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NET WORTH, CREDIT CONSTRAINTS AND  
ECONOMIC DEVELOPMENT

Masaya Sakuragawa

Nagoya City University

June 1993

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussions and critical comments. Professor Sakuragawa was a Visiting Fellow at the Economic Growth Center during the period August 1992 to October 1993 and is a Lecturer at the Faculty of Economics, Nagoya City University, Japan.

I am grateful to Oded Galor, Kazuya Kamiya, Koichi Hamada, David Porteous and Shoichi Royama for their helpful comments.

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## **Abstract**

This paper constructs an overlapping generations model with production in which moral hazard coming from asymmetric information leads to capital market imperfections. In a small open economy setting the initial net worth positions of borrowers affect the long-run state of the economy. An economy that is richer in the early stages is more likely to achieve a long-run rich equilibrium. In a closed economy, the multiple steady states disappear. This result is sharply contrasted with Diamond [1965] because in his model multiple steady states may be viable in a closed economy, but disappear in a small open economy.

Journal of Economic Literature Classification No. D82, 016.

KEY WORDS: Asymmetric Information, Development

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## 1. Introduction

Many development economists stress that a country's economic development is closely linked to the development of financial markets and financial intermediation in that country. Several recent pieces of empirical research also support this view of development.

McKinnon [1973] [1991], Fry [1988], and World Bank [1989] emphasize that informational frictions in financial markets is one important source of the underdevelopment trap in less developed countries. In these countries, ill-defined property leads to severe conflicts of interests among different groups. Moral hazard and adverse selection problems that are closely related to asymmetric information result in greater agency costs in financial markets, and make the bankruptcy proceedings difficult in the event where borrowers face financially distressed. Therefore, the process of economic development could be interpreted as the one of mitigating incentive conflicts in financial markets.<sup>1</sup>

The purpose of this model is to develop a model of overlapping generations with production in the presence of asymmetric information between lenders and borrowers. We make use of Townsend's [1979] costly-state-verification approach, in

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<sup>1</sup> Since the seminal work of Stiglitz and Weiss [1981], many inefficiencies in financial markets have been analyzed in frameworks with asymmetric information. More recently, the theoretical insights have been applied to macroeconomic settings, including Williamson [1987], Greenwood and Williamson [1989], Bernanke and Gertler [1989] [1990], Greenwald and Stiglitz [1993], and etc.

which there is asymmetric information is on borrowers' project returns and monitoring by lenders is costly.<sup>2</sup> With a risk-neutrality setting, a standard debt contract is derived as an optimal contract, and financial intermediation emerges endogenously.<sup>3</sup>

We consider a small open economy where agents can freely have access to foreign safe assets with a constant interest rate. However, our model differs from the *ordinary* small open economy in the strict sense because borrowers can not borrow freely at this rate because of the moral hazard problem. Our model is similar to Bernanke and Gertler ( hereafter, B-G ) [1989] in that the economy is a two-goods economy. Production of the final consumption good takes two steps. First, individual entrepreneurs transform the consumption good into the investment good using their own risky projects. Second, competitive firms produce the consumption good by employing labor and the investment good, according to neoclassical, constant-returns-to-scale, production technology. There is no asymmetric information problem with production of the consumption good, but that of the

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<sup>2</sup> As Greenwald and Stiglitz ( hereafter, G-S ) [1993] emphasize, the costly state verification model is not the only one describing capital market imperfections. Other several kinds of models in which adverse selection and signaling process are important can explain the similar phenomenon. However, I believe that our approach is a useful one to associate capital market imperfections with economic development because the costly-state verification approach focuses attention on the process of bankruptcy. In practice, McKinnon [1973] [1991], Fry [1988], and World Bank [1989] emphasize the role of technology for bankruptcy proceedings as one important source of underdevelopment.

<sup>3</sup> See, for example, Diamond [1984] or Williamson [1986].

investment good involves a moral hazard problem.

Our model differs from B-G in one important respect.<sup>4</sup> Equilibria may be defined by *credit rationing*, as in Stiglitz and Weiss [1981] or Williamson [1987]. Only when borrowers default on their debt, monitoring costs are incurred by intermediaries, and these costs play an important role in hampering capital accumulation through the effect of "credit rationing". However, the extent of credit rationing is mitigated by internal funds available to borrowers. The "net worth effect", as B-G have called it, works to alleviate credit constraints, thus promoting capital accumulation.<sup>5</sup> The effects of credit rationing and the net worth effect are opposite with respect to capital accumulation. The interplay of these two factors generates business fluctuations.

More significantly, the net worth effect has a potential for multiple stationary states.<sup>6</sup> Each stationary state is ordered

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<sup>4</sup> Strictly, another important difference is that, while they employ stochastic monitoring, we employ only deterministic monitoring, where monitoring occurs only with probability 0 or 1. See, Townsend [1988] and Mookherjee and Png [1989] for costly-state-verification models with stochastic monitoring.

<sup>5</sup> Other authors, such as Calomiris and Hubbard [1990], Froot and Stein [1991], and G-S [1993], emphasize the importance of net worth positions of borrowers on investment. Especially G-S extensively investigates the question how net worth positions of borrowers affect macroeconomics through the change in the firm's risk-taking behavior.

<sup>6</sup> B-G showed that the net worth effect leads to the dynamic process in the Diamond's [1965] overlapping generations model even in a small open economy setting, while G-S showed that the net worth effect generates the cycle. However, both of them does not refer to the possibility of multiple steady states.



according to the extent of credit rationing: in some states credit is severely rationed by intermediaries, while in other states credit is expanded by them. Under the neoclassical, constant-returns-to-scale, production technology, the wage rate is an increasing function of the economy's capital-labor ratio. Thus, internal funds of borrowers available to projects, which are equal to the wage rate, are higher as the capital-labor ratio increases. If an economy is initially poor, the long-run equilibrium is poor, where credit is severely rationed and the net worth positions of borrowers are low. Conversely, if the economy is initially rich, the large internal funds of borrowers enable intermediaries to expand the supply of credit, leading to a long-run rich equilibrium.

The model contains an important implication for economic development. Over all, development is closely related to the extent of financial development, which in turn is positively linked to the wealth level of borrowers. The wealth level is the driving force behind development in a world of capital market imperfections. Put another way, poverty generates poverty, while wealth generates wealth. A country that is wealthier in the early stages is more likely to achieve a wealthier state with developed financial intermediation in the long run. Conversely, a country which is poor in the early stages is more likely to fail in development. Hence, the model might give an alternative insight into economic development, as distinguished from much of the other literature which emphasizes the role of accumulation of

knowledge or human capital as an engine of development ( e.g., Romer [1986], Lucas [1988], Matsuyama [1991], and etc ). Recently, Galor and Zeira [1993] show that the initial distribution of wealth affects output in the long run in the presence of indivisibility in investment in human capital together with capital market imperfections. In contrast, our model implies that capital market imperfections are sufficient to explain the long-run divergence in development among countries.

In a closed economy setting, the multiple steady states disappear. This result is sharply contrasted with Diamond [1965] because in his model multiple steady states may be viable in a closed economy, but disappear in a small open economy.<sup>7</sup> When agents cannot have access to foreign safe assets, the net worth effect is not sufficient to generate multiplicity, but instead generates the non-monotonic behavior of the interest rate as the economy develops. This result suggests that too early liberalization of the domestic deposit market to world markets may derive the developing economy down to an underdevelopment trap, even if the world interest rate is less than the steady state rate attained in the closed economy.

The remainder of this paper is structured as follows. Section 2 sets out the basic structure of our model. Section 3 derives financial arrangements under the costly-state-

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<sup>7</sup> Galor and Ryder [1989] demonstrated that, if the elasticity of saving on the real interest rate is sufficiently negative, multiple steady states may arise in the Diamond model in a closed economy.

verification model. Section 4 analyzes the equilibria of the model and demonstrates that there are multiple steady states. Section 5 examines the effects of "financial repression", and Section 6 the effects of income redistribution. Section 7 investigates the effects of the net worth effect in a closed economy. Finally, some concluding remarks are made.

## 2. The Model with Asymmetric Information

Let us consider an overlapping generations economy with intragenerational lending and borrowing.

At each period  $t = 0, 1, \dots, \infty$ , a continuum of agents who live for two periods are born in each country. Agents are either lenders or entrepreneurs, where  $\alpha$  ( $0 < \alpha < 1$ ) is the fraction of agents who are lenders and  $(1 - \alpha)$  the fraction of agents who are entrepreneurs. Equilibrium conditions are written in per capita terms.

We make use of costly state verification approach originated by Townsend [1979], in which there is asymmetric information on borrowers' project returns and monitoring by lenders is costly. Following B-G [1989] or Hamada and Sakuragawa [1992], we assume that the production of the final consumption good take two steps.<sup>8</sup> First, individual entrepreneurs transform the

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<sup>8</sup> This assumption simplifies incentive problems. Alternatively, if we assume an one-sector model using production function with two inputs, labor and the capital good, incentive problems are more complicated because there emerges incentive problems among three types of agents: borrowers, lenders, and workers.

consumption good into capital using their own risky projects, and second, competitive firms produces the consumption good according to neoclassical, constant-returns-to-scale, production technology by employing labor and capital. Production in the consumption good is instantaneous and that in the investment good takes one period. Capital depreciates fully in one period.<sup>9</sup> The consumption good is numeraire.

The economy is a small open economy in which all agents have access to foreign safe assets with a constant interest rate  $r$ . However, our model differs from the *ordinary* small open economy because borrowers can not borrow freely at this rate because of a moral hazard problem.

Because production technology of the consumption-good firm is homogeneous of degree one, output of the consumption good can be, without loss of generality, described in terms of the actions of a single, aggregate, price-taking firm.<sup>10</sup> Denote the per capita aggregate capital available at period  $t$  by  $k_t$ , and the production of the consumption good at period  $t$  by  $y_t$ . The production function is, then, denoted in per capita terms by

$$(1) \quad y_t = f(k_t),$$

where  $f(\cdot)$  is continuously differentiable, strictly increasing

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<sup>9</sup> The model is easily extended to an economy where capital depreciates at a rate  $\delta$  ( $0 \leq \delta < 1$ ).

<sup>10</sup> Under the constant-returns-to-scale technology, the firm optimize only the capital-labor ratio. This fact justifies the specification of a single, aggregate, price-taking firm.

and concave, with  $f(0) = 0$ , and

$$\lim_{k_t \rightarrow 0} f'(k_t) = \infty.$$

The consumption-good firm purchases capital and hires labor in spot markets. Under the assumption of constant returns to scale, factor payments completely exhaust output. From the maximization problem of the firm, each input is paid its respective marginal product. Under the assumption of one-period depreciation for capital, the market price of the investment good is equated to the marginal product of capital. We derive

$$(2) \quad R_t = f'(k_t), \text{ and } W_t = f(k_t) - k_t f'(k_t),$$

where  $R_t$  is the marginal product of capital and  $W_t$  the wage rate.

Each lender born at  $t$  maximizes the expected value of the second period utility  $E_t(c_{t+1} - l_{t+1})$ , where  $c_{t+1}$  is consumption at period  $t+1$ ,  $l_{t+1}$  the quantity of effort expended to observe project returns at period  $t+1$ , and  $E_t$  the expectation operator conditional on information available at period  $t$ . In youth each lender supplies a fixed one unit of labor to the consumption-good firm and receives  $W_t$  as the wage rate. Lenders consume only when old and hence save their income entirely either by lending to some other agents or by investing in foreign assets.

Each entrepreneur also consumes only when old, and hence maximizes the expected second period consumption  $E_t(c_{t+1})$ . Each entrepreneur also supplies one unit of labor to the consumption-good firm and receives the wage rate  $W_t$  when young. Each

entrepreneur has access to only one investment project when young. Each project is not transferable between agents. Each project produces random  $\omega$  units of the investment good at period  $t+1$  with an input of  $\theta$  units of the consumption good at period  $t$ . Returns are independent and identically distributed across entrepreneurs in one generation.  $\omega$  follows a uniform distribution function over  $[0, 2\mu]$  with a mean of  $\mu$ .<sup>10</sup> An entrepreneur who has realized  $\omega$  sells it to a consumption-good firm for a market price  $R_{t+1}$  and receives  $R_{t+1}\omega$ . Entrepreneurs differ in the monitoring cost  $\beta$ , where  $\beta$  follows the probability density function  $g(\beta)$ , which is continuously differentiable on  $[0, \beta^+]$ . Let  $G(\beta)$  denote the associated probability distribution function. All agents can freely identify each entrepreneur by his value of  $\beta$ .

The actual realization of each project is freely observable only to the project owner although all agents know its distribution. Other agents must incur  $\beta$  units of effort to observe the project return of the entrepreneur with  $\beta$ .<sup>11</sup> Lenders are endowed with a unbounded quantity of effort. Both

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<sup>10</sup> The specific assumption on uniform distributions would simplify analysis without much sacrifice of the results derived below. Almost all of our results are derived under a more general distribution function  $H(\omega)$  so long as it satisfies

$$h(\omega) + \beta h'(\omega) > 0.$$

See, for example, Froot and Stein [1991].

<sup>11</sup> Focus attention only on the case in which monitoring deterministic. See Townsend [1988] and Mookherjee and Png [1989] for stochastic monitoring.

types of agents are protected by the limited liability constraints, such that  $c_t \geq 0$  and  $l_t \geq 0$ . The distributions of  $\omega$  and  $\theta$  are independent with each other.

Let  $k_{\max}$  ( $< +\infty$ ) denote the maximum level of capital-labor ratio attainable. The maximum is realized when all entrepreneurs operate their projects, such that  $k_{\max} = \mu (1 - \alpha)$ , where the maximum capital-labor ratio is proportional to the number of all the projects funded under the assumption that capital depreciates fully in one period. We impose two Assumptions:

**Assumption 1**

$$W(k_{\max}) \equiv f(k_{\max}) - k_{\max} f'(k_{\max}) < \theta,$$

and

**Assumption 2**

$$\mu f'(k_{\max}) > \theta r.$$

Assumption 1 implies that all entrepreneurs have to raise outside loans for their risky projects in any state of  $k_t$  because  $W_t$  is increasing in the capital-labor ratio. This assumption makes the incentive problem important. Next, from Assumption 2 and the fact that  $f'(\cdot)$  is decreasing, it follows that

$$(3) \quad \mu R_{t+1} - [\theta - W_t]r > W_t r.$$

The L.H.S. of (3) is the expected profit under perfect

information when any entrepreneur undertakes his project using his wage income as internal funds, while the R.H.S. is consumption when he invests her wage in foreign assets. (3) implies that any entrepreneur would be more willing to fund his project by borrowing than to invest in the safe asset at least under perfect information.

Suppose that there are "many" foreign lenders who can potentially lend to the entrepreneurs with an opportunity rate of return  $r$ .

It is helpful to discuss these issues in a world of symmetric information. In the absence of asymmetric information, the first-best solution is always achieved. By Assumption 2, all entrepreneurs receive loans from lenders and fund their investment projects. At any period  $k_{\max}$  is achievable and credit rationing never occurs.

### 3. Debt Contracts, Intermediation, and Credit Rationing

An entrepreneur born at  $t$  must make a contract with a lender born at  $t$  in order to receive  $(\theta - W_t)$  units for operating his project. Let us characterize the contract by a pair  $\{L(\omega, R_{t+1}), S\}$ , where  $L(\omega, R_{t+1})$  is an integrable, positive payment function, such that  $L(\omega, R_{t+1}) \leq R_{t+1}\omega$  for any  $\omega \in [0, 2\mu]$  and  $S$  is a subset of  $\omega \in [0, 2\mu]$  in which monitoring occurs. We restrict the type of contracts to the set of incentive compatible contracts. Consider an interest payment  $x_t \in [0, 2\mu]$ , such that



$$S = [ \omega : L(\omega, R_{t+1}) < R_{t+1}x_t ].$$

This condition ensures that all contracts under consideration satisfy incentive compatibility. Among the set of all incentive compatible contracts, the contract satisfying the following property is optimal:

$$L(\omega, R_{t+1}) = R_{t+1}\omega \quad \text{for } \omega \in S = [ \omega < x_t ],$$

and

$$= R_{t+1}x_t \quad \text{for } \omega \in S^c = [ \omega \geq x_t ].$$

Let  $r$  denote the certain rate of interest prevailing between period  $t$  and period  $t+1$  for which each lender has to be compensated. The optimal contract is, then, characterized by a pair  $\{ R_{t+1}x_t, R_{t+1}\omega \}$ , which would maximize the expected return of the entrepreneur with a loan of  $(\theta - W_t)$  units required, while giving the lender a level of expected return of at least  $r$  per unit invested:

$$\max_{x_t} \frac{R_{t+1}}{2\mu} \int_{x_t}^{2\mu} (\omega - x_t) d\omega$$

subject to

$$\frac{R_{t+1}}{2\mu} \left[ \int_0^{x_t} \omega d\omega + \int_{x_t}^{2\mu} x_t d\omega \right] - \frac{1}{2\mu} \int_0^{x_t} \beta d\omega = [\theta - W_t]r \quad .$$

The contract form has the property of a standard debt contract which Gale and Hellwig [1985] and Williamson [1986] have described. If  $\omega \geq x_t$ , the entrepreneur pays  $R_{t+1}x_t$  to the lender as interest payment, while, if  $\omega < x_t$ , the entrepreneur defaults, monitoring occurs, and the lender receives the whole return on the project. Thus, the state in which monitoring occurs is interpreted as the state of bankruptcy.

Let us specify the institutional arrangement in which debt contracts are written. As Diamond [1984] and Williamson [1986] have demonstrated, there is a possibility that financial intermediation is formed in equilibria.<sup>12</sup> If  $2W_t \geq \theta$ , each entrepreneur can fund her project from only one lender. Direct lending prevails and hence contracts are written between individual entrepreneur and lender. In contrast, if  $2W_t < \theta$ , the entrepreneur would have to make contracts with more than two lenders in direct lending. In the event where the entrepreneur is insolvent, each lender monitors her independently so that there is a duplication of monitoring in equilibria. In such a case, there may be a room for any other institutional arrangement to be considered as a means of eliminating the inefficient duplication of monitoring.

Let  $\pi^e(x_t)$  denote the expected profit of the entrepreneur born at  $t$ ;

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<sup>12</sup> Recently, several authors have analyzed the endogenous formation of financial intermediation. See, for example, Diamond and Diba [1983], Boyd and Prescott [1986], and Greenwood and Jovanovic [1990].

$$(5) \quad \pi^e(x_t) = \frac{R_{t+1}}{2\mu} \int_{x_t}^{2\mu} (\omega - x_t) d\omega.$$

Let  $\pi(x_t, \beta)$  denote the expected profit of the intermediary lender born at  $t$  from a loan to the entrepreneur with  $\beta$ ;

$$(6) \quad \pi(x_t, \beta) = \frac{1}{4\mu} R_{t+1} x_t^2 + R_{t+1} x_t \left(1 - \frac{1}{2\mu} x_t\right) - \frac{\beta x_t}{2\mu}. \quad 13$$

The first term is the expected return when the entrepreneur is insolvent, the second term the expected interest payments, and the third the expected monitoring costs born by the intermediary, where  $x_t/2\mu$  is the probability of bankruptcy.

For the entrepreneur with a characteristic  $\beta$ ,  $x_t$  satisfies

$$(7) \quad \max_{x_t} \pi^e(x_t)$$

subject to the individual rationality condition of the intermediary, such that

$$(7') \quad \pi(x_t, \beta) = (\theta - W_t)r,$$

where (7') states that the intermediary must be compensated for  $(\theta - W_t)r$  units for lending to the entrepreneur.

Differentiating (6) with respect to  $x_t$  gives

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<sup>13</sup> Strictly, the lender who actually makes loans may be an individual lender or an intermediary lender, depending on parameter values. However, we call him the intermediary lender because the allocation under intermediated economy reproduces the same allocation as achieved in direct lending when direct lending survives.

$$(8) \quad \pi_1(x_t, \beta) = R_{t+1} \left(1 - \frac{1}{2\mu} x_t\right) - \frac{\beta}{2\mu},$$

where  $x_t$  is defined over  $[0, 2\mu]$ .

In order to guarantee positively-valued interest payments over the whole region, we impose the following assumption.

### Assumption 3

$$2\mu f'(k_{\max}) > \beta^+.$$

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For the shape of  $\pi(\cdot)$ , under Assumption 3 three properties hold:

$$(9) \quad \pi_{11}(\cdot) = -\frac{1}{2\mu} R_{t+1} < 0,$$

$$(9') \quad \lim_{x_t \rightarrow 2\mu} \pi_1(\cdot) = -\frac{\beta}{2\mu} < 0,$$

and,

$$(9'') \quad \lim_{x_t \rightarrow 0} \pi_1(\cdot) = R_{t+1} - \frac{\beta}{2\mu} > 0,$$

for any  $0 < \beta < \beta^+$ , and  $0 \leq k_{t+1} \leq k_{\max}$ . As illustrated in Figure 2, these three properties ensure that for any entrepreneur with  $\beta$   $\pi(x_t, \beta)$  is strictly concave in  $x_t$  and reaches a maximum for some

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<sup>14</sup> Without Assumption 3, there exists a range of capital-labor ratio, above which  $\pi(x_t, \beta)$  is monotonic decreasing as depicted at the dotted curve in Figure 2.

interior, and that generates a possibility of *equilibrium credit rationing* as in Stiglitz and Weiss [1981] and Williamson [1987].

Incorporating  $\pi(x_t, \beta) = 0$  in (8) into (6), we obtain the expected maximum profit that any intermediary can earn from loans to the entrepreneur with  $\beta$ , such that

$$(10) \quad \mu \frac{[R_{t+1} - \frac{\beta}{2\mu}]^2}{R_{t+1}},$$

given  $R_{t+1}$ . If (10) exceeds or equals the profit from the alternative investment

$$(11) \quad [\theta - W_t]r,$$

it is attractive for the intermediary to supply loans to the entrepreneur with  $\beta$ , but if (10) is strictly less than (11), it is attractive rather to invest in the safe asset. If there are interior equilibria, there exist cutoffs  $\beta_t$ , such that  $0 < \beta_t < \beta^+$ , satisfying

$$(12) \quad \mu \frac{[R_{t+1} - \frac{\beta}{2\mu}]^2}{R_{t+1}} = [\theta - W_t]r,$$

given  $W_t$  and  $R_{t+1}$ , since (10) is a strictly decreasing function of  $\beta$ . The entrepreneur with  $\beta_t$  is denoted by the *marginal entrepreneur*. If equilibria involve (12), entrepreneurs with monitoring costs  $\beta \leq \beta_t$  can receive loans, while those with  $\beta >$

$\beta_t$  cannot and thus *credit rationed*. The latter class of borrowers do not receive loans in spite of the fact that they would be willing to pay higher than market-determined rates in order to have access to their investment projects. As in Stiglitz and Weiss [1981] and Williamson [1987], the fact that the expected return from a loan of one unit to the borrowers would fall below the opportunity return generates credit rationing. The former class of borrowers who actually receive loans promise to pay loan interest rates which are implicitly determined by

$$\pi(x_t, \beta) = \theta r,$$

given  $W_t$  and  $R_{t+1}$ .

It is useful to calculate the expected profit of the marginal entrepreneur who undertakes his risky project by debt:

$$(13) \quad R_{t+1}\mu - (\theta - W_t)r - \frac{\beta_t}{2\mu} x_t,$$

while the profit from the safe investment is  $W_t r$ . Whether equilibria will be characterized by credit rationing depends on parameter values. If (13) exceeds  $W_t r$ , and hence if

$$(14) \quad \mu R_{t+1} - \beta_t \frac{2\mu R_{t+1} - \beta_t}{2\mu R_{t+1}} > r\theta,$$

the neighboring entrepreneur with a higher monitoring cost is credit rationed, and hence the equilibria are characterized by

credit rationing, defined by (12).<sup>15</sup> Whenever the condition (14) is satisfied, the inequality is also satisfied for the entrepreneur with a higher monitoring cost adjacent to the marginal entrepreneur because entrepreneurs are assumed to be continuously ordered in terms of the monitoring cost. Conversely, if the inequality of (14) is reversed, such that

$$(14') \quad \mu R_{t+1} - \beta_t \frac{2\mu R_{t+1} - \beta_t}{2\mu R_{t+1}} \leq \theta r,$$

equilibria do not involve credit rationing. The marginal entrepreneur is now defined by the borrower who is indifferent between undertaking his risky project or investing his internal funds in the safe asset, such that

$$\pi^e(x_t, \beta_t) = W_t r,$$

or equivalently,

$$(15) \quad R_{t+1}\mu - \theta r - \frac{\beta_t}{2\mu} x_t = 0.$$

The entrepreneur adjacent to the marginal entrepreneur then would be more willing to choose the safe investment rather than the risky project and hence is *never credit rationed*.

We define equilibria where the condition (14) is satisfied as *credit rationing equilibria* (CRE), and those where,

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<sup>15</sup> Combining  $x_t$  satisfying  $\pi_1(x_t, \beta) = 0$  in (8) into (13) yields the expected profit of the marginal entrepreneur. By comparing it with the alternative return  $[\theta - W_t]r$ , we obtain (14).

alternatively, the condition (14') is satisfied as *no credit rationing equilibria* (NCRE). In general, equilibria may involve both regions of CRE and of NCRE simultaneously. However, in this section we proceed with the analysis focusing only on CRE, where (14) is always satisfied. This procedure may be restrictive but it is without loss of generality from two reasons. First, as analyzed below in the Appendix, the qualitative results in NCRE are almost the similar to those in CRE.<sup>16</sup> Second, as demonstrated below, there exist equilibria which are CRE over the whole region.

#### 4. Multiple Steady States

In this section we incorporate the ingredients representing capital market imperfections into a general equilibrium setting.

The aggregate quantity of capital at period  $t+1$  equals the average return of each project  $\mu$  times the number of projects funded at period  $t$ , such that

$$(16) \quad k_{t+1} = \mu(1 - \alpha)G(\beta_t). \quad 17$$

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<sup>16</sup> See the Appendix [1][2] in details.

<sup>17</sup> If the depreciation rate is  $\delta$  ( $0 \leq \delta \leq 1$ ), in general the law of motion of capital is rewritten as

$$k_{t+1} = \mu(1 - \alpha)G(\beta_t) + (1 - \delta)k_t.$$

Then, the dynamic process would be a little complicated, but the qualitative results reproduce the economy with  $\delta = 1$ , represented by (16).



From the application of the implicit function theorem, there exists a continuously differentiable function  $\beta: [0, k_{\max}] \rightarrow [0, \beta^+]$ , such that

$$(17) \quad k_{t+1} = \mu(1 - \alpha)G(\beta(k_{t+1})), \text{ with } \beta'(\cdot) > 0, \text{ and } \beta(0) = 0.$$

Incorporating (2), (2') and

$$(17') \quad \beta_t = \beta(k_{t+1})$$

into (12), we describe the functional relation between the capital level in two successive periods as

$$(18) \quad n(k_{t+1}) = m(k_t),$$

where  $m(k_t)$  is the "cost of lending" and  $n(k_{t+1})$  the "benefit of lending", defined by

$$(19) \quad m(k_t) \equiv r[\theta - W(k_t)],$$

and

$$(20) \quad n(k_{t+1}) \equiv \mu \frac{[f'(k_{t+1}) - \frac{\beta(k_{t+1})}{2\mu}]^2}{f'(k_{t+1})},$$

By (17), Assumption 1 and 3,  $m(\cdot)$  and  $n(\cdot)$  are strictly decreasing and positive-valued, and

**Lemma 1**

$n(k_{t+1}) \rightarrow \infty$  as  $k_{t+1} \rightarrow 0$ .

*Proof.*

$$\lim_{k_{t+1} \rightarrow 0} n(k_{t+1}) = \mu \lim_{k_{t+1} \rightarrow 0} f'(k_{t+1}) - \lim_{k_{t+1} \rightarrow 0} \beta(k_{t+1}) + \frac{1}{4\mu} \lim_{k_{t+1} \rightarrow 0} \frac{\beta(k_{t+1})^2}{f'(k_{t+1})} = \infty,$$

since

$$\lim_{k_{t+1} \rightarrow 0} f'(k_{t+1}) = \infty, \quad \text{and} \quad \lim_{k_{t+1} \rightarrow 0} \beta(k_{t+1}) = 0,$$

Q.E.D.

In economic terms, (19) captures the "net worth effect" as B-G have called. That is, an increase in  $k_t$  gives rise to an increase in internal funds available to entrepreneurs, which tends to reduce the intermediary's cost of lending. (20) captures two reasons why the intermediary's benefit of lending declines as  $k_{t+1}$  increases. First, an increase in  $k_{t+1}$  is associated with loans to borrowers of lower quality. Second, an increase in  $k_{t+1}$  gives rise to a reduction of the price of the investment good. Both elements tend to lead to a reduction of the intermediary's profit.

Figure 2 illustrates one typical case where the economy converges to a unique interior stationary state. The stationary

state exhibits credit rationing. The story behind the dynamic adjustment process is as follows. Given the initial capital level  $k_0$  which is less than  $\hat{k}$ , intermediaries will expand the supply of credit. The credit expansion induces a rise in the wage rate. The rise in the wage rate simultaneously implies an increase in internal funds available to entrepreneurs, which in turn, mitigates credit constraints, leading to further capital accumulation.

**Proposition 1**

If  $m(k_{\max}) > n(k_{\max})$ , there exist interior stationary equilibria.

*Proof.* Both of  $m(\cdot)$  and  $n(\cdot)$  are continuous over  $[0, k_{\max}]$ . Using Lemma 1, if  $m(k_{\max}) > n(k_{\max})$ , there exist interior values to satisfy  $m(\cdot) = n(\cdot)$  from the application of the intermediate value theorem. Q.E.D.

Conversely, if  $m(k_{\max}) < n(k_{\max})$ ,  $k_{\max}$  is a stationary state, where all borrowers can fund their projects in the absence of credit rationing.

Consider an example in which we specify the probability distribution function of the monitoring cost by  $G(\beta) = c\beta^b$ , with  $b > 0$  and  $c > 0$ . Specifically, the case with parameter values of  $\theta = 0.55$ ,  $\alpha = 0.76$ ,  $\mu = 1.25$ , and  $a = 0.5$  qualitatively replicates an economy with a unique interior steady

state, described in Figure 2.<sup>18</sup> Given the parameter values, for any  $k_0 \in (0, 0.3]$ , there is a unique equilibrium which converges to the steady state  $E$ , which is globally stable. If the initial state  $k_0$ , which represents the initial number of projects funded, is smaller (greater) than  $\hat{k}$ , then the initial wage rate  $W_0$  is smaller (greater) than the steady state wage rate  $\hat{W}$ .  $k_t$  and  $W_t$  are monotonically increasing (decreasing) and eventually approach the steady state value.

However, because both of the curves slope downward, there may be multiple intersections. We obtain the following Proposition.

**Proposition 2**

There is a function  $G(\cdot)$  which exhibits multiple steady states.

*Proof.* See the Appendix [4].

See Figure 3. Suppose at first that  $\theta$  is relatively large, for example,  $\theta = \theta_1$ .  $n(k_{t+1})$  and  $m(k_t)$  uniquely intersect at  $A$ . There is only one steady state. When  $\theta$  is decreased and other parameters are held constant,  $m(k_t)$  shifts down while  $n(k_{t+1})$  does not move. There are two steady states  $B_1$  and  $B_2$  for  $\theta_2 = 0.34088$ . For some smaller values than  $\theta_2$ , there are three steady states  $C_1$ ,  $C_2$ , and  $C_3$ . That is, there exists  $\theta_3$  such that 0.3354

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<sup>18</sup> The proof appears in the Appendix [3].

$\theta_3 < \theta_2 = 0.34088$ . Hence, as  $\theta$  becomes smaller, the loan size tends to be smaller, permitting incentive constraints to be more mitigated. Figure 3 illustrates one example that, for relatively small values of  $\theta$ , multiple steady states are more likely to emerge.

As Figure 4 illustrates, if  $k_0 < \bar{k}$ ,  $k_t$  monotonically converges to  $\hat{k}$ , while if  $k_0 > \bar{k}$ ,  $k_t$  reaches the upper bound  $k_{\max}$  after some finite number of periods. There are three steady states,  $\hat{k}$ ,  $\bar{k}$ , and  $k_{\max}$ . The long-run state depends entirely on the initial state of the economy, which is represented by  $k_0$  ( or  $W_0$  ). Two interior steady states are associated with credit rationing in which some borrowers are credit rationed. Among the two equilibria,  $\hat{k}$  is dynamically stable and  $\bar{k}$  is dynamically unstable. The corner solution  $k_{\max}$  is characterized by a state without credit rationing, and is dynamically stable.

The long-run state of the economy is historically determined, depending on the initial wealth level. If the economy is initially poor with only the small numbers of projects funded, the small net worth positions of borrowers prevent lenders from expanding the supply of credit. Credit is severely rationed and only the borrowers of relatively high quality ( small monitoring costs ) can receive loans. In contrast, if the economy is initially rich with large numbers of projects funded, the high net worth positions of borrowers enable intermediaries to expand the supply of credit. Even the borrowers of low quality can receive loans. Eventually there emerges a state in

which all entrepreneurs receive loans and wages are high.

The logic behind the multiple steady states is as follows. At a poor state with a small capital-labor ratio, the small net worth requires large outside loans for funding of their projects. The induced large agency costs prevent intermediaries from expanding the supply of credit. The feedback between small net worth and severe credit constraints reinforces the low income trap. However, the net worth of borrowers is greater as the economy is wealthier because the wage rate is an increasing function of the capital-labor ratio. Thus, we may have another steady state with credit more expanded. If the economy is at a rich state with a higher capital-labor ratio, the agency costs are smaller due to small loans required, mitigating credit constraints. The high net worth positions of borrowers enable intermediaries to expand the supply of credit, which in turn leads to a rise in the wage rate and so on. Now the feedback reinforces the long-run state without credit rationing.

In order to show that the existence of multiple steady states is closely related to the net worth effect, it is useful to examine an alternative economy where only the lenders work in the first period of life. Suppose that entrepreneurs do not work in both periods of life, and that any other assumptions remain unchanged. The only difference from the above model is that all entrepreneurs have to borrow  $\theta$  units from intermediaries at any stage, and thus (18) is replaced by

$$(18') \quad n(k_{t+1}) = r\theta .$$

Since  $n(\cdot)$  is a monotonic decreasing function of  $k_{t+1}$ , (18') determines a unique capital-labor ratio, and the dynamic process disappears.

We could measure the extent of financial intermediation or financial "depth" as the number of risky projects funded. Our model, then, implies the positive relation between internal wealth of borrowers, saving and investment channelled through financial intermediation. In this regard, our analysis is closely parallel to Gurley and Shaw [1967] and to McKinnon [1973], who insists that a country's economic development is closely linked to the "depth" of financial intermediation in that country.

Different initial conditions are associated with different long-run states in our model. Multiple steady states arise also in an one-sector overlapping generations model pioneered by Diamond [1965] in a closed economy, if the elasticity of saving on the real interest rate is sufficiently negative, as has been demonstrated by Galor and Ryder [1989].<sup>19</sup> Our multiple steady states arise in a small open economy setting, the result of which is contrasted with Diamond.<sup>20</sup> The initial wealth level of

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<sup>19</sup> Galor [1992], alternatively, develops a two-sector overlapping generations model, and shows that multiple steady states are consistent in the model when the elasticity of saving on the real interest rate is positive.

<sup>20</sup> At least to my knowledge, there is scarce evidence to support this condition. For example, Hall [1988] shows that consumption and hence saving is almost independent of the real

borrowers dictates the long-run capital-labor ratio through the interaction between credit rationing and net worth effects. An economy with a high initial level of wealth achieves a wealthy state with developed financial intermediation, while an economy with a low level is forced to a stagnant state with less developed financial intermediation in the long run.

Our analysis contains an important implication for the link between economic development and financial development. There is reciprocal causation between both developments, which is channelled through internal wealth positions of borrowers. The initial wealth level itself is the driving force behind development. Put another way, *poverty generates poverty, while wealth generates wealth under capital market imperfections*. A country which is relatively richer in the early stages of development is more likely to succeed in development, whereas a country which is initially poor is more likely to be forced to a low income trap with less developed financial intermediation.<sup>21</sup> Hence, the model might give an alternative insight into economic development, as distinguished from much of the other literature

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interest rate, using the post-war data in the United States.

<sup>21</sup> It may be argued that it is highly artificial to identify equilibria associated with capital market imperfections in terms of credit rationing. According to historical evidence, low income traps associated with less developed financial markets may not be typically featured by credit rationing, but may be described by underinvestment or governments' regulations on financial markets ( see, for example, McKinnon [1973] ). One advantage of our use of credit rationing to describe equilibria is that equilibria are numerically solvable under some parameter values to make it possible to prove the existence of multiple steady states.



which emphasizes the role of accumulation of knowledge or human capital as an engine of development ( e.g., Romer [1986], Lucas [1988], Matsuyama [1991], and etc ). Our model predicts that the difference in the initial wealth levels of countries would be sufficient to derive their long-run divergence under capital market imperfections. Recently, Galor and Zeira [1993] show that the initial distribution of wealth affects output in the long run in the presence of indivisibility in investment in human capital together with capital market imperfections. In contrast, our model implies that capital market imperfections are sufficient to explain the long-run divergence in development among countries.

##### **5. The Effects of "Financial Repression".**

More recently, a number of authors have developed models which associate financial intermediation with growth.<sup>22</sup> As McKinnon [1973] has emphasized, governments in many developing countries have repressed financial markets and hampered the formation of financial intermediation. Following Williamson [1986], we are able to examine the effects of "financial repression" by comparing an economy under intermediated lending with another economy in which financial intermediation is suppressed.

Suppose that financial intermediation is prohibited. The

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<sup>22</sup> See, for example, Bencivenga and Smith [1991] or Greenwood and Jovanovic [1990].

costs of the impossibility of intermediation can be captured in such a manner that monitoring costs may be greater in direct lending than in an intermediated economy. In direct lending, a duplication of monitoring may occur when borrowers are insolvent. If parameter values satisfy  $2 W_t \leq \theta < 3 W_t$ , two lenders must match one entrepreneur in contractual arrangements. In the event where the borrower defaults, two lenders monitor the borrower independently, thus  $2\beta$  units of effort are spent for monitoring. If  $3 W_t \leq \theta < 4 W_t$ ,  $3\beta$  units are spent, and so on.

The duplication of monitoring thus leads to an increase in the monitoring costs incurred by lenders, which tends to force lenders to contract credit supply in order to maintain their reservation utility. The effects are represented by a downward shift of  $n(\cdot)$ . In a regime with a unique interior steady state, the steady state capital-labor ratio becomes smaller. In another regime with multiple steady states, the steady state in which all borrowers receive loans may disappear. Then, the government policy to repress intermediation generates the long-run divergence in income levels among intermediated economies and economies where intermediation is suppressed.

## **6. Income Redistribution and the Long-Run State.**

Let us examine the effects of income redistribution between lenders and borrowers. Suppose that entrepreneurs receive  $\tau W_t$  units of the consumption good as transfer from the government

when young and the government levies

$$\frac{(1 - \alpha)}{\alpha} \tau W_t$$

as a lump-sum tax on each lender to finance the revenue for the transfer. Given this transfer scheme, (18) is written as

$$(18'') \quad n(k_{t+1}) = m(k_t, \tau) ,$$

where

$$m(k_t, \tau) \equiv r[\theta - (1-\tau)W(k_t)] .$$

the redistribution from lenders to borrowers increases the long-run capital-labor ratio through the downward shift of the  $m(\cdot)$  curve. Transfer to borrowers mitigate credit constraints through the net worth effect, leading to an expansion of credit supplied by intermediaries. Furthermore, for some critical value of  $\tau = \tau_0$ , the system bifurcates when the model has multiple steady states. As small change in  $\tau$  around  $\tau_0$  drastically changes the dynamic behavior of the model ( see Figure 5A, 5B, and 5C ), leading to a large change in the long-run capital-labor ratio.

## 7. The Closed Economy and the unique Steady State.

In this section we investigate a closed economy in which any agent can not have access to foreign safe assets with a constant

interest rate. The only riskless asset available to any agent is bank deposits, and then the deposit rate of interest is determined to satisfy the market clearing for the deposit market.

Now the deposit market clears to satisfy

$$(21) \quad W(k_t) = k_{t+1},$$

where the L.H.S. is the aggregate supply of loans and the R.H.S. the aggregate demand for loans. Suppose that the equilibrium is defined by CRE,<sup>23</sup> and thus the deposit rate of interest evolves according to

$$(22) \quad \mu \frac{[f'(k_{t+1}) - \frac{\beta(k_{t+1})}{2\mu}]^2}{f'(k_{t+1})[\theta - W(k_t)]} = r_t,$$

over the whole region. To confine attention on an economy where the steady state is interior, we impose the following Assumption.

**Assumption 4**

$$W(k_{\max}) \leq k_{\max}.$$

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<sup>23</sup> The conditions under which equilibrium is CRE over the whole region are  $\Lambda'(\cdot) \leq 1$ , and  $\Lambda''(\cdot) \leq 0$ , where  $\Lambda(\cdot)$  is defined in the Appendix [2].

Under Assumption 4, there is a unique interior stationary state in a closed economy. As shown in Figure 6A, given any initial state  $k_0 \in (0, k_{\max}]$ , the interior stationary state is unique and dynamically stable.<sup>24</sup> Denoting the L.H.S. of (22) by  $Z(k_t, k_{t+1})$ , we obtain  $Z_1(\cdot) > 0$  and  $Z_2(\cdot) < 0$ . Substituting (21) into (22),  $r_t$  is defined only in terms of  $k_t$ , such that

$$(23) \quad Z(k_t, W(k_t)) = r_t.$$

The first term in the L.H.S. represents the net worth effect. Other things being equal, the deposit rate of interest is higher as capital intensity increases. The second term captures other two effects which state that, other things being equal, the deposit rate of interest rate is lower as capital intensity increases. When the net worth effect is negligible, the deposit rate of interest rate is monotonic decreasing. However, when the effect matters, it may not be monotone. Figure 6B illustrates a case where the deposit rate of interest fluctuates as capital intensity increases.

#### Proposition 4

The sufficient condition for the non-monotonic behavior of the interest rate is that  $dr_t/dk_t > 0$  at the interior steady state capital-labor ratio  $k^* (> 0)$ .

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<sup>24</sup> Without Assumption 4, the steady state is unique, that is,  $k^* = k_{\max}$ .

In contrast with the small open economy, the net worth effect never leads to multiple steady state, but generates the non-monotonic behavior of the interest rate. This implies that the net worth effect is not sufficient to generate multiple steady states. This result is sharply contrasted with Diamond [1965] because in his model multiple steady states may be viable in a closed economy, but disappear in a small open economy. The non-monotonic behavior suggests that, even if the interest rate of foreign asset is less than the steady state interest rate in the closed economy, openness of the domestic deposit market to world markets may not increase the long-run capital intensity. Too hasty liberalization of the domestic capital market may derive the economy down to the underdevelopment trap.

## **8. Conclusion**

The model developed in this paper contains an important implication for the link between economic development and financial development. The initial wealth level is the driving force behind development. Poverty generates poverty, while wealth generates wealth under capital market imperfections. A country that is richer in early stages is more likely to achieve a long-run richer state.

## Appendix

### [1] Characterization of No Credit Rationing Equilibria (NCRE)

If equilibria are NCRE, the marginal entrepreneur is defined by the borrower with  $\beta_t$ , who is indifferent between two investment opportunities, the risky project and the safe asset, given the quoted interest payment  $x_t$ , such that

$$(15) \quad R_{t+1}\mu - \theta r - \frac{\beta_t}{2\mu} x_t = 0,$$

where  $x_t$  is a smallest one to satisfy the intermediary's individual rationality condition,

$$(24) \quad R_{t+1} \left( x_t - \frac{1}{4\mu} x_t^2 \right) - \frac{\beta_t x_t}{2\mu} = [\theta - W_t]r.$$

Using (2), (2'), and (17'), from (24), implicitly we obtain

$$(25) \quad x_t = \Gamma(k_t, k_{t+1}),$$

with

$$(25') \quad \frac{\partial \Gamma(\cdot)}{\partial k_t} = -\frac{rW'}{\pi_x} < 0,$$

and

$$(25') \quad \frac{\partial \Gamma(\cdot)}{\partial k_{t+1}} = -\frac{\pi_R f'' + \pi_\beta \beta}{\pi_x} > 0,$$

where

$$\pi_x = R_{t+1} \left( 1 - \frac{1}{2\mu} x_t \right) - \frac{\beta_t}{2\mu} > 0,$$

$$\pi_R = x_t - \frac{1}{4\mu} x_t^2 > 0,$$

and

$$\pi_\beta = -\frac{x_t}{2\mu} > 0.$$

Denote

$$\frac{\partial \Gamma(\cdot)}{\partial k_t} \equiv \Gamma_1, \quad \text{and} \quad \frac{\partial \Gamma(\cdot)}{\partial k_{t+1}} \equiv \Gamma_2.$$

Using (15) (17'), and (24), from the application of the implicit function theorem, there exists a continuously differentiable function

$$(26) \quad k_{t+1} = \psi(k_t),$$

satisfying



$$\mu f'(\psi(k_t)) - \theta r - \frac{\beta(\psi(k_t))}{2\mu} \Gamma(k_t, \psi(k_t)) = 0,$$

with

$$\psi' = -\frac{\beta\Gamma_1}{X\beta' + \beta\Gamma_2 - 2\mu^2 f''} > 0.$$

[2] Switching between CRE and NCRE

Equilibria are CRE if the expected profit of the marginal entrepreneur exceeds the profit from the alternative safe investment, such that

$$[i] \quad \mu R_{t+1} - \beta_t \frac{2\mu R_{t+1} - \beta_t}{2\mu R_{t+1}} > r\theta,$$

while equilibria are NCRE if otherwise, such that

$$[ii] \quad \mu R_{t+1} - \beta_t \frac{2\mu R_{t+1} - \beta_t}{2\mu R_{t+1}} < r\theta,$$

which are restatements of (14) and (14'). Using (11), conditions [i] and [ii] are replaced by

$$[\text{iii}] \quad \frac{\beta_t^2}{2\mu R_{t+1}} > W_t ,$$

and

$$[\text{iv}] \quad \frac{\beta_t^2}{2\mu R_{t+1}} < W_t ,$$

The L.H.S. of [iii] ( or [iv] ) is increasing in  $k_{t+1}$ , and the R.H.S. is increasing in  $k_t$ , and  $\beta_t$ ,  $W_t$ , and  $R_{t+1}$  are continuous on  $[ 0, k_{\max} ]$ . Thus, there exists a unique continuous function

$$k_{t+1} = \Lambda( k_t ),$$

satisfying

$$\frac{\beta(\Lambda(k_t))^2}{2\mu f'(\Lambda(k_t))} = W(k_t) ,$$

with

$$\text{with } \Lambda'(\cdot) = \frac{W'f^2}{\beta(2\beta'f' - f'')} > 0, \text{ and } \Lambda(0) = 0.$$

Equilibria are CRE if  $k_{t+1} > \Lambda( k_t )$ , while equilibria are NCRE if  $k_{t+1} < \Lambda( k_t )$ . In CRE,  $k_t$  evolves according to  $k_{t+1} = \phi( k_t )$ ,

satisfying

$$\mu \frac{[f'(\phi(k_t)) - \frac{\beta(\phi(k_t))}{2\mu}]^2}{f'(\phi(k_t))} = [\theta - w(k_t)]r,$$

with  $n(\phi(k_t)) = m(k_t)$ , while in NCRE,  $k_t$  evolves according to  $k_{t+1} = \psi(k_t)$ , satisfying

$$\mu R_{t+1} - r\theta - \frac{\beta_t}{2\mu} \Gamma(k_t, k_{t+1}) = 0,$$

**Proposition 5**

(I) In CRE,  $k_{t+1} = \phi(k_t) < \psi(k_t)$ , and (II) in NCRE,  $k_{t+1} = \phi(k_t) < \psi(k_t)$ .

*Proof.* First, prove (I). Suppose to the contrary that, in CRE,

$$\psi(k_t) < \phi(k_t),$$

which implies that, given  $k_t$ , the next capital level  $k_{t+1}$  in CRE is strictly greater than  $k_{t+1}$  in NCRE. Denote variables of NCRE by superscript "N". Since  $k_{t+1} > k_{t+1}^N$ ,

$$(i) \quad \beta_t > \beta_t^N, \quad \text{and} \quad R_{t+1} < R_{t+1}^N,$$

must be satisfied from (17) and (2'), where  $\beta_t^N$  and  $R_{t+1}^N$  satisfy

(ii)

$$\mu R_{t+1}^N - r\theta - \frac{\beta_t^N}{2\mu} \Gamma(k_t, k_{t+1}^N) = 0,$$

given  $k_t$ . The following conditions must be satisfied

(iii)

$$\Gamma(k_t, k_{t+1}^N) \leq \frac{2\mu R_{t+1}^N - \beta_t^N}{R_{t+1}^N} < \frac{2\mu R_{t+1} - \beta_t}{R_{t+1}}.$$

The first inequality arises from the fact that the second term is the feasible maximum interest payment of the borrower with  $\beta_t^N$ , and the second inequality arises from (i). From (i), (ii), and (iii), the expected profit of the marginal entrepreneur must be strictly negative, such that

$$\mu R_{t+1} - \beta_t \frac{2\mu R_{t+1} - \beta_t}{2\mu R_{t+1}} < r\theta,$$

which is a contradiction. Thus, if equilibria are CRE,  $\phi(k_t) < \psi(k_t)$ .

Second, prove (II). Suppose to the contrary that, in NCRE,

$$\phi(k_t) < \psi(k_t),$$

which implies that, given  $k_t$ , the next capital level  $k_{t+1}$  in NCRE

is strictly greater than  $k_{t+1}$  in CRE. Since  $k_{t+1} < k_{t+1}^N$ ,

$$(i) \quad \beta_t < \beta_t^N, \quad \text{and} \quad R_{t+1} > R_{t+1}^N,$$

where a pair  $\{ k_{t+1}^N, \beta_t^N, R_{t+1}^N \}$  must satisfy

$$(ii) \quad \mu R_{t+1}^N - r\theta - \frac{\beta_t^N}{2\mu} \Gamma(k_t, k_{t+1}^N) = 0,$$

given  $k_t$ . Then,

$$(iii) \quad \mu R_{t+1} - r\theta - \frac{\beta_t}{2\mu} \Gamma(k_t, k_{t+1}) > 0,$$

### Lemma 2

$$\Gamma(k_t, k_{t+1}) = \frac{2\mu R_{t+1} - \beta_t}{R_{t+1}}, \quad \text{if} \quad k_{t+1} = \phi(k_t).$$

*Proof.* First, suppose to the contrary that

$$x(k_{t+1}) > \Gamma(k_t, k_{t+1}),$$

where

$$x(k_{t+1}) = \frac{2\mu R_{t+1} - \beta_t}{R_{t+1}},$$

Since  $k_{t+1} = \phi(k_t)$ ,  $k_{t+1}$  in  $\Gamma(\cdot)$  must correspond to the marginal cost of the marginal entrepreneur,  $\beta_t = \beta(k_{t+1})$ . Then, by definition,  $\Gamma(\cdot)$  must be the interest payment which maximizes the intermediary's profit, Because  $\pi(\cdot)$  is strictly concave, this is a contradiction.

Second, suppose to the contrary that

$$x(k_{t+1}) < \Gamma(k_t, k_{t+1}),$$

Then, there exists a smaller interest payment less than  $x(k_{t+1})$  which strictly increases the entrepreneur's profit given that the intermediary's IR condition is satisfied. A contradiction. Q.E.D.

Using Lemma 2, (iii) is replaced by

$$(iv) \quad \mu R_{t+1} - r\theta - \frac{\beta_t}{2\mu} \frac{2\mu R_{t+1} - \beta_t}{R_{t+1}} > 0,$$

which is a contradiction. Q.E.D.

Therefore, the motion of the capital-labor ratio is, in general, described as

$$k_{t+1} = \min \{ \phi(k_t), \psi(k_t) \}.$$

In Figure 7, one typical case involving both regions of CRE and NCRE is illustrated, where the lower boundary of  $\phi(\cdot)$  and  $\psi(\cdot)$  characterizes the equilibria.

[3] The proof of the existence of a unique interior steady state.

Prove this using an example. Assume the parameter values:  $\alpha = 0.76$ ,  $a = 0.5$ ,  $\mu = 0.25$ ,  $b = 2$ ,  $r = 1$ , with  $c = \frac{1}{4(1-\alpha)\mu^3}$ .

Then,

$$m(k_t) = \theta - 0.5 k_t^{0.5},$$

and

$$n(k_{t+1}) = \frac{2.5(0.5k_{t+1}^{-0.5} - k_{t+1}^{0.5})^2}{k_{t+1}^{-0.5}}.$$

By (13), the whole interval is

$$(1-1) \quad 0 \leq k_t \leq k_{\max} = 0.3.$$

Steady states values of  $k$  satisfy  $n(k) = m(k)$ . Denoting  $k^{0.5} \equiv X$ , define  $\Gamma(X)$  by

$$\Gamma(X) \equiv X^4 - 0.8 X^2 - 0.4 \theta X + 0.25,$$

where  $\Gamma(X) = 0$  implies  $n(k) = m(k)$ , with the whole interval  $0 \leq X \leq X_{\max} = 0.3^{0.5}$ .

Then, a sufficient condition that there exists at least one  $k$  satisfying  $n(k) = m(k)$  for  $0 < k < 0.3$  is  $\Gamma(0)\Gamma(X_{\max}) < 0$ . Since  $\Gamma(0) = 0.25$ , it suffices to show that there exist  $\theta$  satisfying

$$(1-2) \quad \Gamma( X_{\max} ) = 0.08 - 0.4 \theta 0.3^{0.5} < 0,$$

given Assumption 1, 3 and (14). Assumption 1 implies that

$$(1-3) \quad 0.5 \cdot 0.3^{0.5} = 0.274 < \theta.$$

Assumption 3 is satisfied since  $0.5 ( 0.3 )^{-0.5} - 0.3^{0.5} > 0$ .

Denote the L.H.S of (14) by  $\Omega( k )$ :

$$\Omega( k ) = 2.5( 0.25 k^{-0.5} - k^{0.5} + 2 k^{1.5} ).$$

The first-derivative gives

$$\Omega'( k ) = 2.5k^{-1.5} \left( k - \frac{1 + \sqrt{7}}{12} \right) \left( k - \frac{1 - \sqrt{7}}{12} \right).$$

For  $0 \leq k \leq 0.3$ ,

$$0.3 = \arg \min_k \Omega( k ),$$

with  $\Omega( 0.3 ) = 0.5934$ . Hence, (14) is

$$(1-4) \quad 0.5934 > \theta.$$

Therefore,  $0.2/0.3^{0.5} < \theta < 0.5934$  satisfy (1-2) (1-3) and (1-4).

[4] The proof of Proposition 2

Prove this using an example. Suppose that  $\alpha = 0.388$ ,  $a = 0.5$ ,  $\mu = \frac{25}{16}$ ,  $r = 1$ ,  $b = 2$ , with  $c = \frac{1}{4(1 - \alpha)\mu^3}$ . We obtain

$$n(k) = \frac{25}{16} k^{-0.5} (0.5 - k)^2,$$

and

$$m(k) = \theta - 0.5k^{0.5}.$$

By (13), the whole interval satisfies



$$(2-1) \quad 0 \leq k \leq k_{\max} = 0.45.$$

Assume that

$$(2-2) \quad n(0.45) > m(0.45),$$

which leads to  $\theta < 0.34088$ . Since  $n(0) > m(0)$ , it suffices to demonstrate that there exists some  $k$  satisfying  $n(k) < m(k)$ , given Assumption 1, 3, and (14), for  $0 < k < 0.45$ .

Assumption 1 implies that

$$(2-3) \quad 0.5 (0.45)^{0.5} = 0.3354 < \theta.$$

Assumption 3 is satisfied since  $0.5 (0.45)^{-0.5} - 0.45^{0.5} > 0$ .

For  $0 \leq k \leq 0.45$ ,

$$\frac{1 + \sqrt{7}}{12} = \arg \min_k \Omega(k), \text{ with } \Omega\left(\frac{1 + \sqrt{7}}{12}\right) = 0.3489.$$

Hence, (14) is

$$(2-4) \quad 0.3489 > \theta.$$

By (2-3) and (2-4), at least  $0.3355 < \theta < 0.3488$  satisfy the three conditions. Suppose that  $\theta = 0.34$ , then we obtain  $n(0.4) < m(0.4)$ . Q.E.D.

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Figure 1

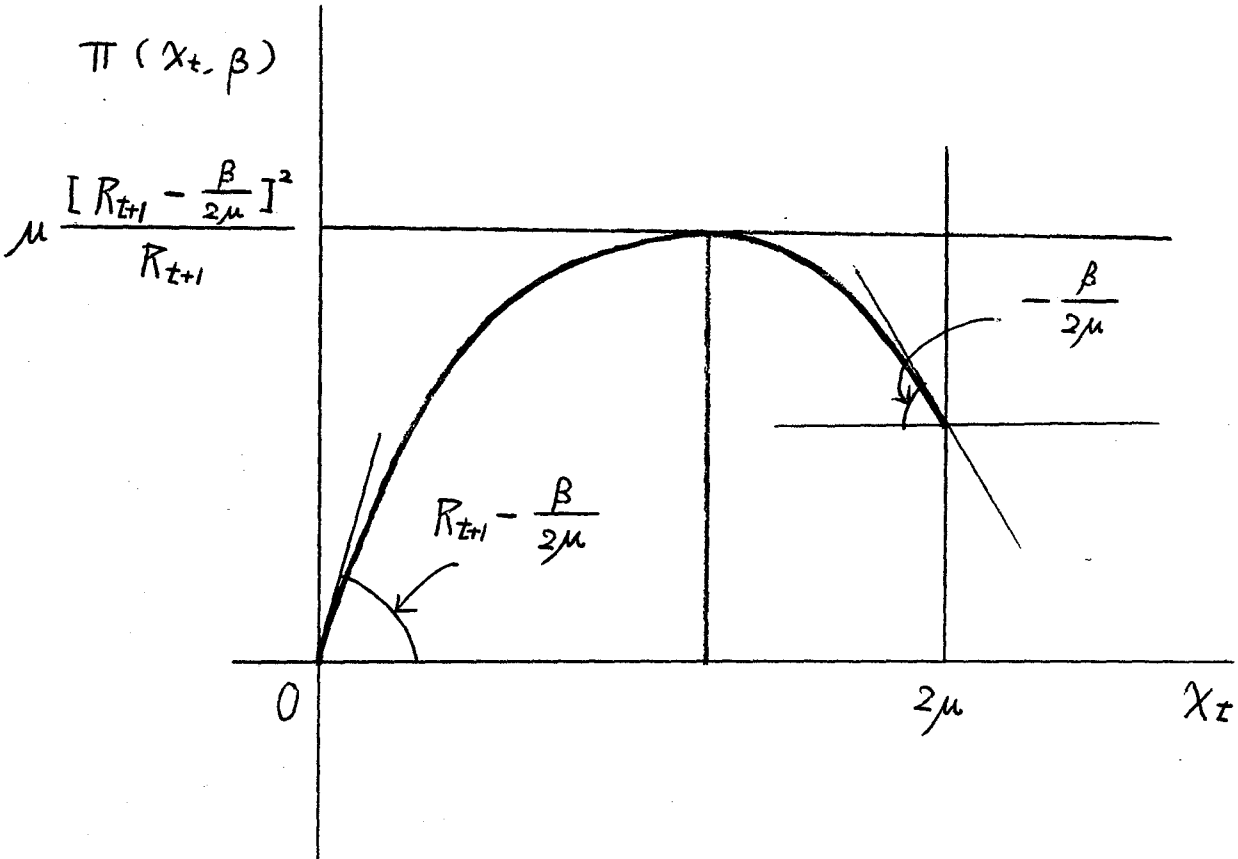


Figure 2

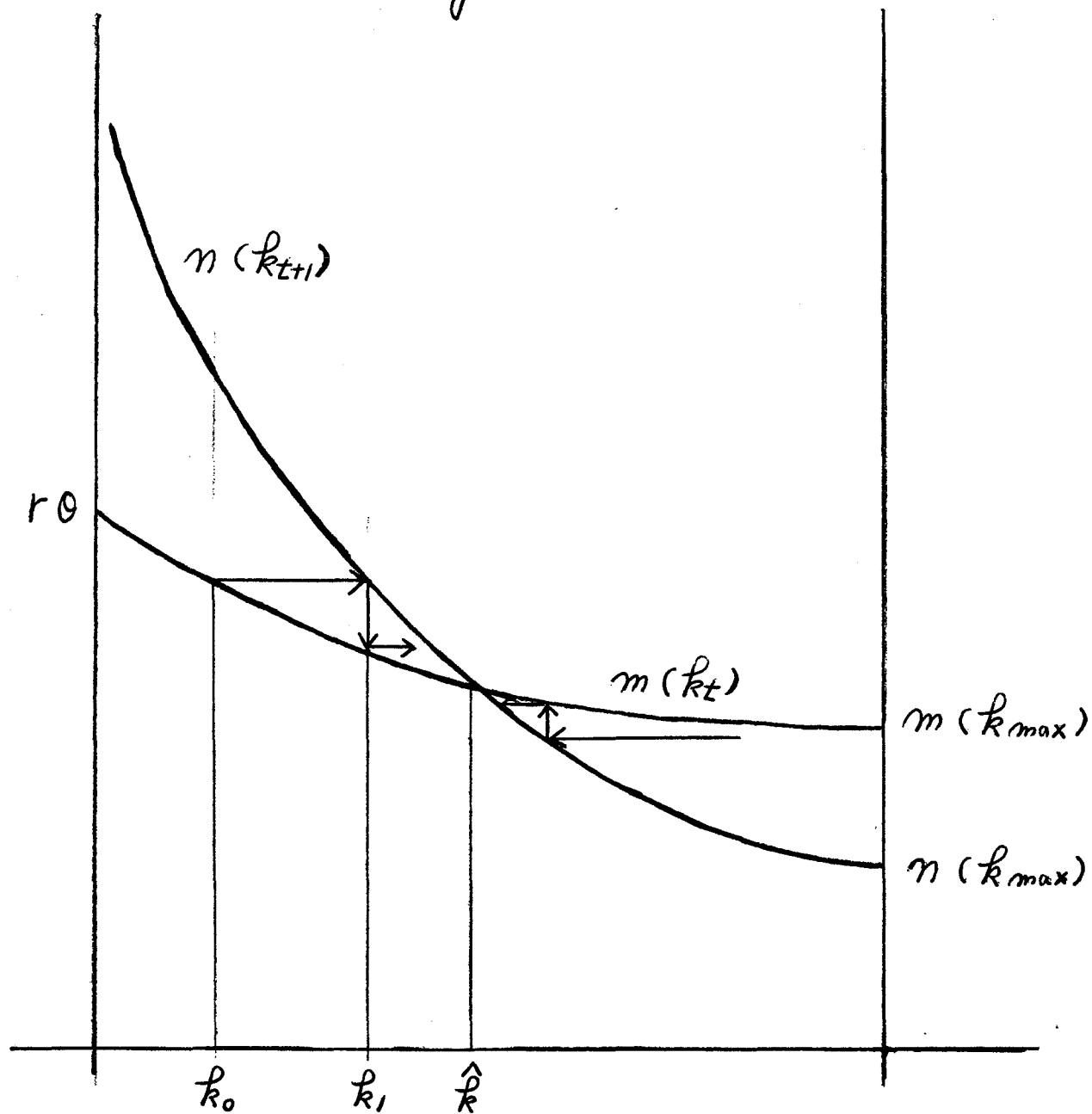


Figure 3

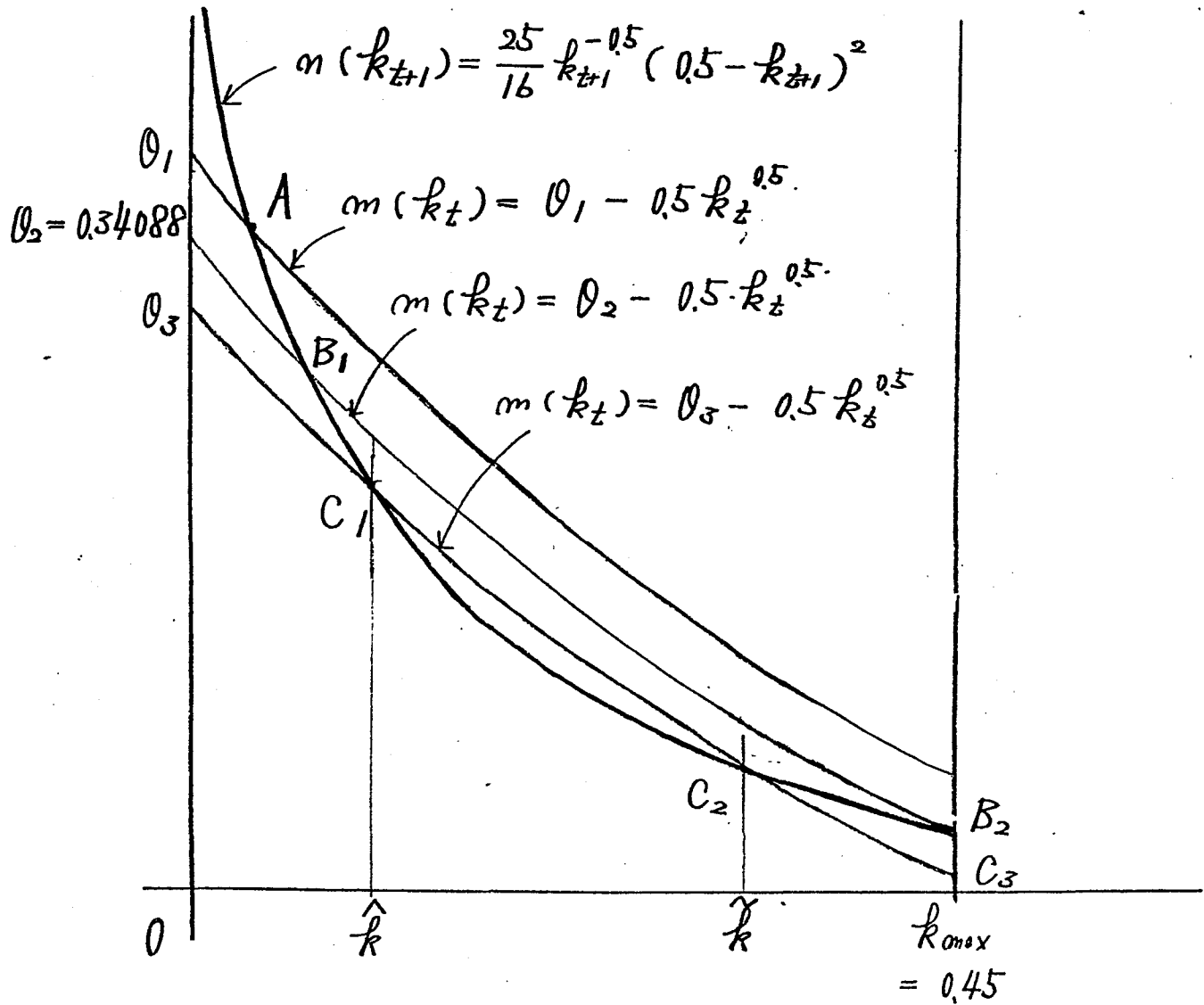


Figure 4

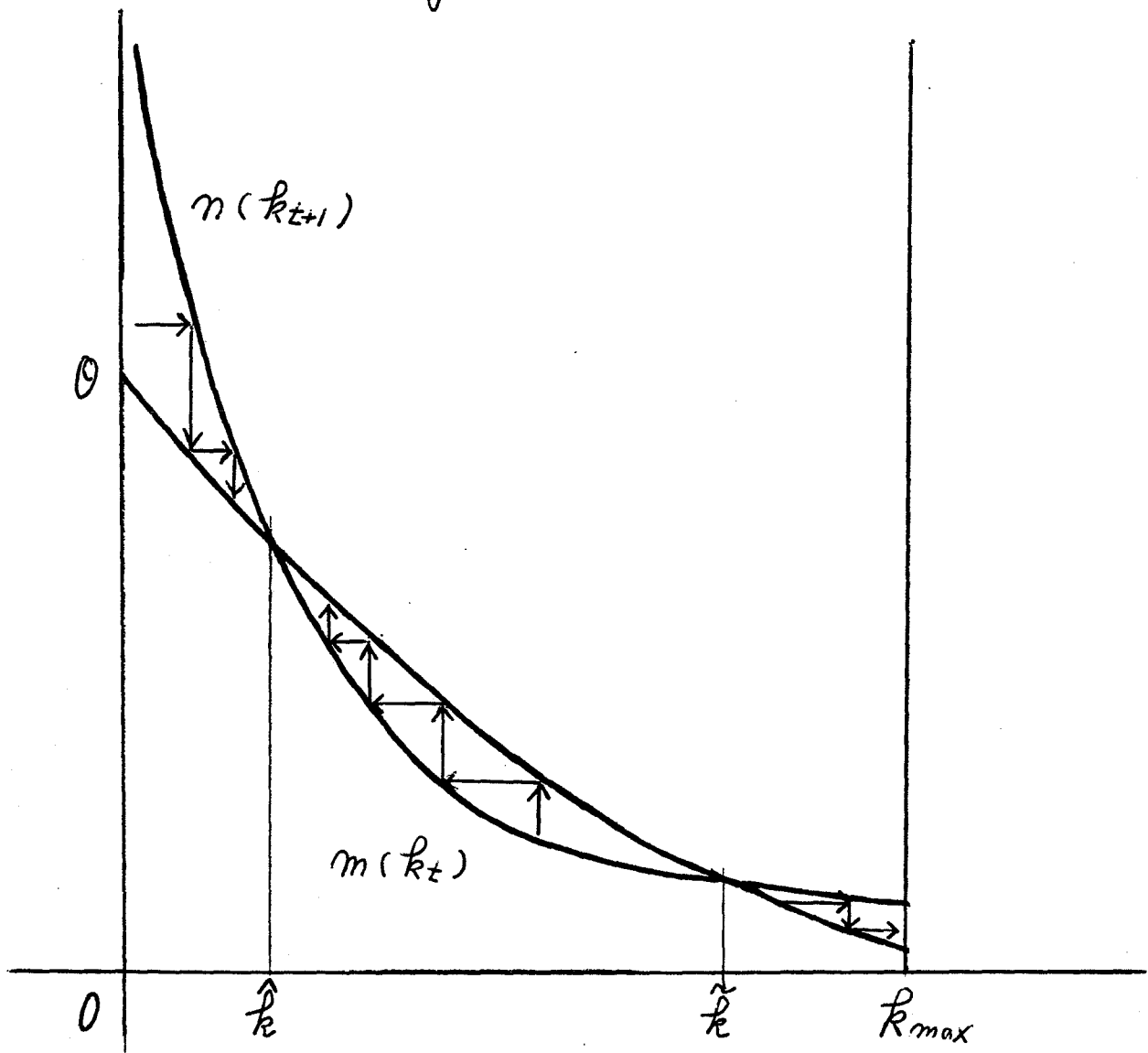


Figure 5A

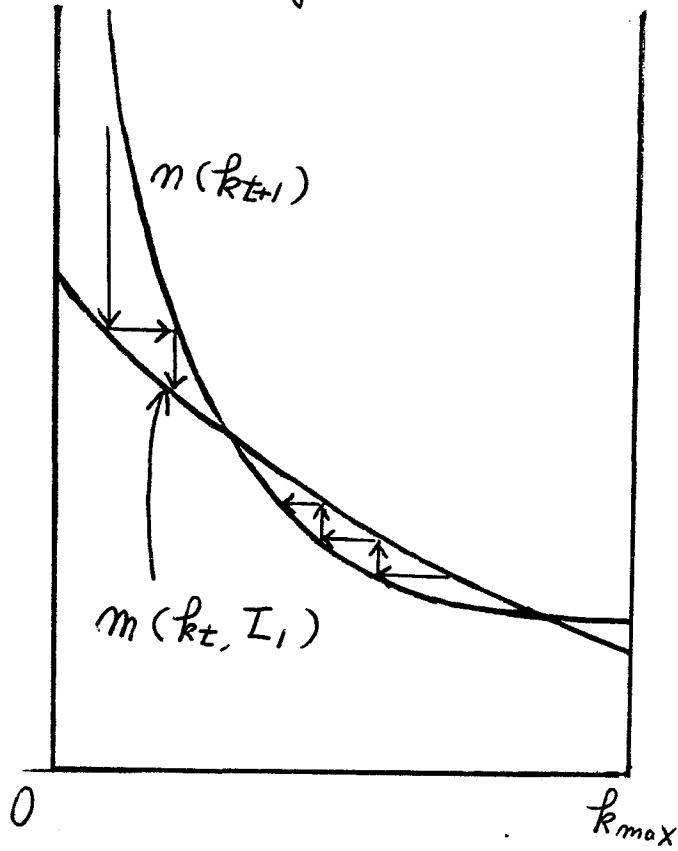


Figure 5B

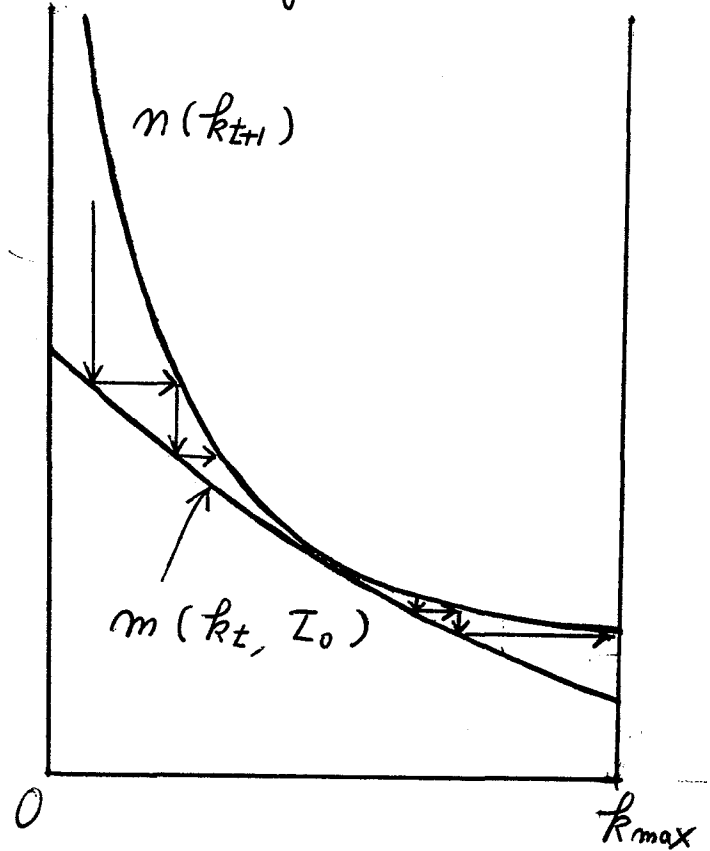


Figure 5C

$$I_1 < I_0 < I_2$$

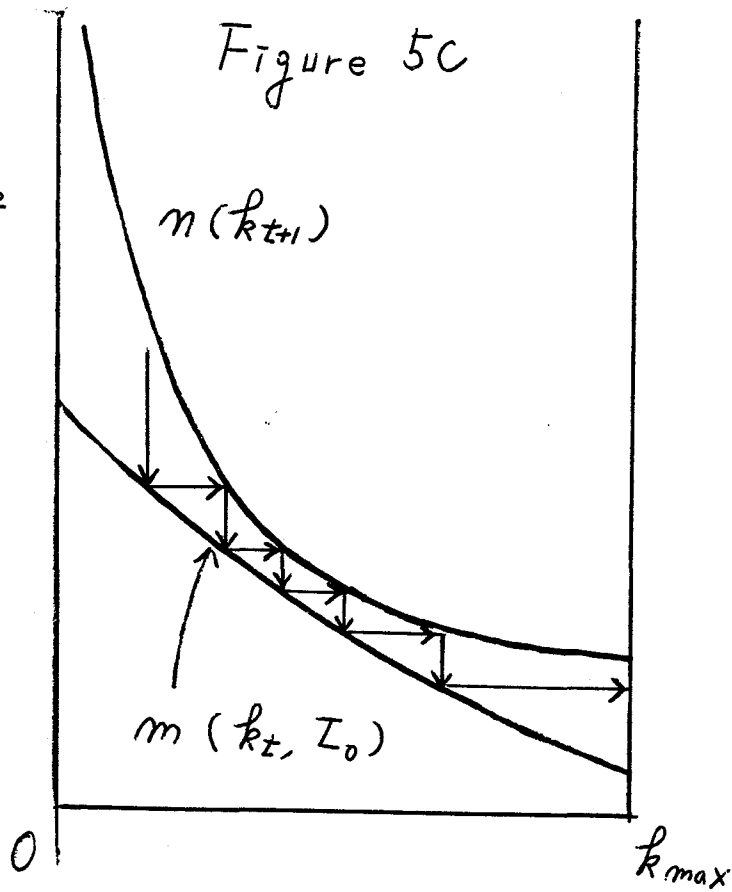




Figure 6A

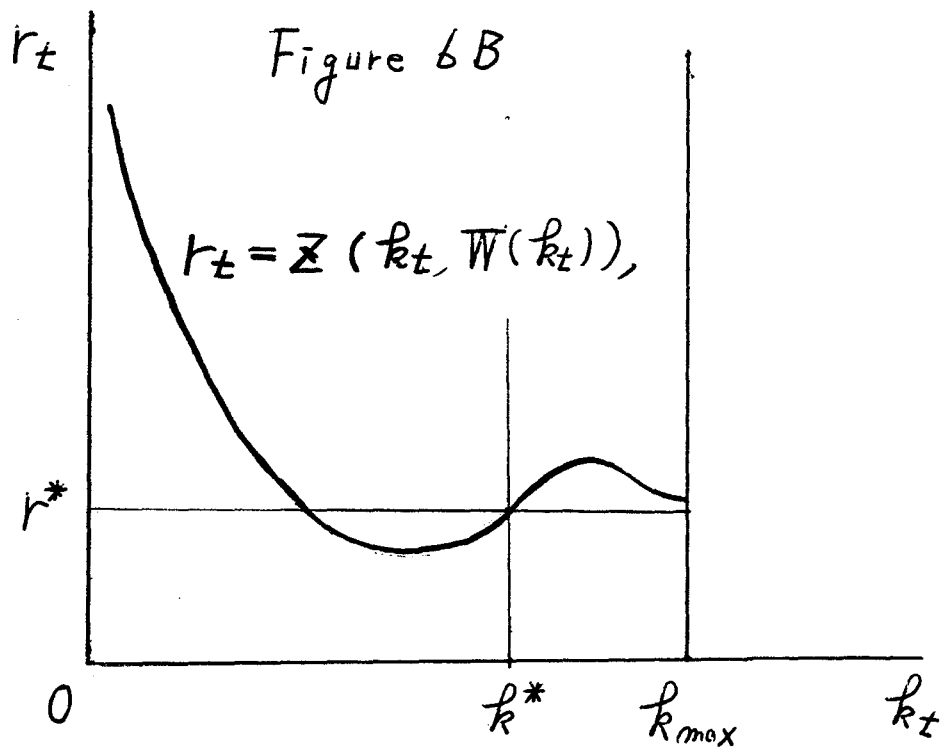
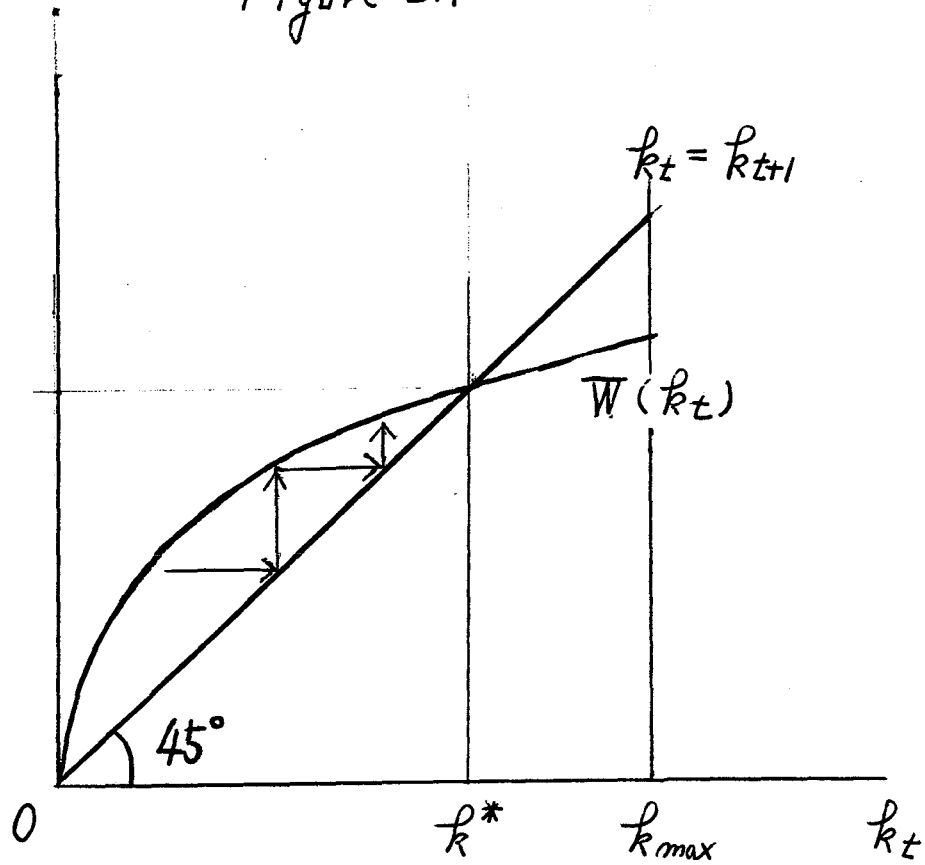


Figure 7

