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CENTER DISCUSSION PAPER NO. 561

FISCAL UNCERTAINTY, INFORMATIONAL EXTERNALITIES AND THE WELFARE COST OF SPECULATION

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Abstract: Fiscal Uncertainty, Informational Externalities and the Welfare Cost of Speculation

This paper analyzes the welfare consequences of stochastic government budgetary policies, using a model in which private agents can devote real resources to learn about future seigniorage. In this environment, if financial markets are not informationally efficient in the strong sense, the private return to information exceeds its social value because informed individuals can redistribute wealth from those who are less informed. Thus stochastic government policies, and seigniorage in particular, may give rise to socially wasteful speculative acquisition of information.

Two specific models are used to explore the welfare costs of the speculative acquisition of information. The first model assumes that agents are identical but nevertheless trade in order to develop a simple framework for expositional purposes and to analyze comparative statics. The second model endogenizes the decision to trade by assuming that individuals differ with respect to the time profiles of their endowments. The principal results of the paper, derived by simulations, are as follows: First, limited heterogeneity of endowments is sufficient to generate trade in markets with heterogeneously informed traders. Second, the welfare costs of speculative acquisition of information can be surprisingly large, close to 1% of real GNP for realistic parameter values. Finally, expected increases of government spending and greater uncertainty about its financing both stimulate potentially wasteful acquisition of information.

The analysis has more general implications: By specifying a channel through which variability of optimal government policies reduces welfare, it points to changes in the theory of optimal intertemporal taxation. Furthermore, a case is made for the analysis of policies to limit wasteful acquisition of information.

1. Introduction

Trade-offs involving inflation play a central role in macroeconomics and international finance. A recent example of particular interest is the study of speculative attacks on the exchange rate. In this literature it is usually assumed that the authorities value a low-inflation environment (i.e. a fixed exchange rate system), but, at the same time, find it necessary to have a high rate of monetary growth. The conflict between these two policies is the source of exchange-rate crises. To gain a deeper understanding of these phenomena it is necessary to model explicitly why the monetary authorities pursue inconsistent objectives. Phelps (1973) has shown that seigniorage may be an essential component of an optimal tax policy. Building on this result Grilli (1988) shows that budgetary policies may be the source of collapses of fixed exchange rates. What is still left to be explained, however, is why the authorites would want to constrain their use of seigniorage by entering a fixed exchange rate system. In this paper we argue that discretionary use of the inflation tax engenders socially wasteful speculation. This analysis lays the foundation for models of incentive-compatible commitment to an exchange rate peg.

We examine the welfare consequences of informational speculation in a simple general equilibrium macro-economic model. Speculation is a controversial issue and one on which public opinion and economic analysis often diverge. Economists generally view speculation either as a beneficial process for the transfer of price risks (the Keynes-Hicks view) or, alternatively, as unprofitable (Milgrom-Stokey (1981), Tirole (1982)). On the other hand, it is commonly claimed that speculation is not desirable on two major grounds. First, it is believed to induce instability and excess

volatility in market prices. Second, it is argued that since it does not generate wealth but only redistributes it, speculation is a wasteful activity from a societal point of view.

Economists have dedicated a lot of attention to the first part of the argument. While Friedman (1953) forcefully defended the view that speculators must induce price stability, more recent results shed some doubts on this optimistic view (e.g. Hart and Kreps (1986)). However, economic analysis has been silent about the second argument which addresses directly some of the potential welfare consequences of speculation. The main reason for the lack of results in this area is the complexity of the models which are suited to address this type of question. We believe, nonetheless, that the issue is of sufficient interest and importance that it may be worthwhile sacrificing the elegance of models with closed form solutions if the reward is a first basic understanding of the problem.

Our strategy in this work was to see whether we could construct a simple macro model capable of producing the popular view that speculation induces welfare losses. Before we describe the details of our analysis, however, it is necessary to be more precise about what we mean by speculation. The definition of speculation is perhaps as vague as its desirability is controversial; we do not intend to settle the argument in this paper.[1] We adopt the Working definition of speculation as trade on the basis of differential information because it allows us to capture the

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[1] General discussion of the nature of speculation can be found in Hirshleifer (1975), Feiger (1976) and Tirole (1982).

popular perception of speculation as wasteful. Thus speculators, in this paper, are individuals who trade in asset markets on the basis of superior information. The collection, processing and evaluation of information is a costly activity. It is socially beneficial if it facilitates a superior allocation of real resources. Even if socially worthless, however, the acquisition of costly information may nevertheless be privately profitable if it allows speculators to redistribute wealth from less well informed individuals. The preliminary results reported here indicate that speculation is likely to occur only if information is surprisingly cheap, but, if it does occur, may be quite costly.

The plan of the paper is as follows: We start with an explanation of the equilibrium concept used, and relate it to other equilibrium concepts for models with heterogeneously informed traders. Next we analyze a simple representative agent model, in which uncertainty derives from stochastic budgetary policies. We show that this model is able to deliver the result that speculation is welfare reducing and analyze how changes in the environment affect speculative activity. We argue, however, that the model can be criticized on the basis of internal inconsistency, following an argument similar to Tirole's (1982). The problem is that in a world of identical agents, there are no gains from trade, so that rational agents should refrain from trading altogether. We then extend the model by introducing heterogeneity in the form of different time profiles of endowments. This generates gains from trade, thus removing the potential inconsistency. We show that the conclusions from the simpler model are not affected: speculation may still be costly in welfare terms.

2. The Modelling Strategy

There exists a considerable body of work on the properties of rational expectations equilibria in markets with heterogeneously informed traders (Green (1973), Grossman (1976, 1977, 1981), Radner (1979) and Jordan and Radner (1982)). An important insight of this literature dating back to Lucas (1972) and explored in detail by Grossman (1977, 1981), Hellwig (1979) and others,[2] is that prices aggregate and convey the heterogeneous information in the economy, thereby providing traders with the opportunity to learn about other agents' private information. Prices need not reveal information perfectly, but if they do profitable speculation is impossible and, as Grossman and Stiglitz (1980) have shown, a rational expectations equilibrium may not exist if the collection of information is costly. Generally, the incentive to acquire costly information is smaller the more perfectly prices reveal information.

In this paper, we assume that informed individuals not only have more precise knowledge on future realizations of at least some random variables in the model, but also are able to derive information from contemporaneous price observations; that is, they form expectations rationally as the concept is commonly understood. Uninformed agents, on the other hand, are assumed to be unable to extract information from currently observed prices and are consequently restricted to forming expectations conditional on their endowments. This scenario crudely captures the notion that interpreting price data is costly, although of course not (as we assume), infinitely so.

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[2] Further references can be found in Grossman (1981), Jordan and Radner (1982) and Admati (1985).

We implicitly assume that the cost of interpreting price data exceeds that of acquiring information about future states of nature directly, otherwise existence problems similar to those noted by Grossman and Stiglitz (1980) would arise. Our model therefore exaggerates the extent of speculation compared to the results from a rational expectations equilibrium model.

These informational assumptions are analytically convenient in that they yield a reasonably tractable general equilibrium model in which asset markets are not informationally efficient in the strong sense, so that individuals will have an incentive to acquire costly information. In this our analysis follows most closely that of Hellwig (1982) who shows that an economy in which agents condition on past prices (but know the model of the economy) approaches full informational efficiency without destroying individuals' incentive to acquire information, as the interval between "market days" shrinks.

The alternative equilibrium concept for models with heterogeneously informed agents, noisy rational expectations equilibrium, as popularized by Lucas (1972), Grossman (1976, 1981) and Grossman and Stiglitz (1980) is difficult to apply in a multi-period general equilibrium setting. In fact, the restrictive assumptions on the distributions of asset returns employed to derive closed-form solutions in single-period CAPM-models fail to hold, thereby robbing these models of their attractive simplicity. Furthermore, the rational expectations equilibrium approach has, as Hellwig (1982) and Dubey, Geanakoplos and Shubik (1987) have pointed out, the undesirable feature of requiring market participants to act on information which is produced by their collective actions.

3. A Representative Agent Model

We consider a two-period economy populated by a continuum of consumers who have identical preferences and endowments but may choose, in the manner outlined below, different information sets. Specifically, individuals derive utility from consumption and real money holdings[3] in both periods of their lives. They receive (exogenously determined) endowments of goods in each period. Money enters the economy as a result of government budget deficits which may be in part attributable to lump-sum nominal transfers to consumers. To simplify the analysis we assume logarithmic utility functions. Finally we assume that agents have free access to a credit market in which nominal bonds are traded.

Before markets open in the first period a consumer may choose to become "informed" at a cost K in terms of first-period goods. That is by becoming informed the individual observes some future random variables with greater precision and obtains the ability to extract information from contemporaneous price observations. To simplify the analysis we shall assume that all informed individuals have the same information set, as do all uninformed agents. The cost of becoming informed, K, can be interpreted as the fixed cost in terms of real income foregone, of time invested in data acquisition and processing. We model the cost of becoming informed as a fixed cost to reflect the set-up costs in acquiring information; in more general models one would expect the cost of information to have both a fixed component and a component which is increasing in the precision of the

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[3] The utility of money holdings is derived, in the manner of McCallum (1983) and Feenstra(1986), from the transactions motive for holding money.

information. In addition we assume that moral hazard problems preclude the sharing of information.

A formal statement of the decision problems of representative informed and uninformed individuals serve to summarize the preceding discussion. Thus, an agent i, $i = U$ (uninformed), I (informed), solves:

$$(3.1) \qquad \underset{c_t^i, m_t^i, b_t^i}{\text{Max}} \; E_t^i\left\{\alpha \log c_t^i + (1-\alpha)\log m_t^i + \beta[\alpha \log c_{t+1}^i + (1-\alpha)\log m_{t+1}^i]\right\}$$

subject to

$$(3.2) \qquad c^i_t + m_t^i + \frac{b_t^i}{P_t} = Y_t - T_t + \frac{M_{t-1}}{P_t} + \frac{V_t}{P_t} - \gamma^i K \,,$$

and

$$(3.3) \qquad c_{t+1}^i + m_{t+1}^i = Y_{t+1} - T_{t+1} + (1 + R_t)\frac{b_t^i}{P_{t+1}} + m_t^i \frac{P_t}{P_{t+1}} + \frac{V_{t+1}}{P_{t+1}} \,,$$

where c^i, m^i, and b^i denote consumption, real money holdings and holdings of private nominal bonds (with face value equal to that of one unit of currency), while P, Y, T, and V denote the price level, the individual's endowment of goods, lump-sum taxes levied by the government and lump-sum nominal transfers from the government. Finally, E_t^U and E_t^I denote expectations conditional on period t endowment information and the information set of an informed individual respectively, while $\gamma^I = 1$, $\gamma^U = 0$

and $\beta \leq 1$. The model is closed by the government budget constraints (in per capita terms):

$$(3.4) \quad G_t + \frac{V_t}{P_t} - T_t = \frac{M_t - M_{t-1}}{P_t}$$

and

$$(3.5) \quad G_{t+1} + \frac{V_{t+1}}{P_{t+1}} - T_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}}$$

and the assumption that the policy variables G (per capita exhaustive government spending), T, and V are exogenously determined. Individuals are assumed to know the joint distribution of the exogenous endowment variables Y_t, Y_{t+1}, M_{t-1} and policy variables.

The conditions which must be satisfied in equilibrium are two-fold. First, those which determine equilibrium prices and allocations for arbitrary values of information costs, K, and proportion of individuals engaged in speculation, λ. Second, that which determines, for given K the equilibrium fraction of the population informed, λ^*, such that the ex ante expected utilities of speculators and uninformed individuals are equal. Formally, for given cost of information, K, and fraction of individuals who speculate, λ, an equilibrium is defined as a vector of prices and allocations, $(R_t, P_t, P_{t+1}, c_t^U, c_{t+1}^U, m_t^U, m_{t+1}^U, c_t^I, c_{t+1}^I, m_t^I, m_{t+1}^I)$ which satisfy equations (3.1) to (3.5) and the market equilibrium conditions:

$$(3.6) \quad \lambda c_t^I + (1-\lambda)c_t^u = Y_t - G_t - \lambda K,$$

$$(3.7) \quad \lambda m_t^I + (1-\lambda)m_t^U = \frac{M_t}{P_t},$$

and

$$(3.8) \quad \lambda c_{t+1}^I + (1-\lambda)c_{t+1}^U = Y_{t+1} - G_{t+1},$$

where we have used Walras' law to eliminate the bond market in period t and the money market in period $t+1$. Let U^I and U^U denote the ex post utility, in equilibrium, of representative informed and uninformed individuals, for given λ and K, i.e.

$$(3.9) \quad U^I = U^I(Y_t, G_t, T_t, M_{t-1}, V_t, Y_{t+1}, G_{t+1}, T_{t+1}, V_{t+1}, K, \lambda),$$

$$(3.10) \quad U^U = U^U(Y_t, G_t, T_t, M_{t-1}, V_t, Y_{t+1}, G_{t+1}, T_{t+1}, V_{t+1}, K, \lambda)$$

and define the ex ante expected net benefit of being informed, ψ, as

$$(3.11) \quad \psi(\lambda, K, \ldots) \equiv E[U^I(\lambda, K, \ldots) - U^U(\lambda, K, \ldots)].$$

Then, for any given K the equilibrium proportion of individuals informed, $\lambda^*(Y_t, G_t, T_t, M_{t-1}, V_t, Y_{t+1}, G_{t+1}, T_{t+1}, V_{t+1}, K)$, satisfies

$$(3.12) \quad \psi(\lambda^*, K, \ldots) = 0, \quad 0 < \lambda^* < 1,$$
$$\lambda^* = 1 \text{ if } \psi(\lambda, K, \ldots) > 0, \quad 0 \le \lambda \le 1, \text{ and}$$
$$\lambda^* = 0 \text{ if } \psi(\lambda, K, \ldots) < 0, \quad 0 \le \lambda \le 1 .$$

We shall from now on simplify the analysis by assuming that the financing of government spending in period $t+1$ is the sole source of uncertainty in the model and that the government does not pay nominal transfers to individuals. This allows us to address a matter of considerable topical interest (as of November 1987): the relationship between uncertainty about the financing of government spending and speculation in financial markets. Also we assume that by becoming informed, an individual removes uncertainty completely. A further assumption, adopted purely for analytical convenience, is that the distribution of future government behavior with respect to taxation and seigniorage is such as to generate a uniform distribution of money growth rates.

4. The Social Cost of Speculation

The nature of the information externality characterizing speculation is illustrated most clearly by the comparison of equilibrium allocations in the polar cases when everyone and no one, respectively, is informed. This comparison yields an upper bound for the social cost of speculation, since it does not take into account the endogenous determination of the number of speculators.

The first-order conditions for expected utility maximization for a representative individual i $(i = I, U)$, are

$$(4.1) \qquad \frac{\alpha}{c_t^i} = \delta_t^i$$

$$(4.2) \qquad \frac{1-\alpha}{m_t^i} = \delta_t^i - E_t^i\left\{\frac{\delta_{t+1}^i P_t}{P_{t+1}}\right\}$$

$$(4.3) \quad \delta^i_t = (1 + R_t)E^i_t\left\{\frac{\delta^i_{t+1}P_t}{P_{t+1}}\right\}$$

$$(4.4) \quad \frac{\beta\alpha}{c^i_{j,t+1}} = \delta^i_{j,t+1}, \text{ and}$$

$$(4.5) \quad \frac{\beta(1-\alpha)}{m^i_{j,t+1}} = \delta^i_{j,t+1},$$

as well as the budget constraints, equations (3.2) and (3.3), where δ^i_t and δ^i_{t+1} are Lagrange multipliers, and j indexes period-$t+1$ states of nature.

Consider first the case where no one speculates ($\lambda = 0$). Since individuals are assumed to form expectations about period $t+1$ outcomes rationally, we first solve for the equilibrium in period $t+1$:

$$(4.6) \quad C^{UU}_{t+1} = Y_{t+1} - G_{t+1} = Y_{t+1}(1-g_{t+1}),$$

$$(4.7) \quad m^{UU}_{t+1} = \left[\frac{1-\alpha}{\alpha}\right]c^{UU}_{t+1} = \left[\frac{1-\alpha}{\alpha}\right](1-g_{t+1})Y_{t+1},$$

$$(4.8) \quad \delta^{UU}_{t+1} = \frac{\beta\alpha}{(1 - g_{t+1})Y_{t+1}}, \text{ and}$$

$$(4.9) \quad P^{UU}_{t+1} = \left[\frac{\alpha}{1-\alpha}\right]\frac{M_{t+1}}{(1 - g_{t+1})Y_{t+1}},$$

or, using the government budget constraint,

$$(4.10) \quad P^{UU}_{t+1} = \frac{\alpha(M_t/Y_{t+1})}{(1-\alpha)(1 - g_{t+1}) - \alpha(g_{t+1} - r_{t+1})},$$

where g and τ are government spending and tax rates and a superscript "UU" indicates the equilibrium value of a variable when everyone is uninformed. The rate of monetary growth resulting from tax and spending rates τ_{t+1} and g_{t+1} is

$$(4.11)\quad \mu_t = \frac{M_{t+1}}{M_t} - 1 = \frac{\alpha(g_{t+1} - \tau_{t+1})}{(1-\alpha)(1 - g_{t+1}) - \alpha(g_{t+1} - \tau_{t+1})}.$$

Similarly, in period t

$$(4.12)\quad c_t^{UU} = (1 - g_t)Y_t,$$

$$(4.13)\quad m_t^{UU} = \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{1+R_t^{UU}}{R_t^{UU}}\right] c_t^{UU}, \text{ and}$$

$$(4.14)\quad P_t^{UU} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{R_t^{UU}}{1 + R_t^{UU}}\right)\left(\frac{M_t/Y_t}{1 - g_t}\right).$$

Using equation (4.3) the equilibrium nominal interest rate in period t is

$$(4.15)\quad R_t^{UU} = \left\{\beta E_t^U\left(\frac{1}{1 + \mu_t}\right)\right\}^{-1} = \left\{\beta E_t^U\left[\frac{(1-\alpha)(1 - g_{t+1}) - \alpha(g_{t+1} - \tau_{t+1})}{(1-\alpha)(1 - g_{t+1})}\right]\right\}^{-1}.$$

These results yield the realized utility of a representative individual when everyone is uninformed as

$$(4.16)\quad U^{UU} = U^{U}(0, K, \ldots)$$
$$= \log Y_t + \beta \log Y_{t+1} + \log(1 - g_t) + \beta \log(1 - g_{t+1})$$
$$+ (1+\beta)(1-\alpha)\log\left[\frac{1-\alpha}{\alpha}\right] + (1-\alpha)\log\left[1 + \beta E_t^U\left(\frac{1}{1+\mu_t}\right)\right].$$

The derivation of equilibrium allocations when everyone is informed about future government financing policies is analogous to the foregoing, and results in ex post utility for a representative individual

$$(4.17)\quad U^{II} = U^{I}(1, K, \ldots) = \log Y_t + \beta \log y_{t+1} + \log(1 - g_t - k)$$
$$+ \beta\log(1 - g_{t+1}) + (1+\beta)(1-\alpha)\log\left[\frac{1-\alpha}{\alpha}\right] + (1-\alpha)\log\left[1 + \beta\left(\frac{1}{1+\mu_t}\right)\right]$$

where $k = K/Y_t$, the cost of information as a fraction of per capita GNP.

It is clear that a social planner who tries to maximize the expected lifetime utility of a representative consumer would prefer the equilibrium in which no one is engaged in speculative information acquisition, since the social cost of speculation in this benchmark case is given by

$$(4.18)\quad E[U^{UU}(0, K, \ldots) - U^{II}(1, K, \ldots)] = \log(1 - g_t) - \log(1 - g_t - k)$$
$$+ (1-\alpha)\log\left[1 + \beta E\left(\frac{1}{1+\mu_t}\right)\right] - (1-\alpha)E\log\left[1 + \beta\left(\frac{1}{1+\mu_t}\right)\right],$$

which is positive because $k > 0$ and by the concavity of $\log(\cdot)$.[4] Society as a whole does not gain from being informed, because the returns to

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[4] For equation (4.18) to hold more generally when additional variables are stochastic, μ_t should be independent of variables known in period t.

speculation are purely redistributive. As a result, the investment in information represents a net loss. Note, however, that even if information about future monetary growth could be obtained costlessly, society would be better off if everyone were uninformed. The reason is that different money growth realizations (μ_t), although neutral in period t+1, induce variations in period t equilibrium per capita real money holdings if they are observed in period t. Since individuals are risk averse, such variations are undesirable ex ante, although optimal ex post. That is, seigniorage is a lump-sum tax only when it is unanticipated. Similar points have previously been made by Weiss (19) and King (1984).

5. Equilibrium Speculation

In this section we analyze in greater detail the equilibrium of the model under the assumption that speculators observe future taxation (and, by implication, seigniorage), while uninformed individuals cannot extract speculators' privileged information from observed market prices.

The derivation of agents' equilibrium allocations for arbitrary λ and K proceeds as before. The commodity market equilibrium conditions and the first-order conditions which govern the allocation of resources to consumption and money holdings yield the equilibrium price level in periods t+1 and t

$$(5.1) \qquad P_{t+1} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{M_{t+1}/Y_{t+1}}{1 - g_{t+1}}\right), \text{ and}$$

$$(5.2) \qquad P_t = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{R_t}{1 + R_t}\right)\left(\frac{M_t/Y_t}{1 - g_t - \lambda k}\right).$$

Thus, the average value of per capita endowments (i.e. consumption good plus real balances) in period t is

$$(5.3) \quad e_t = Y_t(1 - g_t - \lambda k)\left\{1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + R_t}{R_t}\right)\right\};$$

those of speculators and uninformed individuals fall short of and exceed this amount by $(1-\lambda k)Y_t$ and $\lambda k Y_t$, respectively. Denote by θ_t^i the consumption share of per capita disposable income of a representative individual in group i $(i = I, U)$, e.g. $\theta_{t+1}^U = C_{t+1}^U/[Y_{t+1}(1 - g_{t+1})]$. Then the real bond holdings of the informed and uninformed in period t are

$$(5.4) \quad \frac{b_t^I}{P_t} = Y_t\left\{(1 - g_t - \lambda k)(1 - \theta_t^I)\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + R_t}{R_t}\right)\right] - (1-\lambda)k\right\} \quad \text{and}$$

$$(5.5) \quad \frac{b_t^U}{P_t} = Y_t\left\{(1 - g_t - \lambda k)(1 - \theta_t^U)\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + R_t}{R_t}\right)\right] + \lambda k\right\},$$

while consumption shares in period $t + 1$ are given by

$$(5.7) \quad \theta_{t+1}^I = 1 + \frac{R_t}{1 + \mu_t}\left\{1 - \theta_t^I - \frac{(1-\lambda)\alpha k}{1 - g_t - \lambda k}\right\}, \quad \text{and}$$

$$(5.8) \quad \theta_{t+1}^U = 1 + \frac{R_t}{1 + \mu_t}\left\{1 - \theta_t^U + \frac{\alpha\lambda k}{1 - g_t - \lambda k}\right\}.$$

Assuming that per capita income in period t is known to all, the first-order conditions, equations (4.1), (4.3) and (4.4), can be rewritten as

$$(5.9)\quad \left(\theta_t^i\right)^{-1} = \beta R_t E_t^i\left\{\left((1+\mu_t)\theta_{t+1}^i\right)^{-1}\right\},$$

which immediately yields

$$(5.10)\quad \theta_t^I = \frac{1}{1+\beta}\left\{1 + \frac{1+\mu_t}{R_t} - \frac{(1-\lambda)\alpha k}{1 - g_t - \lambda k}\right\},$$

$$(5.11)\quad \theta_{t+1}^I = \frac{\beta R_t}{1+\mu_t}\theta_t^I, \text{ and}$$

$$(5.12)\quad \frac{1}{\theta_t^U} = \frac{\beta R_t}{\bar{\mu}-\underline{\mu}}\left\{\log\left\{1 + \bar{\mu} + R_t\left[1 - \theta_t^U + \frac{\alpha\lambda}{1 - g_t - \lambda k}\right]\right\}\right.$$

$$\left. - \log\left\{1 + \underline{\mu} + R_t\left[1 - \theta_t^U + \frac{\alpha\lambda}{1 - g_t - \lambda k}\right]\right\}\right\}$$

where, to derive equation (5.12), we used the properties of the uniform distribution.[5] Finally, the net private value of information about future government policy can be expressed in terms of ratios of consumption shares:

$$\begin{aligned}(5.13)\quad \psi(\lambda, K, \ldots) &= E\left\{\log\,\theta_t^I(\lambda, k, \ldots) - \log\,\theta_t^U(\lambda, k, \ldots)\right.\\ &\quad \left. + \beta[\log\,\theta_{t+1}^I(\lambda, k, \ldots) - \log\,\theta_{t+1}^U(\lambda, k, \ldots)]\right\}\\ &= \psi(\lambda, k, g_t\, g_{t+1}),\end{aligned}$$

where $\psi(\cdot)$ is contingent upon the particular distributional assumption on μ_t.

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[5] Note that even if future endowments were uncertain it would still be possible to describe agent's decisions solely in terms of policy variables.

To establish the existence of an equilibrium fraction of the population engaged in speculation, λ^*, we only need to show that $\psi(\cdot)$ is continuous in all its arguments for all $\lambda \in [0,1]$. Continuity may fail for three reasons: The first is the free-rider problem that arises when uninformed individuals can perfectly infer the information of informed traders from prices. The resulting discontinuity identified by Grossman and Stiglitz (1980) does not occur in our model since we've ruled out perfect revelation by assumption. Hart (1974) has pointed out that an equilibrium need not exist in models with incomplete asset markets. Existence fails when heterogeneously informed individuals take off-setting infinite long and short positions, with the result that their budget sets are not compact and their excess demands, therefore, not well-defined. Since our model includes only money and bonds this situation could only arise if a single individual were to take an infinite long position in money matched by an infinite short position in bonds. Such a position would violate the individual's intertemporal budget constraint, because bonds dominate money in return. Finally, to ensure feasibility for all possible values of λ, the cost of information is restricted to not exceed per capita disposable GNP, i.e. $k < 1 - g_t$. For any λ and k satisfying this restriction the existence of an equilibrium follows by standard methods.

Since the utility function is continuously differentiable, ex post utilities, and hence $\psi(\lambda, k, \ldots)$, are continuous and differentiable in λ and k (Mas-Collel (1985), pp. 197-201). Consequently there exists at least one λ^* satisfying equation (3.12). Note that this result does not depend on the particular utility function chosen (it would hold for any strictly concave twice continuously differentiable utility function satisfying the

Inada conditions) or the specification of the nature and sources of uncertainty in the model. The restriction that the cost of information be smaller than per capita disposable GNP is also, in our opinion, quite innocuous.[6] The equilibrium social cost of speculation (allowing λ to adjust to its equilibrium value λ^*) is given by:

$$
\begin{aligned}
(5.14)\quad \Phi(k, g_t, g_{t+1}) &= E\Big\{U^U(0,k,\ldots) - [\lambda^* U^I(\lambda^*,k,\ldots) + (1-\lambda^*)U^U(\lambda^*,k,\ldots)]\Big\} \\
&= \log(1 - g_t) - \log(1 - g_t - \lambda^* k) + (1-\alpha)\left\{\log\left[1 + \beta E\left(\frac{1}{1+\mu_t}\right)\right]\right. \\
&\quad \left. - \log\left[\frac{1 + R_t(\lambda^*, k, \ldots)}{R_t(\lambda^*, k, \ldots)}\right]\right\} - \lambda^* E\{\log \theta_t^I(\lambda^*,k,\ldots) + \beta \log \theta_{t+1}^I(\lambda^*,k,\ldots)\} \\
&\quad - (1-\lambda^*)E\{\log \theta_t^U(\lambda^*,k,\ldots) + \beta \log \theta_t^U(\lambda^*,k,\ldots)\}.
\end{aligned}
$$

where $R_t(\lambda^*, k, \ldots)$ is the equilibrium nominal interest rate when λ^* of the population speculates.

The net private value of information when no one is informed, $\psi(0, k, \ldots)$, can be derived explicitly, and provides some insight into the relative cost of information required for speculation not to occur. Substituting $\lambda = 0$ into equation (5.13) and using the fact that a single trader has no influence on the market

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[6] Alternatively one could, by restricting the range of λ, consider conditions for the existence of an equilibrium for values of k exceeding $1 - g_t$.

$$(5.15)\quad \psi(0,\ k,\ \ldots) = E\left\{(1+\beta)\log\left\{1 - \frac{\alpha k}{1-g_t} + \beta(1+\mu_t)E\left(\frac{1}{1+\mu_t}\right)\right\}\right.$$
$$\left. - (1+\beta)\log(1+\beta) - \beta\log(1+\mu_t) - \beta\log E\left(\frac{1}{1+\mu_t}\right)\right\}$$

which is positive for sufficiently small values of k.

Since it is not possible, in general, to derive closed form solutions for the consumption share of uninformed individuals, θ_t^U, and the net value of information, $\psi(\lambda,\ k,\ \ldots)$, we report simulation results below. Alternatively, some intuition can be gained by approximating the equilibrium to derive upper and lower bounds for the consumption share of the uninformed. Thus, by using the "$\log(1+x) \approx x$"-approximation or by ignoring Jensen's inequality in equation (5.12), it can be shown that

$$(5.16)\quad \theta_{1t}^U < \theta_t^U < \theta_{2t}^U, \text{ where}$$

$$(5.17)\quad \theta_{1t}^U = \frac{1}{1+\beta}\left\{1 + \frac{1+\mu}{R_t} + \frac{\alpha\lambda k}{1 - g_t - \lambda k}\right\},$$

is the desired consumption share of an individual who always overestimates the real interest rate by assuming that the rate of monetary growth will equal its lower bound, and

$$(5.18)\quad \theta_{2t}^U = \frac{1}{1+\beta}\left\{1 + \frac{1 + E\mu_t}{R_t} + \frac{\alpha\lambda k}{1 - g_t - \lambda k}\right\}.$$

Corresponding to θ_{1t}^U and θ_{2t}^U are lower and upper bounds for the equilibrium nominal interest rate, R_{1t} and R_{2t}, such that

$$(5.19)\quad \frac{1 + \lambda\mu_t + (1-\lambda)\underline{\mu}}{\beta} = R_{1t} < R_t < R_{2t} = \frac{1 + \lambda\mu_t + (1-\lambda)E\mu_t}{\beta}.$$

As can be expected, if the uninformed underestimate the rate of monetary growth the equilibrium nominal interest rate is lower than it would otherwise have been, while R_{2t} exceeds the true equilibrium interest rate because it is based on an underestimate of the uninformed's expected marginal utility of future consumption. It is clear that speculators redistribute income from the uninformed by borrowing in periods where the rate of monetary growth is high and vice versa. The benefits of redistribution are moderated by the fact that speculators hold more money balances in high inflation periods, that is, they "pay" proportionately more of the seigniorage tax.

The approximations show that increased speculation, as measured by the proportion of the population acquiring information, λ, increases interest-rate volatility. This contradicts Friedman's (1953) assertion that profitable speculation must stabilize prices. The intuition underlying our result is straightforward: If no one is informed, future policy realizations do not affect current outcomes, while, in an economy with speculators, an increase in the proportion of speculators increases the impact of future policies on current prices. Hart and Kreps (1986) have shown that profitable speculation may also destabilize prices in an environment similar to that envisaged by Friedman.

6. Comparative Statics

The response of the equilibrium proportion of the population engaged in speculation to changes in information technology (relative to per capita

income) and in government policies are of considerable interest. To derive these comparative statics results we consider, in turn, the effects of changes in k, λ, g_t, g_{t+1} and $(\bar{\mu}-\underline{\mu})$ (for given g_t) on the net benefit of information. Note that the comparative statics results derived here apply to interior equilibria, such that $0 < \lambda^* < 1$, only.

It is expositionally convenient to start with a change in the relative cost of information, say a decrease brought about by either an improvement in information technology or an increase in per capita income. From the definition of $\psi(...)$ and the commodity market equilibrium condition we have

$$(6.1) \quad \psi_k = \left(\frac{1}{1-\lambda}\right) E\left\{(\theta^U_t \theta^I_t)^{-1} \frac{d\theta^I_t}{dk} + \beta(\theta^U_{t+1} \theta^I_{t+1})^{-1} \frac{d\theta^I_{t+1}}{dk}\right\}.$$

That is, we need to evaluate the effects of changes in k on the equilibrium consumption shares of an informed individual. Since the value of a speculator's endowment in period t relative to the economy-wide average, given by

$$(6.2) \quad \frac{e^I_t}{e_t} = 1 - (1-\lambda)k\left\{\left\{1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + R_t}{R_t}\right)\right\}(1 - g_t - \lambda k)\right\}^{-1},$$

is increased by a reduction in k, his desired consumption shares in both periods increase and conversely for an uninformed individual whose relative endowment is

$$(6.3)\qquad \frac{e^U_t}{e_t} = 1 + \lambda k\left\{\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+R_t}{R_t}\right)\right](1 - g_t - \lambda k)\right\}^{-1}.$$

Thus θ^I_t and θ^I_{t+1} increase while θ^U_t and θ^U_{t+1} decrease if redistribution and intertemporal substitution effects engendered by the change in the equilibrium interest rate do not outweigh the initial wealth effects. This requires stability of equilibrium, that is that aggregate saving be an increasing function of R_t. If this condition is satisfied, a decrease in k increases the net value of being informed in every state of nature, i.e. $\psi_k < 0$.

Next consider an increase in the fraction of the population engaged in speculation. Using the commodity market equilibrium conditions as before:

$$(6.4)\qquad \psi_\lambda = \frac{1}{1-\lambda}E\left\{(\theta^I_t\theta^U_t)^{-1}\cdot\frac{d\theta^I_t}{d\lambda} + \beta(\theta^I_{t+1}\theta^U_{t+1})^{-1}\cdot\frac{d\theta^I_{t+1}}{d\lambda} + \frac{\theta^I_t-\theta^U_t}{\theta^U_t} + \beta\left[\frac{\theta^I_{t+1}-\theta^U_{t+1}}{\theta^U_{t+1}}\right]\right\}$$

An increase in λ has two effects: It raises, ceteris paribus, the value of the endowments (relative to the economy-wide average) of speculators and uninformed individuals alike, thereby raising the desired consumption shares of representative individuals in both groups. In the aggregate the increase in the proportion of the population engaged in speculation reinforces this increase in demand by speculators while reversing, at least in part, the increase in demand by uninformed individuals. As a result θ^I_t and θ^U_t can both increase or either can increase at the expense of the other.

Thus, in sum, since none of the terms of ψ_λ can be signed unambiguously, the existence of multiple equilibria cannot be ruled out on a priori grounds. This result is confirmed by the simulation exercise reported

on in the next section. The equilibria are easily Pareto-ranked, however, with smaller values of λ^* corresponding to higher welfare. The following comparative static statements refer to the equilibrium corresponding to the lowest value of λ^* (which would be stable if λ were to adjust in tattonement-like fashion).

Given the caveat stated we conclude that a decrease in the cost of information would increase the proportion of the population engaged in speculative acquisition of information. The resource cost of speculation, as a fraction of per capita income, $\lambda^* \cdot k$, may increase or decrease depending on the elasticity of λ^* with respect to k. The relevant conceptual experiment in this case is to consider the effect on $\psi(\ldots)$ of a change in k accompanied by a change in λ such that $\lambda \cdot k$ is unchanged. Once again the results are inconclusive, but the simulations prove instructive.

Four different government policy experiments are of interest. These are changes in current government spending (g_t), changes in expected future government spending (g_{t+1}), permanent changes in government spending (g_t and g_{t+1}) and, finally, changes in the distribution of future taxes which increase uncertainty about monetary growth. An increase in current government spending, as can be seen from equations (6.3) and (6.4), is analogous to an increase in k, and consequently lowers the value of information. Intuitively, it reduces both the resources available for information acquisition and for potential redistribution. The other changes in government policies involve changes in the distribution of rates of

monetary growth.[7] It is well known (Gould (1974), Laffont (1976)) that there is no unambiguous relationship between the value of information and changes in the distributions of the underlying sources of uncertainty. The simulation results presented in the next section are nevertheless intuitively appealing.

7. Simulation Results

Simulation results for the model are presented in Figures 7.1 to 7.10.[8] Figures 7.1 to 7.4 illustrate the value of information, ψ, the equilibrium fraction of the population engaged in speculation, λ^*, and the equilibrium output loss due to information acquisition, $\lambda^* \cdot k$, for a benchmark parameterization of the model. The implications of alternative government policies are explored in Figures 7.5 to 7.8, and, finally, Figures 7.9 and 7.10 summarize the effects of variations in the "deep structural parameters" α and β.

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[7] We will generally assume that the distribution of taxes changes as well so that we can limit our attention to changes in the parameters of the distribution.

[8] The methodology is straightforward: For each set of parameter values (α, β, λ, k, g_t, g_{t+1}, $\underline{\mu}$, $\bar{\mu}$) the model, appropriately modified for discrete μ, is solved for a hundred possible values of μ_t using a GAUSS procedure based on Broyden's secant method. The solutions are used to calculate the corresponding net benefit of being informed, ψ. Finally, for each set of parameters (α, β, k, g_t, g_{t+1}, $\underline{\mu}$, $\bar{\mu}$), the equilibrium fraction of speculators, λ^*, is chosen so as to set ψ appropriately close to zero.

The benchmark parameter values were chosen so as to approximate those for western industrial economies. In particular government spending was assumed to equal 25% of GNP ($g_t = g_{t+1} = 0.25$), the maximum government budget deficit to be financed by printing money to equal 2.5% of GNP ($0 \leq g_{t+1} - \tau_{t+1} \leq 0.025$), and the discount factor, β, to equal 0.97. If "money" in this model is interpreted as nominally denominated government liabilities a value of 0.95 would be appropriate for α. Alternatively if "money" is high-powered money α should be about 0.995. We chose the former value for the benchmark model.

Two conclusions are immediately apparent from the simulations of the benchmark model: First, speculation only occurs at relatively low levels of information costs. Individuals speculate only if the cost of acquiring information is less 1.6% of per capita GNP. Second, interior equilibria in which a proper fraction of the population speculates, occur for a small subset of parameter values only; in this case at values of the cost of information between 1.5% and 1.6% of per capita GNP. A direct consequence of this property of the model is that the output loss due to speculative acquisition of information may be surprisingly high. This is illustrated in Figure 7.4 which shows that the output cost of speculation increases in the cost of information to reach a maximum of 1.5% of aggregate output and falls off rapidly at higher levels of information costs as the fraction of the population engaged in speculation declines to zero when K equals 1.6% of per capita income. Also note that multiple interior (in λ) equilibria do arise—the Pareto-dominated equilibria and the associated output costs are shown by the dashed lines in Figures 7.3 and 7.4.

We attribute the result that in most equilibria either every one or no one is engaged in acquisition of information to the assumptions that agents

are identical and that the cost of acquiring information is fixed. The fact that speculation only occurs at very low levels of information costs can be interpreted as pointing to the need for models of the sharing of information. A particularly interesting extension would be to view financial institutions, especially institutional investors, as coalitions formed to spread the costs of information and thereby to avoid redistributive losses in the financial markets.

Figure 7.5 confirms that increases in current government spending, g_t, lowers the net value of information by decreasing the resources available for redistribution and increasing the opportunity cost of becoming informed. An (expected) increase in future government spending, g_{t+1}, on the other hand, if it is not necessarily accompanied by increased taxes, raises the mean rate of monetary growth and widens the range of possible rates of monetary growth, thereby raising the net value of information. This is illustrated in Figure 7.6. Figure 7.7 combines these two conceptual experiments to show that information is more valuable in economies with higher constant rates of government spending out of GNP. The effects of increased uncertainty about the financing of future government spending is shown in Figure 7.8.[9] The diagram confirms our intuition that greater uncertainty about future government policies would raise the value of information.

Figures 7.9 and 7.10 show how the simulation results depend on the deep structural parameters of the model, α and β . α can be interpreted as

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[9] (Specifically the distribution of future tax rates is varied so as to increase $\bar{\mu}$ and decrease $\underline{\mu}$ by equal amounts and thus to generate a mean-preserving spread of possible rates of monetary growth around a mean rate of 0.75.)

parameterizing the transactions technology of the economy, with an increased value of α associated with a more efficient transactions technology. An increase in α has two effects: First, it raises, ceteris paribus, the first-period price level, thereby reducing the value of initial endowments of money and, consequently, raising the relative cost of information and reducing its net value. Second, it raises the rate of monetary growth associated with any second-period government budget deficit, thus extending the right-hand side tail of the distribution of rates of monetary growth resulting from a given level of government spending and distribution of tax rates. This latter effect raises the value of information, and, as seen in Figure 7.9, dominates the first-period endowment effect. Finally, since the cost of acquiring information is incurred in the first period, an increase in the representative consumer's pure rate of time preference (decrease in β) lowers the net value of information as shown in Figure 7.10.

8. A Model with Heterogeneous Agents

In the previous analysis we implicitly assumed that all agents always enter the credit market. Notice, however, that uninformed individuals will always be better off if they do not borrow or lend. This is the case because there are no gains from trade in a representative agent model. In situations like this, as Tirole (1982) has shown, speculative markets do not exist. However, as we will show in this section, the welfare costs of speculation do not crucially depend on uninformed agents behaving against their own interest. We, therefore, see the representative agent model as a useful approximation to a more complete model which explicitly accounts for

gains from trade and uninformed individuals' decision whether to trade or not.

The complete model, which is analyzed in detail in Appendix B, assumes that individuals are heterogeneous with respect to their endowment profiles. Specifically, we separate the population into equal-sized groups of type-A and type-B individuals. Type-A individuals receive endowments of commodities in period t and t+1 which, respectively, exceed and fall short of the economy-wide endowment by a fraction δ. Type-B individuals' endowment profiles are the mirror image of those of type-A's and both groups receive identical endowments of nominal money and pay the same taxes.

Information is socially valuable in this setting because we restrict individuals to trading nominal bonds. That is, information substitutes, albeit imperfectly, for indexed contracts. The social value of information is easily calculated as the average of the premia which type-A and type-B individuals are willing to pay for a change from an uninformed to a fully informed economy: Let $EU^{i,II}$ denote the expected utility of a representative individual of type i if everyone is informed and the individual pays the premium a^i in period t, and let $EU^{i,UU}$ denote the expected utility of a type-i individual when no one is informed. Then the social value of information, measured in terms of period t goods, is given by

$$(8.1) \quad a^* = \frac{1}{2}(a^{A^*} + a^{B^*})$$

where a^{A^*} and a^{B^*} satisfy

$$(8.2)\quad EU^{A,II}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta, a^A, a^B) - EU^{A,UU}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta)$$

and

$$(8.3)\quad EU^{B,II}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta, a^A, a^B) - EU^{B,UU}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta)$$

Equations (8.2) and (8.3) emphasize that our measure of the value of information is a general equilibrium one which requires coordination of individual decisions by a social planner. Clearly speculation is potentially socially beneficial in this model: it moves the equilibrium nominal interest rate towards that which would prevail in a fully informed economy, thereby compensating in part for the inefficiency brought about by the absence of indexed contracts.

The decision whether to trade intertemporally or not, could in principle be separated into decisions pertaining to trade in the bond market and trade in the money market. To simplify the analysis we shall assume that individuals who refrain from intertemporal trade simply hold their initial endowments of money and that the government does not create money in period t.[10]

Let λ^A and λ^B denote the fractions of type-A and type-B individuals who speculate and, similarly, let σ^A and σ^B be the fractions of individuals

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[10] Although individuals who do not trade intertemporally simply consume their endowments of the consumption good, we can justify their holding money by assuming that they trade the constituent parts of a composite consumption good and hold money for that purpose.

who refrain from trading. For given λ^A, λ^B, σ^A, σ^B and cost of information, kY_t, let $EU^{i,S}$, $EU^{i,U}$ and $EU^{i,NT}$ denote the expected utility in equilibrium of representative individuals of type i, $i = A, B$, who speculate, are uninformed but nevertheless trade, and refrain from trade. Define the net private benefit of information relative to uninformed trade as before:

$$(8.4) \quad \psi^i(\lambda^A, \lambda^B, \sigma^A, \sigma^B, k, \ldots) \equiv EU^{i,S} - EU^{i,U}, \; i = A, B,$$

the net benefit of uniformed trade relative to not trading as

$$(8.5) \quad \Omega(\lambda^A, \lambda^B, \sigma^A, \sigma^B, k, \ldots) \equiv EU^{i,U} - EU^{i,NT}, \; i = A, B,$$

and the net private benefit of information relative to not trading, as

$$(8.6) \quad \Gamma^i(\lambda^A, \lambda^B, \sigma^A, \sigma^B, k, \ldots) \equiv EU^{i,S} - EU^{i,NT}, \; i = A, B.$$

An equilibrium is defined as a set of prices, allocations and proportions of individuals of both types who speculate and refrain from trade, λ^{i*} and σ^{i*}, $i = A, B$, such that individuals are maximized on their budget sets, markets clear. That is,

$$(8.7) \quad \lambda^A c_t^{A,S} + (1 - \lambda^A - \sigma^A)c_t^{A,U} + \lambda^B c_t^{B,S} + (1 - \lambda^B - \sigma^B)c_t^{B,U}$$

$$= 2Y_t\{1 - g_t - \lambda k + (\sigma - \sigma^A)\delta\},$$

for the goods market in period t,

$$(8.8)\quad \lambda^A m_t^{A,S} + (1 - \lambda^A - \sigma^A) m_t^{A,U} + \lambda^B m_t^{B,S} + (1 - \lambda^B - \sigma^B) m_t^{B,U} = 2(1-\sigma)\frac{M_{t-1}}{P_t}$$

for the money market in period t (where $\sigma = \frac{1}{2}(\sigma^A + \sigma^B)$ and $\lambda = \frac{1}{2}(\lambda^A + \lambda^B)$)

$$(8.9)\quad \lambda^A c_{t+1}^{A,S} + \sigma^A c_{t+1}^{A,NT} + (1 - \lambda^A - \sigma^A) c_{t+1}^{A,U} + \lambda^B c_{t+1}^{B,S} + \sigma^B c_{t+1}^{B,NT}$$

$$+ (1 - \lambda^B - \sigma^B) c_{t+1}^{B,U} = 2Y_{t+1}(1 - g_{t+1}),$$

for the goods market in period t+1, and

$$(8.10)\quad \psi^i(\lambda^{i*}, \lambda^{j*}, \sigma^{i*}, \sigma^{j*}, \ldots) = 0, \quad 0 < \lambda^{i*} < 1$$

$$\lambda^{i*} = 1 \text{ if } \psi^i(\lambda^i, \lambda^{j*}, 0, \sigma^{j*}, \ldots) > 0 \text{ for all } \lambda^i \in [0,1] \text{ and}$$

$$\Gamma^i(1, \lambda^{j*}, 0, \sigma^{j*}, \ldots) > 0, \text{ and}$$

$$\lambda^{i*} = 0 \text{ if } \psi^i(\lambda^i, \lambda^{j*}, \sigma^{i*}, \sigma^{j*}, \ldots) < 0 \text{ for } \lambda^i \in [0, 1-\sigma^{i*}]$$

$$(8.11)\quad \Omega^i(\lambda^{i*}, \lambda^{j*}, \sigma^{i*}, \sigma^{j*}, \ldots) = 0, \quad 0 < \sigma^{i*} < 1$$

$$\sigma^{i*} = 0 \text{ if } \Omega^i(\lambda^{i*}, \lambda^{j*}, \sigma^i, \sigma^{j*}, \ldots) > 0 \text{ for all } \sigma^i \leq 1 - \lambda^{i*}$$

$$\sigma^{i*} = 1 - \lambda^{i*} \text{ if } \Omega^i(\lambda^{i*}, \lambda^{j*}, \sigma^i, \sigma^{i*}, \ldots) < 0 \text{ for all } \sigma^i \leq 1 - \lambda^{i*}$$

$$\text{and } \Gamma^i(\lambda^{i*}, \lambda^{j*}, 1 - \lambda^{i*}, \sigma^{j*}, \ldots) \geq 0$$

where the inequality holds if $\lambda^{i*} = 0$, for i, j = A, B, i ≠ j.

It is interesting to note that our model provides an explanation for the well-documented excess sensitivity of consumption to current income (Flavin (1981), Hall and Mishkin (1982)). In this case excess sensitivity results not from liquidity constraints, but from both potential borrowers and lenders refraining from trade with possibly better-informed individuals.

To provide an output-based measure of the welfare costs of speculation we consider the amount which individuals would pay to move from a speculative to an uninformed equilibrium. Operationally we compute it by subtracting the previously defined value of information a^* from the average amount, d^*, which individuals in a fully informed equilibrium would be willing to pay to avoid a speculative equilibrium. Formally, d^* is given by

$$(8.12)\quad d^* = \frac{1}{2}(d^{A^*} + d^{B^*}), \text{ where}$$

$$(8.13)\quad EU^{i,II}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta, d^A, d^B) = \lambda^{i*} EU^{i,S}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta, k)$$

$$+ (1-\lambda^{i*}-\sigma^{i*}) EU^{iU}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta, k) + \sigma^{i*} EU^{i,NT}(Y_t, Y_{t+1}, G_t, G_{t+1}, M_{t-1}, \delta, k)$$

$$i = A, B.$$

The most striking result to emerge from simulations of this model is that limited heterogeneity (in the context of a two-period model) is sufficient to produce results which are qualitatively similar to those

obtained in the representative agent model with agents "forced" to trade.[11] For example, if $\delta = 0.1$ (i.e. individuals experience endowments 10% above and below average per capita income) everyone speculates if information costs less than 1.1% of per capita income. If the cost of information exceeds 2.31% of per capita GNP no one speculates and for k between these values interior equilibria (in λ's and σ's) occur. At this level of heterogeneity the social value of information, a^*, is 0.33% of per capita GNP.

Table 8.1 reproduces some of the interior equilibria, expenditures on acquisition of information and the excess burden of speculation, defined as $(d^* - a^*)$ and expressed as a percentage of per capita GNP. As could be expected, the excess burden of speculation exceeds the expenditures on information in interior equilibria because it takes into account, inter alia, the welfare losses engendered when some individuals refrain from trading. Similarly the expenditures on information exceed the excess burden of speculation when everyone speculates.

9. Conclusion

In this paper we have argued that redistributive speculation based on costly information is characterized by an informational externality. We analyzed a simple macroeconomic model in which bond market speculation of this nature occurs because of uncertainty about the financing of government spending.

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[11] The assumed parameter values are as follows: $\alpha = 0.95$, $\beta = 1$, $g_t = g_{t+1} = 0.25$ and τ_{t+1} is assumed to vary between 0.225 and 0.25.

In this setting we demonstrated the existence of an equilibrium in which an endogenously determined fraction of the population is engaged in speculative acquisition of information. We illustrated and characterized the equilibrium social cost of speculation. Comparative statics results are complicated by the existence of multiple equilibria in which a proper fraction of the population speculates.

Simulation exercises designed to complement the analytical results indicate that speculation will only occur if information is relatively cheap, but, if it does occur, may be surprisingly costly. We interpret these results as indicative of the need to model the role of financial institutions as coalitions designed to spread information costs over large numbers of individuals.

The simple representative agent model which we have used for much of the analysis is subject to criticism because it assumes that uninformed individuals trade when it is not in their interest to do so. We have shown, however, that conclusions drawn from it match closely those from a more complete model with heterogeneous agents in which information is valuable and uninformed individuals are free not to trade. We, therefore, interpret the simple representative agent model as a convenient vehicle for comparative statics exercises.

TABLE 8

k	λ^{A*}	σ^{A*}	λ^{B*}	σ^{B*}	$\lambda^{*} \cdot k$	$d^{*} - a^{*}$	a^{*}
2.3	0	0	1.3	0	0.015	0.025	0.33
2.2	0	0	21.83	0	0.24	0.37	0.33
2.1	0	0	52.2	47.8	0.547	0.72	0.33
2.0	0	0	54.5	45.5	0.545	0.71	0.33
1.8	0	0	60.15	39.85	0.541	0.705	0.33
1.5	0	0	71.1	28.9	0.533	0.70	0.33
1.3	65.5	34.5	57.5	42.5	0.80	0.923	0.33
1.1	100.0	0	100.0	0	1.1	0.769	0.33

λ^{A*}, σ^{A*}, λ^{B*} and σ^{B*} are expressed as percentages of the type-A and -B populations. $\lambda^{*} \cdot k$ and $d^{*} - a^{*}$ are expressed as percentages of per capita income in period t.

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APPENDIX A

The derivatives discussed in Section 6 are as follows: That θ_t^I decreasing in k is shown by

$$(A.1) \quad \frac{d\theta_t^I}{dk} = \frac{(1-\lambda)Z}{1+\beta}\left[\frac{\alpha(1-g_t)}{(1-g_t-\lambda k)^2}\right]\left\{\frac{\lambda(1+\mu_t)}{R_t^2}\cdot\frac{A}{A+B} - (1-\lambda)\frac{\partial\theta_t^U}{\partial R_t}\right\} < 0$$

where $Z \equiv \left\{\lambda\cdot\frac{\partial\theta_t^I}{\partial R_t} + (1-\lambda)\cdot\frac{\partial\theta_t^U}{\partial R_t}\right\}^{-1} < 0$, $A \equiv \beta(R_t\theta_t^U)^2$, and

$$B \equiv \left\{1+\bar{\mu}+R_t\left[1-\theta_t^U+\frac{\alpha\lambda k}{1-g_t-\lambda k}\right]\right\}\left\{1+\underline{\mu}+R_t\left[1-\theta_t^U+\frac{\alpha\lambda k}{1-g_t-\lambda k}\right]\right\} > 0$$

Similarly, θ_{t+1}^I is decreasing and θ_{t+1}^U increasing in k, as shown by

$$(A.2) \quad \frac{d\theta_{t+1}^I}{dk} = \left[\frac{\beta}{1+\beta}\right]\left[\frac{Z}{1+\mu_t}\right]\left[\frac{1-g_t}{(1-g_t-\lambda k)^2}\right]\left[\frac{B}{A+B}\right] < 0$$

The effect of a change in λ on θ_t^I is given by

$$(A.3) \quad \frac{d\theta_t^I}{d\lambda} = -Z\left\{\frac{(1-\lambda)\alpha k(1-g_t-k)}{R_t(1-g_t-\lambda k)^2}\left\{\frac{\theta_t^U}{1+\beta} - \left[\theta_t^I + \frac{\alpha k}{1-g_t-\lambda k}\right]\frac{A}{A+B}\right\}\right.$$

$$\left. + \frac{1}{1+\beta}\left[\frac{1+\mu_t}{R_t^2}\right]\left[\theta_t^I - \theta_t^U - \left[\frac{A}{A+B}\right]\frac{(1-\lambda\alpha k^2}{(1-g_t-\lambda k)^2}\right]\right\} \gtrless 0.$$

The response of θ^I_{t+1} is similarly indeterminate:

$$(A.4)\quad \frac{d\theta^I_{t+1}}{d\lambda} = -\left(\frac{\beta}{1+\beta}\right)\left(\frac{Z}{1+\mu_t}\right)\left\{\left(\theta^I_t - \theta^U_t + \frac{\alpha k}{1-g_t-\lambda k}\right)\left(1 - \frac{(1-\lambda)\alpha k}{1-g_t-\lambda k}\right)\right.$$

$$\left. + \left(\frac{A}{A+B}\right)\frac{(1-\lambda)(1-\alpha)\alpha k^2}{(1-g_t-\lambda k)^2}\right\} \gtrless 0,$$

but likely to be positive. In general θ^I_t and θ^I_{t+1} are likely to move in opposite directions in response to a change in λ; an increase in R_t is necessary for θ^I_t to decrease and sufficient for θ^I_{t+1} to increase. Finally,

$$(A.5)\quad \frac{\theta^I_t - \theta^U_t}{\theta^U_t} + \beta\left(\frac{\theta^I_{t+1} - \theta^U_{t+1}}{\theta^U_{t+1}}\right)$$

$$= \frac{1}{\theta^U_t}\left\{(\theta^I_t - \theta^U_t) - \left(\frac{\beta R_t}{1+\mu_t}\right)\left(\frac{\theta^U_t}{\theta^U_{t+1}}\right)\left(\theta^I_t - \theta^U_t + \frac{\alpha k}{1-g-\lambda k}\right)\right\} \gtrless 0$$

Equations (A.3) to (A.5) lead to the statement in the text.

APPENDIX B

The Model with Heterogeneous Agents

Consider the equilibrium which arises when fractions λ^A and λ^B of type-A and type-B agents, respectively, speculate and fractions σ^A and σ^B refrain from trade. The optimization problem of a type-A speculator is

$$\text{(B.1)} \qquad \underset{c_t^{A,S}, m_t^{A,S}, b_t^{A,S}}{\text{Max}} \; EU^{A,S} = \alpha \log c_t^{A,S} + (1-\alpha)\log m_t^{A,S} + \beta E_t^{A,S}\{\alpha \log c_{t+1}^{A,S} + (1-\alpha)\log m_{t+1}^{A,S}\}, \text{ subject to}$$

$$\text{(B.2)} \qquad c_t^{A,S} + m_t^{A,S} + b_t^{A,S} = Y_t(1+\delta) - T_t + \frac{M_{t-1}}{P_t} - K, \text{ and}$$

$$\text{(B.3)} \qquad c_{t+1}^{A,S} + m_{t+1}^{A,S} = Y_{t+1}(1-\delta) - T_{t+1} + (1 + R_t)\frac{P_t}{P_{t+1}} b_t^{A,S} + m_t^{A,S} \cdot \frac{P_t}{P_{t+1}},$$

while that of a type-A uninformed trader is

$$\text{(B.4)} \qquad \underset{c_t^{A,U}, m_t^{A,U}, b_t^{A,U}}{\text{Max}} \; EU^{A,U} = \alpha \log c_t^{A,U} + (1-\alpha)\log m_t^{A,U} + \beta E_t^{A,U}\{\alpha \log c_{t+1}^{A,U} + (1-\alpha)\log m_{t+1}^{A,U}\}, \text{ subject to}$$

$$\text{(B.5)} \qquad c_t^{A,U} + m_t^{A,U} + b_t^{A,U} = Y_t(1+\delta) - T_t + \frac{M_{t-1}}{P_t} \text{ and}$$

$$(B.6)\quad c_{t+1}^{A,U} + m_{t+1}^{A,U} = Y_{t+1}(1-\delta) - T_{t+1} + (1 + R_t)\frac{P_t}{P_{t+1}}b_t^{A,U} + m_t^{A,U}\cdot\frac{P_t}{P_{t+1}} ,$$

where b now denotes real bond holdings. Finally, type-A individuals who refrain from trading solve

$$(B.7)\quad \text{Max } EU^{A,NT} = \alpha \log c_t^{A,NT} + (1-\alpha)\log m_t^{A,NT}$$

$$+ \beta E_t^{A,NT}\{\alpha \log c_{t+1}^{A,NT} + (1-\alpha)\log m_{t+1}^{A,NT}\} \text{ subject to}$$

$$(B.8)\quad c_t^{A,NT} = Y_t(1+\delta) - T_t ,$$

$$(B.9)\quad m_t^{A,NT} = \frac{M_{t-1}}{P_t} , \text{ and}$$

$$(B.10)\quad c_{t+1}^{A,NT} + m_{t+1}^{A,NT} = Y_{t+1}(1-\delta) - T_{t+1} + \frac{M_{t-1}}{P_{t+1}} .$$

The optimization problems for type-B individuals are identical to those of type A individuals, but with $Y_t(1-\delta)$ and $Y_{t+1}(1+\delta)$ replacing $Y_t(1+\delta)$ and $Y_{t+1}(1-\delta)$ in the budget constraints. The government budget constraints are as before.

Let $\lambda = \frac{1}{2}(\lambda^A + \lambda^B)$ and $\sigma = \frac{1}{2}(\sigma^A + \sigma^B)$ and, as before, $K = kY_t$, $G_t = g_tY_t$ and $G_{t+1} = g_{t+1}Y_{t+1}$; then the market-clearing conditions are

$$(B.11)\quad \lambda^A c_t^{A,S} + (1 - \lambda^A - \sigma^A)c_t^{A,U} + \lambda^B c_t^{B,S} + (1 - \lambda^B - \sigma^B)c_t^{B,U}$$

$$= 2Y_t(1 - g_t - \lambda k) - \sigma^A Y_t(1 + \delta - g_t) - \sigma^B Y_t(1 - \delta - g_t)$$

$$= 2Y_t\{1 - g_t - \lambda k + (\sigma - \sigma^A)\delta\},$$

for the goods market in period t,

$$(B.12)\quad \lambda^A m_t^{A,S} + (1-\lambda^A-\sigma^A)m_t^{A,U} + \lambda^B m_t^{B,S} + (1-\lambda^B-\sigma^B)m_t^{B,U} = 2(1-\sigma)\frac{M_{t-1}}{P_t},$$

for the money market in period t, and

$$(B.13)\quad \lambda^A c_{t+1}^{A,S} + \sigma^A c_{t+1}^{A,NT} + (1 - \lambda^A - \sigma^A)c_{t+1}^{A,U} + \lambda^B c_{t+1}^{B,S} + \sigma^B c_{t+1}^{B,NT}$$

$$+ (1 - \lambda^B - \sigma^B)c_{t+1}^{B,U} = 2Y_{t+1}(1 - g_{t+1})$$

for the goods market in period t+1. From the first-order conditions of agents who trade we have:

$$(B.14)\quad m_t^{i,j} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + R_t}{R_t}\right)c_t^{i,j},\ i = A, B, \text{ and } j = S, U,$$

which, along with the money market equilibrium condition yields

$$(B.15)\quad \frac{M_t}{P_t} = \frac{M_{t-1}}{P_t} = Y_t(1 - g_t - \lambda k)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1 + R_t}{R_t}\right)(1 + A), \text{ where}$$

$$A = \frac{\delta(\sigma - \sigma^A) - \sigma\lambda k}{(1-\sigma)(1 - g_t - \lambda k)},$$

since we've made the simplifying assumption that the government does not create any money in period t. Using equation (B.15) the real bond holdings of individuals who trade in period t are

$$(B.17)\quad b_t^{A,S} = Y_t(1 - g_t - \lambda k)\left\{\left\{1 + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]\right\}(1 - \theta_t^{A,S}) + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]A + \frac{\delta - (1-\lambda)k}{1 - g_t - \lambda k}\right\},$$

$$(B.18)\quad b_t^{A,U} = Y_t(1 - g_t - \lambda k)\left\{\left\{1 + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]\right\}(1 - \theta_t^{A,U}) + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]A + \frac{\delta + \lambda k}{1 - g_t - \lambda k}\right\},$$

$$(B.19)\quad b_t^{B,S} = Y_t(1 - g_t - \lambda k)\left\{\left\{1 + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]\right\}(1 - \theta_t^{B,S}) + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]A - \left[\frac{\delta + (1-\lambda)k}{1 - g_t - \lambda k}\right]\right\}, \text{ and}$$

$$(B.20)\quad b_t^{B,U} = Y_t(1 - g_t - \lambda k)\left\{\left\{1 + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]\right\}(1 - \theta_t^{B,U}) + \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1 + R_t}{R_t}\right]A - \left[\frac{\delta - \lambda k}{1 - g_t - \lambda k}\right]\right\},$$

where the θ's are consumption shares as before. From the first-order conditions for allocating resources in period t+1:

$$(B.21)\quad m_{t+1}^{i,j} = \left(\frac{1-\alpha}{\alpha}\right)c_{t+1}^{i,j},\ i = A,\ B,\quad j = S,\ U,\ NT,$$

and the equilibrium conditions, equations (A.13) and (A.15), the (inverse of) the inflation rate is given by

$$(B.22)\quad \frac{P_t}{P_{t+1}} = \left[\frac{R_t}{1+R_t}\right]\left[\frac{M_t}{M_{t+1}}\right]\left\{\frac{Y_{t+1}(1-g_{t+1})}{Y_{t+1}(1-g_t-\lambda k)}\right\}(1+A)^{-1},$$

where $M_{t+1} = (1+\mu_t)M_t$, with the rate of monetary growth, μ_t, determined by equation (4.11) as before. Equations (B.21) and (B.22) enable us to write period-t+1 consumption shares as

$$(B.23)\quad \theta_{t+1}^{A,S} = \left\{1 - \frac{\alpha\delta}{1-g_{t+1}} + \frac{R_t}{(1+\mu_t)(1+A)}\left[1 + (1-\alpha)A - \theta_t^{A,S} + \frac{\alpha(\delta-(1-\lambda)k)}{1-g_t-\lambda k}\right]\right\}$$

$$(B.24)\quad \theta_{t+1}^{A,U} = \left\{1 - \frac{\alpha\delta}{1-g_{t+1}} + \frac{R_t}{(1+\mu_t)(1+A)}\left[1 + (1-\alpha)A - \theta_t^{A,U} + \frac{\alpha(\delta+\lambda k)}{1-g_t-\lambda k}\right]\right\}$$

$$(B.25)\quad \theta_{t+1}^{B,S} = \left\{1 + \frac{\alpha\delta}{1-g_{t+1}} + \frac{R_t}{(1+\mu_t)(1+A)}\left[1 + (1-\alpha)A - \theta_t^{B,S} - \frac{\alpha(\delta+(1-\lambda)k)}{1-g_t-\lambda k}\right]\right\}$$

$$(B.26)\quad \theta_{t+1}^{B,U} = \left\{1 + \frac{\alpha\delta}{1-g_{t+1}} + \frac{R_t}{(1+\mu_t)(1+A)}\left[1 + (1-\alpha)A - \theta_t^{B,U} - \frac{\alpha(\delta-\lambda k)}{1-g_t-\lambda k}\right]\right\}$$

$$(B.27)\quad \theta_{t+1}^{A,NT} = \left\{1 - \frac{\alpha\delta}{1-g_{t+1}}\right\},$$

and finally

$$(B.28)\quad \theta_{t+1}^{B,NT} = \left\{1 + \frac{\alpha\delta}{1-g_{t+1}}\right\}.$$

Equations (B.23) to (B.26) along with the first-order condition governing individuals' actions in the bond market:

$$(B.29)\quad \frac{1}{c_t^{i,j}} = \beta(1 + R_t)E_t^{i,j}\left\{\frac{P_t}{P_{t+1}c_{t+1}^{i,j}}\right\},$$

which can be rewritten as:

$$(B.30)\quad \left[\theta_t^{i,j}\right]^{-1} = \frac{\beta R_t}{1+A} E_t^{i,j}\left\{\left[(1 + \mu_t)\theta_{t+1}^{i,j}\right]^{-1}\right\},$$

yield:

$$(B.31)\quad \theta_t^{A,S} = \frac{1}{1+\beta}\left\{\frac{(1+\mu_t)(1+A)}{R_t}\left\{1 - \frac{\alpha\delta}{1 + g_{t+1}}\right\} + 1 + (1-\alpha)A + \frac{\alpha[\delta-(1-\lambda)k]}{1 - g_t - \lambda k}\right\},$$

$$(B.32)\quad \theta_t^{B,S} = \frac{1}{1+\beta}\left\{\frac{(1+\mu_t)(1+A)}{R_t}\left\{1 + \frac{\alpha\delta}{1 + g_{t+1}}\right\} + 1 + (1-\alpha)A - \frac{\alpha[\delta+(1-\lambda)k]}{1 - g_t - \lambda k}\right\}$$

$$(B.33)\quad \left[\theta_t^{A,U}\right]^{-1} = \frac{\beta R_t}{\bar{\mu}-\underline{\mu}}\left\{\log\left\{(1+\bar{\mu})(1+A)\left\{1 - \frac{\alpha\delta}{1-g_{t+1}}\right\}\right.\right.$$

$$\left.\left. + R_t\left\{1 + (1-\alpha)A - \theta_t^{A,U} + \frac{\alpha(\delta+\lambda k)}{1-g_t-\lambda k}\right\}\right\}$$

$$- \log \left\{ (1+\underline{\mu})(1+A)\left\{1 - \frac{\alpha\delta}{1-g_{t+1}}\right\} + R_t\left\{1 + (1-\alpha)A - \theta_t^{A,U} + \frac{\alpha(\delta+\lambda k)}{1-g_t-\lambda k}\right\}\right\}$$

$$\text{(B.34)} \quad \left[\theta_t^{B,U}\right]^{-1} = \frac{\beta R_t}{\bar{\mu}-\underline{\mu}}\left\{\log\left\{(1+\bar{\mu})(1+A)\left\{1 + \frac{\alpha\delta}{1-g_{t+1}}\right\}\right.\right.$$

$$\left. + R_t\left\{1 + (1-\alpha)A - \theta_t^{B,U} - \frac{\alpha(\delta-\lambda k)}{1-g_t-\lambda k}\right\}\right\}$$

$$\left. - \log\left\{(1+\underline{\mu})(1+A)\left\{1 + \frac{\alpha\delta}{1-g_{t+1}}\right\} + R_t\left\{1 + (1-\alpha)A - \theta_t^{B,U} - \frac{\alpha(\delta-\lambda k)}{1-g_t-\lambda k}\right\}\right\}\right\},$$

$$\text{(B.35)} \quad \theta_t^{A,NT} = 1 + \frac{\delta + \lambda k}{1 - g_t - \lambda k},$$

$$\text{(B.36)} \quad \theta_t^{B,NT} = 1 - \left\{\frac{\delta + \lambda k}{1 - g_t - \lambda k}\right\},$$

and, finally,

$$\text{(B.37)} \quad m_t^{A,NT} = m_t^{B,NT} = Y_t(1 - g_t - \lambda k)\left\{\frac{1-\alpha}{\alpha}\right\}\left\{\frac{1 + R_t}{R_t}\right\}(1+A).$$

The expected utilities of individuals who trade are given, as before, by

$$\text{(B.38)} \quad EU^{i,j} = \log Y_t + \beta \log Y_{t+1} + \log(1 - g_t - \lambda k) + \beta \log (1 - g_{t+1})$$

$$+ (1+\beta)(1-\alpha)\log\left[\frac{1-\alpha}{\alpha}\right] + E\left\{\log \theta_t^{i,j} + \beta\log \theta_{t+1}^{i,j} + (1-\alpha)\log\left[\frac{1+R_t}{R_t}\right]\right\},$$

$$i = A, B, \quad j = S, U,$$

while the expected utility of an individual who does not trade is given by

$$(B.39)\quad EU^{i,NT} = E\left\{\alpha\log c_t^{i,NT} + (1-\alpha)\log m_t^{i,NT}\right\} + \beta\left\{\alpha\log c_{t+1}^{i,NT} + (1-\alpha)\log m_{t+1}^{i,NT}\right\}$$

$$= E\left\{\alpha\log(Y_t(1-g_t-\lambda k)\theta_t^{i,NT}) + (1-\alpha)\log\left[Y_t(1-g_t-\lambda k)\left[\frac{1-\alpha}{\alpha}\right]\left[\frac{1+R_t}{R_t}\right](1+A)\right]\right.$$

$$\left. + \beta\left\{\log Y_{t+1} + \log(1-g_{t+1}) + \log \theta_{t+1}^{i,NT} + (1-\alpha)\log\left[\frac{1-\alpha}{\alpha}\right]\right\}\right\}$$

$$= \log Y_t + \beta\log Y_{t+1} + \log(1-g_t-\lambda k) + \beta\log(1-g_{t+1}) + (1+\beta)(1-\alpha)\log\left[\frac{1-\alpha}{\alpha}\right]$$

$$+ (1-\alpha)\log(1+A) + E\left\{\alpha\log \theta_t^{i,NT} + \beta\log \theta_{t+1}^{i,NT} + (1-\alpha)\log\left[\frac{1+R_t}{R_t}\right]\right\},$$

$i = A, B$. Define the net private benefit of information relative to uninformed trading as before:

$$(B.40)\quad \psi^i(\lambda^A, \lambda^B, \sigma^A, \sigma^B, k, \ldots) \equiv EU^{i,S} - EU^{i,U}$$

$$= E\{\log \theta_t^{i,S} - \log \theta_t^{i,U}) + \beta(\log \theta_{t+1}^{i,S} - \log \theta_{t+1}^{i,U})\}$$

$i = A, B$, the net benefit of uninformed trade relative to not trading as

(B.41) $\Omega^i(\lambda^A, \lambda^B, \sigma^A, \sigma^B, \ldots) = EU^{i,U} - EU^{i,NT}$

$= E\{\log \theta_t^{i,U} - \alpha \log \theta_t^{i,NT} + \beta(\log \theta_{t+1}^{i,U} - \log \theta_{t+1}^{i,NT})\} - (1-\alpha)\log(1+A),$

$i = A, B$, and the net benefit of information relative to not trading as

(B.42) $\Gamma^i(\lambda^A, \lambda^B, \sigma^A, \sigma^B, \ldots) = EU^{i,S} - EU^{i,NT}$

$= E\{\log \theta_t^{i,S} - \alpha \log \theta_t^{i,NT} + \beta(\log \theta_{t+1}^{i,S} - \log \theta_{t+1}^{i,NT})\} - (1-\alpha)\log(1+A)$,

$= \psi^i(\ldots) + \Omega^i(\ldots).$

An equilibrium is defined as a set of prices, allocations, and values of λ's and σ's, λ^{i*} and σ^{i*}, $i = A, B$, such that individuals are maximized on their budget sets, markets clear, and

(B.43) $\psi^i(\lambda^{i*}, \lambda^{j*}, \sigma^{i*}, \sigma^{j*}, \ldots) = 0 \ , \quad 0 < \lambda^{i*} < 1$

$\lambda^{i*} = 1$ if $\psi^i(\lambda^i, \lambda^{j*}, 0, \sigma^{j*}, \ldots) > 0$ for all $\lambda^i \in [0,1]$ and

$\Gamma^i(1, \lambda^{j*}, 0, \sigma^{j*}, \ldots) > 0$, and

$\lambda^{i*} = 0$ if $\psi^i(\lambda^i, \lambda^{j*}, \sigma^{i*}, \sigma^{j*}, \ldots) < 0$ for $\lambda^i \in [0, 1-\sigma^{i*}]$

(B.44) $\Omega^i(\lambda^{i*}, \lambda^{j*}, \sigma^{i*}, \sigma^{j*}, \ldots) = 0, \quad 0 < \sigma^{i*} < 1$

$\sigma^{i*} = 0$ if $\Omega^i(\lambda^{i*}, \lambda^{j*}, \sigma^i, \sigma^{j*}, \ldots) > 0$ for all $\sigma^i \leq 1-\lambda^{i*}$

$$\sigma^{i*} = 1-\lambda^{i*} \text{ if } \Omega^i(\lambda^{i*}, \lambda^{j*}, \sigma^i, \sigma^{i*}, \ldots) < 0 \text{ for all } \sigma^i \leq 1-\lambda^{i*}$$

$$\text{and } \Gamma^i(\lambda^{i*}, \lambda^{j*}, 1-\lambda^{i*}, \sigma^{j*}, \ldots) \geq 0,$$

where the inequality holds if $\lambda^{i*} = 0$, for i, j = A, B, $i \neq j$.

Figure 7.1
Utility Differential k: 0 to 10%

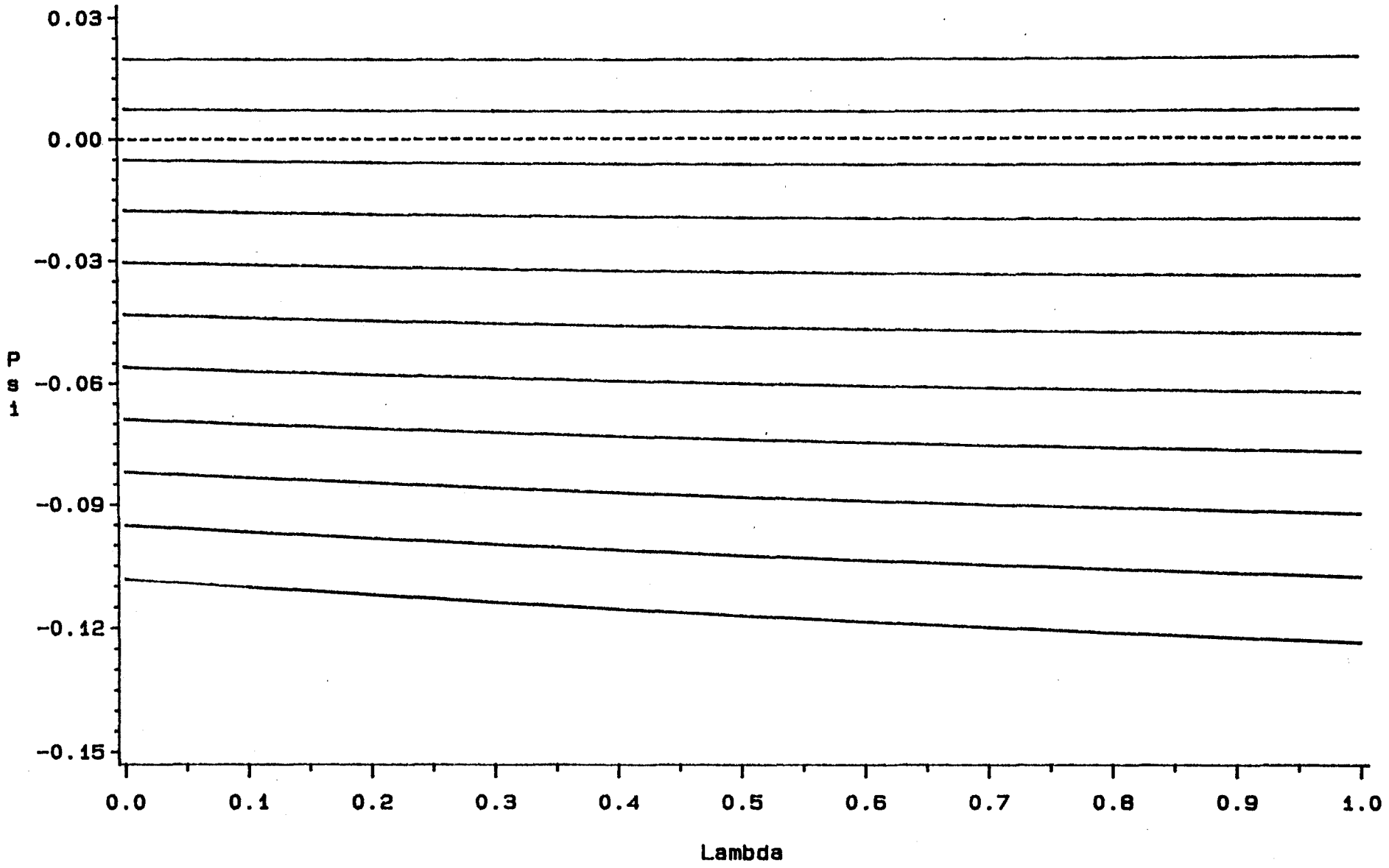


Figure 7.2
Utility Differential k: 1.0% to 1.8%

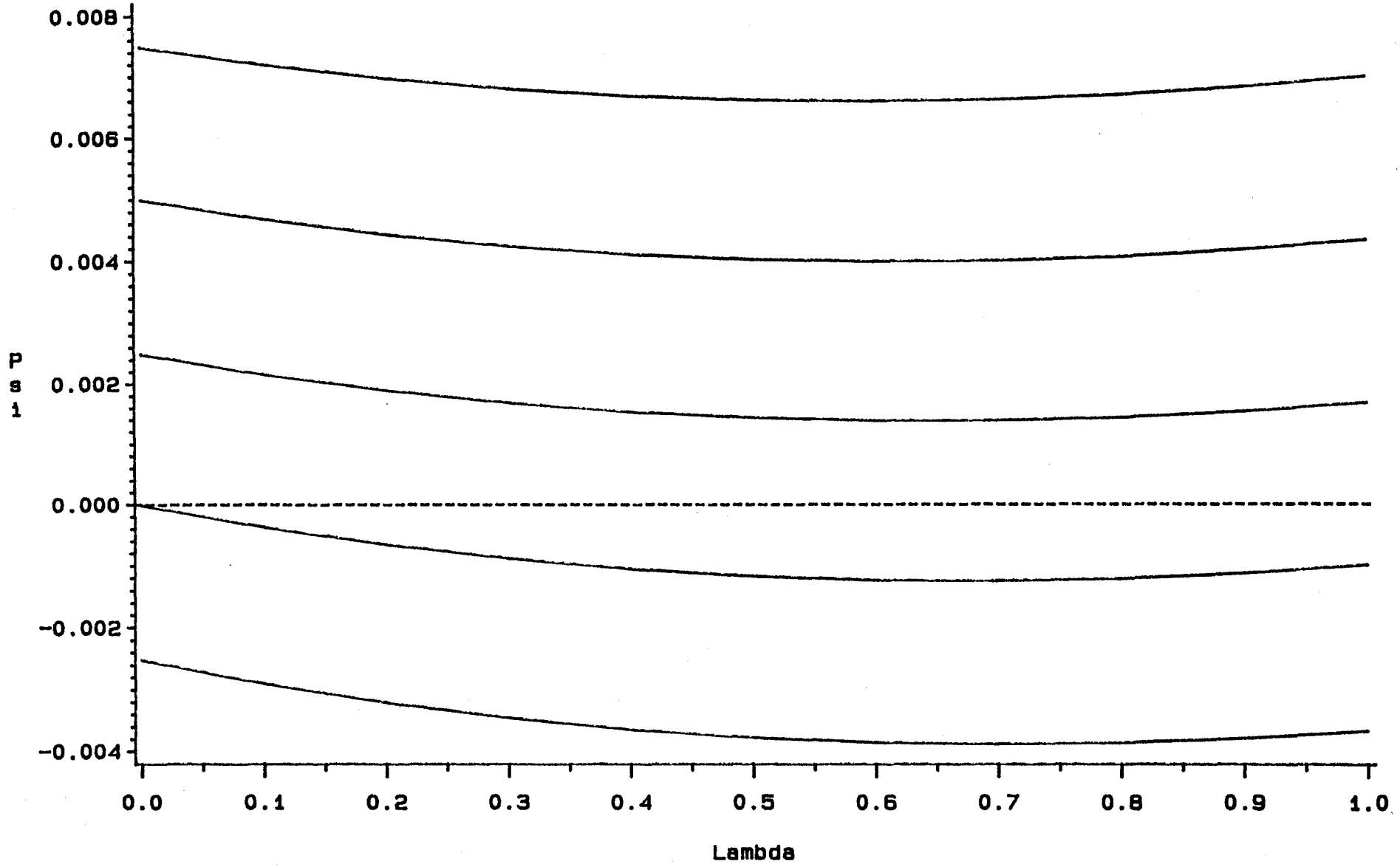


Figure 7.3
Equilibrium Fraction of Speculators

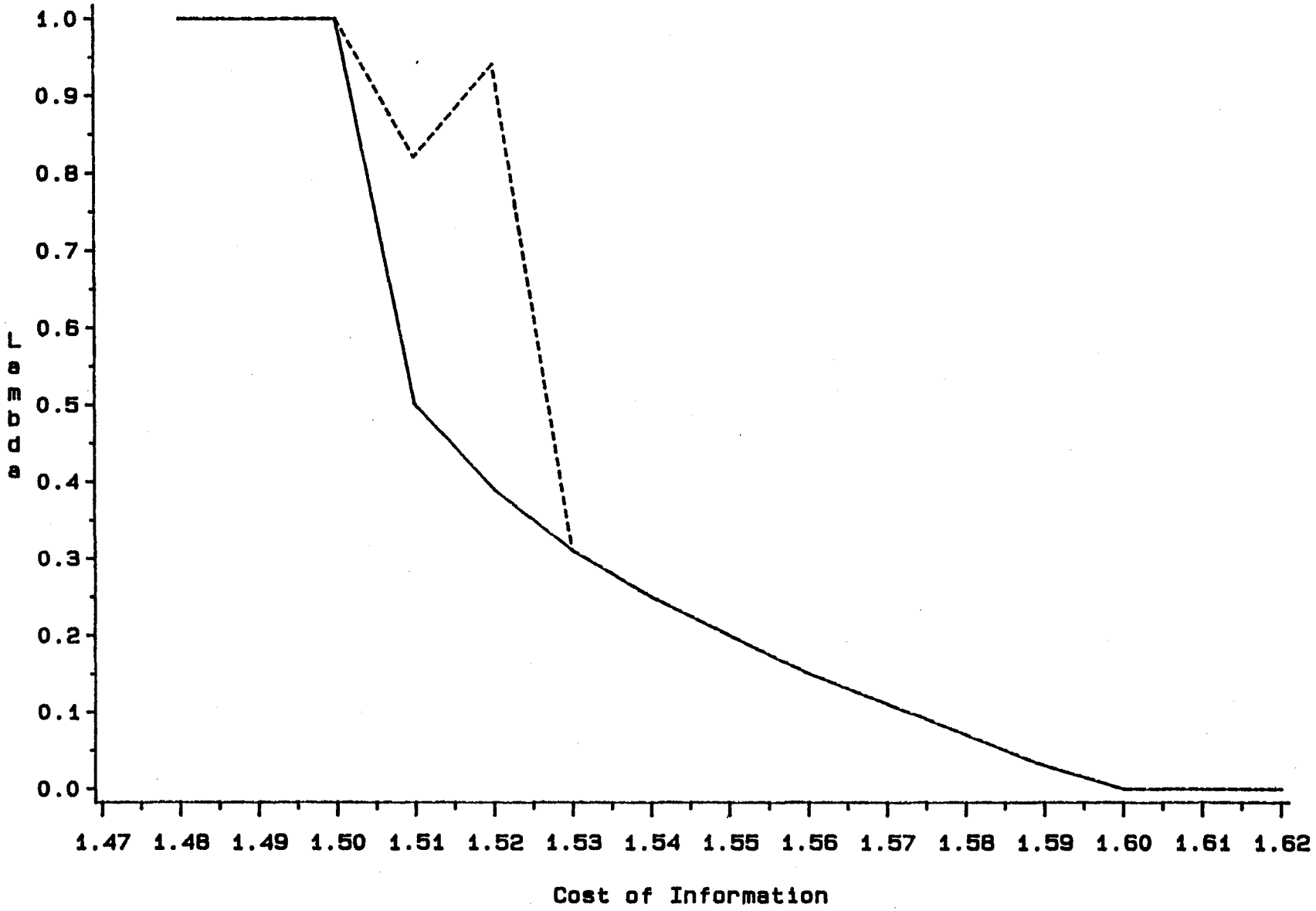


Figure 7.4
Equilibrium Percentage Output Loss

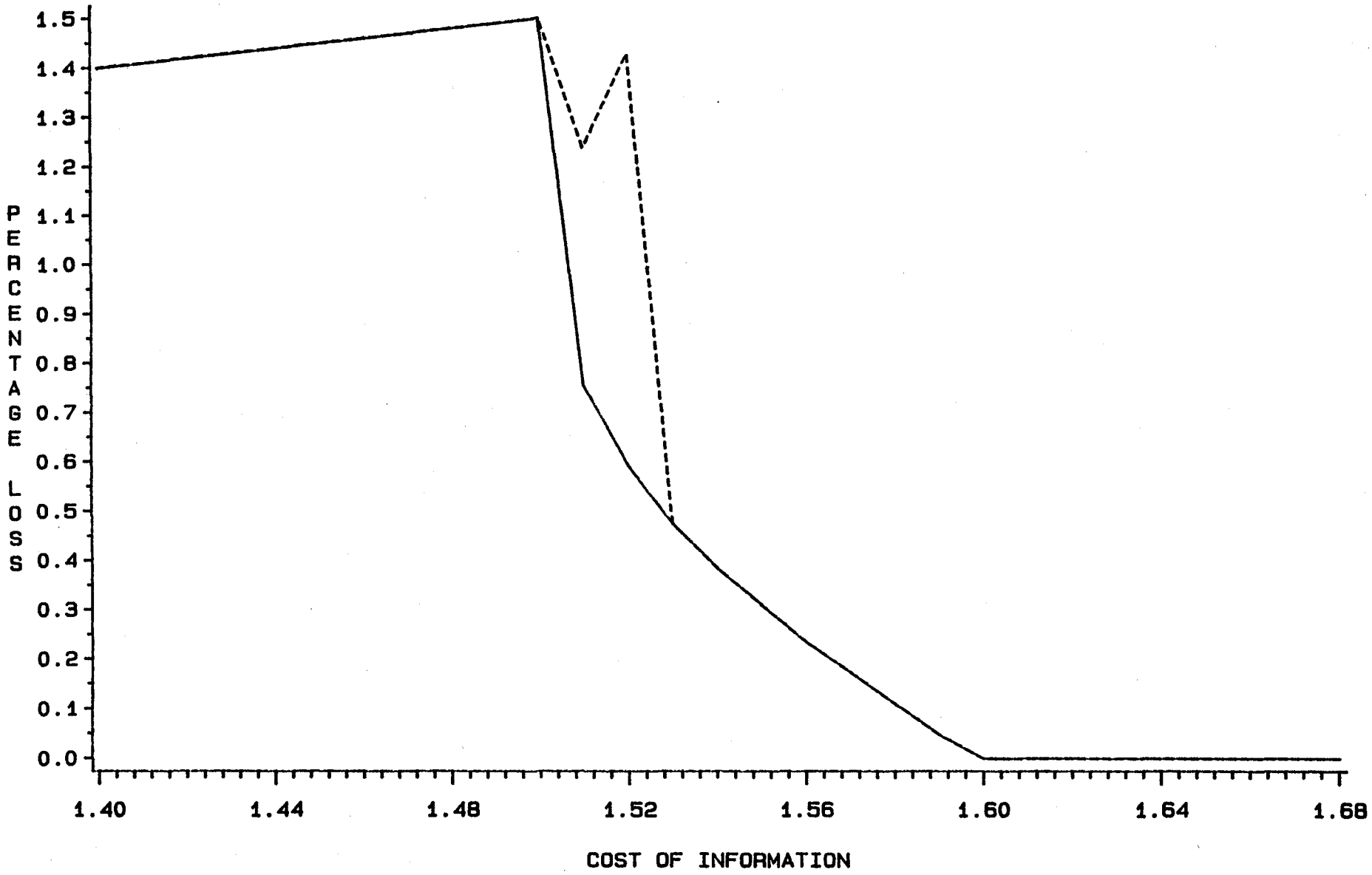
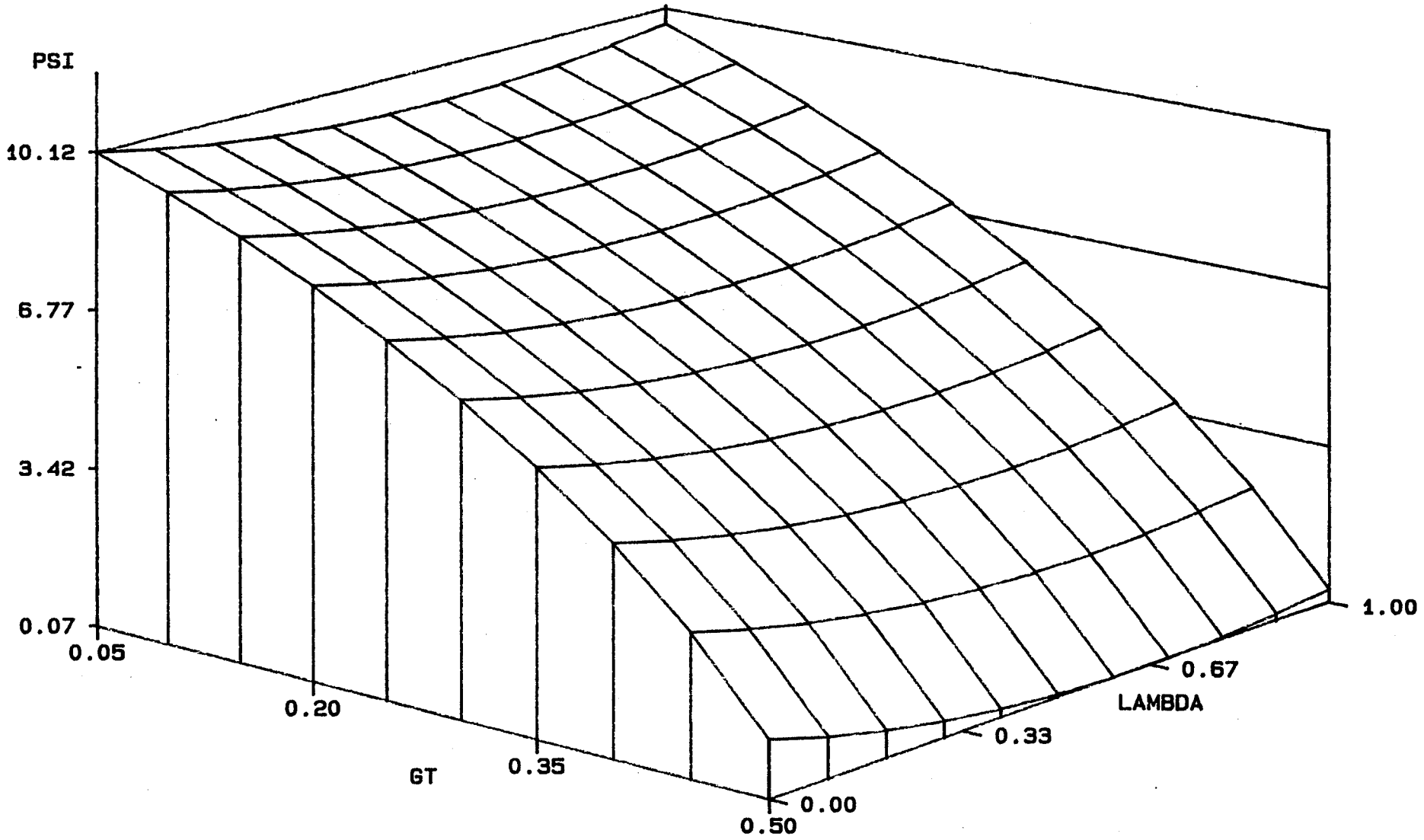
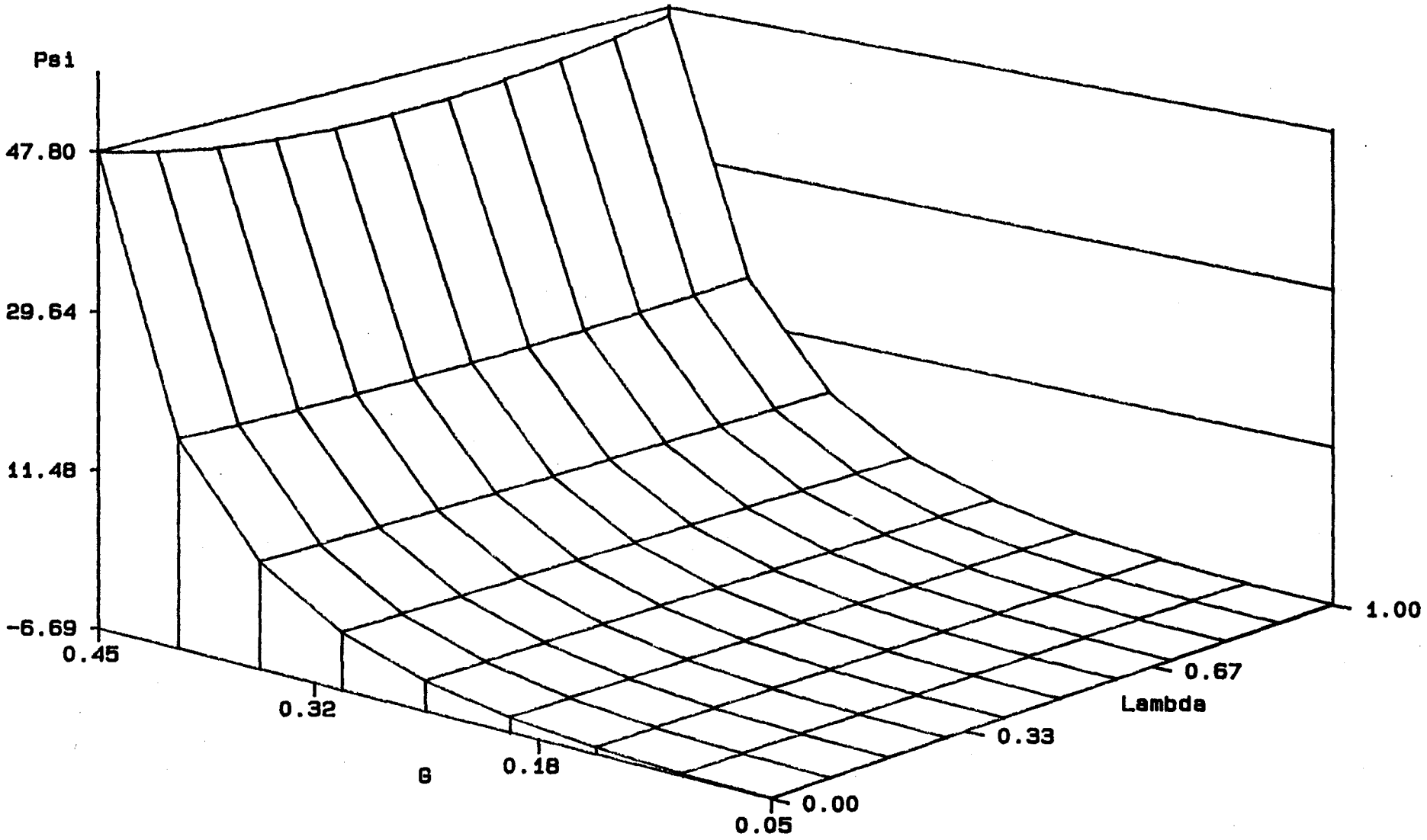


Figure 7.5
Utility Differential As a Function of Lambda and Current Government Spending



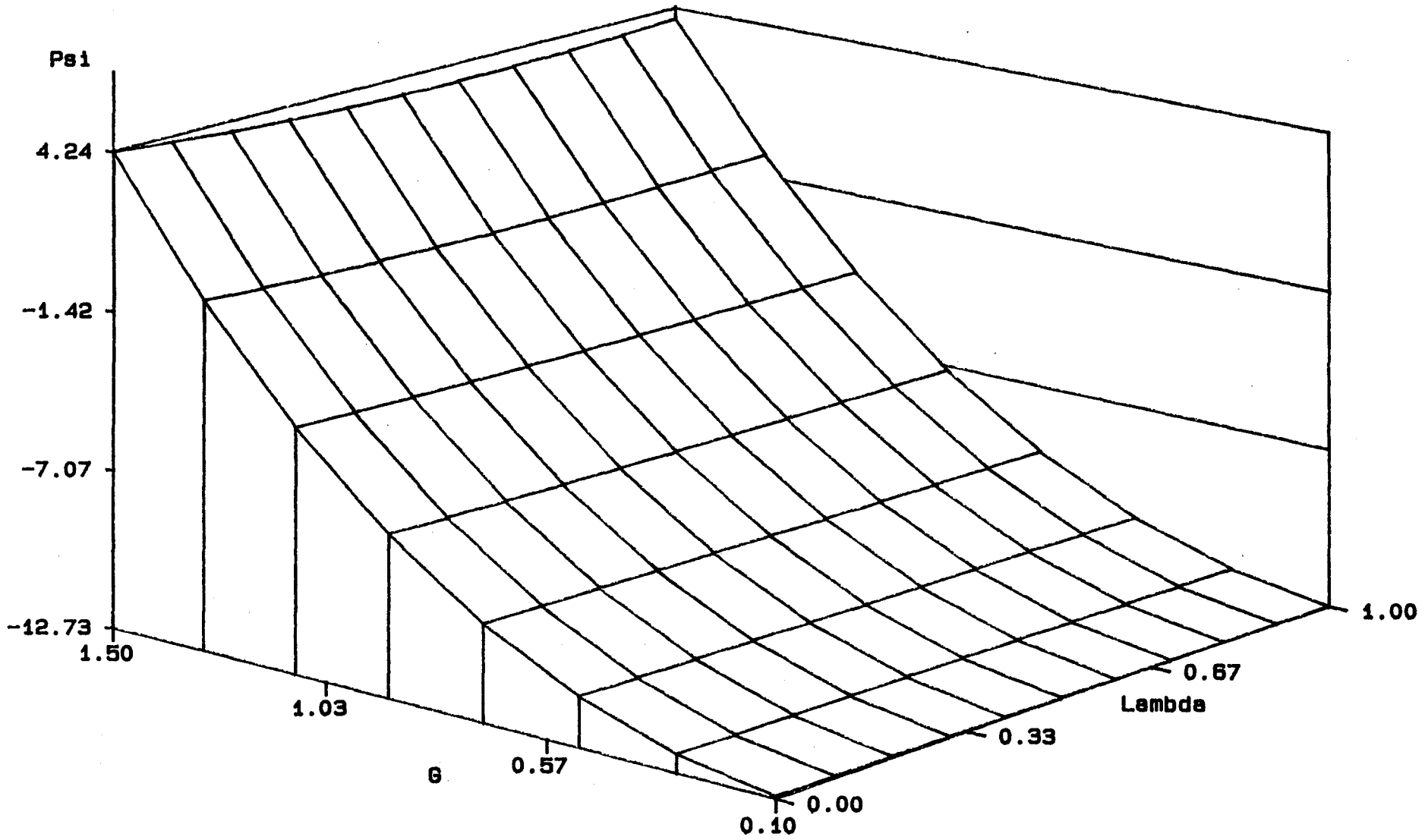
PSI=PSI*1000

Figure 7.6
Utility Differential as a Function of Future Government Spending



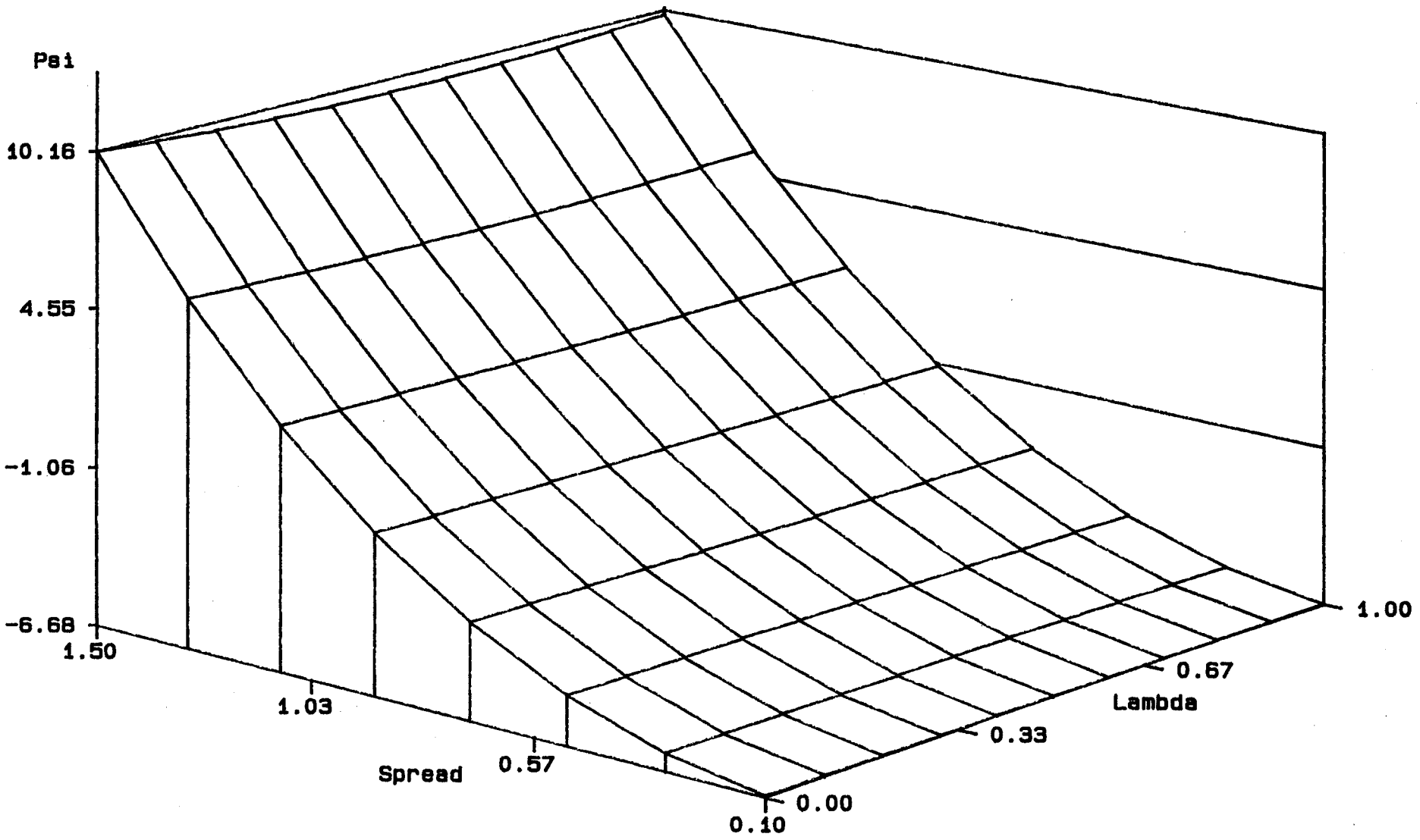
Psi=Psi*1000

Figure 7.7
Utility Differential as a Function of Government Spending (Permanent)



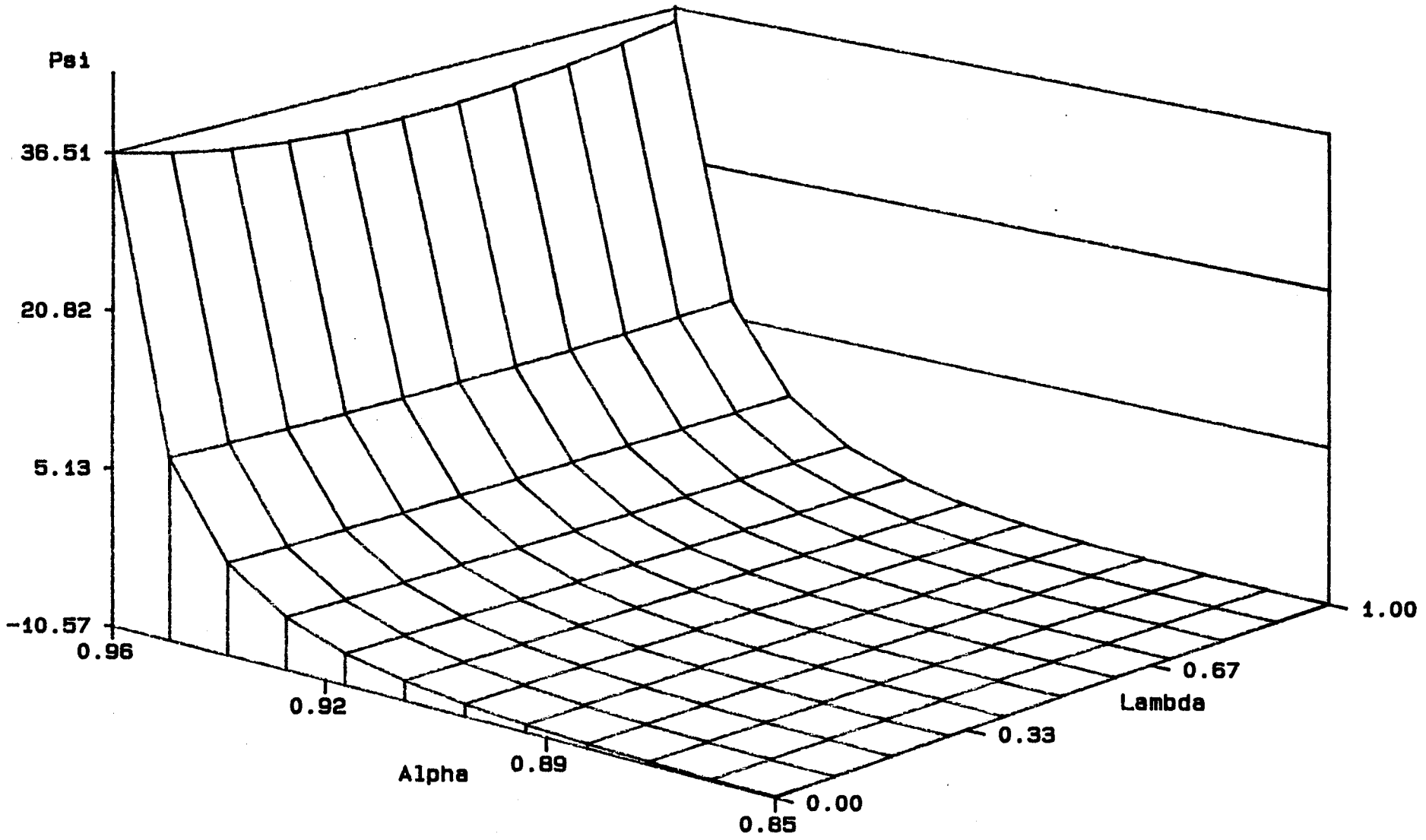
psi=psi*1000

Figure 7.8
Utility Differential As a Function of Lambda and Monetary Uncertainty



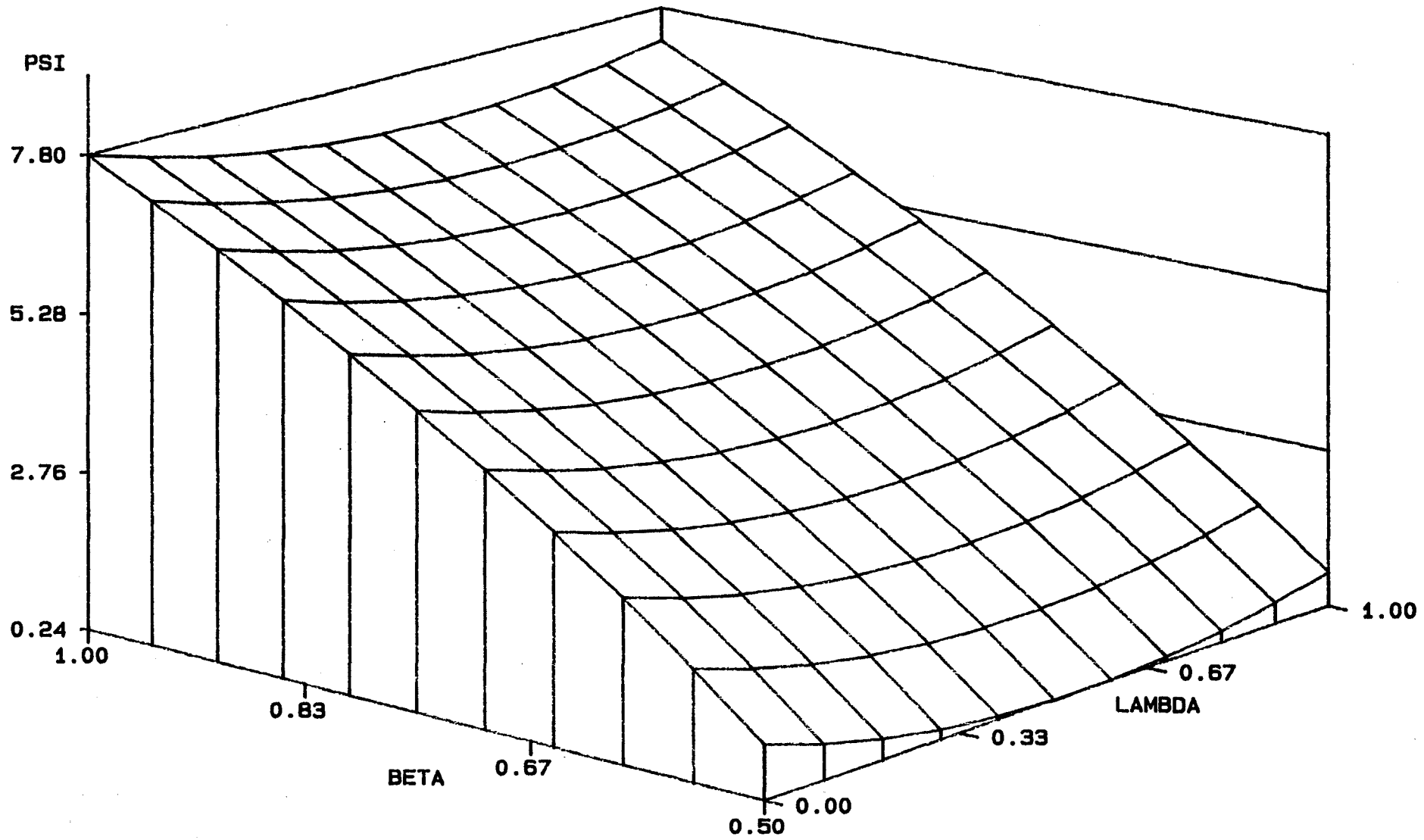
psi=psi*1000

Figure 7.9
Utility Differential As a Function of Lambda and Alpha



psi=psi*1000

Figure 7.10
Utility Differential As a Function of Lambda and Beta



PSI=PSI*1000