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CENTER DISCUSSION PAPER NO. 617

TESTING FOR SOLVENCY OF THE PUBLIC SECTOR:

AN APPLICATION TO ITALY

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TESTING FOR SOLVENCY OF THE PUBLIC SECTOR:  
AN APPLICATION TO ITALY

Abstract

Credible participation in the European Monetary Union requires that member countries pursue sustainable fiscal and monetary policies --- and solvency is a weak prerequisite for sustainability. Solvency tests develop from the idea of verifying whether the public sector intertemporal budget constraint in a dynamically efficient economy would be met, were both the current fiscal and monetary policy and the macroeconomic environment stable over time. This has an empirical counterpart in a zero unconditional expectation for the series of discounted net public debt, which is the null hypothesis for the time series based tests. The empirical application rejects the null hypothesis of solvency for the general government in Italy, with a caveat suggested by the absence of a consolidated public sector accounts including the vast Italian public holding companies.

Key Words: Solvency, Public Debt, Italy

## TESTING FOR SOLVENCY OF THE PUBLIC SECTOR

An application to Italy

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The growth of public debt during the 1980's in Italy has generated widespread worries about the ability of the government to sustain its current fiscal and financial policy without resorting to extraordinary policy measures involving (partial) repudiation, high inflationary monetization or *once and for all* capital levy.

Among the requirements for sustainability, the intertemporal budget constraint imposes only mild restrictions on the behaviour of the public sector. In practice, almost any short run path of revenues and expenditures can be consistent with the intertemporal budget constraint. Large and persistent deficits today can always be offset by running large surpluses some time in the future.

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Indeed, a general discussion of sustainability must take into account broader questions such as possible threats to financial stability and the political acceptability of the distributive effects it may involve. At the same time, a more focused and empirically oriented consideration of solvency may help clarify some of the issues involved in the public sector patterns of debt and deficit.

This paper addresses the issue of testing for Public Sector solvency with an application to Italy by using the framework of analysis developed by Hamilton and Flavin (Hamilton and Flavin 1986) and Wilcox (Wilcox 1989), as modified by Buiter and Patel (Buiter and Patel 1990).

Tests for solvency develop from the basic idea of verifying whether the present value budget constraint of the public sector in a dynamically efficient economy would be satisfied a) *had the fiscal and financial policy in a given time period been pursued indefinitely*, and b) were the relevant features of the macroeconomic environment characterizing the sample period stable over time.

If solvency is not supported by the empirical evidence, a change either in policy or in the relevant macroeconomic variables (growth, inflation, interest rate) must occur at some point in the future time.

It should be obvious that the tests refer to the feasibility rather than the optimality of the fiscal and financial policy.

Recent contributions in the literature have approached solvency-related issues from a different point of view. In the case of Italy, for example, it has been argued that the high level of public debt is associated with a risk premium on government bonds reflecting fears of repudiation.

Important differences can be stressed between this approach and the one

followed in this paper. First, risk premia on government bonds may reflect much more than the violation of a long run budget constraint: distributional and political considerations, for instance. Second there is no need for the solvency constraint to be met in each sample period, provided that expectations of a regime change make the policy within the sample viable. In the first case, we may observe risk premia on government bonds even when the government is solvent. In the second case, insolvency of the public sector with respect to the sample period may not be reflected in the pricing of the public debt.

This paper is organized as follows. Section I briefly reviews the arithmetic of the government budget constraint. Section II discusses the meaning of solvency in the framework of the dynamic efficiency of the economy. Section III addresses the issue of public capital in setting up the test. Sections IV and V present the methodological aspects of the tests and the empirical evidence. The main results of the paper are summarized in the Conclusion.

## I

### THE PUBLIC SECTOR BUDGET CONSTRAINT The arithmetic of solvency

This section develops the arithmetic of the public sector's budget constraint following the framework and notation by Buiter and Patel (Buiter and Patel 1990, Buiter 1990).

The definition of the Public sector includes central and local government, the central bank, public agencies, social security funds and public enterprises. For simplicity, the analysis is carried out in terms of

one period debt, assuming a unique (holding period) interest rate.

The consolidated public sector budget identity is as follows.

$$\begin{aligned} & \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - B_{t-1}}{P_t} + \frac{V_t (\bar{B}_t^* - \bar{B}_{t-1}^*)}{P_t} - \frac{V_t (F_t^* - F_{t-1}^*)}{P_t} - (K_t - K_{t-1}) \equiv \\ & \equiv C_t - T_t + i_{t-1} \frac{B_{t-1}}{P_t} + i_{t-1}^* \frac{V_t}{P_t} (\bar{B}_{t-1}^* - F_{t-1}^*) - (\rho_{t-1} - \delta_{t-1}) K_{t-1} \quad (I.1) \end{aligned}$$

where  $M$  is the nominal stock of monetary base,  $B$  and  $\bar{B}^*$  are the stocks of domestic and foreign currency denominated public debt,  $F^*$  is the stock of foreign reserves,  $K$  the stock of capital owned by the public sector and evaluated at current reproduction costs,  $C$  is consumption by the public sector,  $T$  net current revenue,  $i$  and  $i^*$  are the domestic and foreign nominal interest rates,  $V$  the foreign exchange rate,  $P$  the domestic price level,  $\rho$  the cash rate of return on public sector capital and  $\delta$  the depreciation rate.

The identity (I.1) distinguishes between stock adjustment and current account. Define total interest bearing liabilities net of both real and financial assets as *net debt* (ND). The left-hand side of (I.1) refers to changes in both real and financial asset holdings, i.e. to the evolution of ND, in the period; the right-hand side contains current expenditures and revenues, including net interest payments<sup>2</sup>.

Net investment by the public sector appears on the left-hand side of the identity (I.1). Conventional budget surplus, however, includes capital expenditure. Identity (I.1) can then be rewritten bringing the corresponding term to the right-hand side.

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(2) The gross rate of return on public capital supposedly consists of its cash rate of return plus a component in the form of either net tax revenues or reduction in expenditure.

Let  $A_t = K_t - (1 - \delta_{t-1}) K_{t-1}$ ; (I.1) thus becomes

$$\begin{aligned} & \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - B_{t-1}}{P_t} + \frac{V_t (\bar{B}_t^* - \bar{B}_{t-1}^*)}{P_t} - \frac{V_t (F_t^* - F_{t-1}^*)}{P_t} \equiv \\ & \equiv C_t + A_t - T_t + i_{t-1} \frac{B_{t-1}}{P_t} + i_{t-1}^* \frac{V_t}{P_t} (\bar{B}_{t-1}^* - F_{t-1}^*) - \rho_{t-1} K_{t-1} \end{aligned} \quad (I.2)$$

The left-hand side of the expression now refers to the evolution of *net financial debt* (NFD), defined as the total liabilities of the public sector less the total *financial assets* owned by that sector.

If the public sector owned no real assets, there would be no difference between *net* and *net financial debt*. On the other hand, borrowing in order to finance investment doesn't change the latter but does change the former. The evolution of the two variables may be quite different.

As will be shown below, these considerations will be important in the practical setting up of empirical tests, where sample periods are limited and most of the available information refers to public sector *net financial debt*.

Identity (I.1) and identity (I.2) are both equally valid as the starting point of the analysis undertaken in this section. Since (I.2) provides a notation closer to the available information, we will begin with this second alternative.

For the sake of simplicity, assume a unique interest rate  $i^*$  on both external debt and international reserves, so that  $B^* = \bar{B} - F$  will stand for net official foreign debt.

Expressing the variables in identity (I.2) in terms of their ratio to GDP, some algebraic manipulations lead to the following

$$\begin{aligned} b_t + b_t^* & \equiv c_t + a_t - \tau_t - \frac{\rho_{t-1}}{(1+n_{t-1})} k_{t-1} - \sigma_t + \\ & + b_{t-1} \frac{1 + i_{t-1}}{(1+\pi_{t-1})(1+n_{t-1})} + b_{t-1}^* \frac{(1+i_{t-1}^*)(1+\epsilon_{t-1})}{(1+\pi_{t-1}^*)(1+n_{t-1}^*)} \end{aligned} \quad (I.3)$$



where lower case letters are the corresponding upper case variables as a ratio to GDP,  $\pi_t$  is the inflation rate,  $n_t$  the GDP growth rate,  $\epsilon_t$  the depreciation rate of the foreign exchange rate and  $\sigma_t$  seigniorage as proportion of GDP (i.e.  $\sigma_t \equiv \frac{M_t - M_{t-1}}{P_t Y_t}$ ).

The expression (I.3) can be written more compactly by using the implicit interest rate,  $i_{im}$ , defined as the ratio between total (foreign and domestic) interest bill divided by the total stock of debt outstanding at the beginning of the period.

Let  $d_t$  be the ratio of total (internal and external) *net financial debt* to GDP ( $d_t \equiv b_t + b_t^*$ ) and  $\delta_t$  the *primary (non interest) deficit* in percentage of GDP, i.e.

$$\delta_t \equiv c_t + a_t - r_t - \frac{\rho_{t-1} k_{t-1}}{1+n_{t-1}}. \quad (I.4)$$

Denote by  $r$  the implicit interest rate in real terms and by  $\xi$  its value net of the GDP growth rate ( $\xi \equiv r - n$ ).

The identity (I.3) becomes

$$d_t \equiv (1 + \xi_{t-1}) d_{t-1} + \delta_t - \sigma_t. \quad (I.5)$$

Identity (I.5) tells us that the public sector net financial debt increases relative to GDP if the sum of primary deficit *plus* interest payments, calculated at the inflation-corrected, growth-corrected cost of borrowing, *minus* revenue from seigniorage, is positive.

Solving (I.5) recursively forward in time and denoting with  $E_t$  the expectation operator conditional on information at time  $t$ , we obtain

$$d_t = \sum_{i=0}^{\infty} E_t \prod_{j=0}^i (1 + \xi_{t+j})^{-1} [ -\delta_{t+1+i} + \sigma_{t+1+i} ] + \lim_{i \rightarrow \infty} E_t \prod_{j=0}^i (1 + \xi_{t+j})^{-1} d_{t+1+i}. \quad (I.6)$$

Consider now the definition of solvency.

The public sector is solvent when, in expectation, the present discounted value of primary surplus minus the present value of revenue from seigniorage is at least equal to the value of the outstanding stock of net financial debt.

According to this definition, to obtain solvency we need the last term of (I.6) to be non positive.

Define as  $q_i$  the discount factor between period 0 and period  $t+i$ ,

$$q_i = \prod_{j=0}^i (1+\xi_j)^{-1},$$

and pose  $q_{-1} = 1$ . The terminal condition we need to impose for solvency can be rewritten as

$$\lim_{i \rightarrow \infty} (q_{t-1})^{-1} E_t q_{t+i} d_{t+1+i} \leq 0. \quad (\text{I.7})$$

In other words, the expectation conditional on information at time  $t$  of the present value of future public net financial debt cannot be positive in the limit. The public sector cannot be a net debtor in present value.

Negative values of the limit result in a sort of *supersolvency*, since the expected terminal condition of the public sector is that of net creditor. This circumstance is not plausible but cannot be ruled out *a priori*. In this case, some other sector in the economy must be violating its solvency constraint<sup>3</sup>.

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(3) In part of the literature on empirical testing for solvency, (I.7) is written as an equality. This provides a suitable theoretical framework to adapt testing procedures for "no bubble" conditions in asset pricing (see Hamilton and Flavin 1986:812). In order to rule out the inequality sign, it could be argued that a strict negative inequality would imply a strict *positive* inequality for some other agent in the economy. This agent would then be playing a *Ponzi Game* against the government. If we exclude this case,

Note that the discount factor  $q_1$  is a function of the interest rate effectively paid on the public sector's debt, i.e. the actual cost of borrowing. Therefore the sequences of the debt stock and the discount factor are not independent, and there is no freedom to choose alternative discounting<sup>4</sup>.

In (I.3) through (I.7), the variables are measured in terms of units of GDP and discounted at  $r-n$ . Solvency says that *ultimately* the debt stock cannot grow faster than the interest rate on the debt. The unit with which the variables are measured is irrelevant. The analysis could be carried out in terms of either real or nominal variables, discounted at the real and nominal implicit interest rate, respectively, with any currency chosen as unit of account<sup>5</sup>.

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than (I.7) as an equality sign would refer to a sort of bilateral *no Ponzi Game* conditions for both the government and the private sector.

(4) In this sense, by using  $i_{im}$  in the discount factor, the discounted series of net debt fully captures the fiscal and financial policy pursued by the public sector as reflected by primary deficits, seigniorage revenue and interest payments. Nonetheless, from an accounting perspective, any arbitrary interest rate  $i^*$  could be used in discounting the debt, as long as the primary deficit is *augmented* in order to accommodate for the difference in interest payments stemming from  $(i_{im} - i^*)$ . However, there is a crucial testing problem related to this procedure. The augmented primary deficit will now depend on the stock of debt. In order to test for solvency, it will not be possible to focus only on the series of discounted debt, as we do in this paper. In practice, even if the public sector is not solvent, it will always be possible to find an  $i^*$  such that the discounted debt is zero in the limit, implicitly preventing the series of discounted augmented primary deficit from converging.

(5) Consider the following. Solvency says that *ultimately*  $B_t/B_{t-1} < (1+i_t)$ . Here the variables are measured in nominal terms. In real terms we would have  $(B_t/B_{t-1}) * (P_{t-1}/P_t) < (1+i_t)/(1+\pi_t)$ . Rearranging this, we have  $(B_t/B_{t-1}) < (1+i_t) * (P_{t-1}/P_t) / (1+\pi_t) = (1+i_t)$ , which is equivalent to the nominal variables case. Notice that, as long as the nominal interest rate is positive, we can always solve (I.5) forward. In other words, the solvency criterion (I.7) is independent of  $\xi$  (or  $r$ ) being positive.

## II

### THE PUBLIC SECTOR BUDGET CONSTRAINT

#### The theory of solvency

What is the meaning of (I.7)? Does the public sector face a present value budget constraint as households do? This section has the objective of answering these questions by providing a synthesis of some key results in the literature on public debt.

If the economy were dynamically inefficient, there would be excess capital and a decumulation could improve welfare. In a model *à la* Diamond, the stationary state of such an economy would be characterized by a net marginal productivity of capital - in the absence of uncertainty and taxation, this is equal to the real interest rate - which is lower than the output growth rate. Under these conditions, there would be no theoretical reason for the present value budget constraint of the public sector to hold. *Ponzi Games* would be feasible and the public sector could service the outstanding debt by borrowing more *ad infinitum*. In this economy, the link between debt today and collecting the resources necessary to its repayment through budget surpluses and seigniorage in the future would be severed.

Note that the efficiency condition ( $r > n$ ) refers to the (before tax) net marginal product of capital. Both uncertainty and taxes can make the net real interest rate, which is the relevant cost of debt service, diverge from the rate of return on capital. And in fact, for a long span of time before 1982 the Italian real interest rate on public debt has been below the rate of growth of the economy, while the mean return on capital has been above, as suggested by the fact that gross profits persistently exceed gross investment (see Abel et al 1989).

Therefore, the circumstance that  $r$  is lower than  $n$  even for prolonged

time periods cannot be taken as a test of the dynamic efficiency of the economy.

In a dynamically efficient economy (which we assume Italy to be) primary deficits less current seigniorage today have important implications for the amount of resources the public sector will have to collect in the future. The present value budget constraint must be satisfied and (I.7) must hold.

In practice, (I.7) implies that the expected growth rate of the public debt will be ultimately lower than the interest rate on the debt.

This is a weak criterion to evaluate the sustainability of the public debt. It is well known that (I.7) can be satisfied also when the debt/GDP ratio increases without bound, provided that its rate of growth is less than  $\xi$ . Note that in this case the interest bill on the debt will at some point be larger than the whole GDP. Public sector solvency can then be logically justified by stressing three important elements implied by the approach. First, solvency when the debt/GDP ratio grows unboundedly requires that, at some point, the tax base consists of both GDP and interest payments. Second, tax must be non distortionary (lump sum) and there should be no relevant costs of collection and enforcement. In this case the increasing tax burden has no effect on the availability and (efficient) allocation of resources. Third, the analysis ignores the distributive effects of a growing debt and the political acceptability of those effects. These and similar considerations support the common view that only finite values of the debt/GDP ratio are sustainable.

Therefore, it could be helpful to use a stronger solvency condition than (I.7), and to make a distinction between a *weak* and a *practical (strict)* solvency criterion (Buiter and Patel 1990).

The weak criterion is based on (I.7). As shown above, it requires that ultimately the discounted debt stock grow at the rate lower than the interest on the debt. As will be discussed in Section IV, the discounted debt series must have no trend, either deterministic or stochastic. The practical criterion assumes that only finite debt/GDP ratios are feasible and rules out the presence of trends also in the series of the *undiscounted* debt/GDP ratio  $d_t^6$ .

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(6) Consider also the implications of solvency constraint with respect to various measures of the budget deficit. The question is whether a positive outstanding debt in an efficient economy where  $r > n$  requires a promise by the public sector to run budget surpluses at some date in the future. The *conventional* definition of financial surplus (CFS) can be written as follows.

$$CFS = T + \rho K - [C + A] - \left[ i \frac{B}{P} + i^* \frac{V}{P} B^* \right] \quad (a1)$$

Neither solvency criterion necessarily implies positive values of the CFS at any time, even disregarding seigniorage. This is not to say that a budget surplus cannot be either desirable or optimal in some circumstances. The point is that in principle the solvency constraint in a dynamically efficient economy can be satisfied without resort to seigniorage even in the presence of a series of continuous budget deficits. This point can be developed by extending the analysis to the *operational* and *primary* financial surplus. Correcting the contribution of the interest bill to the deficit (a1) for the inflation rate, the growth rate of GDP and the currency depreciation rate leads to the definition of *operational* surplus (OFS).

$$OFS = T + \rho K - [C + A] - \left[ \frac{1+i}{(1+\pi)(1+n)} - 1 \right] \frac{B}{P} + \left[ \frac{(1+i^*)(1+\epsilon)}{(1+\pi)(1+n)} - 1 \right] \frac{V}{P} B^* \quad (a2)$$

Assume no seigniorage revenue. If no operational surplus were ever achieved, the ratio debt/GDP would grow without limit. In principle, this does not violate the weak solvency criterion, but does violate the practical one. Therefore, in absence of seigniorage, operational surpluses at some point in time are necessary to satisfy the practical solvency criterion. They would not be necessary, though, if seigniorage revenue was allowed for. By the same token, the achievement of primary (non interest) surpluses at some point is a necessary condition for the weak solvency criterion (and a *fortiori* for the practical one) only in the absence of seigniorage.

### III

#### PUBLIC CAPITAL AND INVESTMENT

Consider the bench-mark case of the net return on the public sector capital being equal to the interest rate. *Ceteris paribus*, borrowing in order to finance investment does not affect the net worth of the public sector.

Given short sample periods, the solvency constraint in the presence of a sustained increase in the (discounted) *net financial debt*/GDP ratio does not necessarily imply the need for an *active* change of the fiscal and financial strategy at some point in the future. To the extent that the additional debt finances capital formation, the trend displayed by the debt/GDP ratio will be modified *automatically* as soon as the installed capital becomes productive.

In other words, if the investment projects guarantee a rate of return (in cash or more generally in terms of net reduction in the primary deficit) at least equal to the borrowing rate, budget deficits to support capital expenditure are neutral, even desirable, with respect to the solvency constraint. Possible expansion paths of the public debt used to finance capital expenditure would be followed by periods of increasing primary surpluses.

Note that in principle, regardless of the rate of return on public capital, subtracting the value of public sector's physical assets from the net financial debt rules out the risk of misinterpreting trends in the (discounted) debt/GDP ratio. Indeed, the whole analysis in section I can be recast in terms of *net debt*. Of course, this will lead to quite different results, depending on how we value the public capital: at reproduction cost or at market value, under the assumption that each asset will remain owned by

the public sector or will be sold to the private sector. Valuation under alternative assumptions will possibly give upper and lower boundaries to the net debt which will contain valuable information in order to assess public sector's solvency.

In practice, though, formidable empirical difficulties are present in assessing the value of and the return on the real assets owned by the public sector: currently available information refers mainly to financial debts and assets. This is a practical testing problem with no clear solution. In the case of Italy, for example, gross investment by the public sector (excluding public corporation) has been a rather stable fraction of GDP (Figure 4) in the last 25 years. Nonetheless, this fraction has increased slightly through the 80's, while it has decreased in most OECD countries. As usual, an important caveat regarding the nature of *capital expenditure* in public accounting schemes should be kept in mind when comparing these figures.

#### IV

#### TESTING FOR SOLVENCY

As shown in section I, the present value budget constraint implies that the appropriately discounted series  $D_t$  be non positive in expectation in the limit. This is one testable implication of the solvency constraint: the unconditional expectation of the discounted debt of the public sector must be non positive. An application of time-series methodology offers a direct way to address the issue.

As a preliminary step, the series of net financial debt should be discounted back to some base period. The next step will be to test whether the data generating process (DGP) which describes the behavior of the series



over the sample period is covariance stationary. If it is, one can test whether the unconditional mean of the process is non positive. A positive drift or time trend will eventually imply insolvency.

If non stationarity of the process cannot be rejected, in principle either insolvency or supersolvency of the public sector may obtain. One can, however, dispose of this second possibility by noticing that roots less than or equal to -1 are implausible and that the initial debt will (almost) always be positive. Moreover, determinist components such as a drift can always belong in the DGP of the series.

From the point of view of test implications, therefore, if either a stochastic or a positive deterministic trend is found in the series, insolvency will follow - but only in the absence of structural changes in the process at some time in the future. In other words, either the fiscal and financial policy or the relevant features of the economy must change at some point in time in order for the intertemporal budget constraint of the public sector to be satisfied.

It should be clear by now that the test is not aimed at forecasting *Staatsbankrott*.

From the point of view of the testing procedure, these changes in the DGP may occur within the sample period. However, traditional tests of structural breaks can be carried out only for (covariance) stationary series. If non stationarity cannot be rejected, the stability of the DGP over the whole sample or sub-periods will be posed as a maintained hypothesis.

A general framework for the empirical analysis is provided by Wilcox, who was the first to base the test on the series of the discounted debt (Wilcox 1989). He assumes that the series  $D_t$  is represented by the following

ARIMA model

$$[1-\rho(L)] [(1-L)^d Z_t - \alpha_0] = [1-\theta(L)] e_t \quad (IV.1)$$

where  $\rho(L)$  is a p-th order polynomial,  $\theta(L)$  a q-th order polynomial,  $Z_t$  is a random vector whose first element is  $D_t$ ,  $\alpha_0$  is a vector of constants and  $e_t$  is a vector white noise process.

Assume that the series  $(1-L)^d Z_t$  is covariance stationary, or, equivalently, that  $Z_t$  is integrated of order d. Thus  $(1-\rho(L))$  and  $(1-\theta(L))$  have all their roots outside the unit circle while  $\rho(L)$  and  $\theta(L)$  are assumed to satisfy the conditions for stationarity and invertibility.

The autoregressive representation of the process is therefore

$$[1-\theta(L)]^{-1} [1-\rho(L)] [(1-L)^d Z_t - \alpha_0] = e_t \quad (IV.2)$$

which is operational if the process can be approximated by a finite order autoregressive process. Note that  $\alpha_0$  is the unconditional expectation of  $(1-L)^d Z_t$ .

Testing for solvency involves the following steps. First test whether  $Z_t$  is stationary, i.e. whether its order of integration d is less than 1/2. In a univariate representation of the process, in which  $Z_t$  is a scalar and equal to  $D_t$ , a unit root in the DGP governing the variable makes the process inconsistent with solvency. Further testing for a non zero drift can also be carried out.

If the process is stationary, the second step consists of estimating its unconditional expectation  $\alpha_0$ , or, more simply, in testing whether the first element of  $\alpha_0$  is non positive.

The main advantage of Wilcox's framework is that it points out synthetically a sufficient condition for solvency: a stationary and purely

non deterministic process for the series of discounted debt<sup>7</sup>.

V

EMPIRICAL EVIDENCE

Italian data on public debt consolidate only part of the vast public sector. They do not include major public holding companies such as IRI, ENI and EFIM, and the national electric company. In the absence of a time series for the value of the sector real assets, the test will be carried out by using the series of public *net financial debt*.

The implicit interest rate cannot be calculated on a quarterly basis. In what follows, this series will be approximated by a weighted average of

(7) Hamilton and Flavin's pioneering paper on empirical analysis of solvency focused on the primary deficit, rather than on the discounted debt. Let  $x_t$  be the *cum seigniorage* augmented primary surplus ( $x_t \equiv -\Delta_t + \sigma_t$ ) and  $X_t$  its discounted value at time  $t=0$ ,  $X_t \equiv \sum_{i=0}^{\infty} q_{t-1}^i x_{t+i}$ . Solvency implies  $D_t \leq \sum_{i=0}^{\infty} E_t X_{t+i}$ .

It is important to stress that in this case a purely non deterministic stationary process for the series  $X_t$  will not be sufficient to ensure solvency. The argument is made by considering the above expression with an equality sign:  $D_t$  equals the present value of the infinite series of the cum seigniorage augmented primary surplus. The point is that the infinite sum of stationary processes may be non stationary, producing one of the cases discussed in the text. Moreover, Wilcox shows that even if the sum of  $X$  is stationary, for some classes of stochastic process it will not be equal to  $D_t$  (Wilcox 1989:296). Consider the following example. Suppose  $X_t$  follows a simple stationary autoregressive process,  $X_t = \alpha X_{t-1} + \eta_t$ , with  $|\alpha| < 1$  and  $\eta_t$  white noise. Then  $E_t \sum_{i=0}^{\infty} X_{t+i} = \frac{\alpha}{1-\alpha} X_t$ , which implies  $D_t = \frac{\alpha}{1-\alpha} X_t$  and  $D_t - D_{t-1} = -X_t + \frac{1}{1-\alpha} \eta_t$ . The latter expression is inconsistent with an implication of the solvency constraint when (I.7) is written with an equality sign, which is  $D_t = D_{t-1} + \Delta_t - S_t = D_{t-1} - X_t$ .

Thus, a simple stationary first order autoregressive process for the series of the discounted augmented primary deficit net of seigniorage would not be consistent with solvency.

interest rates on different public financial assets.

Given the features of the available information, the results of the test should be considered with caution. Detailed information about the data set is contained in the Appendix I.

The first class of tests refers to the discounted series  $D_t$ . A consistent series of yearly data is available from 1970 to 1988. Quarterly data are available from the last quarter of 1975 to the last quarter of 1988. The two series, shown in Figure 1 and 2, are labelled  $D(A)$  and  $D(Q)$ , respectively.

The quarterly data series covers 15 years characterized by the emergence of the fiscal crisis in Italy. *A priori*, we would expect a strong rejection of the hypothesis of stationarity and/or absence of deterministic trends in the series.

Nonetheless, the stability of the model's parameters over this period can be challenged with respect to several features of the policies and the evolution of the economy in that time span. We believe that at least one important point of change should be explicitly dealt with in the test. This could be located in the second half of the year 1981, as also suggested by the evolution of the variables in Figure 1. Since that year, the Italian Central Bank has no obligation to buy (residually) Government debt not absorbed by the private sector. Also in 1981, the real interest rate charged on treasury bills has become positive, followed in 1982 by the implicit interest on total debt.

As stated in section IV, the test consists of two sequential parts concerning stationarity and the presence of deterministic components in the series.

Stationarity is tested by using the Phillips-Perron approach on a univariate case of Wilcox Model (IV.1) (Phillips and Perron 1987; Phillips 1987; Perron 1988). The equation to be estimated is

$$D_t = \mu + \beta(t-T/2) + \alpha D_{t-1} + u_t \quad (V.1)$$

where  $\{u_t\}_0^\infty$  is a weakly stationary sequence of random variables satisfying the following:

$$E(u_0) = 0;$$

$$E||u_0||^{\beta+\epsilon} < \infty \text{ for some } \beta > 2;$$

$$\{u_t\}_0^\infty \text{ is a strong mixing with mixing number } \alpha_m \text{ which satisfy } \sum_{m=1}^{\infty} (\alpha_m)^{1-2/\beta} < \infty. \quad (V.2)$$

The conditions imposed on the sequence  $\{u_t\}_0^\infty$  are very general. A wide variety of data generating mechanisms for  $D_t$  are permitted, including virtually any ARMA with a unit root<sup>8</sup>.

The advantage of the Phillips-Perron test is that, without loss of generality, the test statistics require only the estimation of a first order autoregressive model by OLS and a correction factor based on the structure of the residuals from this regression.

The first null hypothesis of interest is  $H_0: \alpha=1$ . Three statistics are possible:  $Z(\alpha)$ , which uses the standardized and centered least squares estimates of  $\alpha$ ;  $Z(t_\alpha)$ , which uses the t-statistic on  $\alpha$ ;  $Z(\Phi_3)$ , which is the regression "F-test" studied by Dickey and Fuller (Dickey and Fuller 1981) for the general class of error process in (V.2)<sup>9</sup>.

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(8) For a non technical discussion of these conditions, see Perron 1988.

(9) It should be kept in mind that any existing test of the unit root hypothesis has very low power against stationary alternatives with a root close to unity.

The limiting distributions of  $Z(\alpha)$  and  $Z(t_\alpha)$ , however, are not invariant with respect to the trend parameter  $\beta$  under the null hypothesis of a unit root. The statistic  $Z(\Phi_3)$  is the proper statistic for the joint null  $H_0: \alpha=1, \beta=0$ , within a maintained hypothesis which permits a possible non zero drift.

If stationarity is rejected, an additional test for a non zero drift in the process is carried out. The Phillips-Perron statistic for the null  $H_0: (\alpha=1, \beta=0, \mu=0)$  is the  $Z(\Phi_2)$ , which is again the corresponding Dickey-Fuller regression "F-tests" for the general class of error (V.2)<sup>10</sup>.

The source of critical values is Fuller (1976:371-73) for  $Z(\alpha)$  and  $Z(t_\alpha)$ , Dickey and Fuller (1981:1062-63) for  $Z(\Phi_3)$ ,  $Z(\Phi_2)$  and  $Z(t_\mu)$ .

Since these statistics are influenced by the choice of a lag truncation number ( $\lambda$ ), cases in which the result of the test depends on the number of lags will be starred in the tables. In general, the tables will report only one statistic for some particular  $\lambda$  ( $\lambda=12$  and  $\lambda=5$  in the case of quarterly and yearly observations, respectively), as well as the confidence level at which the corresponding  $H_0$  can be rejected.

The plan of the remaining part of this section is as follows. First, the tests for non stationarity and a non zero drift are carried out with respect to both the discounted debt and the undiscounted net debt/GDP ratio. This follows from the distinction between the *weak* and the *practical* solvency criterion, discussed in section II.

The same testing procedure will then be replicated for two sub-samples

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(10) The statistic  $Z(t_\mu)$  (for the null  $H_0: \mu=0$ ) is also possible, but it will not be invariant with respect to the initial observation in the sample.

of the quarterly data series.

Some considerations about the stability of the macroeconomic environment and evidence about the path of fiscal deficit will conclude the section.

Table 1 refers to the whole sample period for both the quarterly and the yearly discounted data series. The evidence points out non-stationarity in either case; moreover, the values of  $Z(\Phi_2)$  and  $Z(t_\mu)$  reject the null of a zero drift. We cannot therefore rule out the presence of a deterministic component in the process.

Differencing the data makes the quarterly series stationary; for the yearly data, on the other hand, this result is not unequivocally supported by the statistics.

Table 2 shows the test statistics for the series of the undiscounted net financial debt to GDP ratio. The absence of deterministic or stochastic trends from the series is to be related to a *practical* solvency criterion, reflecting the common view that only finite values of the ratio are sustainable.

As far as the quarterly data series is concerned, non-stationarity is supported by the first two statistics in the table. However, both  $H_0: (\alpha=1, \beta=0, \mu)$  and  $H_0: (\alpha=1, \beta=0, \mu=0)$  can be rejected on the basis of  $Z(\Phi_3)$  and  $Z(\Phi_2)$ . In this case, then, the evidence in favor of the presence of a unit root as opposed to a time trend is not strong. Given the path of the variable (Figure 1), this result reinforces the conclusion of insolvency for the Italian public sector.

The evidence for the yearly data series points out non-stationarity. The hypothesis of a zero drift, cannot, however, be rejected.

In order to address the issue of (possible) structural changes within

the sample period, tests have been carried out with respect to sub-samples of the discounted quarterly series. The presumption is that a structural change has occurred in the second half of 1981. This will be taken as a maintained assumption.

As shown in Table 3, the presence of a unit root cannot be rejected in either subsample (75:4-81:2 and 81:3-88:4). Nonetheless, there is an interesting difference in the value of both  $Z(\Phi_2)$  and  $Z(t_\mu)$ , suggesting a non zero drift only for the second part of the sample.

As expected, all our results agree with the view that the current Italian fiscal and monetary policy is not on a sustainable path.

Nevertheless, the test is conditional on the absence of major changes in the relevant macroeconomic variables, in particular growth, interest rate and inflation - changes which would exempt the government from undertaking fiscal reforms in order to satisfy the intertemporal budget constraint.

Table 4 reports the value of these variables within the sample period.



Table 1

Phillips Perron Test for Unit Roots  
Discounted Net Financial Debt

Statistics:					
	$Z(\alpha)$	$Z(t_{\alpha})$	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\mu})$
-----					
Series:					
$D(Q)$	-4.09	-2.10	4.87	12.08	2.61
	(<90)	(<90)	(<90)	(>99)	(>95, <97.5)
$\Delta D(Q)$	-37.64	-7.05	12.42**	-	-
	(>99)	(>99)	(>99)		
$D(A)$	-5.18	-1.40	1.35	6.46	1.82
	(<90)	(<90)	(<90)	(>99)	(<90)
$\Delta D(A)$	-17.64	-5.56	5.89	-	-
	(<90)	(>99)	(<90)		

Sample period: 1975:4 - 1988:4 for  $D(Q)$   
1970 - 1988 for  $D(A)$

Note: Number in parenthesis indicates the confidence level with which  $H_0$ :Unit Root, is rejected. The annual series includes 17 observations, the quarterly series 53. \*\* indicates that for some  $\lambda$  the confidence level is 95% instead of 99%.

## Variables:

$D(Q)$  is the series of discounted debt, quarterly data;  
 $\Delta D(Q)$  the series of the first differences of  $D(Q)$ ;  
 $D(A)$  is the series of discounted debt, yearly data;  
 $\Delta D(A)$  the series of the first differences of  $D(A)$ .

Table 2

Phillips Perron Test for Unit Roots  
Net Financial Debt/GDP (Undiscounted)

Statistics:					
	$Z(\alpha)$	$Z(t_\alpha)$	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_\mu)$
Series:	-----				
D/GNP(Q)	-3.60	-2.25	9.20	10.00	2.71
	(<90)	(<90)	(>99)	(>99)	(>95, <97.5)
$\Delta(D/GDP)(Q)$	-40.36	-5.69	15.62	-	-
	(>99)	(>99)	(>99)		
D/GDP(A)	-1.64	-0.51	0.91	2.16	0.83
	(<90)	(<90)	(<90)	(<90)	(<90)
$\Delta(D/GDP)(\Delta A)$	-12.90	-3.89	3.40	-	-
	(<90)	(>97.5, <99)	(<90)		

Sample period: 1975:4 - 1988:4 for D(Q)  
1970 - 1988 for D(A)

Note: Number in parenthesis indicates the confidence level with which  $H_0$ : Unit Root, is rejected. The annual series includes 17 observations, the quarterly series 53.

## Variables:

D/GDP(Q) is the series of debt/GDP, quarterly data;  
 $\Delta(D/GDP)(Q)$  the series of the first differences of D/GDP(Q);  
 D/GDP(A) is the series of debt/GDP, yearly data;  
 $\Delta D/GDP(A)$  the series of the first differences of D/GDP(A).

Table 3

Phillips Perron Test for Unit Roots  
Discounted Net Financial Debt for two sub-samples

a) 1975:4 - 1981:2 and b) 1981:3 - 1988:4

Statistics:					
	Z( $\alpha$ )	Z( $t_{\alpha}$ )	Z( $\Phi_3$ )	Z( $\Phi_2$ )	Z( $t_{\mu}$ )
Series:	-----				
D(Q)a	-3.56	-1.41	0.38	0.42	1.47
	(<90)	(<90)	(<90)	(<90)	(<90)
$\Delta$ D(Q)a	-10.88	-4.13	-2.89*	-	-
	(<90)	(>99)	(<90)		
D(Q)b	-13.55	-3.28	2.28	22.96	3.45
	(<90)	(<95)	(<90)	(>99)	(<90)
$\Delta$ D(Q)b	-22.96	-12.16	36.54**	-	-
	(>95,<97.5)	(>99)	(<90)		

Note: Number in parenthesis indicates the confidence level with which  $H_0$ :Unit Root, is rejected.

A \* indicates that for a lower  $\lambda$  the confidence level will be <95%.

A \*\* indicates that for a  $\lambda < 3$  it is possible to reject the null of a unit root at 99% level.

Variables:

D(Q) is the series of debt/GDP, quarterly data;  
 $\Delta$ (D(Q)) the series of the first differences of D(Q);  
"a" and "b" refer to the two sub-samples.

Table 4

## Growth, inflation and interest rate in the sample period

	71-88	76-88	76:1-89:1	76:1-81:2	81:3-89:1
Implicit interest rate	9.88	11.21	-	-	-
Interest rate on 3-6 month bill	-	-	14.50	15.10	13.00
Inflation (GDP deflator)			13.11	18.03	9.62
Growth rate			2.99	2.46	3.75

The striking data in Table 4 is the high level of the real interest rate in the 80's. This has been significantly higher in Italy than in all other G-7 countries since 1987. The basic issue is therefore whether the current Italian commitment to the EMS and the process of European integration will lead to a decline in the real cost of debt, as predicted by those who see the integration process as an opportunity for the Italian government to buy policy credibility and carry out basic fiscal reforms. It should be noted that even in this optimistic scenario, the real interest rate can hardly be expected to go below the growth rate or negative, as in the experience of the 60's and the 70's.

The alternative view tends to stress the fiscal stance as a prerequisite for the success of the integration process. In this case, the lack of credibility of any given program for fiscal reform could keep nominal (and a fortiori real) interest high in the context of the disinflation policy currently pursued by Italian authorities, making the pattern of real interest

rate shown in table 4 persistent over time, while seigniorage revenue will be sharply declining. Ultimately, this may entail doubts about the status of Italy as reliable member of the union.

Under either scenario, it is doubtful that a fundamental reform of the fiscal policy can be avoided. In the first case, the decline of the interest bill must be compared with the reduction in seigniorage revenue due to disinflation as well as the harmonization of the bank reserve requirement. In the second scenario, the fiscal stance will be blamed as the main obstacle towards integration with other European countries.

Figures 3 and 4 show the path of total deficit, public investment, and primary (non-interest) deficit over the sample 1971-1987. All variables are expressed in percentage of GDP.

Throughout the 1980's, the primary deficit has been persistently positive (Figure 3).

Notice the stability of the ratio public investment to GDP (Figure 4), even when, with a debt equation which has become unstable in the 1980's, sustainability considerations may have induced generalized cuts in the expenditure. The discussion in section III provides a framework to address the issue, even if the empirical implementation of test for solvency controlling for capital formation is practically ruled out by the lack of data.

## Conclusions

The solvency of the public sector in Italy has been tested and rejected by focusing on the series of the debt net of financial asset holding and international reserves.

The size of the sample, the quality of the data and the features of the unit root test suggest caution in the assessment of these results. Nonetheless, a simple visual inspection of figures 1 and 2 will reinforce the conclusions.

The testing methodology raises several issues which have been mentioned throughout the paper.

The first is conceptual and relates to the size of the sample. Meeting the intertemporal budget constraint is a long run concern. The test is aimed at assessing whether sample related policies will eventually lead to solvency, if pursued over the distant future. Intuitively, we expect the rejection of solvency to become less likely as the sample size increases.

Past behavior will have more weight and, even in the case of financially troubled governments, can reflect *ex post* solvency. Testing over samples covering a short span, however, decreases the power of the test. In particular, non stationarity of the series will be difficult to reject, biasing the test towards rejection of the hypothesis of solvency. In general, then, it will not be possible to test for structural breaks occurring within the sample using a traditional approach (such as the Chow Test). The test will not be sensitive to structural breaks of unsustainable paths occurring within the sample.

The second issue is the need for an adequate treatment of capital financing *vis a vis* current expenditure. In the case of Italy, for example,

the consolidation of the public sector accounts with the vast public holding companies may change radically the perception of the domestic debt problem with respect to solvency. Developing the analysis in terms of debt net of real assets rather than (to all practical purposes) non monetary debt is a theoretically appealing solution, but cannot be implemented in practice.

#### Appendix I

Quarterly data on debt are taken from the Statistical Bulletin of the Bank of Italy, Table G2, which breaks the total figure into subtotals held by the public and held by the Central Bank-Exchange Office. A consistent series is provided starting 1975:4. All assets are evaluated at par value.

The Bulletin also provides a monthly series of international reserves in liras.

The series of weighted averages of interest rate on new issues of 3- and 6-month treasury bills has been used in the calculation of the rate of discount. This series is net of taxes.

Annual data on public debt are also provided from the Bank of Italy since 1970. The series of the interest rates is obtained from Table E6 in the Bulletin. The source of the annual data on the public sector budget is ISTAT. The series are those in the data set by G.Morcaldò and R.Violi in *Ricerche e metodi per la politica economica*, volume II, Banca d'Italia, 1989.

Annual and quarterly GDP data series are also supplied by ISTAT. The new data series start from 1970. For 1961 to 1969 rates of growth of the old series have been used to project the level of GDP consistent with the new series.

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Figure 1

# discounted & undiscounted debt/gnp

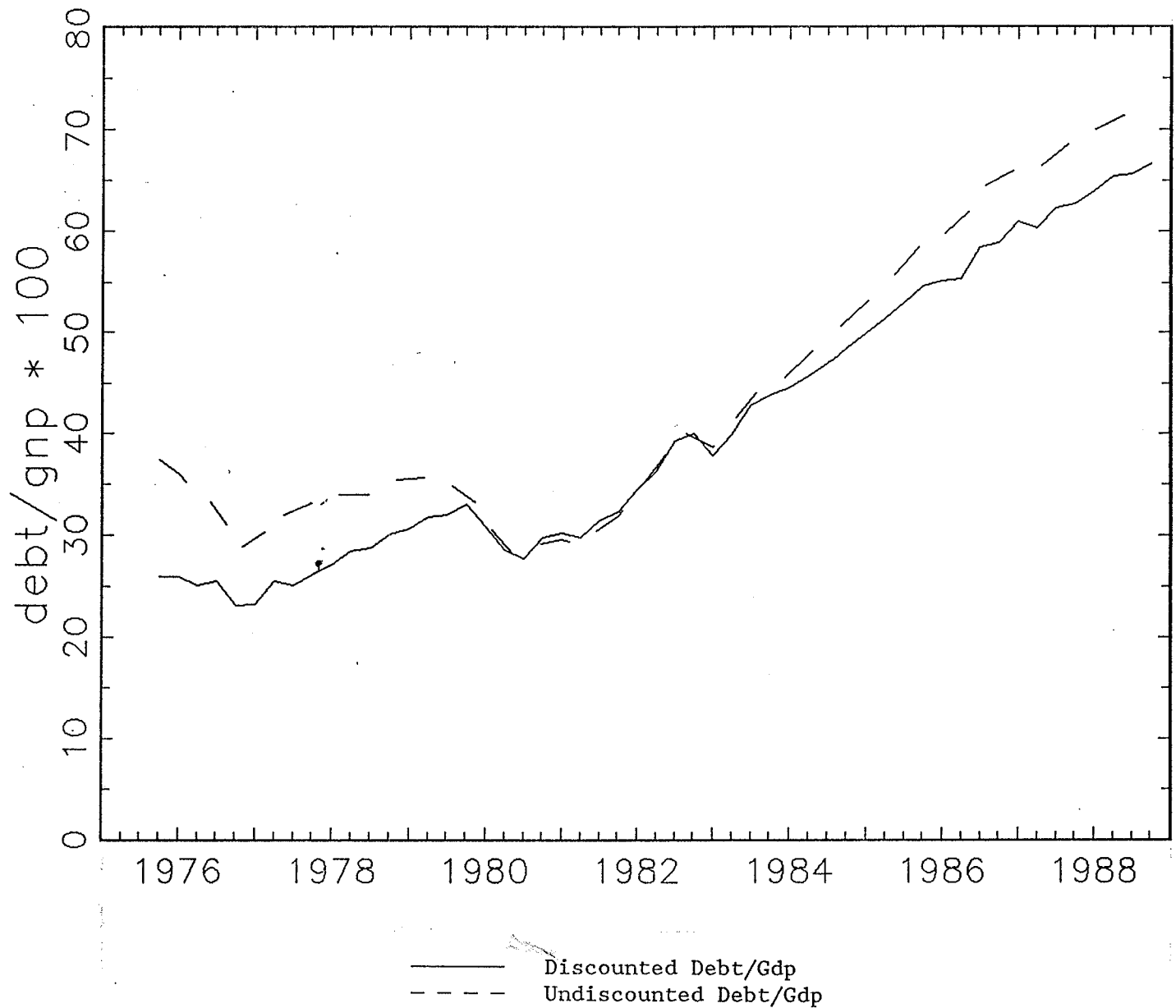


Figure 2

GAUSS Tue May 15 11:36:02 1990

debt/gnp: 1971-88

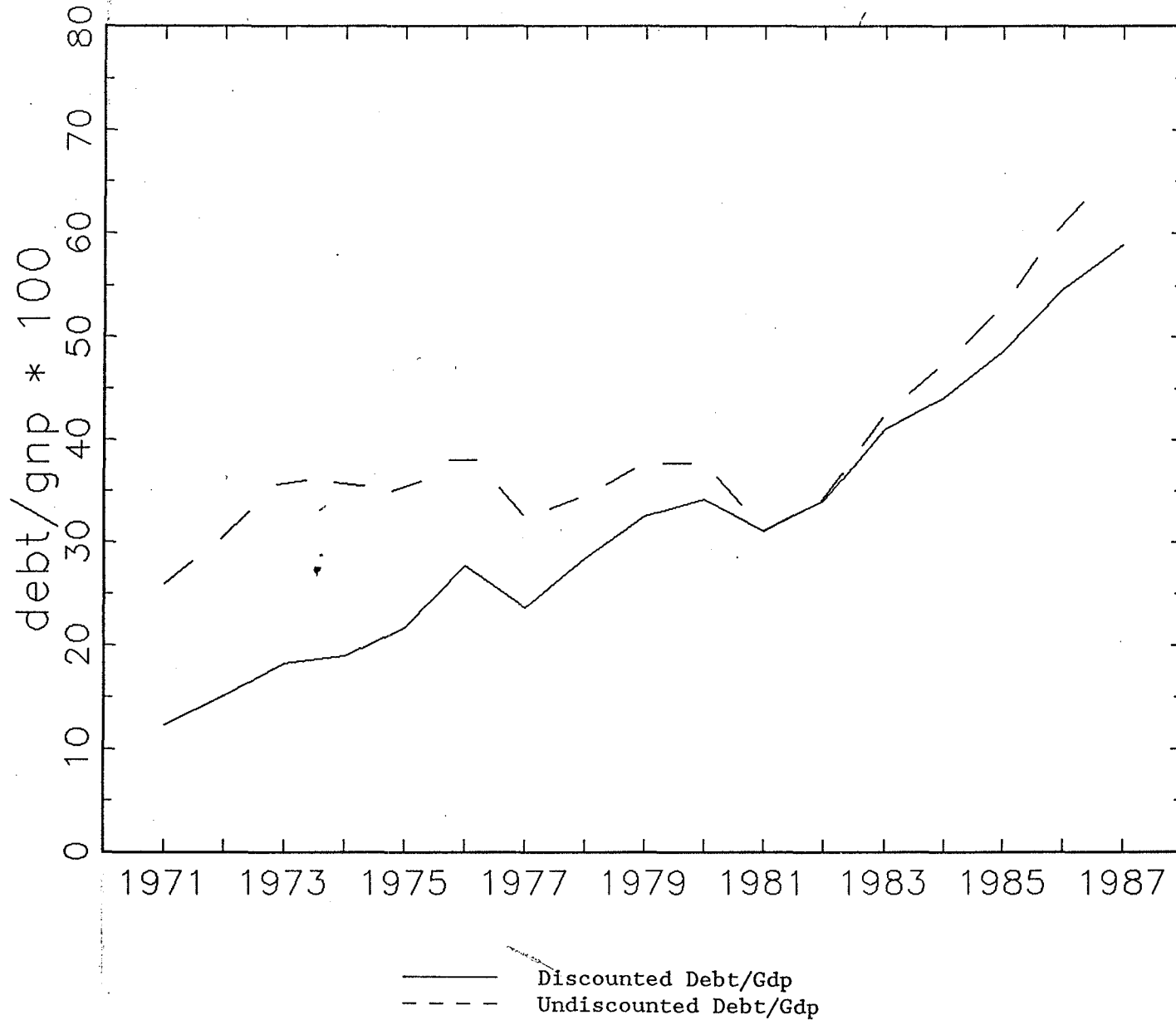


Figure 3

# total and primary deficit

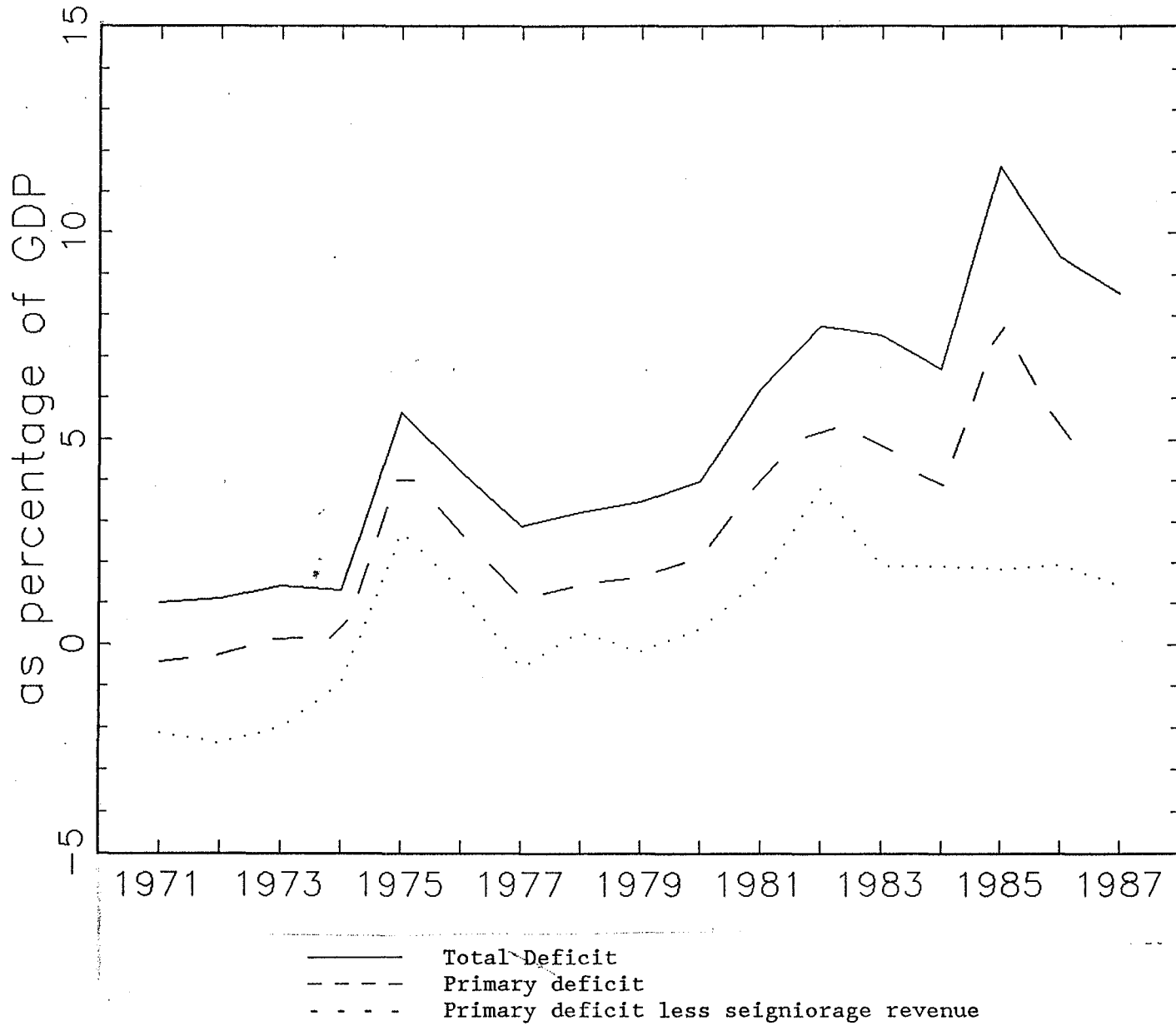


Figure 4

GAUSS Tue May 15 12:08:05 1990

# deficit and public investment

