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## Structural and Stabilization Aspects of Fiscal and Financial Policy in the Dependent Economy, Part I

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CENTER DISCUSSION PAPER NO. 517

STRUCTURAL AND STABILIZATION ASPECTS OF FISCAL  
AND FINANCIAL POLICY IN THE DEPENDENT ECONOMY

Part 1

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Structural and Stabilization Aspects of Fiscal and Financial Policy  
in the Dependent Economy  
Part I

Abstract

The paper considers the response of a small, open dependent economy to a variety of fiscal and financial shocks as well as the influence of alternative budget balancing rules on the response of the system to such external shocks as a change in the world interest rate.

The approach allows for both uncertain individual lifetimes and population growth, using a slightly generalized version of the Yaari-Blanchard model of consumer behavior. Debt neutrality does not prevail unless the sum of the population growth rate and the individual's probability of death equals zero. The government spends on traded and non-traded goods and raises tax revenue both through a lump sum tax and through a distortionary tax on the production of traded goods.

Even though the tax on the production of traded goods is the only conventional distortion in the model, changes in this tax rate will have first order real income effects even when the distortion is evaluated at a zero tax rate, as long as the individual's subjective pure rate of time preference differs from the interest rate. This can occur even in well-behaved steady states of the Yaari-Blanchard model, as long as the population growth rate plus the probability of death differ from zero. This "intrinsic" distortion effectively causes second-best arguments to apply even when there is only one conventional distortion.

Even in the absence of government budget deficits, fiscal choices relating to the composition of public spending and the structure of taxation have important short-term and long-term consequences for the real exchange rate, the sectoral allocation of production, the level and composition of private consumption, the current account (in the short run) and the nation's stock of claims on the rest of the world in the long run.

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I would like to thank Jonathan Eaton for many helpful discussions, especially in connection with the "first-order" real income effects of changes in a distortionary tax rate, when evaluated at a zero level of the distortion, in the Yaari-Blanchard model of consumption.

## 1. Introduction

This is the first of a series of three papers dealing with the macroeconomic effects of fiscal policy in an open economy, a very broad subject area. It covers the implications of alternative public spending, revenue raising and financing actions (or of alternative public spending, revenue raising and financing rules) for a wide range of endogenous macroeconomic variables, both prices and quantities. Among the most important prices are nominal and real interest rates and the cost of capital, the nominal and real exchange rate, the nominal and real wage rate, the terms of trade and the general price level. The most important quantities are the level of domestic output and employment and their composition in terms of traded and nontraded goods sectors, the domestic capital stock and the domestic rate of capital accumulation, the financial assets and liabilities of the private and government sectors and their financial deficits or surpluses. The responses of these prices and quantities can be studied in the short run (i.e. at a point in time), in the timeless long run of the stationary or steady state and, most interestingly, along a "real-time" dynamic adjustment path.

There are many taxonomies for classifying and subdividing this vast subject. The one adopted in these papers involves a two way "real" vs. "monetary" and "balanced-budget" vs. "unbalanced budget" classification. Real models, considered in the first two papers, do not include a monetary sector. Private agents do not hold money and seigniorage or the inflation tax is not a source of government revenue. Governments can borrow, by issuing interest-bearing "real" debt, and financial crowding out can occur, in the short-run and/or the long run. Money is added to the private sector's portfolio of financial assets and money creation to the public sector's sources of revenue in the third paper.

The "balanced" vs. "unbalanced" budget dichotomy emphasizes that there are two kinds of macroeconomic effects associated with fiscal policy, even in "real" models where money financing is not an option. The first kind operates through the public sector debt dynamics associated with unbalanced public sector budgets, i.e. with public sectors financial deficits or surpluses. The presence of these effects is contingent on the absence of "debt neutrality". The second kind, the balanced budget effects, are the consequences of variations in the volume and composition of public spending and in the level and structure of taxation for real and nominal variables that occur even when the budget is balanced continuously. These effects are not contingent on the absence of debt neutrality. They operate through "direct crowding out" (Buiter [1977]), i.e. through the direct complementarity or substitutability of public consumption or investment and private consumption or investment, through the direct (selective or across-the-board) preemption of real resources by government absorption and through the allocative or incentive effects of changes in non-lump sum taxes, transfers, or subsidies.

Recently, the focus of much macroeconomic analysis has been on public sector debt and deficits and their consequences, through "indirect financial crowding out," for private saving, capital formation and the current account of the balance of payments. The second and third papers address some of these issues. The present paper, however, focuses on the important effects of balanced budget fiscal policy on the real exchange rate, the sectoral allocation of productive resources between traded and non-traded goods, private consumption and the nation's net external asset position. It considers the impact, steady state and transitional effects of balanced-budget fiscal policy in a simple open economy macroeconomic model, which is set out in Section 2. The "balanced budget" in this paper is, strictly speaking, a

"real growth corrected" balanced budget. In such an equilibrium the deficit, as a fraction of full employment GDP, equals  $n\beta$ , where  $n$  is the growth rate of full employment real output and  $\beta$  is the public debt outstanding as a fraction of capacity GDP. Such a policy keeps the burden of the debt,  $\beta$ , constant over time. When there is no real growth ( $n=0$ ) the growth-corrected balanced budget and the conventional balanced budget coincide for the model of papers 1 and 2. For the monetary model in paper 3 we define a "real growth, inflation and exchange rate appreciation-corrected balanced budget." This corresponds to a conventionally measured budget deficit (as a fraction of GDP) equal to  $n\beta + \hat{p}(1 - k_1)\beta - \hat{e}k_2 + (n + \hat{p})\mu$ . Here  $n$  is as before the growth rate of capacity real output,  $\beta$  is the ratio of all government interest-bearing public debt to capacity GDP,  $k_1$  is the share of the public debt that is index-linked,  $k_2$  is the share of the public debt that is denominated in foreign currency,  $\mu$  is the monetary base-capacity GDP ratio,  $\hat{p}$  is the rate of inflation and  $\hat{e}$  is the proportional rate of exchange rate depreciation. A deficit or surplus of this magnitude is consistent with maintaining a constant overall debt burden and with constant ratios to capacity GDP of each of the interest-bearing and monetary liabilities of the authorities.

The model developed in Section 2 of this paper bundles familiar ingredients in a somewhat novel fashion. Private consumption behavior is modeled according to the uncertain lifetimes version of the overlapping generations model due to Yaari (1965) and first applied to the analysis of macroeconomic issues by Blanchard (1984, 1985). Uncertain lifetimes result in effective private sector subjective discount rates that exceed the pure rate of time preference and in effective private sector market discount rates that exceed the government's interest rate. This causes absence of debt-neutrality and creates a non-trivial role for financial policy, i.e. for variations in

the taxation-borrowing mix for a given path of real public spending on current goods and services (or "exhaustive" spending), even when taxes are lump-sum.

Even without uncertain lifetimes, population growth alone can, as shown by Weil (1985), cause absence of debt neutrality. Even infinite-lived consumers cannot command the resources of those yet to be born, while the authorities can use them as a tax base. A growing population therefore extends the public sectors' opportunity set beyond that available to the private agents currently alive. (See also Buiter [1986b]).

The production side of the model is the "dependent economy model" of Wilson (1931), Swan (1960) and Salter (1959) in the modern version due to Dornbusch (1974, 1980, Ch. 6, pp. 93-116).

The country is assumed to be a price taker in the world markets for all tradeables, importables and exportables alike. The terms of trade are therefore exogenous. For the issues addressed in these papers they can be ignored. They are aggregated into a composite commodity, traded goods. Non-traded goods are also produced and consumed (by the private and the public sector). The real exchange rate, or the relative price of traded to non-traded goods, is the crucial atemporal or static relative price in this model. Wage and price flexibility and full employment of resources are assumed to hold throughout.

Some of the key conclusions are the following. Balanced budget fiscal policy actions will in general have short-run and long-run effects on the real exchange rate and real private absorption. They will also tend to have transitory effects on the current account and permanent effects on the nation's net foreign asset position. "Balanced-budget multipliers" can be negative in the short run and in the long run when the increase in public spending is financed by higher distortionary tax rates. An increase in the

distortionary tax rate on the production of tradeables may have such strong negative income effects, that the demand for nontraded goods declines sufficiently to lower the producer's relative price of nontraded goods. In that extreme case nontradeables production could contract and tradeables production expand in spite of the increase in the tax rate on traded goods production. This can happen even when the initial level of the distortion is zero, if the (exogenous) real interest rate exceeds the (exogenous) rate of time-preference, something that is consistent with equilibrium given Yaari-Blanchard preferences.

## 2. The Model

### a. Private Consumption

Household consumption behavior is determined by the intertemporal optimization model of Yaari and Blanchard (Yaari [1965], Blanchard [1984, 1985]). Each individual has a constant (age-and time-independent) instantaneous probability of death  $\lambda \geq 0$ . Another interpretation of  $\lambda$  is the probability of dynastic extinction (through childlessness) in a model in which an intergenerational gift or bequest motive is permitted. The details of the derivation of the individual and aggregate consumption rules are given in Appendix 1. The important consequences of introducing uncertain lifetimes are (1) that the effective instantaneous subjective discount rate of households is  $\delta + \lambda$ , where  $\delta$  is the instantaneous pure rate of subjective time preference ( $\delta > 0$ ) and (2) that the effective market discount rate is  $r + \lambda$  where  $r$  is the instantaneous real interest rate. Population growth in addition means that the public sector's tax base grows at a rate  $n$ , unlike the resource base available to those private agents currently alive (even if  $\lambda=0$ ). (See Weil [1985] and Buiter [1986b]).



An individual household discounts its uncertain future after-tax non-interest income using a discount rate  $r + \lambda$ , while the government discounts its certain and growing (at a rate  $n$ ) aggregate future tax revenue using a discount rate  $r$ . Another way of looking at this is that aggregate private consumption and saving behavior are functions of the human capital of those households that are currently alive. The government knows it will be able to tax in the future both those households alive today and those yet to be born that survive into the future. Its tax revenues are therefore not subject to the uncertainty attached to an individual household's future after-tax labor income stream, and unlike the individual's, grow at a rate  $n$ .

With private discount rates exceeding public sector discount rates as long as  $\lambda + n > 0$ , there will be no debt neutrality: for a given program of government spending on real goods and services (or exhaustive spending), the substitution of public borrowing for current taxation will affect private consumption and aggregate private plus public saving, even if the taxes are lump-sum. The debt neutrality case is the special case of this model where  $\lambda = n = 0$  and households live forever.

Aggregate household behavior is characterized in equations (1) through (5).  $q$  denotes per capita private consumption expenditure, measured in terms of traded goods.  $w$  is per-capita non-human or financial household wealth,  $h$  per capita human capital,  $\pi$  per capita non-interest income and  $\tau$  per capita taxes net of transfers.  $w$ ,  $h$ ,  $\pi$  and  $\tau$  are measured in terms of traded goods.  $x_T$  is per capita household consumption of traded goods,  $x_N$  per capita household consumption of non-traded goods and  $p$  the relative price of non-traded goods, i.e. the reciprocal of the real exchange rate.  $r$  is the instantaneous real interest rate measured in terms of traded goods and  $n \geq 0$  the constant instantaneous population growth rate.

$$(1) \quad q = (\delta + \lambda) (w + h) \quad \delta > 0, \lambda \geq 0$$

$$(2) \quad \dot{w} = (r - n) w + \pi - \tau - q \quad r > 0; n \geq 0$$

$$(3) \quad h(t) = \int_t^\infty e^{-\int_t^v (\tau(u) + \lambda) du} [\pi(v) - \tau(v)] dv \quad \underline{1/}$$

or

$$(3') \quad \dot{h} = (r + \lambda) h - (\pi - \tau)$$

$$(4) \quad q = x_T + p x_N$$

$$(5a) \quad x_T = \eta(p) q \quad 0 \leq \eta \leq 1; \eta' \begin{matrix} \geq \\ \leq \end{matrix} 0; 1 - \eta + p\eta' > 0$$

$$(5b) \quad x_N = \frac{(1 - \eta(p))}{p} q$$

Per capita consumption,  $q$ , is a constant,  $\delta + \lambda$  times comprehensive (human plus non-human) per capita wealth. Per capita consumption of the individual commodities,  $x_T$  and  $x_N$  is determined recursively, given  $q$ , by equations (5a, b). As shown in Appendix 1, equations (1) and (5a, b) result when the individual's utility functional is time-additive with a constant risk-adjusted subjective instantaneous discount rate  $\delta + \lambda$ , and instantaneous utility a logarithmic function of a linear homogenous function of the instantaneous consumption vector  $(x_T, x_N)$ .

With homothetic preferences over instantaneous consumption, it is easily checked that, given  $q$ , an increase in the relative price of non-traded goods can either raise or lower consumption of traded goods ( $\eta' \begin{matrix} \geq \\ \leq \end{matrix} 0$ ) while it must lower the consumption of non-traded goods ( $1 - \eta + p\eta' > 0$ ).

In Figure 1, the budget constraint qE represents a higher value of p than qF. With homothetic preferences, indifference curves have a common slope along any ray through the origin such as OA. With strictly convex indifference curves such as KK and K'K', the consumer's equilibrium moves from B to D when p increases.  $x_N$  falls while  $x_T$  either increases or decreases. In the Cobb-Douglas case (5a', b')  $x_T$  remains unchanged.

$$(5a') \quad x_T = \alpha q \quad 0 \leq \alpha \leq 1$$

$$(5b') \quad x_N = \left(\frac{1 - \alpha}{p}\right) q$$

For the CES family, given in (5a'', b'')  $x_T$  increases when the elasticity of substitution ( $\sigma \equiv \frac{1}{1-\rho}$ ) is greater than 1 and falls when it is less than 1.

$$(5a'') \quad x_T = \left[1 + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{\rho}{\rho-1}} p^{\frac{\rho}{\rho-1}}\right]^{-1} q \quad 0 \leq \alpha \leq 1; \rho < 1$$

$$(5b'') \quad x_N = \left(\left(\frac{\alpha}{1-\alpha}\right) p\right)^{\frac{1}{\rho-1}} \left[1 + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{\rho}{\rho-1}} p^{\frac{\rho}{\rho-1}}\right]^{-1} q$$

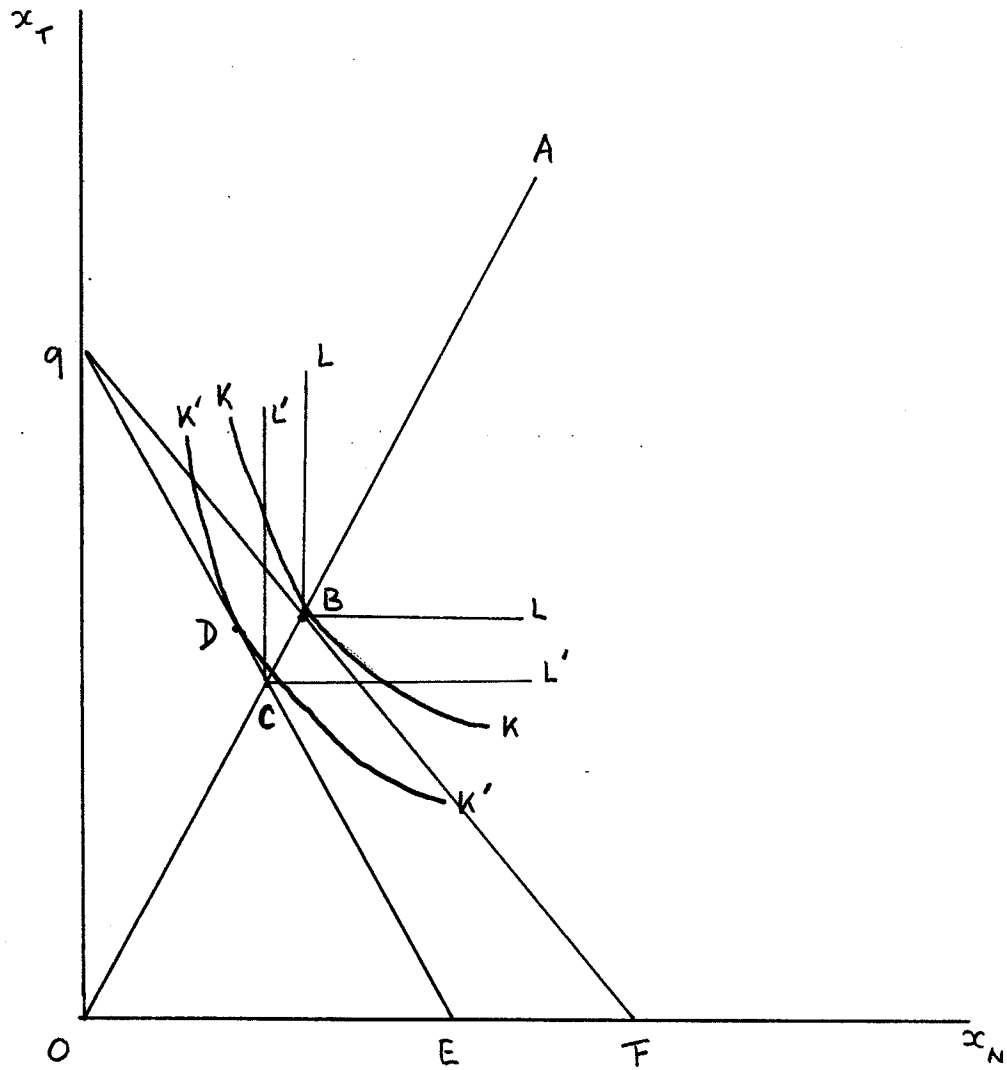
In the extreme case of Leontief indifference curves (shown as LL and L'L' in Figure 1) with a zero elasticity of substitution, consumption of both goods contracts along the ray OA from B to C as p increases.

Note that the consumption function (1) can, using (2) and (3') be rewritten as:

$$(1') \quad \dot{q} = (r - (\delta + n)) q - \lambda (\delta + \lambda) w + n (\delta + \lambda) h$$

or

Figure 1



$$(1'') \quad \dot{q} = (r - \delta) q - (\lambda + n) (\delta + \lambda) w$$

Even though public sector debt and deficits matter in the Yaari-Blanchard model with  $\lambda > 0$ , financial markets are "perfect" in the sense that households can borrow against the security of their expected future after-tax labor income, i.e. that (discounted) future expected disposable labor income is as good a source of purchasing power as current disposable income. One could weaken this e.g. by assuming that a fraction  $\gamma$  of households in each vintage is constrained to spend its current disposable income. The remaining fraction,  $1 - \gamma$ , would follow the Yaari-Blanchard paradigm and would determine its consumption behavior subject only to its intertemporal present value budget constraint. Under this specification of aggregate consumption behavior, equation (1) would be replaced by (1'''), while equations (2), (3), (4), and (5a,b) remain the same.

$$(1''') \quad q = (\delta + \lambda) (w + (1 - \gamma) h) + \gamma (\pi - \tau) \quad \gamma \geq 0$$

In the interest of simplicity the present paper only consider the case where  $\gamma = 0$ . It is easily checked that, if  $r = \delta$ , the steady state properties (but not the dynamics) of the model are independent of  $\gamma$ .

Equation (2) defines the rate of change of non-human wealth per capita as saving per capita ( $rw + \pi - \tau - q$ ) minus the amount of asset accumulation required to offset the effects of population growth ( $nw$ ). For each individual born at time  $s$ , human wealth  $\bar{h}(s,t)$  evolves according to  $\frac{d}{dt} \bar{h}(s,t) = (r + \lambda) \bar{h}(s,t) - (\bar{\pi}(s,t) - \bar{\tau}(s,t))$ , 2/ i.e. it equals the present value of future disposable non-interest income, using the risk-adjusted discount rate  $r + \lambda$ :

$$\bar{h}(s, t) = \int_t^{\infty} e^{-\int_t^v (r(u) + \lambda) du} (\bar{\pi}(s, v) - \bar{\tau}(s, v)) dv$$

Aggregate human wealth  $H$  can be shown to evolve according to  $\dot{H}(t) = (r + \lambda + n) H - \Pi - T$ , where  $\Pi$  and  $T$  are aggregate non-interest income and taxes respectively. Per capita human wealth therefore again evolves according to (3').

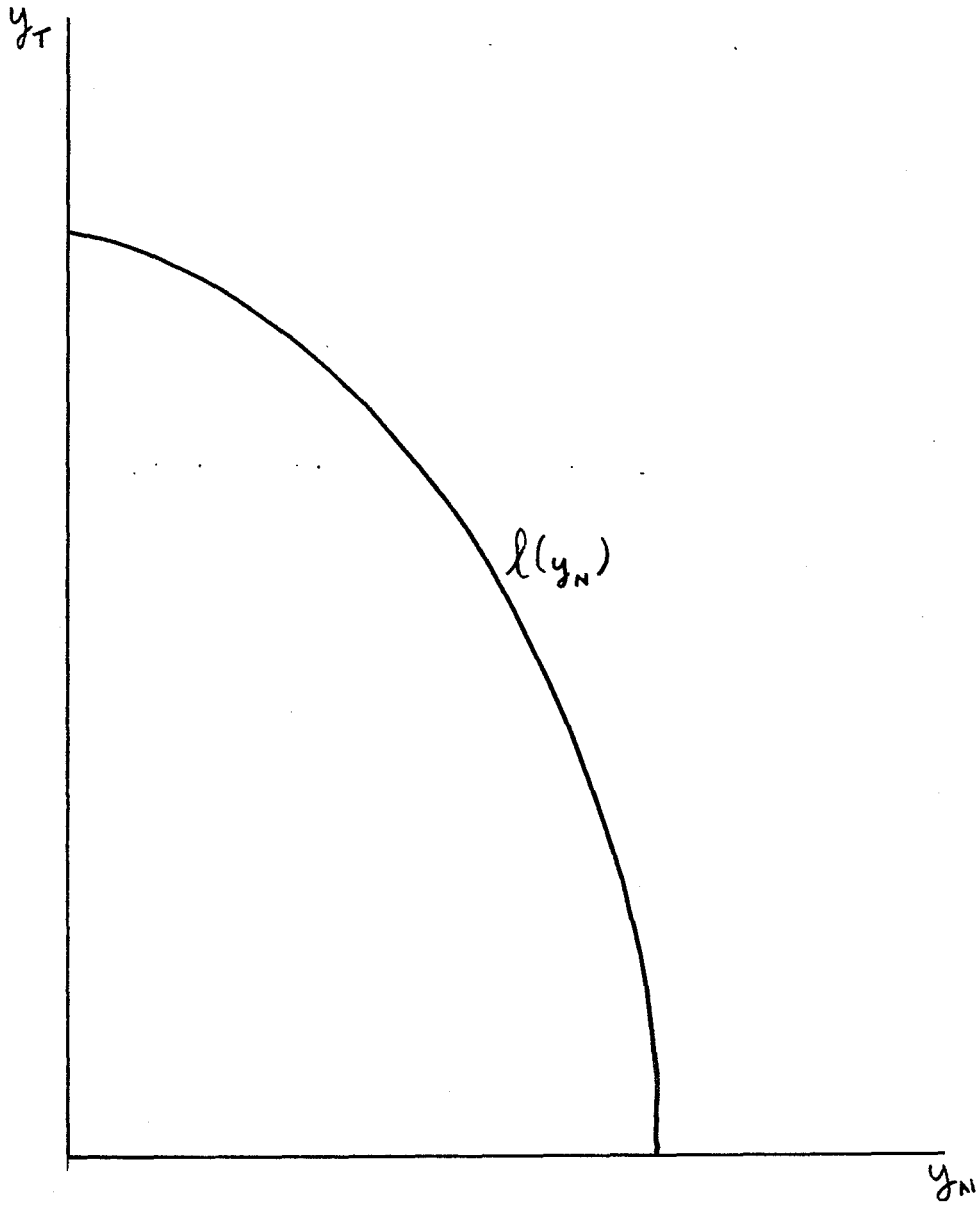
### Production

The production of traded and nontraded goods is modeled through the device of the representative competitive multiproduct firm which maximizes at each instant the value of its production of traded and nontraded goods subject to the technological production possibility frontier  $\ell$  sketched in Figure 2.  $y_T$  and  $y_N$  are the per capita production of traded and nontraded goods, respectively. The firm's problem is

$$\begin{aligned} \max_{y_T, y_N} \pi &= (1 - \theta) y_T + p y_N & \theta < 1 \\ y_T &= \ell(y_N) & \ell' < 0; \ell'' < 0 \end{aligned}$$

$\theta$  is the rate of taxation on the production of traded goods. The world price of traded goods, which is also the price faced by consumers, is unity. The producer's price is  $1 - \theta$ .

The first-order condition for an interior optimum is  $\frac{p}{1-\theta} = -\ell'(y_N)$ . This permits us to write the per capita supplies of traded and nontraded goods as functions of the producer's relative price of traded goods,  $\frac{p}{1-\theta}$ , and to write per capita net domestic product as a function of  $p$  and of  $\theta$ .

Figure 2

$$(8a) \quad y_T = y_T \left( \frac{p}{1-\theta} \right) \quad y_T' = \frac{-\theta}{f''} \leq 0$$

$$(8b) \quad y_N = y_N \left( \frac{p}{1-\theta} \right) \quad y_N' = \frac{-1}{\theta''} \geq 0$$

$$(8c) \quad \pi = \pi(p, \theta) \quad \pi_p = y_N; \quad \pi_\theta = -y_T$$

### The Government

The government spends on traded goods,  $g_T$ , and on non-traded goods,  $g_N$ . It raises revenue by taxing households,  $\tau$  and by taxing production in the traded goods sector,  $\theta y_T$ . It finances any deficit or surplus by issuing or retiring interest-bearing debt  $b$  denominated in terms of traded goods.  $g_T$ ,  $g_N$ ,  $\tau$  and  $b$  are all per capita quantities. From the government's budget identity we therefore obtain:

$$(9) \quad \dot{b} \equiv g_T + pg_N + (r - n)b - \tau - \theta y_T$$

In principle  $g_T$ ,  $g_N$  and  $\tau$  could all be functions of the real exchange rate. Let total per capita public spending measured in terms of traded goods be denoted  $g$ , i.e.

$$(10a) \quad g \equiv g_T + pg_N$$

I'll assume that  $g$  is independent of  $p$  and that spending on the individual commodities is, by analogy with private sector behavior, given by:

$$(10b) \quad g_T = \varepsilon(p; \sigma) g \quad \begin{array}{l} 0 \leq \varepsilon \leq 1; \varepsilon_p \leq 0; 1 - \varepsilon + p\varepsilon_p \geq 0 \\ \varepsilon_\sigma > 0 \end{array}$$



$$(10c) \quad g_N = \left( \frac{1-\varepsilon(p;\sigma)}{p} \right) g$$

$\sigma$  is a shift parameter, with an increase in  $\sigma$  signifying a shift towards public spending on traded goods, given  $p$  and  $g$ . Taxes on households  $\tau$  (measured in traded goods) are assumed to be independent of  $p$ , and so is  $\theta$ .

The government's intertemporal present value budget constraint or solvency constraint is obtained by integrating (9) forward in time and imposing the "no-Ponzi game" transversality condition

$$\lim_{l \rightarrow \infty} b(l) e^{-\int_t^l [r(u)-n] du} = 0.$$

This condition makes sense if, on average, the future instantaneous real interest rate is expected to exceed the future growth rate of the real tax base ( $r > n$ ). The present value government budget constraint is given in (11a and b).

$$(11a) \quad \int_t^{\infty} g(s) e^{-\int_t^s [r(u)-n] du} ds = \int_t^{\infty} (\tau(s) + \theta y_T(s)) e^{-\int_t^s [r(u)-n] du} ds - b(t)$$

or

$$(11b) \quad \int_t^{\infty} G(s) e^{-\int_t^s r(u) du} ds = \int_t^{\infty} (T(s) + \theta Y_T(s)) e^{-\int_t^s r(u) du} ds - B(t)$$

where

$$G(s) = g(s) e^{ns}, \quad T(s) = \tau(s) e^{ns}, \quad Y_T(s) = y_t(s) e^{ns} \quad \text{and} \quad B(t) = b(t) e^{nt}$$

Equation (11b) states that the present value of future planned government revenue should equal the present value of future planned public spending plus

the outstanding stock of public debt. Equation (11a) makes the equivalent statement using per capita quantities and real growth rate adjusted discount rates.

The government's (per capita) revenue  $R$  from the tax on the production of tradeables (or  $y_T$ ) is given by

$$(12) \quad R = \theta y_T \left( \frac{P}{1-\theta} \right)$$

An increase in  $\theta$  increases revenue proportionally at any given level of tradeables production. Given  $p$ , it will increase  $\frac{P}{1-\theta}$ , the producer's relative price of non-traded goods. The volume of tradeables production declines, i.e., the tax base is narrowed. Total revenue will go up or down depending on whether  $\phi$ , the (absolute value of the) elasticity of  $y_T$  with respect to  $\frac{P}{1-\theta}$  is less than or greater than  $\frac{1-\theta}{\theta}$ , i.e.,

$$(13) \quad \left. \frac{\partial R}{\partial \theta} \right|_{p=\bar{p}} = y_T \left( 1 + \frac{\theta}{1-\theta} \phi \right)$$

$$(14) \quad \phi \equiv \frac{y_T'}{y_T} \frac{P}{(1-\theta)} \leq 0$$

We shall see that, in the short run and in the long run, an increase in  $\theta$  lowers  $p$ , the consumer's relative price of non-traded goods. To evaluate the general equilibrium implications of an increase in  $\theta$  for  $R$  we must allow for any endogenous response of  $p$  to the change in  $\theta$ , i.e.

$$(15) \quad \frac{\partial R}{\partial \theta} = y_T \left( 1 + \frac{\theta}{1-\theta} \phi \right) + \frac{\theta}{1-\theta} y_T' \frac{\partial p}{\partial \theta}$$

where  $\frac{\partial p}{\partial \theta}$  will in general be different in the short run and in the long run.

In many cases, the decline in  $p$  as  $\theta$  increases is not enough to reverse the a priori plausible result that a higher value of  $\theta$  will be associated with a higher value of  $p/(1-\theta)$ , i.e., with a lower producer's price for traded goods. It is therefore certainly possible that there is a "Laffer curve" for the traded goods production tax. There is a common presumption that  $\phi$  will be higher (in absolute value) for higher values of  $\theta$  (and of  $p/(1-\theta)$ ). In that case, a reduction in  $\theta$  from a very high level may well increase the revenue from the tax.

#### The rest of the world

The rest of the world provides the domestic economy with perfectly elastic excess demand schedules for traded goods and for financial capital. The instantaneous interest rate in terms of traded goods,  $r$ , is exogenous.

Three equivalent ways of writing the equation of motion for the nation's per capita stock of external assets  $f \equiv w-b$ , are given below in (16a, b,c). The first is based on the identity that equates the current account surplus with net exports of traded goods plus net property income from abroad. The second is based on the current account surplus as the sum of the financial surpluses of the private sector ( $\dot{w}+nw$ ) and the public sector ( $-\dot{b}-nb$ ). The last one represents the current account surplus as the excess of national income over domestic absorption.

$$(16a) \quad \dot{f} = y_T - x_T - g_T + (r-n)f$$

$$(16b) \quad \dot{f} = \dot{w} - \dot{b} = ((r-n)w + \pi - \tau - q) + (\tau + \theta y_T - (r-n)b - g)$$

$$(16c) \quad \dot{f} = (r-n)f + \pi - \theta y_T - (q+g)$$

Equilibrium in the market for non-traded goods

Wage and price flexibility ensure that the non-traded goods market clears at each instant, i.e.,

$$(17) \quad y_N = x_N + g_N \quad \text{or}$$

$$(17') \quad y_N \left( \frac{p}{1-\theta} \right) = \left( \frac{1-\eta(p)}{p} \right) q + \left( \frac{1-\varepsilon(p;\sigma)}{p} \right) g.$$

Equation (17') permits us to solve for the momentary equilibrium value of  $p$  as a function of  $q$ ,  $g$ ,  $\theta$  and  $\sigma$

$$(18a) \quad p = p(q; g, \theta, \sigma)$$

$$(18b) \quad p_q = \Omega \frac{(1-\eta)}{p} \geq 0$$

$$(18c) \quad p_g = \Omega \frac{(1-\varepsilon)}{p} \geq 0$$

$$(18d) \quad p_\theta = -\Omega y'_N \frac{p}{(1-\theta)^2} = \frac{\Omega y'_T}{1-\theta} - \frac{p}{1-\theta} \leq p_\theta \leq 0.$$

$$(18e) \quad p_\sigma = -\Omega \frac{g\varepsilon_\sigma}{p} < 0$$

$$(18f) \quad \Omega = \left[ \frac{-y'_T}{p} + \frac{q}{2} (1-\eta+pn') + \frac{g}{2} (1-\varepsilon+pe_p) \right]^{-1} > 0$$

Unless all spending (at the margin) is on traded goods ( $\eta=1$  or  $\varepsilon=1$ ) higher private or public spending raises  $p$ . A higher tax rate on the

production of traded goods, given  $q$ ,  $g$  and  $\sigma$ , shifts resources towards the production of non-traded goods and lowers the consumer's relative price of nontradeables. The effect of an increase in  $\theta$  on the producer's relative price of nontraded goods  $\frac{p}{1-\theta}$  given  $q$ ,  $g$  and  $\mu$  is given by

$$(18g) \quad \left( \frac{p}{1-\theta} \right)_{\theta} = \frac{p}{(1-\theta)^2} \left( 1 - \frac{y'_N}{1-\theta} \Omega \right) \quad \frac{p}{(1-\theta)^2} \geq \left( \frac{p}{1-\theta} \right)_{\theta} \geq 0$$

Thus, even though the consumer's relative price of traded goods declines as  $\theta$  increases, the producer's relative price rises. The result is an expansion both of the demand for and the supply of non-traded goods and a contraction in the traded goods sector. Only when there is no scope for reallocating production from the traded to the non-traded goods sector ( $y'_N = 0$  or  $\ell'' = -\infty$ ) will  $p/(1-\theta)$  increase by the full amount of the increase in  $\theta$ , with  $p$  constant. When production can be reallocated along a straight-line production possibility frontier ( $y'_N = \infty$  or  $f'' = 0$ ) there will be no change in  $p(1-\theta)$  when  $\theta$  changes. A switch towards public spending on traded goods (an increase in  $\sigma$ , given  $g$ ) lowers  $p$ .

When balanced budget policies are considered, it is only legitimate to treat,  $g$ ,  $\theta$  and  $\sigma$  as exogenous if the lump sum tax  $\tau$  is permitted to adjust passively to ensure budget balance. When unbalanced budget policies are considered,  $\tau$  can be treated as exogenous, together with  $g$ ,  $\theta$  and  $\sigma$ , with  $b$  taking up the slack. Ultimately, however, the requirement of government solvency, summarized in the present value budget constraint (11a or b), will put an upper bound on  $b$  and thereby constrains the paths taken by  $g$ ,  $\theta$ ,  $\sigma$  and  $\tau$  over time. With unbalanced budget policies (at any rate during the transition to steady state) steady state government debt,  $b$ , becomes an endogenous variable. It is easily seen that any steady-state equilibrium that

satisfies (11a, b) has a real growth corrected balanced budget. With unbalanced budget policies outside the steady state, four out of the five long-run fiscal parameters  $g$ ,  $\theta$ ,  $\sigma$ ,  $\tau$  and  $b$  can be chosen independently.

This paper will focus on policies that maintain real growth corrected budget balance continuously, i.e.,  $b$  is treated as constant throughout. Before turning to this in section (4), it is instructive to consider briefly the intertemporal inconsistency of unbalanced budget models that treat all four of the fiscal parameters  $g$ ,  $\theta$ ,  $\sigma$  and  $\tau$  as exogenous.

### 3. Private sector equilibrium and public sector insolvency: a false paradox

For convenience the model is reproduced below:

$$(19a) \quad \dot{q} = (r-\delta)q - (\lambda+n)(\delta+\lambda)w \quad \lambda \geq 0; \delta > 0$$

$$(19b) \quad \dot{w} = (r-n)w + \pi(p, \theta) - \tau - q \quad r > n \geq 0; \theta < 1$$

$$(19c) \quad y_N \left( \frac{p}{1-\theta} \right) = \left( \frac{1-\eta(p)}{p} \right) q + g_N \quad 0 \leq \eta \leq 1; 1-\eta+pn' > 0$$

$$(19d) \quad g_T = \varepsilon(p, \sigma)g$$

$$(19e) \quad g_N = \frac{(1-\varepsilon(p, \sigma))g}{p} \quad 0 \leq \varepsilon \leq 1; 1-\varepsilon+p\varepsilon_p > 0; \varepsilon_\sigma > 0$$

$$(19f) \quad \dot{b} = (r-n)b + g_T + pg_N - \theta y_T \left( \frac{p}{1-\theta} \right) - \tau$$

$$(19g) \quad \dot{f} = (r-n)f + \pi(p, \theta) + \theta y_T \left( \frac{p}{1-\theta} \right) - (q+g).$$



assumed by the exogenous variable. The  $q, w$  subsystem will be a saddlepoint locally if  $\Delta$ , the determinant of the  $2 \times 2$  submatrix of the state matrix is negative, i.e., if

$$(21) \quad \Delta = (r+\lambda)(r-(\delta+n+\lambda)) + (\lambda+n)(\delta+\lambda)y_{N^p q} < 0$$

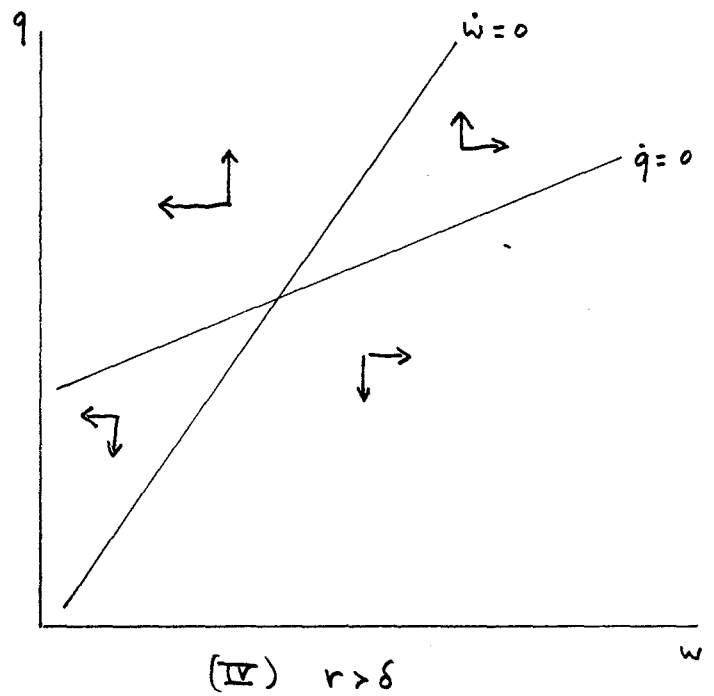
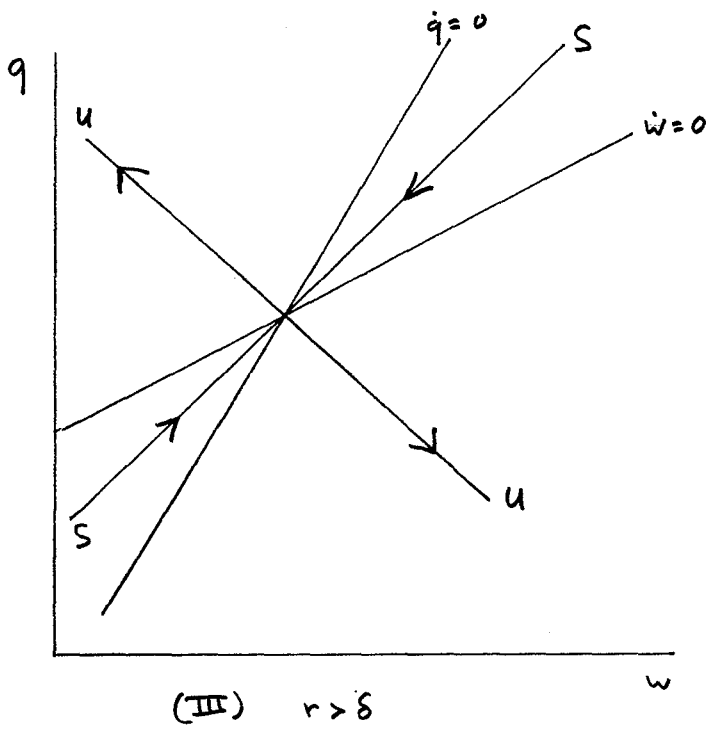
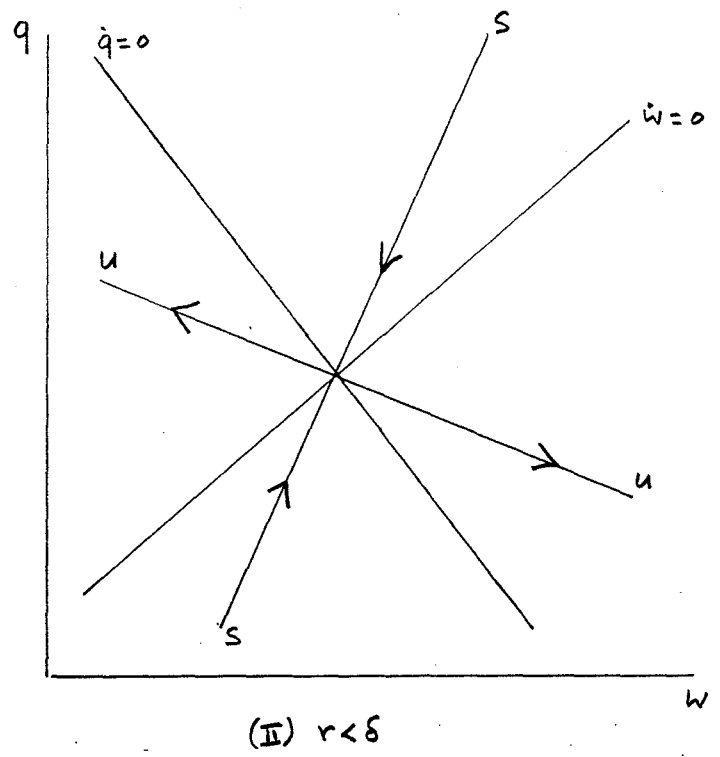
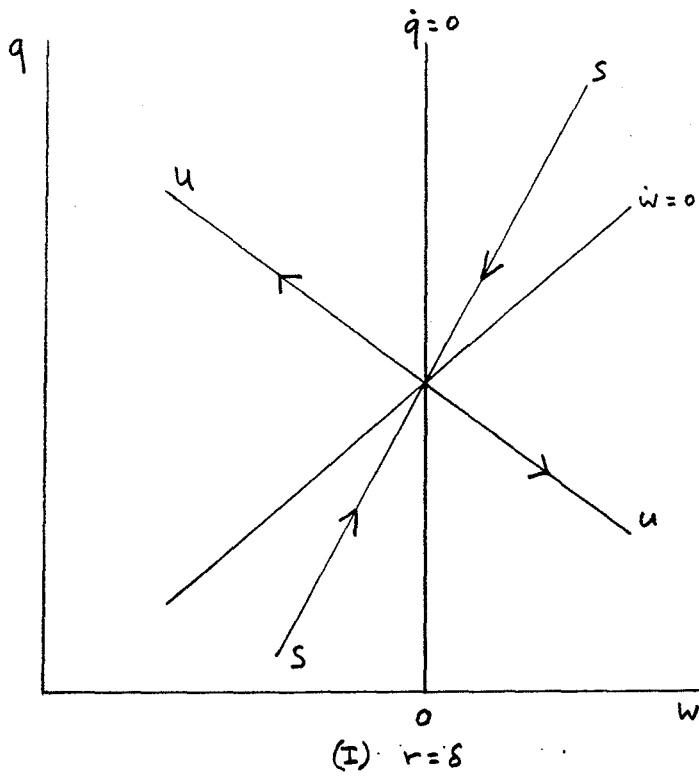
When there is no non-traded goods sector, the second term on the r.h.s. of (21) equals zero. If in addition  $n=0$ , then (21) becomes the saddlepoint equilibrium condition  $(r+\delta)(r-(\delta+\lambda)) < 0$  found in Blanchard [1984, 1985] and Buiter [1986a]. In the model of (21),  $-\lambda < r < \delta+n+\lambda$  is necessary but no longer sufficient for saddlepoint stability. Provided (21) holds, the  $q, w$  subsystem has one stable (negative) and one unstable (positive) eigenvalue. With  $w$  a predetermined and  $q$  a non-predetermined state variable, there will exist a unique continuously convergent solution for  $q$  and  $w$ .

In  $q-w$  space the  $\dot{q}=0$  locus has slope  $\left. \frac{dq}{dw} \right|_{\dot{q}=0} = \frac{(\lambda+n)(\delta+\lambda)}{r-\delta}$  and the  $\dot{w}=0$  locus has slope  $\left. \frac{dq}{dw} \right|_{\dot{w}=0} = \frac{r-n}{1-y_{N^p q}}$ . Throughout the paper, I assume  $r > n$  and  $1-y_{N^p q} > 0$ . The latter condition means that an increase in consumption, part of which falls on non-traded goods, will not raise the relative price of non-traded goods by so much that the value, in terms of tradeables, of non-traded goods production rises by more than the increase in consumption. This still leaves us with 4 qualitatively distinct phase diagram specifications in  $q-w$  space, as shown in Figure 3.

Figures (3I, II and III) have the desired saddlepoint configuration. The unique continuously convergent path for constant values of the exogenous variables is the upward-sloping SS line along which low but rising consumption is associated with positive saving and rising  $w$ , and high but falling consumption is associated with dissaving and declining  $w$ . Saddlepoint



Figure 3



configurations obtain always when  $r=\delta$  or  $r<\delta$ . They may obtain for  $r>\delta$ , but if  $r$  is too far above  $\delta$ , the equilibrium becomes completely unstable. We only consider saddlepoint equilibria in what follows.

The two remaining eigenvalues of the complete system in (20), which govern the public debt and net foreign assets dynamics, both equal  $r-n$ . Since we assume  $r>n$ ,  $b$  and  $f$  will diverge explosively even as  $q$  and  $w$  settle down into stationary equilibrium. The intrinsic dynamics of interest-bearing debt feeds on itself in an explosive manner.

A government whose debt grows indefinitely at a rate faster than its capacity to service that debt violates the solvency constraint given in (11a,b). An explosive net creditor position of the government (given  $w$ ) violates the solvency constraint of the rest of the world. In either case, the assumption that the real interest rate is exogenous becomes untenable. In addition, credit rationing would emerge to constrain the behavior of  $b$  and  $f$ . The model would break down.

Stability of the public debt process requires that, if  $b>0$ , ultimately primary (i.e., non-interest) government surpluses are run to service the debt. The need to run these surpluses will sooner or later affect the behavior of  $q$  and  $w$  through changes in  $g$ ,  $\tau$ ,  $\theta$  or  $\sigma$ . Only in one special case can the dynamic properties of the present  $q,w$  subsystem be consistent with public sector solvency. This is the case when public spending on traded goods,  $g_T$  is adjusted, either continuously or eventually, to ensure continuous or ultimate real growth corrected budget balance, while public spending on non-traded goods is kept constant. In terms of our notation, this amounts to manipulating  $g \equiv g_T + pg_N$  to maintain budget balance and simultaneously varying  $\epsilon$  (the share of traded goods in public spending) to ensure that  $dg_N = 0$ . With this restriction, the long-run steady state equilibrium

of the private sector is given by:

$$(22a) \quad q = (\delta + \lambda)(w + h)$$

$$(22b) \quad q = (r - n)w + \pi(p, \theta) - \tau$$

$$(22c) \quad p = \tilde{p}(q; g_N, \theta)$$

$$\tilde{p}_q = \frac{\tilde{\Omega}(1 - \eta)}{p} > 0; \quad \tilde{p}_{g_N} = \frac{\tilde{\Omega}}{p}; \quad \tilde{p}_\theta = -\tilde{\Omega} y'_N \frac{p}{(1 - \theta)^2} < 0$$

$$(22d) \quad \tilde{\Omega} = \left[ -\frac{y'_T}{p} + q(1 - \eta + p\eta') \right]^{-1}$$

$$(22e) \quad h = (\pi(p, \theta) - \tau)(r + \lambda)^{-1}$$

Substituting (22c,d) into (22a,b) we can solve for long-run  $q$  and  $w$  as functions of  $g_N$ ,  $\theta$ ,  $\tau$  and  $r$ .  $g_T$  will not affect  $q, w, p$  or  $h$  in the short run or in the long run, as long as government solvency is assured, but its behaviour will affect the paths of  $b$  and  $f$ .

The long long-run multipliers are:

$$(23a) \quad \left. \frac{\partial q}{\partial g_N} \right|_{g_T} = -y_N \tilde{p}_{g_N} (n + \lambda)(\delta + \lambda) \Delta_{g_T}^{-1} > 0$$

$$(23b) \quad \left. \frac{\partial w}{\partial g_N} \right|_{g_T} = y_N \tilde{p}_{g_N} (\delta - r) \Delta_{g_T}^{-1}$$

$$(23c) \quad \left. \frac{\partial p}{\partial g_N} \right|_{g_T} = \tilde{p}_q \left. \frac{\partial q}{\partial g_N} \right|_{g_T} + \tilde{p}_{g_N} > 0$$

$$(23d) \quad \left. \frac{\partial q}{\partial \tau} \right|_{g_T} = (n+\lambda)(\delta+\lambda)\Delta_{g_T}^{-1} < 0$$

$$(23e) \quad \left. \frac{\partial w}{\partial \tau} \right|_{g_T} = (r-\delta)\Delta_{g_T}^{-1}$$

$$(23f) \quad \left. \frac{\partial p}{\partial \tau} \right|_{g_T} = \tilde{p}_q \left. \frac{\partial q}{\partial \tau} \right|_{g_T} < 0$$

$$(23g) \quad \left. \frac{\partial q}{\partial \theta} \right|_{g_T} = -(y_N \tilde{p}_\theta - y_T)(n+\lambda)(\delta+\lambda)\Delta_{g_T}^{-1} \leq 0$$

$$(23h) \quad \left. \frac{\partial w}{\partial \theta} \right|_{g_T} = (y_N \tilde{p}_\theta - y_T)(\delta-r)\Delta_{g_T}^{-1}$$

$$(23i) \quad \left. \frac{\partial p}{\partial \theta} \right|_{g_T} = \tilde{p} \left. \frac{\partial q}{\partial \theta} \right|_{g_T} + \tilde{p}_\theta = [\tilde{p}_\theta (r+\lambda)(r-(\delta+n+\lambda)) + \tilde{p}_q y_T (n+\lambda)(\delta+\lambda)] \Delta_{g_T}^{-1} < 0$$

$$(23j) \quad \left. \frac{\partial q}{\partial r} \right|_{g_T} = - [(r-n)q + (\delta+\lambda)(n+\lambda)w] \Delta_{g_T}^{-1}$$

$$(23k) \quad \left. \frac{\partial w}{\partial r} \right|_{g_T} = [(y_N \tilde{p}_q - 1)q + (\delta-r)w] \Delta_{g_T}^{-1}$$

$$(23l) \quad \left. \frac{dp}{dr} \right|_{g_T} = \tilde{p}_q \left. \frac{\partial q}{\partial r} \right|_{g_T}$$

$$(23m) \quad \Delta_{g_T} = (r+\lambda)(r-(\delta+n+\lambda)) + (\lambda+n)(\delta+\lambda)y_N \tilde{p}_q < 0$$

Note that  $\tilde{p}_q$  in (22c) equals  $p_q$  in (18b). Finally with  $i-\epsilon+p\epsilon_p = 0$

$$(23n) \quad \left. \frac{\partial (p(1-\theta)^{-1})}{\partial \theta} \right|_{g_T} = \frac{p}{(1-\theta)^2} \left( \left. \frac{(1-\theta)}{p} \frac{\partial p}{\partial \theta} \right|_{g_T} + 1 \right) =$$

$$\frac{(p+(1-\theta)\tilde{p}_\theta)(r+\lambda)(r-(\delta+n+\lambda)) + \pi \tilde{p}_q (\lambda+n)(\delta+\lambda)}{p \Delta_{g_T}}$$

When evaluating these long-run multipliers, an interesting benchmark is the case when  $r=\delta$ . When  $\lambda=0$ , i.e., with infinite-lived consumers, well-behaved stationary equilibria (with positive but finite per capita private consumption) exist only when  $r=\delta$ . Considering this case permits us to evaluate the economic effects of instability or finite horizons.

Long-run private consumption, measured in traded goods increases when public spending on non-traded goods increases and long-run budget balance is maintained by reducing public spending on traded goods. The reason is that  $p$  increases (the real exchange rate appreciates) and with it real disposable income and human capital measured in traded goods. Financial private wealth  $w$  increases if  $r>\delta$ , decreases if  $\delta>r$ .

Increased lump-sum taxes  $\tau$  (matched by increased  $g_T$  to preserve long-run budget balance) reduce private consumption and raise financial wealth if  $\delta>r$ , lower it if  $\delta < r$ . This will occur even if there is no effect on the relative price of non-traded goods, as in that case

$$\left. \frac{\partial q}{\partial \tau} \right|_{q_T} = \frac{(n+\lambda)(\delta+\lambda)}{(r+\lambda)(r-(\delta+n+\lambda))} < 0.$$

Unless  $\tilde{p}_q$  equals zero, the reduction in consumption, a fraction  $1-\eta$  which falls on non-traded goods, will lower  $p$ .

An increase in  $\theta$ , the tax rate on the production of traded goods directly lowers the after-tax income received by the factors of production ( $\pi_\theta = -y_T < 0$ ). By lowering the consumer's price of non-traded goods ( $\tilde{p}_\theta < 0$ ) it also reduces the value in terms of traded goods of any given amount of domestic production of non-traded goods, and consumption falls. Financial wealth rises if  $\delta > r$ , falls if  $\delta < r$ . It doesn't matter whether  $tpt$  revenue increases or declines, since  $g_T$  is used to balance the long-run budget. The consumer's price of non-traded goods falls. What happens to the producer's price,  $\frac{P}{1-\theta}$  as  $\theta$  increases?

In (18g) we calculated that, given  $q$ , the decline in  $p$  as  $\theta$  increases is not enough to cause  $\frac{P}{1-\theta}$  to decline.  $q$ , however, declines in the long run as  $\theta$  is raised and this further depresses  $p$ . From (23m) it can be checked that with  $\lambda = 0$ ,  $\frac{P}{1-\theta}$  rises when  $\theta$  increases, but that with  $\lambda > 0$ , this could be reversed. Strong negative income effects from the increase in  $\theta$  are necessary to obtain this paradoxical result. The increase in  $\theta$  would, across steady states, be associated with an increase, not a decline in the volume of tradable production and with a corresponding reduction in the volume of nontradable production. The demand for nontradables, however, declines (at a given  $p$ ) even more than the production of nontradables, necessitating the equilibrium decline in  $p$ . We return to this issue in Section 4.

A sufficient condition for an increase in the interest rate to be associated with an increase in long-run consumption is for  $w$  to be non-

negative. Equating (22a) and (22b) we see that

$$(24) \quad w = \frac{(\delta-r)}{(r+\lambda)(r-(\delta+\lambda+n))} (\pi-\tau).$$

Therefore we will be positive (negative) if  $r > \delta$  ( $\delta > r$ ).  $p$  moves in the same direction as  $q$  when  $r$  increases. If  $r > \delta$ , an increase in  $r$  will raise long-run  $w$  if  $y_{N\tilde{p}_g} - 1 < 0$ . As was pointed out during the discussion of the model's stability properties, this (and similar conditions under the different balanced budget regimes), amounts to assuming that a higher value of consumption does not, by raising  $p$ , increase income by more than the increase in consumption.

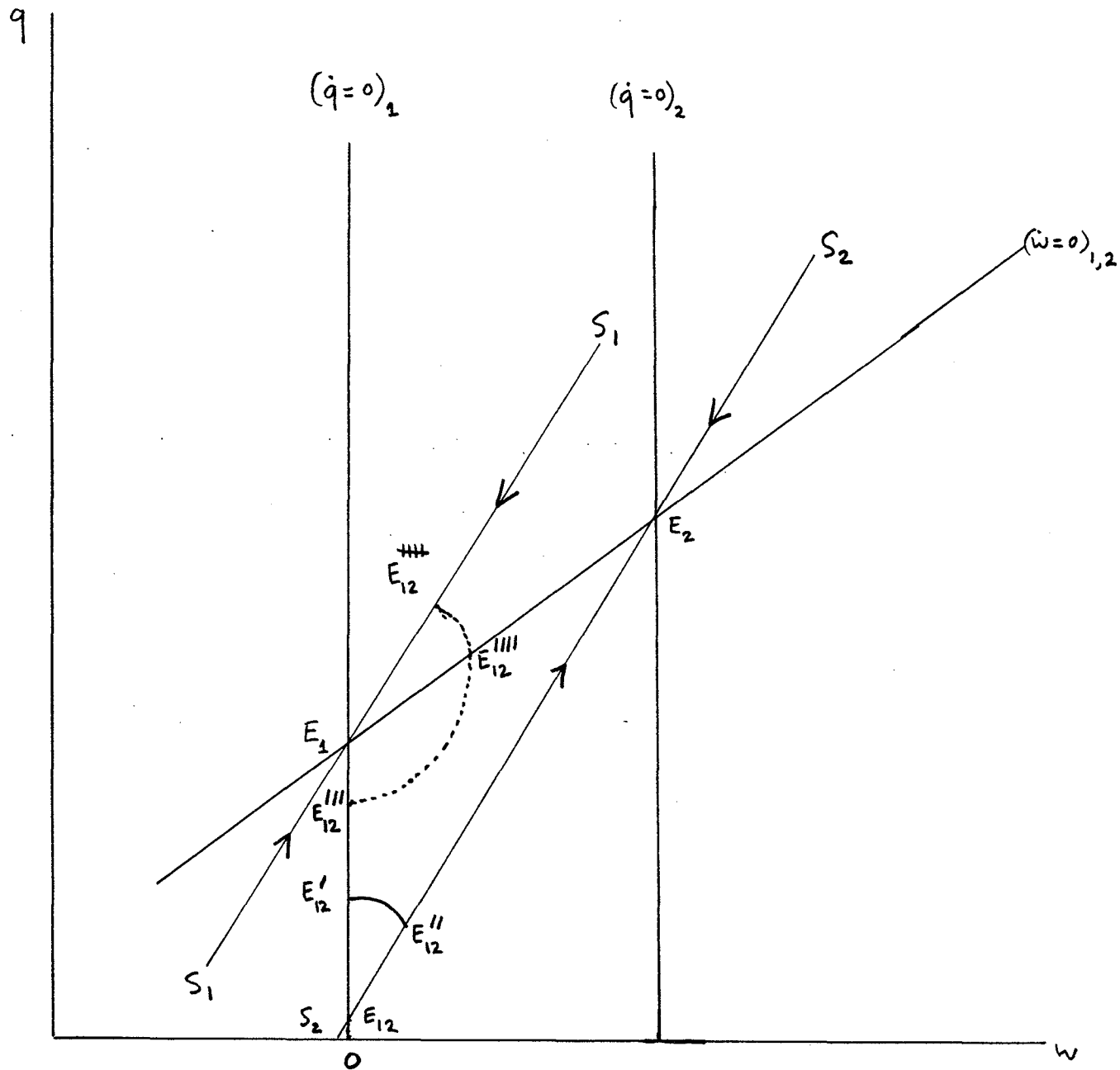
For all shocks other than changes in the interest rate, the  $r = \delta$  special case greatly simplifies the dynamic analysis when the shocks are unanticipated, immediate and permanent (i.e., random walk shocks).  $w$  in that case doesn't change in response to changes in  $g_N$ ,  $\tau$  or  $\theta$ , and the instantaneous response of  $q$  is also the long-run steady state response. Since  $w$  doesn't vary, the current account surplus equals the public sector surplus. For anticipated future shocks in  $g_N$ ,  $\tau$  or  $\theta$  and for transitory shocks, there will be transitional private financial wealth dynamics.

From an initial position with  $r = \delta$ , consider the case of an unanticipated, immediate increase in the world rate of interest which is perceived as permanent. For concreteness assume that the government adjusts  $q_T$  not only in the long-run but at each instant to maintain budget balance.

In Figure 4 an unanticipated permanent increase in  $r$  causes an immediate drop in private consumption from  $E_1$ , the initial long-run

Figure 4:

The Effects of an Increase in  $r$  when  $g_T$  Adjusts to Ensure Government Solvency





equilibrium, to  $E_{12}$ , the point vertically below  $E_1$  on the saddle path through the new long-run equilibrium. From (23 j, k) we know that the long-run effect of this shock is higher values of  $q$  and  $r$ . The impact effect of a higher world rate of interest is therefore in the opposite direction from the long-run effect. After the initial jump down to  $E_{12}$ , consumption rises steadily from  $E_{12}$  to  $E_2$  along  $S_1S_2$ . Private savings rates, however are positive throughout the adjustment process and financial wealth accumulates.

The unexpected news, at  $t_0$ , of a permanent future increase in  $r$  at  $t_1 > t_0$  causes a smaller initial drop in  $q$  (to  $E_{12}'$  say), followed by a further decline in  $q$  between  $t_0$  and  $t_1$  when the system moves along that divergent trajectory (drawn with reference to  $E_1$ ) that will place it on  $S_2S_2$  at  $t_1$ . After  $t_1$  consumption rises smoothly from  $E_{12}''$  to  $E_2$ .

An unanticipated increase in  $r$  at  $t_0$  which is expected to be reversed at  $t_1 > t_0$  is followed by an immediate drop in consumption, to  $E_{12}'''$  say. Saving becomes positive and wealth accumulates until  $t_1' < t_1$ , while consumption rises. Beyond  $E_{12}''''$ , and between  $t_1'$  and  $t_1$ , consumption continues to rise but dissaving takes place. At  $t_1$ , when the interest rate comes down again, consumption and financial wealth are above their initial equilibrium values at  $E_1$ . Both  $q$  and  $w$  then decline gradually back to their initial equilibrium levels.

Assuming continuous budget balance through passive variation in  $q_T$ ,  $f$  exactly mirrors the behavior of  $w$ . Permanent interest rate increases are associated with current account surpluses throughout the adjustment process. A temporary increase in  $r$  is followed by a period of surpluses followed by a period of deficits.

$p$ , the relative price of non-traded goods moves with  $q$ . In all cases there is an immediate decline in  $p$ , permitting resources to be shifted towards

the production of tradables and private consumption to be shifted away from tradables to generate the export surplus that permits the accumulation of foreign assets.

With the permanent unanticipated increase in  $r$ ,  $p$  rises steadily after its initial drop and ultimately achieves a higher value than it had at  $E_1$ . With the anticipated future increase in  $r$ , the price of nontraded goods continues to decline smoothly after the initial discrete drop until it reaches its low at  $t_1$  when  $r$  actually goes up.

In the case of the transitory increase in  $r$ , the initial drop in  $p$  is followed by a gradual increase beyond its initial equilibrium value. When  $r$  resumes its initial value again,  $p$  declines smoothly back to its initial value. The high values of  $p$  after  $t_1'$ , when the solution trajectory crosses the  $\dot{w} = 0$  line at  $E_{12}''''$  is mirrored in the current account deficit.

#### 4. Real Growth Corrected Balanced Budget Policies

In the remaining sections of the paper we consider continuous real growth corrected balanced budget policies. This requires the analysis of the following model:

$$(25a) \quad \dot{q} = (r-\delta)q - (\delta+\lambda)(\lambda+n)w$$

$$(25b) \quad \dot{w} = (r-n)w + \pi(p, \theta) - \tau - q$$

$$(25c) \quad y_N \left( \frac{p}{1-\theta} \right) = \left( \frac{1-\eta(p)}{p} \right) q + \left( \frac{1-\varepsilon(p, \sigma)}{p} \right) g$$

$$(25d) \quad g + (r-n)b - \theta y_T \left( \frac{p}{1-\theta} \right) - \tau = 0$$

Equations (25d) states that the public sector deficit (per capita) equals  $nb$ , exactly sufficient to keep constant the stock of real per capita debt,  $b$ . In what follows (25c and d) will be solved for  $p$  and one of the four fiscal parameters ( $g$ ,  $\tau$ ,  $\theta$  and  $\sigma$ ) as functions of the remaining three fiscal parameters and of  $q$ ,  $r$  and  $b$ . The solutions are then substituted into (25a,b). Since  $\dot{b} = 0$ ,  $\dot{f} = \dot{w}$ . Even though the public sector balances its real growth corrected budget, private sector financial dynamics make for non-trivial current account adjustments in response to virtually any kind of policy or external shock.

#### 4a. $\tau$ responds endogenously to maintain budget balance

In this subsection,  $\tau$  adjusts to satisfy the balanced budget constraint (25d) in response to exogenous variations in  $g$ ,  $r$ ,  $b$ ,  $\theta$  or  $\tau$  and to endogenous variations in  $p$ . The easiest way to proceed is to substitute for  $\tau$  in (25b) using (25d) and to solve (25c) for  $p$  as a function of  $q$ ,  $g$ ,  $\theta$  and  $\sigma$ , which was already done in (18a,f). The resulting linearized dynamic system is given in (26), with  $p_q$  given in (18b) and (18f).

$$(26) \quad \begin{bmatrix} \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} r - \delta & -(\lambda+n)(\delta+\lambda) \\ (y_N + \frac{\theta}{1-\theta} y_T') p_q^{-1} & r-n \end{bmatrix} \begin{bmatrix} q - \bar{q} \\ w - \bar{w} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & q & 0 \\ (y_N + \frac{\theta}{1-\theta} y_T') p_g^{-1} & (y_N + \frac{\theta}{1-\theta} y_T') p_\theta + \frac{\theta}{1-\theta} y_T' \frac{p}{1-\theta} & (y_N + \frac{\theta}{1-\theta} y_T') p_\sigma & f & -(r-n) \end{bmatrix} \begin{bmatrix} g - \bar{g} \\ \theta - \bar{\theta} \\ \sigma - \bar{\sigma} \\ r - \bar{r} \\ b - \bar{b} \end{bmatrix}$$

The saddlepoint equilibrium condition is

$$(27) \quad \Delta_{\tau} = (r+\lambda)(r-(\delta+\lambda+n)) + (\lambda+n)(\delta+\lambda)(y_N + \frac{\theta}{1-\theta} y'_T) p_q < 0$$

The slope of the  $q = 0$  schedule is the same in this case as in the previous case. The slope of the  $\dot{w} = 0$  locus with  $\tau$  endogenous,  $(r-n)[1-(y_N + \frac{\theta}{1-\theta} y'_T) p_q]^{-1}$ , is less, i.e. the schedule is flatter, than with  $g_T$  endogenous,  $(r-n)[1-y_N p_q]^{-1}$ .

The long-run equilibrium values of  $g$  and  $w$  satisfy:

$$(28a) \quad q = (\delta+\lambda) [w + (\pi(p(q;g,\theta,\sigma),\theta) - (g + (r-n)b - \theta y'_T (1-\theta)^{-1} p(q;g,\theta,\sigma)))) (r+\lambda)^{-1}]$$

$$(28b) \quad q = (r-n)w + \pi(p(q;g,\theta,\sigma),\theta) - (g + (r-n)b - \theta y'_T ((1-\theta)^{-1} p(q;g,\theta,\sigma)))$$

The important long-run multipliers are given in (29a to f)

$$(29a) \quad \frac{\partial q}{\partial g} \Big|_{\tau} = (\lambda+n)(\lambda+\delta)(1-(y_N + \frac{\theta}{1-\theta} y'_T) p_g) \Delta_{\tau}^{-1} < 0$$

$$(29b) \quad \frac{\partial w}{\partial g} \Big|_{\tau} = (r-\delta)(1-(y_N + \frac{\theta}{1-\theta} y'_T) p_g) \Delta_{\tau}^{-1}$$

$$(29c) \quad \frac{\partial p}{\partial g} \Big|_{\tau} = p_q \frac{\partial q}{\partial g} \Big|_{\tau} + p_g = [(\lambda+n)(\lambda+\delta)(p_q - p_g) + (r-n)(r-\delta)p_g] \Delta_{\tau}^{-1}$$

$$(29d) \quad \frac{\partial q}{\partial \theta} \Big|_{\tau} = -[(\lambda+n)(\delta+\lambda)(y_N p_{\theta} + \frac{\theta}{1-\theta} y'_T (p_{\theta} + \frac{p}{1-\theta}))] \Delta_{\tau}^{-1} < 0$$

$$(29e) \quad \frac{\partial w}{\partial \theta} \Big|_{\tau} = (\delta-r)(y_N p_{\theta} + \frac{\theta}{1-\theta} y'_T (p_{\theta} + \frac{p}{1-\theta})) \Delta_{\tau}^{-1}$$

$$(29f) \quad \frac{\partial p}{\partial \theta} \Big|_{\tau} = p_q \frac{\partial q}{\partial \theta} \Big|_{\tau} + p_{\theta} = [-(\lambda+n)(\delta+\lambda) \frac{\theta}{1-\theta} y'_T \frac{p}{1-\theta} p_q + (r+\lambda)(r-(\delta+\lambda+n)) p_{\theta}] \Delta_{\tau}^{-1} < 0$$

$$(29g) \quad \left. \frac{\partial q}{\partial \sigma} \right|_{\tau} = -[(\lambda+n)(\delta+\lambda)(y_N + \frac{\theta}{1-\theta} y_T')] p_{\sigma} \Delta_{\tau}^{-1}$$

$$(29h) \quad \left. \frac{\partial w}{\partial \sigma} \right|_{\tau} = (\delta-r)(y_N + \frac{\theta}{1-\theta} y_T') p_{\sigma} \Delta_{\tau}^{-1}$$

$$(29i) \quad \left. \frac{\partial p}{\partial \sigma} \right|_{\tau} = (r+\lambda)(r-(\delta+\lambda+n)) \Delta_{\tau}^{-1} < 0$$

$$(29j) \quad \left. \frac{\partial q}{\partial r} \right|_{\tau} = -[(r-n)q + (\lambda+n)(\delta+\lambda)f] \Delta_{\tau}^{-1}$$

$$(29k) \quad \left. \frac{\partial w}{\partial r} \right|_{\tau} = -[q(1-(y_N + \frac{\theta}{1-\theta} y_T') p_q) + (r-\delta)f] \Delta_{\tau}^{-1}$$

$$(29l) \quad \left. \frac{\partial p}{\partial r} \right|_{\tau} = p_q \left. \frac{\partial q}{\partial r} \right|_{\tau} = -p_q [(r-n)q + (\lambda+n)(\delta+\lambda)f] \Delta_{\tau}^{-1}$$

$$(29m) \quad \left. \frac{\partial q}{\partial b} \right|_{\tau} = (\lambda+n)(\delta+\lambda)(r-n) \Delta_{\tau}^{-1} < 0$$

$$(29n) \quad \left. \frac{\partial w}{\partial b} \right|_{\tau} = (r-n)(r-\delta) \Delta_{\tau}^{-1}$$

$$(29o) \quad \left. \frac{\partial p}{\partial b} \right|_{\tau} = p_q \left. \frac{\partial q}{\partial b} \right|_{\tau} = p_q (\lambda+n)(\delta+\lambda)(r-n) \Delta_{\tau}^{-1} < 0$$

$\Delta_{\tau}$ , defined in (27) is negative.

The long-run effects of fiscal policy and exogenous shocks are quite intuitive. An increase in exhaustive public spending means with a balanced budget that total taxes  $g+(r-n)b$  increase by the amount of the increase in public spending. Private consumption falls as a result. Financial wealth increases if  $\delta > r$ , falls if  $\delta < r$ . If  $r = \delta$  the price level will be boosted if the share of non-traded goods in public spending is higher than that of

non-traded goods in private spending  $p_g > p_q$  or  $1-\varepsilon > 1-\eta$ ). This effect will be reinforced (weakened or offset) if  $\delta > r$  if ( $\delta < r$ ).

A balanced budget increase in  $\theta$  and reduction (or increase) in  $\tau$  will always lower the relative price of nontraded goods. Since, from (18d),  $p_\theta + \frac{p}{1-\theta} \geq 0$ , consumption falls as  $\theta$  increases. Financial wealth will increase if  $\delta > r$ , fall if  $\delta < r$ . All this is independent of whether the increase in the tax rate on the production of tradables increases or lowers the revenues from that tax, as the total tax revenue from both taxes remains unchanged at  $g+(r-n)b$ .

While it is likely that the producer's relative price of non-traded goods  $\frac{p}{1-\theta}$  rises as  $\theta$  increases, it is again possible that strong income effects reverse this conclusion. The paradox of an increase in  $\theta$  reducing the production of non-traded goods and increasing the production of traded goods by sufficiently immiserizing the private sector remains a logical possibility. In the numerator of (29f), both  $y_T' p_q$  and  $p_\theta$  are increasing in absolute value, and  $\left. \frac{\partial p}{\partial \theta} \right|_\tau$  therefore increases in absolute value as the production possibility frontier becomes less concave and the efficiency loss from distortionary production taxes increases. This possibility exists even when taxes are raised from an initial value of  $\theta = 0$ . The reason is that the long-run equilibrium value of aggregate consumption,  $q$ , need not equal the long-run equilibrium value of domestic product, after taxes, because of the possibility of holding net foreign assets, i.e.,

$$q = (1 - \theta)y_T + py_N - \tau + (r - n)w$$

or, given a balanced budget,

$$q = y_T + py_N - g - (r-n)b + (r-n)w = y_T + py_N - g + (r-n)f.$$

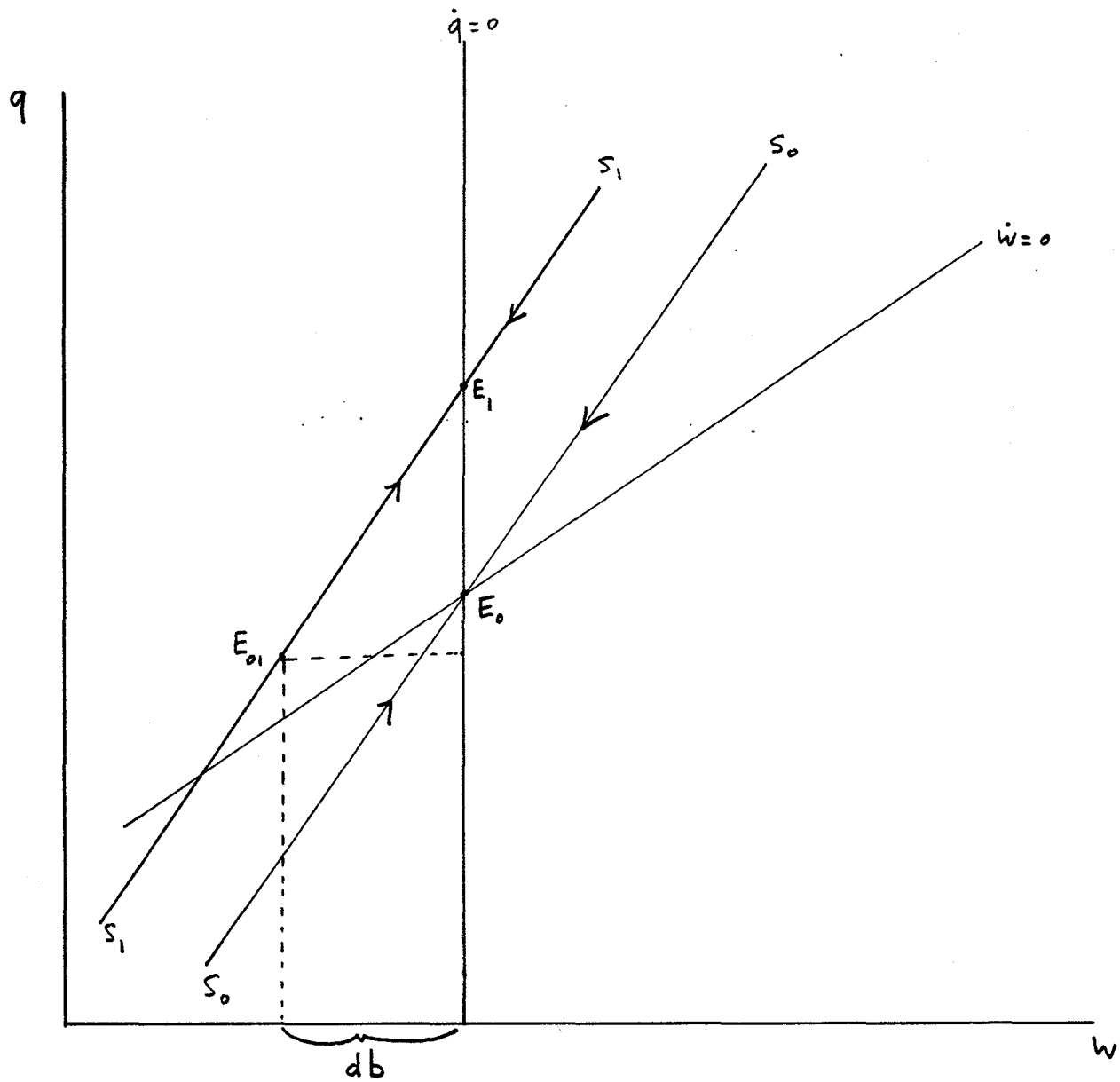
Note, however, that when both  $\theta = 0$  and  $r = \delta$  in the initial equilibrium, then this paradoxical movement in  $\frac{p}{1-\theta}$  cannot occur. The reason is that when  $\delta = r$ ,  $w = 0$ , and  $\left. \frac{\partial w}{\partial \theta} \right|_{\tau} = 0$ , so  $q$  will equal  $y_T + py_N - g - (r-n)b$ . It is only when  $r > \delta$  [i.e. when  $\left. \frac{\partial w}{\partial \theta} \right|_{\tau} < 0$ ] that the paradox occurs.

A switch towards public spending on traded goods for a given total public spending level reduces the price of non-traded goods. Note that since, given  $\theta$ , this raises the revenue from the tpt,  $\tau$  will fall to balance the budget. The effect on  $q$  depends on the sign of  $y_N + \frac{\theta}{1-\theta} y_T'$ . This is the effect of an increase in  $p$  on  $\pi - \tau$  when the balanced budget rule is in effect. If the concavity of the production possibility frontier is sufficiently low, real disposable non-interest income could actually increase as  $p$  falls because of an increase in  $\sigma$ . For lower (absolute) value of  $y_T'$  (and for low values of  $\theta$ )  $q$  will fall as  $\sigma$  rises.

An increase in the world rate of interest will, as in the previous case where  $g_T$  balanced the long-run budget, raise long-run  $q$  unless the initial position is characterized by a sufficiently large foreign debt, i.e., a large negative value of  $f$ . The long-run price of non-traded goods will move in the same direction as aggregate consumption  $q$ . If  $r = \delta$ , financial wealth will increase.

Equations (29m, n, and o) can be used to have a first pass at the issue of debt repudiation. Note that a reduction in  $b$  increase long-run  $q$  and  $p$  and raises (lowers)  $w$  depending on to whether  $r > \delta$  (or  $r < \delta$ ). The long-run effects, but not the dynamic adjustment, are independent of whether the repudiation involves domestic or foreign debt. Figure 5 shows the response of

Figure 5:

Debt Repudiation in a Forgiving Environment



the system, when  $r = \delta$ , to an unexpected, immediate (and permanent) debt repudiation.

In the case where the debt that is repudiated is foreign debt, the authorities cut taxes ( $\tau$ ) by the amount of foreign debt service that is no longer required. The new long-run equilibrium is at  $E_1$ , and since  $w$  is not affected by the default, consumption immediately jumps to its new equilibrium value at  $E_1$ . If the default is on domestic debt, the initial financial wealth of domestic residents falls by the amount of debt that is repudiated ( $db < 0$ ). The initial level of consumption will be on the new saddlepath  $S_1 S_1$  at  $E_{01}$ , and will always be below the level without default of  $E_0$ . <sup>4/</sup> After the default, saving becomes positive and financial wealth augments through current account surpluses. The long-run equilibrium is again at  $E_1$ .

This account assumed that there was no external response to the default, through increased interest rates for future external borrowing (which now includes a default risk premium), through external credit rationing (or enforced financial autarky) after the default or through other penalties imposed through trade sanctions or in any other way. It is the story of the government that defaults but is believed (with subjective certainty) never to do so again.

The long-run effects of an increase in  $r$  on  $q$ ,  $w$  and  $p$  with  $\tau$  endogenous are given in (29j, k, l). Consider the case where  $\delta = r$  and, consequently,  $w = 0$ . Comparing (29j, k, l) with the corresponding expressions when  $g$  is endogenous (33j, k, l) we note the following. (It is assumed that  $1 - \epsilon + p\epsilon_p = 0$  so that  $p_q = \tilde{p}_q$ ). While  $q$  will increase under the  $g_T$  endogenous policy, it may decrease under the  $\tau$  endogenous policy, if  $f$  is sufficiently negative, i.e., if the government's stock of debt is sufficiently large. Under the endogenous taxes, higher debt service by the government

comes straight out of personal disposable income. Even if  $f = w = 0$ ,  $\frac{\partial q}{\partial r}$  will be smaller with  $\tau$  endogenous than with  $g_T$  endogenous. The reason is that the increase in  $p$  will, at a given tpt rate  $\theta$ , reduce tpt receipts. Consequently  $\tau$  will increase when it is endogenous. This will limit the increase in  $q$  and thus in  $p$  below what happens when  $g_T$  is endogenous. 5/

4b.  $\theta$  responds endogenously to maintain budget balance

We now consider the case where  $\theta$  adjusts endogenously to satisfy the balanced budget constraint (25d) at each instant. We proceed by solving (25c, d) for  $p$  and  $\theta$  as function of  $q, g, \tau, \sigma, r$  and  $b$  and substituting these solutions into (25b).

$$(30a) \quad p = p^\theta(q; g, \tau, \sigma, r, b)$$

$$(30b) \quad \theta = \theta(q; g, \tau, \sigma, r, b)$$

$$(31a) \quad p_q^\theta = \frac{(\eta-1)}{p} y_T \left(1 + \frac{\theta}{1-\theta} \phi\right) M_\theta^{-1} > 0$$

$$(31b) \quad p_g^\theta = \left[ \frac{(\epsilon-1)}{p} y_T \left(1 + \frac{\theta}{1-\theta} \phi\right) - \frac{y_T'}{1-\theta} \right] M_\theta^{-1}$$

$$(31c) \quad p_\tau^\theta = \frac{y_T'}{1-\theta} M_\theta^{-1} > 0$$

$$(31d) \quad p_\sigma^\theta = \frac{g\epsilon}{p} y_T \left(1 + \frac{\theta}{1-\theta} \phi\right) M_\theta^{-1} < 0$$

$$(31e) \quad p_r^\theta = -b \frac{y_T'}{1-\theta} M_\theta^{-1} < 0$$

$$(31f) \quad p_b^\theta = -(r-n) \frac{y_T'}{(1-\theta)} M_\theta^{-1} < 0$$

$$(31g) \quad \theta_q = \frac{\theta}{1-\theta} y'_T \frac{(1-\eta)}{p} M_\theta^{-1} > 0$$

$$(31h) \quad \theta_g = [-\Omega^{-1} + \left(\frac{\theta}{1-\theta}\right) y'_T \frac{(1-\varepsilon)}{p}] M_\theta^{-1} > 0$$

$$(31i) \quad \theta_\tau = \Omega^{-1} M_\theta^{-1} < 0$$

$$(31j) \quad \theta_\sigma = \frac{-\theta}{1-\theta} y'_T \frac{g}{p} \varepsilon_\sigma M_\theta^{-1} < 0$$

$$(31k) \quad \theta_r = -\Omega^{-1} b M_\theta^{-1} > 0$$

$$(31l) \quad \theta_b = -(r-n) \Omega^{-1} M_\theta^{-1} > 0$$

$$(31m) \quad M_\theta = -\Omega^{-1} \left(1 + \frac{\theta}{1-\theta} \phi\right) y'_T - \theta \left(\frac{y'_T}{1-\theta}\right)^2 < 0$$

$$= y'_T \left\{ \frac{y'_T}{p} - \left[ \left(\frac{1-\eta+pn'}{p^2}\right) q + \frac{(1-\varepsilon+p\varepsilon)}{p^2} g \right] \left(1 + \frac{\theta}{1-\theta} \phi\right) \right\}$$

To sign these multipliers, I assumed that  $1 + \frac{\theta}{1-\theta} \phi > 0$  i.e., that an increase in  $\theta$  raises tpt revenue at a given price level. This is sufficient (but not necessary) for  $M_\theta < 0$ . Interesting issues arise when the tpt rate has been pushed to the inefficient side of the tpt Laffer curve, but they cannot be addressed here.

The linearized dynamic system is given in (32), the condition for local saddlepoint stability in (33)

$$(32) \quad \begin{bmatrix} \dot{g} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} r-\delta & \\ y_N p^\theta & -y_T \theta q^{-1} \end{bmatrix} \begin{bmatrix} q - \bar{q} \\ w - \bar{w} \end{bmatrix} \quad \begin{matrix} -(\lambda+n)(\delta+\lambda) \\ r-n \end{matrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 9 & 0 \\ y_N^p g^\theta - y_T^\theta g & y_N^p T^\theta - y_T^\theta \tau^{-1} & y_N^p \sigma^\theta - y_T^\theta \sigma & y_N^p r^\theta - y_T^\theta r^{+\omega} & y_N^p b^\theta - y_T^\theta b \end{bmatrix} \begin{bmatrix} g-g \\ \tau-\tau \\ \sigma-\sigma \\ r-r \\ b-b \end{bmatrix}$$

$$(33) \quad \Delta_\theta \equiv (r + \lambda)(r - (\delta + \lambda + n)) + (\lambda + n)(\delta + \lambda)(y_N^p q^\theta - y_T^\theta q) < 0$$

or

$$\equiv (r + \lambda)(r - (\delta + \lambda + n)) + (\lambda + n)(\delta + \lambda) \left( \frac{n-1}{pM_\theta} \right) (y_N y_T + \frac{\theta}{1-\theta} y_T' (\frac{p}{1-\theta} y_N + y_T)) < 0 \quad \underline{7/}$$

Qualitatively, the saddlepoint equilibrium in the  $\theta$  endogenous case is similar to those in the  $g_T$  endogenous and  $\tau$  endogenous cases. The long run multipliers are given in (34a to o). They are obtained by solving (25a, b, c, d) with  $\dot{q} = \dot{w} = 0$ .

$$(34a) \quad \left. \frac{\partial q}{\partial g} \right|_\theta = -(\lambda + n)(\delta + \lambda)(y_N^p g^\theta - y_T^\theta g) \Delta_\theta^{-1}$$

$$(34b) \quad \left. \frac{\partial w}{\partial g} \right|_\theta = (\delta - r)(y_N^p g^\theta - y_T^\theta g) \Delta_\theta^{-1}$$

$$(34c) \quad \left. \frac{\partial p}{\partial g} \right|_\theta = p_q^\theta \left. \frac{\partial q}{\partial g} \right|_\theta + p_g^\theta = [(r + \lambda)(r - (\delta + \lambda + n)) p_g^\theta - (\lambda + n)(\delta + \lambda) y_T (p_g^\theta q - p_q^\theta g)] \Delta_\theta^{-1}$$

$$(34d) \quad \left. \frac{\partial q}{\partial \tau} \right|_\theta = -(\lambda + n)(\delta + \lambda)(y_N^p \tau^\theta - y_T^\theta \tau^{-1}) \Delta_\theta^{-1} > 0 \quad \underline{8/}$$

$$(34e) \quad \left. \frac{\partial w}{\partial \tau} \right|_\theta = (\delta - r)(y_N^p \tau^\theta - y_T^\theta \tau^{-1}) \Delta_\theta^{-1}$$

$$(34f) \quad \left. \frac{\partial p}{\partial \tau} \right|_\theta = p_q^\theta \left. \frac{\partial q}{\partial \tau} \right|_\theta + p_\tau^\theta = [(r + \lambda)(r - (\delta + \lambda + n)) p_\tau^\theta + (\lambda + n)(\delta + \lambda)(y_T (p_q^\theta \tau - p_\tau^\theta q) + p_q^\theta)] \Delta_\theta^{-1}$$

$$(34g) \quad \left. \frac{\partial q}{\partial \sigma} \right|_\theta = -(\lambda + n)(\delta + \lambda) \frac{g^\epsilon \sigma}{pM_\theta} (y_N y_T + \frac{\theta}{1-\theta} y_T' (\frac{p}{1-\theta} y_N + y_T)) \Delta_\theta^{-1}$$

$$(34h) \quad \frac{\partial w}{\partial \sigma} \Big|_{\theta} = (\delta - r) \frac{g \varepsilon_{\sigma}}{p M_{\theta}} \left( y_N y_T + \frac{\theta}{1-\theta} y_T' \left( \frac{p}{1-\theta} y_N + y_T \right) \right) \Delta_{\theta}^{-1}$$

$$(34i) \quad \frac{\partial p}{\partial \sigma} \Big|_{\theta} = p_q^{\theta} \frac{\partial q}{\partial \sigma} \Big|_{\theta} + p_{\sigma}^{\theta} = (r - (\delta + \lambda + n))(r + \lambda) p_{\sigma}^{\theta} \Delta_{\theta}^{-1} < 0$$

$$(34j) \quad \frac{\partial q}{\partial r} \Big|_{\theta} = -[(r-n)q + (\lambda+n)(\delta+\lambda)(w + y_N p_r^{\theta} - y_T \theta_r)] \Delta_{\theta}^{-1}$$

$$(34k) \quad \frac{\partial w}{\partial r} \Big|_{\theta} = -[q(1 - (y_N p_q^{\theta} - y_T \theta_q)) + (r-\delta)(w + y_N p_r^{\theta} - y_T \theta_r)] \Delta_{\theta}^{-1}$$

$$(34l) \quad \frac{\partial p}{\partial r} \Big|_{\theta} = p_q^{\theta} \frac{\partial q}{\partial r} \Big|_{\theta} + p_r^{\theta} = -p_q^{\theta} (r-n)q + (\lambda+n)q + (\lambda+n)(\delta+\lambda)w + (\lambda+n)(\delta+\lambda)y_T (p_q^{\theta} \theta_r - p_r^{\theta} \theta_q)$$

$$= [-p_q^{\theta} ((r-n)q + (\lambda+n)(\delta+\lambda)w) + (\lambda+n)(\delta+\lambda)y_T (p_q^{\theta} \theta_r - p_r^{\theta} \theta_q)$$

$$+ p_r^{\theta} (r+\lambda)(r - (\delta+\lambda+n))] \Delta_{\theta}^{-1}$$

$$(34m) \quad \frac{\partial q}{\partial b} \Big|_{\theta} = -(\lambda+n)(\delta+\lambda)(y_N p_b^{\theta} - y_T \theta_b) \Delta_{\theta}^{-1} < 0$$

$$(34n) \quad \frac{\partial w}{\partial b} \Big|_{\theta} = (\delta - r)(y_N p_b^{\theta} - y_T \theta_b) \Delta_{\theta}^{-1}$$

$$(34o) \quad \frac{\partial p}{\partial b} \Big|_{\theta} = p_q^{\theta} \frac{\partial q}{\partial b} \Big|_{\theta} + p_b^{\theta} = [p_b^{\theta} (r - (\delta + \lambda + n))(r + \lambda) + (\lambda + n)(\delta + \lambda)y_T (p_q^{\theta} \theta_b - p_b^{\theta} \theta_q)] \Delta_{\theta}^{-1} < 0$$

Note from (31b) that even given  $q$ , the effect of an increase in  $g$  on  $p$  is ambiguous. This is because while cet.par. an increase in  $g$  will raise  $p$  (if some of the additional spending falls in non-traded goods (i.e.,  $\varepsilon < 1$ )) the increase in  $\theta$  required to keep the budget balanced (see 31h) will tend to lower  $p$ , both by reducing private demand for non-traded goods and by increasing their supply. Clearly, if all the additional spending falls on

traded goods ( $\epsilon=1$ ), then  $p_g^\theta < 0$ . Even if  $\epsilon=0$ , on the other hand, an increase in  $p$  is not certain. If the tax effect dominates, ( $p_g^\theta < 0$ ) then the long-run effect of an increase in public spending is to lower  $q$  and  $p$ , while  $w$  increases if  $\delta > r$ , falls if  $\delta < r$ .

An increase in  $\tau$ , with budget balance maintained by lowering  $\theta$ , will, given  $q$ , tend to create excess demand for non-traded goods.  $p$  rises to clear the market, i.e.  $p_\tau^\theta > 0$ . Aggregate consumption in terms of traded goods therefore increases in the long run and so does the long-run price of non-traded goods.  $w$  will rise if  $r > \delta$ , fall if  $r < \delta$ .

Switching public spending towards traded goods, holding  $g$  constant will, given  $q$ , reduce  $p$ . This will expand the production of trade goods and the tpt receipts at a given value of  $\theta$ . The tpt rate is therefore lowered to prevent a budget surplus from emerging. For small value of  $\theta$ , long-run  $g$  falls and the price level unambiguously declines in the long run. For large values of  $\theta$ , however, the boost to real income associated with lowering the distortion may be large enough to raise  $q$ . The long-run value of  $p$  of course declines.  $w$  moves in the same direction as  $q$  if  $r > \delta$ , in the opposite direction if  $r < \delta$ .

An increase in  $r$  will, if  $b > 0$ , require a higher value of  $\theta$  to balance the budget. Given  $q$ ,  $p$  falls to equilibrate the non-traded goods market. Long-run aggregate consumption in terms of traded goods increases unless the negative effects on human capital (measured in traded goods) of a lower value of  $p$  and a higher value of  $\theta$ , i.e.  $y_N p_r^\theta - y_T \theta_r$ , are very strong. If  $q$  were to fall, the long-run effect on  $p$  is negative, otherwise it is ambiguous.

A higher value of  $b$  also requires a higher tax rate  $\theta$  to take care of the increased government interest bill. Given  $q$ , the price of nontraded goods

declines. Long-run  $q$  declines, as does long-run  $p$ .  $w$  increases if  $\delta > r$ , decreases if  $\delta < r$ . To the attractions of repudiation is now added the lower distortionary tax rates that it permits.

4c.  $g$  responds endogenously to maintain budget balance

When  $g$  responds endogenously to ensure budget balance in response to variations in  $\tau$ ,  $\theta$ ,  $\sigma$ ,  $r$ , and  $b$  we solve the nontraded goods market equilibrium condition (25c) and the real growth corrected balanced budget condition (25d) for  $p$  and  $g$  as functions of  $g$ ,  $\tau$ ,  $\theta$ ,  $\sigma$ ,  $r$  and  $b$ . This yields

$$(35a) \quad p = p^g(q; \tau, \theta, \sigma, r, b)$$

$$(35b) \quad g = g(q; \tau, \theta, \sigma, r, b)$$

where

$$(36a) \quad p_q^g = \frac{(1-\eta)}{p} M_g^{-1} > 0$$

$$(36b) \quad p_\tau^g = \frac{(1-\varepsilon)}{p} M_g^{-1} > 0$$

$$(36c) \quad p_\theta^g = \left[ \left( \frac{1-\varepsilon}{p} \right) y_T + \frac{y_T'}{(1-\theta)^2} (1+(1-\varepsilon)\theta) \right] M_g^{-1}$$

$$(36d) \quad p_\sigma^g = - \frac{g\varepsilon}{p} M_g^{-1} < 0$$

$$(36e) \quad \left( \frac{\varepsilon-1}{p} \right) b M_g^{-1} \quad (\text{assuming } b \geq 0)$$

$$(36f) \quad p_b^g = (\varepsilon-1)(r-n) M_g^{-1} < 0.$$

$$(36g) \quad g_q = \frac{\theta}{1-\theta} y_T' M_g^{-1} < 0$$

$$(36h) \quad g_\tau = \Omega^{-1} M_g^{-1} \quad 0 \leq g_\tau \leq 1$$

$$(36i) \quad g_\theta = \left[ \Omega^{-1} y_T (1 + \frac{\theta}{1-\theta} \phi) + \theta \left( \frac{y_T'}{1-\theta} \right)^2 \right] M_g^{-1}$$

$$(36j) \quad g_\sigma = - \frac{\theta}{1-\theta} y_T' g \frac{\epsilon_\sigma}{p} M_g^{-1} > 0$$

$$(36k) \quad g_r = -\Omega^{-1} b M_g^{-1} \leq 0 \quad (\text{assuming } b \geq 0)$$

$$(36l) \quad g_b = -(r-n) \Omega^{-1} M_g^{-1} < 0$$

$$(36m) \quad M_g = \Omega^{-1} + \frac{\theta}{1-\theta} y_T' \left( \frac{\epsilon-1}{p} \right) > 0$$

Substituting (35a,b) into (25a,b) and linearizing around a stationary equilibrium yields

$$(37) \quad \begin{bmatrix} \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} r-\delta & -(\delta+\lambda)(\lambda+n) \\ y_N^p q^{g-1} & r-n \end{bmatrix} \begin{bmatrix} q-\bar{q} \\ w-\bar{w} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & q & 0 \\ y_N^p \tau^{g-1} & y_N^p \theta^{g-1} y_T & y_N^p \sigma^g & y_N^p r^{g+w} & y_N^p b^g \end{bmatrix} \begin{bmatrix} \tau-\bar{\tau} \\ \theta-\bar{\theta} \\ \sigma-\bar{\sigma} \\ r-\bar{r} \\ b-\bar{b} \end{bmatrix}$$

For the system given in (37) to have a saddlepoint configuration, (38) must hold:



$$(38) \quad \Delta_g = (r+\lambda)(r-(\delta+\lambda+n))+(\lambda+n)(\delta+\lambda)y_N p_q^g < 0$$

Assuming as before that the  $\dot{w} = 0$  locus is upward-sloping, it can be seen that the slope of the  $\dot{w} = 0$  locus is the same when  $g$  is endogenous as when  $\tau$  is endogenous if  $\theta = 0$ . If  $0 < \theta < 1$ , then the  $\dot{w} = 0$  locus is steeper when  $g$  is endogenous than when  $\tau$  is endogenous. 9/ The  $\dot{q} = 0$  locus always has the same slope, regardless of the financing mode. The shape of the  $\dot{w} = 0$  locus with  $\theta$  endogenous 10/ cannot in general be ranked relative to that of the  $\dot{w} = 0$  locus with  $g$  endogenous.

The long-run effects of changes in  $\tau$ ,  $\theta$ ,  $\sigma$ ,  $r$  and  $b$  on  $q$ ,  $w$  and  $q$  are given below in (39a to o). Note that the special case  $\epsilon=1$ , no public spending on (and no changes in public spending on) non-traded goods, gives the multipliers already obtained in (23a,n) for the  $g_T$  endogenous case.

$$(39a) \quad \left. \frac{\partial q}{\partial \tau} \right|_g = (\lambda+n)(\delta+\lambda)(1-y_N p_\tau^g) \Delta_g^{-1} < 0$$

$$(39b) \quad \left. \frac{\partial w}{\partial \tau} \right|_g = (r-\delta)(1-y_N p_\tau^g) \Delta_g^{-1}$$

$$(39c) \quad \left. \frac{\partial p}{\partial \tau} \right|_g = p_q^g \left. \frac{\partial q}{\partial \tau} \right|_g + p_\tau^g = [(\lambda+n)(\delta+\lambda)(p_q^g - p_\tau^g) + (r-n)(r-\delta)p_\tau^g] \Delta_g^{-1}$$

$$(39d) \quad \left. \frac{\partial q}{\partial \theta} \right|_g = -(\lambda+n)(y_N p_\theta^g - y_T) \Delta_g^{-1}$$

$$(39e) \quad \left. \frac{\partial w}{\partial \theta} \right|_g = (\delta-r)(y_N p_\theta^g - y_T) \Delta_g^{-1}$$

$$(39f) \quad \left. \frac{\partial p}{\partial \theta} \right|_g = p_q^g \left. \frac{\partial q}{\partial \theta} \right|_g + p_\theta^g = [(\lambda+n)(\delta+\lambda)y_T p_q^g + (r+\lambda)(r-(\delta+\lambda))p_\theta^g] \Delta_g^{-1}$$

$$(39g) \quad \left. \frac{\partial q}{\partial \sigma} \right|_g = -(\lambda+n)(\delta+\lambda)y_N p_\sigma^g \Delta_g^{-1} < 0$$

$$(39h) \quad \left. \frac{\partial w}{\partial \sigma} \right|_g = (\delta - r) y_N p_\sigma^g \Delta_g^{-1}$$

$$(39i) \quad \left. \frac{\partial p}{\partial \sigma} \right|_g = p_q^g \left. \frac{\partial q}{\partial \sigma} \right|_g + p_\sigma^g = (r + \lambda)(r - (\delta + \lambda + n)) p_\sigma^g \Delta_g^{-1} < 0$$

$$(39j) \quad \left. \frac{\partial q}{\partial r} \right|_g = -[(r - n)q + (\lambda + n)(\delta + \lambda)(w + y_N p_r^g)] \Delta_g^{-1}$$

$$(39k) \quad \left. \frac{\partial w}{\partial r} \right|_g = [-q(1 - y_N p_q^g) + (\delta - r)(w + y_N p_r^g)] \Delta_g^{-1}$$

$$(39l) \quad \left. \frac{\partial p}{\partial r} \right|_g = p_q^g \left. \frac{\partial q}{\partial r} \right|_g + p_r^g = [-p_q^g[(r - n)q + (\lambda + n)(\delta + \lambda)w] + p_r^g(r + \lambda)(r - (\delta + \lambda + n))] \Delta_g^{-1}$$

$$(39m) \quad \left. \frac{\partial q}{\partial b} \right|_g = -(\lambda + n)(\delta + \lambda) y_N p_b^g \Delta_g^{-1} < 0$$

$$(39n) \quad \left. \frac{\partial w}{\partial b} \right|_g = (\delta - r) y_N p_b^g \Delta_g^{-1}$$

$$(39o) \quad \left. \frac{\partial p}{\partial b} \right|_g = p_q^g \left. \frac{\partial q}{\partial b} \right|_g + p_b^g = p_b^g (r + \lambda)(r - (\delta + \lambda + n)) \Delta_g^{-1} < 0$$

The long-run effects of an increase in  $\tau$  with  $g$  endogenous, given in (38a,b,c) are proportional to the long-run effects of an increase in  $g$  with  $\tau$  endogenous, given in (29a,b,c). This is because  $\left. \frac{\partial q}{\partial \tau} \right|_g = \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial g} \cdot \frac{\partial g}{\partial \tau}$  and  $\left. \frac{\partial q}{\partial g} \right|_\tau = \frac{\partial q}{\partial g} + \frac{\partial q}{\partial \tau} \frac{\partial \tau}{\partial g} \frac{11/}{}$ . Therefore  $\left. \frac{\partial g}{\partial \tau} \right|_g = \left. \frac{\partial q}{\partial g} \right|_\tau \cdot \frac{\partial g}{\partial \tau}$ . Similarly,  $\left. \frac{\partial w}{\partial \tau} \right|_g = \left. \frac{\partial w}{\partial g} \right|_\tau \frac{\partial g}{\partial \tau}$  and  $\left. \frac{\partial p}{\partial \tau} \right|_g = \left. \frac{\partial p}{\partial g} \right|_\tau \cdot \frac{\partial g}{\partial \tau}$ . Since, quite obviously,  $\frac{\partial g}{\partial \tau} > 0$ ,  $q$  falls when  $\tau$  is raised,  $w$  falls if  $r > \delta$  (rises if  $r < \delta$ ) and  $p$  will rise (when  $\delta = r$ ) if  $\epsilon > \eta$ , i.e. if the increase in public spending adds more to public spending on non-traded goods than the increase in taxes subtracts from private spending on non-traded goods.

The ambiguity of the long-run effects of an increase in  $\theta$  with  $g$  endogenous (equations [39d,e,f]) are the same as those of the long-run effects of an increase in  $g$  with  $\theta$  endogenous (equations 34a,b,c) discussed in the previous Section (note that, since we assume that we aren't on the wrong side of the tpt Laffer curve,  $\frac{\partial g}{\partial \theta} > 0$ ).

An increase in the share of traded goods in public spending again lowers long-run  $p$  and  $q$ , and lowers  $w$  if  $r > \delta$ , raises it if  $\delta > r$ . Note that with  $p$  lower, tpt tax receipts will increase (if  $\theta > 0$ ) and  $g$  will rise to preserve budget balance.

A higher interest rate requires a lower volume of public spending to balance the budget. This will depress  $p$ , given  $q$ . The long-run effect in  $q$  is ambiguous. If it increases, the long-run effect on  $p$  is ambiguous. If  $q$  falls,  $p$  must also go down. A larger stock of public debt requires a cut in public spending.  $p$  and  $q$  fall.

The case where the budget is kept balanced by varying the composition of public spending, keeping the total constant in terms of traded goods is omitted for reasons of space and left as an exercise. Much of it (e.g. the long-run effects of changes in  $g$ ,  $\tau$  and  $\theta$ , with  $\sigma$  endogenous) can be derived straightforwardly from the analysis in Sections 4a,b, and c (e.g. from the long-run effects of a change in  $\sigma$  with  $\tau$ ,  $\theta$  or  $g$  endogenous).

## 5. Conclusion

The volume and composition of exhaustive public spending and the structure of taxation are important influences on the composition of production between traded and non-traded goods, the real exchange rate, the net foreign asset position and the volume of private consumption in the short-run and in the long-run. Even when the problem of public debt stabilization

is not present, as in section 4 of the paper where the public sector was assumed to maintain continuous real growth corrected budget balance, important fiscal options remain. Two examples serve to illustrate this.

Consider a government that has decided to reduce the bias against traded goods in its fiscal programme by reducing  $\theta$ , the tpt rate. If tax revenue decreases as  $\theta$  is reduced, the government has several options, even assuming the budget is kept balanced. If it raises the other (lump sum) tax in its arsenal, the following will happen. In the long run (equations 29d,e,f) the consumer's relative price of non-traded goods,  $p$ , rises, i.e. the consumer's real exchange rate depreciates. The producers' relative price of non-traded goods  $\frac{P}{1-\theta}$  should decline as  $\theta$  is reduced, unless the increase in  $p$  more than offsets the reduction in the tax rate on traded goods. This perverse result can happen only if there are "first-order" real income effects on private consumption demand from the reduction in the distortionary tax rate. Conventional intertemporal optimizing models such as the representative infinite-lived household, have the property that there are no "first-order" real income effect if the distortionary tax is the only distortion and the tax rate change is evaluated at  $\theta = 0$ . In the Yaari-Blanchard model of consumer behavior, tax rate changes evaluated at  $\theta = 0$  will have "first order" real income effects even if there are no other conventional distortions, as long as  $r$ , the market interest rate and  $\delta$ , the consumer's pure rate of time preference, differ from each other. Specifically, if  $r > \delta$ , a reduction in  $\theta$  will have a positive first order real income effect, even at  $\theta = 0$ , which may boost private spending in general and private demand for non-traded goods in particular by so much that  $\frac{P}{1-\theta}$  rises when  $\theta$  is lowered. In that case the movement of resources will be out of the traded goods sector and into the non-

traded goods sector, in spite of the reduction in the tax rate on tradeables production.  $r \neq \delta$  is operationally equivalent to another distortion.

Without wishing to advance the perverse case as the likely one, it illustrates the importance of even minor concessions to realism in modelling private consumption and saving behavior. The Yaari-Blanchard model with its effective private discount rate in excess of the marginal cost of government borrowing, generates equilibria than can be characterized by  $r$  below, equal to or above  $\delta$ , in the short run and in the long run. The absence of debt-neutrality in the Yaari-Blanchard model is another important ingredient in the analysis of real-world fiscal and financial policy issues.

If instead of raising its other tax, the government accompanies its cut in  $\theta$  by a cut in public spending, the consequences for the real exchange rate and the sectoral allocation of production can be quite different.

From equations (39d,e,f) it can be seen that the effect on the real exchange rate is ambiguous. The cut in  $\theta$ , which will cet. par. (i.e. given  $q$  and  $g$ ) be associated with an increase in  $p$ , is accompanied by a spending cut which, if at least some of it falls on non-traded goods, will tend to depress their relative price. Unlike the previous case when  $\tau$  was raised, total taxes now are down. Private consumption demand will be boosted and, to the extent that this falls on non-traded goods, will tend to raise  $p$ . If  $p$  declines,  $p(1-\theta)^{-1}$  declines a-fortiori and production and resources more towards the traded goods section. Even if  $p$  increases,  $p(1-\theta)^{-1}$  will fall unless the increase in  $p$  is very large. This is less likely the smaller  $\theta$  and the excess of  $r$  over  $\delta$ .

It can be shown that even if  $\tau$  and  $g$  are kept constant, the government can balance the budget when it reduces the tpt rate by shifting the composition of public spending towards traded goods. 12/ (provided  $\theta > 0$ ).

This will reduce  $p$  and restore total tpt tax receipts to their previous level in spite of the lower rate by reducing  $\frac{P}{1-\theta}$  and further expanding the production of traded goods. With this budget balancing strategy,  $p$  definitely declines and resources unambiguously move into the traded goods sector.

The second example concerns alternative balanced budget responses to an increase in world real interest rates which increases the interest cost of servicing the public debt. If the government raises lump-sum taxes to offset the increased interest bill, the long-run effects will depend on the net foreign asset position of the country (equations 29, j, k, l). Consider for simplicity the case where  $r = \delta$ . In that case the country's net foreign asset position equals  $-b$ , the public debt. For a small negative value of  $f$  (for a small public debt), the long-run response to an increase in the interest rate involves the accumulation of private non-human wealth and an increase in the level of consumption. The short-run response of consumption to an unanticipated permanent rise in  $r$  of course goes in the opposite direction. During the adjustment process a low but rising level of consumption generates the current account surpluses that accumulate into a higher long-run stock of net foreign assets which permit the long-run increase in consumption. The response of  $p$  is an immediate sharp drop followed by a steady rise and an eventual higher long-run equilibrium level.

If the initial position is that of a heavily indebted government and nation, however, all these results can be reversed, with a long-run decline in  $w$ ,  $q$  and  $p$  being preceded by a short-run consumption boom and current account deficit.

If the incipient budget deficit resulting from an increase in  $r$  is avoided by an increase in the tpt rate, the result is much more ambiguous (see equations 34j, k, l). Given  $q$ , the increase in  $\theta$  required to pay for

the increase in debt service will tend to reduce  $p$ , so  $p_r^\theta < 0$ . Two economies starting off with the same moderate external debt position  $((r-n)q+(\lambda+n)(\delta+\lambda)f > 0)$  could therefore have qualitatively different responses of  $p$  and of the composition of production in the short run and in the long run, to an increase in  $r$ , depending on whether  $\tau$  or  $\theta$  is raised to balance the budget. Specifically, the short-run decline in  $p$  and  $q$  followed by a long-run increase in both under the  $\tau$  endogenous rule in response to an unanticipated permanent increase in  $r$  could become, in the  $\theta$  endogenous case, a short-run increase in  $q$  and  $p$  followed by a long-run decline. Of course, the increase in  $\theta$  is, unlike the increase in  $\tau$ , associated with a dead-weight loss if  $\theta > 0$ .

If public spending is cut to balance the budget following an increase in  $r$ , the effect on  $p$ , given  $q$ , is again negative, if at least some of the spending cuts fall on non-traded goods. For the same (moderate) value of the foreign debt, the response of  $q$  and  $p$  can again be in the opposite direction from what it would be if  $\tau$  were adjusted to balance the budget. If the government's share of non-traded goods in total public consumption  $1-\varepsilon$ , exceeds the private sector's share  $1-\eta$ , a long run decline in  $p$  is more likely when  $g$  is endogenous.

All the ingredients necessary for the consideration of alternative balanced-budget adjustment processes following a variety of internal and external shocks are contained in the previous sections. In a subsequent paper the preliminary unbalanced (but solvent) budget analysis of section (3) will be extended to allow for more general borrowing strategies. Finally, the option of money financing, in practice often the authorities' first resort, will be added to the policy menu.

Footnotes

1. When integrated forward in time, equation (3') yields equation (3) when we impose the transversality condition

$$\lim_{\ell \rightarrow \infty} h(\ell) e^{-\int_t^\ell (r(u)+\lambda) du} = 0.$$

2.  $\bar{\pi}(s, t)$  is non-interest income, at time  $t$ , of a household born at time  $s \leq t$ .  $\bar{\tau}(s, t)$  is taxes net of transfers for that same household.
3. Consider, e.g., the simple 2-sector model with one scarce factor,  $L$ , which is subject to diminishing returns in a Cobb-Douglas technology and is perfectly mobile between sectors, i.e.,

$$Y_N = L_N^{\alpha_N} \quad 0 < \alpha_T, \alpha_N < 1; L_N \geq 0; L \geq L_N$$

$$y_T = (L - L_N)^{\alpha_T}$$

Since  $y_T = (L - y_N)^{\frac{1}{\alpha_N} \alpha_T} = \ell(y_N)$ , we have

$$\ell' = \frac{-\alpha_T}{\alpha_N} (L - y_N)^{\frac{1}{\alpha_N} \alpha_T - 1} \frac{1}{y_N} < 0$$

$$\ell'' = \frac{\alpha_T}{\alpha_N} \frac{1}{y_N} \frac{1}{\alpha_N} - 2 \quad (L - y_N)^{\frac{1}{\alpha_N} \alpha_T - 2} \left[ \left( \frac{\alpha_N - 1}{\alpha_N} \right) (L - y_N)^{\frac{1}{\alpha_N}} + \left( \frac{\alpha_T - 1}{\alpha_N} \right) y_N^{\frac{1}{\alpha_N}} \right]$$

$$= - \frac{\alpha_T}{\alpha_N} (L - y_N)^{\frac{1}{\alpha_N} \alpha_T - 1} \frac{1}{y_N} \frac{1}{\alpha_N} - 1 \left[ \left( \frac{1 - \alpha_N}{\alpha_N} \right) y_N^{-1} + \left( \frac{1 - \alpha_T}{\alpha_N} \right) y_N^{\frac{1}{\alpha_N} - 1} (L - y_N)^{-1} \right] < 0$$



Therefore,

$$y_T' \equiv \frac{-\ell'}{\ell''} = \frac{-1}{\left(\frac{1-\alpha_N}{\alpha_N}\right)y_N^{-1} + \left(\frac{1-\alpha_T}{\alpha_N}\right)y_N^{\frac{1}{\alpha_N}-1} (L-y_N)^{-1}} < 0$$

and

$$\phi \equiv \frac{y_T'}{y_T} \frac{p}{1-\theta} = \frac{(\ell')^2}{\ell''\ell} = \frac{-\alpha_T}{\alpha_N^{-\alpha_T} + (1-\alpha_N)y_N^{-\frac{1}{\alpha_N}L}}$$

When  $y_N = 0$ ,  $\phi = 0$ . When  $y_N$  assumes its highest possible value,  $y_N = L^{\alpha_N}$ ,  $\phi = \frac{-\alpha_T}{1-\alpha_T}$ . Therefore, with  $\alpha_T > .5$ ,  $\phi$  can exceed 1 in absolute value and a Laffer curve for the tpt exists.

4. It is comforting that governments cannot put the economy on a Pareto-superior aggregate consumption path through a default on internally held debt. [Note that the traded goods only economy is the special case of our economy when  $p_q = 0$ . In such an economy a path of  $q$  that dominates another path must be Pareto-superior].

It is easily checked [see Buiter (1984)] that the slope of the saddlepaths in Figure 5 is

$$\left. \frac{dq}{dw} \right|_{SS} = \frac{(r-n) + \sqrt{(r-n)^2 + 4(\delta+\lambda)(\lambda+n)(1-p_q(y_N + \frac{\theta}{1-\theta} y_T'))}}{2 [1 - p_q(y_N + \frac{\theta}{1-\theta} y_T')]}$$

$x$ , the horizontal distance at  $E_0$  to the new saddlepath  $S_1S_1$  through  $E_1$  is given by

$$x = db \frac{2(r-n)(1-p_q(y_N + \frac{\theta}{1-\theta} y_T'))}{(r-n) + \sqrt{(r-n)^2 + 4(\delta+\lambda)(\lambda+n)(1-p_q(y_N + \frac{\theta}{1-\theta} y_T'))}} < db$$

Therefore a default of magnitude  $db$  will put the economy on a point such as  $E_{01}$  on  $S_1S_1$  at a value of  $q$  below that at  $E_0$ .

5. What can one say of the "balanced budget multiplier" in this model, i.e. the effect of a tax-financed increase in public spending on real domestic output? Real domestic product (in terms of traded goods) at factor cost is  $(1-\theta)y_T + py_N$ . Since  $(1-\theta)y'_T + py'_N = 0$  by profit maximization, the effect of an increase in  $g$  on domestic value added is simply given by  $y_N \frac{\partial p}{\partial g} \Big|_{\tau}$ . From (29d) if  $r = \delta$ , this will be positive if  $1 - \epsilon > 1 - \eta$ , negative if  $1 - \epsilon < 1 - \eta$ . In terms of non-traded goods, value added at factor cost changes by

$$- y_T \frac{(1-\theta)}{p^2} \frac{\partial p}{\partial g} \Big|_{\tau}.$$

The effect on domestic income at market prices (in terms of traded goods) is given by

$$\frac{\partial}{\partial g} (y_T + py_N) \Big|_{\tau} = (y_N + \frac{\theta}{1-\theta} y_T) \frac{\partial p}{\partial g} \Big|_{\tau},$$

the effect on domestic income at market prices (i.t.o non-traded goods) is given by

$$\frac{\partial}{\partial g} \left( \frac{y_T}{p} + y_N \right) \Big|_{\tau} = - \left( \frac{y_T}{p^2} - \frac{\theta}{1-\theta} \frac{y'_T}{p} \right) \frac{\partial p}{\partial g} \Big|_{\tau}.$$

In terms of the consumer price index  $\tilde{p} = p^{1-\eta}$  which is appropriate when preferences are Cobb-Douglas, the change in real domestic income  $\tilde{y}$  is given by

$$\frac{\partial}{\partial g} \tilde{y} \Big|_{\tau} \equiv \frac{\partial}{\partial g} \left( \frac{y_T + p y_N}{p^{1-\eta}} \right) \Big|_{\tau} = [p^{\eta-2} [\eta p y_N^{-(1-\eta)} y_T] + p^{\eta-1} \left( \frac{\theta}{1-\theta} \right) y_T'] \frac{\partial p}{\partial g} \Big|_{\tau}$$

If  $\delta = r$  and  $b=g=0$ , then  $q = y_T + p y_N$  and the expression reduces to:

$$\frac{\partial}{\partial g} \tilde{y} \Big|_{\tau} = p^{\eta-1} \frac{\theta}{1-\theta} y_T' \frac{\partial p}{\partial g} \Big|_{\tau}$$

This disappears if  $\theta = 0$ . However, if  $\delta \neq r$ , real income will change even when  $\theta = 0$ . In that case, again with  $g_T = g_N = b = 0$ , and therefore  $w = f$ ,

$$\frac{\partial \tilde{y}}{\partial g} \Big|_{\tau} = p^{\eta-2} [(r-n)w + \frac{\theta}{1-\theta} p y_T'] \frac{\partial p}{\partial g} \Big|_{\tau}$$

Since  $w > 0$  when  $r > \delta$ , this expression will be positive even if  $\theta=0$ .

6. Algebraically,  $|\Delta_{\tau}| > |\Delta_{g_T}|$

7. Note that  $y_N p_q^{\theta} - y_T \theta_q^{-1} = \frac{y_T}{p} \left[ \left(1 + \frac{\theta}{1-\theta} \phi\right) (y_N^{\eta+n} q + \epsilon_p g) - y_T' \left(1 + (1-\eta) \frac{\theta}{1-\theta}\right) \right] M^{-1}$

8. Since  $y_N p_{\tau}^{\theta} - y_T \theta_{\tau}^{-1} = \frac{y_T \Omega^{-1} - y_N y_T' (1-\theta)^{-1}}{y_T \Omega^{-1} + \frac{\theta}{(1-\theta)^2} y_T' \left[ \frac{q}{p} (1-\eta + p \eta') + \frac{q}{p} (1-\epsilon + p \epsilon_p) \right]} - 1 > 0$

9. From (37), when  $g$  is endogenous,  $\frac{dq}{dw} \Big|_{\dot{w}=0} = \frac{r-n}{1 - y_N p_q^g}$

$$= (r-n) \left[ 1 - y_N \frac{(1-\eta)}{p(\Omega^{-1} + \frac{\theta}{1-\theta} y_T' (\frac{\epsilon-1}{p}))} \right]^{-1}$$

From (26), when  $\tau$  is endogenous,

$$\frac{dq}{dw} \Big|_{\dot{w}=0} = 0 = (r-n) \left[ 1 - y_N \frac{(1-n)}{p\Omega} - \left( \frac{\theta}{1-\theta} \right) y_T' \frac{(1-n)}{p\Omega} \right]^{-1}$$

10. From (32), when  $G$  is endogenous,

$$\begin{aligned} \frac{dq}{dw} \Big|_{\dot{w}=0} &= \frac{r-n}{1 - y_N p^{\theta} + y_T \theta} \\ &= \frac{(r-n) \left[ \left( 1 + \frac{\theta}{1-\theta} \phi \right) (y_N + n'q + \epsilon_p g) - y_T' \right]}{\left[ \left( 1 + \frac{\theta}{1-\theta} \phi \right) (n y_N + n'q + \epsilon_p g) - y_T' \left( 1 - (1-n) \frac{\theta}{1-\theta} \right) \right]} \end{aligned}$$

11.  $\frac{\partial q}{\partial \tau}$  is the long-run effect on  $q$  of an increase in  $\tau$ , holding  $g$  constant.

$\frac{\partial q}{\partial g}$  is the long-run effect on  $q$  of an increase in  $g$ , holding  $\tau$  constant.

$\frac{\partial g}{\partial \tau}$  is the change in  $g$  required, in the long run, to balance the budget when  $\tau$  is increased and similarly for  $\frac{\partial \tau}{\partial g}$ .

12. Given  $q$ , the change in  $\sigma$  that balances the budget when  $\theta$  is increased is

$$\text{given by: } \sigma_{\theta} = \left[ \Omega^{-1} y_T \left( 1 + \frac{\theta}{1-\theta} \phi \right) + \theta \left( \frac{y_T'}{1-\theta} \right)^2 \right] \left[ \left( \frac{\theta}{1-\theta} \right) y_T' \frac{g \epsilon_{\sigma}}{p} \right]^{-1} < 0$$

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## APPENDIX

### The Yaari-Blanchard Model of Consumer Behavior

The only new element in this Appendix is in the subsection on aggregation where population growth is allowed for.

#### The individual's problem

Each individual born at time  $s$  maximizes the following utility functional at  $t \geq s$ .

$$(1) \quad \max E_t W(s, t) = \max E_t \int_t^{\infty} e^{-\delta(v-t)} u(\bar{z}(s, v)) dv \quad ; \delta > 0$$

$\delta$  is the instantaneous pure rate of time preference.

$$(2) \quad u(\bar{z}) = \begin{cases} \frac{1}{\mu} \bar{z}^\mu & \mu < 1, \mu \neq 0 \\ \ln \bar{z} & (\mu = 0) \end{cases}$$

$1-\mu$  is the coefficient of relative risk aversion.

$E_t$  is the mathematical expectation operator, conditional on the information available at time  $t$ .

During his or her lifetime each consumer faces a common, instantaneous probability of death  $\lambda \geq 0$ . The probability, at time  $t$ , of surviving until time  $v \geq t$  is therefore given by  $e^{-\lambda(v-t)}$ .

Equation (1) can therefore be rewritten as

$$(1') \quad \max \int_t^{\infty} e^{-(\delta+\lambda)(v-t)} \frac{1}{\mu} \bar{z}(s, v)^\mu dv$$

We define the following.  $p_t$  is the domestic currency price of traded goods,  $p_n$  the domestic currency price of non-traded goods,  $\bar{m}$  the nominal money stock,  $\bar{x}_T$  consumption of traded goods,  $\bar{x}_N$  consumption of non-traded goods and  $i$  the nominal interest rate. In addition:

$$(3a) \quad \pi(v) \equiv p_N(v)/p_T(v)$$

$$(3b) \quad \bar{c}(s,v) \equiv \bar{x}_T(s,v) + \pi(v)\bar{x}_N(s,v)$$

$$(3c) \quad \bar{q}(s,v) \equiv \bar{c}(s,v) + i(v)\bar{m}(s,v)/p_T(v).$$

The instantaneous utility function  $\bar{z}$  is further characterized in equations (4a, b).

$$(4a) \quad \bar{z}(s,t) = f^q(\bar{\psi}(s,t), p_T(t), p_N(t), \bar{m}(s,t))$$

$$(4b) \quad \bar{\psi}(s,t) = f^c(\bar{x}_T(s,t), \bar{x}_N(s,t))$$

The instantaneous utility function  $\bar{z}$  is weakly separable between money and goods.  $f^c$  is linear homogeneous in  $\bar{x}_T$  and  $\bar{x}_N$ .  $f^q$  is linear homogeneous in  $\bar{\psi}$  and  $\bar{m}$ . It is also homogeneous of degree zero in  $p_T$ ,  $p_N$  and  $\bar{m}$ .  $f^c$  is increasing in  $\bar{x}_T$  and  $\bar{x}_N$ , strictly quasi-concave, three times continuously differentiable and satisfies Inada-type condition that ensure strictly positive solutions for  $\bar{x}_T$  and  $\bar{x}_N$ , for positive  $\bar{c}$ .  $f^q$  is increasing in  $\bar{\psi}$  and  $\bar{m}$ , decreasing in  $p_T$  and  $p_N$  for  $\bar{m} > 0$ , strictly quasi-concave, three times continuously differentiable and satisfies Inada-type conditions ensuring positive solutions for  $\bar{\psi}$  and non-negative solutions for  $\bar{m}$ , for positive  $\bar{g}$ .



Thus, preferences over consumption of traded goods  $\bar{x}_T$ , nontraded goods  $\bar{x}_N$  and real money balances are homothetic. In addition there is a subutility function  $f^c$  generated by homothetic preferences over  $\bar{x}_T$  and  $\bar{x}_N$ .

Homogeneity of degree zero of  $f^q$  in  $p_T$ ,  $p_N$  and  $\bar{m}$  implies that  $\bar{z}$  can be written as:

$$(4a') \quad \bar{z} = f^q(\bar{\psi}, 1, \pi, \frac{\bar{m}}{p_T})$$

Homogeneity of degree 1 of  $f^q$  in  $\bar{\psi}$  and  $\bar{m}$  implies that (4a') can be rewritten as:

$$(4a'') \quad \bar{z} = \bar{q} f^q(\bar{\psi} \bar{q}^{-1}, 1, \pi, \frac{\bar{m}}{p_T} \bar{q}^{-1})$$

Homogeneity of degree one of  $f^c$  in  $\bar{x}_T$  and  $\bar{x}_N$  implies that (4a'') can be rewritten as:

$$(4a''') \quad \bar{z} = \bar{q} f^q(f^c(\frac{\bar{x}_T}{\bar{q}}, \frac{\bar{x}_N}{\bar{q}}), 1, \pi, \frac{\bar{m}}{p_T} \bar{q}^{-1})$$

The instantaneous flow budget identity of the consumer is given in (5). There are 5 financial assets among which the consumer allocates his financial wealth: non-interest bearing domestic money,  $\bar{m}$ , interest-bearing short bonds denominated in domestic currency,  $\bar{b}$ , interest-bearing short bonds denominated in foreign currency,  $\bar{b}^*$ , interest-bearing short bonds whose value is index-linked to the price of traded goods,  $\bar{j}$ , and claims to real reproducible capital  $\bar{k}$ . The instantaneous domestic nominal interest rate is  $i$ ; the instantaneous domestic real interest rate (in terms of traded goods) is

$r$ , the instantaneous foreign nominal interest rate is  $i^*$  and the domestic currency price of a unit of installed capital is  $p_k$ .  $\epsilon$  is the spot foreign exchange rate,  $\eta$  the nominal value of divided payments per unit of capital,  $\bar{w}$  nominal wage income, and  $\bar{\tau}$  tax payments net of transfers.

$$\begin{aligned}
 (5) \quad & \frac{d}{dt} [\bar{m}(s,t) + \bar{b}(s,t)] + p_T(t) \frac{d}{dt} \bar{j}(s,t) + \epsilon(t) \frac{d}{dt} \bar{b}^*(s,t) + p_k(t) \frac{d}{dt} \bar{k}(s,t) \\
 & \equiv (\lambda + \eta)(\bar{m}(s,t) + \bar{b}(s,t)) + p_T \bar{j}(s,t) + \epsilon(t) \bar{b}^*(s,t) + p_k \bar{k}(s,t) + i(t) \bar{b}(s,t) \\
 & + r^*(t) p_T(t) \bar{j}(s,t) + i^*(t) \epsilon(t) \bar{b}^*(s,t) + \eta(t) \bar{k}(s,t) + \bar{w}(s,t) - \bar{\tau}(s,t) \\
 & - p_T(t) \bar{x}_T(s,t) - p_N(t) \bar{x}_N(s,t)
 \end{aligned}$$

Equation (5) is the identity that saving equals disposable income minus consumption.

The first term on the r.h.s of (5) reflects the operation of efficient life insurance or annuities markets. Each private consumer enters into the following contract with an insurance company. As long as he lives the consumer receives a rate of return  $\rho$  on his total financial asset holdings at each instant. When he dies, the portfolio becomes the property of the insurance company (conversely, if the consumer's net financial asset position is negative, he pays a rate of return  $\rho$  to the company on his net debt, with the debt being cancelled when he dies). The insurance industry is competitive with free entry. There is a large population and  $\lambda$  is not only the instantaneous probability of death for an individual but also the fraction of each vintage and therefore of the total population that dies at each instant. It is obvious that the competitive (zero expected profit) rate of

return paid by or to insurance companies is  $\rho = \lambda$ . (Not  $\rho = n + \lambda$ ; a fraction  $\lambda$  of the entire population owning a fraction  $\lambda$  of all nonhuman assets dies each period. This is all that can be paid out by the insurance companies to the surviving agents).

Let real financial wealth in terms of traded goods be denoted  $\bar{a}$ , i.e.

$$(6) \quad \bar{a} = \frac{\bar{m} + \bar{b} + p_T \bar{j} + \epsilon \bar{b}^* + p_k \bar{k}}{p_T}$$

We assume that all financial assets other than money earn the same expected rate of return i.e.

$$(7) \quad r = i - \frac{\dot{p}_T}{p_T} = i^* + \frac{\dot{\epsilon}}{\epsilon} - \frac{\dot{p}_T}{p_T} = \frac{\eta}{p_K} + \frac{\dot{p}_k}{p_k} - \frac{\dot{p}_T}{p_T}$$

(6) and (7) permit us to rewrite (5) as

$$(8) \quad \frac{d}{dt} \bar{a}(s,t) \equiv (r(t)+\lambda)a(s,t) + \frac{\bar{w}(s,t)}{p_T(t)} - \frac{\bar{r}(s,t)}{p_T(t)} - (\bar{x}_T(s,t) + \pi(t)\bar{x}_N(s,t) + i(t) \frac{\bar{m}(s,t)}{p_T(t)})$$

Integrating (8) forward in time we obtain the household's present value budget constraint, intertemporal budget constraint or solvency constraint:

$$(9) \quad \int_t^{\infty} e^{-\int_t^u (r(u)+\lambda) du} \bar{q}(s,v) dv \equiv \bar{a}(s,t) + \bar{h}(s,t)$$

where

$$(10) \quad \bar{h}(s,t) \equiv \int_t^{\infty} e^{-\int_t^u (r(u)+\lambda) du} \left( \frac{\bar{w}(s,v) - \bar{r}(s,v)}{p_T(v)} \right) dv$$

$\bar{h}$  is the consumer's human capital, the present discounted value (using the private discount rate  $r+\lambda$ ) of expected future after-tax labour income.

In order to obtain (9) we imposed the no-Ponzi game transversality condition:

$$(11) \quad \lim_{t \rightarrow \infty} e^{-\int_t^{\infty} (r(u)+\lambda) du} \bar{a}(s, \bar{t}) = 0$$

From the first-order conditions for an interior optimum we obtain the following consumption functions:

$$(12a) \quad \bar{x}_T(s, t) = \eta^T(\pi(t)) \bar{c}(s, t) \quad 0 < \eta^T < 1$$

$$(12b) \quad \bar{x}_N(s, t) = \frac{(1-\eta^T(\pi(t))) \bar{c}(s, t)}{\pi(t)} \quad 1-\eta^T + \pi \eta^{T'} > 0$$

$$(13a) \quad \bar{c}(s, t) = \eta^C(i(t)) \bar{q}(s, t) \quad 0 < \eta^C \leq 1$$

$$(13b) \quad \frac{\bar{m}(s, t)}{p_T(t)} = \frac{(1-\eta^C(i(t))) \bar{q}(s, t)}{i(t)} \quad 1-\eta^C + i \eta^{C'} > 0$$

$$(14a) \quad \bar{q}(s, t) = \theta(t) (\bar{a}(s, t) + \bar{h}(s, t))$$

$$(14b) \quad \theta(t) = \begin{cases} \gamma(t)^{\frac{\mu}{1-\mu}} \left[ \int_t^{\infty} e^{-[\frac{\mu}{\mu-1} \int_t^u r(u) du + (v-t)(\lambda + \frac{1}{1-\mu} \delta)]} \gamma(v)^{\frac{\mu}{1-\mu}} dv \right]^{-1} & \text{if } \mu \neq 0 \\ \delta + \lambda & \text{if } \mu = 0 \end{cases}$$

$$(14c) \quad \gamma(v) = f^q \left( f^C(\eta^T(\pi(v)) \eta^C(i(v))), \left( \frac{1-\eta^T(\pi(v))}{\pi(v)} \right) \eta^C(i(v)), 1, \pi(v), \frac{1-\eta^C(i(v))}{i(v)} \right)$$

Aggregation

At each instant a new age cohort composed of many agent is born. The size of the cohort born at time  $t$  is  $(n+\lambda)e^{nt}$ ,  $n, \lambda \geq 0$ . The constant instantaneous probability of death of an agent,  $\lambda$ , is also taken to be the proportion of agents in each cohort which die at each instant. The size of the surviving cohort at time  $t$  which was born at time  $s \leq t$  is therefore  $(n+\lambda)e^{ns} e^{-\lambda(t-s)}$ . Total population at any instant  $t$  is given by

$$(n+\lambda)e^{-\lambda t} \int_{-\infty}^t e^{(\lambda+n)s} ds = e^{nt}$$

For any individual agent's stock or flow  $\bar{v}(s,t)$  we define the corresponding aggregate  $V(t)$  to be

$$(15) \quad V(t) = (n+\lambda)e^{-\lambda t} \int_{-\infty}^t \bar{v}(s,t) e^{(n+\lambda)s} ds .$$

We assume that each agent, regardless of age, earns the same wage income and pays the same taxes net of transfers, i.e.,

$$\bar{w}(s,t) = \bar{w}(t) \text{ for all } s \leq t$$

$$\bar{\tau}(s,t) = \bar{\tau}(t) \text{ for all } s \leq t .$$

It follows that each surviving agent has the same human capital:

$$\bar{h}(s,t) = \bar{h}(t) \text{ for all } s \leq t.$$

Aggregate consumption is, by direct computation, given by

$$(16a) \quad Q(t) = \theta(t)(A(t) + H(t))$$

$$(16b) \quad \dot{A}(t) = r(t) A(t) + W(t) - T(t) - Q(t) \quad \underline{3/}$$

$$(16c) \quad \dot{H}(t) = (r(t) + \lambda + n)H(t) + T(t) - W(t) \quad \underline{4/}$$

$\theta(t)$  is as defined in (14b, c).

For any aggregate  $V$ , the corresponding per-capita magnitude is defined by

$$(17) \quad v(t) \equiv V(t)e^{-nt}$$

Per capita consumption is therefore given by:

$$(18a) \quad q(t) = \theta(t) (a(t) + h(t))$$

$\theta(t)$  is as defined in (14b,c).

$$(18b) \quad \dot{a}(t) = (r(t) - n)a(t) + w(t) - \tau(t) - q(t)$$

$$(18c) \quad \dot{h}(t) = (r + \lambda)h + \tau(t) - w(t)$$

$$(18d) \quad c(t) = n^c(i(t))q(t)$$

$$(18e) \quad \frac{m(t)}{p_T(t)} = \left( \frac{1 - n^c(i(t))}{i(t)} \right) q(t)$$

$$(18f) \quad x_T(t) = \eta^T(\pi(t))c(t)$$

$$(18g) \quad x_N(t) = \left(\frac{1-\eta^T(\pi(t))}{\pi(t)}\right)c(t)$$

From (18a,b,c) an alternative representation of the behavior of  $q$  is:

$$(18h) \quad \dot{q}(t) = (r(t)-\delta)q(t) - (\lambda+n)(\delta+\lambda)a(t)$$

or

$$(18i) \quad \dot{q}(t) = (r(t) - (\delta+n))q(t) - \lambda(\delta+\lambda)a(t) + n(\delta+\lambda)h(t).$$

If labor-augmenting (Harrod-neutral) technical change at a constant proportional rate  $\mu$  is allowed for, the behavior of private consumption is shown in 19. For any per capita variable  $v$ ,  $\tilde{v}$  denotes the corresponding magnitude per unit of efficiency labor.

$$(19a) \quad q = \theta(a+h)$$

$\theta$  is as defined in (14b,c)

$$(19b) \quad \dot{\tilde{a}} = (r - (n+\mu))\tilde{a} + \tilde{w} + \tilde{\tau} - \tilde{q}$$

$$(19c) \quad \dot{\tilde{h}} = (r + \lambda - \mu)\tilde{h} + \tilde{\tau} - \tilde{w}$$

$$(19d) \quad \tilde{c} = \eta^c \tilde{q}$$

$$(19e) \quad \frac{\tilde{m}}{p_T} = \left(\frac{1-\eta^c}{i}\right)\tilde{q}$$

$$(19f) \quad \tilde{x}_T = \eta^T \tilde{c}$$

$$(19g) \quad \tilde{x} = \frac{(1-\eta^T)}{\pi} \tilde{c}$$

Using (19a,b,c), the consumption function can be written as:

$$(19a') \quad \dot{\tilde{q}} = (r - (\delta + \mu + \lambda))\tilde{q} - (\delta + \lambda)\tilde{n}a + (\delta + \lambda)\lambda\tilde{h}$$

or

$$(19a'') \quad \dot{\tilde{q}} = (r - (\delta + \mu))\tilde{q} - (\delta + \lambda)(\lambda + n)\tilde{a}.$$

In Buiter [1986b] it is shown that  $\lambda = n = 0$  is necessary for debt neutrality, but that  $\mu = 0$  is not necessary. The result that population growth alone can destroy debt neutrality is due to Weil [1985].



Footnotes

1. To obtain (13a,b), I assumed that the following first-order condition could be solved for  $\frac{\bar{m}/p_T}{\bar{c}}$  as a function of  $i$  only:

$$\frac{f_4^q(f^c(\eta^T(\pi), \frac{1-\eta^T(\pi)}{\pi}), 1, \pi, \frac{\bar{m}}{p_T \bar{c}})}{f_1^q(f^c(\eta^T(\pi), \frac{1-\eta^T(\pi)}{\pi}), 1, \pi, \frac{\bar{m}}{p_T \bar{c}}) f_1^c(\eta^T(\pi), \frac{1-\eta^T(\pi)}{\pi})} = i$$

i.e.,

$$\frac{f_4^q(f^c(\eta(\pi), \frac{1-\eta^T(\pi)}{\pi}), 1, \pi, \frac{\bar{m}}{p_T \bar{c}})}{f_1^q(f^c(\eta^T(\pi), \frac{1-\eta^T(\pi)}{\pi}), 1, \pi, \frac{\bar{m}}{p_T \bar{c}}) f^c(\eta^T(\pi), \frac{1-\eta^T(\pi)}{\pi})} = i .$$

This will be the case if  $f_{43}^q - f_4^q (f_1^q)^{-1} f_{13}^q - (f_{41}^q - f_4^q (\frac{1}{f_1^c} + \frac{f_{11}^q}{f_2^q})) f_2^c \frac{(1-\eta^T)}{\pi} = 0$ .

Unless this is satisfied,  $\eta^c$  will be a function of both  $i$  and  $\pi$ .

2. If  $r$  and  $\gamma$  are constants, then

$$\theta(t) = \frac{\mu}{\mu-1} r + \lambda + \frac{\delta}{1-\mu}$$

3. We use the fact that  $\bar{a}(t, t) = 0$ .

4. We use the fact that  $\bar{h}(t, t) e^{nt} = \bar{h}(t) e^{nt} = H(t)$ .

Note also that  $W(t) = \bar{w}(t) e^{nt}$  and  $T(t) = \bar{\tau}(t) e^{nt}$ .