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CENTER DISCUSSION PAPER NO. 715

QUALITY IMPROVEMENTS IN MODELS OF GROWTH

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Quality Improvements in Models of Growth*

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Abstract

Technological progress takes the form of improvements in quality of an array of intermediate inputs to production. In an equilibrium that is standard in the literature, all research is carried out by outsiders, and success means that the outsider replaces the incumbent as the industry leader. The equilibrium research intensity involves three considerations: leading-edge goods are priced above the competitive level, innovators value the extraction of monopoly rents from predecessors, and innovators regard their successes as temporary. We show that, if industry leaders have lower costs of research, then the leaders will do all the research in equilibrium. However, if the cost advantage is not too large, then the equilibrium research intensity and growth rate depend on the existence of the competitive fringe and take on the same values as in the standard solution. We discuss the departures from Pareto optimality and analyze the determination of the economy's rate of return and growth rate.

Keywords: Quality Ladders, Technological Progress, Endogenous Growth, Innovation.

JEL: O31, O32, O33, O40, O41.

Some recent contributions to the endogenous-growth literature model technological progress as either an increase in the number of types of products (Romer [1990], Grossman and Helpman [1991, Ch. 3]) or an improvement in the quality of products (Aghion and Howitt [1992], Grossman and Helpman [1991, Ch. 4]). We can think of increases in the number of types, N , as basic innovations that amount to dramatically new kinds of goods or methods of production. In contrast, increases in the quality or productivity of the existing goods involve a continuing series of improvements and refinements of products and techniques.

Figure 1 shows the basic setup. We consider intermediate inputs, as in Ethier (1982) and Romer (1990), and assume that these come in N varieties, arrayed along the horizontal axis. In models of expanding variety, N can increase over time, but we treat N as fixed in the present analysis. The leading-edge quality of each type of intermediate good is currently at the level shown on the vertical axis. We specify later the precise meaning of the ladder numbers indicated on this axis. Since the process of quality improvement turns out to occur at different rates (and in a random manner), the figure shows that the levels currently attained vary in an irregular way across the sectors.

For analyses of basic innovation, a common assumption is that the new types of intermediate inputs do not interact directly with the old ones. In particular, Romer (1990) uses the Spence (1976)/Dixit-Stiglitz (1977) functional form in which inputs enter into the production function in an additively separable manner. This specification turns out to imply that the introduction of a new kind of good does not tend to make any old goods obsolete. (Young [1993] takes a different approach, in which the newly discovered goods are complements of some existing goods and substitutes for others.)

In contrast, when a product or technique is improved, the new good or method tends to displace the old one. That is, it is natural to model different quality grades for a good of a given type as close substitutes. We follow Aghion and Howitt (1992) and

Grossman and Helpman (1991, Ch. 4) and make the extreme assumption that the different qualities of a particular type of intermediate input are perfect substitutes; hence, the discovery of a higher grade turns out to drive out completely the lower grades. For this reason, successful researchers along the quality dimension tend to eliminate the monopoly rentals of their predecessors, the process of "creative destruction" described by Schumpeter (1942) and Aghion and Howitt (1992). This feature of the quality-improvement model is the key distinction from the models of expanding variety, as formulated by Romer (1990) and Grossman and Helpman (1991, Ch. 3).

A. Sketch of the Model

Before we get into the technical details, we provide a sketch of the structure of the model that we shall develop to analyze improvements in quality. Producers of final product use N varieties of intermediate inputs, but N is constant. Each type of intermediate good has a quality ladder along which improvements can occur. Improvements build on the currently best technology and derive from efforts by researchers. A successful researcher retains exclusive rights over the use of his or her improved intermediate good.

At each point in time, the knowledge exists to produce an array of qualities of each type of intermediate good. In the equilibrium, however, only the leading-edge quality is actually produced in each sector and used by final-goods producers to generate output.

The researcher who has a monopoly over the use of the latest technology receives a flow of profit. We begin with a standard model in which the latest innovator is a different person from the previous innovator, so that a research success terminates the predecessor's flow of profit. Therefore, in considering how much resources to devote to research, entrepreneurs consider the size of the profit flow and its likely duration. This

duration is random, because it depends on the uncertain outcomes from the research efforts by competitors.

The temporary nature of an inventor's monopoly position brings in two important considerations: first, the shorter the expected duration of the monopoly the smaller the anticipated payoff from R&D, a distortion because the advances are permanent from a social perspective, and, second, part of the reward from successful research is the creative-destruction effect that involves the transfer of monopoly rentals from the incumbent innovator to the newcomer. Since this transfer has no social value, this second force constitutes an excessive incentive for R&D. We show that the second element is larger than the first, because the two terms are basically the same, except that the second element comes earlier in time. Hence, the net effect is an increase in the private return from research relative to the social return.

In a later section, we assume that the industry leader has a cost advantage in research. If the cost advantage is large enough, then the leader carries out all the research; in particular, the leader regards innovations as permanent and does not give any credit for the expropriation of his or her own monopoly rentals. If the leader's cost advantage in research is smaller, then the leader still carries out all the research in equilibrium, but the intensity of this research is the amount required to deter entry by outsiders. The rate of return and growth rate then turn out to coincide with the values that prevail when research is done by outsiders. We comment at the end about implications for policy.

B. Behavior of Firms

1. Levels of Quality in the Production Technology

The production function for firm i is

$$(1) \quad Y_i = AL_i^{1-\alpha} \cdot \sum_{j=1}^N (\bar{X}_{ij})^\alpha,$$

where $0 < \alpha < 1$, L_i is labor input, and \bar{X}_{ij} is the quality-adjusted amount employed of the j th type of intermediate good. The potential grades of each intermediate good are arrayed along a quality ladder with rungs spaced proportionately at interval $q > 1$.¹ We normalize so that each good begins—when first invented—at quality 1. The subsequent rungs are at the levels q , q^2 , and so on. Thus, if κ_j improvements in quality have occurred in sector j , then the available grades in the sector are $1, q, q^2, \dots, (q)^{\kappa_j}$. Increases in the quality of goods available in a sector—that is, rises in κ_j —result from the successful application of research effort, to be described later. These improvements must occur sequentially, one rung at a time.

Let X_{ijk} be the quantity used by the i th firm of the j th type of intermediate good of quality rung k . The rung k corresponds to quality q^k , so that $k=0$ refers to quality 1, $k=1$ to quality q , and so on. Thus, if κ_j is the highest quality level available in sector j , then the quality-adjusted input from this sector is given by

$$(2) \quad \bar{X}_{ij} = \sum_{k=0}^{\kappa_j} (q^k \cdot X_{ijk}).$$

¹This setup follows the models of Aghion and Howitt (1992) and Grossman and Helpman (1991, Ch. 4), except that we follow Romer (1990) in modeling the goods as intermediate inputs to production, rather than final products.

The assumption in equation (2) is that the quality grades within a sector are perfect substitutes as inputs to production. The overall input from a sector, \bar{X}_{ij} , is therefore the quality-weighted sum of the amounts used of each grade, $q^k \cdot X_{ijk}$.

In models of expanding variety, such as Romer (1990), $\kappa_j=0$ applies in each sector, and technological advances arise in equation (1) from increases in N . Since N is fixed here, we are assuming implicitly that all of the existing types of intermediate goods were discovered sometime in the (distant) past. But we allow κ_j to evolve over time in each sector in response to the R&D effort aimed at quality improvement in that sector.

Figure 2 shows a possible path for the evolution of the leading-edge quality in sector j . The best quality available equals 1 at time t_0 , rises to q (rung 1) at time t_1 , to q^2 (rung 2) at time t_2 , to q^k (rung k) at time t_k , and so on. Thus, $t_{k+1}-t_k$ is the interval over which the best quality is q^k for $k = 0, 1, \dots, \kappa_j - 1$. The figure shows intervals of differing length for each value of k ; these lengths are random in the model developed below.

The researcher responsible for each quality improvement in sector j retains the monopoly right to produce the j th intermediate good at that quality level. In particular, if the quality rungs $k = 1, \dots, \kappa_j$ have been reached, then the k th innovator is the sole source of intermediate goods with the quality level q^k .² (The results would be the same if the innovator instead licensed production to competitive producers of goods.)

The intermediate good is nondurable and entails a unit marginal cost of production (in terms of output, Y). That is, the cost of production is the same for all qualities q^k , where $k = 0, \dots, \kappa_j$. Thus, the latest innovator has an efficiency advantage over the prior

²Since this model does not consider the initial discovery of a type of product, we have to assume that goods of quality 1 (rung 0) can be produced by anyone. The treatment of these lowest quality goods will not be an issue if substantial quality improvements have already occurred in each sector.

innovators in the sector, but will eventually be at a disadvantage relative to future innovators.

Suppose for the moment that only the best existing quality of intermediate good j —with quality level $(q)^{\kappa_j}$ —is available currently for production. (The other grades will turn out not to be used in equilibrium.) The marginal product of this good can be computed from equations (1) and (2) as

$$(3) \quad \partial Y_i / \partial X_{ij\kappa_j} = A\alpha \cdot L_i^{1-\alpha} \cdot (q)^{\alpha\kappa_j} \cdot (X_{ij\kappa_j})^{\alpha-1}.$$

If units of the leading-edge good are priced at $P_{j\kappa_j}$ and if no other quality grades of good j are available, then the implied demand function (from the aggregate of profit-maximizing final-goods producers) can be written as

$$(4) \quad X_{j\kappa_j} = L \cdot [A\alpha \cdot (q)^{\alpha\kappa_j} / P_{j\kappa_j}]^{1/(1-\alpha)}.$$

The leading-edge producer acts as a monopolist in this environment, and profit maximization leads to the markup formula,

$$(5) \quad \text{monopoly pricing} \Rightarrow P_{j\kappa_j} = P = 1/\alpha.$$

Hence, the monopoly price is constant over time and across sectors.

The aggregate quantity produced of the j th intermediate good—all of leading-edge quality—can be determined from equation (4) as

$$(6) \quad \text{monopoly pricing} \Rightarrow X_{j\kappa_j} = LA^{1/(1-\alpha)} \cdot \alpha^{2/(1-\alpha)} \cdot (q)^{\kappa_j \alpha / (1-\alpha)}$$

The evolution of κ_j over time in each sector and the divergences of the κ_j across the sectors will lead to variations in $X_{j\kappa_j}$ over time and across sectors.

Suppose now that goods from quality rungs below κ_j are also available for production in sector j . We assume in this section that the κ_j th innovator, who has the rights to produce the best known quality, was not also the $(\kappa_j - 1)$ th innovator, who can produce the next best quality. If the leading-edge producer charges the monopoly price shown in equation (5) and if this price is high enough, then the producer of the next lowest grade will be able to make positive profits by producing.

Recall from equation (2) that the different quality grades are perfect substitutes, but are weighted by their respective grades. Thus, each unit of the leading-edge good is equivalent to $q > 1$ units of the next best good. It follows that if the highest grade is priced at $P_{j\kappa_j}$, then a good of the next lowest grade could be sold at most at the price $(1/q) \cdot P_{j\kappa_j}$, the one below that at the price $(1/q^2) \cdot P_{j\kappa_j}$, and so on. If $(1/q) \cdot P_{j\kappa_j}$ is less than the unit marginal cost of production, then the next best grade (and, moreover, all of the lower quality grades) cannot survive.

Equation (5) shows that the leading-edge producer's monopoly price is $1/\alpha$, a price that would allow the next best producer to price at most at $1/(\alpha q)$, the one below that at $1/(\alpha q^2)$, and so on. If $1/(\alpha q)$ is less than one, then the next best producer (and all lower quality producers) cannot compete against the leader's monopoly price.

Therefore, the condition $\alpha q > 1$ implies that monopoly pricing will prevail. This inequality will hold if q , the spacing between quality improvements, is large enough; the lower grades are then immediately driven out of the market even though the leading good is priced at the monopoly level. In this case, only the best available quality, κ_j , of

each type of intermediate good is produced and used as an input by final-goods producers. The price and quantity of type j are then given by equations (5) and (6).

If $\alpha q \leq 1$, then we can follow Grossman and Helpman (1991, Ch. 4) by assuming that the providers of intermediate goods of a given type engage in Bertrand price competition. In this case, the quality leader employs a limit-pricing strategy, that is, the leader sets a price that is sufficiently below the monopoly price so as to make it just barely unprofitable for the next best quality to be produced.³ This limit price is given by

$$(7) \quad \text{limit pricing} \Rightarrow P_{j\kappa_j} = q.$$

If the leader prices at $q - \epsilon$, where ϵ is an arbitrarily small positive amount, then the producer of the next best quality can charge at most $1 - \epsilon/q$, a price that results in negative profit. The lower quality goods are therefore again driven out of the market. A comparison of equations (7) and (5) shows that, if $\alpha q \leq 1$ —the condition for limit pricing to prevail—then the limit price is no larger than the monopoly price.

The total quantity produced (of the highest quality) when limit pricing applies is given by

$$(8) \quad \text{limit pricing} \Rightarrow X_{j\kappa_j} = LA^{1/(1-\alpha)} \cdot (\alpha/q)^{1/(1-\alpha)} \cdot (q)^{\kappa_j \alpha / (1-\alpha)}$$

³Grossman and Helpman (1991, Ch. 4) effectively assume $\alpha=0$, so that the magnitude of the elasticity of demand is 1, and the monopoly price shown in equation (5) is infinite. Since the inequality $\alpha q \leq 1$ must hold in this situation, monopoly pricing cannot apply in their model.

A comparison of equations (8) and (6) shows that, if $\alpha q < 1$, then the quantity produced under limit pricing is larger than the amount that would have been produced under monopoly.

The monopoly formulas in equations (5) and (6) apply if $\alpha q \geq 1$, and the limit-pricing formulas in equations (7) and (8) hold if $\alpha q < 1$. Either way, price is a fixed markup on the marginal cost of production, and only the best available quality of each type of intermediate good is actually produced in each sector and used by final-goods producers. We assume in the main discussion that $\alpha q \geq 1$, so that the monopoly formulas in equations (5) and (6) apply. The main results are similar, however, if $\alpha q < 1$, so that limit pricing prevails.

We can use the results to rewrite the production function from equation (1) as

$$(9) \quad Y_i = A L_i^{1-\alpha} \cdot \sum_{j=1}^N (q)^{\alpha \kappa_j} \cdot (X_{ij\kappa_j})^\alpha,$$

where $X_{ij\kappa_j}$ is the i th firm's input of the j th intermediate from the highest available quality rung, κ_j . We can, in other words, ignore any goods of less than leading-edge quality.

If we substitute the quantity $X_{ij\kappa_j}$ from equation (6) (with L_i appearing instead of L) and aggregate over the firms i , then we get an expression for aggregate output:

$$(10) \quad Y = A^{1/(1-\alpha)} \cdot \alpha^{2\alpha/(1-\alpha)} \cdot L \cdot \sum_{j=1}^N q^{\kappa_j \alpha / (1-\alpha)}.$$

Since L and N are constants, the key to growth of Y in this model is expansions of the quality-ladder positions κ_j in the various sectors.

We can define an aggregate quality index,

$$(11) \quad Q \equiv \sum_{j=1}^N q^{\kappa_j \alpha / (1-\alpha)},$$

so that

$$(12) \quad Y = A^{1/(1-\alpha)} \cdot \alpha^{2\alpha/(1-\alpha)} \cdot LQ.$$

The index Q is a combination of the various κ_j 's, and increases in the κ_j 's affect aggregate output to the extent that they raise Q .

We also note from aggregation of equation (6) across the sectors that total spending on intermediates, denoted by X , is proportional to Q :

$$(13) \quad X = A^{1/(1-\alpha)} \cdot \alpha^{2/(1-\alpha)} \cdot LQ.$$

We now consider the determinants of changes in the κ_j 's.

2. The Incentive to Innovate

a. The Flow of Monopoly Profit

Innovation in a sector takes the form of an improvement in quality by the multiple q . The κ_j th innovator in sector j raises the quality from $q^{\kappa_j - 1}$ to q^{κ_j} . This innovator will be able to price in accordance with equation (5) and sell the quantity of intermediate goods given by equation (6). The flow of profit associated with quality

rung κ_j equals $(P-1) \cdot X_{j\kappa_j}$ and is therefore given by

$$(14) \quad \pi_{j\kappa_j} = LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \cdot q^{\kappa_j \alpha / (1-\alpha)}$$

The profit shown in equation (14) applies when a sector's highest quality rung is κ_j . Thus, this profit accrues from the time of the κ_j th quality improvement, t_{κ_j} , until the time of the next improvement by a competitor, t_{κ_j+1} . The interval over which the κ_j th innovation is in the forefront is therefore

$$T_{j\kappa_j} = t_{\kappa_j+1} - t_{\kappa_j}$$

If the interest rate is the constant r —as will be true in equilibrium—then the present value (evaluated at time t_{κ_j}) of the profit from the κ_j th innovation in sector j is given by

$$(15) \quad V_{j\kappa_j} = \pi_{j\kappa_j} \cdot [1 - \exp(-rT_{j\kappa_j})] / r.$$

This present value, which represents the prize for the κ_j th innovation, depends positively on $\pi_{j\kappa_j}$ and $T_{j\kappa_j}$. Since we know $\pi_{j\kappa_j}$, we now have to determine the duration, $T_{j\kappa_j}$.

b. The Duration of Monopoly Profit

Denote by $Z_{j\kappa_j}$ the flow of resources (in units of Y) expended by the aggregate of

potential innovators in sector j when the highest quality-ladder number reached in that sector is κ_j . The higher $Z_{j\kappa_j}$ the larger the probability, $p_{j\kappa_j}$, per unit of time of a successful innovation, that is, an increase in the ladder number from κ_j to κ_j+1 . Specifically, we assume

$$(16) \quad p_{j\kappa_j} = Z_{j\kappa_j} \cdot \phi(\kappa_j),$$

so that, for given κ_j , the probability of success is proportional to the overall research effort, $Z_{j\kappa_j}$.⁴ We assume also that the probability of success declines for given effort with the complexity of the project, represented by the ladder number, κ_j , that is, $\phi'(\kappa_j) < 0$.

The randomness of R&D success implies that progress will occur unevenly in a single sector; usually nothing happens, but on rare occasions the productivity jumps by a discrete amount. We assume, however, that individual sectors are small and that the probabilities of research success across sectors are independent. The Law of Large Numbers then implies that the jumpiness in microeconomic outcomes is not transmitted to the macroeconomic variables: the adding up across a large number of independent sectors N leads to a smooth path for the aggregate quality index Q , shown in equation (11), and therefore for aggregate economic growth. (This result would be exact if we

⁴The linearity in $Z_{j\kappa_j}$ means that the marginal contribution of R&D effort to the probability of success, $\partial p / \partial Z_{j\kappa_j}$, equals the average contribution, $p / Z_{j\kappa_j}$. That is, the research process is *not* being modeled as a congestible resource, like a fishing pond, in which an individual's expected return declines with the aggregate level of investment. For this reason, a researcher is indifferent to entry by additional researchers or to changes in the effort levels by his or her competitors. The model therefore does not have the property of some patent-race formulations in which—for congestion reasons—the overall level of research tends to be too high from a social perspective (see Reinganum [1989] for a survey of these models).

treated N as continuous, that is, if sectors were of infinitesimal size.) Thus, the analysis abstracts from the aggregate fluctuations that are the focus of real business-cycle models.⁵

Define $G(\tau)$ to be the cumulative probability density function for $T_{j\kappa_j}$. The derivative of $G(\tau)$ is

$$(17) \quad dG/d\tau = [1-G(\tau)] \cdot p_{j\kappa_j},$$

where we assume that $p_{j\kappa_j}$ is constant over the interval, $(t_{\kappa_j}, t_{\kappa_j+1})$. In other words, the research effort, $Z_{j\kappa_j}$, and, hence, the probability of success, $p_{j\kappa_j}$, do not vary over time between innovations in a sector. (These conditions hold in equilibrium.) Since $p_{j\kappa_j}$ is constant over time, we can readily solve the differential equation (17). If we use the boundary condition, $G(0)=0$, then the result is

$$G(\tau) = 1 - \exp(-p_{j\kappa_j} \tau).$$

The probability density function can then be found from differentiation of the cumulative density:

$$(18) \quad g(\tau) = G'(\tau) = p_{j\kappa_j} \cdot \exp(-p_{j\kappa_j} \tau).$$

⁵We could get similar results for aggregate growth if we assumed that innovation occurred deterministically in a single sector, that is, the application of a given level of R&D effort generated a quality improvement after a known interval of time. The aggregation would then depend, however, on how the R&D effort and, hence, the innovations were synchronized across the sectors. The framework with a Poisson success probability in each sector is much more manageable.

We can use equations (15) and (18) to compute the expected present value of profit, evaluated at time t_{κ_j} :

$$E(V_{j\kappa_j}) = (\pi_{j\kappa_j}/r) \cdot p_{j\kappa_j} \cdot \int_0^{\infty} (1 - e^{-r\tau}) \cdot \exp(-p_{j\kappa_j}\tau) \cdot d\tau = \pi_{j\kappa_j} / (r + p_{j\kappa_j}).$$

If we substitute for $\pi_{j\kappa_j}$ from equation (14), then we get

$$(19) \quad E(V_{j\kappa_j}) = LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \cdot [q^{\kappa_j \alpha / (1-\alpha)}] / (r + p_{j\kappa_j}).$$

c. Determination of R&D Effort

We now consider how the prize for successful research, $E(V_{j\kappa_j})$, determines the quantity of R&D effort, $Z_{j\kappa_j}$, and thereby the probability of success, $p_{j\kappa_j}$. We assume that potential innovators care only about the expected value shown in equation (19) and not about the randomness of the return. This assumption can be satisfactory even if individuals are risk averse because each R&D project is small and has purely idiosyncratic uncertainty.⁶

The cost of research per unit of time is $Z_{j\kappa_j}$, and this effort results in the probability $p_{j\kappa_j}$ per unit of time of success, where $p_{j\kappa_j} = Z_{j\kappa_j} \cdot \phi(\kappa_j)$ from equation (16). The expected reward per unit of time for pursuing the (κ_j+1) th innovation is $p_{j\kappa_j} \cdot E(V_{j,\kappa_j+1})$. Hence, the expected flow of net profit, $\Pi_{j\kappa_j}$, from research in a sector that is currently at quality rung κ_j is $p_{j\kappa_j} \cdot E(V_{j,\kappa_j+1}) - Z_{j\kappa_j}$, which equals

⁶We have to assume that research projects are carried out by syndicates that are large enough to diversify the risk. The syndicates cannot be so large, however, that they would internalize the distortions that are present in the model.

$$(20) \quad \Pi_{j\kappa_j} = Z_{j\kappa_j} \cdot \left[\phi(\kappa_j) \cdot LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \cdot [q^{(\kappa_j+1) \cdot \alpha/(1-\alpha)}] / (r+p_{j,\kappa_j+1}) - 1 \right].$$

We assume free entry into the research business. Hence, if $Z_{j\kappa_j} > 0$, then $\Pi_{j\kappa_j} = 0$ must hold. The term $Z_{j\kappa_j}$ then cancels out in equation (20), and the free-entry condition with positive R&D can be written as

$$(21) \quad r+p_{j,\kappa_j+1} = LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \cdot \phi(\kappa_j) \cdot [q^{(\kappa_j+1) \cdot \alpha/(1-\alpha)}].$$

Note that the right-hand side depends on κ_j , but does not differ otherwise across sectors.

Equation (21) implies that the probability of innovation would generally vary with κ_j because of two offsetting effects. The term $q^{(\kappa_j+1) \cdot \alpha/(1-\alpha)}$ appears because the expected reward from an innovation is increasing in κ_j (see equation [19]). This effect arises because the quantity of intermediates sold (equation [6]) increases with quality and, hence, with κ_j . The second effect reflects the assumption that innovations in a sector are increasingly difficult, that is, $\phi'(\kappa_j) < 0$.

If the first effect dominates, then the rate of return to R&D is higher the more advanced a sector. The more advanced sectors will then tend to grow faster than less advanced sectors, and the growth rate of the overall economy will rise over time as the average value of κ_j increases. In other words, R&D features a form of increasing returns, and this property creates a pattern of divergence for growth rates.

In contrast, if the second effect dominates, then more advanced sectors will tend to grow relatively slowly, and the growth rate of the overall economy will fall over time. In this case, R&D exhibits a form of decreasing returns, and this relation generates the

kind of convergence behavior that appears in the neoclassical growth model (see Barro and Sala-i-Martin [1992]).

Finally, if the two forces exactly offset, then all sectors will tend to grow at the same rate, and the growth rate of the overall economy will be constant over time. In this case, R&D exhibits constant returns. This case therefore features endogenous, steady-state growth of the sort that arises in models with constant returns to a broad concept of capital (Rebelo [1991]). We focus the subsequent discussion on this case, that is, we deal with the situation in which rates of return to R&D are constant.

Equation (21) shows that the balance between the two forces depends on the form of $\phi(\kappa_j)$. A specification that makes the two effects exactly offset is

$$(22) \quad \phi(\kappa_j) = (1/\zeta) \cdot q^{-(\kappa_j+1) \cdot \alpha / (1-\alpha)},$$

where the parameter $\zeta > 0$ represents the cost of research. A higher ζ lowers the probability of success for given values of $Z_j \kappa_j$ and κ_j in equation (16).

The term on the far right-hand side of equation (22), $q^{-(\kappa_j+1) \cdot \alpha / (1-\alpha)}$, indicates the negative effect of a project's complexity (represented by κ_j+1) on the probability of success. This particular form cancels out the term on the far right-hand side of equation (21). For that reason, p_{j, κ_j+1} is invariant with κ_j .

If we substitute from equation (22) into equation (21), then the free-entry condition becomes

$$(23) \quad r+p = (L/\zeta) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)},$$

where we substituted $p=p_{j,\kappa_j+1}$ because the probability is constant⁷ (across sectors and over time in a given sector).⁸ The right-hand side of equation (23) represents the rate of return from research (the expected flow of profit per unit of research effort). The key, however, is that a successful researcher maintains this return only until the time of the next innovation. The rate of return must therefore cover the ordinary rate of return, r , plus the premium for the probability, p , per unit of time that a competitor will succeed and thereby drive the incumbent out of business.

Equation (23) implies that the probability of an innovation per unit of time is

$$(24) \quad p = (L/\zeta) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} - r.$$

If r is constant over time, then p is also constant (as well as the same for all sectors).

The amount of resources devoted to R&D in sector j is $Z_{j\kappa_j} = p/\phi(\kappa_j)$. If we use equations (22) and (24) to substitute for $\phi(\kappa_j)$ and p , then we get

$$(25) \quad Z_{j\kappa_j} = q^{(\kappa_j+1) \cdot \alpha/(1+\alpha)} \cdot \left[LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} - r\zeta \right].$$

⁷Equation (21) determines the probabilities that correspond to the *next* quality rung in each sector, p_{j,κ_j+1} . In the equilibrium, the current relative spending on R&D across sectors

can differ from the amount that corresponds to the constant p (equation [24] below) by a random term.

⁸For the case of limit pricing (equations [7] and [8]), the result in equation (23) is modified to

$$r+p = (L/\zeta) \cdot A^{1/(1-\alpha)} \cdot (q-1) \cdot (\alpha/q)^{1/(1-\alpha)}.$$

This result applies if $\alpha q \leq 1$ and reduces to equation (23) if $\alpha q = 1$.

Hence, more advanced sectors—with higher κ_j —have a larger quantity of research devoted to them. The probability of success is, however, independent of κ_j because equation (22) implies that correspondingly more effort is required in a more advanced sector to generate the same probability.

The aggregate of R&D spending, denoted by Z , is

$$(26) \quad Z \equiv \sum_{j=1}^N z_j \kappa_j = Q \cdot q^{\alpha/(1-\alpha)} \cdot \left[LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} - r\zeta \right],$$

where Q is the aggregate quality index, as shown in equation (11). Hence, Z is proportional to Q for a given value of r .

3. The Behavior of the Aggregate Quality Index

The level of aggregate output, Y in equation (12), the aggregate resources expended on intermediates, X in equation (13), and the total expenditure on R&D, Z in equation (26), are all constant multiples of the aggregate quality index, Q . (We assume here that r is constant, a condition that will hold in equilibrium.) Hence, the growth rates of these quantities are all equal to the growth rate of Q :

$$\gamma_Y = \gamma_X = \gamma_Z = \gamma_Q,$$

where the symbol γ denotes the growth rate of the variable designated by the subscript. To understand growth in this model, we have to explain the changes over time in Q .

Recall the definition of Q from equation (11):

$$(11) \quad Q \equiv \sum_{j=1}^N q_j^{\alpha/(1-\alpha)}$$

In sector j , the term $q_j^{\alpha/(1-\alpha)}$ does not change if no innovation occurs, but rises to $(\kappa_j+1) \cdot q_j^{\alpha/(1-\alpha)}$ in the case of a research success. The proportionate change in this term due to a success is $q_j^{\alpha/(1-\alpha)} - 1$, and the probability per unit of time of a success is the value p shown in equation (24). Since p is the same for all sectors, the expected proportionate change in Q per unit of time is given by

$$(27) \quad E(\Delta Q/Q) = p \cdot (q^{\alpha/(1-\alpha)} - 1).$$

If the number of sectors, N , is large, then the Law of Large Numbers implies that the average growth rate of Q measured over any finite interval of time will be close to the expression shown on the right-hand side of equation (27). We assume, in particular, that N is large enough to treat Q as differentiable, with \dot{Q}/Q non-stochastic and equal to the right-hand side of equation (27). (This result holds exactly if N approaches infinity, and the size of each sector tends to zero.) If we substitute for p from equation (24), then we get the growth rate of Q :

$$(28) \quad \gamma_Q = \left[(L/\zeta) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} - r \right] \cdot [q^{\alpha/(1-\alpha)} - 1].$$

Equation (28) shows that, to determine the growth rate of Q —and, hence, the growth rates of Y , X , and Z —we have to pin down the rate of return, r . To determine

r , we have to bring in the behavior of households as consumers. We first analyze the market value of firms, then consider the choices of households (who own the firms), and finally deal with the equilibrium conditions that allow us to determine r and γ_Q .

4. The Market Value of Firms

Since goods below leading-edge quality are not produced, the only firm with market value in each sector is the one that possesses the rights over the latest (κ_j th) innovation. The market value of this innovation, $E(V_{j\kappa_j})$, is given by equation (19). If we substitute for $r+p$ from equation (23), then the formula becomes

$$(29) \quad E(V_{j\kappa_j}) = \zeta \cdot q_j^{\kappa_j \alpha / (1-\alpha)}$$

The expression on the right-hand side represents the (expected) cost of generating the κ_j th innovation (taking account of equation [22]). Note that the more advanced a sector—the higher κ_j —the greater is the market value of the leading-edge firm.

The aggregate market value of firms, denoted by V , is the sum of equation (29) over the N sectors:

$$(30) \quad V = \zeta \cdot \sum_{j=1}^N q_j^{\kappa_j \alpha / (1-\alpha)} = \zeta Q.$$

The total market value of firms is therefore a constant multiple of Q .

C. Households and Market Equilibrium

We assume that each household maximizes a familiar expression for utility over an

infinite horizon:

$$(31) \quad U = \int_0^{\infty} \left(\frac{c^{1-\theta} - 1}{1-\theta} \right) \cdot e^{-\rho t} dt,$$

where c is consumption per person, the rate of population growth is zero, $\theta > 0$ is the constant elasticity of marginal utility with respect to c , and $\rho > 0$ is the rate of time preference. Households earn the rate of return r on assets and receive the wage rate w (equal to the marginal product of labor) on the fixed aggregate quantity L of labor. In a closed economy, the total of households' assets equals the market value of firms, ζQ , as shown in equation (30).⁹ The key condition that we need from household optimization is the standard one for the growth rate of consumption:

$$(32) \quad \gamma_C = (1/\theta) \cdot (r - \rho),$$

where C is aggregate consumption.

To apply the result in equation (32), we have to use our previous analysis to derive an expression for C , the level of consumption. The economy's overall resource constraint is

$$C = Y - X - Z,$$

⁹The wage rate equals the marginal product of labor, which can be computed from equation (1). We can use this condition to show that the households' aggregate income, $wL + r\zeta Q$, equals a concept of net product that takes account of capital losses, $Y - X - p\zeta Q$, where X is aggregate spending on intermediates (equation [13]) and $p\zeta Q$ is the capital loss from destruction of the market value of superseded innovations. Household saving, $\zeta \dot{Q}$, equals income less consumption, $Y - X - p\zeta Q - C$. The economy's resource constraint is $Y = C + X + Z$, where Z is aggregate spending on R&D. Household saving therefore equals $Z - p\zeta Q$, that is, net investment equals R&D spending less the capital loss due to destruction of existing market value.

where Y is given in equation (12); X is given in equation (13); and Z is given in equation (26). Since Y, X, and Z are all constant multiples of Q, C is also a constant multiple of Q. In particular, if we make the substitutions for Y, X, and Z, then we get¹⁰

$$(33) \quad C = \left[A^{1/(1-\alpha)} \cdot (1-\alpha^2) \cdot \alpha^{2\alpha/(1-\alpha)} \cdot L - p\zeta \cdot q^{\alpha/(1-\alpha)} \right] \cdot Q.$$

Since the expression in brackets is constant (if p is constant), C grows at the same rate as Q.

The variables Y, X, Z, C, and Q all grow at the same rate, which we can denote by γ . Equation (28), which derived from the behavior of firms as producers, gives one expression for γ as a function of r . Equation (32), which came from the behavior of households as consumers, provides another expression for γ as a function of r . In a market equilibrium, the value of r is such as to equate the two expressions for the growth rate. The resulting values of r and γ are constants and therefore constitute the steady-state values. The solutions are

$$(34) \quad r = \frac{\rho + \theta \cdot [q^{\alpha/(1-\alpha)} - 1] \cdot \left[(L/\zeta) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \right]}{1 + \theta \cdot [q^{\alpha/(1-\alpha)} - 1]}$$

$$(35) \quad \gamma = \frac{[q^{\alpha/(1-\alpha)} - 1] \cdot \left[(L/\zeta) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} - \rho \right]}{1 + \theta \cdot [q^{\alpha/(1-\alpha)} - 1]}$$

We assume that the parameters are such that γ is positive (so that the free-entry condition in equation [21] actually holds with equality), and $r > \gamma$ applies (to satisfy the

¹⁰The transversality condition, $r > \gamma$ in the following, ensures that the expression for C in equation (33) is positive.

transversality condition).¹¹ Equation (27) implies that the equilibrium value of p is the expression for γ in equation (35) divided by the term $[q^{\alpha/(1-\alpha)} - 1]$:

$$(36) \quad p = \frac{[(L/\zeta) \cdot A^{1/(1-\alpha)} \cdot (\frac{1-\alpha}{\alpha}) \cdot \alpha^{2/(1-\alpha)} - \rho]}{[1 + \theta \cdot (q^{\alpha/(1-\alpha)} - 1)]}$$

This model exhibits no transitional dynamics.¹² The single state variable is the aggregate quality index, Q . Given an initial value, $Q(0)$, the variables Q , Y , X , Z , and C all grow at the constant rate γ shown in equation (35). The interest rate, r , is the constant value shown in equation (34).

Although the mean growth rate of output in each sector j is also γ , the realized growth depends on the random outcomes of research efforts. In particular, the relative quality positions of the sectors and, hence, the relative amounts spent on intermediate goods and R&D evolve in a random-walk like fashion. At a point in time, the realized quality positions across the sectors will therefore exhibit an irregular pattern, as suggested by Figure 1.

The solution in equation (35) implies that the economy's growth rate is higher if people are more willing to save (lower ρ and θ), if the technology for producing goods is better (higher A), if the cost of doing research is lower (lower ζ), and if q , the step size between innovations, is larger. The model also contains a scale effect: a larger labor

¹¹Equations (34) and (35) imply that the condition for $r > \gamma$ is $\rho > (1-\theta) \cdot [1 - q^{-\alpha/(1-\alpha)}] \cdot [(L/\zeta) \cdot A^{1/(1-\alpha)} \cdot (\frac{1-\alpha}{\alpha}) \cdot \alpha^{2/(1-\alpha)}]$. The condition for $\gamma > 0$ in equation (35) is $\rho < (L/\zeta) \cdot A^{1/(1-\alpha)} \cdot (\frac{1-\alpha}{\alpha}) \cdot \alpha^{2/(1-\alpha)}$.

¹²We have shown here only that an equilibrium exists with no transitional dynamics. We can show that any other proposed solution violates a condition of optimality (either the transversality condition does not hold or consumption hits zero in finite time). The proof is the same as the one provided in Barro and Sala-i-Martin (1994, Ch. 4).

endowment, L , raises the growth rate.¹³ The model has this property because a quality improvement can be used in a nonrival manner across the entire economy. The larger the economy—represented by L —the lower the cost of an invention per unit of L (or Y). For that reason, an increase in L has the same effect on γ in equation (34) as an equiproportionate decrease in the R&D cost parameter, ζ .

Scale effects are not supported empirically if we identify scale with the size of a country's population or economic activity. Countries may, however, not be the proper unit for measuring scale in the present context. The scale that matters in the model has two aspects: first, it involves the total of production over which an improved input can be used in a nonrival manner, and, second, it measures the scope of the inventor's property rights. If ideas flow readily across borders, then countries do not define the proper units in the first context. Countries may also be inappropriate in the second context if patent protection applies internationally or if a monopoly position can be sustained worldwide by secrecy.

If the world operated as a single unit with respect to the flow of ideas and the maintenance of property rights, then L would be identified with world population or an aggregate of world economic activity. The model would then predict a positive relation between world per-capita growth and the levels of world population or the aggregate of world output. Kremer (1993) argues that this hypothesis may be correct.

D. Innovation by the Leader

The results predict a continual leapfrogging in leadership positions in an industry. Since the incumbent does no research, he or she is replaced on top at the time of the

¹³Because of this effect, the economy does not tend toward a steady state with constant per-capita growth if we allow for growth in population, L , at a positive rate.

next quality improvement by an outside competitor, who is subsequently replaced by another outsider, and so on.

In the real world, most improvements in the quality of existing products seem to be made by industry leaders. This outcome likely arises because the leaders typically have the best information about the current technology and other advantages that effectively reduce their research costs.¹⁴ We therefore want to investigate whether a change in specification about research costs will improve the model's predictions about research by insiders.

We begin with a setting in which outsiders are precluded from research; hence, the industry leader acts as a monopolist with respect to the choice of research intensity. Next we allow for research by outsiders, but at a cost that exceeds that for insiders. We show that the monopoly research outcome applies if the leader's cost advantage in R&D is sufficiently great. Otherwise, the probability of research success coincides with the value from our previous analysis. However, we now predict—in contrast with Grossman and Helpman (1991, Ch. 4)—that the research will be carried out by the (low-cost) insider, rather than the (high-cost) outsiders.

1. The Leader as a Monopoly Researcher

Suppose that the incumbent's research technology also takes the form of equations (16) and (22):

$$(37) \quad p_{j\kappa_j} = (Z_{j\kappa_j} / \zeta_\ell) \cdot q^{-(\kappa_j+1) \cdot \alpha / (1-\alpha)},$$

¹⁴Current technological leaders—companies or countries—are less likely to have a cost advantage for the discovery of entirely new products. See Brezis, Krugman, and Tsiddon (1993) for this argument.

where ζ_l is the cost parameter for the industry leader. We assume that the leader's cost may be smaller than that for competitors, $\zeta_l \leq \zeta$.

Suppose, for the moment, that the leader is the *only* source of research in a sector, that is, outsiders are prohibited from conducting research. This setup differs from the previous one in two respects: the leader regards all quality improvements as permanent and does not value the transfer of monopoly rentals from his or her predecessor (that is, from himself or herself). We therefore have to compute the present value of the leader's net receipts in this new environment.

Let $Z_{j\kappa_j}$ be the level of research effort, $p_{j\kappa_j}$ the resulting probability of success per unit of time, and $\pi_{j\kappa_j}$ the flow of monopoly profit, which is still given by

$$(14) \quad \pi_{j\kappa_j} = LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \cdot q^{\kappa_j \alpha / (1-\alpha)}$$

Let $V_{j\kappa_j}$ be the present value of the leader's net receipts. The expectation, $E(V_{j\kappa_j})$, can be broken into two parts. The first part is the present value of the net earnings, $\pi_{j\kappa_j} - Z_{j\kappa_j}$, up to the time of the next quality improvement. These earnings accrue, as before, over an interval of random length $T_{j\kappa_j}$. The present value of this flow has the same form as equation (15):

$$(\pi_{j\kappa_j} - Z_{j\kappa_j}) \cdot [1 - \exp(-rT_{j\kappa_j})] / r.$$

Since the probability density for $T_{j\kappa_j}$ is still given by equation (18), the expected present value of the first part of $E(V_{j\kappa_j})$ is

$$[(\pi_{j\kappa_j} - Z_{j\kappa_j})/\tau] \cdot p_{j\kappa_j} \cdot \int_0^{\infty} (1 - e^{-\tau}) \cdot \exp(-p_{j\kappa_j} \tau) \cdot d\tau = (\pi_{j\kappa_j} - Z_{j\kappa_j})/(\tau + p_{j\kappa_j}).$$

The second part of $E(V_{j\kappa_j})$ covers the period after the time of the next quality improvement, $T_{j\kappa_j}$. The expected present value starting from that date is $E(V_{j\kappa_{j+1}})$, but we have to discount this term by the factor $\exp(-\tau T_{j\kappa_j})$. Therefore, if we again use the probability density for $T_{j\kappa_j}$ from equation (18), then we can evaluate this second part as

$$E(V_{j\kappa_{j+1}}) \cdot p_{j\kappa_j} \cdot \int_0^{\infty} e^{-\tau T_{j\kappa_j}} \cdot \exp(-p_{j\kappa_j} \tau) \cdot d\tau = p_{j\kappa_j} \cdot E(V_{j\kappa_{j+1}})/(\tau + p_{j\kappa_j}).$$

If we combine the two parts, then we get

$$(38) \quad E(V_{j\kappa_j}) = \left(\frac{1}{\tau + p_{j\kappa_j}} \right) \cdot [\pi_{j\kappa_j} - Z_{j\kappa_j} + p_{j\kappa_j} \cdot E(V_{j\kappa_{j+1}})].$$

We can use equation (37) to substitute out for $Z_{j\kappa_j}$ in equation (38). The result is

$$E(V_{j\kappa_j}) = \left(\frac{1}{\tau + p_{j\kappa_j}} \right) \cdot \left[\pi_{j\kappa_j} - \zeta \ell^q \cdot (\kappa_{j+1}) \cdot \alpha / (1 - \alpha) \cdot p_{j\kappa_j} + p_{j\kappa_j} \cdot E(V_{j\kappa_{j+1}}) \right].$$

Thus, $E(V_{j\kappa_j})$ depends on $p_{j\kappa_j}$ and some other terms, including $E(V_{j\kappa_{j+1}})$, that are independent of $p_{j\kappa_j}$. The monopolist would choose $p_{j\kappa_j}$ (by selecting the R&D effort, $Z_{j\kappa_j}$) to maximize $E(V_{j\kappa_j})$. If we set the derivative of $E(V_{j\kappa_j})$ with respect to $p_{j\kappa_j}$ to zero to get the first-order condition, then the result can be written as

$$(39) \quad E(V_{j,\kappa_j+1}) - E(V_{j\kappa_j}) = \zeta_\ell \cdot q^{(\kappa_j+1) \cdot \alpha / (1-\alpha)} = Z_{j\kappa_j} / p_{j\kappa_j},$$

where the last equality uses equation (37).

The result in equation (39) differs from the free-entry condition in the previous setup ($\Pi_{j\kappa_j} = 0$ in equation [20]) in two respects. First, the term $Z_{j\kappa_j} / p_{j\kappa_j}$ is now equated to the increment in present value, $E(V_{j,\kappa_j+1}) - E(V_{j\kappa_j})$, rather than to the full present value, $E(V_{j,\kappa_j+1})$, because the leader does not value the expropriation of his or her own monopoly profit. Second, the term $E(V_{j\kappa_j})$ is calculated differently from before, because it considers that the leadership position is permanent, rather than temporary.

To see this last property, substitute the result for $E(V_{j,\kappa_j+1})$ from equation (39) into equation (38) and also substitute for $Z_{j\kappa_j}$ from equation (37) to get

$$(40) \quad E(V_{j\kappa_j}) = \pi_{j\kappa_j} / r.$$

The term on the right-hand side is the present value yielded by a permanent stream of profit of size $\pi_{j\kappa_j}$. (Since the stream is permanent, the discount rate is r , rather than $r + p_{j\kappa_j}$.)

If we substitute from equation (40) into equation (39) and use equation (14) to substitute out for $\pi_{j\kappa_j}$, then we get a condition for r . The resulting value, denoted r_ℓ is the equilibrium rate of return for an environment in which the research in all sectors is carried out by the industry leader:¹⁵

¹⁵If $r < r_\ell$ where r_ℓ is given in equation (41), then the derivative of $E(V_{j\kappa_j})$ with respect to

$$(41) \quad r_\ell = (L/\zeta_\ell) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} \cdot [1-q^{-\alpha/(1-\alpha)}].$$

The corresponding growth rate (of Q and the other quantities) is given, as usual, by

$$\gamma_\ell = (1/\theta) \cdot (r_\ell - \rho).$$

The rate of return in the previous model satisfies the condition (from equation [23]),

$$(42) \quad r = (L/\zeta) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{2/(1-\alpha)} - p.$$

This expression includes p on the right-hand side, although we could also substitute the equilibrium value for p from equation (36). The result for r_ℓ in equation (41) differs from the solution for r in equation (42) in three ways. First, $\zeta_\ell < \zeta$ tends to make $r_\ell > r$. Second, r falls with p in equation (42) because the private return to an innovation is temporary. This force tends to make $r_\ell > r$. Finally, equation (41) includes the term $[1-q^{-\alpha/(1-\alpha)}] < 1$, because the leader weighs only the increment in present value from a research success. This term tends to make $r_\ell < r$.

If we use equation (36) to substitute out for p in equation (42), then we get the equilibrium value for r , as expressed in equation (34). If $\zeta_\ell = \zeta$, so that the leader has no cost advantage in research, then we can use equations (41) and (34) to show $r_\ell < r$.¹⁶ The difference between the rates of return involves two offsetting forces: $r_\ell < r$ because no weight is given to the expropriation of the existing monopoly rentals, but $r_\ell > r$ because

$p_{j\kappa_j}$ is positive, so that the leader would like to carry out an infinite amount of research. If $r > r_\ell$, then the derivative is negative, so that no research is carried out, and the economy does not grow. An equilibrium with positive growth therefore requires $r = r_\ell$.

¹⁶The proof requires the transversality condition, $r > \gamma$, given in n. 12.

innovations are viewed as permanent. The net effect is unambiguous because the two forces are essentially the same, except that they differ in sign and one comes earlier than the other. The extraction of the monopoly rent is the amount taken from one's predecessor. The treatment of an innovation as temporary is equivalent to ignoring the rents that will be taken by one's followers. The terms are the same in magnitude, except for two considerations: the later term is higher because of growth of the economy at the rate γ , but is smaller in present value because of discounting at the rate r . The relation $r > \gamma$ —the transversality condition—implies that the first term dominates, so that $r_\ell < r$ must hold.

2. Research by Outsiders

Suppose now that we allow outside competitors, as well as the leader, to carry out research. If $\zeta = \zeta_\ell$, then equations (41) and (34) imply $r_\ell < r$, that is, outsiders view research more favorably than the leader. The probability of success, p , and the expected growth rate of quality in each sector are therefore higher than the values that would be determined by a leader who had exclusive rights to do research.

These results do not mean that all research will be conducted by outsiders. Given the competitors' willingness to carry out enough research to generate a probability of success p , the leader would have to accept this probability of an innovation as a constraint given by the existence of the outside competition. (The constraint is effective here because leaders would otherwise determine a lower probability.) For a given success probability—and, hence, a given expected duration of the currently leading technology—the rate of return from research for the leader is exactly the same as that for outsiders (when $\zeta = \zeta_\ell$). Although the leader does not consider the extraction of the existing monopoly rentals as part of the return from successful research, he or she does count as a return the prevention of the loss of these rentals to an outsider. Thus, when

$\zeta = \zeta_p$, the solutions that we obtained before are valid—for r in equation (34), γ in equation (35), and p in equation (36)—but it is a matter of indifference whether the research is done by leaders or outsiders.¹⁷

Now we make the more realistic assumption $\zeta_l < \zeta$, that is, incumbents have a cost advantage in improving and refining the existing types of products. We noted before that $r_l < r$ applies when $\zeta_l = \zeta$, but equation (41) shows that a reduction in ζ_l raises r_l . There exists a critical value $\bar{\zeta}$ such that $\zeta_l < \bar{\zeta}$ implies $r_l > r$. If the leader's cost advantage in research in each sector is large enough so that $\zeta_l < \bar{\zeta}$, then the existence of the outside competition does not constrain the incumbent's choice of research intensity. Hence, $\zeta_l < \bar{\zeta}$ implies that the equilibrium rate of return equals the value r_l shown in equation (41), and the growth rate is given correspondingly by $\gamma_l = (1/\theta) \cdot (r_l - \rho)$.¹⁸ Note that the computations that underlie these solutions for r_l and γ_l treat innovations as permanent and do not attach any value to the taking of the existing monopoly rentals.

Consider now the range $\bar{\zeta} < \zeta_l < \zeta$. In this case, the cost advantage for leaders is not sufficient to ignore the outside competition. The equilibrium is then an analog to limit pricing—research intensities and the corresponding probability of success are just sufficient to deter outsiders from entering the research business. In particular, the limit success probability is the value p shown in equation (36). In this equilibrium, the industry leaders conduct all the research, but the solutions for r and γ are the same as those that arise when outsiders do all the research (equations [34] and [35], respectively).¹⁹

¹⁷This statement is correct if monopoly pricing prevails in either case.

¹⁸This equilibrium determines the aggregate spending on R&D and the economy's overall growth rate. The allocation of R&D across the sectors is indeterminate, however, because all rates of return to research by leaders equal r_l and are independent of the amount invested. See n. 7 for an analogous indeterminacy in the model in which outsiders carry out all of the research.

¹⁹The only difference from before is that the amount spent on research, Z , is smaller because it depends on the research cost for leaders, ζ_l , rather than that for outsiders, ζ . The level of consumption, C , in equation (33) is correspondingly higher.

Thus, on the one hand, we no longer predict the pattern of leapfrogging in which every innovation goes along with the replacement of the industry leader by an outsider. But, on the other hand, the values of r and γ are the same as those predicted by the leapfrogging model. The results are as if researchers were seeking the incumbent's rentals and anticipating that their successes would only be temporary.

Another result in the range where $\bar{\zeta} < \zeta < \zeta$ is that the existence of the competitive fringe for research is important for the equilibrium. Research intensity and the economy's growth rate are higher than they would be if this fringe did not exist (because $r < r$ applies in this range).

E. Pareto Optimality

We can assess the Pareto optimality of the decentralized equilibria by comparing them with the solution to the social-planner's problem. The social planner seeks to maximize the expression for utility in equation (31), subject to the economy's resource constraint,

$$(43) \quad Y = AL^{1-\alpha} \cdot \sum_{j=1}^N (q^{\kappa_j} \cdot X_{j\kappa_j})^\alpha = C + \sum_{j=1}^N (X_{j\kappa_j} + Z_j) = C + X + Z.$$

The first part of the equation says that total output depends on the quality levels, q^{κ_j} , and the quantities employed, $X_{j\kappa_j}$, of the leading-edge intermediates in each sector. (We have already used the result here that the optimizing social planner would not produce and use any intermediate goods of less than leading-edge quality.) The next

part of the equation indicates that output can be used for consumption, C ; intermediates, X ; and R&D effort, Z .

The planner's problem is also constrained by the R&D technology. We assume that the probability of a research success in sector j , which has attained the quality rung κ_j , is again given from equation (37) by

$$(37) \quad p_{j\kappa_j} = (1/\zeta_\rho) \cdot Z_{j\kappa_j} \cdot q^{-(\kappa_j+1) \cdot \alpha / (1-\alpha)}$$

We enter the leader's research cost, ζ_ρ which we assume is no larger than the cost for outsiders, because the social planner would assign the research activity to the low-cost researcher.

It is convenient first to work out the planner's choice of $X_{j\kappa_j}$ —which is a static problem—and then use the result to write out a simplified Hamiltonian expression. We can show that the first-order condition for maximizing U with respect to the choice of $X_{j\kappa_j}$ implies

$$(44) \quad X_{j\kappa_j} \text{ (social planner)} = LA^{1/(1-\alpha)} \cdot \alpha^{1/(1-\alpha)} \cdot q^{\kappa_j \alpha / (1-\alpha)}$$

Recall that the choice in a decentralized economy is

$$(6) \quad X_{j\kappa_j} = LA^{1/(1-\alpha)} \cdot \alpha^{2/(1-\alpha)} \cdot q^{\kappa_j \alpha / (1-\alpha)}$$

Hence, because of monopoly pricing of the intermediate inputs, the privately chosen quantity is smaller than the socially chosen amount (by the multiple $\alpha^{1/(1-\alpha)}$).

Substitution for $X_{j\kappa_j}$ from equation (44) into equation (43) gives an expression for aggregate output:

$$(45) \quad Y \text{ (social planner)} = A^{1/(1-\alpha)} \cdot \alpha^{\alpha/(1-\alpha)} \cdot LQ,$$

where $Q = \sum_{j=1}^N q_j^{\kappa_j \alpha / (1-\alpha)}$ is the same aggregate quality index that we considered for the decentralized economy in equation (11). The level of output for a decentralized economy is

$$(12) \quad Y = A^{1/(1-\alpha)} \cdot \alpha^{2\alpha/(1-\alpha)} \cdot LQ.$$

Therefore, for given Q , the social-planner's level of output exceeds the decentralized value. This result reflects the decentralized economy's failure to achieve static efficiency by choosing a high enough quantity of intermediate goods, $X_{j\kappa_j}$, in each sector.

Equation (45) also implies that the social planner's growth rate of Y equals the growth rate of Q .

If the social planner applies the research effort $Z_{j\kappa_j}$ to sector j , then the expected change in Q per unit of time is given by

$$E(\Delta Q) = \sum_{j=1}^N p_{j\kappa_j} \cdot \left[q_j^{(\kappa_j+1) \cdot \alpha / (1-\alpha)} - q_j^{\kappa_j \cdot \alpha / (1-\alpha)} \right].$$

Substitution for $p_{j\kappa_j}$ from equation (37) simplifies the expression to

$$(46) \quad E(\Delta Q) = [1-q^{-\alpha/(1-\alpha)}] \cdot (Z/\zeta_\ell).$$

Thus, the expected change in Q —and, hence, in Y —depends only on the aggregate of R&D spending, Z , and not on the manner in which this spending is spread across the sectors. We again assume that the number of sectors is large enough so that we can treat Q as differentiable; hence, we use equation (46) to represent the actual change, \dot{Q} .

We can use the results to write the social-planner's Hamiltonian expression as

$$(47) \quad J = \left(\frac{c^{1-\theta}-1}{1-\theta}\right) \cdot e^{-\rho t} + \nu \cdot \left[LA^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{1/(1-\alpha)} \cdot Q - Z - cL \right] \\ + \mu \cdot [1-q^{-\alpha/(1-\alpha)}] \cdot (Z/\zeta_\ell).$$

The Lagrange multiplier ν applies to the resource constraint. This constraint comes from equation (43) after substitution for Y from equation (45). The shadow-price μ attaches to the expression for \dot{Q} from equation (46).

Note that the Hamiltonian in equation (47) depends on the aggregate outlay for R&D, Z , but not on the distribution of this spending across the sectors. This property means that the relative allotments of R&D across the sectors are indeterminate.²⁰

We now use familiar methods to derive the dynamic-optimization conditions for the choices of c and Z in equation (47). The first-order conditions and the transition equation for Q lead to the social-planner's growth rate:

$$(48) \quad \gamma \text{ (social planner)} = (1/\theta) \cdot \left\{ \left(\frac{L}{\zeta_\ell}\right) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{1/(1-\alpha)} \cdot [1-q^{-\alpha/(1-\alpha)}] - \rho \right\}.$$

²⁰This indeterminacy reflects the lack of diminishing returns to research in each sector. This kind of indeterminacy also applied in the decentralized frameworks; see n. 7 and n. 18.

The implicit social rate of return, which corresponds to the expression in the large brackets that precedes the term $-\rho$, is therefore

$$(49) \quad r \text{ (social planner)} = \left(\frac{L}{\zeta_\ell}\right) \cdot A^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{1/(1-\alpha)} \cdot [1-q]^{-\alpha/(1-\alpha)}.$$

The planner's rate of return can be readily compared with the return r_ℓ (from equation [41]) that applies when industry leaders have a monopoly in research. The rate r_ℓ is lower than the social rate of return by the multiple $\alpha^{1/(1-\alpha)}$ because of the effect from the monopoly pricing of the intermediate goods. (Recall that the decentralized quantity of intermediates, X_{jm_j} , in equation [6] falls short of the social-planner's quantity in equation [44] by the factor $\alpha^{1/(1-\alpha)}$.) The gap in rates of return corresponds to an excess of the planner's growth rate over the decentralized growth rate. An appropriate subsidy on the purchases of intermediate goods would, if financed by a lump-sum tax, eliminate the discrepancy in rates of return and growth rates. This subsidy also removes the static inefficiency that results from the economy's failure to employ a sufficient quantity of intermediate goods.

The rate of return r_ℓ prevails in the decentralized economy if leaders have a sufficient cost advantage in research ($\zeta_\ell < \zeta$ in the previous discussion). Otherwise, the rate of return is the value r shown in equation (34). The spread between the social rate of return and r adds $r_\ell - r$ to the gap between the social rate and r_ℓ , a gap that we have already discussed.

Recall that $r_\ell < r$ because of two offsetting effects: the rate r is higher because it counts the expropriation of the predecessor's monopoly profit, but is lower because it views the benefits from an innovation as temporary. We discussed before why the first effect was larger, so that $r_\ell < r$ on net. This result implies that the difference between

the social rate of return and r is smaller than the difference between the social rate and r_t . Hence, the gap between the planner's and decentralized growth rates is also not as large as before. It is even possible that the privately determined rate of return and growth rate would exceed the planner's values. This result applies if the effect from the monopoly pricing of the intermediates is less important than the gap between r and r_t (which reflects the net effect from the seeking of monopoly profit).

We already mentioned that the appropriate subsidy to the purchase of intermediates would remove the distortions from monopoly pricing. The additional distortions that arise when the private rate of return is r (given in equation [34]) can be eliminated if a scheme is implemented—in the spirit of Coase (1960)—that effectively endows industry leaders with property rights over their monopoly profits. This scheme would require innovators to compensate their immediate predecessor for the loss of rental income. An innovator in sector j then raises the cost of innovation to include the required compensation to the current leader, but also raises the prospective reward to include the anticipated compensation from the next innovator. The first part of the scheme causes the innovator to count only the net change in the flow of monopoly rentals as a contribution; that is, the incentive to seek the existing rents is eliminated. The second part motivates the innovator to view his or her contribution as lasting forever, rather than just until the next innovation. As usual, however, the successful implementation of this kind of policy becomes problematic in a richer model; for example, in contexts where quality improvements are hard for a policymaker to evaluate.

The internalization just described occurs automatically in the model if the leaders have a monopoly position in research, so that the private rate of return is the value r_t . Thus, one way to reach the first best in this framework is to preclude research by outsiders! This provision reduces the incentive to innovate, but only to the appropriate

extent. This method works, however, only if the effects of monopoly pricing have already been neutralized through the appropriate tax-subsidy policy. If these tax-subsidy policies are infeasible, then the prevention of research by followers is likely to worsen the outcomes in this model.

We can summarize the model's conclusions about welfare as follows:

1. The decentralized rate of return and growth rate coincide with the social-planner's choices if the effects of monopoly pricing are eliminated (for example, by implementing the appropriate subsidy to the purchase of intermediates) and if innovators are forced to compensate their immediate predecessor for the loss of monopoly rentals. This compensation scheme effectively institutes the claim on these monopoly rentals as a formal property right.
2. If the effects of monopoly pricing are eliminated, but no compensation is awarded to predecessors, then the decentralized values for the rate of return and the growth rate *exceed* the social-planner's values. The failure to compensate one's predecessor makes the private rewards to innovation too high, whereas the failure to receive compensation later goes the other way. The net effect from this rent seeking is unambiguous, however, because the distortions are essentially the same, except that the second one occurs later.
3. If there are no interventions—so that the effects of monopoly pricing are not eliminated and no compensation is paid to one's predecessor—then the decentralized values for the rate of return and the growth rate may be higher or lower than the social-planner's values. Monopoly pricing causes the decentralized rate of return and growth rate to fall short of the socially optimal values, but this effect is offset by the net effect from rent seeking, a force that makes the private rate of return too high.

F. Summary Observations about Growth

The quality improvements studied in this paper represent ongoing refinements of products and techniques, whereas the expansions of variety considered in other models describe basic innovations. From a modeling standpoint, one distinction between the two kinds of technological progress is that goods of higher quality are close substitutes for those of lesser quality, so that quality enhancements tend to make the old goods obsolete. In contrast, discoveries of new kinds of products may not, on average, be direct substitutes or complements for the existing types. Therefore, basic innovations may not drive out the old varieties. One consequence of this distinction is that, in a decentralized economy, the R&D effort aimed at quality improvements may be too high (because of the incentive to seek the monopoly rents of incumbents), whereas the effort aimed at basic innovations tends to be too low.

Another difference from the varieties models is that the costs of quality improvements for insiders tend to be smaller than those for outsiders. Hence, we argued that the insiders would tend, in equilibrium, to carry out the research that underlies the regular process of product refinement. In contrast, insiders are unlikely to have a cost advantage in breakthrough research; basically because there are no insiders for this activity. Therefore, dramatically new innovations are unlikely to come from existing industry leaders.

The two types of technological progress have similar predictions about the determination of growth rates. In both cases, growth is higher if the willingness to save is greater, the level of technology is higher, and the cost of R&D is lower. Both formulations also predict scale effects, represented in the models by the quantity of a fixed factor like raw labor or human capital.

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Figure 1: Quality Ladders and Discoveries of New Products

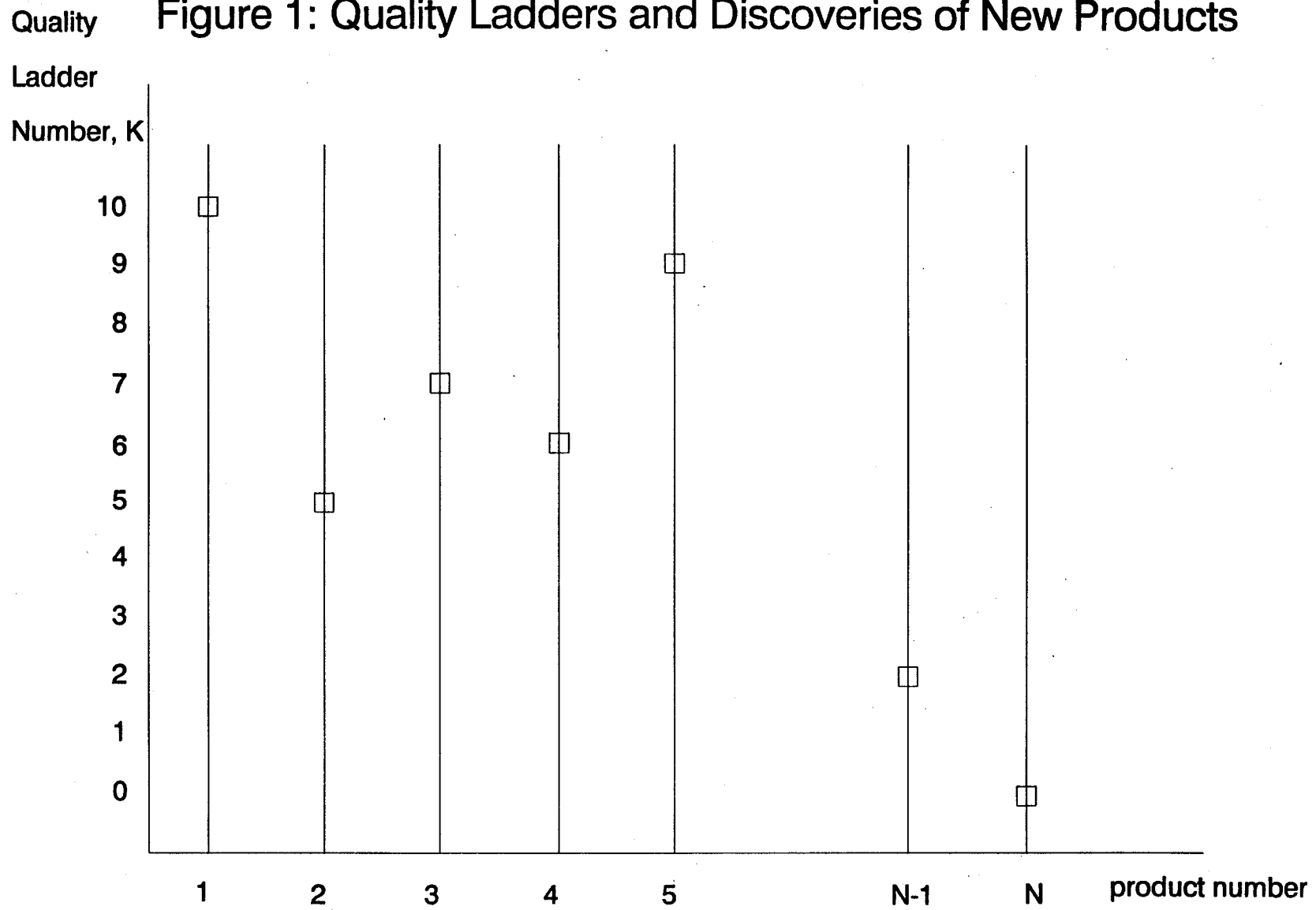


Fig 2. A Quality Ladder in a Single Sector

