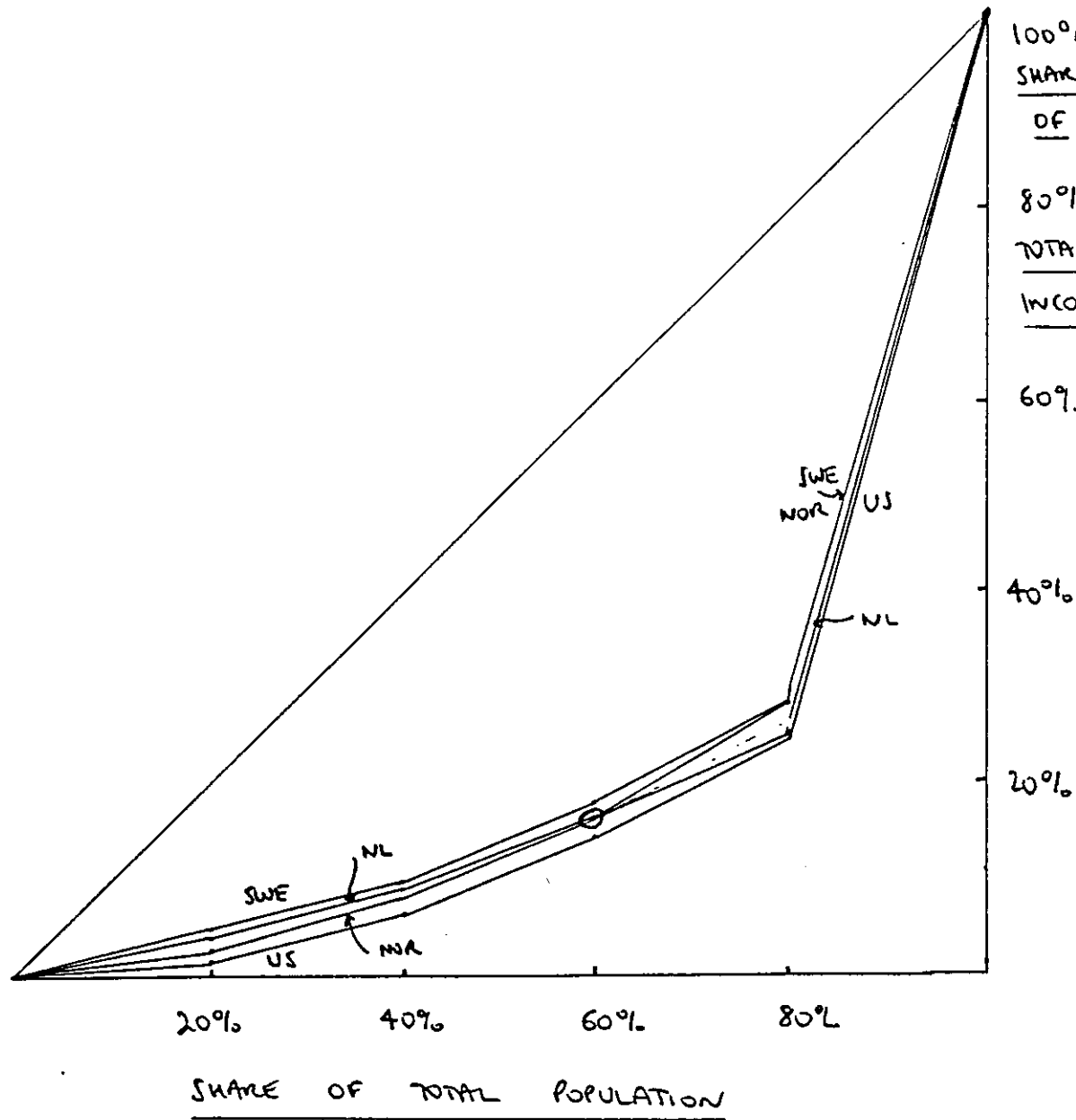
incomes up or down proportionately. This assumption is considered further in the next section.

If, for the moment, we are concerned only with the relative position as measured by the conventional Lorenz curve, then the LIS data for quintile shares in Table 1 allow us to draw certain conclusions. Plotting the Lorenz curves reveals that the Lorenz condition is satisfied for quite a number (31) of the 45 possible comparisons. For example, the curve for the Netherlands lies above that for all countries except Norway (where there is an intersection) and Sweden (whose curve is superior). The curves for these three countries and the US are shown in Figure 3. As is often the case with actual data, the differences between the curves would not be easily discerned if they were drawn to scale; the differences have therefore been exaggerated in the diagram. (Graphically, the Lorenz curve is less useful as an empirical tool than as a conceptual device.)

The LIS example suggests that the partial ranking is quite extensive. At the same time, this example may be a little deceptive, in that it is based on broad income ranges (so that the curves may for example intersect within the top 20%); and we should note that Buhmann, Rainwater, Schmaus and Smeeding (1988), using LIS data for the same 10 LIS countries, report that the Lorenz curves intersect in 36 out of 45 cases. (One difference from the data used here is that they weight each income unit by the number of persons.)

Use of the Lorenz dominance criterion is widespread, but it raises a number of questions. Does it in fact matter that we are comparing countries with quite different mean incomes? What do we do about differences in family size? What do we do when the Lorenz curves cross? These questions are taken up in the next three sections.

Figure 3 Lorenz Curves for Netherlands, Norway, Sweden and the US



4. Differences in Mean Incomes and Generalised Lorenz Curves

The fact that countries have different mean incomes clearly has to be taken into account in any overall social judgment. One way to proceed - and that implicit in much work on international comparisons - is to suppose that we consider separately "efficiency" and "equity" and then bring them together in a final judgment. If, further, we assume that measures of equity should be *scale invariant*, that is they should be unchanged by a proportionate increase in all incomes, then this provides a justification for the use of the standard Lorenz curve. It should immediately be noted that the assumption of scale invariance is a controversial one, and that Kolm (1976a and b) has explored the alternative assumption that measures of inequality should be invariant with respect to absolute increases in all incomes, as well as an intermediate possibility where people consider inequality to be unchanged by a mix of proportionate and absolute increases in income (the mix being determined by a parameter ξ , about which people may have different views).

In practice, most empirical studies of income inequality have assumed scale invariance. If we accept this for the present argument, then there remains the problem of the trade-off between equity and efficiency. Considerations of dominance allow some conclusions to be drawn. If we assume that the overall social evaluation prefers a higher mean income, then if a country (e.g. Sweden) has both a higher mean income than another (eg Britain) *and* a superior Lorenz curve, then it dominates overall. (From (4) and the definition of $H_2(Y)$, it may be seen that the requirement on the mean corresponds to $H_2(Y^{\max}) \leq 0$.) Taking the relative mean income figures given in Table 2 below, we can see that this dominance condition is satisfied for 13 out of the 45 possible comparisons. The question of the trade-off between mean incomes and relative inequality is discussed further

in Section 8.

A second approach to the introduction of differences in mean incomes - perhaps the most natural one in a welfarist context - is to assume that the social welfare function combines both equity and efficiency concerns. The consequences have been explored by Shorrocks (1983), who introduces the notion of a *generalised Lorenz curve*. This replaces the

relative percentage of total income

on the vertical axis by

absolute total income (per head)

so that it is now denominated in currency, rather than per cent. This amounts to multiplying by the mean the share that is taken in the standard Lorenz curve. It may be seen that the end point when all the population is covered is equal to the mean income. (We continue to assume that the population is expressed in relative terms.)

The link with the social welfare function is provided by the following result proved by Shorrocks (1983):

If the distributions A and B have the same total population, and the generalised Lorenz curve for B lies everywhere above, or co-incides with, that for A, then the distribution B is preferred for all social welfare functions in W_2 .

It follows that if the mean is higher and the standard Lorenz curve is also better (as in the 13 cases referred to above) then the distribution is preferred. But it is also possible that B may be preferred even if the standard Lorenz curves intersect or even its standard Lorenz curve is worse. Compare for example the 3-person example of B with (2,3,4) and the alternative A of (1,1,1). The latter has a Lorenz curve which coincides with the diagonal - complete equality - but it would be rejected in favour of B by all increasing welfare functions.

To apply the generalised Lorenz curve to cross-country comparisons, we

need to apply an appropriate rate of conversion. The illustrative figures below are based on the quintile shares from the LIS data-set (as used earlier) and the mean income per capita in US dollars adjusted by the OECD purchasing power parities. The resulting figures for the generalised Lorenz curves are (the mean incomes are normalised, with that of the US equal to 100, so that if everyone had an income equal to the US mean the figures would read 20%, 40%, 60%, 80% and 100%):

Table 2 Generalised Lorenz Curves for 10 Countries

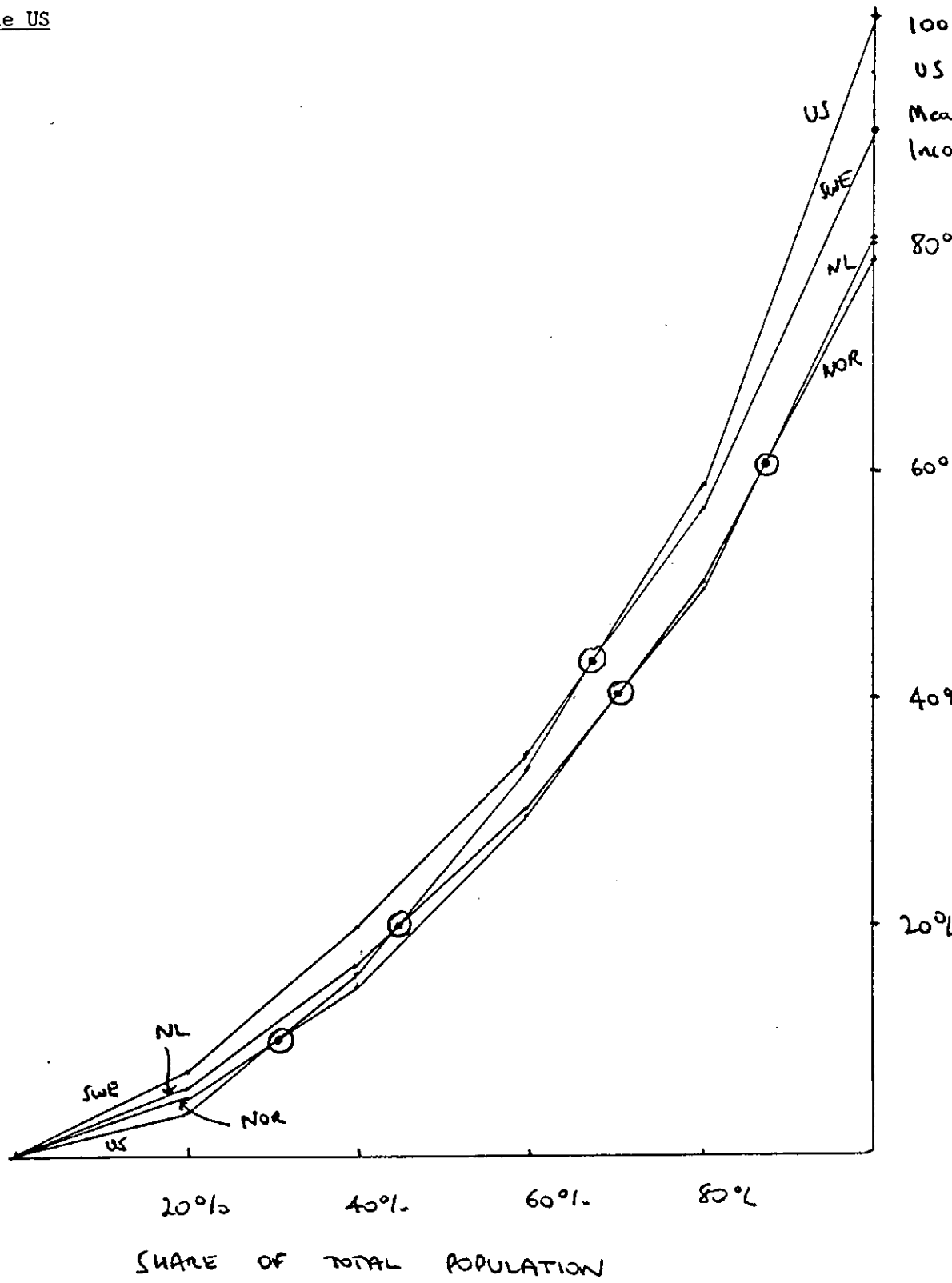
Country	Cumulative total income relative to US mean income					
	Bottom	20%	40%	60%	80%	100%
USA		4.5	15.8	33.6	59.1	100.0
Canada		5.1	16.3	33.4	56.8	93.7
Sweden		7.3	19.3	34.9	57.0	90.0
Switzerland		5.1	16.5	32.4	53.2	88.5
Australia		4.5	14.3	29.4	50.4	83.9
West Germany		5.2	14.8	27.7	45.3	80.4
Netherlands		5.9	16.4	30.4	49.2	80.3
Norway		4.9	14.9	29.7	49.5	78.2
UK		3.7	11.0	22.7	38.9	64.6
Israel		3.4	10.1	20.2	33.9	56.0

Note: the countries are ordered according to mean income in US dollars.

Sources: mean income per capita in US dollars calculated from GDP per capita in national currency for 1979, or 1979/80, from ILO, The Cost of Social Security 1978-80, 1985, Appendix Table) adjusted by the OECD purchasing power parities (with the exception of Israel, where I have used the figure of 56% of US per capita GDP in 1977 from Summers, Kravis and Heston, 1980, p 64).

Since we cannot rule out the possibility that people have a vanishingly small concern about equity, so that they are really only concerned with mean income, then the country with the highest mean income - the US according to these figures - cannot be dominated. The US does not however dominate Canada, Sweden, Switzerland, West Germany, Netherlands and Norway, as is illustrated by the generalised Lorenz curves for four of these countries in Figure 4. In those cases where it is dominant (Australia,

Figure 4 Generalised Lorenz Curves for Netherlands, Norway, Sweden and the US



UK and Israel), the differences in mean income outweigh the differences in the distribution (those 3 countries all have superior standard Lorenz curves). Considering all the pair-wise comparisons, there are more intersections than in the case of Table 1 (18 compared with 14).

This example is intended only to illustrate the methodology, but it suggests that the generalised Lorenz curves may show a different picture from the conventional comparison which abstracts from differences in the mean incomes. The finding that there are more intersections than with the standard Lorenz curves is in contrast to the findings of Shorrocks (1983), but this is likely to depend on the case in hand, particularly the range of differences in mean income (in his data set, covering countries like Tanzania as well as Sweden, it was more than 10:1).

5. Differences in Family Composition

In comparing different countries, we clearly have to take account of the fact that the family composition, and its relation to income, may well be different. As evidence from the LIS data-base brings out, the composition of the bottom 20% is rather different: for example, if we consider families ranked according to their equivalent income, 25% of those in the bottom quintile have children in Sweden, compared with over 40% in the US (Smeeding, Rainwater, Rein, Hauser and Schaber, forthcoming, Table 3.2).

The standard procedure to deal with such differences in family size is to use an equivalence scale. This is what was done in the LIS calculations just quoted, where the authors take a simple scale:

1 for the first person in the family + 0.5 for all subsequent persons with a value of 6 for all families with more than 10 persons. So that on this LIS scale an income of \$30,000 for a couple is equivalent to \$20,000

for a single person. The trouble is that we have to justify the choice of equivalence scale and here there seems to be wide disagreement. That this should be so is not altogether surprising. Where scales are based on "expert" judgment about dietary and other needs, then there is obviously room for disagreement. In the case of studies based on household budget data, there has been a great deal of debate about the methods used. Moreover, the ethical status of scales derived from expenditure decisions is open to question. For example, it is not obvious that parental decisions necessarily fully reflect the interests of children. The diversity of scales used in practice is demonstrated by the survey in Buhmann, Rainwater, Schmaus and Smeeding (1988, Table 2). They summarise the scale in terms of a power function of family size:

$$y = c x^e$$

so that an exponent $e = 1$ corresponds to a per capita calculation and $e = 0$ corresponds to no adjustment being made. They distinguish between scales developed for statistical purposes (including that described above as the LIS scale), where e ranges between 0.70 and 0.84, scales used to assess the adequacy of benefit programmes, where e ranges from 0.35 to 0.67, scales based on observed consumer spending, where e ranges from 0.23 to 0.57, and scales derived from subjective evaluations, where e ranges from 0.12 to 0.36. As this makes clear, there is considerable variation. Buhmann et al go on to examine the sensitivity of cross-country comparisons to the choice of equivalence scale. They conclude that "as the exponent value implicit in each equivalence scale [increases], differences in measured inequality at first decrease and then increase in a systematic way" (1988, p 129).

The advantage of this parameterisation of the equivalence scales is that it provides a simple way of taking account of differences in judgments about the needs of families of different sizes. As such, it has

similarities to the use of the iso-elastic measures of inequality discussed in Section 7 below. An alternative approach has been described by Atkinson and Bourguignon (1987). This requires that there be some degree of agreement about the needs of different families but does not assume that there is complete agreement. Like the Lorenz criterion, it will only give a partial rather than a complete ranking, and it does not give a numerical measure. But it does allow us to say something.

In order to illustrate the Atkinson-Bourguignon approach, let us suppose that there are n family types. In the diagrammatic illustration in Figure 5, it is assumed that n is 3: single persons, couples, and couples with children. What is more we are agreed that these differences in family size are the only relevant differences. The families may differ in other respects, such as where they live, or what they consume, but these are not deemed to be relevant to the social judgment. The overall social welfare function is assumed to be additively separable over individual families. For each group we have a welfare function that is non-decreasing and s -concave, but the welfare function may differ between family types, so that the total level of social welfare may be written:.

$$W = \sum_{i=1}^n p_i \int w^i(Y) f^i(Y) dY \quad (6)$$

where p_i is the proportion of the population in the i -th group, $f^i(Y)$ is the density function for the distribution of income within that group, normalised so that the integral over the total range of income is unity, and W belongs to W_2 .

Now if we know nothing more about the social values, other than that specified above, then the condition for the distribution B to be preferred to A is that the generalised Lorenz curve for B be above (or co-incide with) that for A for each of the family types taken individually. This is

clearly a demanding condition, and it rules out any redistribution between family types. A policy which lowered the mean income for single persons but raised that for couples could not dominate (since the mean has to be higher, or equal, for all family types separately).

Suppose however that we could agree that, at a given income level, an extra £1 of income is worth more to a family of type n than to type $n-1$: ie it is worth more to a couple with children than to a couple without children and more to couples than to single persons. In other words, there is agreement that, in this sense, type n is the "most deserving", that type $(n-1)$ is the "next most deserving", and so on. What is more, suppose that we are agreed that the differences in the social marginal valuation of income become no greater as we move to higher income levels. That is, we become less concerned about differences in needs at higher incomes. In formal terms, we are assuming that the marginal valuation of income for a family of type i may be written:

$$w_i^Y(Y) = \sum_{j=i}^n \epsilon_j(Y) \quad \text{for } i = 1, \dots, n \quad \text{where } \epsilon_j(Y) \geq 0 \text{ for all } Y \text{ and } j$$

(7)

where

$$\partial \epsilon_j(Y) / \partial Y \leq 0 \text{ for all } Y \text{ and } j$$

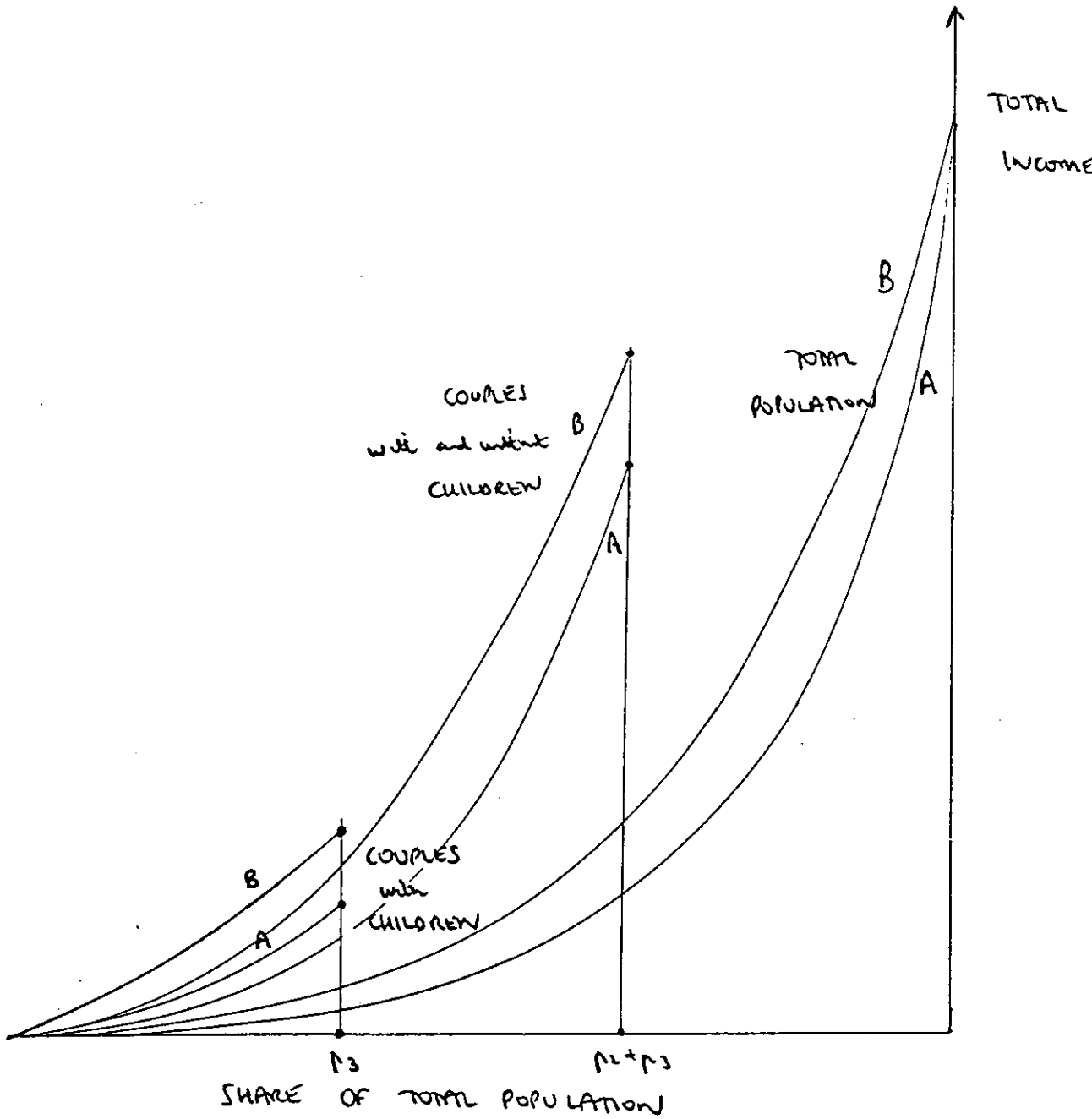
If we can agree on these two aspects, and if there is the same proportion of each family type in the two distributions (a qualification discussed further below), then Atkinson and Bourguignon show that there is a procedure to check whether there is agreement that one distribution, B , is regarded as an improvement for all social judgments.

First, we construct the generalised Lorenz curves for the type n which is regarded as the "most deserving", with the horizontal axis showing the percentage of the total population (ie the curves end at p_n). In our $n = 3$

example in Figure 5, these are the curves for couples with children. We require that the curve for the distribution B lie everywhere above. There is no avoiding this condition, since a person may attach all the weight to the most deserving group: ie regard the marginal value of income for the other groups as zero. This is an extreme position, but one that is not ruled out by our assumptions. Secondly, we calculate the generalised Lorenz curves for couples with and without children *combined*, again expressed as a proportion of the total population (ie the curves end at $P_{n-1} + P_n$). This gives a second pair of curves and again we need the curve for the preferred distribution to lie above. This does not however require that the curve for couples alone be "better"; this may lie in part below, providing this is offset by a corresponding better part of the curve for couples with children. Next we construct the curve for groups n , $n-1$ and $n-2$. In the case $n = 3$, this is the total generalised Lorenz curve and we require that the curve for B be everywhere above or not below. This would obviously be the relevant criterion if people treated the three groups as equally deserving.

What I have described is a quite practicable procedure, which allows us to reach definite conclusions in some but not all cases. Moreover, it may be applied even where there is not a complete agreement about the ranking of family types. In that case, one can carry out the test for each of the possible orderings, and require that the conditions hold for each ordering. At the same time, we should note that the results only apply to the case where there is the same proportion of each family type in the two distributions, which does not typically apply to cross-country comparisons, such as those using the LIS data set. This is an aspect which needs further consideration, but the procedure described above should certainly still be illuminating.

Figure 5 Dominance Conditions for Differing Family Types



6. Where Lorenz Curves Cross

The introduction of the assumption of s -concavity restricted the class of welfare functions from W_1 to W_2 and allowed us to extend the range of rankings. This suggests that we might make progress by making further assumptions about the form of W , thereby further extending the range of distributional rankings. Suppose for example that we were willing to accept that equalising transfers are more beneficial at low incomes than at high incomes; or, equivalently, that a "good" transfer at the bottom can offset an equally-sized "bad" transfer at the top. This has been called the *principle of diminishing transfers* by Kolm, who describes it as preferring

"to transfer one pound from a person who earns 500 pounds a month to another who earns only 100, than to transfer one pound from a 900 pounds earner to a person who already earns 500 pounds" (1976b, p 87).

He notes that the effect of a marginal transfer from person i to person j on the standard deviation is proportional to $(Y_j - Y_i)$, so that the pair of marginal transfers described above may be seen as "variance-preserving". (He also points out that a more egalitarian concept would measure the distance in *relative* terms, so that £500 in the example is replaced by $900/5 = 180$.)

In the case of the Dalton transfer principle, we have a three-way interconnection between the principle, preference for all W belonging to W_2 , and second-degree dominance. In Atkinson (1973), the parallel with the literature on risk-taking (particularly Whitmore, 1970), was used to show the interconnection between the principle of diminishing transfers, third-degree dominance, and preference for all W belonging to the class:

W_3 : non-decreasing, s -concave and transfer sensitive

Attention was limited in Atkinson (1973) to the additively separable social welfare function

$$W = \int w(y) f(y) dy \quad (8)$$

where w is differentiable. The class W_3 is then defined in terms of the derivatives: $w'(y) \geq 0$ (non-decreasing), $w''(y) \leq 0$ (concave), and $w'''(y) \geq 0$ (transfer sensitive), from which the progression in terms of assumptions is evident. (As noted earlier, we allow for the possibility that these derivatives may be zero, so that $w(y) = y$ belongs to the class.) The third-degree dominance condition is that, where the mean incomes are the same:

$$H_3(Y) \geq \int_0^Y H_2(y) dy \quad \text{for all } Y \in [0, Y^{\max}] \quad (III)$$

The progression of the sub-scripts again makes clear the weakening which additional assumptions permit.

This approach was independently developed by Foster and Shorrocks (1987), and Davies and Hoy (1987). The former generalise the concept of a pair of transfers as used by Kolm, defining a *favourable composite transfer* as one of an amount Δ from person j to person i , and of $\$$ from person k to person l , where the people are ranked i, j, k and l in increasing order of income both before and afterwards, such that the variance is unchanged: ie it is a *variance-preserving* pair of transfers. It may be noted that this allows the transfers to be of different size, but that the constant variance requirement means that $\$ \geq \Delta$ implies that Y_k exceeds Y_l by more than Y_j exceeds Y_i , and vice versa. (So that this does not allow for the more egalitarian concept suggested by Kolm, an aspect which could be further examined.) From Foster and Shorrocks (1987), one can conclude that:

If the distributions A and B have the same total population and the same mean income, the distribution B is preferred for all social welfare functions in W_3

if and only if

B can be obtained from A by a sequence of mean-preserving equalising transfers and/or favourable composite transfers.

Moreover, this condition is equivalent to third-degree dominance.

How can this result be applied in practice? Suppose that the distributions have the same mean (as where the data have been normalised by dividing by the mean), then Atkinson (1973) gave a result for the case where the Lorenz curves intersect once and an algorithm where there are multiple intersections. The following applies where the Lorenz curves have a single crossing:

If the distributions A and B have the same total population and the same mean income, and (i) the Lorenz curve for B cuts that for A from above, and (ii) the variance for B is no greater than that for A, then it is preferred for all social welfare functions in W_3 .

This gives a role for the coefficient of variation in the ranking of distributions. Applied to data on 12 countries (as used in Atkinson, 1970), this allowed the ranking to be extended from 16 out of 66 pair-wise comparisons where the Lorenz curves did not intersect to a further 13 cases where there was a single intersection satisfying the variance condition. A further 1 case satisfied the condition for multiple intersections. (Multiple intersections are discussed further by Davies and Hoy, 1987.)

This treatment applies where we abstract from differences in mean incomes. The extension of the results to generalised Lorenz curves is discussed by Dardanoni and Lambert (1988). It is immediately evident that if the permissible social welfare functions include both the Rawlsian concern with the least advantaged and distributional indifference (ie concern solely with mean income), as is the case with the class W_3 , then we require that the generalised Lorenz curve should start above and end above. Dominance can only apply where there is an even number of intersections (leaving aside situations where the curves are coincident for part of the

income range). Inspection of Table 2 shows that there are only 2 cases where this could be helpful.

7. Summary Measures of Inequality

A complete ranking can of course be achieved by applying a summary measure of inequality, like the Gini coefficient with which we began. The trouble is that this presumes a degree of agreement about social judgments which does not seem to be found in reality. Put another way, the adoption of a single summary measure does not allow for differences of view regarding distributional justice. How then can one proceed if one wants to go further than permitted by the dominance results described in earlier sections?

The answer given in Atkinson (1970) was to seek to *parameterise* differences of judgment, giving numerical measures of the degree of inequality conditional on the choice of a parameter which incorporated the distributional judgment. Starting from the additive social welfare function (8), and making the assumption that the measure is scale-invariant, we are led to the iso-elastic inequality measure considered in Atkinson (1970), here expressed in discrete form:

$$I_{\epsilon} = 1 - [(1/n)\sum(Y_i/Y)^{1-\epsilon}]^{1/(1-\epsilon)} \quad \text{where } \epsilon \neq 1$$

$$\text{or } I_1 = 1 - \Pi(Y_i/Y)^{1/n} \quad \text{where } \epsilon = 1 \quad (9)$$

where i is summed from 1 to n . It is the parameter ϵ which varies with different judgments about inequality, ranging from $\epsilon = 0$, where the person attaches no weight to inequality to $\epsilon \rightarrow \infty$, which corresponds to the Rawlsian case where one is concerned solely with the lowest income, inequality being measured by $(1 - Y_{\min}/Y)$.

The term in square brackets in the definition of I_{ϵ} is a measure of the *equally distributed equivalent* income, or the income which if equally

divided would generate the same level of social welfare, expressed as a proportion of the total income. This interpretation may be seen by referring back to Figure 2. Whereas the total income is that shown by the Y_1+Y_2 line, the point on the 45° line indicates that the same level of welfare could be generated with a smaller total income, Y_{ede} , which is the equally distributed equivalent income. If this is 90% of the mean income, then $I = 0.1$.

From a more general standpoint, it may be shown that the requirements of additive decomposability, scale invariance, and invariance with respect to population replication lead to the *generalised entropy class* (Cowell and Kuga, 1981, and Shorrocks, 1980):

$$\begin{aligned} I_c &= [1 - (1/n)\Sigma(Y_i/Y)^{1-c}] / c(1-c) && \text{where } c \neq 0 \text{ or } 1 \\ \text{or } I_1 &= -\log_e \{\Pi(Y_i/Y)^{1/n}\} && \text{where } c = 1 \\ \text{or } I_0 &= 1/n \{\Sigma(Y_i/Y)\log_e(Y_i/Y)\} && \text{where } c = 0 \end{aligned} \quad (10)$$

This class is ordinally equivalent to the iso-elastic class, with $c = \epsilon$, where c is strictly greater than 0. As c tends to 0, we have the Theil measure; and $c = -1$ gives the square of the coefficient of variation. These measures are based on the assumption of scale invariance. Kolm (1976a and b) has proposed the class of measures

$$I_k = 1 - \left\{ (1/n)\Sigma \left[\frac{Y_i + \xi}{\bar{Y} + \xi} \right]^{1-\epsilon} \right\}^{1/(1-\epsilon)} \quad \text{where } \epsilon \neq 1 \quad (11)$$

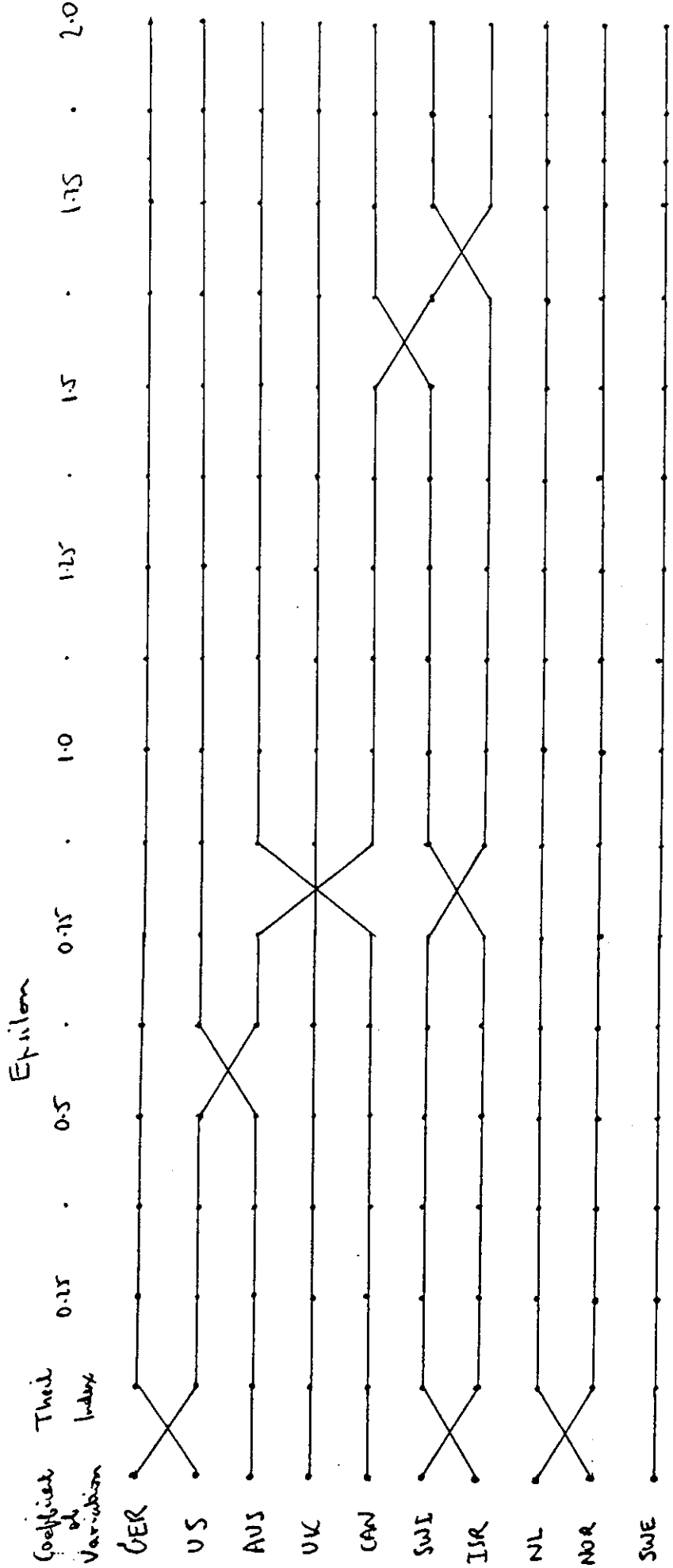
which are invariant with respect to an equi-proportionate increase in $Y_i + \xi$ (so that $\xi = 0$ corresponds to the case of scale invariance).

The use of summary measures belonging to the generalised entropy class is illustrated in Figure 6. The calculations are based on the aggregated quintile data in Table 1, and do not necessarily correspond to those which would result from using the full data-set. For this reason, the results are quoted simply in terms of rankings. These are given first for the

coefficient of variation ($c = -1$), where the ranking is the same as that with the Gini coefficient. The large share of the top 20% in Germany is given particular weight by this measure. As we move to the Theil indicator ($c = 0$), the US moves to the top, and Israel and Switzerland, and Norway and the Netherlands, change places. The remaining values correspond to ϵ ranging from 0.25 to 2.0. It is interesting to note that the ranking does not change greatly: of the 45 possible pair-wise comparisons, only 7 change ranking over this range. The ranking with $\epsilon = 2.0$ agrees with that according to the Rawlsian criterion, except that Germany and Norway change places and that Australia and Canada rank equally.

The application of this class of summary measures provides valuable information about the relation between the ranking of different countries and different distributional judgments as parameterised by ϵ or c . Moreover, it gives a numerical measure of the extent of inequality. But their use raises again the issue of the trade-off with mean incomes.

Figure 6 Rankings According to the Generalised Entropy Measure



8. Differences in Mean Incomes Again

With the use of summary measures such as those in the iso-elastic class, there remains the question as to how equity should be combined with efficiency as represented by the mean income, \bar{Y} .

The simple answer in the case of the measure I_{ϵ} is to multiply by mean income, or, more accurately, to consider $\bar{Y}(1-I)$ as a measure of overall welfare. This corresponds to the procedure of Sen (1976), for example, who compares states in India according to $\bar{Y}(1-G)$, where G is the Gini coefficient. Taking the mean incomes from Table 2, and the Gini coefficients quoted at the start of the paper, this would give for the LIS data-set:

	\$US 1979
USA	7,236
Canada	7,037
Sweden	7,295
Switzerland	6,677
Australia	6,273
West Germany	5,781
Netherlands	6,298
Norway	6,169
UK	4,830
Israel	4,244

The countries are listed in order of their mean income, so that it can be seen that the equity adjustment does indeed change the ranking: Canada falls below Sweden on account of its greater inequality, the Netherlands rises above Australia, and West Germany falls below Norway.

Simple multiplication of $(1-I)$ by the mean implies in the case of the iso-elastic measure taking the equally distributed equivalent income, since $\bar{Y}(1-I) = Y_{ede}$. The implications for the rankings of different countries are shown in Figure 7. With a value of $\epsilon = 0.25$, we have a ranking identical to that by mean income, except that the Netherlands are above Germany. As may be seen, as ϵ increases, a number of countries change ranking; and there are more intersections than in the case where we

considered only I_{ϵ} . With a value of $\epsilon = 2.0$, Sweden, Canada, Switzerland and the Netherlands have all overtaken the US.

The simple multiplication approach is a natural one if we are adopting a straightforward welfare function approach; if instead the evaluation is assumed to involve separate efficiency and equity considerations, then people may attach different weights. One way of taking these into account is to follow Graaff (1977) and consider

$$Y \cdot (1-I)^{\sigma}$$

where the parameter σ allows differing weight to be placed on the equity component. Graaff considers the range of σ from 0 to 1, with 0 corresponding to the view of a person with no concern about equity and 1 corresponding to the simple multiplication considered above. We may calculate for example that with the Gini coefficient the greater inequality in Canada than in Sweden is exactly offset by the higher mean income where $\sigma = 0.53$. On the other hand, we may want to consider σ in excess of 1. For example, with $\sigma = 2$ and ϵ tending to infinity, this procedure is equivalent to ranking according to Y_{ede}/\sqrt{Y} . In other words, we consider the position of the least advantaged *relative* to a standard that rises with average incomes but less than proportionately. In the limit, as σ tends to infinity, we are concerned solely with the relative position.

At this point, it should be noted that we have moved beyond our starting assumption that the evaluation of a social state is solely a function of individual welfare levels. This may, of course, be true of the function $W\{Y_1, \dots, Y_i, \dots, Y_n\}$, in that the way in which individual incomes enter may depart from the evaluation of individuals: for example, because in making a social judgment we decide to bypass individual preferences (as for example in an egalitarian form of the social welfare function concerned with distance) or because we lack information about

these preferences (Sen, 1973, p 51). But the introduction of non-welfare information may lead the social welfare function to have a different form, and this is an aspect which clearly needs further consideration. As Sen has commented, "most actual public judgments make extensive use of non-utility information, varying from relative incomes and ownerships to the description of who is doing what to whom" (1982, p 18). The form discussed in the previous paragraph may be seen as one way of introducing notions of relativity.

9. Conclusions

The main focus of this paper has been on differences in social judgments concerning the measurement of inequality. Where people disagree about the weight to be attached to different aspects of inequality, what can be said? I have argued that one should neither conceal such differences, nor despair. An intermediate approach has been suggested, making use of two main instruments to identify common ground.

First, I have argued that dominance conditions may provide tools which are both powerful and easy to apply in empirical research. The best-known of these is the Lorenz curve and this is in widespread use. But I have also shown how the generalised Lorenz curve (Section 4), and the analogue of the Lorenz curve where families differ in needs (Section 5), may be illuminating. Secondly, we may seek to parameterise differences in social judgments, examples being the exponent ϵ or c of the iso-elastic/generalised entropy inequality measures (Sections 6 and 7), or the trade-off σ between equity and efficiency (Section 7). Worthy of further examination are the parameter s relating equivalence scales to family size (Section 5) and the parameter ξ referred to briefly in Sections 4 and 7.

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