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Persistent Differences in National Productivity Growth Rates with a Common Technology and Free Capital Mobility. The Roles of Private Thrift, Public Debt, Capital Taxation and Policy toward Human Formation

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CENTER DISCUSSION PAPER NO. 627

PERSISTENT DIFFERENCES IN NATIONAL PRODUCTIVITY GROWTH RATES  
WITH A COMMON TECHNOLOGY AND FREE CAPITAL MOBILITY.  
THE ROLES OF PRIVATE THRIFT, PUBLIC DEBT, CAPITAL TAXATION  
AND POLICY TOWARDS HUMAN CAPITAL FORMATION.

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January 1991

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PERSISTENT DIFFERENCES IN NATIONAL PRODUCTIVITY GROWTH RATES

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AND POLICY TOWARD HUMAN CAPITAL FORMATION.

Abstract

The paper develops a two-country endogenous growth model to investigate possible causes for the existence and persistence of productivity growth differentials between nations despite a common technology, constant returns to scale and perfect international capital mobility. Private consumption is derived from a three-period overlapping generations specification.

The source of productivity (growth) differentials in our model is the existence of a non-traded capital good ('human capital') whose augmentation requires a non-traded current input (time spent by the young in education rather than leisure).

We consider the influence on productivity growth differentials of private thrift, public debt, the taxation of capital and savings and of policy towards human capital formation.

KEY WORDS: Endogenous Growth, Convergence, Capital Mobility, Non-Traded Goods

## (1) INTRODUCTION.

This paper develops a simple two-country growth model to investigate possible causes for the existence and persistence of differences in average labor productivity<sup>1</sup> between countries and regions despite a common global technology, non-increasing returns to scale and unrestricted international mobility of financial capital. The model can either be viewed as an endogenous growth version of the two-country overlapping generations (OLG) neoclassical growth model of Buiter [1981, 1989], Frenkel and Razin [1987], and Buiter and Kletzer [1990a,b]<sup>2</sup>, or as a two-country, OLG<sup>3</sup>, constant returns to scale version of one of the models belonging to the class discussed in Lucas [1988].

Our choice of an OLG consumption structure is motivated by our desire to include deficit financing and intergenerational redistribution among the fiscal policies we analyze. Our OLG consumption structure implies that there is no first-order debt neutrality. Government borrowing with debt service financed through taxes on the younger generations will therefore tend to crowd out domestic saving. In our model it will also be shown to influence domestic human capital formation, although generally in the opposite direction from its effect on saving. The OLG structure also permits one to consider the implications of constant but internationally different time preference rates, without this inevitably implying asymptotic ownership of all global financial wealth by the residents of a single country. Alogoskoufis and van der Ploeg [1991], independently developed a two-country endogenous growth OLG model with perfect international capital mobility.

Even with a balanced budget the government can influence domestic physical capital formation directly by varying the marginal tax rates on the

capital income of firms. To the extent that this influences the interest rate, such policies will also indirectly influence household saving and human capital formation. The government can also influence the domestic accumulation of human capital directly (and with a balanced budget) by varying the marginal tax rate on non-human asset income earned by households<sup>4</sup>.

Many current models investigate differences in long-run economic performance between national economies. In much of the endogenous growth literature the same technology (generally summarized in a production function) is assumed to be available to each national economy. With the exception of Alogoskoufis and van der Ploeg, who assume perfect international financial capital mobility, that part of the endogenous growth literature that explicitly models a multi-country world assumes complete financial autarky: no international lending and borrowing (or direct international investment) occur. In addition, labor (and population in its other aspects, that of consumers and portfolio holders) is also assumed to be internationally immobile<sup>56</sup>. We maintain the assumption of immobile labor in our model. With financial autarky, differences in national saving rates will be translated into long-run differences in national productivity *levels* for exogenous growth models and into differences in national productivity *levels and growth rates* for endogenous growth models.

The recent literature on international trade and growth considers cases in which there may or may not be direct technology transfer (instantaneous or gradual) and trade in intermediate inputs<sup>7</sup>.

The technological assumptions made in this paper (constant returns to reproducible factors) permit "endogenous growth". The key points of the paper would, however, hold true even if the global technology only permitted exogenous (long-run) growth as is the case in much of the growth theory of the

fifties and sixties (see e.g. Solow [1956, 1970]). In that case (long-run) equalization of productivity growth rates would be assured automatically, and the "bite" of the assumptions of a shared global technology and perfect financial capital mobility would be in the prediction of convergence of *levels* of output per worker.

Under the assumptions of free international technology transfer, free international financial capital mobility and no non-traded productive inputs, many of the existing exogenous and endogenous growth models would imply global convergence of output per worker <sup>8</sup>. Differences in national savings rates would not account for differences in national rates of accumulation of augmentable factors. In the simplest version of the model (absent adjustment costs) convergence would be immediate: any country could import Japanese-style levels and growth rates of output per person-hour overnight. The class of models to which this global convergence result applies includes all those for which the common aggregate national (or regional<sup>9</sup>) production function exhibits constant or decreasing returns<sup>10</sup> to internationally mobile factors of production <sup>1112</sup>.

Our model is a complete, two-country dynamic general equilibrium model which determines (among other things) both the levels and the growth rates of output and output per worker in each country. For reasons of space and because (with the help of one further assumption) it permits a major reduction in the amount of tedious algebra inflicted on the reader, we restrict ourselves to the analysis of the determinants of *productivity growth differentials* between the two countries. This is an important subject in its own right.

**Can Japanese-level productivity be imported "off-the-shelf"?**

The prediction that, with a common global technology and free

international mobility of financial capital, levels and growth rates of output per worker should be equalized across the globe is a source of empirical embarrassment. This remains true even if its sharp edges are dulled somewhat by allowing for real world restrictions on the international mobility of financial capital and for adjustment costs in the accumulation of augmentable factors of production.

It makes no sense to have as one's maintained hypothesis the notion that Bolivia or Togo can order to go, off-the-shelf and overnight (or subject to a few years delay to allow for the presence of the familiar strictly convex adjustment costs of the neoclassical theory of investment), the highest productivity levels of the last decade of the twentieth century, simply by opening themselves completely to international financial capital mobility and international transfer of technology. There are important "local" or national essential complementary inputs into the production process that cannot be imported but have to be "home-grown". We are referring to the social, political, cultural, legal and educational infrastructure without which modes of production and economic organization conducive to high productivity cannot be realized.

In our formal model, we try to capture some of the essence of these "home-grown" inputs by including in the production function a non-traded capital good ("human capital") whose production requires a non-traded current input (efficiency units of labor time) that has an alternative use in consumption as intrinsically valued leisure. The model can easily be extended to include a second alternative use for labor time, by allowing current labor time to be used in the production of goods other than human capital. This second good, which can be consumed privately or publicly or used for physical capital accumulation, would be the traded output of our current model.

The notion that the current endowment of labor time can be used either for accumulating human capital or for the production of marketable output is a standard feature of many endogenous growth models (see e.g. Lucas [1988] and Romer [1990a]). For expository simplicity only, we exclude "raw" labor (in our formal model the time endowment of the young generation) from use as an input in the current production of the traded good. Only physical (traded) capital and human (non-traded) capital (in our formal model the endowment of time (in efficiency units) of the middle-aged) are arguments in the production function of traded output<sup>13</sup>.

We realize that our non-traded human capital good whose production requires the input of a non-traded, non-produced input that has alternative uses as a consumption good (or as both a consumption good and a productive input into the production of traded goods), captures but very partially our notion of "home-grown" infrastructure. Some elements of the home-grown infrastructure (the rule of law, the clear definition and defense of property rights, the enforcement of contracts and general popular attitudes towards entrepreneurship, business and private profit) are variables with a bounded range of variation rather than resembling capital-like inputs whose quantity can be varied (given time and effort) without upper bound.

Other "home-grown" inputs such as a skilled and educated labor force fit more easily into our formal straight jacket. It is true that countries can send their citizens abroad to advance their education and that the processes of education and training within a country can make use of imported inputs. This, however is, and has been historically, of second-order importance. In our formal model human capital cannot be traded at all, but in one version of the model we permit the production of human capital goods to make use of a



traded input in addition to non-traded domestic labor time. There is, however, imperfect substitutability between the home-grown and the tradable inputs to human capital accumulation. Our formal model should, however, be viewed only as a first stab at characterizing "difficult" and uneven economic growth and development. We believe that, when attempting to map from our formal model to the real world, the non-tradedness of human capital and labor services should be interpreted as due to more basic factors (technological, economic, social, political and cultural) than just restrictive trade policies. Models that imply that, but for perverse government policies, Japanese levels of productivity could be achieved everywhere within a relatively short period of time cannot, in our view, be taken seriously.

#### Increasing returns.

It is extremely simple, analytically, to account for persistent (and even widening) differentials in national or regional productivity levels and/or growth rates by working with models that exhibit increasing returns to mobile factors of production at the level of the national or regional economy. Both the command optimum and the competitive equilibrium (if one exists<sup>14</sup>) of many models with increasing returns to scale will, with international or interregional mobility of non-labor factors of production and immobile labor, lead to widening differentials between national levels or growth rates of labor productivity. Accumulation and production will often tend to become concentrated in the nation or region that started off, through historical accident, with the largest scale of production.

Even in models without persistent endogenous or exogenous growth, increasing returns to scale and factor mobility may lead to regional (including national) divergence in production patterns (see e.g. the

"economies of agglomeration" studied by Arthur [1983, 1988 and 1989] and Krugman [1990]).

In the current paper we assume constant returns to scale and competitive output markets, although we intend to investigate in future work the consequences of (bounded and unbounded) increasing returns and of non-competitive behavior by firms.

How important are increasing returns to scale? Views on the empirical significance of increasing returns (and other sources of non-convexities in the production possibility set) seem to move in irregular cycles (see e.g. Smith [1775, especially the famous Volume 1, Chapter 3], Young [1928], Arrow [1962], Kaldor [1966, 1975, 1981] for a range of early formal or informal arguments in support of the proposition that increasing returns matter for the explanation of aggregate economic growth).

The issue can of course only be settled empirically. There is no evidence we know of that convincingly supports the existence of unbounded increasing returns to scale <sup>1516</sup>.

Testing empirically for the presence or absence of increasing returns to scale is complicated by the absence of any clear consensus on what the relevant scale variable is, that is on where the increasing returns are located. Some theories seem to locate increasing returns at the level of the individual enterprise, establishment or plant. Others suggest industry output, often without specifying whether it is the regional, national or global industry that matters. Current output and cumulative output have both been suggested as scale variables. In Lucas [1988], the aggregate (national?) stock of human capital generates the key externality. While externalities and increasing returns are in principle quite distinct, in Lucas [1988] the human capital externalities do create increasing returns, in the sense that

activities which absent the externality would have constant returns to scale, experience increasing returns with the externality. A very different picture emerges if it is not the total but the average level of human capital that generates the externality. The stock or flow of R&D (or of "knowledge") is often assumed to generate externalities and increasing returns to scale in activities that, absent the externality would have constant returns to scale. Whether this occurs at the level of the industry (national, regional or global) or of the global economy as a whole is left open by the theory.

A recent statement of the view that nonconvexities matter greatly both at the level of the individual firm and at the aggregate level is in Paul Romer [1990b]. Romer's views on these issues carry weight because of his central role in launching the "endogenous growth" research program (see especially Romer [1986, 1987]; for further developments see Lucas [1988], Romer [1989a,b; 1990a], Murphy, Shleifer and Vishny [1989]). It is therefore important to point out that the arguments in Romer [1990b] leading to his *linking of non-rivalness and increasing returns* are based partly on an a-priori assertion or postulate, partly on a non-sequitur and partly on interesting and suggestive illustrations, but not on systematic empirical investigation.

The a-priori assertion, which we shall refer to as the "replication postulate"; comes in the form of the following bold statement:

**"The most basic premise in our scientific reasoning about the physical world is that it (is) possible to replicate any sequence of events by replicating the relevant initial conditions. (This is both a statement of faith and a definition of relevant initial conditions)." (Romer [1990b, p. 3]).**

Internally consistent Postulates are neither right nor wrong. If they are to be useful to an empirical science, the primitive terms they contain must be interpretable unambiguously in terms of real world counterparts.

(This must be where faith comes in). The replication postulate, however, is not supplemented with any hints as to what "relevant initial conditions" might mean in any empirical scientific application. Until a rule is provided for determining and verifying "relevant initial conditions", the replication postulate is without empirical content, an empty box.<sup>17</sup>

Romer goes on to fill the empty box with the following assertion:

"For production theory, this means that it is possible to double the output of any production process by doubling all of the rival inputs" (Romer [1990b, p. 3]).

This, of course, is a non-sequitur unless we accept the *empirical statement* that the relevant initial conditions are limited to the rival inputs<sup>18</sup>. No arguments are given to support this claim.

If instead one were to make the (no less plausible) assertion that the relevant initial conditions consist of the rival inputs and the nonrival inputs, there would be constant returns to rival and non-rival inputs combined.

One could go further and quite plausibly take the rival inputs (or both the rival and nonrival inputs combined) together with the moment of their application to constitute the relevant initial conditions. In that case there cannot be constant returns to rival inputs (or to both rival and nonrival inputs combined) if the replication of the rival inputs (or of both the rival and nonrival inputs) takes place at a different date from their initial or benchmark application: we pass this way but once.

By far the simplest way of underlining the vacuousness of the replication postulate, however, is by permitting *scale* (of production or of application of inputs) to belong to the set of relevant initial conditions. Assume for

concreteness that the list of rival and non-rival inputs completes the set of relevant initial conditions. Consider two carefully controlled experimental settings, A and B. Replication in experimental setting B of the productive process in experimental setting A involves applying the same quantities of all (rival and non-rival) inputs in the two experimental Settings. Application of the replication postulate implies that the same levels of output will be produced in both A and B.

Doubling the quantity of all inputs applied in A (doubling the scale of production) and doubling the quantity of all inputs applied in B again results (given the replication postulate and our list of relevant initial conditions) in equal levels of output in the two experimental settings. Nothing is implied, however, about increasing, constant or decreasing returns to scale "within" the two settings. The output levels in A and B following the doubling of the input levels of all rival and non-rival inputs may have doubled, quadrupled, fallen by half or remained constant.

Like Romer's application of the replication postulate to production theory, our last four paragraphs do not contain any serious reference to the only kind of evidence that could resolve the returns to scale issue: empirical evidence from the physical and bio-medical sciences, from production engineering, from management science and even from the social sciences on indivisibilities and other possible sources of fixed costs.

Non-rival inputs clearly exist (Starrett's example of information as a productive input is an obvious one (Starrett [1988, p. 74])) and may well be important empirically, although no systematic evidence has as yet been collected. Their existence and significance is a totally separate issue from the existence and significance of increasing returns to scale.

We conclude this introduction with a brief outline of the rest of the

paper. Section 2 develops the model. Section 3 analyzes a very simple special case in which human capital accumulation requires no traded inputs. Traded inputs into human capital accumulation are added in Section 4, which considers the effects of changes in distortionary tax rates and exhaustive public spending. Section 5 studies the effects of deficit financing and lump-sum intergenerational redistribution on growth rate differentials. Section 6 concludes.

## (2) THE MODEL.

### a. Household behavior.

The decisions concerning consumption, labor supply, human capital formation and financial portfolio allocation are taken by households—consumers. The household sector in each country is modeled through a three period overlapping generations model. We only derive the household decision rules for the home country. The corresponding decision rules for the foreign households are obtained attaching the superscript <sup>\*</sup> to foreign taste parameters and household choice variables. In the first period of its life ('youth'), the  $j^{\text{th}}$  consumer born in period  $t$  has an endowment of time,  $j^0_h$  when measured in efficiency units, which she can either choose to consume as leisure  $j^l_t$  in period  $t$  or to allocate to an alternative use, which we shall call education  $j^e_t$ . This education process during the first period of the household's life adds to the endowment of labor time in efficiency units  $j^1_h$  during the second period ('middle age'), that is during period  $t+1$  for a household born in period  $t$ .

While young the household can also choose to spend private resources other than time on human capital formation. Such private spending on education  $j^m_t$  will have to be financed by borrowing, since the household is born without financial endowment and does not earn any income in the first period of its life. Public spending on the education of household  $j$ ,  $j^g_t$  also boosts  $j^1_h$ . For simplicity the young are assumed not to pay any taxes or to receive any transfer payments other than the benefits from the "transfer in kind"  $j^g_t$ , which cannot be resold by the recipient.

There is a key externality in the process of human capital formation. Formally we model this by assuming that  $j^0_h$ , the amount of time measured in efficiency units (henceforth human capital) that the  $j^{\text{th}}$  household of

generation  $t$  starts off with is given by the average amount of human capital achieved by the previous generation during middle age, that is, letting  $N_t$  denote the number of households-consumers in period  $t$ ,

$$j_t^h{}^0 = \frac{1}{N_{t-1}} \sum_{i=1}^{N_{t-1}} i_t^h{}^1$$

Each member of a new generations stands, as regards its starting level of human capital (knowledge, education), on the shoulders of the average member of the previous generation. Note that  $j_t^h{}^1$  is *non-rival* with respect to the levels of human capital achieved in period  $t$  by members of generation  $t$ ,  $i_t^h{}^0$ ,  $i = 1, \dots, N_t$ . If generation  $t$  is larger (say because of a higher rate of population growth), more members of generation  $t$  will benefit from the higher (average) level of education achieved by the previous generation. We also assume that the effect of  $j_t^h{}^1$  on  $i_t^h{}^0$  is *non-excludable*. Those in generation  $t$  who benefit from the knowledge accumulated by generation  $t-1$  cannot be made to pay for these benefits. Such complete *non-excludability* will almost surely result in Pareto-inefficiency of the unaided competitive equilibrium. This human capital accumulation mechanism is an obvious extension of the one developed by Lucas [1988], following Uzawa [1965]. Apart from our use of an OLG structure instead of Lucas' representative infinite-lived agent, our human capital accumulation technology differs by permitting the use of purchased produced inputs in addition to time.

Note that if the OLG structure without voluntary private intergenerational transfers were to be replaced by one which permitted an operative intergenerational gift and bequest motive, part of the intergenerational externality would be internalized. If private intergenerational transfers were motivated by concern about the welfare of ancestors or descendants (rather than by the joy of giving), each generation



would allow for and value appropriately the positive effect of its own education expenditures ( $j^e_t$  and  $j^m_t$  for household  $j$  of generation  $t$ ) on the human capital of its children, grandchildren and subsequent descendants. The beneficial effects of  $j^h_{t-1}$  are however, in our specification, not limited to the lineal descendants of household  $j$  but are shared by all born in period  $t$  and later. Unless, through vigorous intermarriage a la Bernheim and Bagwell [1988], all of society effectively constitutes one big happy family, the human capital formation externality, whose domain is both intergenerational and across families or dynasties, will not be fully internalized even if one assumed universal operative intergenerational gift motives. During the second period (middle age) the only household choice concerns how much to consume,  $j^c_t$ . The entire endowment of labor time services in efficiency units  $j^h_t$  is supplied inelastically in the labor market.

In the last period of life ('old age' or 'retirement') households do not work or educate themselves. The old consume  $j^c_t$ , which equals the value of the resources they carried into old age through saving in the first two period of their lives, minus any taxes  $j^{\tau}_t$  paid in their last period.

Formally, each household  $j$ , ( $j = 1, \dots, N_t$ ) of generation  $t$ , ( $t \geq 0$ ) maximizes the following:

$$(1) \quad \text{Max}_{\{j^{\ell}_t, j^h_t, j^e_t, j^m_t, j^c_t, j^c_t\}} j^U_t = \beta^2 u(j^c_t) + \beta u(j^c_t) + v(j^{\ell}_t)$$

$u(\cdot)$  and  $v(\cdot)$  are increasing, strictly quasi-concave, twice continuously differentiable and satisfy the Inada conditions

$$\lim_{x \rightarrow \infty} u(x) = \lim_{x \rightarrow \infty} v(x) = 1 / \lim_{x \rightarrow 0} u(x) = 1 / \lim_{x \rightarrow 0} v(x) = 0; \quad \beta > 0.$$

subject to

$$(2) \quad -(1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1})_j m_t + j^h_1 w_{t+1} - j^c_1 - j^r_1 \\ - [j^c_2 + j^r_2](1 + r_{t+2} - \theta_{t+2})^{-1} \geq 0 \quad 19$$

$$(3) \quad j^l_t = j^h_0 - j^e_t$$

$$(4) \quad j^h_0 = \frac{1}{N_{t-1}} \sum_{i=1}^{N_{t-1}} i^h_1$$

$$(5) \quad j^h_1 = j^h_0 \left[ 1 + \psi \left( \frac{j^e_t}{j^h_0}, \frac{j^m_t}{j^h_0}, \frac{j^g_t}{j^h_0} \right) \right]$$

At the initial date,  $t = 0$ , we have  $j^h_0 > 0$ .

In equation (1),  $\beta$  is the constant one-period subjective discount factor. It need not be less than unity.

Equation 2 is the lifetime budget constraint of member  $j$  of generation  $t$ .  $w_{t+1}$  the wage paid per unit of efficiency labor in period  $t+1$ . The before tax interest factor on loans from period  $t$  to period  $t+1$  is  $1 + r_{t+1}$ .  $\theta_t$  is the period  $t$  residence-based tax rate on all non-human asset income in the home country. It is therefore also the subsidy rate to all domestic borrowing, including borrowing by the young. We also wish to consider the subsidization of "student loans" (expenditures on human capital formation) as a policy instrument.  $\varphi$  is the subsidy rate on these loans. Households also pay lump-sum taxes when middle aged ( $j^r_1$ ) and when old ( $j^r_2$ ). These can of course be negative. We assume that the authorities do not tax any generation "into the ground", that is  $j^r_1$  and  $j^r_2$  are such that (2) can be satisfied for positive values of  $j^c_1$ ,  $j^c_2$  and  $j^m_t$ .

$\psi(\dots)$ , the (proportional) return function for the household's human capital formation process, is increasing in its three arguments, strictly concave, twice continuously differentiable and satisfies

$$\psi(0, \dots) = 0$$

This means that current education  $j e_t$  is essential for a positive rate of growth of human capital. Population grows at a constant proportional rate:

$$(6) \quad N_t = (1 + n)N_{t-1} \quad n > -1; N_0 > 0.$$

We also have:

$$j h_t^0, j h_t^1, j c_t^1, j c_t^2, j m_t \geq 0$$

$$0 \leq j \ell_t, j e_t \leq j h_t^0$$

Since  $j g_t \geq 0$ , our specification of the human capital investment function implies that per capita human capital never declines. It would be easy to allow for human capital depreciation, "forgetting", and other snags in the intergenerational transmission of knowledge, without this affecting the key qualitative properties of our model.

Household  $j$  of generation 2 behaves competitively (takes  $w_{t+1}$ ,  $r_{t+1}$  and  $r_{t+2}$  as given) and also takes  $j h_t^0$ ,  $j \tau_t^1$ ,  $j \tau_t^2$ ,  $\theta_t$ ,  $\varphi_t$  and  $j g_t$  as given. We assume that each household from a given generation within a country are identical. We can therefore drop the  $j$  subscript and write the solution to the household optimization problem as follows:

$$(7) \quad u'(c_t^1) = (1 + r_{t+2} - \theta_{t+2})\beta u'(c_t^2)$$

$$(8) \quad v'(\ell_t) = \beta u'(c_t^1) w_{t+1} \psi_1 \left( \frac{e_t}{h_t^0}, \frac{m_t}{h_t^0}, \frac{g_t}{h_t^0} \right)$$

$$(9) \quad 1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1} = w_{t+1} \psi_2 \left( \frac{e_t}{h_t^0}, \frac{m_t}{h_t^0}, \frac{g_t}{h_t^0} \right)$$

$$(10) \quad -(1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1})m_t + [h_t^1 w_{t+1} - c_t^1 - \tau_t^1] \\ - [c_t^2 + \tau_t^2](1 + r_{t+2} - \theta_{t+2})^{-1} \geq 0$$

$$(11) \quad \ell_t = h_t^0 - e_t$$

$$(12) \quad h_t^1 = h_t^0 \left[ 1 + \psi \left( \frac{e_t}{h_t^0}, \frac{m_t}{h_t^0}, \frac{g_t}{h_t^0} \right) \right]$$

$$(13) \quad h_t^0 = h_{t-1}^1 \quad ; h_0 > 0$$

To gain the advantage of closed-form solutions we shall consider the special case where the single-period utility is of the constant elasticity of marginal utility form, that is

$$(14a) \quad u(x) = v(x) = \frac{1}{1-\gamma} x^{1-\gamma}; \quad \gamma \geq 0.$$

In addition we'll assume that the human capital growth rate function  $\psi$  takes the constant returns Cobb-Douglas form given in (14b)

$$(14b) \quad \psi\left(\frac{e_t}{h_t^0}, \frac{m_t}{h_t^0}, \frac{g_t}{h_t^0}\right) = \eta \frac{e_t^a (m_t + g_t)^{1-a}}{h_t^0} \quad 0 \leq a \leq 1; \eta > 0$$

We have chosen the special case where the public sector inputs into the education process,  $g$ , are perfect substitutes for the private sector traded inputs,  $m$ . It would be easy to extend the analysis to the case where they are imperfect substitutes or where the private traded input is strictly more or less productive in the education process than the public input<sup>20</sup>. Note from our specification of the private budget constraint (10) that public expenditure on education benefiting the  $i^{\text{th}}$  individual can only be enjoyed by the  $i^{\text{th}}$  individual: it is excludable and rival. It cannot be resold. This of course will matter only if the  $m \geq 0$  constraint is binding.

The household decision rules for the foreign country are completely analogous to those for the home country given in equations (7) to (13) and will not be reproduced here. Parameters, variables and functions with the superscript \* will characterize the foreign country. Note that while all taste and policy parameters can differ between the two countries, the production function (introduced below) and the human capital accumulation technology, represented by the  $\psi$  function are the same in both countries.

#### b. Firm behavior.

Firms are competitive and maximize profits. Both factors of production

(labor and capital) can be varied costlessly at parametric factor prices. Output,  $y_t$  is sold in a competitive world market. For the moment we assume that the production function is linear homogeneous in the two factors of production that can be varied at the discretion of the firm. In the home country, firm  $i$ ,  $i = 1, \dots, I$  faces the production function

$$(15) \quad y_t = F(k_t, h_t)$$

This function is linear homogeneous in the  $i^{\text{th}}$  firm's two private inputs, capital  $k_t$  and efficiency units of labor  $h_t$ , with  $F(0, \cdot) = F(\cdot, 0) = 0$ , increasing in both its arguments, strictly concave, twice continuously differentiable and satisfies the Inada conditions:  $\lim_{k \rightarrow 0} \frac{\partial F}{\partial k} = 1 / \lim_{k \rightarrow \infty} \frac{\partial F}{\partial k} = \infty$ . Capital depreciation is ignored.

The linear homogeneity of the production function permits us to write:

$$y_t / h_t = F(k_t / h_t, 1) = f(k_t / h_t)$$

The first-order conditions for optimal use of capital and labor are:

$$r_t = f'(k_t / h_t)$$

$$w_t = f(k_t / h_t) - (k_t / h_t) f'(k_t / h_t)$$

We assume all firms in the domestic economy to be identical. Dropping the subscript  $i$  we write the aggregate production function for the home country and the representative domestic firm's first order conditions as follows:

$$(16) \quad Y_t = F(K_t, H_t) = H_t f(K_t / H_t)$$

$$(17) \quad r_t = f'(K_t / H_t)$$

$$(18) \quad w_t = f(K_t / H_t) - (K_t / H_t) f'(K_t / H_t) = \omega(K_t / H_t),$$

$$\omega' = -(K_t / H_t) f''(K_t / H_t) > 0.$$

The derivation of foreign country output, interest rate and wage rate is analogous. Note that the two countries have identical production technologies

$f(\cdot)$  .

We also assume that at the initial date,  $t = 0$ ,  $K_0 + K_0^* > 0$ .

**(c) The government.**

In both countries the government spends on the education of its young, levies lump-sum taxes on the middle aged and the old, taxes all asset income of its residents, pays interest on its debt and borrows to finance any excess of current outlays over current revenues. Government debt is single-period debt denominated in the traded output. The outstanding stock of home country government debt outstanding is  $B_t$ . Foreign government debt is denoted  $B_t^*$ . The home country government budget identities is given in equation (19). The conventional solvency constraints, given in (20) is assumed to apply. The foreign country counterparts are obvious and have been omitted.

$$(19) \quad B_{t+1} = (1 + r_t)B_t + g_t N_t - \tau_{t-1}^1 N_{t-1} - \tau_{t-2}^2 N_{t-2} \\ - \theta_t \left\{ \left[ \frac{c_{t-2}^2 + \tau_{t-2}^2}{1 + r_t - \theta_t} \right] N_{t-2} - m_{t-1} N_{t-1} \right\} + \varphi_t m_{t-1} N_{t-1}$$

$$(20) \quad \lim_{T \rightarrow \infty} \prod_{i=0}^T (1 + r_{t+i})^{-1} B_{t+1+T} \leq 0$$

$B_0$  and  $B_0^*$  are given.

**(d) Market equilibrium.**

There is perfect international mobility of financial capital. In the absence of distortionary source-based taxes on capital income, the domestic and foreign before-tax interest rates and rates of return on fixed capital will be equalized.

$$(21) \quad r_t = r_t^* = f'(K_t/H_t) = f'(K_t^*/H_t^*)$$

The after-tax rates of return to private saving,  $1 + r_t - \theta_t$  in the home country and  $1 + r_t - \theta_t^*$  in the foreign country, however, can differ. These policy-induced distortions are quite separate from the distortions due to the externalities in human capital formation.

From the production function, equalization of capital-human capital ratios in the two economies implies that the wage rates (of efficiency labor) in the two countries are also equalized, although labor itself is not traded internationally.

$$(22) \quad w_t = w_t^*$$

The fact that both countries' labor markets clear each period means that

$$(23a) \quad H_t = h_{t-1}^1 N_{t-1}$$

$$(23b) \quad H_t^* = h_{t-1}^{*1} N_{t-1}^*$$

The condition for equilibrium in the world capital market is given in equation (24), where equalization of domestic and foreign interest rates and wage rates has already been imposed.

$$(24) \quad K_{t+1} + K_{t+1}^* + B_{t+1} + B_{t+1}^* = \\ [w_t h_{t-1}^1 - c_{t-1}^1 - \tau_{t-1}^1 - (1 + r_t - \theta_t - \varphi_t) m_{t-1}] N_{t-1} - m_t N_t \\ + [w_t h_{t-1}^{*1} - c_{t-1}^{*1} - \tau_{t-1}^{*1} - (1 + r_t - \theta_t^* - \varphi_t^*) m_{t-1}^*] N_{t-1}^* - m_t^* N_t^*$$

The total stock of non-human assets at the end of period  $t$  (the beginning of period  $t+1$ )  $K_{t+1} + K_{t+1}^* + B_{t+1} + B_{t+1}^*$  has to be willingly held by the private sectors of the two countries. The old (those born in period  $t-2$ ) will not be holding any assets: they have at the end of period  $t$  just exhausted the last of their lifetime savings. The savings of the middle aged (those born in period  $t-1$ ) will be the sum of their primary (non-interest) current surpluses during middle age ( $[w_t h_{t-1}^1 - c_{t-1}^1 - \tau_{t-1}^1]$  per person of generation  $t-1$  in the case of the home country) and their compounded primary current surpluses from their youth ( $-(1 + r_t - \theta_t - \varphi_t) m_{t-1}$  per person of generation  $t-1$  in the case

of the home country). The young at the end of period  $t$  (those born in period  $t$ ) will have negative savings equal to the value of their borrowing to finance their education (student loans). In the case of the home country this amounts to savings of  $-m_t$  per person of generation  $t$ . Home country private financial wealth at the beginning of period  $t+1$ ,  $S_{t+1}$  is given by (25).  $Z_{t+1}$  denotes the net foreign assets of the home country at the beginning of period  $t+1$ .

Note that  $Z = -Z^*$ .

$$(25) S_{t+1} \equiv [w_t h_{t-1}^1 - c_{t-1}^1 - \tau_{t-1}^1 - (1 + r_t - \theta_t - \varphi_t)m_{t-1}]N_{t-1} - m_t N_t$$

Equation (24) can be obtained from the global goods market equilibrium condition in (26), the two private sector life-time budget constraints (10 in the case of the home country), the two public sector budget identities (19) for the home country and (20), the "exhaustion of output by factor payments" conditions (27a) and (27b) and the condition given in (28) that what the old in period  $t$  spend on consumption and taxes must be equal to the principal plus (net of tax) interest they carried forward from the previous period.

$$(26) Y_t + Y_t^* = K_{t+1} - K_t + K_{t+1}^* - K_t^* \\ + m_t N_t + c_{t-1}^1 N_{t-1} + c_{t-2}^2 N_{t-2} + m_t^* N_t^* + c_{t-1}^{*1} N_{t-1}^* + c_{t-2}^{*2} N_{t-2}^* \\ + g_t N_t + g_t^* N_t^*$$

$$(27a) Y_t = r_t K_t + w_t h_{t-1}^1 N_{t-1}$$

$$(27b) Y_t^* = r_t K_t^* + w_t h_{t-1}^{*1} N_{t-1}^*$$

$$(28) \frac{(c_{t-2}^2 + \tau_{t-2}^2)N_{t-2}}{1 + r_t - \theta_t} + \frac{(c_{t-2}^{*2} + \tau_{t-2}^{*2})N_{t-2}^*}{1 + r_t - \theta_t^*} =$$

$$K_t + K_t^* + B_t + B_t^* + m_{t-1} N_{t-1} + m_{t-1}^* N_{t-1}^*$$

Defining the notation:  $k \equiv K/H$ ;  $k^* \equiv K^*/H^*$ ;  $b \equiv B/H$  and  $b^* \equiv B^*/H^*$ ,

equation (24) can be rewritten as:



$$\begin{aligned}
(29) \quad & \left[ 1 + \frac{h_t^{*1} N_0^*}{h_t^1 N_0} \left( \frac{1+n^*}{1+n} \right)^t \right] k_{t+1} + b_{t+1} + \frac{h_t^{*1} N_0^*}{h_t^1 N_0} \left( \frac{1+n^*}{1+n} \right)^t b_{t+1}^* \\
& = \frac{(c_{t-1}^{*2} + \tau_{t-1}^2)}{1 + f'(k_{t+1}) - \theta_{t+1}} [(1+n)h_t^1]^{-1} - \frac{m_t}{h_t^1} \\
& + \left[ \frac{(c_{t-1}^{*2} + \tau_{t-1}^{*2})}{1 + f'(k_{t+1}) - \theta_{t+1}^*} [(1+n^*)h_t^{*1}]^{-1} - \frac{m_t^*}{h_t^{*1}} \right] \frac{h_t^{*1} N_0^*}{h_t^1 N_0} \left( \frac{1+n^*}{1+n} \right)^t
\end{aligned}$$

To achieve an equilibrium for this 2-country economy that is Pareto efficient, the two governments are required to subsidize human capital formation in order to internalize the externality, to forswear the use of distortionary taxes and possibly to choose their human capital accumulation inputs  $g$  and  $g^*$  to achieve productive efficiency. In addition one of the governments may have to use lump-sum taxes and transfers to ensure dynamic efficiency<sup>21</sup>. The first-best policy to internalize the externality is to subsidize time spent by the young in education,  $e_t$ . In our model such an education subsidy is equivalent either to a subsidy on the wage earned by the middle aged or to a tax on leisure.

If in our model we also permitted the young to work (in addition to choosing between leisure and education), and if work did not produce a human capital externality, then the equivalence between a subsidy to education, a tax on leisure and a wage subsidy to the middle-aged would break down. Efficiency would then require a subsidy to education or a tax both on leisure and on time spent working while young. The equivalence between an education subsidy and a tax on leisure would also break down when the middle-aged can choose leisure, unless age discrimination can be built into the leisure tax. A subsidy to borrowing by the young for educational expenditures is not needed in the first best. If a tax on leisure or a wage subsidy are not feasible, then subsidizing student loans will be a second-best policy. Subsidizing

private borrowing in general will be next best.

**(e) International productivity differentials.**

Note that  $Y_t = H_t f(k_t) = h_{t-1}^1 N_{t-1} f(k_t)$ . Output per worker  $\Pi$  in the home country is given by

$$\Pi_t \equiv Y_t/N_{t-1} = h_{t-1}^1 f(k_t)$$

The rate of growth of output per worker,  $\pi$  is given by

$$\pi_t \equiv \frac{\Pi_{t+1}}{\Pi_t} - 1$$

Similarly, with a common technology and free capital mobility, we have for the foreign country:

$$\Pi_t^* \equiv Y_t^*/N_{t-1}^* = h_{t-1}^{*1} f(k_t)$$

$$\pi_t^* \equiv \frac{\Pi_{t+1}^*}{\Pi_t^*} - 1$$

It follows that the differences in the growth rate of output per worker are due solely to (and are (almost) equal to) differences in the growth rate of human capital per worker<sup>22</sup>:

$$(30) \quad \pi_t - \pi_t^* = \left[ \frac{h_t^1}{h_{t-1}^1} - \frac{h_t^{*1}}{h_{t-1}^{*1}} \right] \frac{f(k_{t+1})}{f(k_t)}$$

Indeed, in steady state, the labor productivity growth differential is given by:

$$(31) \quad \pi - \pi^* = \frac{h_t^1}{h_{t-1}^1} - \frac{h_t^{*1}}{h_{t-1}^{*1}}$$

**(f) Investment shares and growth rate differentials.**

Our model has some strong implications concerning growth rate differentials and investment share differences. The causation, however, runs

from the exogenous or policy-determined growth differentials to the endogenous investment shares. In our model the home-country investment share is given by

$$\kappa_t \equiv \frac{K_{t+1} - K_t}{Y_t}. \text{ Considering only steady states for simplicity, we have that}$$

$$\kappa = [(1 + \pi)(1 + n) - 1] \frac{k}{F(k)}$$

The excess of the home country steady-state share of fixed capital formation in growth rate GDP over the foreign share is therefore given by

$$\kappa - \kappa^* = [(1 + \pi)(1 + n) - (1 + \pi^*)(1 + n^*)] \frac{k}{F(k)}$$

The exogenous growth differential therefore drives the endogenous investment share difference<sup>23</sup>.

#### (g) The global equilibrium.

As the focus of this paper is on productivity growth rate *differentials* rather than on the levels in the individual countries, it will turn out to be possible, under conditions to be stated below, to ignore the evolution (and endogenous response) of such global variables as the capital-human capital ratio  $k$  and the two public debt-human capital ratios  $b$  and  $b^*$ . A few words about one feature of the long-run or steady-state equilibrium are however in order.

Consider the case where the fiscal variables  $\theta$ ,  $\theta^*$ ,  $\tau_t^1/h_t^1$  and  $\tau_t^{*1}/h_t^{*1}$ ,  $g_t/h_t^1$ ,  $g_t^*/h_t^{*1}$  are constants and the two national public debt-human capital ratios are also constants, that is  $b_t = b$  for all  $t$  and  $b_t^* = b^*$  for all  $t$ . Lump-sum taxes on the old  $\tau_t^2/h_t^1$  and  $\tau_t^{*2}/h_t^{*1}$  adjust so as to satisfy the public sector budget identities.

There are two different kinds of steady state solutions. In the first, the long run growth rate of aggregate human capital differs between the two countries. In the second they are the same.

Consider first the case of differential long-run growth rates of the aggregate national human capital stocks. Without loss of generality we consider the case where the relative weight of the foreign country in the global capital market  $\frac{h_t^{*1} N_0^*}{h_t^1 N_0} \left[ \frac{1+n^*}{1+n} \right]^t \equiv \sigma_t$  decreases steadily, that is the growth rate of aggregate human capital in the home country is higher than in the foreign country. Note that this does not necessarily require that home country per capita human capital  $h_t^1$  grows at a faster rate than  $h_t^{*1}$ ; population growth rates matter as much as the growth rates of per capita human capital. Which country dominates the capital markets in the long run depends on relative size (measured by population in efficiency units) alone.

From (29) we notice that the long run or steady-state global capital-human capital ratio  $k$  is governed by the parameters describing private sector and government behavior in the home country alone according to

$$k + b = \frac{c^2 + \tau^2}{h^1(1+n)(1+f'(k) - \theta)} - \frac{m}{h^1}$$

The second case is where there exists a steady state value of relative foreign country size  $\sigma_t$  that is neither zero nor infinite but some positive constant value  $\sigma^{24}$ . In this case the appropriate steady-state condition for global capital market equilibrium will continue to reflect the parameters characterizing private sector and government behavior in both countries according to their relative weight, as shown in the following equation:

$$(1 + \sigma)k + b + \sigma b^* = \frac{c^2 + \tau^2}{h^1(1+n)(1+f'(k) - \theta)} - \frac{m}{h^1} + \sigma \left[ \frac{c^{*2} + \tau^{*2}}{h^{*1}(1+n^*)(1+f'(k) - \theta^*)} - \frac{m^*}{h^{*1}} \right]$$

### (3) A SIMPLE EXAMPLE.

Consider the following special case where the single-period utility functions in both countries are logarithmic ( $\gamma = 1$ ), the growth rates of human capital depend only on education ( $\alpha = 1$ ) and  $g_t = g_t^* = 0$ . From  $\alpha = 1$  it follows immediately that  $m_t = m_t^* = 0$ .

It is well-known that in OLG models such as ours, absent distortionary taxes, debt financing of public expenditure matters for the real equilibrium only to the extent that it redistributes lifetime resources between generations. For instance, issuing public debt when debt service is financed with taxes on the middle-aged, is equivalent to balanced-budget intergenerational redistribution from the middle-aged to the old, that is, it is equivalent to an increase in the scale of an unfunded social security retirement scheme. As long as there are no distortionary taxes, we could therefore restrict the analysis of taxes, transfers and public debt to balanced budget redistribution between the middle-aged and the old.

Clearly, when there are distortionary taxes such as our asset income taxes, the ability to alter the time profile of distortionary tax rates by unbalancing the budget will have real consequences.

The home country household equilibrium for this special case is given by

$$(32) \quad \frac{c_t^2}{c_t^1} = (1 + r_{t+2} - \theta_{t+2})\beta$$

$$(33) \quad \frac{c_t^1}{\ell_t} = \beta w_{t+1} \eta$$

$$(34) \quad \ell_t = h_t^0 - e_t$$

$$(35) \quad h_t^1 = h_t^0 + \eta e_t = (1 + \eta)h_t^0 - \eta \ell_t$$

$$(36) \quad h_t^1 w_{t+1} - c_t^1 - \tau_t^1 - \frac{c_t^2 - \tau_t^2}{1 + r_{t+2} - \theta_{t+2}} = 0$$

In addition we have the equation for the home country intergenerational transmission of human capital (37) and the home country government budget identity (38):

$$(37) \quad h_t^0 = h_{t-1}^1$$

$$(38) \quad b_{t+1} = [1 + f'(k_t)](1+n)^{-1} \frac{h_{t-1}^1}{h_t^1} b_t - \tau_{t-1}^1 \frac{1}{(1+n)h_t^1} - \tau_{t-2}^2 \frac{1}{(1+n)^2 h_t^1} \\ - \theta_t \left[ \frac{c_{t-2}^2 + \tau_{t-2}^2}{(1 + f'(k_t) - \theta_t)(1+n)^2 h_t^1} \right]$$

Obvious counterparts to each of the last seven equations exist for the foreign country. The world-wide capital market equilibrium condition is equation (29) with  $m_t = m_t^* = 0$  for all  $t$ .

We obtain the following solution for  $c_t^2$ :

$$(39) \quad c_t^2 = \frac{\beta^2(1 + r_{t+2} - \theta)}{1 + \beta + \beta^2} \left\{ (1 + \eta) h_t^0 w_{t+1} - \left[ \tau_t^1 + \frac{\tau_t^2}{1 + r_{t+2} - \theta_{t+2}} \right] \right\}$$

Since  $\ell_t = \frac{c_t^2}{\beta^2(1 + r_{t+2} - \theta)w_{t+1}}$ , it follows that the growth of human

capital in the home country is governed by equation (40), with a parallel equation for the foreign country:

$$(40) \quad h_t^1 = \frac{\beta(1 + \beta)(1 + \eta)}{[1 + \beta + \beta^2]} h_{t-1}^1 \\ + \frac{1}{(1 + \beta + \beta^2)w(k_{t+1})} \left[ \tau_t^1 + \frac{\tau_t^2}{1 + f'(k_{t+2}) - \theta_{t+2}} \right]$$

From equation (40) it is clear that, at a given value of  $k$ , the rate of growth of human capital per capita between period  $t$  and period  $t+1$  depends on three factors. Two of these can differ between the countries. They are the rate of time preference  $(1 - \beta)/\beta$  and the present discounted value of the net

lifetime transfer of resources away from members of generation  $t$  through the

government budget  $\tau_t^1 + \frac{\tau_t^2}{1 + r_{t+2} - \theta_{t+2}}$

The third is part of the technology, assumed to be common to both countries. It is the productivity or efficiency of the human capital accumulation process, measured by  $\eta$ .

At a given value of the ratio of non-human to human capital  $k$ , a lower time preference rate, that is a larger subjective discount factor  $\beta$  means a relatively higher valuation of future vs. current consumption. It implies a higher rate of accumulation of human capital by the young (as well as an increased desire for financial saving by the middle-aged).

Similarly, at a given value of  $k$ , a net increase in lifetime resources of a generation increases consumption of all normal goods. Leisure is a normal good. Education or human capital formation will decline as a result. Anything that reduces the present discounted value of the lifetime taxes paid by a generation will reduce human capital formation by that generation. For instance, the anticipated (in period  $t$ ) substitution of borrowing for taxes on the middle-aged in period  $t+1$  (a reduction in  $\tau_t^1$ ) or for taxes on the old in period  $t+2$  (a reduction in  $\tau_t^2$ ) will reduce the human capital accumulation effort of the generation born in period  $t$  and thus the period  $t$  growth rate of human capital.

An increase in  $\eta$  will stimulate human capital accumulation at a given value of  $k$ , even though it will, through the effect it has on the demand for leisure, reduce the time spent on education in natural units. Education measured in efficiency units increases.

Using equations (30), (40) and the foreign counterpart of (40), the

difference between the national growth rates of output per worker can be written as:

$$(41) \quad \pi_t - \pi_t^* = \left[ (1 + \eta) \left\{ \frac{\beta(1 + \beta)}{[1 + \beta + \beta^2]} - \frac{\beta^*(1 + \beta^*)}{[1 + \beta^* + \beta^{*2}]} \right\} \right. \\ \left. + \frac{1}{(1 + \beta + \beta^2)\omega(k_{t+1})} \left[ \tau_t^1/h_{t-1}^1 + \frac{\tau_t^2/h_{t-1}^1}{1 + f'(k_{t+2}) - \theta_{t+2}} \right] \right. \\ \left. - \frac{1}{(1 + \beta^* + \beta^{*2})\omega(k_{t+1})} \left[ \tau_t^{*1}/h_{t-1}^{*1} + \frac{\tau_t^{*2}/h_{t-1}^{*1}}{1 + f'(k_{t+2}) - \theta_{t+2}^*} \right] \right] \frac{f(k_{t+1})}{f(k_t)}$$

It is clear from (41) that, given zero net lifetime expected fiscal transfers to generation  $t$  in both countries ( $\tau_t^1/h_{t-1}^1 + \frac{\tau_t^2/h_{t-1}^1}{1 + f'(k_{t+2}) - \theta_{t+2}} = \tau_t^{*1}/h_{t-1}^{*1} + \frac{\tau_t^{*2}/h_{t-1}^{*1}}{1 + f'(k_{t+2}) - \theta_{t+2}^*} = 0$ ), the more patient country will have the higher productivity growth rate in period  $t$ :  $\beta > \beta^*$  implies  $\pi_t > \pi_t^*$ . Also, if  $\beta = \beta^*$ , then the country that is expected to make the larger life time fiscal transfer to generation  $t$  will have the lower growth rate of productivity in period  $t$ .

Also from (41), a set of sufficient conditions for being able to perform productivity growth differential comparative dynamics while ignoring the effects of exogenous shocks on  $k$ ,  $b$  and  $b^*$  is that only steady states are considered ( $k_{t+1} = k_t$ ) and that the initial values of the four lump-sum tax parameters are zero ( $\tau_t^1 = \tau_t^2 = \tau_t^{*1} = \tau_t^{*2} = 0$ )<sup>25</sup>.

Another set of sufficient conditions for being able to analyze the effect on the productivity growth differential of parameter and policy changes, without consideration of the effect of such changes on the economy-wide state variables  $k$ ,  $b$  and  $b^*$ , is also immediately apparent from (41). It is that all shocks are evaluated at an initial *symmetric* equilibrium; this means that in the initial equilibrium all private sector behavioral parameters and policy



instrument values are identical in the two countries:  $\beta = \beta^*$ ;  $\theta_t = \theta_t^*$ ;  $\tau_t^1/h_{t-1}^1 = \tau_t^{*1}/h_{t-1}^{*1}$ ;  $\tau_t^2/h_{t-1}^2 = \tau_t^{*2}/h_{t-1}^{*2}$ . We shall refer to this as the *symmetric* case.

Note that in neither of these cases do distortionary asset income tax rates affect the growth rate differential. In the general case, they can, from equation (41), affect the differential by altering the present discounted value of life time lump-sum transfers to members of generation  $t$  (for a given value of  $k_{t+1}$ ) and by altering  $k_{t+1}$ . When, as in the next Section, the young borrow to finance the use of produced inputs in the accumulation of human capital ( $0 < a < 1$ ), there is another channel through which distortionary taxes influence the rate of growth differential: the after-tax interest rate (which is a function of  $\theta$ ,  $\varphi$ ,  $\theta^*$  and  $\varphi^*$  even at a given value of  $k$ ) influences the demand by the young for the produced human capital accumulation input.

Under either of these two sets of sufficient conditions, the effects of private parameter changes or policy changes on home country productivity growth at a given value of  $k$  are the same as the effects on the productivity growth *differential* discussed earlier in this Section.

#### (4) TRADED PRIVATE AND PUBLIC INPUTS IN HUMAN CAPITAL ACCUMULATION.

When we consider the general constant elasticity of marginal utility single-period utility function ( $\gamma \geq 0$ ) and traded inputs are permitted as an argument in the function for the growth rate of human capital ( $0 \leq a \leq 1$ ), equations (32) and (33) become

$$(32') \quad \left[ \frac{c_t^2}{c_t^1} \right]^\gamma = (1 + r_{t+2} - \theta_{t+2})\beta$$

$$(33') \quad \left[ \frac{c_t^1}{\ell_t} \right]^\gamma = \beta w_{t+1} \eta a e_t^{a-1} (m_t + g_t)^{1-a}$$

Privately optimal choice of  $m_t$  may involve (for sufficiently large  $g_t$ ) a corner solution at  $m_t = 0$ . For the case where the constraint  $m_t \geq 0$  is not binding, we have the additional first order condition:

$$(42) \quad 1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1} = w_{t+1} \eta (1-a) e_t^a (m_t + g_t)^{-a}$$

Finally, the human capital accumulation function is now

$$(35') \quad h_t^1 = h_t^0 + \eta e_t^a (m_t + g_t)^{1-a}$$

For positive values of  $a$ , the growth of human capital in the home country is now governed by:

$$(43) \quad h_t^1 = h_{t-1}^1 \left[ \frac{a w_{t+1} (1 + \Omega_2 \Omega_1^{\frac{1-\gamma}{\gamma}}) + \Omega_2 \Omega_1^{\frac{1}{\gamma}} - w_{t+1}}{a w_{t+1} (1 + \Omega_2 \Omega_1^{\frac{1-\gamma}{\gamma}})} \right] \\ + \frac{\tau_t^1 + (1 + r_{t+2} - \theta_{t+2})^{-1} \tau_t^2 - (1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}) g_t}{a w_{t+1} (1 + \Omega_2 \Omega_1^{\frac{1-\gamma}{\gamma}})}$$

$$\Omega_1 = a \eta w_{t+1} \left[ \frac{1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}}{\eta w_{t+1} (1-a)} \right]^{\frac{a-1}{a}} > 0$$

$$\Omega_2 = \beta^{\frac{1}{\gamma}} + \beta^{\frac{2}{\gamma}} (1 + r_{t+2} - \theta_{t+2})^{\frac{1-\gamma}{\gamma}} > 0$$

The interpretation of this is clearer when we write down the solutions for the two inputs in the human capital accumulation process

$$(44) \quad e_t = \Omega_3 \left[ h_{t-1}^1 (\Omega_2 \Omega_1^{\frac{1}{\gamma}} - w_{t+1}) + \tau_t^1 + \frac{\tau_t^2}{1 + r_{t+2} - \theta_{t+2}} - (1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}) g_t \right]$$

$$\begin{aligned}
(45) \quad m_t &= \left[ \frac{1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}}{\eta(1-a)^w_{t+1}} \right]^{\frac{-1}{a}} e_t - g_t \\
&= \left[ \frac{1-a}{a} \right] \left[ (1+r_{t+1}-\theta_{t+1}-\varphi_{t+1}) (1+\Omega_2 \Omega_1^{\frac{1-\gamma}{\gamma}}) \right]^{-1} \\
&\quad * \left[ h_{t-1}^1 (\Omega_2 \Omega_1^{\frac{1}{\gamma}-w}_{t+1}) + \tau_t^1 + \frac{\tau_t^2}{1+r_{t+2}-\theta_{t+2}} \right] \\
&\quad - \left[ \left( \frac{1-a}{a} \right) (1 + \Omega_2 \Omega_1^{\frac{1-\gamma}{\gamma}}) \right]^{-1} + 1 \Big] g_t \\
\Omega_3 &= \left[ \Omega_1 (1 + \Omega_2 \Omega_1^{\frac{1-\gamma}{\gamma}}) \right]^{-1} > 0.
\end{aligned}$$

Equation (45) holds of course only when the constraint  $m_t \geq 0$  is not binding.

For the foreign country, equations analogous to (43), (44) and (45) can be derived. Noting that  $a$ ,  $\eta$ ,  $w$  and  $r$  are the same in the two countries, the foreign equations are obtained by attaching the superscript  $*$  to each  $\gamma$ ,  $\beta$ ,  $g$ ,  $\tau^1$ ,  $\tau^2$ ,  $\theta$  and  $\varphi$  in equations (43) to (45). We again are considering only the effect of variations in the parameters describing private sector behavior and fiscal policy on productivity growth *differentials*. For simplicity we also focus on perturbations of an initial *symmetric* equilibrium, which now amounts to the restriction that in the initial equilibrium we have  $\beta = \beta^*$ ;  $\gamma = \gamma^*$ ;  $\theta_t = \theta_t^*$ ;  $\varphi_t = \varphi_t^*$ ;  $\tau_t^1/h_{t-1}^1 = \tau_t^{*1}/h_{t-1}^{*1}$ ;  $\tau_t^2/h_{t-1}^1 = \tau_t^{*2}/h_{t-1}^{*1}$  and  $g_t/h_{t-1}^1 = g_t^*/h_{t-1}^{*1}$ . The technologies are, as always, constrained to be identical. In this Section the net lifetime fiscal transfer to generation  $t$  is assumed to be equal to zero,

$$\begin{aligned}
\text{that is } & \tau_t^1 + \frac{\tau_t^2}{1+r_{t+2}-\theta_{t+2}} - (1+r_{t+1}-\theta_{t+1}-\varphi_{t+1})g_t \\
& = \tau_t^{*1} + \frac{\tau_t^{*2}}{1+r_{t+2}-\theta_{t+2}^*} - (1+r_{t+1}-\theta_{t+1}^*-\varphi_{t+1}^*)g_t^* = 0
\end{aligned}$$

It is easily checked that under these conditions an increase in the value of  $\beta$  raises the growth rate of per capita human capital in the home country relative to the growth rate of per capita human capital in the foreign country. This result is independent of the value of  $\gamma$ . Greater home country patience achieves this relatively higher rate of growth of productivity by raising both  $e_t$  and  $m_t$  relative to the foreign country.

Note that with borrowing by the young to finance the purchase of traded inputs into the human capital accumulation process, the effect of a lower rate of time preference (a higher value of  $\beta$ ) on home country relative private financial wealth is ambiguous. This is true even if there is no government debt. While a higher value of  $\beta$  will cause the middle-aged to save more, it will also cause the young to dissave more by taking out more "student loans" (see equation (25)). Since the increased value of the human capital assets acquired by the young is not counted in conventionally measured saving, the net effect of an increase in  $\beta$  on conventionally measured private financial wealth is ambiguous.

The analysis of the effects of a change in  $\phi$ , the subsidy rate on student loans, is straightforward. Consider the case where the value of  $g_t$  equals zero<sup>26</sup>. The reduction in cost of borrowing by the young increases  $m_t$ . The effect on time spent on education,  $e_t$ , is ambiguous, since, if the initial value of  $m_t$  is positive, the increase in the subsidy rate will have a positive income effect which *cet. par.* would increase the demand for leisure. Since consumption during middle age and during old age are both normal goods, however, the effect on the total amount of resources transferred from youth to middle age is positive. The relative growth rate of productivity in period  $t$  therefore increases in the home country.

The effect of an increase in  $\theta$  on relative home country productivity

growth (in a symmetric equilibrium) is the same as that of an increase in  $\varphi$  if in addition  $\tau_t^2 = 0$ . There is of course a further impact on global savings through the effect of an increase in  $\theta$  on the intertemporal terms of trade of the middle-aged.

As long as the non-negativity constraint on private spending on education ( $m_t \geq 0$ ) is not binding and (43), (44) and (45) hold, an increase in public spending on education ( $g_t$ ) will, *cet. par.* lead to a reduction in  $m_t + g_t$ , the total amount spent on education by the private and public sectors combined. Time spent on education,  $e_t$ , will also be reduced and the growth rate of human capital will decline unambiguously.

The intuition is clear: as a profit maximizing *firm* facing a given wage and interest rate, the young worker would respond to the in-kind free gift of  $g_t$  by reducing his private input of  $m_t$  one for one. The free gift of  $g_t$ , however, also has an income effect on the young worker as a consumer (see equation (43) in which  $g_t$  enters (properly discounted) as a net transfer to the household on a par with  $-\tau_t^1$  and  $-\tau_t^2$ . The net result is the more than 100 percent crowding out of private education spending by public spending on education apparent in the last term on the R.H.S. of equation (45).

If the increase in public spending on the education of a member of generation  $t$ ,  $g_t$ , is financed during period  $t$  through borrowing and if the additional public debt thus incurred is serviced and repaid in periods  $t+1$  and/or  $t+2$  through increased lump-sum taxes on members of that same generation  $t$ , (that is through increases in  $\tau_t^1$  and/or in  $\tau_t^2$ ) then of course there will not be any change in  $-g_t(1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}) + \tau_t^1 + \tau_t^2(1 + r_{t+2} - \theta_{t+2})^{-1}$ . Absent any income effect from the increase in public spending on education, the "direct crowding out" (Buiter [1977]) of private by public spending is of course exactly one-for-one.

If the policy aim is to boost human capital formation, this model suggests that increasing public spending on education while the private sector still engages in private spending on education, would not be very smart<sup>27</sup>. An obviously superior policy is the one actually pursued (up to a point) by most governments in the real world: the removal of the education decision from the realm of private decision making. Compulsory school attendance up to a certain age is indeed the rule in most societies. It can be checked easily that with administrative assignment of  $e$  and of  $g$  and access to non-distortionary taxes, Pareto-efficient equilibria can be supported.

When the  $m_t \geq 0$  constraint is binding (which will only be the case when  $g_t > 0$ ), the private optimum is characterized by the following equations:

$$\left[ \frac{c_t^2}{c_t^1} \right]^\gamma = (1 + r_{t+2} - \theta_{t+2})\beta$$

$$\left[ \frac{c_t^1}{h_{t-1}^1 - e_t} \right]^\gamma = \beta w_{t+1} \eta a e_t^{a-1} g_t^{1-a}$$

$$w_{t+1} [h_{t-1}^1 + \eta e_t^a g_t^{1-a}] - c_t^1 - \tau_t^1 - \frac{c_t^2 + \tau_t^2}{1 + r_{t+2} - \theta_{t+2}} = 0$$

The equilibrium amount of time spent on education can be solved from

$$\tau_t^1 + \frac{\tau_t^2}{1 + r_{t+2} - \theta_{t+2}} - w_{t+1} h_{t-1}^1 = w_{t+1} \eta g_t^{1-a} e_t^a \left\{ 1 + a\beta [1 + \beta^\gamma (1 + r_{t+2} - \theta_{t+2})^{\frac{1-\gamma}{\gamma}}] \left( 1 - \frac{h_{t-1}^1}{e_t} \right) \left[ a\beta \eta w_{t+1} e_t^{a-1} g_t^{1-a} \right]^{\frac{1-\gamma}{\gamma}} \right\}$$

The effect of an increase in public spending on education,  $g_t$ , on private time spent on education is ambiguous. The increase in the quantity of the complementary factor of production  $g_t$  raises the marginal return to another hour spent on education. The income effect, however, goes the other way and suggests an increase in the demand for leisure. Even when  $e_t$  declines,

however, the net effect of the growth rate of human capital is positive. The intuition for this is that the positive income effect of the increase in public spending also raises the demand for  $c_t^1$  and  $c_t^2$ . The net effect of an increase in  $g_t$  on  $h_t^1$  is therefore positive when the non-negativity constraint on  $m_t$  is binding.

Inter-country differences in productivity growth rates disappear when all inputs into human capital accumulation are tradable. We could analyze this case by considering the case where  $a = 0$  and non-traded time is not an input into human capital accumulation. The problem with this is that, unless by chance  $1 + r_{t+1} - \theta_{t+1} = \eta w_{t+1}$ , we will have a corner solution for the traded human capital accumulation input. The home country private sector equilibrium conditions are for this case:

$$-m_t(1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}) + w_{t+1}h_t^1 - c_t^1 - \tau_t^1 - \frac{c_t^2 + \tau_t^2}{1 + r_{t+2} - \theta_{t+2}} = 0$$

$$h_t^1 = h_t^0 + \eta(m_t + g_t)$$

$$\left[ \frac{c_t^2}{c_t^1} \right]^\gamma = (1 + r_{t+2} - \theta_{t+2})^\beta$$

$$l_t = h_t^0$$

$$m_t = 0 \text{ if } 1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1} < \eta w_{t+1}$$

$$= \infty \text{ if } 1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1} > \eta w_{t+1}$$

This knife-edge solution reflects the fact that when  $a = 0$ , our education technology is linear in an input which can be purchased at a parametric price and which does not enter directly into the strictly concave utility function. While in general equilibrium finite values of  $m_t$  (and  $m_t^*$ ) will still be ensured through the endogeneity of the wage rate and the interest rate, it is convenient to respecify the human capital accumulation function for the case where only traded inputs enter as follows:

$$h_t^1 = h_t^0 + \eta \lambda^{-1} (m_t + g_t)^\lambda \quad 0 < \lambda < 1.$$

With this strictly concave accumulation function the first order condition for optimal private choice of  $m_t$  becomes, when the non-negativity constraint on  $m_t$  is not binding

$$(46) \quad 1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1} = \eta (m_t + g_t)^{\lambda-1} w_{t+1}$$

With perfect international mobility of financial capital and no differential source-based taxes on capital rentals, the before-tax interest rate will be equalized in the world economy. With a common production function the wage rate (per unit of efficiency labor) will also be equalized throughout the world economy. With a common human capital accumulation technology (common values of  $\eta$  and  $\lambda$  in this example), the equilibrium values of  $m_t + g_t$  will be the same throughout the world economy. Taste parameters (such as  $\beta$  and  $\beta^*$ ) therefore no longer matter for differences in productivity growth rates. Neither do redistributive lump-sum taxation or public sector deficits. The only aspect of fiscal policy in our model that matters for growth differentials are the tax rates on non-human asset income and student loan subsidy rates ( $\theta$ ,  $\theta^*$ ,  $\varphi$  and  $\varphi^*$ )<sup>28</sup>. Different source-based capital rental tax rates would *cet. par.* cause different wages to be generated in the parts of the world where they apply. By raising the return to human capital accumulation a higher home country relative real wage would *cet. par.* increase  $m_t$  and thus the relative growth rate of home country human capital.

Note from equation (46) that a permanently higher subsidy rate on student loans in the home country (a higher value of  $\varphi$ ) will *cet. par.* be associated with a permanently higher home country relative growth rate of human capital and a permanently higher relative rate of growth of output per worker. An increase in the home country tax rate on all saving (subsidy rate on all borrowing)  $\theta$  will have the same effects.



Also from equation (46), higher public spending on education would *cet. par.* (i.e. without allowing for possible consequences for the world rate of interest and the wage rate of the financing decisions associated with higher public spending) only crowd out private spending on education one-for-one:  $d(m_t + g_t) = 0^{29}$ .

If the non-negativity constraint  $m_t \geq 0$  on private expenditure on education is binding, the government can of course boost the growth rate of human capital simply by raising  $g_t$ , its own expenditure on education.

Returning to the general case of  $0 \leq a < 1$ , it is easy to consider the difference made by the existence of national source-based taxation (at a rate  $\hat{\theta}$  in the home country and  $\hat{\theta}^*$  in the foreign country) of the rental income from capital instead of national residence-based taxation of the income from all non-human wealth. Student loan subsidies are also omitted. With free international mobility of financial capital we now have equalization of after-tax rates of return to physical capital, that is

$$(47) \quad r_t = r_t^* = (1 - \hat{\theta}_t)f'(k_t) = (1 - \hat{\theta}_t^*)f'(k_t^*) \quad \hat{\theta}, \hat{\theta}^* < 1$$

With source-based capital taxation, perfect capital mobility and a common technology, the home country wage rate will be above the foreign wage rate if and only if  $\hat{\theta}$  is below  $\hat{\theta}^*$ . Even if all other private and policy parameters are identical, different wage rates will be associated with different productivity growth rates. In the logarithmic case ( $\gamma = \gamma^* = 1$ ) it is very easy to see that the country with the higher real wage will have the higher growth rate of productivity <sup>30</sup>.

(5) GOVERNMENT BORROWING AND LUMP-SUM INTERGENERATIONAL REDISTRIBUTION:  
MUST WHAT HELPS SAVING HURT HUMAN CAPITAL FORMATION?

The budget identity and the solvency constraint (assumed to hold with strict equality) of the home government (equations (19) and (21a)) together imply the following present value budget constraint:

$$(48) \quad b_t = \sum_{i=0}^{\infty} \delta_{t+i} \left\{ \frac{\tau_{t+i-1}^1}{h_{t+i}^1 (1+n)} + \frac{\tau_{t+i-2}^2}{h_{t+i}^1 (1+n)^2} + \theta_{t+i} \left[ \frac{c_{t+i-2}^2 + \tau_{t+i-2}^2}{(1+f'(k_{t+i}) - \theta_{t+1})(1+n)^2 h_{t+i}^1} \right] \right. \\ \left. - (\theta_{t+i} + \varphi_{t+i}) \frac{m_{t+i-1}}{h_{t+i}^1 (1+n)} - \frac{g_{t+i}}{h_{t+i}^1} \right\}$$

$$\delta_{t+i} = \prod_{j=0}^i \frac{h_{t+j}^1 (1+n)}{h_{t+j-1}^1 (1+f'(k_{t+j}))}$$

All this says is that the outstanding value of the public debt should be equal to the present discounted value of the future primary (non-interest) public sector budget surpluses. Holding constant the path of public spending on goods and services, a tax cut this period will require a future tax increase of equal present discounted value. The required tax increase may of course be spread out over many future periods.

While the competitive equilibria of OLG models such as the one we are considering may be dynamically inefficient <sup>31</sup>, we shall consider the consequences of a cut in lump-sum taxes during period  $t$  when the interest rate is above the growth rate of human capital in each period. Any government, acting unilaterally, could issue debt or vary lump-sum taxation to achieve a national Pareto improvement if dynamic inefficiency prevailed (see Buiter and Kletzer [1990a, 1990b]). For simplicity, the distortionary tax rates  $\theta$  and  $\varphi$  and their foreign counterparts  $\theta^*$  and  $\varphi^*$  are set equal to zero in what follows, as are domestic and foreign exhaustive public spending  $g$  and  $g^*$ .

Again all policy changes are perturbations of an initial symmetric equilibrium.

In period  $t$  the government can only change  $\tau_{t-1}^1$ , the tax on the middle aged, and  $\tau_{t-2}^2$ , the tax on the old. Period  $t$  human capital formation is performed by the young in that period, that is by generation  $t$ . Human capital formation in period  $t$  will only be a function of expectations at time  $t$  concerning  $\tau_t^1$  and  $\tau_t^2$ . The behavior of members of generation  $t$  during period  $t$  is therefore only affected by tax changes in period  $t$  to the extent that such changes in  $\tau_{t-1}^1$  and  $\tau_{t-2}^2$  carry *announcement effects* concerning  $\tau_t^1$  and  $\tau_t^2$ , the taxes they will pay when middle-aged and old. Of course, if the changes in  $\tau_{t-1}^1$  and  $\tau_{t-2}^2$  are news with respect to the information set of period  $t-1$ , then the saving behavior of the middle-aged in period  $t$  will be affected. The scope for time-inconsistent policy behavior in a model like ours is clearly considerable. For reasons of space these issues will not be considered further.

Any change in the government's policy concerning borrowing and lump-sum taxes and transfers that increase (reduces) the net life-time fiscal transfer to generation  $t$ , that is  $T_t = -[\tau_t^1 + \frac{\tau_t^2}{1 + r_{t+2} - \theta_{t+2}}]^{32}$ , will reduce human capital formation by that generation that period (see equation (43)). Will all policies that achieve an increase in  $T_t$  have an unambiguous effect on national saving in that period or beyond?

Let national financial wealth at the beginning of period  $t$  be denoted  $J_t$ . It follows that

$$(49) \quad J_{t+1} \equiv S_{t+1} - B_{t+1} \equiv K_{t+1} - B_{t+1} + Z_{t+1} = \left[ \frac{c_{t-1}^2 + \tau_{t-1}^2}{1 + r_{t+1}} \right] N_{t-1} - B_{t+1} \\ - m_t N_t = [w_t h_{t-1}^1 - c_{t-1}^1 - \tau_{t-1}^1 - (1 + r_t) m_{t-1}] N_{t-1} - B_{t+1} - m_t N_t$$

It is clear from equation (45) that an increase in  $T_t$ , the net life-time fiscal transfer to generation  $t$  will, by raising life-time resources, increases the demand for leisure, reduces human capital formation and therefore reduces  $m_t$ , the amount of the complementary human capital accumulation input demanded. Since the resources to pay for  $m_t$  have to be borrowed (student loans), national saving *cet. par.* increases as a result of the same fiscal action that reduces human capital formation. To determine what will happen in periods beyond  $t$  to national saving, we shall consider two canonical special cases.

The first change in the borrowing-lump-sum tax/transfer mix is a permanent balanced budget tax cut in favor of the middle-aged financed by an increase in taxes on the old. For simplicity let  $b_t = 0$  for all  $t$ . What this amounts to is a permanent reduction in the scale of the unfunded social security retirement scheme. When middle-aged each person has a tax cut of amount  $\mu$  and when old a tax increase of  $(1+n)\mu$ . That is:

$$d\tau_{t+i}^1 = -(1+n)^{-1} d\tau_{t+i-1}^2 = \mu < 0 \text{ for all } i \geq -2.$$

The present discounted value of lifetime taxes falling on generation  $t+i$  changes by  $\mu \left[ \frac{r_{t+2+i} - n}{1 + r_{t+2+i}} \right]$  which is negative if the interest rate exceeds the rate of growth of population, as we assume. With leisure a normal good, this policy therefore reduces forever more the home country allocation of time to education and thus the rate of growth of the stock of human capital relative to that in the rest of the world. From equation (43) it will forever lower the growth rate of home country labor productivity relative to the growth rate of labor productivity in the foreign country.

As noted before, the increase in  $T_t$  (for all  $t$  in our example) will reduce  $m_t$  together with  $e_t$ . This reduction in financial dissaving by the

young is reinforced by the middle-aged, for familiar life-cycle reasons. The reduction in the size of the home country unfunded social security retirement scheme will increase saving by the middle aged and therefore raise the total national stock of non-human assets held by domestic residents (this does not require the interest rate to be above the population growth rate).

What we have here is an example of the general point that in our model, *permanent changes in the intergenerational distribution of income brought about by changes in government borrowing, lump-sum taxes and transfers that boost domestic saving will tend to reduce human capital formation and conversely.*

A second example of a similar (negative) permanent effect on human capital formation and (positive) permanent effect on financial saving is that of a debt-financed quasi-permanent tax cut for the young: in period  $t$  an (unexpected) one-time tax is levied on the old or the middle-aged. The revenues from this one-time levy are used to retire public debt. For simplicity assume the outstanding public debt is reduced to zero following the levy. The present discounted value of all net future tax receipts too is therefore reduced to zero. This "present value tax dividend" can be distributed (in many different ways) across generations in such a manner that the present discounted value of lifetime taxes for all current and future generations (except the unfortunate current old or middle-aged) is lower. For instance, one could give the middle-aged each period after  $t$  the same size tax cut, with the value of the tax cut determined by the requirement that their present discounted value be equal to the original tax levy in period  $t$ .

This policy would clearly raise the permanent income (at given wages and interest rates) of all generations born in period  $t$  or later. It would therefore reduce their expenditure of resources on human capital formation and their borrowing while young to finance human capital formation. The future

middle-aged would all increase their saving for life-cycle purposes. Again, relative human capital formation and national saving would move in opposite directions.

(6) CONCLUSION.

The presence of a non-traded ("home-grown") human capital good which is an essential input in its own accumulation is sufficient for the existence of persistent international differentials in levels and growth rates of labor productivity. This is true even though there is perfect international mobility of financial and physical capital and technologies are identical across the world.

A higher subjective rate of time preference will lower a country's relative rate of growth by reducing the relative rate of accumulation of human capital.

A higher public debt burden will, to the extent that it represents a net intergenerational redistribution towards the old (which will be the case if taxes are typically paid by the working generation and public debt is typically owned by the old), increase the relative growth rate of human capital and output. That is, deficit financing policies and lump-sum intergenerational redistribution policies that boost financial saving will reduce the relative rate of human capital accumulation and the relative growth rate of labor productivity.

In the tradition of Uzawa [1965] and Lucas [1988], the human capital accumulation process involves positive external effects. A first-best policy towards human capital accumulation requires subsidies to education or, in our model, a tax on leisure or a wage subsidy. The same result can of course also

be achieved command-style, that is through administrative assignment of time and resources spent on schooling, overriding individual choice. Improvements over the unassisted decentralized equilibrium that fall short of full Pareto efficiency are achieved by subsidizing private borrowing for educational expenditure (student loans).

When public spending on education is neither more nor less efficient than private spending, an increase in public spending on education will crowd out private spending more than one-to-one, unless the income effect of the public transfer in kind for the generation receiving it is offset by increases in taxes.

A higher subsidy rate to student loans will raise the relative growth rate of productivity and a higher source-based tax on capital income will lower it. Residence-based taxes will in addition affect world-wide productivity growth through their effect on the world rate of interest. They affect the productivity growth *differential* only through its differential effect on the cost of borrowing to finance private educational expenditures. Changes in source-based taxes also affect global saving. They have a differential effect on productivity growth by altering relative wages.

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## FOOTNOTES

<sup>1</sup>The terms labor productivity, output per worker, output per person hour and output per capita will be used interchangeably.

<sup>2</sup>These models themselves are 2-country versions either of Diamond's famous 2-period Samuelsonian OLG model with a neoclassical production function (Diamond [1965]) or of the Yaari-Blanchard-Weil OLG model. Another example is Chang [1990].

<sup>3</sup>Alogoskoufis and van der Ploeg [1990a,b] contain the first applications (that we know of) of the OLG private consumption specification to endogenous growth. Unlike our model which uses the (3-period variant of the) Samuelson [1958] OLG model, Alogoskoufis and van der Ploeg use the Yaari-Blanchard-Weil version of the OLG model. At the same Tokyo Conference at which an early version of this paper was presented, Alogoskoufis and van der Ploeg also presented a two-country OLG endogenous growth model with perfect international capital mobility. Because their model does not have an essential non-traded growth input, it implies convergent growth rates (Alogoskoufis and van der Ploeg [1991]).

<sup>4</sup>Other papers analyzing the consequences of the use of distortionary taxes in (closed) endogenous growth models are Rebelo [1990], King and Rebelo [1990] and Barro and Sala i Martin [1990a]. The latter also consider productive public spending. Jones and Manuelli [1990] analyze an infinite-lived representative agent version of the open economy endogenous growth model with distortionary taxes.

<sup>5</sup>Examples include Grossman and Helpman [1989a,b,c,d; 1990] and Feenstra [1990].

<sup>6</sup>In the macroeconomic literature Lucas [1988, pp.14-17] recognizes and emphasizes the importance of factor mobility assumptions for the predictions of neo-classical growth theory. It is equally important for endogenous growth theory with non-increasing returns (of which our paper is an example) and for endogenous growth theory with increasing returns.

<sup>7</sup>We should note that Grossman and Helpman [1990] do allow trade in intermediate goods in a model in which technological knowledge concerning new types of intermediate goods is costlessly and instantaneously transferable between countries. The externality associated with the creation of this new knowledge is also global. However, the technologies for producing intermediate and final goods are not identical. The concept of comparative advantage therefore has meaning in their model. In their other work dealing with two-country endogenous growth there also is assumed to be a difference between the technologies for producing new knowledge. Knowledge transfer is non-instantaneous and sometimes costly. See Grossman and Helpman [1989a, b, c and d]. Feenstra [1990] develops a two-country endogenous growth model in which there is no financial capital mobility or technology transfer. Quah and Rauch [1990] also consider trade in intermediate goods in an endogenous growth model. Young [1989] considers a two-country endogenous growth model in which immobile labor is the only factor of production in both countries and in which there is no international borrowing and lending. Technologies differ initially between the two countries and will not necessarily converge, even asymptotically.

<sup>8</sup>What we say about convergence or divergence of *national* productivity

levels can apply to any sub-unit of the global economy. In most of international trade theory the label "nation" does double duty. It defines the set of agents under the political jurisdiction of a particular government (and thus subject to its taxes, regulations, tariffs etc); it is at the same time used to define the domain of mobility of certain economic agents, factors of production and their owners and other inputs (e.g. cultural attitudes towards lying, bribery, shirking, finking etc.). It is clear that the relevant domain of a particular agent, factor of production or cultural trait need not coincide with the political definition of the nation state. The relevant domain or region can be a subset of the national economy, can contain the national economy or can have an intersection with the national economy that is non-empty but less than the union of the two.

<sup>9</sup>What we say about convergence or divergence of *national* productivity levels can apply to any sub-unit of the global economy. In most of international trade theory the label "nation" does double duty. It defines the set of agents under the political jurisdiction of a particular government (and thus subject to its taxes, regulations, tariffs etc); it is at the same time used to define the domain of mobility of certain economic agents, factors of production and their owners and other inputs (e.g. cultural attitudes towards lying, bribery, shirking, finking etc.). It is clear that the relevant domain of a particular agent, factor of production or cultural trait need not coincide with the political definition of the nation state. The relevant domain or region can be a subset of the national economy, can contain the national economy or can have an intersection with the national economy that is non-empty but less than the union of the two.

<sup>10</sup>We shall use the short-hand expression "decreasing returns to reproducible factors" to mean that the production function is homogeneous of degree less than unity in reproducible factors of production.

<sup>11</sup>By models with constant returns to reproducible or augmentable inputs we mean those endogenous growth models that do not have scale effects entering into the determination of the equilibrium growth rate. Endogenous growth can of course be obtained in such models even without constant returns to the reproducible inputs. All that is required is that the Inada conditions be violated in such a way that the marginal product of reproducible inputs be bounded sufficiently far away from zero even when the ratio of reproducible to non-reproducible inputs increases without bound. The endogenous growth models we shall consider achieve this despite the Inada conditions being satisfied, by having constant returns to reproducible inputs at the level of the aggregate national production function. By models with decreasing returns to reproducible inputs we mean models for which the Inada conditions are satisfied and the production function is homogeneous of degree less than unity in reproducible inputs. The steady state growth rate of labor productivity (if a steady state exist) is exogenous and equal to the exogenous rate of labor-augmenting (or Harrod neutral) technical change.

<sup>12</sup>Recent examples of studies that investigate national differences in per capita output levels and growth rates using as (one of) the technological maintained hypotheses the constant or decreasing returns to augmentable factors of production model and the common global technology of production include the empirical studies of Barro [1989a,b], King and Rebelo [1989], Benhabib and Jovanovic [1989] and Cohen [1990]. For more on the facts on convergence see Baumol [1986] and Baumol, Blackman and Wolff [1987]. Easterly [1989] has a technology that can exhibit increasing returns to scale but

focuses on the case of constant returns to reproducible factors and either a constant value for the irreproducible factor or independence of output from the irreproducible factor in steady state. In Easterly [1990] the model is simplified to exhibit constant returns to reproducible factors. Irreproducible factors play no role. Finally, Edwards [1989] develops and tests a simple model of growth in developing countries in which the assumption of access to a common global technology is abandoned. It is replaced by one of gradual catching up by a technologically backward nation to the higher external level of technology. The rate at which a country catches up is postulated to be an increasing function of the degree of external orientation in the country's international trade relations.

Barro and Sala-i-Martin [1990] use a model without factor mobility to analyze convergence of growth rates among regions within a nation state (the states of the USA). They recognize that this framework is unrealistic for countries and especially for the U.S. states and note that extensions of the neoclassical growth model that allow for features of an open economy tend to speed up the predicted rate of convergence.

<sup>13</sup>In a companion paper to the current one, we consider a Yaari-Blanchard-Weil OLG model to study the same range of issues analyzed in this paper. In that model, the current stock of human capital can be allocated to leisure, to current production or to further human capital accumulation. See Buiter and Kletzer [1991]. The closed economy version of an endogenous growth model with the Yaari-Blanchard-Weil OLG structure is analyzed in Alogoskoufis and van der Ploeg [1990a,b].

As pointed out to us by Olivier Blanchard, dynamic inefficiency can never occur, regardless of the specification of the consumption side of the model, when there is a production function such as the one used in Alogoskoufis and van der Ploeg, in which aggregate output is linear in the aggregate physical capital stock. When  $Y = AK$ ,  $A > 0$ , the marginal product of capital, MPK, is constant and equal to  $A$ , which is also the average product of capital. In a closed system, output per unit of capital is obviously never less than capital formation per unit of capital, which equals output per unit of capital minus the sum of private and public consumption per unit of capital. A necessary condition for the occurrence of dynamic inefficiency is obviously that the marginal product of capital be able to fall below the average product. This is ruled out when output is linear in the physical capital stock. Note that  $A$  need not be constant for this argument to hold, but only independent of  $K$ .

Let  $C$  denote aggregate private consumption and  $G$  aggregate public consumption. Without loss of generality, capital consumption is ignored. It follows that, in a closed economy, since  $C, G \geq 0$ , we have

$$\frac{\dot{K}}{K} = A - \frac{C}{K} - \frac{G}{K} \leq A = \text{MPK}$$

Our specification of the production function has constant returns to the two reproducible factors (physical and human capital) together but less than unitary returns to each of the two factors of production separately. The marginal product of each of the two factors of production is below its average product and is endogenous. Dynamic inefficiency is therefore in principle possible in our model.

<sup>14</sup>If increasing returns are internal to the individual firm (that is if returns to scale are increasing in productive inputs that can be varied at the level of the individual firm) and if they are unbounded, then no competitive equilibrium can exist.

<sup>15</sup>In an interesting recent paper, Caballero and Lyons [1989] find limited evidence in US manufacturing industry of internal increasing returns (at the two-digit level), but strong evidence of external economies, that is economies external at the two-digit level but internal at the level of the US economy.

<sup>16</sup>For a recent opposing view see Romer [1986].

<sup>17</sup>It should be noted that Romer's "replication postulate" may not be universally accepted by physical scientists as "the most basic premise of our scientific reasoning". It has a distinct pre-Quantum Theory flavor. While economics is still pretty much Newtonian as a science (or even pre-Newtonian, that is Keplerian as regards theory and pre-Tycho Brahean as regards empirical observation) one should be careful in one's assertions about what real scientists believe.

A fundamental property of the quantum world is that one cannot, even in principle, measure precisely both the momentum and the position of a particle at the same time. It is often explained in physical terms, that the act of measuring the position of a particle disturbs its momentum and vice versa. One implication is what Einstein referred to as a "spooky action at a distance", a communication between two particles even when they are far apart, so that particle A can be disturbed by the measurement made on particle B. Einstein had trouble with this, as it seemed to mean that the communication traveled faster than light. However, it is consistent with the experimental evidence (the test of Bell's inequality). It also makes a hash of the replication postulate.

<sup>18</sup>"An input in production is rival if its use by one person or firm precludes its use by another" (Romer [1990b]). A completely non-rival good is one whose use by one person or firm in no way precludes its use by another. A good can be partly rival or even super non-rival: its use by one firm or person may enhance its availability for use by others. The use of a certain class of objects as a medium of exchange and means of payment may have this feature: my willingness to accept payment in a certain potential medium of exchange increases when the currency is used more widely. Starrett [1988] contains a very clear discussion of the subject.

<sup>19</sup>We assume that  $1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}$  and  $1 + r_{t+2} - \theta_{t+2}$  are positive. From our assumptions about the production technology (given below) it follows that  $r > 0$ . The restriction that gross after-tax rates of return be positive is therefore a restriction on permissible fiscal policy.

<sup>20</sup>An example of such a function would be the following generalization of equation (14)

$$\psi\left(\frac{e_t}{h_t^0}, \frac{m_t}{h_t^0}, \frac{g_t}{h_t^0}\right) = \eta \frac{e_t^a (m_t + g_t)^{1-a-\delta} g_t^\delta}{h_t^0} \quad 0 \leq a \leq 1; 0 \leq \delta < 1; \eta > 0$$

Note that this permits the marginal productivity of public spending on education to be greater than ( $\delta > 0$ ), equal to ( $\delta = 0$ ) or less than ( $\delta < 0$ ) the marginal productivity of private spending on education.

<sup>21</sup>While with perfect international capital mobility a single government can achieve world-wide dynamic efficiency, both governments can choose (balanced or unbalanced-budget) intergenerational redistribution schemes using lump-sum taxes and transfers to achieve this (Buiter and Kletzer [1990b]).

<sup>22</sup>In continuous time  $\pi_t = \text{dln}\Pi_t = \text{dln}h_t^1 - \text{dln}f(k_t)$  and  $\pi_t^* = \text{dln}\Pi_t^* = \text{dln}h_t^{*1} - \text{dln}f(k_t^*)$ . Therefore  $\pi_t - \pi_t^* = \text{dln}h_t^1 - \text{dln}h_t^{*1}$ .

<sup>23</sup>The assumptions which have been made explicitly or implicitly, that population growth rates and labor force participation rates are exogenous, are of course not the strongest point of the paper.

<sup>24</sup>Note that  $\sigma$  will be a function of the parameters of the model.

<sup>25</sup>Note that without intergenerational redistributive policy ( $\tau_t^1 + \tau_t^2(1 + r_{t+2} - \theta_{t+2})^{-1} = \tau_t^{*1} + \tau_t^{*2}(1 + r_{t+2} - \theta_{t+2}^*)^{-1} = 0$ ), the growth rate of home country output per worker exceeds that of the foreign country if and only if the home country discount factor  $\beta$  exceeds that of the foreign country  $\beta^*$ . This is true in and out of steady state.

<sup>26</sup>We assume that  $1 + r_{t+1} - \theta_{t+1} - \varphi_{t+1}$  and  $1 + r_{t+2} - \theta_{t+2}$  are positive. From our assumptions about the production technology (given below) it follows that  $r > 0$ . The restriction that gross after-tax rates of return be positive is therefore a restriction on permissible fiscal policy.

<sup>27</sup>Clearly, one would have to take a disaggregated view of the matter and allow for heterogeneity within generations in the real world. Even if average private spending on education is positive, there may be many individuals spending nothing on education.

<sup>28</sup>The conclusion that, with common values of  $\theta$  and  $\varphi$  in the two countries, there would be convergence (in our model instantaneous equality) of productivity growth rates when  $a = 0$  remains correct also if  $\lambda = 1$  (when the growth rate of per capita human capital is linear in  $m + g$ ).

<sup>29</sup>In the framework of this paper, any consequences of lump-sum financing of, say, increased home country public spending on education would affect domestic and foreign interest rates and wage rates equally. This would therefore not alter productivity growth differentials.

<sup>30</sup>This result follows through also for the constant elasticity of marginal utility case.

<sup>31</sup>See footnote 12 for a discussion of how certain other endogenous growth models can never exhibit dynamic inefficiency.

<sup>32</sup>An alternative definition of the net fiscal transfer to generation  $t$  would be  $-\left[\tau_t^1 + \tau_t^2(1 + r_{t+2} - \theta_{t+2})^{-1} - g_t(1 + r_{t+1} - \theta_{t+1})\right]$ . When  $a < 1$  (and the non-negativity constraint on private educational expenditures ( $m_t \geq 0$ ) is not binding), the transfer of educational services to a member of generation  $t$ ,  $g_t$ , can be viewed as income-in-kind, but it will clearly not be a lump-sum transfer.