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## Smoothing Primary Exporters' Price Risks: Bonds, Futures, Options and Insurance

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CENTER DISCUSSION PAPER NO. 647

SMOOTHING PRIMARY EXPORTERS' PRICE RISKS:

BONDS, FUTURES, OPTIONS AND INSURANCE

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## **Abstract**

The costs of primary commodity price instability are reviewed, and can be significant. Even with full commitment on both sides and stationarity of prices, international lending leads to nonstationary consumption. One-period futures improve smoothing, and a rollover plan is quite effective under first-order serial correlation. With sovereign (exporter) risk the above instruments are infeasible. But packages of simple bonds and put options can achieve smoothing qualitatively similar to, but less efficient than, the constrained optimal state-contingent contracts for Markovian price processes. Bonds and options have the practical advantage of greater potential liquidity than more complex contracts.

JEL Classification (New style): F34; G13; D81; Q17; O19

Key words: default risk, futures, hedging, options, commodity bonds, commodity price instability, persistence of shocks, stabilization

**SMOOTHING PRIMARY EXPORTERS' PRICE RISKS:  
BONDS, FUTURES, OPTIONS AND INSURANCE**

By

Kenneth M. Kletzer, David M. Newbery and Brian D. Wright

Nearly three fourths of the total wealth of Norway, by a recent estimate, consisted of oil reserves.<sup>1</sup> Her net oil income varies much more than the oil price, because it is net of the substantial but relatively more stable costs of extraction. Clearly, Norway's national income is highly sensitive to the price of oil. Most other developed economies have a much more diverse production mix, making internal diversification of commodity price risk a possibility. But many less developed countries, for example Mexico, Nigeria and Zambia, are each highly dependent on one primary commodity that provides the majority of their export earnings. Given this specialization and the great instability of primary commodity prices relative to other goods and services, such countries are likely to experience unusually sharp fluctuations in export earnings and gross national product. In such countries domestic diversification of income risk still leaves consumers with substantial commodity price risk exposure. If these price fluctuations are reflected in consumption they impose risk costs on consumers. Assuming optimal intra-temporal domestic diversification, this paper considers what further alternatives might be available to reduce the cost associated with price fluctuations in a major commodity. To be useful, such measures must recognize the fact those for most primary commodities, prices are serially

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<sup>1</sup>Aslaksen and Bjerkholt (1985, Table 5). The assumed price of oil was roughly the real price of the period 1975-1980.

correlated, and so price shocks tend to be persistent, making intertemporal smoothing more difficult.

The paper is organized as follows. First we review the costs of income variability caused by export price instability, with some reference to the empirical magnitudes. Then in section 2 we consider the role of conventional instruments, including loans and futures contracts. Particular attention is paid to the potential use of futures rollovers for price protection when disturbances are serially correlated. All of these instruments encounter difficulties in the presence of sovereign risk and/or capital shortages. In section 3 we discuss the nature of sovereign default risk and its implications when borrowers are anxious to smooth consumption. Dynamic consumption smoothing, with and without a default constraint, are addressed in section 4, for i.i.d. and Markovian disturbances. We use the constrained optimal fully state contingent contract as a benchmark for evaluation of simpler contracts using familiar, and more liquid, option and loan contracts.

## **1. The Costs of Consumption Variability**

Newbery and Stiglitz (1981) have set out the theory of measuring the costs of risky consumption, and provided estimates of the extent of risk in the export of six primary commodities from a number of developing countries. Before reviewing that evidence it is perhaps useful to distinguish between the costs of a risky, as yet unknown, income stream, and that of variable but predictable income stream. According to the standard expected utility model, there is no difference in the present value of utility of a stream of consumption,  $c_t$ , which varies in a predictable way, and one which takes the same values at each date, but whose values were not known initially. There is however, an important difference between two identical but variable income streams,  $y_t$ , one known in advance, and the other unknown. If income is

predictable, then it is possible to compute its present value, and to determine the optimum time path of consumption permitted by this certain value and the opportunities available for lending and borrowing. If income is uncertain, then such precise consumption smoothing is not possible, and the achievable time path of consumption will have lower value than in the former case. More generally, knowing the future value of income will allow more efficient decision making generally, not just of consumption paths, and thus be more valuable.

Granted this, it should also be remarked that the economic value of the additional information of the future time path of income may be small compared with the costs of variability, and if, as Newbery and Stiglitz (1981) argued, the latter are typically small, the distinction may not be so important.<sup>2</sup>

With this in mind, consider the problem of calculating the cost of risk for a country that has economically unresponsive production ('zero supply elasticity') and which seeks to maximize the expected utility of its representative consumer

$$V_t = E \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where E is the expectations operator,  $c_t$  is consumption in period t, and u is felicity, concave in consumption. The rate at which utility is discounted, that is, the rate of pure time preference, is r. and the discount factor is  $\beta = 1 / (0 + r)$ .<sup>3</sup>

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<sup>2</sup>The psychological costs of uncertainty as opposed to variability may not be so small, for a variety of reasons, not least of which is the frequent failure of the expected utility theory to predict behaviour by failing to accurately model perceptions of risk. Whether these costs are real or based on mis-perception is of course the key issue in determining the welfare basis of the expected utility hypothesis (EUH). The maintained hypothesis here is that the EUH properly measures the actual welfare costs.

<sup>3</sup>Since Newbery and Stiglitz (1981) the experimental evidence against the expected utility hypothesis has continued to accumulate, and several interesting alternatives have been proposed. At this stage, however, no alternative has replaced the expected utility approach for general analysis of the costs of risk and the value of its reduction.

Using the standard formulas (given for example in Newbery and Stiglitz, 1981) if the coefficient of (partial) relative risk aversion, is  $R$ , (defined for annual variations in consumption), and if the CV of consumption is  $\sigma_c$ , then the annual cost of the risk,  $\rho$ , is defined implicitly by  $u(\bar{c} - \rho) = Eu(c_t)$ , where a bar over a variable indicates its expected value, and the relative cost,  $\rho / \bar{c}$ , is approximately  $\frac{1}{2} R \sigma_c^2$ .

Let us now consider what these formulas imply for the magnitudes of the costs of market risk borne by the consumer and the benefits of market risk reduction.

Newbery (1990) recently updated estimates of the variability of 7 'soft' commodities, and found that price variability had increased somewhat when comparing the period 1970-86 with the period 1950-69 and found the unweighted coefficient of variation (CV) of prices to be 22%.<sup>4</sup> The costs of price instability increase as the *square* of the CV, which means that if we consider the period 1970-86, coffee price instability (CV = 24%) is four times as costly as that of jute (CV = 12%), not twice as costly, as the figures might otherwise suggest.

A consumer completely specialized in production of a commodity with "average" CV of price of 0.22, and with no output uncertainty and no variable inputs, has a CV of income of 0.22. If income is entirely consumed each period, the cost of risk to this consumer, assuming for purposes of illustration a coefficient of relative risk aversion  $R = 2$ , is about 5 percent of income. Were the product coffee, with a CV just a little higher at .24, the cost of risk is higher by about one fifth, that is around 6 percent. If the export had a mean share  $\alpha$  of the consumer's income, the rest of which was essentially deterministic, the cost would be  $5\alpha^2$  percent. We can begin to explore the practical magnitudes of the cost of risk and its reduction in four countries highly dependent on a single primary export, using the data in Table 1.

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<sup>4</sup>Calculated as the root mean square of the average of the squared percentage deviation from the centered five year moving average of prices.

The exercise addresses the following rather specific question. What would be the effect on the variability of commodity export revenue and of total export revenue, assuming that the price of the commodity received by the country was equal to a price stabilized at a weighted average of the current and previous four year's real world price level, with linearly declining weights.<sup>5</sup> The levels of exports are assumed to be unchanged, and the export unit values are assumed to be proportional to the world commodity price level (and so would change by the same proportion when stabilized). There are various ways in which such a scheme might work, but the simplest would be the commodity stabilization manager (operating in some international organization) would first calculate the stabilized price (which is a mechanical exercise as it is based on available data). The difference between the stabilized and spot price would be transferred to the country (rather as is done under STABEX but not as a loan, and with the contractual implication that if the spot price exceeded the stabilized price the country would transfer money to the stabilization manager). The crucial feature of this stabilization scheme is that it can be implemented using observed data. Whether it would be feasible raises the same kinds of questions as lending and borrowing, considered below.

#### Insert Table 1

The table shows that stabilizing the price of copper halves the variability of copper export revenue (and thus lowers the cost of that variability to one-quarter), and has a somewhat smaller effect on the variability of total export revenue. (Variability in this case is measured as the standard deviation (SD) of percentage deviations from the *centred* five year real moving average export revenue, (effectively the CV). It is an *ex post* measure of the welfare benefits.) Figs. 1 and 2 show the

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<sup>5</sup>Calculated as the root mean square of the average of the squared percentage deviations from the centred five year moving average of prices.



effect of stabilizing the price of copper for Zambia and coffee for Brazil. (The former is an extreme example of a country heavily dependent on one primary commodity, the latter shows the large effect of spreading the risk over all exports.) The graphs give the percentage deviations from the centred 5 year moving average, and the graphs of deviations of commodity exports and total exports are displaced vertically to make it easier to see what is going on—commodity deviations are shown on the left-hand scale and total exports are on the right-hand scale. The graphs have been scaled to be directly comparable, and the prices have been deflated by the index of Manufacture Unit Value exports to LDCs. Powell (1990) comes to a similar conclusion that price stabilization has a rather smaller effect than might be expected for Zambia.

Stabilizing the price of coffee reduces the variability of Brazil's coffee export revenue by one third (and almost halves the cost) but has a rather small effect on the variability of Colombia's coffee revenue. It has a smaller effect still on the variability of total export revenue for both countries (though the levels of total export variability are not particularly high in any case). The final two columns give the *ex post* risk benefits of this stabilization policy, relative to commodity revenue and total export revenue, for the value of R of 2. They are surprisingly small. Col (1) gives the share of the commodity in total GDP, and if stabilization had no effect on the other components of GDP (a strong assumption), then spreading commodity price risks evenly over the whole economy would greatly reduce their cost (to between 1 and 10% of the values of col (7)). Evidently, if producers could easily diversify their risk by spreading their assets across the various productive activities in place within their country, they should have surprisingly little reason to be interested in any alternative means of risk reduction considered below, unless these measures raised mean income as well as reducing its risk.

However, for various reasons we do not observe full domestic diversification, and in any case for the kind of heavily indebted countries we have in mind, we are

interested in the benefits of stabilizing the flow of imports. We therefore concentrate on cases where other means of reducing the cost of risk in commodity exports are required. We further simplify the discussion by assuming that the country's production of the export is non-stochastic. This reduces the problem to one of price risk, and eliminates moral hazard in production as an impediment to smoothing arrangements discussed below. These assumptions tend to bias the analysis in favour of such arrangements.

## 2. International Transactions for Buffering Export Price Instability

If a small country produces a commodity with a highly variable revenue stream that cannot be diversified internally, a natural alternative is to attempt to use international transactions to smooth consumption of its producers.

For simplicity, assume that the country in question has a negligible part of world income, and that competitive investors exist beyond its borders who are market risk-neutral. Further, the rate of time preference is identical for all countries and constant over time and equal to the opportunity cost of capital in the world market. Except where otherwise stated, there are no transaction costs, no domestic saving, and no storage<sup>6</sup>.

Given these assumptions, all smoothing arrangements involve a transfer from the exporter to other countries when price is high, and vice versa when price is low. The exchange is intertemporal rather than contemporaneous, so the ability of both sides to credibly commit to a given arrangement is crucial in determining the amount of smoothing that is possible.

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<sup>6</sup>Storability of primary commodities has strong implications for the nature of the price process that are not considered here. See Wright and Williams (1982), and Williams and Wright (1991) for a more extended discussion.

Consider, first, the case where all parties have unlimited commitment ability. In that case a very simple contract achieves complete smoothing: sell the natural resource industry to foreigners, in return for a perpetuity of the same net present value. Note that continuing commitment is needed on both sides, to continue to pay the perpetuity, on the one hand, and to continue to export and pay the net revenues to foreigners, on the other. Note that this arrangement is optimal for any price process for which a mean present value exists.

At this juncture even the uninitiated reader may be experiencing some misgivings regarding the political realism of wholesale disposal of the national patrimony of a commodity exporter to foreigners. What about using the familiar strategy employing international borrowing and lending?

## 2.1 Conventional Bonds under Commitment

Following the line of argument of a model in Newbery and Stiglitz (1981 pp. 201-203), assume that consumption  $c_t$  is chosen to maximize (1), subject to the asset accumulation equation

$$D_{t+1} = (D_t + y_t - c_t)(1+r) \quad (2)$$

where  $D_t$  is predetermined and  $y_t$  is the realization in period  $t$  of the stochastic income process, assumed without trend or drift. To distinguish the loan arrangement from a Ponzi scheme, add the condition that  $D$  is not expected to change, given the current debt service obligation is met (or, if  $D_t$  is positive, given that the current return on investment is received):

$$E_t(D_{t+1}) - D_t = 0. \quad (3)$$

The optimal choice of  $c_t$  in this problem satisfies  $u'(c_t) = Eu'(c_{t+1})$ .

In the simplifying case of quadratic utility,

$$c_t = E_t(c_{t+1}), \quad i = 1, 2, \dots \quad (4)$$

Next year's expected total income including asset income is

$$E_t(y_{t+1}) + r\beta D_{t+1} = E_t(y_{t+1}) + r(D_t + y_t - c_t) \quad (5)$$

Given (3), expected consumption equals expected total income,

$$c_t = E_t(c_{t+1}) = E_t(y_{t+1}) + r(D_t + y_t - c_t) \quad (6)$$

Solving for  $c_t$ ,

$$c_t = r\beta (D_t + y_t) + \beta E_t(y_{t+1}) \quad (7)$$

Substituting (6) in (2),

$$D_{t+1} + y_{t+1} = D_t + y_t + (y_{t+1} - E_t(y_{t+1})) \quad (8)$$

$$D_t + y_t = D_0 + y_0 + \sum_{i=0}^{t-1} (y_{t-i} - E_{t-i-1}(y_{t-i})) \quad (9)$$

If  $D_0 = 0$ , (7) and (8) imply

$$c_t = \beta E_t(y_{t+1}) + r\beta \left( \sum_{i=0}^{t-1} (y_{t-i} - E_{t-i-1}(y_{t-i})) + y_0 \right) \quad (10)$$

Thus consumption varies directly with accumulated balances, which follow a martingale process. How does this compare with the best conceivable consumption pattern?

If income (other than interest on D),  $y$ , follows a stationary i.i.d. process, then intuition correctly indicates that the best consumption pattern that has an expected present value that does not exceed that of the representative individual's endowments is to consume the same amount each period:

$$c_{t+i} = E_{t+i+1}(y_{t+i}) = E_t(y_{t+1}) = c_t, \quad i=0,1,2,3, \quad (11)$$

This prescription is consistent with a conception of smoothing as an intertemporal tradeoff between good and bad realizations. But what if  $y$  is serially correlated? In the extreme case of a random walk, no such intertemporal tradeoff can be anticipated. Consider the following prescription, suggested by a referee:

"[S]ay income follows a random walk then consumption should follow a random walk if there are no adjustment costs. If there are quadratic costs of adjustment then actual consumption should follow a partial adjustment or the more general error-correction scheme."

For a country embarking on an export stabilization scheme, the above might seem consistent with (10) for  $i=t$ . But it is not. A borrower fully capable of commitment will set

$$c_t = y_t = E_t(y_{t+1}) = c_{t+1} = c_{t+i}, \quad i=2,3,\dots \quad (12)$$

As in the case of stationary  $y$ , the optimal consumption path is fully smoothed at year  $t$ ; consumption is constant thereafter. With probability one the representative consumer will in some later period  $t+j$  find this constant consumption level is less than  $y_{t+j}$  and wish that the previously agreed consumption were renegotiable; but full commitment means that no changes in the agreement that maximized (1) in year  $t$  are possible.

As noted, loans do not achieve the constant consumption level that maximizes (1) at time  $t$ , even if we accept the heroic assumption that there is no (negative or positive) level of  $B$  beyond which the scheme is infeasible. The fundamental problem is that the borrower's income remains state-contingent, while her repayment received by the lender are not. To attain greater consumption stabilization, a still stronger assumption is needed: Lenders must be willing and able to commit to future consumption-smoothing disbursements in some states that will not generate future repayments of equivalent expected present value. That is, such repayments not only violate (3) but reduce the net worth of the payer. Hence they would not be made by an unconstrained lender.

For more effective smoothing we must go beyond simple borrowing and lending to contracts with commitment of the smoothing party to some state-contingent future payments that will render the net receipts of the "borrower" independent of the state. We now turn to alternative financial arrangements that offer payments of this kind, maintaining, for now, the assumption of full bilateral commitment.

## **2.2 Contracts for Insurance with Full Commitment**

Futures and forward contracts can be used by exporters to reduce the price contingency of the commodities they sell. In practice futures and forwards are similar, but not identical, forms of contracts. The commitment mechanism peculiar to futures (daily adjustment of margin payments and receipts in response to price changes) introduces an analytical difference from forward contracts. But given the full commitment assumption we ignore this difference here.

Forward contracts can be written so that what is hedged is exactly what the exporter will produce. For this advantage, they sacrifice the liquidity of a futures contract, which is written in standardized units of a specified standard commodity, (with, perhaps, fixed differentials for quality variations). A forward contract may

therefore have a large bid-ask spread, implying impaired flexibility of adjustment subsequent to commitment.

For a discussion of futures contracts, it is useful to begin by defining some notations:

$p_t$  Spot price at harvest in year  $t$

$F_{t,j}$  Futures price for delivery after harvest in year  $t$  at date  $j < t$ .

$b_t = p_t - F_{t,t}$  Contemporaneous basis

$f_t \equiv F_{t,t-1}$  futures price at start of year  $t$

$f_{t+1} - f_t$  Intertemporal basis

Trading in futures markets exposes producers to two different kinds of risk, both confusingly called basis risk. Contemporaneous basis risk arises because the producers who have sold futures to hedge output typically liquidate this by buying them back in the terminal month, and selling their output. If the terminal futures price were equal to the spot price of what is hedged there would be no risk, but in general this is not true, due to (often subtle) differences in grade, location, or other characteristics. So ex ante the producer faces the risk that the two prices will not be the same—that is, the basis risk at the point of sale. While this basis is the stuff on which futures markets survive or perish, the risk involved is small compared to the risk of not hedging for most producers (i.e. those for which the futures market offers an appropriate contract). Here we shall therefore ignore this type of risk, and assume in what follows that  $b_t = 0$ , or  $F_{t,t} = p_t$ . We shall also adopt the convention for any random variable  $x$  that  $E x_t \equiv E_{t-1} x_t$ , and assume that the futures market is unbiased, so that  $f_t = E_{t-1} P_t$ . Under these assumptions, we treat futures and forwards as essentially identical. Newbery and Stiglitz (p. 186) show that, in the case of stationary, uncorrelated output and price disturbances, the ratio of income variance

with and without optimal forward hedging, is roughly  $1/(1+k^2)$ , where  $k$  is the ratio of the CVs of price and output.

If prices and output are both i. i. d. and equally variable,  $k$  equals 1 and a forward hedge can halve the cost of risk in the case of quadratic utility. Here we simplify the discussion (and favor the use of futures or forwards) by ignoring output uncertainty. Then with credible commitment on both sides, full smoothing is achieved by selling all production forward one period, via futures or forward contracts. If the hedging is starting de novo in year  $t$ , this full smoothing starts in year  $t+1$ . To advance the onset of full smoothing to year  $t$ , one could, assuming perfect capital markets with loans as described above, take a loan on the full proceeds of the futures (forward) sale. The combination of the loan and the forward hedge is equivalent to a simple "commodity bond" (Priovolos 1991 ch. 1).

### 2.3 Hedging with Serial Correlation

Thus far, the smoothing arrangements seem simple and efficient. But the evidence shows that the prices of many of the major traded primary commodities exhibit significant annual serial correlation which dramatically affects the costs and the benefits, and the feasibility of price stabilization. Deaton and Laroque (1989) have studied this problem, using annual data on 24 commodity prices for the period 1900-1987, deflated by an index of manufacturing unit values. Their measures of persistence are the sum of all autocorrelation coefficients (whether significant or not), with the sums being linearly declining weighted averages over the window widths of 20 or 40 years.<sup>7</sup> Their results suggests that about one-quarter of price shocks are permanent, that three-quarters or more of the price shock will persist for at least a

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<sup>7</sup>The theoretical and econometric basis for these estimates is given in Chochran (1988) and Campbell and Mankiw (1987). The wider the window, the more data is allowed to influence the estimate, but at lower reliability. The wider window may thus give a downward biased estimate



year, and even after two years typically 60 percent of the price shock will persist.<sup>8</sup>

First-order correlation coefficients average 0.8 with a standard deviation of 0.08.

The evidence suggests, therefore, that serial correlation is prevalent for the world prices of primary commodities, and this fact should be taken into account in designing methods for consumption smoothing. If income is serially correlated because prices are serially correlated, a fall in current income signals lower than anticipated income next year, and hence lower  $y_{t+1}$  and lower  $c_{t+1}$  (other things being equal). This will raise  $u'(c_{t+1})$  and hence lower current consumption. If the autoregression coefficient is near unity, consumption may be depressed almost as much as current income and little smoothing will take place if only the annual harvest is hedged. The volumes that must be hedged for significant consumption smoothing become so much larger that transactions costs can no longer be ignored.

Suppose for example that prices follow the simple autoregressive scheme:

$$\tilde{p}_t = \alpha p_{t-1} + (1 - \alpha)\bar{p} + \tilde{u}_t, \quad (13)$$

where  $\tilde{u}_t$  is i.i.d. with zero mean, and is the forecast error at the start of year  $t$ . Given our assumptions on the futures price, this can be written as.

$$\tilde{p}_t = f_t + \tilde{u}_t, \quad f_t = E_{t-1}\tilde{p}_t = \alpha p_{t-1} + (1 - \alpha)\bar{p}.$$

Again,  $f_t$  is the expected price, equal to the futures price at the start of period  $t$  in an unbiased market. But now the second type of basis, "intertemporal basis,"  $f_{t+1} - f_t = \alpha(p_t - p_{t-1})$ , becomes significant because it will fluctuate from year to year, possibly substantially. Even if futures markets extend only one year ahead and incur transactions cost, it is possible to roll over hedges to provide additional income

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<sup>8</sup>Cuddington and Urzúa (1988) used the same data set and somewhat less sophisticated techniques, claiming the improbable result that real commodity prices are I(1). Others have claimed likewise, though the power of the statistical tests leaves much to be desired, especially given the leptokurtic nature of the price distributions.

smoothing to that achievable within the crop year. In the Appendix we show how to construct a sequence of rollover hedges in the futures market that provides considerable risk reduction and insurance against this basis risk.

The way the roll-over works is to sell more futures initially than needed for one-period hedging, and then use the surplus futures sales to finance the next year's futures transactions. This is not perfect, for the amount of hedging required next year will depend on production, and that will depend on the futures price prevailing next year, which is not yet known. Consequently, despite the absence of production risk, future output cannot be perfectly hedged, and there remains some residual risk (as there would be if there were output risk). Nevertheless, because the costs of risk increase with the square of the deviation, reducing the risk by a given fraction reduces the cost of risk by more than that fraction and can be worthwhile.

The appendix shows how to construct a rolling  $n$ -period hedge for the special case of no output risk, but supply responsive to futures prices. The model has a linear supply schedule (linear in the futures price, which is the action certainty equivalent price in the absence of output risk). In year  $t$ , production  $q_t$  is planned, and at the start of the year  $q_t[1 + \alpha\beta + \dots + (\alpha\beta)^{n-1}]$  hedges are sold on the futures market. Hedging for longer periods reduces risk, but requires additional purchases of hedges, which of course involve additional transactions costs. The Appendix derives a formula for the value of the additional risk benefit derived per extra present value of hedge, as increasing the current number of rollovers involves a stream of future transaction costs as well as a flow of future risk benefits. The formula for the marginal benefit/cost ratio from increasing the period of hedging from  $n-1$  to  $n$  (and the number of hedges by  $(\alpha\beta)^{n-1}$ ) when each extra futures contract costs  $\mu$  is

$$\frac{(\alpha\beta)^{n-1} (1 + \alpha\beta) R\sigma^2}{1 + \beta - \alpha\beta \mu}$$

The optimal length of the hedge,  $n$ , is given by

$$n = 1 + \ln \left( \frac{\mu(1+\beta)(1-\alpha\beta)}{(1+\alpha\beta)R\sigma^2} \right).$$

Clearly, as the time horizon of the hedge increases, the marginal benefit also falls. Table 3 relates the optimal hedge length to the value of the serial correlation coefficient,  $\alpha$ , the rate of interest,  $r$ , and the forecast error,  $\sigma$ , when coefficient of relative risk aversion is 1, and the transaction costs as a fraction of the value of the hedge is 0.3 of 1%-a figure taken from Gardner (1989).

Thus the table shows that if  $\alpha = 0.7$ ,  $\sigma = 0.10$ , and  $r = 15\%$ , then it would be worth seeing  $n = 5$ , and at  $\alpha = 0.8$ ,  $n$  should be 8. But it is clear that the value of such hedging (on the favourable assumption of no output risk) is quite low, as transaction costs are low and the benefit-cost ratio is in terms of these transaction costs. Higher transactions costs would shorten the horizon over which hedging was cost-effective.

Thus the graph shows that if  $\alpha = 0.8$ , then it would be worth setting  $n = 4$ , and at  $\alpha = 0.9$ ,  $n$  should be 8. But it is clear that the value of such hedging (on the favorable assumption of no output risk) is quite low, as transaction costs are low and the benefit-cost ratio is in terms of these transaction costs. Higher transactions costs would shorten the horizon over which hedging was cost-effective.

The other point to make is that the number of hedges rises with the horizon, which would increase the risk of performance default if the contracts did not require payment of margin calls as the futures price changes, to cover any change in the value of the contract. The transaction costs calculated by Gardner include the foregone interest rate differential on the money left on deposit to cover margin calls, and this can be thought of as ensuring contract performance. A two-period version of this arrangement would be in effect a commodity bond, a combination of a futures hedge and a loan on the proceeds. The loan would be used as a performance bond to ensure that the hedger delivers on the contract, were the latter not liquidated by an offsetting

trade.<sup>9</sup> But this just shifts the commitment issue back one step. What ensures repayment of the loan? Until recently the financial literature on commodity bonds (Brennan and Schwartz 1980, Schwartz 1982, O'Hara 1984; Priovolos and Duncan, eds., 1991 chapters 2 - 7), like the literature on futures markets, has neglected the commitment issue; but that is central to the discussion of intertemporal international transactions.

### 3. The Structure of Commitments and Exporters' Smoothing Contracts

Here we focus on income-smoothing financial transactions between investors in developed countries (DCs) and a less-developed country (LDC) heavily dependent on a single commodity subject to substantial revenue fluctuations. In smoothing transactions the net financial flow in any period is in general unbalanced; there is none of the mutuality characteristic of balanced real trade. If transactions are considered period by period, there is in general an incentive for one party or the other to repudiate its current obligation.

The case with pure price uncertainty is illustrated in Figure 3, in which the world spot price  $P_t$  is on the horizontal axis and the exporter's contract payment per unit committed are shown on the vertical axis. If all sales are spot, then payment per unit and  $P_t$  are related by the 45° line OA.

Under a *forward* contract, the borrower's incentive to default is the difference between the spot price at maturity,  $P_t$ , and the forward price to be paid on delivery. The latter equals the expected price  $\bar{P}$  as of the signing of the contract, under the assumptions of risk neutrality, competitive buyers, and credible seller commitment to

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<sup>9</sup>In continuous time, the additional adjustment of variation margin as the futures price changes is necessary to ensure performance.

deliver. The short-run temptation to default (to be weighed against any effects on future smoothing opportunities) is  $P_t - \bar{P}$ ; the higher the spot price, the greater the temptation. The short-run default incentive of the buyer of the contract (the "long" side) is, symmetrically,  $\bar{P} - P_t$ . The simple (non-contingent) *commodity bond* can be considered as a combination of a one-period loan and a forward contract of the same duration. In such a contract, the borrower incurs at the outset a repayment obligation of  $\bar{P}$  per unit of exports (from a loan of  $\bar{P}/(1+r)$  per unit in the previous period) in addition to the delivery obligation. This adds the amount of the loan repayment under compliance,  $\bar{P}$ , to the short-run incentive to default. The temptation to default (more precisely, to repudiate one's obligations) is thus  $P_t$ .

These default temptations at time  $t$  must be balanced against the opportunity cost of defaulting. For full commitment the latter must dominate. In domestic transactions these default incentives are counterbalanced by legal constraints on the smoothing party (the "lender" or "insurer" and by the threat of loss of collateral of the smoothed party ("the borrower" or "the insured") if it does not fulfil its obligations.

Lack of such collateral constitutes the key distinction between such domestic transactions and their international counterparts, as recognized by many commentators and described in masterly fashion by Keynes (1924). Respect for sovereign immunity precludes the forced seizure or destruction of assets within the borders of a sovereign state by foreigners. In the absence of attachable collateral, incentives for repayment other than the threat of foreclosure must be found to support smoothing transactions in equilibrium. Interference with some external trading opportunities, such as denial of future access to loans or disruption of current commodity trade, is the sanction most widely recognized as providing repayment incentives (see Eaton and Gersovitz (1981), Eichengreen and Portes (1989a and 1989b) and Bulow and Rogoff (1989), for example). Lenders (including all effectively risk-neutral suppliers of smoothing financial flows) can disrupt intratemporal

transactions because they have access to courts to enforce debt seniority clauses against all other potential lenders, so that the borrower cannot repudiate a debt and repay subsequent debts or acquire foreign assets. Repudiation leads to the possible elimination of foreign borrowing or lending opportunities. The cost of repudiation is the loss of future consumption smoothing that could have been available. The amount that the borrower is willing to pay to her current creditors depends on her expectations of future lending conditional on her current behavior.

In the analysis below, we assume that the sanctions that support international financial transactions are the threat of disruption of intertemporal foreign trade, that is access to international borrowing and lending. We further assume that lenders in developed countries can commit themselves to make payments in some future states to the borrower. That is, they can write contracts binding them to make contingent payments that may not be profitable to make ex post (as in the case of futures and options contracts). Lenders are also competitive and offer loans that are expected to be profitable given the equilibrium behavior for the borrower. But given our commitment assumption, lenders can write ex ante expected profitable loan contracts with state-contingent repayment schedules such that repayments are negative in some states.<sup>10</sup>

Other commitment structures would lead to very different smoothing relationships. Kletzer and Wright (1990a) assume that lenders also cannot commit their future actions beyond respect for the seniority of other lenders' claims.<sup>11</sup> Thus

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<sup>10</sup>The assumption of one-sided creditor commitment is also made in the sovereign lending literature by Atkeson (1991), Bulow and Rogoff (1989b) and Worrall (1990), and is implicit in others. Because equilibrium contracts often lead to negative forward-looking expected profits for lenders, these are not models of renegotiation of repayments, as are Bulow and Rogoff (1989a) and Kletzer and Wright (1990a).

<sup>11</sup>In a sequel, Kletzer and Wright (1990b) prove that if no party can commit their future actions, then respect for seniority can be sustained in a noncooperative equilibrium. Therefore, some smoothing of the borrower's consumption is feasible under anarchy (that is, without external enforcement of contracts).

lenders are able, if they wish, to refuse to provide any smoothing flows in any period.

Both parties are free to renegotiate the relationship each period. With renegotiation of repayments, one-period simple loan contracts are shown to achieve the constrained optimum. Repayment obligations can be renegotiated down to zero but are never negative in equilibrium as they can be under the assumptions here.

Bulow and Rogoff (1989a) model renegotiation of repayments by a sovereign when foreigners can credibly threaten to disrupt the commodity trade of the debtor in each period and there is no consumption-smoothing motive for borrowing (all parties are risk-neutral). Each period the borrower and lender bargain over the amount to be paid by the borrower so that she can trade that period. This is equivalent to the situation in which the lender acquires monopoly rights to supply the imports desired by the borrower, and the terms of trade are determined by the equilibrium for the bilateral monopoly bargaining game. The amount of repayment just equals the difference between this equilibrium price and the lender's opportunity cost (for example, the world price). One could readily add contemporaneous trade sanctions (exactly as in Bulow and Rogoff) to our model. All that changes is that the price,  $p$ , received by the exporter is not the world price but instead the equilibrium bilateral monopoly price. The focus of this paper is on the use of external financial markets for consumption-smoothing; lending does not serve to smooth consumption in the Bulow and Rogoff trade sanctions set-up.

For simplicity, we concentrate on the case of pure price uncertainty so that income is  $y = p\bar{q}$ , where  $\bar{q}$  is fixed output. The exporter is tempted to default whenever her expected utility from consuming her income in the current and every future period (after she observes the current price) exceeds the utility she expects to receive from maintaining access to consumption-smoothing by paying her creditors. Efficient equilibrium borrowing and lending constrained by sovereign immunity is characterized below. Since the borrower is risk-averse, she will be willing to pay

more to preserve access to consumption-smoothing from abroad when her income is high than when it is low. In equilibrium, the borrower is willing to pay nothing in the lowest income state to avoid permanent exclusion from credit markets, but in the equilibrium smoothing relationship she is never expected to do so. As her income rises, her willingness to pay increases, so that in this sense the costs of default increase with income, as one commentator has noted. But so does the temptation to default: The high-income states are the states in which she is expected to pay for the smoothing she receives in lower states.

The commodity's price is assumed to be distributed over a finite number of possible states according to a Markov chain. That is, the probabilities for each possible income state in period  $t$  depend only on the income realized in period  $t-1$ . This includes the case of a simple autoregressive process (with disturbance distributed over a finite support). There are  $S$  states, ordered  $y^1 < y^2 < \dots < y^S$ , and the probability that  $y^j$  occurs in period  $t+1$  is a function of  $y_t$ , denoted  $\pi_j(y_t)$ .

#### **4. Overview of Dynamic Smoothing Strategies**

Depending on the degree of price risk, the intensity of risk aversion, the discount rate and other factors, the otherwise optimal smoothing plan may or may not induce the exporter to default. Let us consider the alternate cases in turn.

##### **4.1 Default Constraint Nonbinding**

From an initial uncovered situation, the availability of commodity bonds adds to the short-run resources represented by initial income  $y_1$ . Assume that the sovereign starts with no savings, but that she can save overseas in the countries that host the international lenders. Assume also that these lender countries collectively enforce financial contracts within their borders. In particular, they cooperatively enforce claims



by foreigners on domestic assets, and senior claims of domestic lenders on sovereign borrowers are enforced with respect to all inflows from sovereign borrowers, including savings deposits as well as loan repayments.<sup>12</sup> If so, one description of the optimal infinite horizon smoothing plan for implementation in period 1, given current income  $y_1$  (assumed for this exposition to be entirely from export of one commodity at price  $p$ ), and the discount rate equal to the interest rate is as follows: Invest  $\beta y_1$ , where  $\beta \equiv 1/(1+r)$ , overseas for a certain periodic rate of return of  $r$ , issue a simple c-bond payable in units of the commodity to cover all output, with current sale price  $\beta \bar{y}$ , and consume  $r\beta y_1 + \beta \bar{y}$  in each period 1, 2, 3,.... Full consumption smoothing is immediately achieved forever: consumption is the same for all periods and states.

The opportunities for legally protected overseas investment at the (certain) market interest rate and for trade in c-bonds (or, equivalently, in futures and conventional bonds) at unbiased prices are all the financial facilities needed for this plan. Furthermore, note that, if the initial income  $y_1$  is invested where it can be collateralized for the c-bond loan (for example in the lending country), the default constraint is relaxed relative to the comparative static analysis above that assumed all income was from sales of c-bonds and none of the current income in the period in which c-bonds were introduced was saved. So, even if full c-bond coverage seemed infeasible in that analysis, the above strategy may work.

If one ignores transactions costs, as we do here, a number of different combinations of contracts could replicate the above arrangement, given the assumption of a nonbinding default constraint. One example is a short forward contract plus a loan on the anticipated proceeds of the contract.

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<sup>12</sup>Both types of enforcement together support the dynamic smoothing contracts that follow. Bulow and Rogoff (1989) show that if the former type alone is effective, the smoothing strategies formulated below do not work.

## **4.2 Default Constraint Binding**

If the default constraint binds on hedging with commodity bonds or forward contracts, the full smoothing described above is infeasible. The alternative of using futures markets is precluded because the variation margin requirements that make default unattractive cannot be met by a liquidity-starved borrower. Nor will the margin calls be loaned by a third party lender because of the induced incentive of the borrower to default on those loans. What kinds of consumption smoothing contracts are feasible in such cases? The common type of commodity bond, (see Priovolos 1991 for a list of recent issues), with a call option for the buyer, is clearly inappropriate for this type of smoothing. True, the premium associated with the option would increase the lower consumption levels if the contract were feasible. But by selling the call to the lender, the borrower places herself under great temptation of default when price is high, and gets very inefficient low-end protection.

A more promising strategy is to limit the maximum temptation for the borrower by giving her some share in the marginal gain from increases in high prices, while limiting her maximum losses. A fully state-contingent loan contract would be ideal.

### **4.2.1 The optimal state-contingent, default-proof insurance contract**

Before presenting the commodity bond package, we consider as a standard of comparison the optimal consumption plan for a risk-averse sovereign commodity exporter, "the borrower." Assume that the risk-neutral lender must achieve non-negative expected profits at the onset of a long-term contract and that there is free entry. The exporter, unlike the lender, can repudiate the long-term relationship at any time, and will do so whenever permanent autarky is superior to continuation of the consumption-smoothing relationship.

The problem is to maximize

$$u(c_1) + E_1 \sum_{t=2}^{\infty} \beta^{t-1} u(c_t), \quad (14)$$

with respect to state-contingent consumption plans  $\{c_t\}_{t=1}^{\infty}$ , where  $c_t = c_t(y_1, \dots, y_t)$ ,

subject to the no-default constraint for the borrower

$$u(c_t) + E_t \sum_{i=1}^{\infty} \beta^i u(c_{t+i}) \geq u(y_t) + E_t \sum_{i=1}^{\infty} \beta^i u(y_{t+i}) \quad (15)$$

for every  $t = 1, 2, \dots$ , and the profitability constraint for the lender,

$$(y_1 - c_1) + E \sum_{t=2}^{\infty} \beta^{t-1} (y_t - c_t) \geq 0, \quad (16)$$

where  $E_t$  is the expectation operator conditional on date  $t$  information.

This is similar to the problem of finding an optimal implicit long-term wage contract, as in Holmstrom (1983). We can rewrite it as a dynamic programming

problem. Define the history of states as  $w_t$ , where  $w_t \equiv (y_0, \dots, y_t)$  and  $w_{t+1} \equiv (w_t, y_{t+1})$ . Let  $V_1(V_2^t, y_t)$  represent the maximal surplus that the exporter gets at time  $t$  from the consumption smoothing plan over permanent autarky when the risk-neutral "lender" receives profit  $V_2^t$ . At time  $t$  the lender's profit  $V_2^t$  will be a function of  $w_t$ , that is  $V_2^t \equiv V_2(w_t)$ . The function,  $V_1(\cdot)$ , is given by the optimality equation:

$$V_1(V_2^t, y_t) = \max \left\{ u(c_t) - u(y_t) + \beta E_t V_1(V_2(w_{t+1}), y_{t+1}) \right\}, \quad (17)$$

with respect to  $c_t$  and  $\{V_2(w_t, y_{t+1})\}_{y_{t+1}=y^1}$

subject to

$$V_2^t \leq (y_t - c_t) + \beta E_t V_2(w_t, y_{t+1}), \quad (18)$$

and

$$0 \leq V_1(V_2(w_t, y_{t+1}), y_{t+1}), \forall y_{t+1} \in \{y^1, \dots, y^s\} \quad (19)$$

and  $0 = V_2(y_0)$ ,

Solving (14) - (16) for the state-contingent infinite horizon consumption plan is equivalent to solving problem (17) at each date  $t$ , in each event  $w_t$ , for  $c_t$  and the (promised) profit to the lender in each state of nature,  $y_{t+1}$ , for the next period, by Bellman's principal. A sufficient condition for a solution to (17) to exist is that the global endowment is bounded in each period (so that  $c_t$  is bounded).

Because  $u(c)$  is strictly concave and continuously differentiable, the function  $V_1(V_2^t, y_t)$  can be shown to be strictly concave and continuously differentiable in  $V_2^t$  as well. (The constraints define a convex choice set; with bounded global endowment, it is also compact.) We form a Lagrangian for problem (17) and assigning the multiplier  $\lambda(w_t)$  to constraint (18) and multipliers  $\pi_1(y^1)\beta\phi(w_t, y^1), \dots, \pi_s(y_t)\beta\phi(w_t, y^s)$  to the constraints (19), we obtain from the first order conditions and envelope condition:

$$u'(c_t) = (1 + \phi(w_t, y_{t+1})) u'(c_{t+1}), \quad (20)$$

$$\phi(w_t, y_{t+1}) \geq 0 \text{ and } \phi_{t+1} \cdot V_1(V_2(w_{t+1})) = 0, \quad (21)$$

for every  $y_{t+1} = y^1, \dots, y^s$ .

The Euler condition (20) implies that consumption is monotonically increasing over time until consumption for the next period is the same for all states, in which case it equals consumption in the current period and a steady state is reached. Consumption is smoothed across states in period  $t+1$  for which the default constraint

is not binding and is higher for states in which the constraint is binding. Higher consumption in such states, at the expense of lower consumption in earlier, lower states, is necessary to match the utility from repudiation;  $\phi(w_t, y_{t+1})$  is not zero only if  $V_1(V_2^{t+1}, y_{t+1})$  is equal to zero. In general, consumption will not be fully smoothed across time beginning in date 1 in contrast with the case without sovereign risk (compare (15) with (20)).

Since income follows a Markov chain, the utility possibility set depends only on the current resources available,  $y_{t+1}$ , so that when  $V_1^t$  is zero the borrower's consumption defined by

$$u(\underline{c}^i) - u(y^i) + \beta E_t V_1(V_2^{t+1}, y_{t+1}) = 0 \quad (22)$$

depends only on  $y_t$ . Equations (9) and (10) imply that  $\phi(w_{t+1})$  and  $c_{t+1}$  depend on  $c_t$  and  $y_{t+1}$  everywhere along the equilibrium path.

We make the additional assumption that the distribution of income displays first-order stochastic dominance in previous period income, that is,  $\sum_{j=k}^S \pi_j(y_t) \geq \sum_{j=k}^S \pi_j(y_t^1)$ , whenever  $y_t > y_t^1$  for every  $1 < k < S$ . The mean-reverting autoregressive process, used above in the discussion of hedging with full commitment under serial correlation, satisfies this assumption. It can be proved (recursively over  $y$  using (22) starting with  $y^1$ ) that there is a series of consumption levels  $\underline{c}^i$  such that

$$y^1 = \underline{c}^1 < \underline{c}^2 < \dots < \underline{c}^S < y^S. \text{ }^{13}$$

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<sup>13</sup>If  $y$  is distributed i.i.d., then the  $\underline{c}^j$  are given by

$$u(\underline{c}^j) = (1 - \beta)u(y^j) + \beta \left[ \sum_{i=1}^{j-1} \pi_i u(y^i) + u(y^j) \sum_{i=j}^S \pi_i \right].$$

Under free entry, the profit for lenders in period 1 is zero, so that first period consumption solves

$$(y_1 - c_1) + \beta E_1 V_2(w_2) = 0.$$

The dynamics for borrower consumption can now be described. There exists a state,  $1 \leq n \leq S$ , which depends on all of the parameters of the problem, such that if  $y_1 \geq y^n$ , then consumption is immediately and fully smoothed for all dates and states from period 1 on. That is, if  $y_1 \geq y^n$  then there is a  $c_1$  that satisfies

$$c_1 = y_1 + E_1 \sum_{t=2}^{\infty} \beta^{t-1} (y_t - c_1)$$

and

$$c_1 \geq \underline{c}^S.$$

If  $y_1 < y^n$ , for  $n > 1$ , these two conditions cannot be satisfied. In that case there is a state  $j$ ,  $1 \leq j < S$ , such that

$$c_1 \geq \underline{c}^j \text{ and } c_1 < \underline{c}^{j+1}.$$

Consumption in period 2 equals  $c_1$  if  $y_2$  is less than or equal to  $y_1$ . If  $y_2 = y^i > y_1$ , then consumption rises to  $c_2 = \underline{c}^i$ . In general,  $c_t = \underline{c}^i$  where  $y^i = \max\{y_1, \dots, y_t, \text{ and } y^i\}$ .

Whenever  $c_1 < \underline{c}^S$ , the steady state consumption is equal to  $\underline{c}^S$ . Long run consumption only exceeds  $\underline{c}^S$  if the borrower's consumption is fully smoothed across all dates, that is, if the constrained first-best is an unconstrained Pareto optimum.

#### 4.2.2 Implementation with one-period state-contingent loan contracts

The constrained first-best can be achieved using one-period loan contracts incorporating state-contingent repayment schedules under free entry by competitive

risk-neutral financiers, equivalently "lenders" "investors" or "insurers", assuming seniority is enforced. The loan contract specifies a loan made in period  $t$  to the borrower,  $\ell(w_t)$  and a repayment schedule,  $R(w_t, y_{t+1})$  for  $t+1$ . The zero expected profit condition is

$$-\ell(w_t) + \beta E_t R(w_t, y_{t+1}) = 0$$

Using the solution to the first-best problem (6), simply set

$$\begin{aligned} R(w_{t-1}, y_t) &= V_2^t \\ R(w_t, y_{t+1}) &= V_2(w_t, y_{t+1}) \\ \text{and} \quad \ell(w_t) &= c_t + V_2^t - y_t. \end{aligned}$$

Given that the lenders' surplus in period 1 is zero and constraint (18), we have that

$$-\ell(w_t) + \beta E_t R(w_t, y_{t+1}) = 0, \quad \forall t \geq 1.$$

A consequence of the solution to the dynamic programming problem, (17)-(19), is that  $V_2(w_t, y_{t+1})$  is increasing in  $y_{t+1}$  for all  $w_t$ . This implies that  $\beta E_t V_2(w_t, y_{t+1})$  is non-decreasing in  $y_t$  when the distribution of income is a Markov chain displaying first-order stochastic dominance. The one-period state-contingent loan repayment,  $R(w_t)$ , is increasing in  $y_t$  for all  $y_t$  since it equals  $V_2(w_t)$ . For  $y_t \leq \max \{y^j, y_2, \dots, y_{t+1}\}$ , the repayment is increasing at least one for one with  $y_t$  because consumption in period  $t$  is constant for these states and  $\beta E_t V_2(w_t, y_{t+1})$  is non-decreasing in  $y_t$ . If income is i.i.d., then  $\beta E_t V_2(w_t, y_{t+1})$  is constant for different realizations of  $y_t$  no greater than the  $\max \{y^j, y_2, \dots, y_{t-1}\}$ , so that  $R$  is increasing one for one with income in this range. By our definition of  $\ell(w_t)$ , it also follows that  $\ell(w_t, y_{t+1}) \leq \ell(w_t)$  if  $y_{t+1} \leq y_t$ .

The long-term optimum subject to sovereign immunity for the borrower can be implemented using short-term contracts since these lenders can commit themselves

to any level of surplus ( $R(w_t, y_{t+1})$ ) for each state in the next period, replicating what commitment does in the long-term contract yielding the constrained first-best. This is an extension of the argument given by Worrall (1990) for the case of i.i.d. incomes.

Because lenders can commit themselves in period 1 to the long-term first-best contract,  $V_2^1$  can be negative in general for some states and periods after the first. This implies that  $R(w_t)$  can be negative for some  $w_t$ . In fact  $R(w_t)$  must be negative for some  $y_t$  if first period consumption is less than steady state consumption or if  $y_1 > y^1$  and consumption is smoothed for all dates. Suppose that repayments are non-negative for all states for each contract chosen in the steady state and that first period consumption is less than steady state consumption. The repayment schedule for a contract chosen in the steady state when state  $y_1 = y^k$ ,  $0 \leq k \leq S$ , could be offered by a free entrant in the first period and earn non-negative profits, a contradiction.  $R(w_t)$  must also be negative in at least the lowest state  $y^1$  if consumption is smoothed immediately and initial income  $y_1$  exceeds  $y^1$ . This is because the borrower receives a net transfer of  $c^* - y^1$  in the lowest state, but the steady state loan he receives in that state is no greater than the loan she receives in the first period  $c^* - y_1$ , which is less than the net transfer in the lowest state.

It is useful to separate the state-contingent one-period loan contract into two parts, a loan that is repaid with certainty in equilibrium and an insurance contract for the next period. The amount lent is the same,  $\ell(w_t)$ , and the repayment is just  $\frac{1}{\beta} \ell(w_t)$ . The insurance contract is actuarially fair and specifies a premium net of indemnity of

$$R(w_t, y_{t+1}) + \frac{1}{\beta} \ell(w_t)$$

If  $y$  is distributed i.i.d., then the loan and insurance contracts chosen in period  $t$  are the same for all states  $y_t \leq \max \{y_1, \dots, y_{t-1}, y^j\}$ . When the distribution of  $y$  is not



i.i.d. (but is Markov satisfying our assumptions), then the insurance premium depends on  $y_t$  even though planned consumption for each state  $y_{t+1}$  is the constraint for all  $y_t \leq \max \{y_1, \dots, y_{t-1}, y^j\}$  (since the probabilities  $\pi_t$  depend on  $y_t$ ). This implies that the loan taken out in period  $t$  varies with  $y_t$  even if  $y_t \leq \max \{y_1, \dots, y_{t-1}, y^j\}$ . The borrower's debt follows an autoregressive process, but it does satisfy the conventional solvency constraint, and the current debt is always paid with certainty.

#### 4.2.3 Smoothing with commodity bonds

We now discuss how a commodity bond constructed as a combination of a one-period simple loan and a put option for the exporter can achieve feasible smoothing for the LDC. This combination of contracts provides a pattern of consumption over time and states similar to that achieved by the one-period state-contingent loan contracts. The commodity bond generally fails to provide the same level of surplus for the exporter as attained in the constrained first-best. It has, however, the offsetting advantage of simplicity and the associated potential for more liquid trading as a relatively standardized instrument. Our discussion here extends and makes more precise the analysis of commodity bonds in Wright and Newbery (1989) by allowing for autocorrelation in the income process and characterizing the welfare sacrifice of using simple instruments.

The 1-period put option on the commodity has strike value  $y_t^*$ . The exporter exercises the put if  $y_t \leq y_t^*$ , receiving income  $y_t^*$ . The option premium,  $z_t^*$ , is given by the zero profit condition:

$$z_t^* = E_t(\max\{y_t^* - y, 0\}).$$

This must be paid in every state. We let it be paid at date  $t$ , the same date that the put is exercised or expires. Therefore, the exporter gets from this option the net income:

$$\max\{y_t, y_t^*\} - z_t^*.$$

Now, let the exporter also have access to a loan market with standard non-state-contingent bonds (one-period). The penalty for non-repayment is removal of all opportunities for smoothing in the future, whether by borrowing or via options. In period  $t$ , she chooses an option contract with strike value  $y_{t+1}^*$  for the next period and a loan  $\ell_t$  (positive or negative). The repayment due at  $t+1$  is  $\ell_t / \beta$ , so that her consumption at  $t+1$  is

$$c_{t+1} = [\max\{y_{t+1}, y_{t+1}^*\} - z_t^* - \ell_t / \beta] + \ell_{t+1}$$

To find the equilibrium path in c-bonds, define the state variable:

$$D_t = \max\{y_t^*, y_t\} - z_t^* - \ell_{t-1} / \beta.$$

At time  $t$ , the exporter chooses  $\ell_t$  and  $y_t^*$ .

The equilibrium problem is the same as the following dynamic programming problem:

$$V(D_t, y_t) = \max \{u(c_t) - u(y_t) + \beta E_t V(D_{t+1}, y_{t+1})\}$$

with respect to  $(\ell_t, y_t^*)$

subject to

$$D_{t+1} = \max \{y_{t+1}, y_{t+1}^*\} - z_{t+1}^* - \ell_t / \beta$$

and  $V(D_{t+1}, y_{t+1}) \geq 0, \quad \forall y_{t+1},$

where  $c_t = D_t + \ell_t$ .

Let  $\pi_k(y_t)\phi_k(w_t)$  be the multipliers for each of the constraints

$$V(D_{t+1}, y^k) \geq 0, \quad k=1, \dots, S.$$

$$\pi_k(y_t)\phi_k(w_t)V(D_{t+1}, y^k) = 0, \quad \phi_k(w_t) \geq 0.$$

The first order conditions with respect to  $\ell_t$  can be written as

$$u'(c_t) - \beta \sum_{k=1}^S \pi_k(1 + \phi_k)V'_k(D_{t+1}, y^k) \frac{1}{\beta} = 0,$$

and the first order condition with respect to  $y_{t+1}^*$  is

$$\beta \sum_{k=1}^S \left[ \pi_k(1 + \phi_k)V'(D_{t+1}, y^k) \cdot \frac{d}{dy_{t+1}^*} [\max\{y_{t+1}^*, y_{t+1}\} - z_{t+1}^*] \right] = 0$$

Because  $z_{t+1}^* = E_t[\max\{y_{t+1}^* - y^k, 0\}]$ , the first-order condition with respect to  $y_{t+1}^*$  becomes

$$\sum_{k \leq j} [\pi_k(1 + \phi_k)V'(D_{t+1}, y^k)] - \sum_{k=1}^S \pi_k(1 + \phi_k)V'(D_{t+1}, y^k) \sum_{k \leq j} \pi_k = 0$$

where  $y^{j-1} < y_{t+1}^* \leq y^j$  defines  $j \in \{1, \dots, S\}$

Thus,

$$E_t[u'(c_{t+1}) \cdot (1 + \phi_k(w_{t+1})) | y_{t+1} \leq y_t^*] = u'(c_t).$$

The pattern of smoothing is qualitatively similar to that when *unrestricted* state-contingent loans are available. *But the insurance contract implicit in that*

scheme has a premium net of indemnity payment that increases with income levels such that consumption is higher than in the previous period. Here, the insurance part of the contract is the put option, for which the borrower's payment to the importer is constant for income realizations above the strike value. Another way of stating this difference is that the net income available to the borrower in  $t$  is given by  $[y_t - R(w_t)]$  in the constrained first-best and by  $B_t$  in the commodity bonds scheme.  $R_t$  varies with  $y_t$  to keep the default constraint binding in states for which consumption goes up; in the commodity put case,  $B_t$  is constant for similar states. The two net payment schedules for the borrower are illustrated in Figure 4. Both coincide only when consumption is fully smoothed (as it will be in the steady state) or if there are only two states.

In the constrained first-best, when consumption cannot be completely smoothed from period 1 on, the no default constraint is binding whenever consumption rises. This is not true, in general, for the solution to the commodity bond problem; the multipliers may not all be zero for states above the strike state.

Because this problem is more complicated than that for the constrained first-best, we first note how the contract choices and consumption evolve when income is i.i.d..

insert figure 4

From the first order conditions,

$$u'(c_t) \geq u'(c_{t+1}) \text{ for } y_{t+1} \geq y_{t+1}^*$$

and  $\phi_k = 0$  for  $k < n$  where

$$n = \max\{k: y^k \leq y_{t+1}^*\}$$

For  $y^k < y^n$ , consumption  $c_{t+1}$  is monotone increasing in  $y_{t+1}^*$ .

We start by describing the dynamics for the put option and non-contingent bond scheme in the i.i.d. case. Initial income is  $y_1$ . The exporter takes a loan  $\ell_1$  and consumes  $c_1 = y_1 + \ell_1$  in the first period. She also contracts for a put option with strike income and premium.

$$z_2^* = E[\max\{y_2^* - y_2, 0\}].$$

Next period, if  $y_2 \leq y_2^*$ ,

$$c_2 = (y_2^* - z_2^* - \ell_1 / \beta) + \ell_2.$$

$y_2^*$  is chosen so that in equilibrium the exporter does not default in any states at  $t = 2$ , given  $\ell_1$ , and  $c_2 = c_1$  if  $y^2 \leq y_2^*$ . If  $y_2^* < y^s$ , then there is a state such that the exporter is indifferent between being able to continue smoothing her consumption and permanent autarky in period 2. Choosing a higher strike income than  $y_2^*$  to obtain more insurance for period 2 would lower first period consumption. There is a trade-off between smoothing across states of nature at date 2 and between dates 1 and 2.

If  $y_2 \leq y_2^*$ , the Euler equation implies that the exporter chooses the same put option and non-contingent loan repayment ( $\ell_1 / \beta$ ) for period 3 as for period 2. In the first period  $t$  in which  $y_t$  exceeds  $y_1^*$ , her choice of strike income rises to a level sufficient to smooth her consumption in the following period for all  $y_{t+1}^* \leq y_t$ . Her choice of loan also changes. The new put is exercised for all  $y_{t+1} \leq y_t$ , and the borrower chooses a new loan to make  $c_t$  as large as possible without causing her to choose to default in any state in the next period,  $t+1$ , in equilibrium.

Once the highest state  $y^s$  occurs, consumption is smoothed across all states and remains constant thereafter. In equilibrium for the commodity bond scheme there

is a monotonically rising consumption floor similar to that for the first-best (for the i.i.d. case).

Steady state consumption is given by

$$c^* = E y + (1 - 1/\beta) \ell^*$$

since

$$z^S = E \max\{y^S - y, 0\} = y^S - E y.$$

To avoid default in the steady state,  $c^*$  must be at least as great as  $\underline{c}^S$ . This implies that there is an upper bound on  $\ell^k$  for conventional solvency to hold.<sup>14</sup> The commodity bond scheme achieves the constrained first-best if initial income is high enough to allow complete smoothing at  $\underline{c}^S$ .<sup>15</sup> The two also coincide if  $y^S$  is the only state above the initial strike value for income. While the consumption level such that the no default constraint is binding in the steady state is  $\underline{c}^S$ , consumption levels such that the no default constraints are binding, in general, will exceed  $\underline{c}^j$ , for all  $j < S$ .

In the optimum for the borrower, the steady state loan,  $\ell^*$ , is equal to  $\ell_{t+1}$  where  $y^S$  occurs for the first time in period  $t$ . Recall that the non-contingent bond chosen in any period is the same for all  $y_t \leq y_t^*$  in the i.i.d. case. The amount borrowed in period  $t$  depends on the sequence of strike incomes that have been chosen in all previous periods. For example, if  $y_t \leq y_2^*$  for every  $j < t$ , steady state consumption will be different than it would be if for some  $j$ ,  $1 < j < t$ ,  $y_2^* < y_j^* < y^S$ . The commodity

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<sup>14</sup>For a given distribution of  $y$  and concave function  $u(c)$ , if the discount rate is large,  $\underline{c}^S$  will exceed  $E y$  so that  $\ell^*$  must be negative. The exporter must invest some of her current resources abroad and use the interest to assure that steady state consumption exceeds  $\underline{c}^S$ . (Domestic investment is ruled out by assumption, and foreign investment is subject to seizure by creditors if there is a default.)

<sup>15</sup>Full smoothing is not possible given sovereign risk if the initial state satisfies:  
 $\underline{c}^S > (1 - \beta)y_1 + \beta E y.$

bond scheme results in less efficient smoothing over time than occurs in the first-best. That is, the initial consumption will be lower and the steady state consumption higher, in general. This welfare cost is due to using a simpler financial contract that does not provide as much insurance across states exceeding the strike state.

An example with i.i.d. income and three states is used to illustrate the commodity bond dynamics. Suppose that initial income  $y_1 = y^1$ . We assume that full smoothing over all dates is infeasible and that the parameters of the problem are such that period 1 strike income is less than  $y^2$ . This will be the case if  $c_1 < \underline{c}^2$  in the constrained first-best (which is possible).

With i.i.d. income the consumption level once  $y^3$  first occurs depends upon whether or not  $y^2$  is realized earlier. If  $y^2$  occurs before  $y^3$  first occurs, then the equilibrium contracts satisfy:

$$c_1 = y^1 + \ell_1 = (y^{*1} - z^1) - \frac{\ell_1}{\beta} + \ell_1,$$

$$c_2 = (y^2 - z^1) - \ell_1 / \beta + \ell_2 = (y^{*2} - z^2) - \ell_2 / \beta + \ell_2,$$

and

$$c_3 = (y^{*3} - z^2) - \ell_2 / \beta + \ell_3 = (y^{*3} - z^3) - \ell_3 / \beta + \ell_3,$$

$$z^1 = \pi_1(y^{*1} - y^1)$$

where

$$z^2 = \pi_1(y^{*2} - y^1) + \pi_2(y^{*2} - y^2),$$

and

$$y^{*3} - z^3 = E y.$$

We also have that  $c_3 \geq \underline{c}^3$  and  $c_2 \geq \underline{c}^2$ .

If  $y^3$  first occurs before  $y^2$  does, then  $c_1$  is as above and

$$c'_3 = (y^3 - z^1) - \ell_1 / \beta + \ell'_3 = (y^{*3} - z^3) - \ell'_3 / \beta + \ell'_3.$$

These can all be solved to show that  $\ell_3 \geq \ell_3'$ , so that  $c_3' \geq c_3$ , so that with equality only if  $y^{*2} = y^2$ . We also have that  $c_3 \geq \underline{c}^3$  but that  $c_2 \geq \underline{c}^2$ . Therefore we also must have that  $c_1$  is lower than it would be in the first-best.

Figure 5 displays the four possible consumption levels and strike incomes  $y^{*1}$  and  $y^{*2}$ .

When the commodity's price is autocorrelated, then the pattern of consumption smoothing can be more complicated because the loan contract chosen with the put option will depend on the state even if the state realization is below the strike income. This happens when income is distributed according to a Markov chain (other than i.i.d.) because the probability of states above the strike income occurring in period t+1 depends on  $y_t$  so that the option premium for a given strike,  $y_{t+1}^*$ , depends on  $y_t$ :

$$z_{t+1} = \sum_{i=1}^S \pi_i(y_t) \max \{y_{t+1}^* - y^i, 0\}.$$

To keep consumption in period t+1 equal to consumption in period t,  $z_{t+1} - \ell_{t+1} / \beta$  needs to be constant across different realizations of the state,  $y_t$ . That is, compare two values for  $y_t$  below the strike income chosen in t-1 for period t and choose the same strike income,  $y_{t+1}^*$  for period t+1 for the two potential period t realizations. Now, for  $c_{t+1}$  to be the same as  $c_t$ ,  $\ell_{t+1}$  must be larger for the higher  $y_t$  (since we assume that the distribution displays first-order stochastic dominance in  $y_t$ ).

If the steady state is reached, the non-contingent loan will evolve according to

$$\ell_t = \frac{1}{\beta} \ell_{t+1} + (c^* - E_{t-1} y_t),$$

where  $c^*$  is steady state consumption.



Since steady state consumption must be at least  $c^S$ , the restriction that these loans always be repaid with certainty imposes a constraint on the amount of debt the borrower has outstanding when state S is reached. This implies (from the Euler condition) that the non-contingent loan and put option selected at date t in equilibrium depends on the entire history of income realizations,  $w_t$ . The strike income will rise monotonically from  $y_2^*$  to  $y^S$  as in the i.i.d. case, but may rise even if  $y_t < y_t^*$  in the Markov case. The consumption floor rises over time, as in the first-best case, but the consumption floor might rise even if the strike income  $y_t^*$  exceeds  $y_t$ . Furthermore if  $y_t < y_t^*$ , consumption can be rising with  $y_t$ . (But note that  $c_t$  is always at least as great as  $c_{t-1}$ . This behavior contrasts with the consumption dynamics for the first-best non-contingent loan and insurance contract scheme for the Markovian case (discussed in Section 4: 2. 2. above.).

In the constrained first-best, the exporter can insure herself as much as possible subject to the no default constraints. Given the initial realization for income, she can trade all of the risk in her permanent income with the foreign lenders for a consumption plan that respects those constraints. Even when the set of instruments is restricted to simple bonds and options, lender commitment still results in a monotonically rising consumption for the exporter, even when income is serially correlated. The cost of simplicity is reflected in the fact that the consumption path starts at a lower level if the initial income state is low. But these costs must in practice be weighted against the reduction in transactions cost associated with reduced contractual complexity and greater market liquidity.

## 5. Conclusions

Price risks may be significantly costly for exporters specialized in primary commodities. Even if there were no problem of contractual commitment, international loans of the conventional type would not provide optimal risk-sharing between risk-averse exporters and a capital market risk-neutral with respect to the prices of the exports in question.

Futures or forward contracts offer the insurance necessary to improve upon the risk-sharing achieved by loans with full commitment. Indeed futures contracts with maturities of one production period (roughly the maximum maturity observed in practice) can achieve substantial long-term protection, even if one recognizes the empirical fact that prices are positively serially correlated. The marginal net benefits of lengthening the horizon beyond one production period (roughly what is observed in practice) depend upon transactions cost, the degree of serial correlation, and the discount factor. The extra benefits of a substantially longer hedging horizon may often be rather small.

If production responds to incentives with a one-period lag, the rollover strategy does not provide perfect protection at the time the hedge is made. This is true even if production response to inputs is non-stochastic, in contrast to the case of one-period hedging.

In cases where a sovereign exporter can offer no collateral, and is short of liquid resources, the use of futures is precluded by the need to furnish the margins that guard against default. Standard loans and buffer funds have the disadvantage that they will with probability one reach crisis states in which the resolution of the crisis is ill-defined; recognition of this by potential lenders no doubt dampens their enthusiasm for such contracts somewhat.

In this context with full lender commitment commodity bonds with a put for the seller (borrower-exporter) offer at least part of the benefits of using fully state-

contingent contracts constrained only by sovereign immunity. In fact when initial conditions are sufficiently good the two are identical. A straight commodity bond suffices for fully smoothing consumption. When the initial state is bad, commodity bonds combining a put and a loan for the exporter can achieve some degree of consumption smoothing in the face of random export prices for commodity-dependent countries that cannot offer credible collateral for foreign loans. This is true even if prices are Markovian rather than i.i.d. Consumption is nondecreasing over time and becomes fully smoothed if and when the highest income state is visited.

Though a bond with a put option does not in general achieve a constrained efficient consumption path, it has the significant practical advantage of comprising two similar and simple instruments that might have more liquid markets, and hence reduced transaction costs, relative to a constrained efficient contract with country-specific state contingencies. Moreover, in equilibrium the bonds are always repaid in full; there is no prospect of costly loan renegotiations. This commodity bond package contrasts with commonly observed commodity bond contracts, which generally attach a call option for the buyer to the loan. The consumption-smoothing achieved reduces downside exposure of the seller, while leaving her a sufficiently large share of high realizations that she is not tempted to default.

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Table 1. Effects of Completely Stabilizing Export Revenue  
(percentages)

County/ Commodity	GDP Share	Export Share	Commodity Revenue	Export Revenue	One-Year Risk Cost (percent of income) if Consumer		
					(a) Specialized in Commodity	(b) Diversified Across Exports	(c) Diversified Across Economy
	(1)	(2)	CV (3)	CV (4)	(5)	(6)	(7)
<b>Copper Exporters</b>							
Chile	(14)	62	18	17	(3)		(.15)
Zambia	(34)	91	21	22	(4)		(.66)
<b>Coffee Exporters</b>							
Brazil	(4)	26	22	14	(4)		(.03)
Colombia	(7)	58	15	10	(2)		(.00)
	1984 figs from 1986 WDR						

Source: IMF (1991), World Bank Commodities Division for commodity price data

- Notes:
- (2) is the unweighted average of the annual shares of commodity exports in total export revenue
  - (3) is the SD of percentage deviations from centred 5-yr MA commodity export revenue
  - (4) as for (3) but for total export revenue



**Table 1 Commodity price instability, 1950-1986**

Coefficient of variation, percentages

Commodity	1950-69		1970-86	
	5-yr MA	price change	5-yr MA	price change
Cocoa	21	25	22	28
Coffee	12	16	24	35
Tea	7	11	19	23
Sugar	35	39	38	47
Cotton	6	13	13	19
Jute	20	22	12	18
Rubber	16	24	18	23

Source: Newbery (1990, Table 5.1)

Notes: Price change is the standard deviation of  $2(p_t - p_{t-1}) / (p_t + p_{t-1})$ .  
5-year MA is the CV of deviations from the 5-year moving average.

**Table 2 Squared coefficient of variation of prices, 1950-1986**

percentages				
Commodity	1950-69		1970-86	
	5-yr MA	price change	5-yr MA	price change
Cocoa	4.4	6.3	4.4	7.8
Coffee	1.4	2.6	5.8	12.3
Tea	.05	1.2	3.6	5.3
Sugar	12.3	15.2	14.4	22.1
Cotton	0.4	1.7	1.7	3.6
Jute	4.0	4.8	1.4	3.2
Rubber	2.6	5.8	3.2	5.3

Source: Table 1

**Table 3 Effects of stabilizing copper and coffee prices, 1961-1986**

percentages

Country	Coefficients of variation				
	Average export share (1)	Revenue unstabilized (2)	Revenue stabilized (3)	Exports unstabilized (4)	Exports stabilized (5)
<b>Copper</b>					
Chile	50	18	17	12	8
Zaire	39	16	16	31	24
Zambia	88	18	16	18	12
Papua New Guinea	20	33	13	14	13
<b>Coffee</b>					
Brazil	22	22	22	17	16
Colombia	43	19	15	10	10

Source: World Bank data

- Notes: (1) is average share of exports in total export revenue  
 (2) is the CV of deviations from 5-yr MA export revenue  
 (3) is the CV of deviations from 5-yr export revenue valuing the exports at prices stabilized at their 5-yr MA level  
 (4) is the CV of deviations from 5-yr MA total export revenue  
 (5) is the CV of deviations from 5-yr MA total export revenue valuing the exports at prices stabilized at their 5-yr MA level

All export revenues were deflated by the Index of Manufacturing Unit Value (MUV).

**Table 4 Persistence of price shocks, 1900-1987**

Commodity	Persistence Autocorr measure	Deaton PER20	Longest Lag years
Rice	0.11	0.18	9
Palm oil	0.13	0.13	5
Coffee	0.38	0.17	11
Bananas	0.45	0.59	10
Wheat	0.46	0.24	10
Sugar	0.52	0.11	6
Cocoa	0.65	0.29	2
Tea	0.72	0.37	2
Beef	1.0		0
Maize	1.1	0.19	10
Lamb	1.30		4
Average	0.61		6.28
Timber	0.1		8
Wool	0.35		2
Jute	0.4	0.19	5
Hides	0.43		2
Cotton	0.67	0.39	3
Tobacco	0.73		4
Rubber	1.0		0
Average	0.51		3.43
Oil	0.51		11
Silver	0.65		8
Tin	0.65	0.43	5
Lead	0.73		3
Aluminium	0.93		5
Zinc	1.0		0
Copper	1.0	0.31	0
Coal	1.0		0
Average	0.81		4.0

Source: Cuddington and Urzúa (1987), Deaton and Laroque (1989, Table 2)

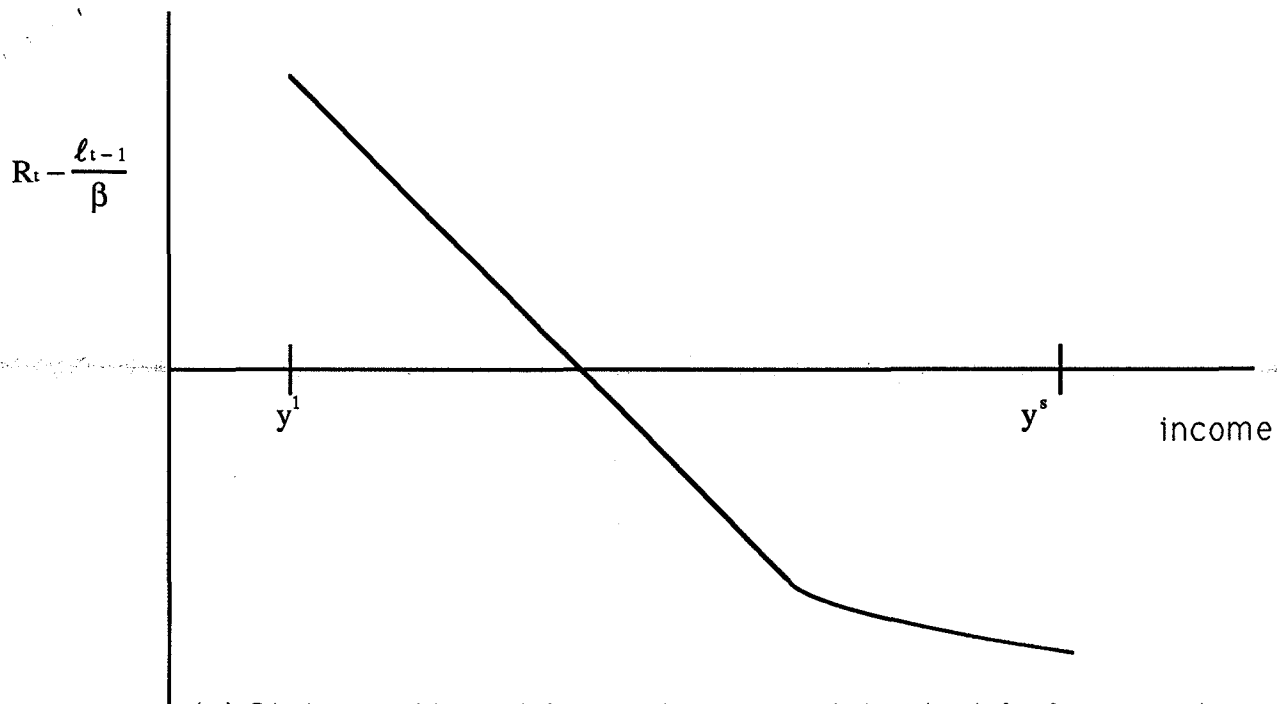
Notes: Annual data. The first measure is the sum of the statistically significant autocorrelation coefficients, as calculated by Cuddington and Urzúa and explained in the text. Deaton and Laroque's measure of persistence is PER20, given in from Table 5 below, and explained therein. The longest lag is the highest order statistically significant lag.

**Table 5 Variability and persistence of annual commodity prices, 1900-1987**

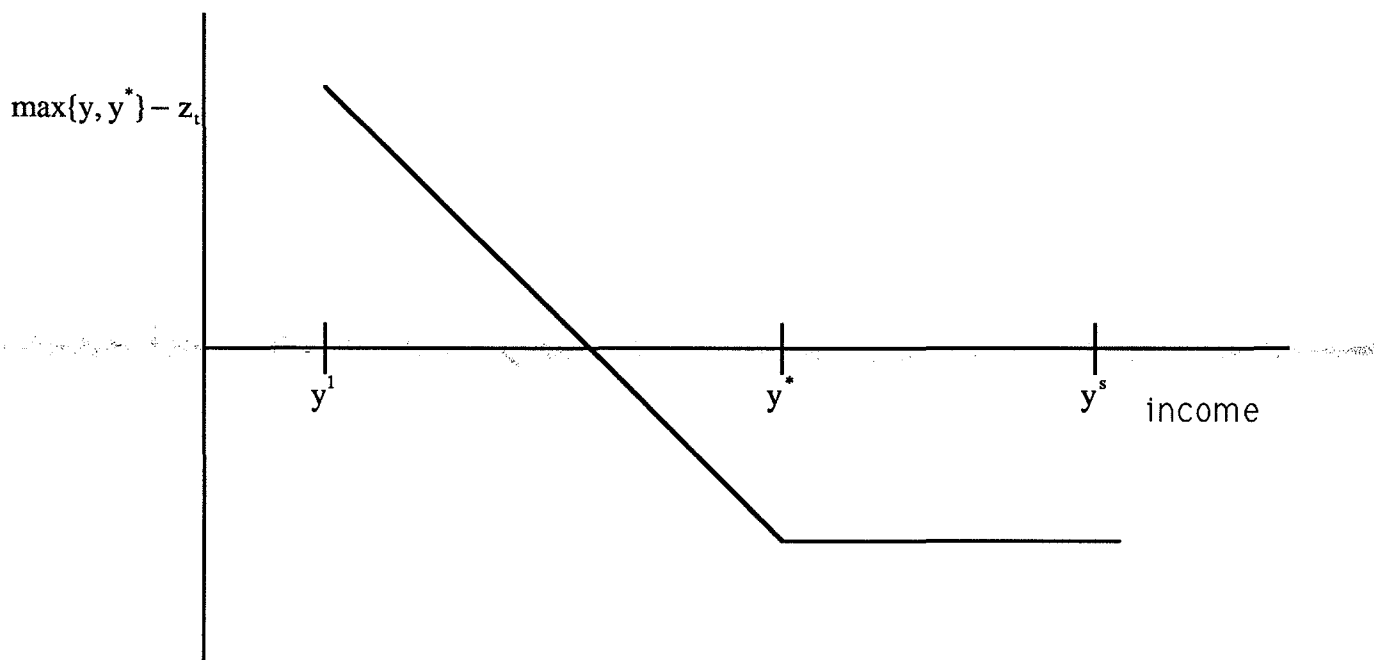
Commodity	CV	AR1	AR2	PER20	PER40
Bananas	0.17	0.91	0.82	0.59	0.52
Cocoa	0.54	0.83	0.66	0.29	0.24
Coffee	0.45	0.80	0.62	0.17	0.11
Copper	0.38	0.84	0.64	0.31	0.22
Cotton	0.35	0.88	0.68	0.39	0.13
Jute	0.33	0.71	0.45	0.19	0.09
Maize	0.38	0.76	0.53	0.19	0.10
Palm oil	0.48	0.73	0.48	0.13	0.05
Rice	0.36	0.83	0.61	0.18	0.08
Sugar	0.60	0.62	0.39	0.11	0.06
Tea	0.26	0.78	0.59	0.37	0.28
Tin	0.42	0.90	0.76	0.43	0.18
Wheat	0.38	0.86	0.68	0.24	0.11

Source: Deaton and Laroque (1989, Table 2).

Notes: CV is the coefficient of variation. AR1 and AR2 are the first and second order autocorrelation coefficients of the deflated series of prices. PER20 and PER40 are the Campbell/Mankiw-Cochrane measures of persistence with window widths of 20 and 40 years.



(a) State-contingent loan scheme receipt schedule for exporter



(b) Put option receipt schedule for exporter

Figure 4: Full Insurance and the put option compared

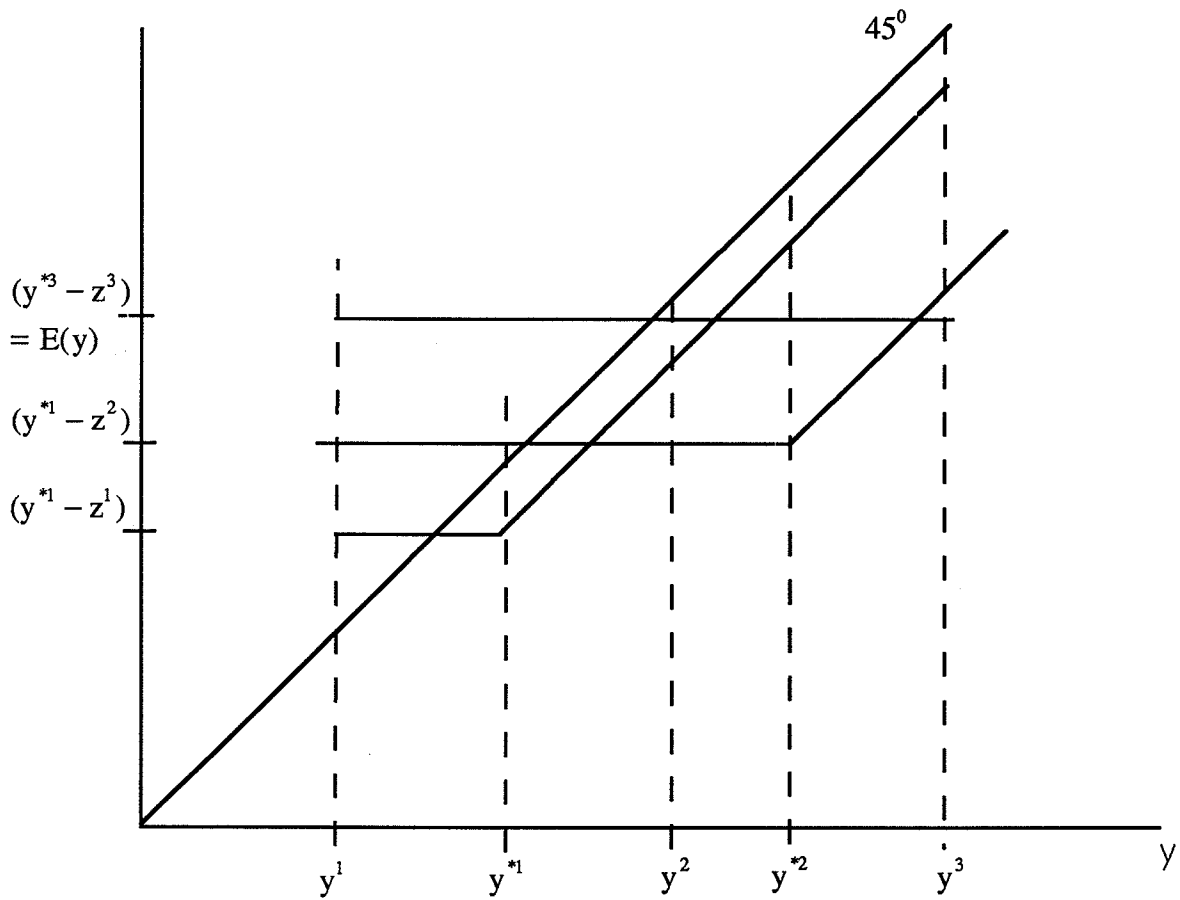
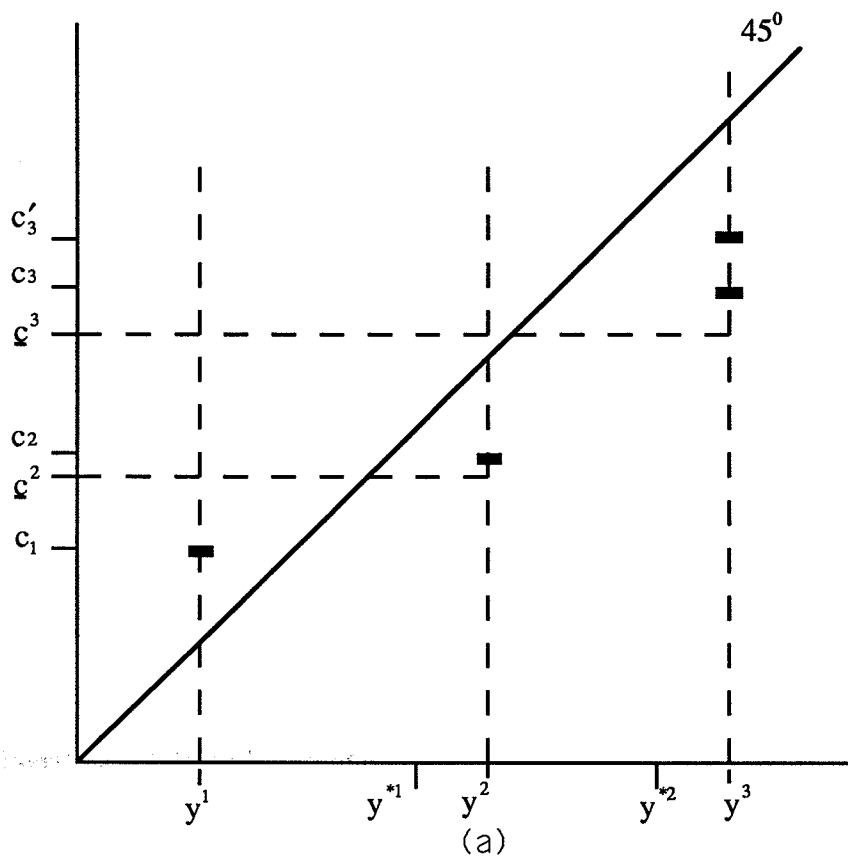


Figure 5: Examples of consumption-smoothing using a bond and a put option : i. i. d. case