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Working Paper

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Center Discussion Paper, No. 678

Provided in Cooperation with:

Yale University, Economic Growth Center (EGC)

Suggested Citation: Corsetti, Giancarlo (1992) : A Portfolio Approach to Endogenous Growth: Eaton's Model Revisited, Center Discussion Paper, No. 678, Yale University, Economic Growth Center, New Haven, CT

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<https://hdl.handle.net/10419/160600>

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CENTER DISCUSSION PAPER NO. 678

A PORTFOLIO APPROACH TO ENDOGENOUS GROWTH:
EATON'S MODEL REVISITED

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December 1992

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussions and critical comments. Professor Corsetti worked on this project while at Yale University. This is a revised version of the paper originally written January 1991.

I thank Willem Buiter, Paolo Pesenti and T.N. Srinivasan for their helpful comments and suggestions.

ABSTRACT

A few years before the development of endogenous growth theory, Eaton's article in the *Review of Economic Studies* provided a suitable framework of analysis to study policy-related issues in stochastic endogenous growth model. In this paper, we highlight the analytical core of the model, completing the original discussion and showing directions for possible extensions and generalizations. The main advantage of a portfolio approach to growth and policy analysis consists of pointing out the equilibrium relations between fiscal policy, rates of return and portfolio composition. The financial counterpoint of alternative fiscal policies highlighted by Eaton's model thus makes us aware of additional degrees of freedom in fiscal and financial engineering, which logically complement the traditional analysis of intertemporal optimal taxation.

KEY WORDS: Endogenous Growth, Risk, Asset Pricing

1. Introduction.

More than ten years ago, from the pages of the Review of Economics and Statistics, Jonathan Eaton proposed an interesting way of analyzing long-run effects of fiscal policies in a Merton-type stochastic growth model including both private capital and government bonds (Eaton [1981]). Since then, growth theory has evolved in such a way that an appropriate development of this original contribution may come in handy as a suitable framework to analyze both policy-related and financial issues in the process of capital accumulation. Our generalization of Eaton's model has already proven to be insightful in a number of applications. A series of companion papers explores issues in government spending and growth (Corsetti [1991a]), taxation and risk-taking (Corsetti [1991b]) as well as in inflation and growth (Corsetti and Pesenti [1991c]). Related work by Turnovsky can also be considered part of the same literature (see for example Turnovsky [1990]).

In view of its potential usefulness, it is unfortunate that Eaton's original contribution does not provide an explicit treatment of a number of model-related issues. First, it provides little intuition about the analytical core of the model. Second, it does not carry out welfare analysis, nor does it address the question of determining an optimal policy. Finally, it does not explore some important properties of the class of competitive equilibria under consideration. The goal of this paper is to complete Eaton's discussion of the model, re-casting it from the vantage point of the current debate in growth theory, as well as to provide some suggestions for possible extensions and generalizations.

The most apparent simplifying assumption of Eaton's model was certainly not greatly welcomed at the time of its publication. The production function is assumed to be linear in capital, while capital is the only factor of production. That is, output is produced with a so-called AK technology, where A is a linear coefficient and K is the capital stock outstanding in the economy. The fundamental objection to this kind of model used to be that it does not account for

the distribution of income between capital and labor, as we perceive it empirically. Nonetheless, what has happened in the field of growth theory since the publication of Eaton's article has certainly weakened the above criticism. Beginning with the contributions by Romer [1986] and Lucas [1988], the debate on growth issues has increasingly focused on the role of human capital, sometimes modelled as a close substitute for physical capital. In this sense, the meaning of capital has been progressively extended so as to include embodied and disembodied knowledge as well. Moreover, as is well known, this shift in focus has been historically associated with a strong statement regarding production in the growth process. The research agenda under the name of endogenous growth (EG) theory relies on production functions in which the marginal product of accumulated inputs is sufficiently bounded away from zero, so that the Inada conditions do not hold (Kurz [1968], Jones and Manuelli [1990] and, for the case of two capital goods, see Mulligan and Sala-i-Martin [1991]). Since a linear technology is the simplest specification with this characteristic, AK models have become increasingly popular in the endogenous growth literature. They permit the construction of clear-cut analytical schemes especially suited to study long-run effects of fiscal and financial policies, which was exactly the main goal pursued by Eaton in his 1981 paper (see Rebelo [1991]). Having come full circle, we believe that it is worthwhile to revisit Eaton's construction and try to understand the scope of its applicability to relevant policy issues in the current debate.

In Eaton's model the linear coefficient in the production function is not constant, but is distributed as a brownian motion with drift. The analysis thus belongs to a class of rational expectation, stochastic models that, together with Eaton [1981], have also been developed by Gertler and Grinols [1982], Cox, Ingersoll and Ross [1985], and Stulz [1986]. While the logical link of all these models with the EG literature is in the form of the production function, modelling shocks in the form of brownian motion ideally relates them to Merton's stochastic growth analysis. However, there is an important difference. In the early literature on stochastic

growth, the central focus was the existence and uniqueness of a steady state distribution for capital and consumption per worker. In the EG models, the steady state is characterized in terms of stationary growth rates, rather than stationary levels of these variables.

Eaton analyzes the consequences of alternative ways of financing government spending when this is some *given* linear stochastic function of the capital stock. In the absence of lump-sum taxes, tax policies directly affect the expected yield and riskiness of private capital relative to government debt, changing incentives to invest and the rate of capital accumulation. As the model assumes that government spending is some *given* linear function of capital, such a specification could model either a policy rule or a Leontief technology in capital and public goods: there will be some minimum expenditure on public goods per unit of capital which is needed to run the production process. Under both interpretations, it is apparent that the design of an optimal policy will require the availability of non-lump-sum taxes, to complement the spending-induced distortions in the economy. When there is a fixed required ratio of public spending to capital, the decision to invest one additional unit of output by the private agents leads to higher government spending. If private agents take public spending as independent of their own decisions, a tax on capital income can make them assess correctly the implications of their investment behavior. So, an important (but originally unstated) conclusion of Eaton's analysis is the optimality of non-lump sum taxes¹. This, as well as many other results in our paper, refers to the logical core of the literature on optimal fiscal and financial policy in growth models (see for example Buiter [1990,1991], Chamley [1986], Chari, Christiano and Kehoe [1991], Jones, Manuelli and Rossi [1991] King and Rebelo [1990], Lucas [1990] and Zhu [1990]). However, it should be stressed that the major point of interest consists in the framework of analysis. In the tradition of Merton's

¹ The case of a Leontief technology is discussed extensively in Jones, Manuelli and Rossi [1991]. Eaton's original formulation, which we follow, better describes the case of a policy rule.

[1969,1975] seminal contributions, modeling productivity shocks as brownian motions allows us to exploit the theoretical body of modern financial theory to address macroeconomic issues in general equilibrium. In particular, many endogenous growth models discuss specifications including public goods as well as external effects in production. The last section of this paper will provide an example of how Eaton's construction can be applied to these models.

This paper is organized as follows. Section 2 and 3 will present the structure of the economy and characterize an equilibrium allocation as in Eaton [1981]. The following two sections are devoted to the comparative statics of tax reforms and to the determination of an optimal policy. These sections will highlight some properties of the model which are not discussed in Eaton's paper, such as the existence of an equivalence class of policies which support the same real allocation, the existence of an optimal policy based on non-lump-sum taxation which supports the first-best allocation, and the optimality of a zero debt to capital ratio. Section 7 will present a graphical interpretation of the main results of the analysis. Finally, the flexibility of the analytical framework will be illustrated by introducing an external effect of capital on labor productivity. Some concluding remarks will close the paper.

2. Eaton's Model of Stochastic Endogenous Growth.

2.1 Preferences and Technology.

The economy is populated with many identical, infinitely-lived households, characterized by a time-separable, power utility function in consumption only

$$(2.1.1) \quad E_0 \int_0^{\infty} \frac{C(t)^{1-R}-1}{1-R} \exp(-\delta t) dt \quad \delta > 0, R \in (0, \infty)$$

where E_0 is the expectation operator conditional on information at time $t=0$, $C(t)$ is the instantaneous consumption rate at time t , R is the elasticity of marginal utility and δ the rate of time preference, positive by assumption.

The production function is assumed to be linear in one reproducible factor, hereafter $K(t)$. This is some broad measure of capital which includes disembodied knowledge, non-human and human capital, the latter being the result of past investment (education and training, for example), to the same extent that physical capital is.

The linear coefficient of $K(t)$ in the production function is not constant, but follows a Brownian motion with drift: instantaneous output is subject to an economy-wide productivity shock entering the production function in a multiplicative way. For the sake of notational consistency, denote instantaneous stochastic flows with $dZ(t)$, where $Z(t)$ will indicate the cumulative value of the corresponding variable. Thus, denoting with $Y(t)$ cumulative net output, the instantaneous output flow net of depreciation can be expressed as:

$$(2.1.2) \quad dY(t) = [\eta dt + \sigma d\omega(t)] K(t)$$

where $d\omega(t)$ is a standard Wiener process with zero mean and unit variance, η and σ are positive constants which denote the instantaneous drift and the instantaneous standard deviation of productivity shocks, respectively. As in Cox, Ingersoll and Ross [1985], depreciation is stochastic and we cannot rule out the possibility of observing *negative* and unbounded instantaneous output flows². In each instant, the mean and variance of the conditional distribution of dY depends on the existing capital stock. Therefore, current shocks will have long lasting effects on the output process to the extent that they affect capital accumulation over time. However, the linearity of the production function implies that the rate of return on capital will be i.i.d. with mean η and variance σ^2 . Finally, investment is assumed to entail no adjustment costs, and the capital good can be consumed.

² However, a relatively large ratio between the drift and the standard deviation of the process makes the probability of observing negative realization of the output flows very small.

2.2 The Government.

The government is assumed to consume a time-invariant, deterministic fraction \tilde{g} of the outstanding capital³:

$$(2.2.1) \quad G(t)dt = \tilde{g}K(t)dt \equiv g\eta K(t)dt$$

where, for convenience, we define $\tilde{g} \equiv g\eta$ ⁴. Government expenditure is financed either by tax revenues or by issuing government bonds. Assume an instantaneous *net* tax function of the form

$$(2.2.2) \quad dT(t) = \tau dY(t) + \alpha \sigma K(t) d\omega(t) = [\tau \eta dt + (\tau + \alpha) \sigma d\omega(t)] K(t).$$

$T(t)$ denotes cumulative net taxes, τ is a time-invariant tax rate and α is a time-invariant rate at which the government subsidizes (for $\alpha < 0$) or taxes (for $\alpha > 0$) capital owners contingent on the realization of the output process⁵. Notice that, when $\tau + \alpha = 0$, net tax revenues will be totally insulated from productivity shocks, while capital income will be taxed at a flat rate when $\alpha = 0$.

³ In this paper, government expenditure has been linked to the existing capital stock in the economy. Alternatively, it could have been specified as a function of output (in the absence of uncertainty, the two specifications would be equivalent). For example, Eaton [1981] poses

$$dG(t) = g\eta K(t)dt + g'\sigma K(t)d\omega(t).$$

In this case, however, the presence of a contingent component of government spending (at the rate g') raises the possibility of a negative $dG(t)$ to complement the negative $dY(t)$.

⁴ In Eaton [1981], the government consumption process is given an institutional interpretation. However, in the spirit of the renewed interest in the literature about the effects of government spending on production, we could have similarly modeled a Leontief technology in capital and public goods, where $G(t)dt$ is the minimum provision of public goods needed to run a given amount of capital in the economy. As an example of such public good, which by assumption is produced with the same technology as the private good, we could consider government activities to enforce contracts and property rights.

⁵ "Such a differential tax rate will arise if, for instance, the government explicitly coinsures risky undertakings in a degree beyond that implied by the average tax rate" Eaton [1981]: p.437.

When a given policy involves budget deficits, the government finances them by issuing consols paying an instantaneous coupon rate of u units of output. Denote with $B(t)$ and $q_B(t)$ the number of consols in the economy and their price in terms of consumption goods. The government budget identity⁶ can then be written as

$$(2.2.3) \quad d[q_B(t)B(t)] = G(t)dt - dT(t) + q_B(t)B(t) \left[\frac{u dt}{q_B(t)} + \frac{dq_B(t)}{q_B(t)} \right]$$

with $\frac{dq_B(t)}{q_B(t)}$ denoting capital gains on bond holding. By the solvency constraint of the public sector, the value of $q_B(t)B(t)$ will equal the present discounted value of future primary surpluses (i.e. spending *minus* tax revenue). Differing from Eaton [1981], in our analysis we do not restrict $B(t)$ to being exclusively positive, i.e. the government can be a net creditor *vis-a-vis* private agents. In this case, public expenditure will be partly financed through income from government financial assets, instead of using revenue from taxation.

Throughout this paper, we also assume that the government is able to precommit itself to some given policy, announced and immediately effective at $t=0$, so that we will not address policy-related time-consistency issues.

3. The Competitive Equilibrium.

In this section, we will characterize the competitive equilibrium allocation conditional on *given* policy parameters. The decentralized economy is characterized by two perfectly competitive financial markets: the capital market and the government bond market. With the consumption good being the *numeraire*, Eaton writes private financial wealth in real terms⁷ as

⁶ The government budget identity can be derived from the discrete case as shown in Merton [1969] (in Merton [1990]:124-126). See also Ingersoll [1987], p.272.

⁷ It is important to keep in mind that in Eaton's model taxes are non lump-sum.

$$(3.1) \quad W(t) \equiv K(t) + q_B(t)B(t)$$

and characterize its evolution over time as follows. Each rate of return, including both income and capital gains, can be broken down into an anticipated and an unanticipated component. Since by assumption technological shocks are the only source of uncertainty in the economy, the unanticipated component of returns will be function of these shocks. In the case of government consols, we can conjecture that in equilibrium they will be characterized by the following linear additive form

$$(3.2) \quad \frac{z dt}{q_B(t)} + \frac{dq_B(t)}{q_B(t)} \equiv r_B(t)dt + \sigma_B(t)d\omega(t)$$

with $r_B(t)$ and $\sigma_B(t)$ ⁸ being endogenously determined; by the same token, we define the after-tax return on equity shares dy_s ⁹ as

$$(3.3) \quad dy_s \equiv r_s(t)dt + \sigma_s(t)d\omega(t)$$

Denoting with $n(t)$ the share of capital in wealth and assuming that agents consume at the non-stochastic rate $C(t)$ (Merton [1969]), the process of wealth accumulation can then be described as

Consistently, relative to the value of bonds, $K(t)$ is the present discounted value of the current and future flow of *after-tax* income accruing from this asset.

⁸ Throughout the paper, we will denote by σ_i the covariance between the variable i and the Wiener process $d\omega$.

⁹ In the case of capital shares, the underlying assumptions of our model imply that dividends and capital gains are taxed at the same rate τ while unanticipated output is taxed or subsidized at the rate α . Under these conditions r_s and σ_s are independent of the dividend policy adopted by the representative competitive firm.

$$(3.4) \quad dW(t) = n(t)W(t) [r_s(t)dt + \sigma_s(t)d\omega(t)] + \\ + [1-n(t)]W(t) [r_B(t)dt + \sigma_B(t)d\omega(t)] - C(t)dt.$$

Therefore, the *consumer* problem is to choose both an optimal consumption plan and an optimal portfolio plan, solving

$$(3.5) \quad \text{Max}_{\{C, n\}} E_0 \int_0^{\infty} \frac{C(t)^{1-R}}{1-R} \exp(-\delta t) dt$$

subject to (3.4), to the initial conditions $K(0) = \bar{K} > 0$ and $B(0) = \bar{B}^{10}$, as well as to the non-negativity constraint $C(t) \geq 0$. In addition, private sector solvency will require $W(t) \geq 0$, while feasibility implies $K(t) \geq 0$. Assuming that (3.4) is bounded and that an interior solution exists, this well known problem¹¹ yields the following first order conditions

$$(3.6) \quad C(t) = \chi(t) W(t)$$

$$(3.7) \quad (r_s(t) - r_B(t)) - R(\sigma_s(t) - \sigma_B(t)) [n(t)\sigma_s(t) + (1-n(t))\sigma_B(t)] = 0$$

According to the first condition, at any point in time agents consume at the non stochastic rate $\chi(t)dt$ out of wealth. The second expression is the first-order condition for the optimal portfolio share in a standard intertemporal asset pricing model, where all returns are perfectly correlated.

In a perfectly competitive market, *firms* will equate the rental price of capital

¹⁰ Since we allow for both positive and negative B's, we implicitly assume that the sign of $B(0)$ is consistent with the solvency constraint, so that the price of consols is non negative.

¹¹ See Merton (1969) or a textbook in Financial Economics, such as Ingersoll (1987), for the derivation of the solutions in the text. Malliaris and Brock (1982) is also a standard reference.

to its after-tax marginal product. Therefore, by (2.2.2) and (2.3.2), we will have

$$(3.8) \quad r_s(t) = r_s = (1-\tau)\eta$$

$$(3.9) \quad \sigma_s(t) = \sigma_s = (1-\tau-\alpha)\sigma$$

We have now all the elements we need in order to characterize the macro-economic equilibrium, except one: the rates of return on government bonds expressed as a function of other variables and/or parameters in the model. Notice that, as consumers can be described by a constant relative risk aversion utility function *and* all the parameters of the model are time-invariant, we can conjecture that there exists a steady state equilibrium where portfolio shares are independent of wealth and therefore constant. With time-invariant portfolio shares, each component of financial wealth will grow at the same stochastic rate, equal to the rate of capital accumulation, as follows from differentiating the definition of both capital share and government debt in wealth *and* dividing through by wealth

$$(3.10) \quad \frac{dW(t)}{W(t)} = \frac{d[q_B(t)B(t)]}{q_B(t)B(t)} = \frac{dK(t)}{K(t)}$$

In our (conjectured) rational expectation equilibrium with constant tax and spending rates, the market value of government debt will be instantaneously scaled up to the size of the economic activity. This particular point is best illustrated by replacing the government intertemporal budget identity (2.2.3) in (3.10), and rearranging the expression so as to obtain

$$(3.11) \quad r_B(t) = E_t \frac{dK(t)}{K(t)dt} + \eta(g-\tau) \frac{n(t)}{1-n(t)}$$

$$(3.12) \quad \sigma_B(t) = \left[\text{Var} \left[\frac{dK(t)}{K(t)} \right] \right]^{1/2} + \sigma(\tau+a) \frac{n(t)}{1-n(t)}$$

The equilibrium rate of return on bonds can be written as the sum of the rate of growth of the economy and the income accruing from tax revenue in excess of spending, in terms of capital goods. The first component is demand-driven. It accounts for capital gains and losses accruing to investors when they try to adjust their portfolios *vis-a-vis* output shocks as they are reflected in the rate of capital accumulation. The second component is simply the income flow from the security. Note also that, for a given equilibrium level of $C(t)/K(t)$, the rate of return on public debt is independent of the coupon rate w ; that is, for given tax and spending rates, an increase in w leads to higher rates of debt issue, such that capital losses on consols will exactly offset the change in the coupon. Moreover, by comparing σ_s in (3.6) with σ_B in (3.9), it is apparent that the rate of return on capital can be either more or less variable than that on bonds, depending on the sign of $\tau+a$. When $\tau+a=0$, both rates are equally variable and the supply of bonds is deterministic. The government will run a budget surplus (deficit) to the extent that tax revenue, $\tau\eta Kdt$, exceeds (falls short of) spending, $g\eta Kdt$. Nonetheless, as argued above, randomness in the rate of return on consols is induced by private agents' demand in the financial market (see Eaton [1981], p. 437-438)¹².

As a final step towards the analytical characterization of the equilibrium, we can substitute the mean and variance of each rate of return (r_B , r_s , σ_B and σ_s) in the first-order condition for the optimal consumption policy and optimal portfolio plan, solving for n and χ . For reasons which will become clear later, define γ as a particular combination of fiscal and technological parameters of the form

$$(3.13) \quad \gamma \equiv \eta\tau - R\sigma^2(\tau+a).$$

¹² Adding taxes on income from bonds does not modify the analysis. It may help to consider the rate of return on bonds as after-tax.

The optimal consumption and portfolio plans can then be expressed as follows

$$(3.14) \quad \frac{C(t)}{W(t)} = R^{-1} \{ (R-1)[\eta - \frac{1}{2}R\sigma^2 - \gamma] + \delta \} \equiv \chi$$

$$(3.15) \quad n(t) = n = \frac{\chi}{\chi + \gamma - \eta g}$$

Indeed, for constant policy parameters, the optimal portfolio shares will not change with the level of wealth: in equilibrium $n(t)=n$ is time-invariant and the consumption rate is a constant fraction of the existing capital stock. By the same token, with constant portfolio shares, both anticipated and unanticipated components of returns on assets are independent of calendar time.

While the amount of risk for each asset depends crucially on fiscal parameters, the economy as a whole face an exogenously-given, technology-related social risk which cannot be diversified. Thus, when considering the market portfolio held by the representative individual (combining bonds and capital shares with weight $1-n$ and n , respectively), its rate of return

$$(3.16) \quad r_{\epsilon} dt + \sigma_{\epsilon} d\omega(t) = [\eta - \gamma] dt + \sigma d\omega(t)$$

will have a constant variance, independent of policy variables. Its expected return, however, will be inversely related to γ ¹³. These features also characterize the growth rate for the economy. Since portfolio shares are constant, the stochastic growth rate for the economy is time-invariant

$$(3.17) \quad \frac{dK(t)}{K(t)} = R^{-1} [\eta - \delta - \frac{1}{2} R(1-R)\sigma^2 - \gamma] dt + \sigma d\omega(t)$$

¹³ Note also that all rates of return in the economy (bonds, equities and market) coincide when $\tau + \alpha = 0$.

and distributed as a brownian motion with drift. As for the return on the market portfolio, the variance of this process reflects the social risk in the economy, while the deterministic component is inversely related to γ . Finally, the characterization of the competitive equilibrium is completed with the determination of shadow riskless rate associated with the equilibrium allocation. Following the argument in Eaton [1981], it can be shown that this rate is equal to

$$(3.18) \quad r = [\eta - R\sigma^2 - \gamma].$$

which is clearly a function of both technological and policy-related parameters.

4. Comparative Statics: Crowding Out, Crowding In and An Equivalence Class of Policies.

The previous section has characterized the equilibrium allocation conditional on given policy parameters closely following Eaton's original scheme. In this and the next section, devoted to the comparative statics of policy reform and to the definition of an optimal policy, we take a major departure from his analysis. A first important result, which is not stated in Eaton's original contribution, is the existence of an equivalence class of policies which support the same real allocation. By (3.14) and (3.15), it is apparent that changing policy parameters without affecting γ will not alter either the portfolio shares or the consumption rate. As an index of government fiscal policy, γ completely captures the effects of alternative policies on consumption, capital accumulation, real holding of debt as well as social welfare as measured by the representative agent's indirect utility of wealth. By the definition of γ in (3.13), trading-off anticipated and contingent tax rates according to

$$(4.1) \quad da = \frac{\eta - R\sigma^2}{R\sigma^2} d\tau$$

will determine a continuum of policies supporting the same equilibrium. Notice that the rate of return on the market portfolio, the shadow riskless rate and the debt to capital ratio also depend on tax parameters only through γ .

Nonetheless, whether or not γ varies, *any* change in the tax rates will be reflected in both the budget process and the return on financial assets. Consider some equilibrium-preserving, *iso-growth* and *iso-welfare* tax reform conditional on a given γ . In our economy, the change in tax rates will affect the return on equities. For the optimal portfolio not to change, the rate of return on government security must vary too, appropriately offsetting the rise (fall) in the demand for the alternative asset. Given the coupon rate u , this will be reflected in the equilibrium path for the price of consols $\{q_B(t)\}_{t=0}^{\infty}$. Since for a fixed γ the net financial position of the government over time $\{q_B(t)B(t)\}_{t=0}^{\infty}$ does not respond to policy reforms, the rate of bond issue must adjust accordingly.

As both the rate of growth (3.17) and the return on the market portfolio (3.16) are monotonically decreasing in γ , fiscal reforms can have crowding out as well as crowding in effects. Policies increasing γ will lower the holding of portfolio share in capital n , raise consumption for a given level of capital and reduce accumulation. Policies decreasing γ will make the economy grow faster. Each value of γ will also correspond to a particular debt to capital ratio $H[\gamma]$, where $H \equiv (1-n)/n$. It is interesting to note that an increase in the public debt to capital ratio is not necessarily associated with lower growth. The derivative of H with respect to γ

$$(4.2) \quad \frac{\partial H}{\partial \gamma} = \frac{1}{\chi} [R(1+H)-H].$$

will be positive if parameter values are such that $R \geq \frac{H[R]}{1+H[R]} = 1-n[R]$, negative if otherwise. In some limiting cases¹⁴, it is possible to obtain an *inverse* crowding out

¹⁴ It is apparent that, since a positive private sector wealth requires $H \geq -1$, the condition

effect: for lower values of γ , capital accumulation could be increasing *vis-a-vis* a rising debt to capital ratio¹⁵.

5. Optimal policy.

Consider now a benevolent government whose objective is to maximize private agents' expected utility. In terms of the well-known Ramsey problem, an optimal policy consists in choosing tax rates which achieve this goal consistent with government consumption and with market determination of prices and quantities. One way to characterize a solution is to find a feasible policy which maximizes the representative agent's indirect utility of wealth. Considering the equilibrium closed-form expression for this function, we can write

$$(5.1) \quad \text{Max}_{\gamma} V[W(0),0] = \text{Max}_{\gamma} \frac{(\chi[\gamma] + \gamma g \eta)^{1-R}}{\chi[\gamma]} \frac{K(0)}{1-R}$$

By the first-order condition for this problem, the welfare maximizing policy γ_0 will be

$$(5.2) \quad \gamma_0 = \eta g.$$

Notice that the equilibrium demand for government securities can be written as

$$(5.3) \quad 1-n = \frac{\gamma - \eta g}{\chi - \gamma - \eta g}.$$

$R \geq 1-n$ is always satisfied for $R \geq 1$. However, for values of the coefficient of relative risk aversion lower than unity, it is possible that parameter values are such that $(1-n)$ is larger than R . Notice that in this case the coefficient of relative risk aversion is a lower boundary for the share of wealth in public debt. The demand for government bonds cannot go below this boundary without inducing a violation of the transversality condition for the consumer problem.

¹⁵ Also, the effect of an increase in the level of technological uncertainty on capital accumulation will depend on both the degree of risk aversion and the contingent tax and subsidies scheme by the government. See Eaton [1981] for a discussion.

It is evident that γ_0 will make the private demand for government bonds equal to zero. No government deficit or surplus will then be feasible at an optimum, so that there will be no freedom to choose among tax parameters consistent with γ_0 but implying a non zero $(\tau+a)$. For, while a non zero $(\tau+a)$ would make it impossible to balance the budget instant by instant, the market will not allow deficit financing. Thus, we can write the optimal policy as $(g, \tau=g, a=-g, u)$, where the rate of bond issue will be residually determined¹⁶. This policy consists in taxing only the *expected* capital income at a positive rate, insulating government revenues from instantaneous productivity shocks.

An alternative and informative way to characterize an optimal policy consists of comparing the allocation in a command economy (*given* the spending rule (2.2.1)) with the allocation in the market economy. It can be shown that a social planner trying to maximize consumer's utility subject to the resource constraint would prescribe agents to consume at a deterministic fraction out of capital equal to

$$(5.4) \quad \frac{C(t)}{K(t)} = R^{-1} \{ (R-1)[\eta(1-g) - .5R\sigma^2] + \delta \}$$

¹⁶ In the text, we have proceeded by assuming that the fiscal parameters τ and α are predetermined so that the process governing the issue of government bonds is endogenously determined as a function of parameters and technological shocks. Define the rate of bond issue as

$$\frac{dB(t)}{B(t)} \equiv \mu_B dt + \Sigma_B d\omega(t)$$

By differentiating the definition of a portfolio share n using Ito's lemma, we can write the parameters for this process as

$$\begin{aligned} \mu_B &= \Gamma_E - r_B - \text{cov} \left[\frac{dB(t)}{B(t)}, \frac{dq_B(t)}{q_B(t)} \right] / dt + \frac{u}{q(t)} = \\ &= \frac{n}{1-n} (\eta g - \gamma) + \left[\frac{n}{1-n} \right]^2 (\tau + \alpha)^2 \sigma^2 + \frac{u}{q(t)} \\ \Sigma_B &= \sigma - \sigma_B = \frac{-n}{1-n} \sigma (\tau + \alpha) \end{aligned}$$

Given the coupon rate on consols and the rate of government consumption, it can be shown that there exists a one-to-one relationship between any fiscal policy (τ, α) and the rate of debt issue, as defined by (μ_B, Σ_B) . Therefore, two out of the four policy variables $(\tau, \alpha, \mu_B, \Sigma_B)$ are residually determined.

If some policy supporting the command optimum exists, it must be such that the consumption rate out of capital $\frac{C(t)}{W(t)n}$ in the market economy coincides with (5.4). By evaluating (3.14) and (3.15) at $\gamma=\gamma_0$, it is easy to verify that $(g,\tau=g,a=-g,u)$ is indeed the policy we are looking for.

We then conclude that there exists a policy which can sustain the command optimum, and that this policy is associated with a zero debt to capital ratio, so that any tax parameters implying either a positive or a negative net financial position for the government will produce equilibria which are sub-optimal^{17 18}.

7. A Graphical Analysis

The results developed so far can be given an intuitive graphical representation in the mean-variance space of asset returns. Consider first a command economy, corresponding to Figure 1. If we project the rate of return on capital net of capital spending in the mean-variance or, more precisely, in the (E,σ) space, this rate will correspond to a point K^G with coordinates $(\eta(1-g),\sigma)$. Since the rate of return on capital follows a brownian motion, there is no loss of generality in assuming that preferences are locally quadratic. In the (E,σ) space, non-satiated risk-averse consumers will be characterized by a map of upward sloping indifference curves. Now, because in this economy the optimal portfolio obviously consists of capital only, the equilibrium will correspond to the representative agent indifference curve passing through K^G . The equilibrium *riskless rate* will correspond to the point at which

¹⁷ The specification of the spending function adopted by Eaton [1981]

$$dG(t) = [g\eta dt + g'\sigma d\omega(t)] K(t)$$

differs from our because of a linear contingent component. In this case it is easy to show that the optimal policy is $(g,g',\tau=g,a=g'-g)$, still involving a balanced budget rule.

¹⁸ As a final remark, cases could be take into account where the the choice of tax rates may not be free of constraints. For instance, suppose that for some reason the only feasible policy is a flat-rate tax on income with $\alpha=0$. In this case, there is no constant tax rate which permits the economy to attain a command optimum. With $\alpha=0$, it is impossible to balance the budget instant by instant, so that the optimal value of γ , which makes the private demand for government securities zero, does not correspond to a feasible policy.

the tangent to the indifference curve at K^g crosses the axis of the expected returns. This will determine the only rate such that, if a riskless asset existed, it would be neither demanded nor supplied by the representative investor. In Figure 1, the riskless rate is denoted by ρ ¹⁹.

Figure 2 and 3 refer to the competitive economy of Section 3. Consider the first figure. With only one source of uncertainty in the economy, all rates of return are perfectly correlated. In the space of mean/standard deviation of returns we can therefore represent portfolios combining different assets with straight segments. Define the line connecting any pair of rates of return a frontier $F(\cdot)$. It can be easily verified that a frontier $F(\cdot)$ has the following properties: in equilibrium, all assets will lie on it; the position of the frontier in the space of Figure 3 is uniquely determined by a particular macroeconomic equilibrium; the slope of the frontier is $(R\sigma)^{-1}$ and does not change across different equilibria. For future reference, focus on the debt to capital ratio H and denote a frontier by $F(H)$.

The position of the rate of return on the market portfolio on $F(H)$ is easy to identify: since the social risk is exogenous in our economy, its variance will be identically equal to σ . By construction, the market rate will lie on the portfolio line connecting the rates of return on bonds and equities. However, it is worth noticing that these rates will not necessarily lie on opposite sides with respect to the market, as we allow for a negative government debt (i.e. private agents can go short in government bonds).

Figure 3 refers to the crowding-out and crowding-in effects of alternative policies. In the figure, we project $F(H[\gamma_0])$, which coincides with the command optimum frontier, together with two possible frontiers for $\gamma > \gamma_0$ and $\gamma < \gamma_0$. By the

¹⁹ The following argument illustrates why ρ is unique in this economy. Suppose that the representative investor faces a riskless rate different from ρ . Then, she would consider a new efficient portfolio frontier connecting A to K^g and would try to move onto the new frontier, i.e. to form a portfolio containing a positive amount of the riskless asset. However, this cannot be an equilibrium portfolio for the representative agent in an economy where the riskless asset does not exist in positive supply. The unique sure rate compatible with a portfolio including capital shares only is ρ .

result of the comparative static exercise, growth will be slower in the first case, faster in the second one; $F(H[\gamma > \gamma_0])$ will lie to the left, $F([\gamma < \gamma_0])$ will lie to the right of $F(H[\gamma_0])$. Note that, while the standard deviation of the market return reflect the given level of social risk σ , its mean is endogenously determined²⁰.

8. A simple Extension of the Model: Learning by Doing.

The goal of this section is to show how the basic structure of the model can be used to explore these growth- and policy-related issues in models with externalities in the production function. By way of a representative example, consider a simplified version of an Arrow-Romer growth model, where the amount of capital in the economy has a positive external effects on the productivity of labor (see Romer [1986]). The private firm will now face a production function with constant return to scale in capital and labor, the latter measured in efficiency units:

$$(8.1) \quad dY = [\xi dt + \psi d\omega(t)] K^{1-\beta} J^\beta$$

The measure of labor in efficiency units captures the external effect of capital on the productivity of workers. Denoting with $L(t)$ the physical units of labor and with \mathbb{C} the total amount of capital in the economy, we can write $J(t) = L(t)\mathbb{C}$. Assuming identical firms, this identity can also be written as $J(t) = L(t)N(t)K(t)$, where $N(t)$ is the number of firms in the economy, exogenously given. Thus, the above specification implies the presence of scale effects on the nation-wide productivity, a feature which is well known in the literature. While at a private firm level the return on capital is perceived as decreasing, it is quite apparent that from a social

²⁰ The metric of the indifference curve map in the (E, σ) space is conditional on a given level of wealth. For a given level of wealth, utility increases when moving South-East in the diagram. However, in our model wealth is endogenous, and the three frontiers in figure 3, drawn for different policies, correspond to different levels of wealth. Therefore, we will have a different metric of the map of indifference curve for each particular frontier.

point of view we are facing exactly the same linear technology as in section 2.

Posing $\eta \equiv NL\xi$ and $\sigma \equiv NL\psi$, and substituting the definition of J in (8.1), we obtain exactly (2.1.2). For the sake of simplicity, we will try to deviate from the framework of the analysis laid down in the preceding sections as little as possible. We will assume identical preferences as in (2.1.1), which implies that raw labor is inelastically supplied. The government is assumed to tax output according to (2.2.2), where, however, the spending rate will be normalized to zero. Thus, posing $G(t)=0$, the government intertemporal budget identity (2.2.3) still holds.

Abstracting from the normalization of government spending, the main difference with respect to Eaton's specification consists of, first, privately perceived decreasing return on capital and, second, the presence of labor income in the market economy. Because of the external effects of capital on labor productivity, we expect a market allocation without public intervention to be suboptimal. In particular, investment in productive capital will be too low. Provided that the government can raise revenue in a non-distortionary way, a policy of investment subsidies can then enhance welfare and growth. In what follows, we will re-examine these simple propositions in our stochastic economy.

Given the presence of privately perceived decreasing return on capital, the extension of Eaton's solution strategy requires an additional analytical step. In particular, consider the present discounted value of the after-tax firm output. This can be viewed as the sum of two assets, the first being a claim to the present and future flow of after-tax capital income, the second a claim to the present and future flow of after-tax labor income. Denoting these assets with S (equity shares) and E, respectively, and assuming the existence of competitive markets for both assets, private wealth in terms of the capital good can now be expressed as

$$(8.2) \quad W(t) = K(t) + q_E E + q_B B$$

where, as before, q 's denote asset prices. From this point on, we can simply follow the solution strategy outlined in the previous sections. First, with only one source of uncertainty in the economy, once again we can conjecture an equilibrium where the rates of return on these assets will be in the form $r_i dt + \sigma_i d\omega(t)$ (for $i=S,E,B$). The above specification of wealth and rates of return will allow us to write the consumer problem as in section 3, including an additional asset (E) in the budget constraint. The first order condition for this problem can be written as

$$(8.3) \quad C = \zeta W$$

$$(8.4) \quad (r_s - r_e) - R(\sigma_s - \sigma_e)[n_s \sigma_s + n_B \sigma_B + (1 - n_B - n_s) \sigma_e]$$

$$(8.5) \quad (r_B - r_e) - R(\sigma_B - \sigma_e)[n_s \sigma_s + n_B \sigma_B + (1 - n_B - n_s) \sigma_e]$$

where n_s , n_B and n_e denote the shares of the three assets in wealth. By the profit maximization conditions of a competitive firm, the rate of return on capital will be equal to its marginal product:

$$(8.6) \quad r_s(t) = (1 - \tau)(1 - \beta)\eta$$

$$(8.7) \quad \sigma_s(t) = [1 - (\tau + a)](1 - \beta)\sigma$$

while, by the same argument as in section 3, the other rates of return can be easily specified by equating the rate of growth of different wealth components. We therefore obtain

$$(8.8) \quad r_B(t) = \eta - \frac{C}{W} + \tau\eta(n_s/n_B)$$

$$(8.9) \quad \sigma_B(t) = \sigma[1+(\tau+a)(n_s/n_B)]$$

$$(8.10) \quad r_e(t) = \eta - \frac{C}{W} + \frac{(1-\tau)\beta\eta n_s}{1-n_s-n_B}$$

$$(8.11) \quad \sigma_e(t) = \sigma[1+(1-\tau-a)\frac{n_s}{1-n_s-n_B}]$$

The system of equations provided by the above expressions allows us to solve for the equilibrium allocation. In particular, recalling the definition of γ in (4.13), we will have:

$$(8.12) \quad n_s = \frac{(1-R)[\beta(\eta-R\sigma^2)+(1-\beta)\gamma+\delta/(1-R)-\eta+.5R\sigma^2]}{\beta(\eta-R\sigma^2)+(1-\beta)\gamma+[\delta/(1-R)-\eta+.5R\sigma^2]}$$

$$(8.13) \quad n_B = \frac{R\gamma}{\beta(\eta-R\sigma^2)+(1-\beta)\gamma+[\delta/(1-R)-\eta+.5R\sigma^2]}$$

$$(8.14) \quad \chi = \frac{n_s(1-R)}{n_s-(1-R)} \left[\frac{\delta}{1-R} - \eta + .5R\sigma^2 \right]$$

As in Eaton's model, we can easily verify that real allocation will respond only to changes in γ , generating the possibility of neutral tax reforms, i.e. variations in the fiscal parameters τ and a that leave the equilibrium portfolio shares and the consumption rate unaffected. The fiscal variable γ measures the magnitude of the tax-distortions in the economy. Increasing γ makes capital less attractive with respect to bonds, depressing capital formation. Decreasing γ fosters investment and growth.

Because of the capital externality, it is clear that welfare-enhancing policies should create incentives for faster capital accumulation. In this sense, it will be not

be wise to choose tax rates corresponding to a positive γ (i.e. to a positive certainty equivalent tax rate). Rather, a desirable policy of capital income subsidies will require negative γ 's. However, how can these subsidies be financed? The model suggests a solution well in the tradition of the intertemporal generalization of the Ramsey optimal taxation scheme.

From (8.13), it is apparent that negative γ 's make the equilibrium demand for government share negative. This implies that the government is a net financial creditor in the economy, i.e. it can finance its subsidies by using income accruing from its financial assets rather than raising revenue through distortionary taxation. The structure of returns ensures that the private sector will be perfectly happy to go short on government debt. By the equilibrium conditions, then, the market portfolio will reflect the social risk, exogenously given in this economy, and the appropriate expected return on the market.

It could be observed that a negative public debt is effectively equivalent to lump-sum taxes. Both allow the government to raise revenue in a non-distortionary way. Indeed, a positive net financial position of the government is what the solution to the intertemporal taxation problem prescribes. In the Ramsey tradition, a positive net financial position is achieved by means of an initial "capital levy", through which the government becomes a share holder. In our framework, the government is similarly endowed with part of the return on capital, but not as a share holder. In principle, negative debt can be issued against a reduction of future tax liabilities at a price reflecting the present discounted value of the tax relief. If taxes were lump-sum, such a financial transaction would be neutral. Outside the Ricardian world, policy neutrality fades away as the tax burden is reduced in trade, with clear effects on the real allocation.

Note that in a command economy, the rate of consumption out of capital is still equal to (5.4). An optimal policy can then be identified by equating this expression to (8.14). It is apparently verifiable that $n_s=1$ is the condition we are

looking for, corresponding to

$$(8.15) \quad \gamma_0 = \frac{-\beta}{(1-\beta)} (\eta - R\sigma^2)$$

In this first-best equilibrium, the external effect of capital on labor productivity is completely internalized, as private wealth coincides with the capital stock. It is also easy to verify that the equilibrium demand for government bonds conditional on γ_0 is negative, according to our previous discussion.

One may wonder what would be the equilibrium and policy implications of alternative tax policies. For example, taxes may be levied on assets income, rather than output. Two cases need to be distinguished. In the first one, tax rates do not vary across assets. Then, the basic results of the above analysis would still be valid, except for the fact that now, for a given capital to wealth ratio, the equilibrium shares of B and E in wealth would marginally respond to equilibrium-preserving tax reforms. Yet, an optimal policy would require public debt be negative. However, if tax rates can differ, it is clear that, as the social marginal contribution to output by raw labor is zero, it would be optimal to tax all labor income and use the proceeding to finance capital income subsidies (as modeled, for example, by Corsetti and Pesenti [1992]).

9. Conclusions.

A few years before the development of endogenous growth theory, Eaton's article in RES provided a convenient framework of analysis to study policy-related issues in stochastic endogenous growth model. In this paper, we have highlighted the analytical core of the model, completing the original discussion and showing directions for possible extensions and generalizations. The main advantage of a Merton-type approach to growth and policy analysis consists of pointing out the equilibrium relations between fiscal policy, rates of return and portfolio composition. Wealth

allocation analysis can thus provide an important complement to alternative models, focusing on real variables.

The fiscal issue underlying Eaton's original contribution is the financing of government spending when this is in some given (deterministic or stochastic) proportion to the existing capital stock in the economy. In contrast to the classical Ramsey result (as in Chamley [1986]), a policy of balance-budgets with a non zero tax rate on capital income can support a first-best allocation in the market economy. Also, any non-zero amount of public debt is suboptimal, because of the tax-related distortions associated with its servicing. However, as we introduce externalities into the production function, the composition of financial wealth in the economy may reflect the public sector need of revenue to finance the appropriate policy of subsidies. If lump-sum taxes cannot be resorted to, a positive net financial position by the government (negative public debt) may be welfare-enhancing.

The financial counterpoint of alternative fiscal policies highlighted by Eaton's model thus makes us aware of additional degrees of freedom in fiscal and financial engineering, logically complementing the traditional analysis of intertemporal optimal taxation.

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Figure 1

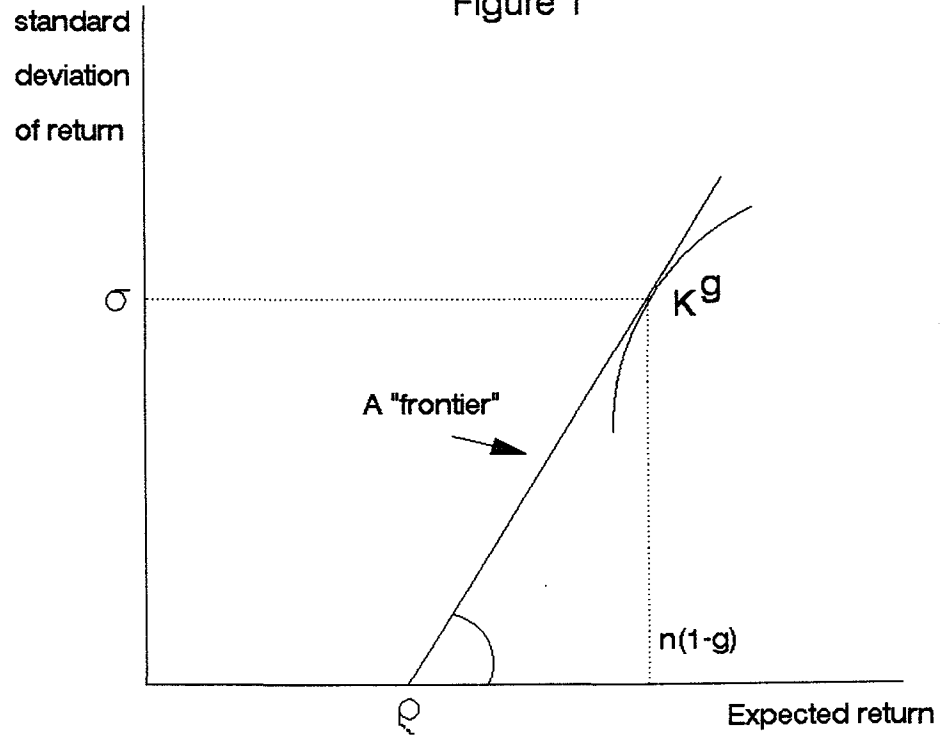


Figure 2

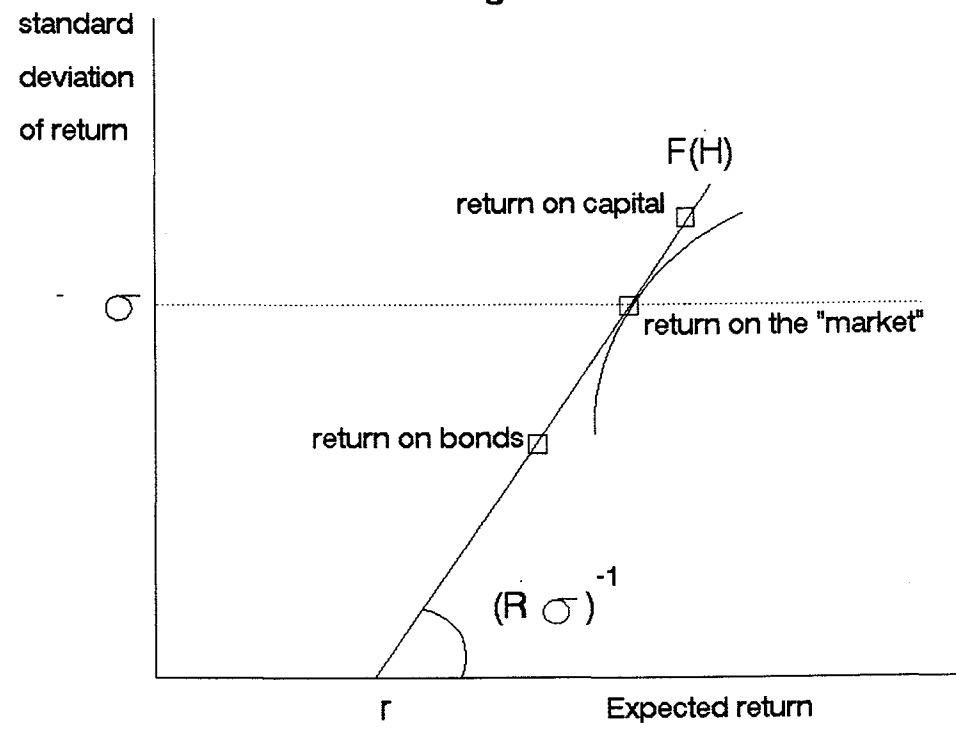


Figure 3

