

Corsetti, Giancarlo; Cavallari, Lilia

**Working Paper**

## Policy Making and Speculative Attacks in Models of Exchange Rate Crises: A Synthesis

Center Discussion Paper, No. 752

**Provided in Cooperation with:**

Yale University, Economic Growth Center (EGC)

*Suggested Citation:* Corsetti, Giancarlo; Cavallari, Lilia (1996) : Policy Making and Speculative Attacks in Models of Exchange Rate Crises: A Synthesis, Center Discussion Paper, No. 752, Yale University, Economic Growth Center, New Haven, CT

This Version is available at:

<https://hdl.handle.net/10419/160657>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

ECONOMIC GROWTH CENTER

YALE UNIVERSITY

P.O. Box 208269  
27 Hillhouse Avenue  
New Haven, CT 06520-8269

CENTER DISCUSSION PAPER NO. 752

**POLICY MAKING AND SPECULATIVE ATTACKS IN  
MODELS OF EXCHANGE RATE CRISES:  
A SYNTHESIS**

Giancarlo Corsetti  
University of Rome III

Lilia Cavallari  
University of Rome "La Sapienza"

February 1996

**Note:** Center Discussion Papers are preliminary materials circulated to stimulate discussions and critical comments. This paper was originally written June 1995. Professor Corsetti was a Visiting Lecturer in the Economics Department at Yale University.

# Policy Making and Speculative Attacks in Models of Exchange Rate Crises: A Synthesis\*

*Lilia Cavallari*

University of Rome "La Sapienza"

*Giancarlo Corsetti*

University of Rome III

February 1996

## Abstract

*This paper discusses within a common analytical framework the logical and analytical links between theories of exchange rate crises that model the abandonment of a peg as an optimizing decision by rational policy makers, and theories that focus on the dynamics of speculative attacks when policies are incoherent with the indefinite defense of the current nominal parity. We show that, in both cases, the condition for a crisis to occur can be expressed by a simple arbitrage rule on exchange rates. We analyze the different economic mechanisms underlying the common arbitrage rule. We also discuss a common mechanism raising the possibility of multiple instantaneous equilibria and self-fulfilling speculative attacks.*

KEY WORDS: Speculative Attacks, Exchange Rate Crises

## 1 Two styles of modelling currency crises

This paper reconsiders the literature on exchange rate crises by contrasting two main approaches to modelling a currency collapse. According to the first approach, the crisis is triggered by the behavior of private agents: the

---

\*Correspondence to: Giancarlo Corsetti, Dip. Economia, Universita' degli Studi Roma Tre, Via Ostiense 139, 00145 Roma, Italy. E-mail: corsetti @ elektra.econ.yale.edu.

analysis is focused on the dynamics and timing of speculative attacks in the foreign exchange markets. According to the alternative approach, abandoning the defense of an exchange rate parity (or target zone) is ultimately a policy decision. The analysis is therefore focused on policy makers' preferences and behavior. Differently from other contributions (see Obstfeld [1994] and Buitier, Corsetti and Pesenti [1996] among others), we contrast these two modelling styles within a *common* analytical framework. Starting from the specification of a small open economy, we presents two analyses of an exchange rate crisis, each representing one modelling style, and contrast the arbitrage mechanism that triggers the crisis.

The first approach, pioneered by Krugman [1979], takes speculative attacks as the proximate cause of an exchange rate crisis. Speculative attacks are the optimal market reaction to policies that are inconsistent with the indefinite survival of a fixed nominal parity. The simplest version of the model assumes a lower bound on international reserves beyond which the parity is no longer defended, while domestic credit expansion persistently exceeds the growth of money demand. Under *both* these assumptions, the exchange rate regime must eventually collapse, even in the absence of a speculative attack: for a given money demand in a credible peg, international reserves shrinks at the same rate of domestic credit expansion, and reach their lower bound in finite time. The event of an exchange rate crisis without a speculative attack, defined "natural collapse" by Grilli [1986], is associated with jumps in both the interest and the exchange rates. In competitive markets, however, rational expectations rule out the possibility of anticipated jumps in prices providing clear arbitrage opportunities. The actual collapse of the parity must therefore occur before the natural collapse. In each period, private agents can attack the currency, drive reserves to their lower bound and force the policy maker to abandon the exchange rate target. An exchange rate crisis will thus occur as soon as a speculative attack on the currency yields non-negative profits.

The exact timing of the crisis of a fixed exchange rate regime may nonetheless be indeterminate. A seminal paper by Obstfeld (Obstfeld [1986]) makes this point by assuming that the rate of domestic credit expansion becomes inconsistent with the defense of the nominal parity only conditional on the realization of a speculative attack. For the same fundamentals of the economy (say, for the same level of wages and the same realization of exogenous shocks to productivity and velocity), it is possible to have two *instantaneous equilibria*: one is characterized by a run on foreign exchange reserves, that

triggers a policy switch and therefore leads to the collapse of the exchange rate regime; in the other equilibrium, no attack occurs and the current parity is maintained. As a result, the timing of the attack depends on the coordination of the private sector expectations on a particular regime. In this paper, we offer a different characterization of multiple instantaneous equilibria in a Krugman-style model, that stresses the conceptual and analytical analogy with the second approach.

The second approach is championed by Buitier, Corsetti and Pesenti [1995, 1996], De Kock and Grilli [1993], Drazen and Masson [1993], Flood, Bhandari and Horne [1989], Obstfeld [1991, 1994] and Ozkan and Sutherland [1994] among others. As in Buitier, Corsetti and Pesenti [1996], we will refer to it as the "endogenous policy approach" to modelling currency crises. The analysis focuses on rational policy makers who use the exchange rate as an optimization instrument. The peg of the exchange rate can be used, for example, as a means to gain anti-inflationary credibility. The policy maker decides whether to defend or to abandon the current parity weighting social costs and benefits of each regime.

As the commitment to defend the nominal parity is only imperfectly credible, the expectations game between the private sector and the policy maker may generate multiple equilibria. In one equilibrium, the private sector develops expectations of exchange rate stability, reducing nominal wages and interest rates. This lowers the benefits from a devaluation and strengthens the policy-maker ability to defend the exchange rate parity. In a second equilibrium, agents assign a high probability to a realignment, driving up wages and interest rates and making the regime more vulnerable to domestic and foreign shocks. Thus, the timing of the crisis is indeterminate, insofar as the stability of the exchange rate may be affected by a sudden shift in the regime of expectations.

This paper aims to explore the logical and analytical links between the Krugman's approach and the endogenous policy approach. Section 2 presents a model of a small open economy. Within this common specification of the economy, we will describe the mechanics of speculative attacks leading to the collapse of a fixed exchange rate regime in Section 3, and analyze rational policy making resulting in an exchange rate crisis in Section 4. In each section, we will begin with a characterization of monetary policy; we will then move to the analysis of the arbitrage condition underlying the crisis; we will analyze the mechanics of the crisis; finally we will explore the possibility of multiple instantaneous equilibria and discuss the role of self-fulfilling

speculative attacks. Section 5 presents our conclusions.

## 2 A common analytical framework

### 2.1 The structure of the economy

This section describes the structure of a small open economy that includes an aggregate supply function and a monetary sector. Unless otherwise specified, all variables (except interest rates) are expressed in logarithms. The real sector is as in Obstfeld [1994]. The aggregate supply is a function of the real wage and of a productivity shock:

$$y_t = \alpha (p_t - w_t) - u_t \quad (1)$$

where  $p_t$  is the price level,  $w_t$  the nominal wage and  $u_t$  a productivity disturbance that is identically and independently distributed with a zero mean. The model is characterized by nominal wage rigidity: all wage contracts are signed at the beginning of each period, and are valid for one period only. Workers' objective function is quadratic in the deviation of income from its equilibrium value ( $y_t = 0$ ). With rational expectations, the optimal wage is thus equal to the expected price level<sup>1</sup>:

$$w_t = E_{t-1} (p_t) \quad (2)$$

where  $E_{t-1}$  is the expectation operator conditional on information available in period  $t - 1$ .

Money demand is a function of income, nominal interest rate,  $i_t$  and of a velocity shock,  $\lambda_t$  :

$$m_t^d - p_t = \phi y_t - \delta i_t + \lambda_t \quad (3)$$

Money supply differs in the two approaches to currency crises under consideration. It will be specified in Sections 3 and 4 below.

We assume both uncovered interest parity (UIP)

---

<sup>1</sup>The workers' objective function is

$$\min_w \frac{1}{2} E_t (y_t)^2$$

The expression in the text derives from the first order condition  $E_t y_t = 0$ .

$$i_t = i_t^* + E_t[e_{t+1}] - e_t \quad (4)$$

and purchasing power parity (PPP)

$$e_t = p_t \quad (5)$$

where the foreign nominal interest rate  $i_t^*$  is stationary and the foreign price level is constant and equal to one, so that the nominal exchange rate coincides with the domestic price level.

## 2.2 The semi-reduced form

Substituting the aggregate supply (1), the demand for money (3) and the PPP condition (5) in the uncovered interest parity condition (4), we can describe the dynamics of the exchange rate in terms of the following differential equation:

$$e_t - \frac{\delta}{1 + \delta + \alpha\phi} E_t[e_{t+1}] = \frac{1}{1 + \delta} [m_t + \alpha\phi w_t + \phi u_t + \delta i_t^* - \lambda_t] \quad (6)$$

Solving this differential equation forward under the appropriate transversality condition, the equilibrium exchange rate can be expressed as a function of four variables: the current wage level  $w_t$ , that is predetermined during the period; a linear combination of the current and the expected future money supply; a contemporaneous productivity shock,  $u_t$ ; and an index of nominal shocks  $\epsilon_t$ , that includes exogenous uncertainty regarding velocity and the foreign interest rate

$$e_t = \frac{1}{1 + \delta + \alpha\phi} \left[ \alpha\phi w_t + E_t \sum_{s=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^s m_{t+s} + \phi u_t + \epsilon_t \right] \quad (7)$$

where the index of nominal shocks  $\epsilon_t$  is a forward-looking variable:

$$\epsilon_t \equiv E_t \sum_{s=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^s [\delta i_{t+s}^* - \lambda_{t+s}]$$

Note that the expression (7) incorporates the equilibrium condition in the labor market (2): with one-period contracts, the difference between expected future wages and exchange rates is zero. The equilibrium exchange rate in

semi-reduced form (7) together with the labor market equilibrium condition (2) will be our main analytical tools in Sections 2 and 3 below. In these sections, we will analyze a currency crisis following the two alternative modelling styles. As the specification of monetary policy is very different in the two models, we will begin our discussion from a description of monetary policy in relation to the policy makers' macroeconomic goals.

### 3 Krugman-style models of exchange rate crises

Krugman-style models of exchange rate crises focus on the optimizing behavior of private markets during a crisis. The public sector behavior is generally reduced to a policy function that is informally related to (rather than formally derived from) specific policy objectives. Spotlights are on private agents, that determine a crisis arbitrating profits across exchange rate regimes.

#### 3.1 A policy function

The basic ingredients of Krugman [1979] analysis are well known. The first one is an exogenously given process driving domestic credit, such as

$$d_{t+1} = \eta d_t + \xi_t \quad \eta \geq 1 \quad (8)$$

where  $d_t$  is domestic credit and  $\xi_t$  is white noise. Informally, the process of domestic credit expansion (8) is sometimes related to the need for seigniorage that arises in the presence of persistent budget deficits. The second basic ingredient is a lower limit to the stock of international reserves, below which the monetary authorities stop defending the exchange rate parity  $\bar{e}_t$ . Some contributions in the literature have related this limit to public sector solvency assuming a high opportunity cost of borrowing reserves (for a discussion see Buitier [1987]). For the sake of simplicity, we normalize the lower threshold on reserves to zero

$$R_t \geq \underline{R} = 0 \quad (9)$$

where  $R_t$  is the level of reserves. Note that combining the two assumptions (8) and (9) is equivalent to imposing a lower bound on money supply:

$$m_t \geq \ln(D_t + \underline{R}) \quad (10)$$

where  $D_t$  denotes the level of domestic credit. Expressions (8) and (9) characterize our policy function.



### 3.2 Speculative attacks and arbitrage in the foreign exchange market

Using a Krugman-style specification, the analyst is interested in modelling markets' behavior around a crisis. Rational agents know that, given the policy function defined by (8) and (9), no fixed parity  $\bar{e}_t$  can be sustained indefinitely. This point can be illustrated by means of a simple argument. Suppose that markets consider the parity  $\bar{e}_t$  perfectly credible so that the wage level coincides with the exchange rate parity. Then, money demand is stationary, depending only on exogenous shocks

$$m_t^d - \bar{e}_t = -\phi u_t - \delta i_t^* + \lambda_t \quad (11)$$

Against a stationary money demand, domestic credit is trended. Over time, the process of domestic credit expansion tends to drive international reserves towards their lower limit. As a result, the current parity is bound to be abandoned in finite time with probability one. The question is then: should rational agents wait for such a "natural collapse" of the exchange rate regime, or act before that?

Private agents can, in each period, attack the currency, drive international reserves to the lower limit and cause the abandonment of the peg. A speculative attack will be (weakly) profitable if and only if the exchange rate does not appreciate as a consequence of the attack, that is

$$\text{attack iff } \Lambda_t = (R_t - \underline{R})(e_t - \bar{e}_t) \geq 0 \quad (12)$$

where  $\Lambda_t$  is aggregate net profit from a speculative attack. Loosely speaking, informed agents will not attack the currency "too early", that is, before the domestic credit expansion has gone far enough to result in an exchange rate depreciation when the current parity is abandoned. A rational decision thus requires agents to compute the post-attack level of the exchange rate.

The key feature of the model that allows agents to pick up the right time for an attack is the stability of the monetary policy regime described by (8) and (9). After a speculative attack has driven international reserves to their lower limit, the policy maker gives up the exchange rate target. From then on, the level of money supply is exogenously determined and coincides with domestic credit

$$m_{t+s}^R = \ln(D_{t+s} + \underline{R}) = d_{t+s} \quad s \geq 0 \quad (13)$$

where a superscript  $R$  means "conditional on the abandonment of the peg". Therefore, the current and expected future money supply after a speculative attack can be calculated based on the knowledge of the current stock of domestic credit

$$E_t \sum_{s=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^s m_{t+s}^R = \frac{1+\delta}{1+\delta-\eta\delta} d_t \quad (14)$$

The exchange rate conditional on a speculative attack, denoted by  $\tilde{e}_t$  will be given by the semi-reduced form (7) evaluated using (14)

$$\tilde{e}_t = \frac{1}{1+\delta+\alpha\phi} \left[ \alpha\phi w_t + \frac{(1+\delta)}{1+\delta-\eta\delta} d_t + \phi u_t + \epsilon_t \right] \quad (15)$$

Note that the above expression is valid for any wage rate, including the equilibrium one, to be discussed in the next section. Flood and Garber [1984] define  $\tilde{e}_t$  as the *shadow exchange rate*. The condition for an exchange rate crisis to occur can be re-defined in terms of the shadow exchange rate

$$\text{attack iff } \tilde{e}_t - \bar{e}_t \equiv \Delta\tilde{e}_t \geq 0 \quad (16)$$

where, for future reference, we define  $\Delta\tilde{e}_t$  as the *shadow depreciation rate (SDR)*. In each period, the exchange rate parity will be maintained if the *SDR* is negative, abandoned otherwise

$$\begin{aligned} e_t &= \bar{e} & \text{if } \Delta\tilde{e}_t < 0 \\ e_t &= \bar{e} + \Delta\tilde{e}_t & \text{if } \Delta\tilde{e}_t \geq 0 \end{aligned} \quad (17)$$

The shadow exchange rate is a linear function of both real and nominal shocks,  $u_t$  and  $\epsilon_t$ . Figure 1 plots the fixed parity and the shadow exchange rate against the support of the combined shock  $v_t = \phi u_t + \epsilon_t$ , for given wages.

Figure 1 here

The actual exchange rate will coincide with the parity to the left of the threshold value  $\bar{v}_t$ , where the *SDR* is below  $\bar{e}_t$ . It coincides with the shadow exchange rate to the right of  $\bar{v}_t$ .

The basic mechanism underlying Krugman's analysis is private arbitrage. Profit-maximizing agents arbitrate profits across the two *exchange rate regimes*. It is important to stress that there is no corresponding switch in the monetary policy regime. On the contrary, it is the stability of the policy rule

defined by (8) and (9) that gives speculators the option to force a crisis by launching a speculative attack before the point of a "natural collapse". In equilibrium, the payoff of an optimal exercise of such an option is

$$\Lambda_t = \text{Max} \{0, (R_t - \underline{R})\Delta\tilde{e}_t\}$$

Note that Figure 1 can also be reinterpreted in terms of such a payoff per unit of international reserve money.

As well known, Krugman's original specification does not allow for uncertainty. The basic mechanism of an exchange rate crisis following a speculative attack is a deterministic one. Including uncertainty in the model is nonetheless a meaningful exercise, insofar as it accounts for positive interest differentials emerging with expectations of a devaluation. Note also that, in our framework, output is not fixed, but depends on the exchange rate policy. A crisis raises output and employment, with a positive effect on the demand for real balances. During the crisis, the sharp contraction in the demand for money that determines the speculative attack on reserves is exclusively driven by the increase in the interest rate anticipating a future depreciation.

### 3.3 Multiple instantaneous equilibria

The case for multiple equilibria in a Krugman-style model of exchange rate crisis is first presented by Obstfeld [1986], who modifies the assumption of a stable monetary policy regime. The domestic credit component of the policy function (8) becomes conditional on the occurrence of a speculative attack

$$\begin{aligned} d_{t+1} &= \eta d_t + \xi_t && \text{if speculative attack} \\ d_{t+1} &= \bar{d} && \text{otherwise} \end{aligned} \quad (18)$$

The stability of the exchange rate then depends on what the private sector does<sup>2</sup>. For a given level of wages, over some range of the support of the exogenous shocks  $v_t$  two instantaneous equilibria are possible: if agents attack the currency, the policy maker switches monetary regime and the fixed exchange rate regime collapses; if there is no speculative attack, the policy maker maintains a behavior consistent with the survival of the peg. Such a

---

<sup>2</sup>Obstfeld [1986] carefully restricts the parameters as to rule out the possibility of a natural collapse in the absence of a speculative attack.

specification runs into the problem of defining what determines the coordination of private agents on one of the two possible equilibria. The coordination mechanism is usually assumed to be driven by exogenous uncertainty.

It is worth stressing that, in our specification, there is an additional channel through which multiple instantaneous equilibria may emerge, different from the one proposed by Obstfeld [1986]. This channel is the expectations game between the private and the public sectors underlying the determination of nominal wages, to be further discussed at the end of Section 4.

Consider again the process of domestic credit expansion (8) and focus on the determination of nominal wages given by expression (2). By writing out this expression, the nominal wage will be equal to the current parity plus the expected depreciation rate

$$w_t = \bar{e}_t + \pi_t E_{t-1} (\Delta e_t \mid \Delta \bar{e}_t \geq 0) \quad (19)$$

where  $\pi_t$  is the probability of a devaluation in period  $t$ . By (16),  $\pi_t$  coincides with the probability that the shadow depreciation rate be positive

$$\pi_t = \text{prob} [\Delta \bar{e}_t \geq 0]$$

What raises the possibility of multiple equilibria is the fact that the shadow depreciation rate is itself a function of nominal wages. It turns out that, for a large class of distribution of shocks, the equilibrium nominal wage may not be unique. In appendix I we solve for the rational expectations level of nominal wage, and present a simple example where multiple equilibria may arise with a uniform distribution of the shock  $v_t$ .

In the presence of multiplicity of equilibria, anticipations of exchange rate instability leads to high nominal wages, that reduce output and determine a drop in money demand. For a given level of domestic credit, the drop in money demand translates into a contraction of the stock of international reserves. This is the channel through which high nominal wages increase the probability of a devaluation in a self-fulfilling way. Conversely, low nominal wages are associated with a higher output, that raises money demand. The rate of depletion of international reserves is slowed down. As a result, low wages decrease the probability of a devaluation, making the current parity less vulnerable to nominal and real shocks. The stability of the exchange rate parity during the current period depends on which equilibrium wage setters coordinate their expectations when contracts are signed.

## 4 The "endogenous policy approach"

A different approach to exchange rate crises stresses the fact that abandoning a fixed exchange rate regime is ultimately a policy choice. Policy makers are now the main actors on stage: in the analysis, private agents arbitrating profits are replaced by policy makers arbitrating social utility across exchange rate regimes. In order to highlight analogies and differences with the previous approach, we will use the same layout of the model as in section 3. Our analysis thus begins with the description of monetary policy in relation to the policy makers' objective function.

### 4.1 Policy preferences

In the context of the endogenous policy approach, a policy function is derived from "first principles" rather than posited by assumption. In our specification, a policy maker lacking anti-inflationary credibility uses the exchange rate as an imperfect commitment device. At the beginning of each period, *before* wage contracts are signed, she commits to pursue a unilateral peg. Such a commitment is credible only to the extent that the incentive to renege on it *after* wage contracts are signed is offset by some cost of abandoning the peg. Her objective function thus includes two components. The first one is the usual quadratic loss function that expresses the inflation-employment trade-off. The second component of the objective function is a lump-sum welfare cost, paid by the policy maker in the event of a devaluation.

The policy maker minimizes the following loss function:

$$L_t = \sum_{s=0}^{\infty} E_t [\beta^s \ell_{t+s} + c_{t+s} z_{t+s}] = \quad (20)$$

$$\sum_{s=0}^{\infty} E_t \beta^s \left\{ \theta (p_{t+s} - p_{t+s-1})^2 + (y_{t+s} - y^*)^2 + c_{t+s} z_{t+s} \right\}$$

where  $\beta$  is the discount factor,  $\ell_t$  is the one-period loss function,  $y^*$  is the target level of output, greater than the market equilibrium level ( $y_t=0$ ),  $\theta$  measures the aversion towards inflation,  $c_t$  is the lump sum welfare cost of reneging on the exchange rate commitment, and  $z_t$  is a dummy variable assuming the value of one in case of realignment and zero otherwise:

$$z_t = 0 \text{ if } e_t = \bar{e}_t \quad (21)$$

$$z_t = 1 \text{ if } e_t \neq \bar{e}_t$$

In the tradition of Kydland and Prescott [1977] and Barro and Gordon [1983], the policy maker's preference for a target output greater than the equilibrium one<sup>3</sup> induces an inflationary bias in policy making. With rational expectations, private agents are aware of the *ex-post* incentive for the policy maker to resort to a surprise inflation-devaluation. Expected inflation can be contained only if such an incentive is reduced or eliminated altogether. In our specification, the fixed exchange rate is at least imperfectly credible because abandoning the peg entails welfare costs. We adopt the simplifying assumption of treating this cost as a *political cost* associated to a devaluation, exogenously given, reflecting the loss of prestige due to a surprise devaluation. Consistently, we assume that the private sector does not behave strategically over time and current expectations are independent of past policy choices. Alternatively, the realignment costs could be endogenously modelled as reputation costs, emerging from the repeated game between the private and the public sector.<sup>4</sup>

The timing of the model is as follows. At the beginning of each period, the policy maker announces that she will defend the current level of the exchange rate  $\bar{e}_t = e_{t-1}$ . Given such an announcement, workers sign wage contracts prior to the realization of current shocks – so that prices are *ex-ante* flexible –. During the period, given nominal wages, the public sector optimally chooses its monetary policy contingent on the realization of the shocks.

---

<sup>3</sup>The preference for a level of output larger than the equilibrium one is generally motivated by referring to the existence of distortions that make the market equilibrium suboptimal.

<sup>4</sup>In a more general specification of our model, the variable  $c_t$  can be reinterpreted as a synthetic indicator of all intertemporal implications of a surprise abandonment of the exchange rate. In this case, the dummy variable  $z_t$  would express the intertemporal strategies of the private sector in a repeated game with the public sector. For example, we may assume that, when the policy maker reneges on her commitments, the private sector will not believe her announcement for  $S$  periods. During this time span, a unilateral peg would completely lose credibility:  $z_{t+s} = 1$ , for  $s=1,2,\dots,S$ . The only possible equilibrium would be one with floating exchange rates and a discretionary monetary policy. In our paper, we rule out the possibility that private agents coordinate their expectations over time. Thus,  $z_t$  has no memory of the past. Using Obstfeld's [1994] terminology, agents form memoryless expectations.

## 4.2 Realignments and social welfare arbitrage

In each period, the policy maker can choose between defending and abandoning the exchange rate parity. If she chooses to defend the current parity, money supply must accommodate all monetary and real shocks hitting the economy at the time. Money supply conditional on defending the peg, denoted by  $m^F$ , will be as follows:

$$m_t^F = (1 + \delta + \alpha\phi) \bar{e}_t - \left[ \phi\alpha w_t + \phi u_t + \epsilon_t + E_t \sum_{s=1}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^s m_{t+s} \right] \quad (22)$$

If the policy maker chooses to abandon the exchange rate target, it is rational to pursue a discretionary optimal monetary policy. In general, a multi-period loss function such as (20) makes the identification of an optimal policy rather complicated, because current actions may affect the future state of the economy. This is not the case in our analysis: the model has no proper state variables, wage contracts last only one period and private sector expectations do not depend on past behavior of the policy makers. Because of these features of the model, monetary policy only affects the current state of the economy, and the optimal policy can be identified by minimizing the current period loss function  $\ell_t$ . The optimal supply of money conditional on abandoning the nominal parity,  $m^R$ , is determined by the first order condition:

$$\frac{\partial \ell_t}{\partial m_t} = 0 \quad (23)$$

Consistently with our analysis of the Krugman's approach, we define the *shadow depreciation rate*  $\Delta \bar{e}_t$  as the devaluation rate that follows the abandonment of the peg. In the present context, the *SDR* corresponds to the monetary policy implicitly determined by the condition (23).

The policy maker will rationally choose between the two possible monetary policy regimes ( $m^F$  and  $m^R$ ) comparing their relative costs and benefits in terms of the loss function (20). The exchange rate parity will be given up if and only if social welfare in a flexible exchange rate regime ( $-\ell_t^R$ ) exceeds social welfare in a fixed exchange rate regime ( $-\ell_t^F$ ) at least by the fixed cost  $c_t$ :

$$\text{devalue iff } \ell_t^F - \ell_t^R \geq c_t \quad (24)$$

Note that, since  $m^R$  is the money supply that minimizes the one period loss function, by construction the left hand side of the above inequality cannot be negative (i.e.  $\ell_t^F - \ell_t^R \geq 0$ ). Without a positive realignment cost  $c_t$ , the policy maker's commitment to a fixed exchange rate would never be credible.

The condition (24) is the analog of the private arbitrage condition (12) in section 3. There are two differences. First, the main actor is not the market, but the policy-maker, who has the option of switching between monetary policy regimes: what is maximized is social welfare, rather than private profits. Second, there are fixed costs in exercising the regime switch, that are absent in Section 3.<sup>5</sup> Notwithstanding these differences, both conditions ((24) and (12)) can be expressed in terms of the shadow depreciation rate only.

To begin with, note that the shadow depreciation rate is a function of wages, real shocks and policy targets. By using (23), we have

$$\Delta \tilde{e}_t = -\frac{\alpha}{\theta + \alpha^2} (y_t^F - y^*) = -\frac{\alpha (\alpha (\bar{e}_t - w_t) - y^* - u_t)}{\theta + \alpha^2} \quad (25)$$

It is instructive to interpret the difference between output in a peg and target output ( $y_t^F - y^*$ ) as a measure of the deflationary gap due to the defense of the current nominal parity. The above expression makes us aware that there is a linear relationship between the *SDR* and the deflationary gap associated with the defense of the peg. Because of this property of the model, we can eliminate output from the social arbitrage condition (24), as to obtain

$$\text{devalue iff } |\Delta \tilde{e}_t| \geq \sqrt{\frac{2c_t}{\alpha^2 + \theta}} \equiv c_t^e \quad (26)$$

The policy maker will optimally switch from defending the peg to pursuing a discretionary monetary policy when the shadow depreciation rate (in absolute value) exceeds a critical threshold  $c_t^e$ . The value of such a threshold is positively related to the welfare cost of abandoning the peg  $c_t$ . Clearly, there is a formal equivalence between the above expression and the devaluation condition in the Krugman-style model of Section 3 (16). The analogy is best understood by considering that expression (26) translates the welfare costs and benefits of a devaluation into the metric of the exchange rate. This is the same metric chosen by Flood and Garber in which to cast the Krugman

---

<sup>5</sup>Of course, we could include transaction costs in the Krugman-style model of speculative attacks. This would delay the crisis up to the point in which the shadow exchange rate exceeds the current parity by at least the magnitude of these transaction costs.



model<sup>6</sup>. Note that, differently from (16), the above condition is consistent with both an optimal devaluation and an optimal revaluation following the abandonment of the peg. This is due to assuming a quadratic loss function.

With optimizing policy makers, the exchange rate behavior can be summarized by a scheme that is analogous to (17):

$$e_t = \bar{e}_t + \Delta \tilde{e}_t \quad \text{if } |\Delta \tilde{e}_t| \geq c_t^e \quad (28)$$

$$e_t = \bar{e}_t \quad \text{if } |\Delta \tilde{e}_t| \leq c_t^e$$

Figure 2 plots the actual and the shadow exchange rate against the support of the real shock, given nominal wages.

Insert figure 2

In order to facilitate the comparison with Figure 1, in plotting Figure 2 we have restricted the support of the shock in such a way that a re-valuation will never be optimal. An apparent difference with Figure 1 is due to the presence of lump-sum costs of a realignment, that accounts for the "jump" in the actual exchange rate.<sup>7</sup> A second, more crucial, difference with Figure 1 is that  $\tilde{e}_t$  is not conditional on monetary shocks: this variable is plotted against the support of the real shock only. As opposed to Krugman's model, in the present context there is no constraint on monetary policy. In other words, there is neither an exogenously given process driving domestic credit, nor a lower bound on international reserves. In pursuing her employment and inflation targets, the policy maker will always and fully accommodate monetary shocks.

<sup>6</sup>We could also express the switching rule in terms of the output gap:

$$\text{devalue iff } |y_t^F - y_t^*| \geq \sqrt{\frac{2c_t(\alpha^2 + \theta)}{\alpha^2}} \equiv c_t^y \quad (27)$$

where  $c_t^y$  expresses the opportunity cost of defending the exchange rate in terms of percentage points of output (with respect to the target) sacrificed to exchange rate stability.

<sup>7</sup>Transaction costs in Krugman-style models would drive a wedge between the shadow exchange rate and the defended parity at the threshold value  $\bar{v}_t$ , similar to the one in Figure 2.

### 4.3 Multiple instantaneous equilibria and the role of speculative attacks

This section analyzes the expectations game between the private and the public sector underlying the determination of wages. Such a game plays a more important role here than in the Krugman-style model. In the Krugman's model, once monetary authorities have set their policy, private agents can "force" the government to devalue by launching a speculative attack on the currency that drives reserves to their lower limit. In the endogenous policy approach, once wage contracts are signed, there is nothing that the private sector can do to "force" a devaluation. In the architecture of the model, abandoning the peg is an optimal policy choice. Nonetheless, the private sector can affect the policy outcome by changing the conditions based on which policy makers decide their conduct.

In the context of the endogenous policy approach, the optimal policy decision is contingent on the real shocks hitting the economy during the period as well as on the level of nominal wages. When multiple instantaneous equilibria are possible, over time the policy maker may face different equilibrium levels of the wage rate. The stability of the peg is thus affected by switches in the market coordination among possible equilibria. This mechanism provides the theoretical foundations for a theory of self-fulfilling "speculative" behavior in the modelling style under consideration.

In determining the rational expectations level of the wage rate, multiple instantaneous equilibria arise for exactly the same reason discussed in section 3. Re-define the probability of devaluation as:

$$\pi_t = \text{prob} [|\Delta\tilde{e}_t| \geq c_t^e] \quad (29)$$

Then the computation of equilibrium nominal wage combines expectations of the exchange rates in the two possible regimes, weighted by the corresponding probabilities:

$$w_t = E_{t-1}e_t = \bar{e}_t + \pi_t E_{t-1} [\Delta\tilde{e}_t \mid |\Delta\tilde{e}_t| \geq c_t^e] \quad (30)$$

As before, multiple instantaneous equilibria stem from the fact that the shadow depreciation rate is itself a function of nominal wages. Details are given in the Appendix II.

With multiple equilibria, the stability of the exchange rate regime depends on the equilibrium on which private agents coordinate their expectations.

We carry out a simulation of the model using the same parameter values as in Obstfeld [1994].<sup>8</sup> In the simulation, we obtain two equilibria: a *vicious* equilibrium, characterized by high expected inflation and a high ex ante probability of devaluation; and a more stable, *virtuous* equilibrium, where expected inflation and therefore the wage rate are low.

Figure 3 here

Figure 3c plots the differential of the loss function  $\ell_t$  in the two regimes (given by the left hand side of (24)) as well as a particular value of the cost  $c_t$  against the support of the shock  $u_t$ . The loss differential draws a non-linear function that crosses the cost line twice, identifying two possible equilibrium thresholds  $\bar{u}_t$ .<sup>9</sup> The lower trigger point is associated with high equilibrium wage inflation. At the current parity, this implies a sharp decline in competitiveness and stagflation. Abandoning the peg turns out to be optimal for a wide range of productivity shocks, including favorable but small ones. Figure 3b draws the shadow exchange rate in such an equilibrium. As apparent from the graph, the stability of the exchange rate parity is possible only for large favorable shocks (on the left side of the plot). Conversely, the higher trigger point is associated with moderate inflation expectations. The average deflationary gap due to the defense of the peg is small, and a devaluation is optimal only for sizeable negative shocks.<sup>10</sup> The graph in Figure 3a plots the corresponding shadow exchange rate. It is worth stressing that expectations are self-fulfilling in both equilibria: as the optimal policy is conditional on nominal wages, rationally formed expectations are confirmed *on average* by actual policy making.

Modelling speculative attacks as self-fulfilling prophecies requires a move from the identification of multiple instantaneous equilibria to the analysis

---

<sup>8</sup>The graph is drawn for the following parameters values :  $\alpha = 1$ ,  $\theta = .15$ ,  $y^* = .01$ ,  $\mu = .03$ .

<sup>9</sup>Clearly, for sufficiently low values of the political cost, there could be no interior solution. The threshold level of  $u_t$  would correspond to the lower bound of the support so that the peg would not be credible. We have a unique solution when the cost line is tangent to the non linear function. If the support of the shock is bounded to the right, a sufficiently high value of the cost can assure perfect credibility to the exchange rate regime.

<sup>10</sup>A simple exercise of comparative statics shows that an increase in the cost of realignment enhances the credibility of the commitment to a fixed exchange rate, but only in the *virtuous* equilibrium. The opposite is true in the *vicious* equilibrium.

of a dynamic economy in real time. Even when the economy is in the *virtuous* equilibrium of Figure 3a – the argument goes – one cannot rule out that a sudden shift in the regime of expectations will reduce the stability of the exchange rate parity one period ahead, by moving the economy to the *vicious* equilibrium of Figure 3b. Unfortunately, the literature has not (yet) provided a convincing theory of how expectations coordinate on a particular regime over time. As pointed out in Section 3, the coordination mechanism is generally modelled by resorting to *exogenous* uncertainty, that is, by specifying an *ex ante* arbitrary probability distribution defined over all possible equilibria.

Insofar as such a shift in coordination increases the probability of a devaluation and the equilibrium interest rate, it also causes a contraction in money demand at the time when expectations are formed. The shift from the *good* to the *bad* equilibrium leads to a speculative attack on the currency. Such a speculative attack, however, has no direct consequence on the stability of the exchange rate. The shadow depreciation rate does not depend on monetary shocks, and a shift in the regime of expectations affects policy making only through the level of wages.

## 5 Conclusions

This paper has contrasted two styles of modelling exchange rate crises within a common analytical framework. The first style is initiated by Krugman [1979], who models speculative attacks as the proximate cause of a currency collapse. The focus is on market behavior. According to the second style – the “endogenous policy approach” – the abandonment of a peg is ultimately a policy decision. The focus is on optimal exchange rate policy by rational policy makers. Building on our common framework, we have discussed two analyses of a currency crisis, each representative of a particular modelling style.

In Section 4, we have shown that, in the model we have taken as representative of the endogenous policy approach, the defense of the exchange rate during the period does not depend on the realization of monetary shocks. Provided that maintaining the peg is the optimal strategy for the policy maker, money supply will fully accommodate shocks to velocity and foreign interest rate. This characteristic of the model can be related to the well known result in the literature on the optimal choice of exchange rate regimes

first shown by Poole [1970].

The model of section 4 does not include any constraint on monetary policy, in the form of either lower bounds on foreign reserves or domestic credit requirements. This is to stress the point that the abandonment of the exchange rate parity is a policy choice, rather than an event due to technical reasons (such as the depletion of the stock of international reserves). After all, as Obstfeld and Rogoff [1995] put it, the exchange rate is the price of one currency in terms of another: the monetary authorities can always defend a given exchange rate parity, provided they are willing to reduce the supply of the domestic currency to an appropriately low level. Even if domestic credit is fixed, it is still possible to defend the peg through repeated sales of borrowed international reserves (see Buitier, Corsetti and Pesenti [1996] for a discussion).

These considerations are particularly important in the presence of multiple instantaneous equilibria. At the time when expectations are formed, a shift from a *virtuous* to a *vicious* equilibrium increases the probability of abandoning the parity and, *via* the increase in the interest rate, leads to a contemporaneous contraction of money demand. However, even if the drop in demand drives down foreign exchange reserves, the speculative attack *per se* does not play any role in the exchange rate crisis. As a matter of facts, an interest sensitive demand for money is not even a necessary feature of models following the endogenous policy approach (see Obstfeld [1995] and Buitier, Corsetti and Pesenti [1996] among others). The differences with the Krugman-style model of section 3 are apparent. There, for a crisis to occur, money demand has to be interest-sensitive; moreover, both an exogenous process driving domestic credit and a lower bound on reserves are necessary elements in the analysis.

Nevertheless, it would be misleading to conclude that financial shocks and international reserves can only play a role in Krugman-style models of exchange rate crises. For instance, a more general specification of the public sector in Section 4 could stress the social welfare implications of acquiring reserves, by assuming (realistically) that reserves can be borrowed in the international markets at an increasing marginal cost (as in Buitier [1987]). In this case, costs and benefits of defending the peg would also depend on the opportunity costs of intervening in the foreign exchange market, with the stock of international reserves playing an important role in the dynamics of the crisis.

The analysis in this paper has stressed the fact that multiple instanta-

neous equilibria are not a specific feature of a particular modelling style. We have discussed a common kind of multiplicity that arise in the expectations game between the private and the public sectors when the exchange rate policy is not perfectly credible. While we have focused on nominal wage rigidity, Obstfeld [1994] provides an example based on nominal interest rates.

In conclusion, there is a question of whether different modelling styles apply to different kinds of currency crises or, rather, they tell the same story from a different vantage point. Although a definite answer to this question may not exist, we find more useful to think of them as complementing each other in offering a theoretical perspective on exchange rate crises. We also believe that the modelling patterns discussed in this paper provide an insightful and efficient classification scheme of the literature so far developed.

## Appendix I

This appendix discusses the determination of the equilibrium nominal wage in the Krugman-style model. Substituting (19) in the semi-reduced form of the exchange rate conditional on the occurrence of a speculative attack (15), the equilibrium nominal wage must satisfy

$$w_t = \frac{(1 - \pi_t)(1 + \delta + \alpha\phi)\bar{e}_t}{1 + \delta + (1 - \pi_t)\alpha\phi} + \frac{\pi_t}{1 + \delta + (1 - \pi_t)\alpha\phi} \left[ E_{t-1}(v_t \mid \Delta\bar{e}_t \geq 0) + \frac{(1 + \delta)m^R}{1 + \delta - \eta\delta} \right]$$

For a given money supply, the computation of the equilibrium nominal wage requires the identification of a threshold value of the shock  $\bar{v}_t$  such that, for any realization of the shock larger than this threshold, the private sector will launch a speculative attack leading to a currency depreciation. The threshold value  $\bar{v}_t$  can be computed by including the above expression in the switching rule (16). We obtain

$$-\bar{e}_t + \frac{m_t^R}{1 + \delta - \eta\delta} + \frac{\alpha\phi}{1 + \delta + \alpha\phi} \frac{\pi_t E_{t-1}(v_t \mid v_t \geq \bar{v}_t)}{1 + \delta} + \frac{1}{1 + \delta + \alpha\phi} \left[ 1 + \frac{(1 - \pi_t)\alpha\phi}{1 + \delta} \right] v_t \geq 0$$

A solution for  $\bar{v}_t$  is derived by evaluating this expression with the equality sign. In general, the resulting equation will be non linear. For example, assume that  $v_t$  is uniformly distributed between  $-\mu$  and  $\mu$ . Then, our equation becomes quadratic in  $\bar{v}_t$

$$\frac{1}{1 + \delta} \left[ \frac{\alpha\phi}{1 + \delta + \alpha\phi} \right] \frac{1}{4\mu} \bar{v}_t^2 + \frac{1}{1 + \delta + \alpha\phi} \left[ 1 + \frac{1}{2} \frac{\alpha\phi}{1 + \delta} \right] \bar{v}_t + \left[ -\bar{e}_t + \frac{m_t^R}{1 + \delta - \eta\delta} + \frac{\alpha\phi}{1 + \delta + \alpha\phi} \frac{\mu}{4(1 + \delta)} \right] = 0$$

Provided that the current parity has at least some credibility, the interior solution for the threshold value of the shock - when it exists - is not necessarily unique.

## Appendix II

This appendix describes the determination of the equilibrium nominal wage in the model of Section 4. In the endogenous policy approach, expectations of the exchange rate are determined considering the two possible monetary regimes,  $m_t^F$  and  $m_t^R$ . Using (2) and (25), and restricting the support of the shock as to rule out equilibrium re-valuation, equilibrium nominal wages can be expressed as follows

$$w_t = \bar{e} + \frac{\pi_t \alpha [y^* + E_{t-1}(u_t | u_t \geq \bar{u}_t)]}{\theta + \alpha^2 (1 - \pi_t)}$$

With rational expectations, nominal wages must be consistent with the policy makers' switching rule; we thus substitute the above expression in (26) as to obtain

$$\frac{\alpha}{\theta + \alpha^2} \left( \frac{-(\alpha^2 + \theta)y^* - \alpha^2 \pi_t E_{t-1}[u_t | u_t \geq \bar{u}_t]}{\theta + \alpha^2 (1 - \pi_t)} - u_t \right) \geq c_t^e$$

As in Appendix I, it is possible to characterize the equilibrium condition in terms of a threshold value of the real shock,  $\bar{u}_t$ , such that the policy makers will optimally devalue for any realization of the shock larger than this threshold. The equilibrium value of  $\bar{u}_t$  can be computed by finding the roots of the inequality just specified. In general, we will have to solve a non-linear equation, so that, if an interior solution exists, the equilibrium value of  $\bar{u}_t$  will not be unique. For example when  $u_t$  is uniformly distributed between  $-\mu$  and  $\mu$ , we solve the following quadratic equation

$$\frac{\frac{1}{2}\alpha^2 + \theta}{2\mu} \bar{u}_t^2 + \frac{\alpha^2 + \theta + c_t^e \frac{\alpha^3}{\mu} + c_t^e \alpha \frac{\theta}{\mu}}{2} \bar{u}_t + \left[ y(\alpha^2 + \theta) + \frac{1}{4}\alpha^2 \mu + c_t^e \left( \frac{3\alpha\theta}{2} + \frac{\alpha^3}{2} + \frac{\theta^2}{\alpha} \right) \right] = 0$$

Note that there cannot be multiple equilibria if the devaluation cost is either too low (so that the parity is not credible at all) or very high (so that the exchange rate regime is perfectly credible). Multiple equilibria only arise if the credibility of the exchange rate regime imperfect.



## References

- [1] Agénor P.R., Flood R. and S. Bhandari, "Speculative Attacks and Model of Balance of Payment Crises", National Bureau of Economic Research *working paper* n.3919, November 1991
- [2] Barro R.J. and D.B.Gordon, "A Positive Theory of Monetary Policy in a Natural Rate Model", *Journal of Political Economy*, 91, 589-610, 1983
- [3] Blackburn K. and M. Sola, "Speculative Currency Attacks and Balance of Payment Crises", *Journal of Economic Surveys* vol.7, n.2, 119-144, 1993
- [4] Buitier W. H., Corsetti G. and P. Pesenti, "A Center-Periphery Model of Monetary Coordination and Exchange Rate Crises", *National Bureau of Economic Research, Working Paper* No. 5140, June 1995  
Buitier W. H., Corsetti G. and P. Pesenti, *International Monetary Coordination and Financial Market Regulation: The Lessons from the 92-93 crisis*, Cambridge University Press, 1996, forthcoming
- [5] Canzoneri M.B. and D.W.Henderson, *Monetary Policy in Interdependent Economy*, Mit Press, 1991
- [6] Drazen A. and P. Masson, "Credibility of Policies versus Credibility of Policymakers", National Bureau of Economic Research *working paper* n.4448, September 1993
- [7] Flood R.P. and P. Garber, "Collapsing exchange rate regimes: some linear examples", *Journal of International Economics*, 17, pp. 1-13, 1984
- [8] Flood R.P., J. Bhandari and J. Horne, "Evolution of exchange rate regimes", *IMF Staff Papers*, 36, pp. 810-835, 1989
- [9] Grilli V., "Buying and Selling Attacks on Fixed Exchange Rate Systems", *Journal of International Economics*, 17, 1-13, 1986
- [10] Kydland F.E. and E.C. Prescott, "Rules Rather than Discretion: the Inconsistency of Optimal Plans", *Journal of Political Economy*, 85, 473-491, 1977

- [11] Krugman P. "A Model of Balance of Payment Crises", *Journal of Money, Credit and Banking*, 11, 311-325, 1979.
- [12] Obstfeld M., "Rational and Self-fulfilling Balance of Payments Crises", *American Economic Review*, Vol. 76, No. 1, 676-81, 1986.
- [13] Obstfeld M., "The Logic of Currency Crises", *National Bureau of Economic Research working paper no.4640*, February 1994.
- [14] Obstfeld M. and K. Rogoff, "The Exchange Rate in Monetary Policy", Working paper. Princeton University Press.
- [15] Ozkan F. and A. Sutherland, "A Model of ERM Crises", *Center for Economic Policy Research discussion paper n.879*, January 1994.
- [16] Poole W., "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model", *Quarterly Journal of Economics*, May, 1970
- [17] Rogoff K., "The Optimal Degree of Commitment to an Intermediate Monetary Target", *Quarterly Journal of Economics*, 1169-1190, 1985
- [18] Sargent T. and N. Wallace, "Some Unpleasant Monetarist Arithmetic", *Federal Reserve Bank of Minneapolis Quarterly Review*, 5, n. 3, pp. 1-17, 1981

Figure 1  
Krugman-style model

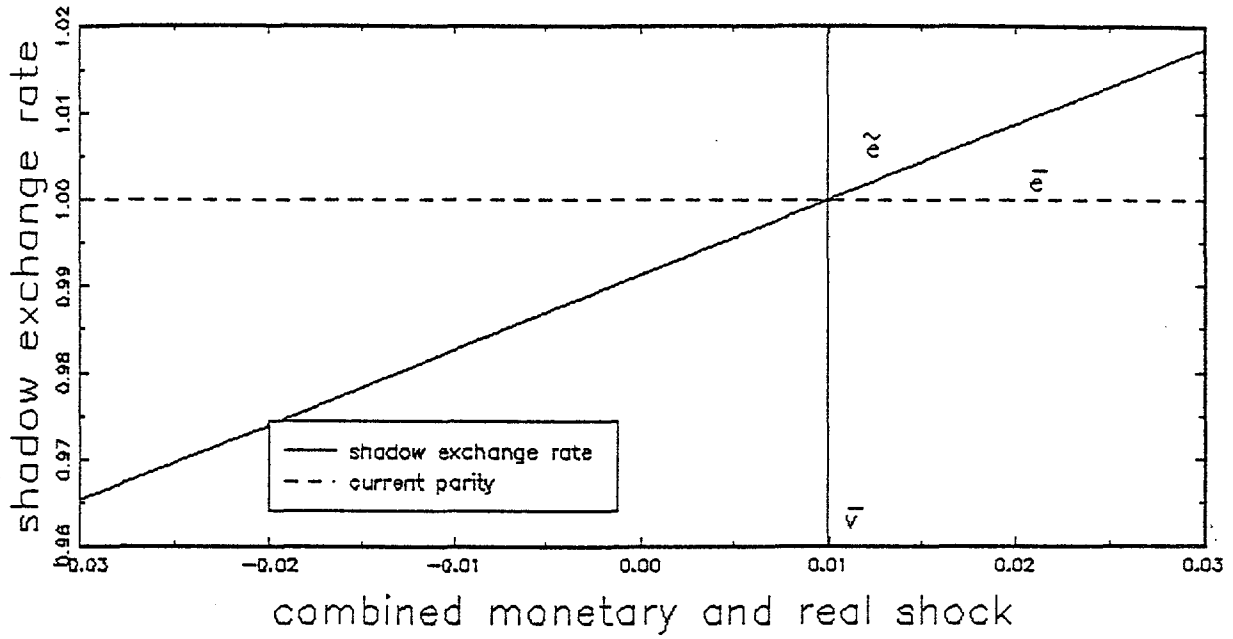


Figure 2  
Endogenous policy approach

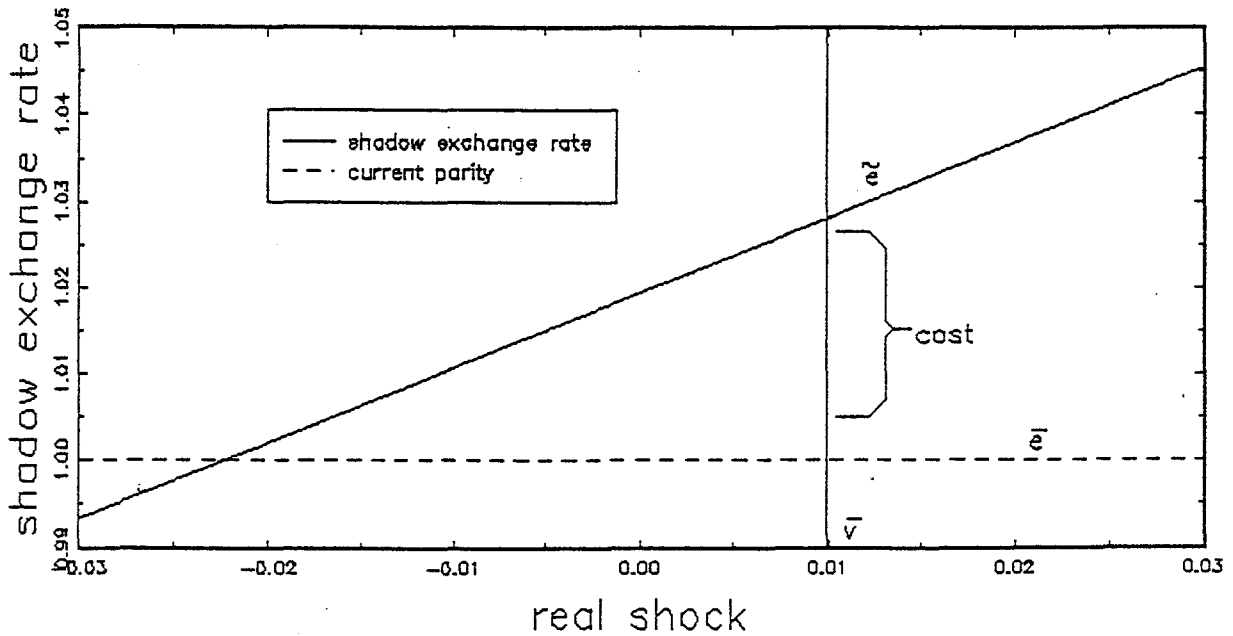


Figure 3a  
Good equilibrium

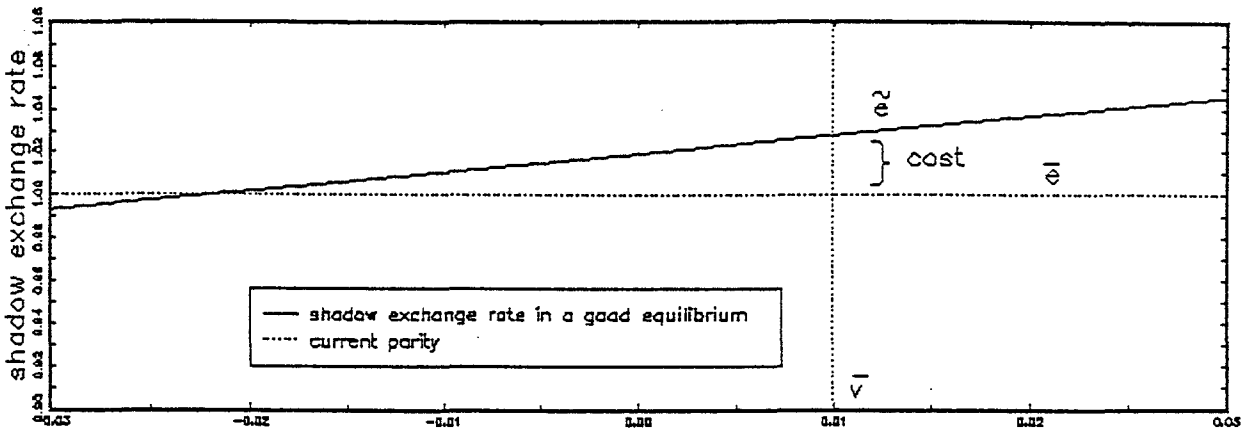


Figure 3b  
Vicious equilibrium

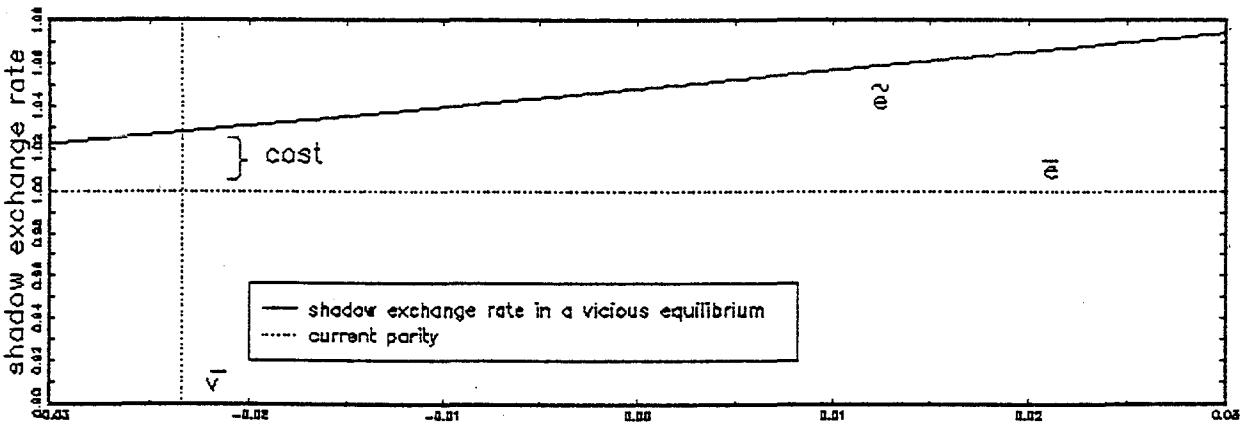


Figure 3c

